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13. ABSTRACT (Maximum 200 words) The activities under this grant fall in three main categories: laboratory set-up and calibration studies, interface software and image feature algorithms, and theoretical investigations of PDEs and algorithm development. As part of the Laboratory Analysis we have been comparing two techniques for contact free measuring, feature-based monocular and stereo. Calibration and spatial resolution have been studied as part of obtaining an effective stereo measurement system. The development of interface software has focused on obtaining object recognition problems and have employed Differential Geometric Techniques. Algorithms and data structures have been obtained which are particularly well suited for extracting fundamental geometric quantities such as principal curvatures, normal fields, etc. The theoretical analysis has resulted in significant progress in several different directions including a deep understanding of Maxwell's equations on nonsmooth domains, wavelet analysis on nonsmooth sets and on surfaces, improved performance of smooth local trigonometric bases, numerical solutions of PDEs using wavelet techniques, sensor fusion based on wavelets, and interactive visual design and surface modeling based on wavelets.				
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Final Report

Bjorn Jawerth

February 3, 1996

1 Summary

The activities under this grant fall in three main categories: laboratory set-up and calibration studies, interface software and image feature algorithms, and theoretical investigations of PDEs and algorithm development.

2 Laboratory Analysis

We have been comparing two techniques for contact-free measuring: feature-based monocular and stereo. There are two issues that must be addressed in the use of a high resolution stereo measurement system. These are calibration and spatial resolution. The primary emphasis in the development of the stereo system has been (a) to obtain surface displacement and strain fields from both planar and non-planar structures, (b) to obtain these field quantities with optimal accuracy, (c) to quantify the accuracy of the method through performance of a carefully chosen set of experiments which includes 3-D translation, 3-D rotation and uniaxially strain.

Calibration of the current low resolution (512×512, 8bit) stereo camera system was performed by imaging of a rectangular grid with known grid spacing. The grid accuracy is +/- 4 microns but the glass substrate quality was not reported by the manufacturer. Camera calibration has been performed both on individual cameras and on the "camera system" with less than 0.50 micron difference in the measured displacements.

For the translation tests, the displacement error was approximately 4.1 microns with a standard deviation of 0.80 microns for each component of displacement. The error in rotation tests was 0.09° with a standard deviation of .15°.

Some of the critical issues for making the stereo process viable, have been addressed, at least partially. First, the stereo process from image acquisition through image analysis is computationally intensive because of subpixel and subset image correlation; this clearly requires higher performance computers and parallel processing to improve turnaround. This still remains to be carried out. Secondly, the process of converting the discrete displacement data (with noise) into a smooth functional form remains an important and difficult problem; wavelet-based algorithms for smoothing the data offer the potential for both speed and accuracy. The necessary theoretical background in terms of so called second generation wavelets is now in place, see below.

3 Interface software

We have developed a technique based on Differential geometric method for object recognition problem by finding three important parameters which uniquely defines an object at different points of a given object. These parameters are the minimum and maximum curvature and the angle subtended along a particular direction. We use three color strips to solve the Euler's equation and the angle subtended is the angle the red strip makes with minimum curvature of the object. We also give algorithms and the data structures developed which are particularly suitable for such applications.

For a detailed presentation we refer to Progress Report for Year 2 of this effort.

4 Theoretical developments: PDEs and related topics

The research carried out as part of this effort span a broad spectrum from theoretical research in PDEs, Differential Geometry, and Wavelet Theory to applied results concerning sensor fusion, wavelet denoising, numerical PDEs, and other areas. Brief summaries of the corresponding papers are given below.

Ref. 1 In the paper [3] we use wavelets for fusing information from different types of sensors. We design a fusion method based on the size of the wavelet coefficients and show that this approach is computationally very efficient.

Ref. 2 The paper [10] contains a survey of some results concerning the connection between Clifford algebras and elliptic boundary value problems on nonsmooth domains. A natural setting for the Cauchy integral operator in higher dimensions is a certain type of Hardy spaces, hence we summarize some of the basic properties of these spaces. The Cauchy operator serves as a tool to extend functions defined on the boundary of a Lipschitz domain to Clifford monogenic functions in the domain.

