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# EFFECT OF MORTALITY ON
# SEARCH EFFECTIVENESS

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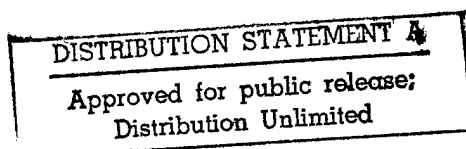1082 Shennecossett Road
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16. Abstract

Survival is an important consideration in assessing the effectiveness of a search for people at sea.

A methodology is proposed in this report for including the effect of mortality on Coast Guard Search and Rescue Effectiveness. This approach can be of assistance in developing Aspects of Survival in the U.S. Coast Guard Addendum to the National SAR Manual and in conducting cost-benefit analyses of existing and proposed systems.

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# METRIC CONVERSION FACTORS

## Approximate Conversions to Metric Measures

| Symbol | When You Know | Multiply By | To Find | Symbol |
|---|---|---|---|---|
| **LENGTH** | | | | |
| in | inches | *2.5 | centimeters | cm |
| ft | feet | 30 | centimeters | cm |
| yd | yards | 0.9 | meters | m |
| mi | miles | 1.6 | kilometers | km |
| **AREA** | | | | |
| in² | square inches | 6.5 | square centimeters | cm² |
| ft² | square feet | 0.09 | square meters | m² |
| yd² | square yards | 0.8 | square meters | m² |
| mi² | square miles | 2.6 | square kilometers | km² |
| | acres | 0.4 | hectares | ha |
| **MASS (WEIGHT)** | | | | |
| oz | ounces | 28 | grams | g |
| lb | pounds | 0.45 | kilograms | kg |
| | short tons (2000 lb) | 0.9 | tonnes | t |
| **VOLUME** | | | | |
| tsp | teaspoons | 5 | milliliters | ml |
| tbsp | tablespoons | 15 | milliliters | ml |
| fl oz | fluid ounces | 30 | milliliters | ml |
| c | cups | 0.24 | liters | l |
| pt | pints | 0.47 | liters | l |
| qt | quarts | 0.95 | liters | l |
| gal | gallons | 3.8 | liters | l |
| ft³ | cubic feet | 0.03 | cubic meters | m³ |
| yd³ | cubic yards | 0.76 | cubic meters | m³ |
| **TEMPERATURE (EXACT)** | | | | |
| °F | Fahrenheit temperature | 5/9 (after subtracting 32) | Celsius temperature | °C |

*1 in = 2.54 (exactly).

## Approximate Conversions from Metric Measures

| Symbol | When You Know | Multiply By | To Find | Symbol |
|---|---|---|---|---|
| **LENGTH** | | | | |
| mm | millimeters | 0.04 | inches | in |
| cm | centimeters | 0.4 | inches | in |
| m | meters | 3.3 | feet | ft |
| m | meters | 1.1 | yards | yd |
| km | kilometers | 0.6 | miles | mi |
| **AREA** | | | | |
| cm² | square centimeters | 0.16 | square inches | in² |
| m² | square meters | 1.2 | square yards | yd² |
| km² | square kilometers | 0.4 | square miles | mi² |
| ha | hectares (10,000 m²) | 2.5 | acres | |
| **MASS (WEIGHT)** | | | | |
| g | grams | 0.035 | ounces | oz |
| kg | kilograms | 2.2 | pounds | lb |
| t | tonnes (1000 kg) | 1.1 | short tons | |
| **VOLUME** | | | | |
| ml | milliliters | 0.03 | fluid ounces | fl oz |
| l | liters | 0.125 | cups | c |
| l | liters | 2.1 | pints | pt |
| l | liters | 1.06 | quarts | qt |
| l | liters | 0.26 | gallons | gal |
| m³ | cubic meters | 35 | cubic feet | ft³ |
| m³ | cubic meters | 1.3 | cubic yards | yd³ |
| **TEMPERATURE (EXACT)** | | | | |
| °C | Celsius temperature | 9/5 (then add 32) | Fahrenheit temperature | °F |

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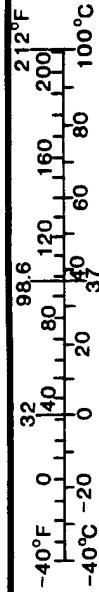
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# ABBREVIATIONS AND ACRONYMS

CASP   Computer Aided Search Planning
OPS    Ocean Prediction System
POA    Probability of Area
POD    Probability of Detection
POS    Probability of Success
SAR    Search and Rescue
SARMIS   Search and Rescue Management Information System

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# 1.0 INTRODUCTION

Ocean Prediction System (OPS) is under development to provide estimates of ocean surface current fields and surface wind fields for use by search planners in real time.

These estimates will be used by CASP 2.0 to develop search plans. These searches should be more efficient and more effective because in-situ small scale dynamic data will be used instead of large scale historical data.

Search effectiveness is expected to increase due to:

- a greater likelihood that the search region will contain the search object, and

- a reduction in the growth of the search area.

Reference [a] provides a framework for conducting a cost/benefit analysis of Ocean Prediction System effectiveness. However, Reference [a] assumes the subject of the search survives. Survivability can significantly affect the cost/benefit analysis, due to the high cost of a human life.

This report presents a methodology for treating survivability in the cost/benefit analysis. This approach can be used in developing Section 2.I, Aspects of Survival, in Reference [b].

The report is organized in the following manner. Factors affecting mortality are discussed in Section 2.0. In Section 3.0, the concept of conditional probability, which is used to combine dependent events, is described. A statistical model that can be used to estimate survivability in SAR cases is outlined in Section 4.0. An example calculation of probability of survival is given in Section 5.0. In Section 6.0, we describe how to combine probability of detection with probability of survival in order to estimate the probability of detecting survivors. A report summary and recommendations are given in Sections 7.0 and 8.0, respectively.

This report is preliminary in that a number of assumptions were made in order to facilitate sample calculations using the proposed methodology. These assumptions should be checked with data where possible. In addition, a simplified model for cumulative probability of detection was used.


# 2.0 FACTORS AFFECTING MORTALITY

The probability of a person surviving an incident at sea depends upon a variety of factors (Reference [c]).

Factors include:

> type of incident,
> person's physical characteristics,
> environmental conditions,
> use of flotation devices and protective clothing,
> shark attack, and
> food and water reserves.

1

Incident type can be used to determine how likely the person is to survive the initial incident. It can also give a perspective on how likely the person is to have survival gear and whether or not the subject was injured during the incident. Examples of incident types include a ship sinking due to weather, a ship fire, and a person falling overboard.

Physical characteristics such as age, weight, and fitness can affect how long a person could survive an incident at sea. In addition, training, background, and personality can affect survivability and can also be modeled. However, these factors will not be addressed in Sections 4.0 and 5.0 in order to simplify the discussion.

Environmental conditions can influence the likelihood of the person surviving the incident, as well as the chance of survival over time. Environmental factors include temperature (air and water), wind, sea state, and precipitation.

Different types of flotation aids can affect the probability of survival. For example, the subject may be floating unaided, clinging to floating debris, or in a life raft. Use of these aids may result in different probabilities of survival. These devices may also alter the probability of detection by increasing visibility. Heavy clothing or a survival suit can increase the chance of survival against hypothermia.

Food and water reserves can affect survivability if the person survives other immediate effects (such as hypothermia and shark attacks) until a time at which thirst and hunger become critical factors. Reserves are a combination of the provisions available at the time of the incident, plus any additional provisions that may be obtained, such as fish and rainwater. Survival equipment can be used to augment and extend food and water reserves, and to favorably impact other survivability factors.

At water temperatures above $68°F$, shark attack becomes a significant risk factor. This assumption is supported by the graph in Figure 1-3 of Volume II in Reference [c] which is based on U.S. Navy data.

# 3.0 MATHEMATICAL PRELIMINARIES

Some of the mortality factors depend on one another. Therefore, a statistical concept, conditional probability that can be used to analyze these dependent events.

## 3.1 Conditional Probability

Conditional probability is used to compute the probability of events that are dependent on each other. The conditional probability of event $A$ occurring, given that event $B$ occurs is denoted by

$$P(A|B). \qquad (3.1)$$

For an event $A$ and a discrete or continuous collection of possible scenarios, $S$, affecting $A$, the probability of $A$ occurring is given by equations (3.2) and (3.3), respectively.

For a discrete set of scenarios,

$$P(A) = \sum_{i=1}^{n} P(A|S_i)P(S_i) \qquad (3.2)$$

where $S_i$ are the different scenarios that can occur.

For a continuous set of scenarios,

$$P(A) = \int_R P(A|S = s)dF_{S(s)} \qquad (3.3)$$

$$= \int_R P(A|S)dF_S \qquad (3.4)$$

where $P(A|S = s)$ is the probability of $A$ occurring given that $S$ is at level $s$ and $dF_{S(s)}$ is the probability that $S$ is at level $s$ and we integrate the possible range $R$ of values of $s$. Equation (3.4) is a commonly-used shorthand notation.

## 3.2 An Example

Suppose that survival only depends on whether the person is wearing a life jacket or not. Let $L$ be the event that a person survives. We are interested in the probability, $P(L)$, that a person survives. We condition on the fact that a person is either wearing a life jacket $J$ or not. That is, the event that a person is living can be partitioned into the event that a person is living and wearing a life jacket and the event that a person is living and not wearing a life jacket. Mathematically,

$$L = \{L \text{ and } J\} \text{ or } \{L \text{ and no } J\} . \qquad (3.5)$$

The two events on the right side of the equality are mutually exclusive in the sense that one cannot be both wearing a life jacket and not wearing life jacket at the same time. This means that the probability of the left hand side is the sum of the probabilities on the right hand side:

$$P(L) = P\{L \text{ and } J\} + P\{L \text{ and no } J\} . \qquad (3.6)$$

Now further assume that we know that a person has a 20% chance of wearing a life jacket. We also assume that if the person is wearing a life jacket, then he or she will survive 95% of the time and that if the person is not wearing a life jacket, then he or she will survive only 25% of the time.

We can combine this information using the statistical method of conditioning. Conditioning says that the likelihood of an event and a scenario is the likelihood of the scenario times the likelihood of the event given that scenario. In our example,

$$P(L \text{ and } J) = P(L|J)P(J). \tag{3.7}$$

This means that the probability that a person survives and is wearing a life jacket is the probability that the person is wearing a life jacket, $P(J)$, times the probability that the person survives when he is wearing the life jacket. The scenario is whether the person is wearing a life jacket and the event is whether the person survives. We similarly obtain for the other term

$$P(L \text{ and } J^c) = P(L|J^c)(P(J^c)). \tag{3.8}$$

where "$c$" means complement or "not."

Therefore,

$$P(L) = P(L \text{ and } J) + P(L \text{ and } J^c) \tag{3.9}$$

$$= P(L|J)P(J) + P(L|J^c)P(J^c) \tag{3.10}$$

$$= (0.95)(0.20) + (0.25)(0.80) \tag{3.11}$$

$$= 0.19 + 0.20 \tag{3.12}$$

$$= 0.39. \tag{3.13}$$

This example is for a discrete distribution, because the scenario has only two possibilities: either a person is wearing a life jacket or not. If the scenario has an infinite number of possibilities (e.g., water temperature is in a range from $50°F$ to $60°F$), then a continuous distribution would be used.

# 4.0 STATISTICAL METHODOLOGY

In this section, we describe a statistical methodology that can be used to model the likelihood of survival based on the factors described in Section 2.0.

We consider the following factors: water temperature, shark attack, use of a survival suit and food and water reserves. We do not explicitly model the effect of a person's physical characteristics on survivability, or address incident types. The incident considered in this report is one in which the person survives the initial incident and enters the water without injuries (i.e., person in the water).

We use the following notation:

$L(t)$ : the person is alive at time $t$,

$T$ : water temperature,

$S$ : the person has a survival suit,

$A(t)$ : a shark attack occurs by time $t$, and

$R(t)$ : resource (food and water) reserves at time $t$.