Then we study some Rellich type identities for Clifford monogenic functions in Lipschitz domains, and, by exploiting the connections between the higher dimensional Cauchy singular integral operator and the classical layer potentials, we establish how these can be used to treat the (classical) Dirichlet as well as Neumann problem on Lipschitz domains in a unified manner. We produce uniqueness and existence results using the Clifford algebra machinery we have developed earlier. The case of more general operators is also considered.

Ref. 3 The main aim of the note [11] is to present some applications of the discrete wavelet transform in the Clifford algebra context to the study of some singular integral operators arising in connections with partial differential equations on non-smooth domains.

Ref. 4 The paper [12] can be considered as a natural continuation of the work of Marius Mitrea, a student of Jawerth's. The main results of his concerning Maxwell's equations in three dimensions are extended to higher dimensions and to more general differential operators.

The Maxwell boundary value problem we consider in arbitrary Lipschitz domains Ω in R^m , $m \geq 3$, is to determine a $(l+1)$ -form E and a l -form H , smooth in Ω and such that

$$\begin{aligned}\delta E + ikH &= 0 && \text{on } \Omega, \\ dH + ikE &= 0 && \text{on } \Omega, \\ E^*, H^* &\in L^p(\partial\Omega), \\ n \wedge E &= A.\end{aligned}$$

The exterior problem is defined similarly, plus a decay condition.

The main result of this paper asserts that, for any bounded Lipschitz domain Ω in R^m , there exists a small positive ϵ such that, for any $2-\epsilon \leq p \leq 2+\epsilon$, these problems have unique solutions. Moreover, these solutions can be expressed in the form of layer potentials and optimal

estimates are obtained. In the case in which Ω is actually a bounded C^1 domain, then these results hold for the full range $1 < p < \infty$.

We actually treat slightly more general problems, namely the interior electric boundary value problem and its counterpart, the interior magnetic boundary value problem. In fact, these two problems are dual to each other via the hodge $*$ -operator; hence, it suffices to treat only one of them.

The main ingredient of the proof is a family of certain Rellich type identities for differential forms, generalizing the ones by Mitrea for the three dimensional case. We also present a brief outline of an extension of this theory for certain first order elliptic complexes on manifolds.

Ref. 5 In the paper [13] we study the integral equations associated to the boundary value problems for the scalar Helmholtz equation (H) and the Maxwell system (M) on Lipschitz domains.

The Galerkin approximation scheme for (H) can be proven to converge under suitable conditions on the meshsize. also, for the regular data case, error Sobolev-Besov space estimates can be obtained.

Furthermore, for the case of a bounded convex domain, by estimating the spectral radius of the double layer acoustic potential operator we show that its inverse can be expressed as a Neumann series strongly convergent in the operator norm.

The case of (M) is more subtle, as the corresponding spaces for the boundary data have a more geometrical nature for which the usual triangulation process is not readily applicable.

Ref. 6 The paper [14] deals with a number of boundary value problems for the higher dimensional version of the Maxwell equations on Lipschitz domains in R^m . The main focus is the spectral analysis of the boundary singular integral operators which naturally arise in the treatment of these problems. Somewhat more specifically, we give estimates for the spectral radii, the point spectra and present some duality results. Several applications are also made.

The techniques of proof are akin those of [12], relying on Rellich estimates for differential forms on Lipschitz domains and the fundamental results of Coifman, McIntosh and Meyer.