We use the following probabilistic notation:

$F_X$ : the cumulative probability distribution (cdf) of random variable $X$,

$dF_X$ : the probability density function (pdf) of random variable $X$,

$P(E)$ : the probability of event $E$, and

$E^c$ : is the complement of event $E$, where

$$P(E^c) = 1 - P(E) . \tag{3.14}$$

The quantity of interest is the probability that a person is alive at time $t$, or equivalently, $P(L(t))$.

## 4.1 Water Temperature $T$

To estimate the effect of temperature on survivability, we express the probability of survival as a function of the temperature distribution, and of the conditional probability of survival at a given temperature. We condition on water temperature $T$ first, because some of the other events depend upon the water temperature $T$ (e.g., shark attacks rarely occur in cold water).

Treating $T$ as a continuous random variable we have that,

$$P(L(t)) = \int P(L(t)|T)dF_T \qquad (4.1)$$

where the integration is over the range of the random variable of $T$.

We need to know the distribution of $T$. We can assume a distribution for $T$ or we can estimate it.

### 4.1.1 Estimation

In order to estimate the distribution of $T$, we need data on the sea temperature for SAR cases. Using this data, we can estimate the distribution parametrically or non-parametrically.

We can also use historical data to obtain information for a given season and region and then estimate the distribution from this data. For example, Table 1 presents mean sea surface temperatures for selected seasons and locations off of the coastal United States. This data was extracted from Naval Oceanographic Office databases.

**Table 1. Average Sea Surface Temperature ($^\circ$F)**

| Location | Season | | | |
|---|---|---|---|---|
| | Jan - Mar | Apr - Jun | Jul - Sep | Oct - Dec |
| Cape Cod Area | 45 | 50 | 65 | 52 |
| Virginia Area | 66 | 70 | 80 | 70 |
| Florida Atlantic Coast | 72 | 76 | 83 | 78 |
| Gulf of Mexico | 73 | 79 | 85 | 79 |
| Gulf of Alaska | 40 | 43 | 52 | 46 |
| Southern California Area | 60 | 58 | 63 | 63 |
| Washington Area | 46 | 50 | 60 | 55 |

### 4.1.2 Examples of Continuous Distributions for $T$

If a continuous function for the temperature distribution is available, then a continuous formulation of the conditional probability should be used. Possible assumptions on the distribution for water temperature $T$ are:

- $T$ is distributed uniformly within $\pm a$ degrees (F or C) tolerance of its mean value $\hat{T}$ for a given region and season,

$$T \sim \text{Unif}[\hat{T} - a, \hat{T} + a] . \qquad (4.2)$$

In this case, the pdf of $T$ is

$$dF_T = \begin{cases} 1/(2a), & \text{if } |T - \hat{T}| \le a, \\ 0, & \text{otherwise.} \end{cases} \tag{4.3}$$

- $T$ is normally distributed about its region and season mean $\hat{T}$ with a standard deviation $s$,

$$T \sim N(\hat{T}, s) \tag{4.4}$$

In this case, the pdf of $T$ is

$$dF_T = \frac{1}{s\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(T-\hat{T})^2}{s^2}} \tag{4.5}$$

for any real $T$.

We then have

$$P(L(t)) = \int P(L(t)|T) dF_T \quad . \tag{4.6}$$

## 4.1.3 Effect of Water Temperature on Survivability

Figure 1 (taken from Reference [c]) presents survival time as a function of water temperature for a person in water without an anti-exposure suit. Data are shown for three body types; fast cooler, average cooler and slow cooler.

If we had information about the physical characteristics of the person, we would use this information here and only use the curve for that person's body type and a distribution for dying before this time (e.g., a shifted exponential with some instantaneous death rate).

For each body type shown in Figure 1, we use a non-linear least squares method with an exponential model

$$f(T) = P_0 + P_1 e^{P_2(T)} \tag{4.7}$$

to estimate the survival time curve at each temperature, $T$, using the following procedure: At equally spaced temperature intervals, determine where each curve intersects a given temperature. This creates a data point for each curve. Make a data set for each curve at several temperatures (at least 10) and then estimate the curves.
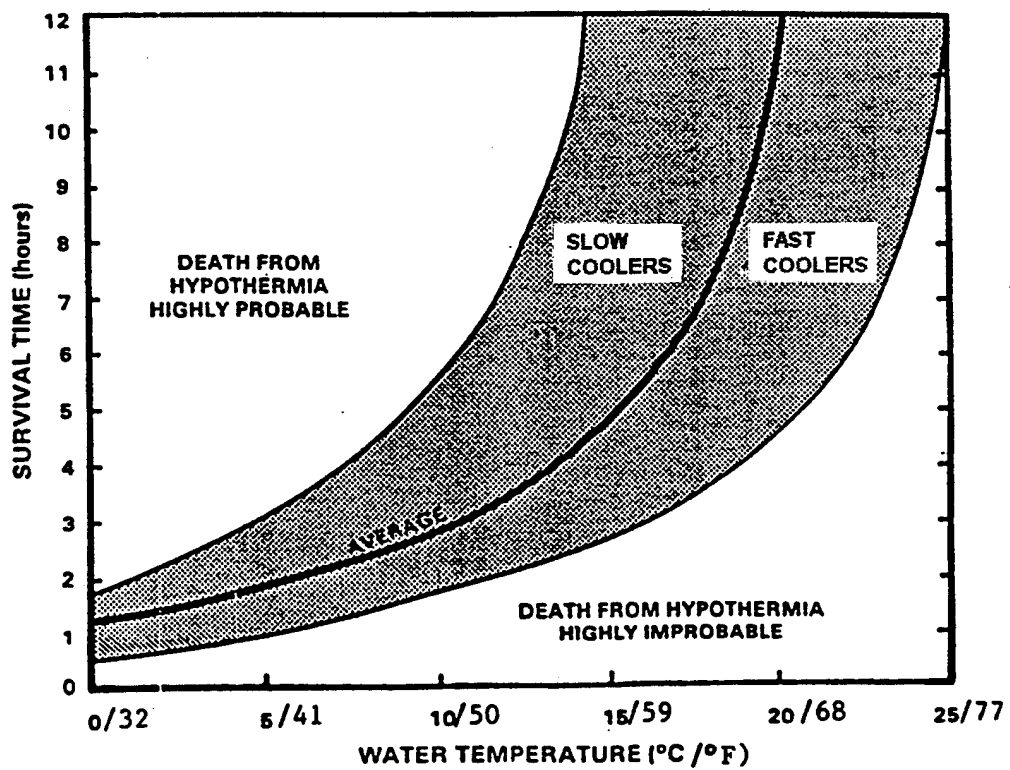
**Figure 1. Survival Time versus Water Temperature** (Figure 4-1 p. 4 - 5 Reference [c])

Once the curves are estimated, label them as $f_{f(T)}$, $f_{a(T)}$, and $f_{s(T)}$ for fast, average, and slow coolers, respectively. For all $T$, the curves are ordered as $f_{f(T)} < f_{a(T)} < f_{s(T)}$.

For a given water temperature, $T$, the set of three curves provides a range of survival times, $[f_{f(T)}, f_{s(T)}]$.

From Figure 1, we see that:

- It is highly likely that a person will survive up to time $f_{f(T)}$,
- It is highly unlikely that a person will survive longer than time $f_{s(T)}$.
- The probability that a person will survive up to time $t$ in between the two bounds decreases as we move from $f_{f(T)}$ to $f_{s(T)}$.

Because the curve for the average cooler, $f_a(T)$, is not perfectly in between the other two bounding curves, a uniform decrease for survival time between $f_{f(T)}$ and $f_{s(T)}$ is not warranted. However, we assume that a uniform decrease occurs for survival time between the two pairs of curves

$$[f_{f(T)}, f_{a(T)}] \quad \text{and} \quad [f_{a(T)}, f_{s(T)}].$$

That is, we assume that $L(t)|T$ is distributed as

$$0.5(\text{Unif}(f_{f(T)}, f_{a(T)}) + \text{Unif}(f_{a(T)}, f_{s(T)})). \tag{4.8}$$

which gives the following probabilities,

$$P(L(t)|S^c, T) = \begin{cases} 1 & , \quad \text{if } t \le f_{f(T)}, \\ 1 - 0.5\dfrac{t - f_{f(T)}}{f_{a(T)} - f_{f(T)}} & , \quad \text{if } f_{f(T)} \le t \le f_{a(T)}, \\ 0.5 - 0.5\dfrac{f_{a(T)} - t}{f_{a(T)} - f_{s(T)}} & , \quad \text{if } f_{a(T)} \le t \le f_{s(T)}, \text{ and} \\ 0 & , \quad \text{if } f_{s(T)} \le t. \end{cases} \tag{4.9}$$

We can use this assumption to obtain the distribution for $L(t)|S, T$ by making the assumption that when wearing a survival suit, one will survive $k$ times longer than if one were not wearing a survival suit. A typical range for $k$ is between 2 and 10 (p. 4 - 6 Vol. I Reference [c]). This means that

$$L(t)|S, T \sim 0.5(\text{Unif}(kf_{f(T)}, kf_{a(T)}) + \text{Unif}(kf_{a(T)}, kf_{s(T)})) \tag{4.10}$$

which gives the following probabilities,

$$P(L(t)|S^c, T) = \begin{cases} 1 & , \quad \text{if } t/k \leq f_{f(T)}, \\ 1 - 0.5\dfrac{t/k - f_{f(T)}}{f_{a(T)} - f_{f(T)}} & , \quad \text{if } f_{f(T)} \leq t/k \leq f_{a(T)}, \\ 0.5 - 0.5\dfrac{f_{a(T)} - t/k}{f_{a(T)} - f_{s(T)}} & , \quad \text{if } f_{a(T)} \leq t/k \leq f_{s(T)}, \text{ and} \\ 0 & , \quad \text{if } f_{s(T)} \leq t/k. \end{cases} \quad (4.11)$$

## 4.2 Shark Attack

We next determine the conditional probability of whether or not there is a shark attack by time $t$. That is,

$$P(L(t)|T) = P(L(t)|A(t), T)P(A(t)|T) + P(L(t)|A^c(t), T)P(A^c|T) \quad (4.12)$$

We assume that if there is a shark attack, the person does not survive. That is,

$$P(L(t)|A(t), T) = P(L(t)|A(t)) = 0. \quad (4.13)$$

Using this information,

$$P(L(t)|T) \quad = \quad P(L(t)|A(t), T)P(A(t)|T) + P(L(t)|A^c(t), T)P(A^c(t)|T) \quad (4.14)$$

$$= \quad P(L(t)|A^c(t), T)P(A^c(t)|T) \quad (4.15)$$

which gives

$$\dot{P}(L(t)) \quad = \quad \int P(L(t)|T)dF_T \quad (4.16)$$

$$= \quad \int P(L(t)|A^c(t), T)P(A^c(t)|T)dF_T. \quad (4.17)$$

### 4.2.1 Distribution for A(t)

Assumptions are needed in order to determine $P(A(t)|T)$. Because sharks are unlikely to be in cold water, we condition on water temperature $T$. We assume an exponential distribution on $A(T)$; that is,

$$P(A(t)|T) = 1 - e^{-t(p(T))} \qquad (4.18)$$

where

$p(T)$ is the instantaneous probability of a shark attack at water temperature $T$.

Use of an exponential distribution means that sharks are equally likely to appear at any time and that one can expect a shark attack every $1/(p(T))$ time periods.

$p(T)$ can be continuous or discontinuous (that is, having a jump discontinuity at a temperature $T$ where it is known that sharks are not present). We assume that $p(T) = 0$ for water temperatures below some temperature $\hat{T}_A$. Note that $\hat{T}_A$ and $p(T)$ may change depending on the region and season.

The distribution of $A(t)$ can be estimated by knowing when sharks are in a given region and how likely they are to attack.

### 4.3 Survival Suit

We next condition on whether the person has a survival suit (i.e., event $S$). We assume that having a survival suit is independent of water temperature and animal attack; that is, you either have one or you don't.

We also assume for simplicity that the probability of having a survival suit $P(S)$ is constant. We then have

$$\begin{aligned} P(L(t)|T, A^c(t)) &= P(L(t)|S, T, A^c(t))P(S|T, A^c(t)) + \\ &\quad P(L(t)|S^c, T, A^c(t))P(S^c|T, A^c(t)) \end{aligned} \qquad (4.19)$$

This gives

$$P(L(t)) = \int P(L(t)|T)dF_T \qquad (4.20)$$

$$= \int P(L(t)|T, A^c(t))P(A^c(t)|T)dF_T \qquad (4.21)$$

$$= \int \left[ P\left(L(t)|S, T, A^c(t)\right)P(S) + P\left(L(t)|S^c, T, A^c(t)\right)P(S^c) \right]P(A^c(t)|T)dF_T \qquad (4.22)$$

We need distributions for

$$L(t)|S, T, A^c(t) \quad \text{and}$$

$$L(t)|S^c, T, A^c(t) \ .$$

This type of estimation is somewhat complicated for the following reasons:

The data (hours of survival) for a given temperature and body type are both right and left censored. That is, if a person is rescued, then you only know that the person lived at least that long; the person could have lived longer. This is called right censoring. Also, if you recover a dead body, then the person died before this time. This is called left censoring. Data should be analyzed using statistical techniques based upon censored data.

It is unlikely that enough observations will be collected at a specific temperature to get a good estimate of the distribution. The data should be smoothed over a range of temperatures.

It is also unlikely that enough observations for an exact body type (because it is continuous) will be collected to get a good estimate of the distribution. The data should be smoothed over a range of body types or people should be grouped into a few body types. Training, background, and personality factors could be treated in a similar fashion.

## 4.4 Food and Water Reserves

In this section, we describe a model that can be used to estimate the effect of food and water reserves on the probability of survival.

Let $R(t)$ be the amount of the resource, either food or water, at time $t$. We condition on whether we have the resource or not.

That is,

$$P(L(t)|S, T, A^c(t)) = P(L(t), R(t) > 0|S, T, A^c(t)) + \tag{4.23}$$
$$P(L(t), R(t) = 0|S, T, A^c(t))$$

$$= P(L(t)|S, T, A^c(t), R(t) > 0)P(R(t) > 0|S, T, A^c(t)) +$$
$$P(L(t)|S, T, A^c(t), R(t) = 0)P(R(T) = 0|S, T, A^c(t)) \tag{4.24}$$

We need to determine the distributions of:

$$(L(t)|S, T, A^c(t), R(t) > 0), \tag{4.25}$$

$$(L(t)|S, T, A^c(t), R(t) = 0), \text{ and} \tag{4.26}$$

$$(R(t)|S, T, A^c(t)); \tag{4.27}$$

and for $P(L(t)|S^c, T, A^c(t))$, the distribution of:

$$(L(t)|S^c, T, A^c(t), R(t) > 0), \tag{4.28}$$

$$(L(t)|S^c, T, A^c(t), R(t) = 0), \text{ and} \tag{4.29}$$

$$(R(t)|S^c, T, A^c(t)). \tag{4.30}$$

Recall that conditioning on $A^c(t)$ means that there has been no shark attack up to time $t$.

### 4.4.1 Assumptions

In order to determine $L(t)|S, T, A^c(t), R(t) > 0$, we assume that if you have some of the resource at time $t$, $R(t) > 0$, then the person is alive. That is,

$$P(L(t)|S, T, A^c(t), R(t) > 0) = 1. \tag{4.31}$$

Note that, even with a survival suit and no shark attacks, the person may die from exposure before the reserves are exhausted. So the conditioning is done on temperature first as discussed in Section 4.1. Assumption (4.31) is valid at temperatures for which $P(L(t)|T = 1)$.

For $L(t)|S, T, A^c(t), R(t) = 0$, we assume that the person will automatically live $h$ hours before there is any effect due to not having the resource; that is,

$$P(L(t)|S, T, A^c(t), R(t) = 0) = 1 \text{ for } t < h.$$

Note that $h$ depends upon such factors as the person's physical condition, water temperature, and whether the person is wearing a survival suit or not.

For times $t > h$, we assume that there is an instantaneous probability of death, $l(R)$, due to the absence of the resources. That is, the distribution is a shifted exponential distribution where:

$$P(L(t)|S, T, A^c(t), R(t) = 0) = \begin{cases} 1 & , \text{ if } t < h, \text{ and} \\ e^{(-(t-h))l(R)}, & \text{ if } t > h. \end{cases} \tag{4.32}$$

Note that this probability only depends on $t$. So that if $t < h$, then there is no impact, otherwise, we weight the probability by $e^{-(t-h)l(R)}$.

We also assume that after $H$ hours without food, the probability that the person is still living is some small probability $p_R = p_R(S, T)$. That is,

$$P(L(t + H)|S, T, A^c(t), R(t) = 0) = p_R. \tag{4.33}$$

This means that $P(L(H)|S, T, A^c(t), R(H) = 0) = p_R$. Using these conditions for $L(t)|S, T, A^c(t), R(t) = 0$ determines the instantaneous death probability $l(R)$.

To determine $R(t)|S, T, A^c(t)$, we assume that there are no reserves at time 0 and there is no possibility of obtaining more reserves after time 0.

This means that $P(R(t) = 0) = 1$ and $P(R(t) > 0) = 0$ for all $t$.

We now have, for $t < h$:

$$P(L(t)) = \int P(L(t)|T)dF_T \tag{4.34}$$

$$= \int P(L(t)|T, A^c(t))P(A^c(t)|T)dF_T \tag{4.35}$$

$$= \int [P(L(t)|S, T, A^c(t))P(S) + P(L(t)|S^c, T, A^c(t))P(S^c)]P(A^c(t)|T)dF_T \tag{4.36}$$

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$$= \int [P(L(t)|S, T, A^c((t), R(t) = 0))P(S) +$$

$$P(L(t)|S^c, T, A^c((t), R(t) = 0))P(S^c)]P(A^c(t)|T)dF_T \tag{4.37}$$

and, for $t \geq h$:

$$P(L(t)) = e^{-(t-h)l(R)}\int P(L(t))dF_T \tag{4.38}$$

$$= e^{-(t-h)l(R)}\int P(L(t)|T, A^c(t))P(A^c(t)|T)dF_T \tag{4.39}$$

$$= e^{-(t-h)l(R)}\int [P(L(t)|S, T, A^c(t))P(S) +$$

$$P(L(t)|S^c, T, A^c(t))P(S^c)]P(A^c(t)|T)dF_T \tag{4.40}$$

$$= e^{-(t-h)l(R)}\int [P(L(t)|S, T, A^c((t), R(t) = 0))P(S) +$$

$$P(L(t)|S^c, T, A^c((t), R(t) = 0))P(S^c)]P(A^c(t)|T)dF_T \tag{4.41}$$

A more realistic model for determining $R(t)$ would be to assume that $R(t)$ is a non-homogeneous Poisson process. This type of model includes the following features:

- The reserves start at a given level, $R_0$,

- For each region, season, and water temperature, additional reserves (food and water) arrive at a Poisson rate $l_R$, and

- For each region, season, and water temperature, the subject consumes the reserves at a rate, $m_R$, that can depend upon having a survival suit and specific body type. For example, if it is cold, you eat more to stay warm. However, if you have a survival suit, the suit keeps you warm; so, you do not eat as much as you would, if you did not have a survival suit.

The equations for this type of model are not presented in this report.

# 5.0 PROBABILITY OF SURVIVAL: AN EXAMPLE

In this section, we illustrate the combined effect of the models presented in Section 4.0 using an example.

Figure 2 shows probability of survival curves versus time for a person in water without a survival suit for three average water temperatures, 50, 65 and 80° F.

The following assumptions were made in calculating these probability curves:

- The person is alive at time $t = 0$.

- Water temperature. Water temperature is uniformly distributed with a $\pm10°F$ tolerance.

- Shark attack. Probability of shark attack is 0.5 per day, if water temperature is above 68° F, and 0 if water temperature is below 68° F.

  We assume that the instantaneous probability of shark attack, $p(T)$, is constant after a specified temperature $T_A^* = 68°F$. If the probability of a shark attack in a 24 hour period is 0.5, then

  $$P(A(24)|T) = 1 - e^{-t(p(T))} \tag{5.1}$$

  $$= 0.5 \tag{5.2}$$

  Solving for $P(T)$, we have that

  $$p(T) = (-\ln(0.5))/24 \tag{5.3}$$

  $$= 0.03. \tag{5.4}$$

  So the instantaneous probability of shark attack is 0.03.

- Reserves. Reserves have no effect on survivability for up to 12 hours, and the person dies after 48 hours without food or water. That is, $h = 12$ hours, $H = 48$ hours and $p_R = 0.001$.
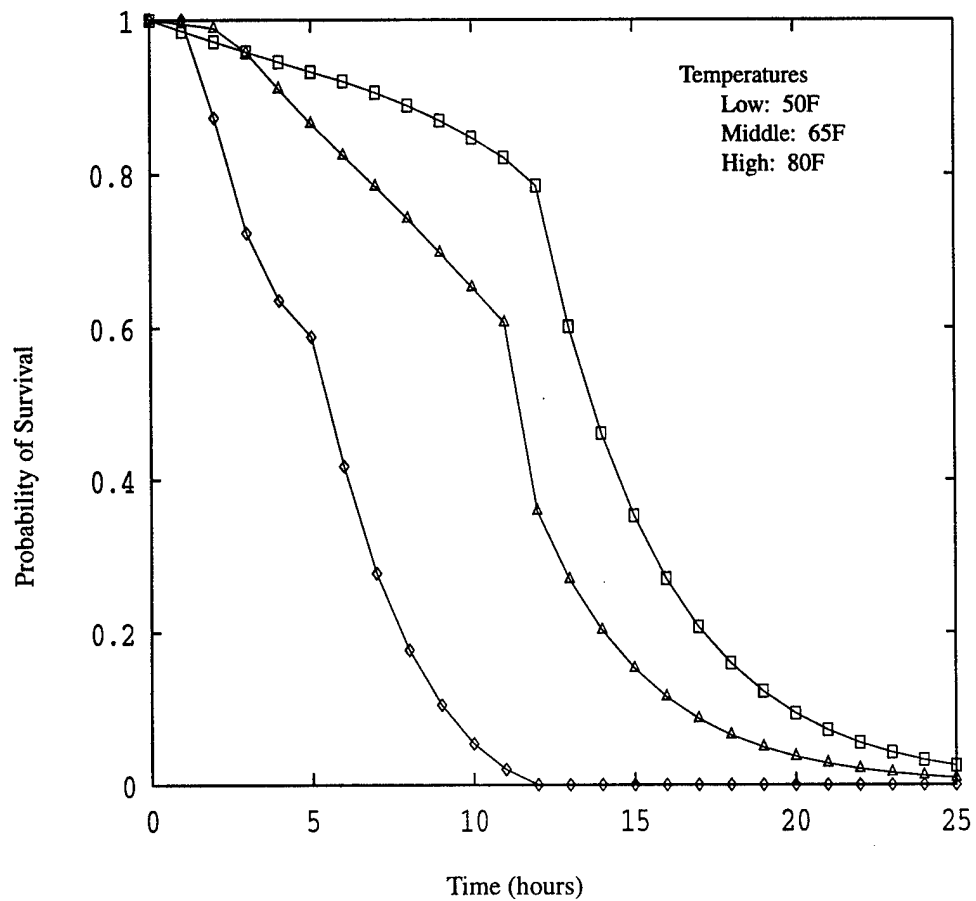
# EXAMPLE



Temperatures
Low: 50F
Middle: 65F
High: 80F

**Figure 2. An Example of Probability of Survival versus Time for a Person in Water at Three Temperatures**

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# 6.0 PROBABILITY OF SUCCESS

We now calculate the probability of success (see Section 2.H, Reference [b]) with the constraint that the person be alive at the time of detection.

The probability of success at time $t$ is given by

$$POS(t) = \int_0^t POD(t')POA(t')dt' \qquad (6.1)$$

where

$$POD(t) = Pr\{\text{Detection at time } t'|\text{person is in the search area}\}, \qquad (6.2)$$

and

$$POA(t) = Pr\{\text{Person is in the search area at time } t'\}. \qquad (6.3)$$

## 6.1 An Example

We illustrate how to calculate the probability of success in detecting a survivor using an example.

We assume that the instantaneous probability of detection, $POD(t)$, is independent of the probability of surviving, $P(L(t))$. That is, the probability of detection does not depend on whether the person is alive. Then the instantaneous probability of detecting a survivor, $POD_s(t)$, is given by

$$POD_s(t) = POD(t-t_0)P(L(t)) \quad, \quad t \geq t_0. \qquad (6.4)$$

where $t_0$ is the time at which the search begins. We assume for simplicity that $t_0 = 0$; that is, the search begins at the time of the incident.

The probability of success in detecting a survivor by time $t$ is given by

$$POS_s(t) = \int_0^t POD(t')P(L(t'))POA(t')dt \qquad (6.5)$$

We now assume, for simplicity, that $POA(t) = 1$; that is, the person is in the search area.

We use, as an example, a search conducted in an area that contains the person.

If the incident position uncertainty is given by a bivariate normal probability density function, then the probability of success as a function of time for a random search is given by

$$POS(t) = 1 - e^{-z} \qquad (6.6)$$

where

$$z = \frac{VWt}{2\pi\sigma_x\sigma_y}, \qquad (6.7)$$

$V$ is the search speed, and $W$ is the sweep width. The instantaneous probability of detection is given by

$$POD(t) = \frac{d}{dt}POS(t) = \frac{VW}{2\pi\sigma_x\sigma_y}e^{-z} \qquad (6.8)$$

## 6.2 Sample Calculation

We illustrate calculation of the probability of detecting a survivor using the following example.

We assume that a person is in the water without a survival suit and that the person enters the water at the time that the search begins. For simplicity, we ignore the effects of shark attacks and food reserves.

For an HH-65 search, we assume that search speed, $V$, is 90 knots, and sweep width, $W$, is 0.1 nm. Also, we assume that the search area uncertainty is given by a bivariate normal distribution with $\sigma_x = \sigma_y = 10nm$, and that we are searching in the correct area.

Suppose also that SAR cases are evenly distributed in the January to March time period across the three Atlantic Coast regions shown in Table 1. Thus, $P(T) = 0.33$ for $T$ in $\{45, 66, 72\}$. The non-linear least squares models for the three hypothermia fatality curves, corresponding to fast, average and slow coolers as shown in Figure 1, are given by

$$f_{f(T)} = 0.14 + 0.030e^{0.078T}, \qquad (6.9)$$

$$f_{a(T)} = 0.75 + 0.026e^{0.089T}, \text{ and} \qquad (6.10)$$

$$f_{s(T)} = 0.97 + 0.035e^{0.099T}. \qquad (6.11)$$

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The temperature set is $\{45, 66, 72\}$. Using equations (6.9), (6.10), and (6.11), we have the following values:

$$f_{f(45)} = 1.14, \qquad f_{a(45)} = 2.18, \qquad f_{s(45)} = 3.98,$$

$$f_{f(66)} = 5.30, \qquad f_{a(66)} = 10.0, \qquad f_{s(66)} = 25.1, \text{ and}$$

$$f_{f(72)} = 8.38, \qquad f_{a(72)} = 16.5, \qquad f_{s(72)} = 44.6. \qquad (6.12)$$

Figure 3 shows $P(L(t))$, the probability of survival until time $t$, using the expression for $P(L(t)|T)$ given by equation (4.9).

The Probability of Success Curves for $POS(t)$ and $POS_s(t)$ are shown in Figure 4 where $POS(t)$ is given by equation (6.6) and $POS_s(t)$ is given by equation (6.5) with $POD$ given by equation (6.8), $P(L(t))$ given by equations (4.9) and (6.12) and $POA = 1$.

Figure 4 illustrates certain effects that result from including mortality:

- The two probability curves $POS$ and $POS_s$ are the same for $t \leq 5$ hours because the probability of surviving that long is high,

- There is little increase in probability of successfully detecting survivors after time $t = 20$ hours because the probability of surviving is very small by that time, and

- The $POS$ curve continues to increase because it doesn't include the effect of mortality.

## 7.0 SUMMARY

A methodology has been presented that can be used to estimate the probability of survival, $P(L(t))$.

Estimates for $P(L(t))$ can then be used with a model for probability of success to estimate the probability of detecting survivors in SAR cases for use in the OPS cost-benefit analysis.
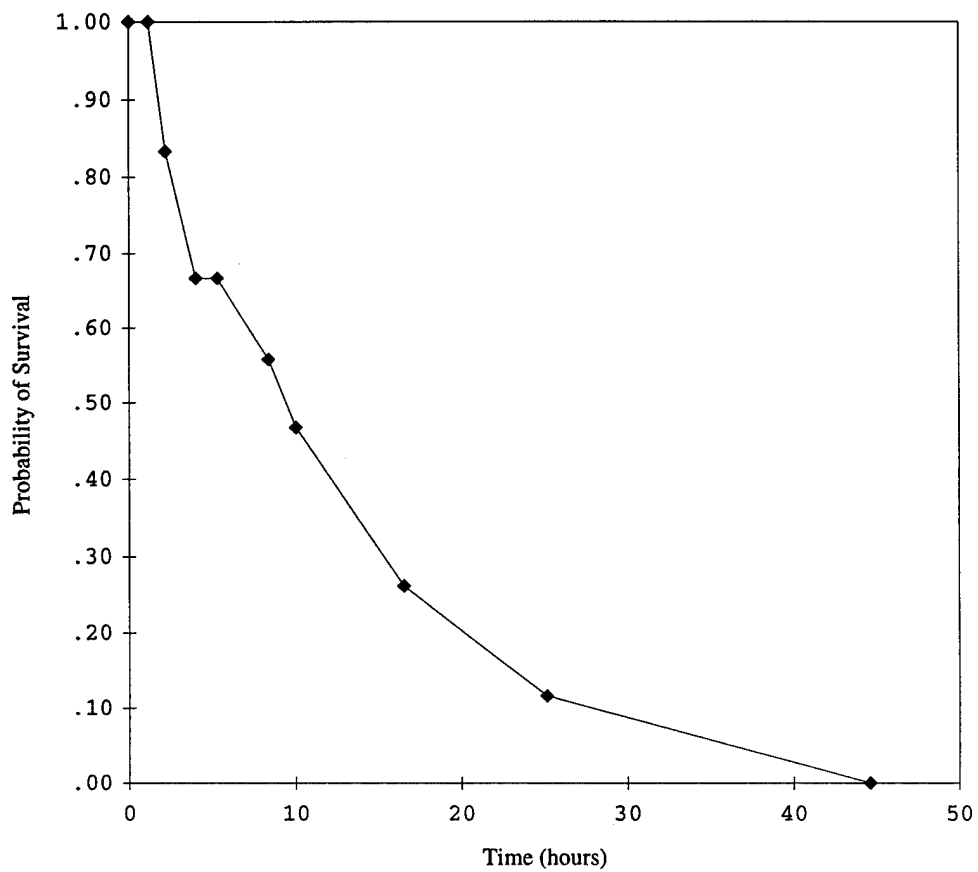
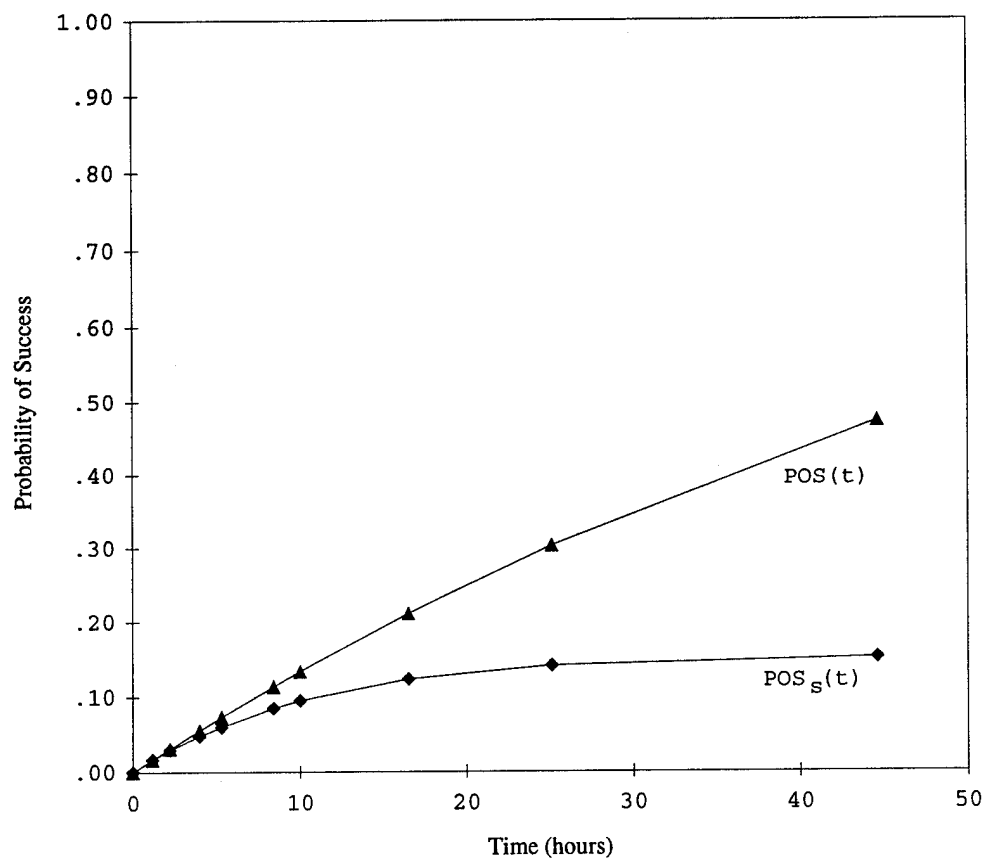**Figure 3. An Example of Probability of Survival versus Time**

**Figure 4. An Example of Probability of Success versus Time**

## 8.0 RECOMMENDATIONS

We recommend that:

- The assumptions made in this report in developing the model for $P(L(t))$ should be verified with data,

- Data should be collected in SAR cases to assist in estimating these parameters (see Appendix A), and

- The *POS* model described in Section 6.0 should be extended to include the effects of the time difference between the incident occurrence and the start of the search and the movement and/or growth in search area due to location uncertainty and motion. Such a model should be used to estimate OPS impact on search effectiveness.

## REFERENCES

a. "Ocean Prediction System (OPS) Cost/Benefit Analysis" (DRAFT), LCDR B. Perkins, U.S. Coast Guard, 18 May 1995.

b. "U.S. Coast Guard Addendum to the National SAR Manual", U.S. Coast Guard COMDTINST M16130.2B, 17 August 1995.

c. "National Search and Rescue Manual", U.S. Department of Transportation, U.S. Coast Guard, COMDTINST M16120.5A, U.S. Joint Chiefs of Staff, JOINT PUB 3-50, 1 February 1991.

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# APPENDIX A

The following SARMIS data should be collected for each SAR case and included in the CASP database to assist in modeling search and rescue effecctiveness.

## A.1 Survival

> time of incident
> type of incident
> initial location of incident
> location at time of rescue/recovery
> temperature (air and water) - estimated at intervals
> wind and sea conditions - estimated at intervals
> use of flotation devices and survival suits
> availability of survival equipment
> time/cause of death
> time of rescue/recovery
> physical condition at time of rescue
> presence of sharks
> subject's survival training and environment experience level (high, medium, low)

## A.2 Search

> search assets - number and type (include speed, altitude, even fatigue)
> time late - time between incident and search assets on scene
> time of detection
> time of rescue/recovery
> search probability area size and location as function of time
> estimated sweep width
> actual search track
> visibility in search areas - estimated at intervals