- Ref. 7** In this paper we present an overview of wavelet based multiresolution analyses. First, we briefly discuss the continuous wavelet transform in its simplest form. Then, we give the definition of a multiresolution analysis and show how wavelets fit into it. We take a closer look at orthogonal, biorthogonal and semiorthogonal wavelets. The fast wavelet transform, wavelets on an interval, multidimensional wavelets and wavelet packets are discussed. Several examples of wavelet families are introduced and compared. Finally, the essentials of two major applications are outlined: data compression and compression of linear operators.
- Ref. 8** We present ideas on how to use wavelets in the solution of boundary value ordinary differential equations. Rather than using classical wavelets, we adapt their construction so that they become (bi)orthogonal with respect to the inner product defined by the operator. The stiffness matrix in a Galerkin method then becomes diagonal and can thus be trivially inverted. We show how one can construct an $O(N)$ algorithm for various constant and variable coefficient operators.
- Ref. 9** In this paper we show how an algorithm developed in a paper by some of the authors, on wavelets on subsets of the real line, can be used for data segmentation. The basic idea is to split the data into smooth segments that can be compressed separately. Relying on results from the earlier paper, a fast algorithm that uses wavelets on closed sets and wavelet probing is numerically tested
- Ref. 10** In this paper an attempt is made to give an overview of some existing wavelet techniques. The continuous wavelet transform and several wavelet-based multiresolution techniques leading to the fast wavelet transform algorithm are briefly discussed. Different families of wavelets and their construction are discussed and compared. The essentials of two major applications are outlined: data compression and compression of linear operators.
- Ref. 11 and 12** We discuss bases formed by smooth local trigonometric functions and their applications to image compression. It is known that these bases can reduce the blocking effect that occurs in JPEG. We present and compare two generalizations of the original construction of Coifman and Meyer: biorthogonal and equal parity folding. They have the advantage that constant and linear components can be

represented efficiently. We show how they reduce the blocking effect and improve the mean square error.

Ref. 13 We discuss smooth local trigonometric bases and their applications to signal compression. In image compression, these bases can reduce the blocking effect that occurs in JPEG. We present and compare two generalizations of the original construction of Malvar, Coifman and Meyer: biorthogonal and equal parity bases. These have the advantage that constant and linear components, respectively, can be represented efficiently. We show how they reduce blocking effects and improve the signal to noise ratio.

Ref. 14 The basic idea of wavelets and multiresolution analysis is to use the dyadic translates and dilates of one function as a basis of L^2 . These wavelets form bases for spaces of functions defined on the whole Euclidean space are usually (bi)orthogonal with respect to the classic L^2 inner product. In this paper we consider a construction of wavelets defined on almost arbitrary sets and adapted to more general pairings. We do not impose any geometrical constraints on the sets such as smoothness, connectivity or convexity; they only have to be nested in a fairly general way. Our construction is based on combining ideas from Alpert and Mitrea. Wavelets on arbitrary sets are useful in applications such as the numerical solution of partial differential and integral equations, and in image processing tasks such as segmentation, compression, motion estimation, stereo matching, and shape from shading.

Ref. 15, 16 For quite some time it has not been clear how to define multiresolution analysis on more general sets in higher dimensions. Similarly, the construction of corresponding wavelets on general sets has been an open problem for several years. In the papers [27] and [23] we solve these problems and present a construction of wavelets on general nonsmooth domains. The basic idea is to extend earlier results concerning biorthogonal wavelets on closed subsets of the real line by the authors and combine those results with a tensor product type argument adapted to the particular set.

Ref. 17 This report contains a detailed description of the Differential Geometric Method for object recognition problems mentioned above.

Ref. 18 Our recent wavelet based fusion research concentrated on the analysis of static images. Although there is a substantial savings in the time needed to create a fused image, there are still problems with the extraction of connected regions from the image. This is due to the ambiguities associated with the choice of the wavelet channels to use for reconstruction. Multiple images derived from multiple sensors can be used to assist this selection process, as well as to derive object characteristics through dynamic scene analysis. Some of our earlier work with epipolar image (EPI) analysis of fused image sequences indicated that the technique was able to successfully act as a navigation aid in highly cluttered, dynamic environments. In in [4] paper, we present a system that combines wavelet analysis with the EPI technique. Frame by frame integration of information from the sensors is done within the wavelet coefficient space, followed by an EPI analysis of features derived from the fused coefficients. We also report the results of a preliminary experiment with a laboratory sequence.

Ref. 19 The need for robust target/background segmentation has led to the use of multiple band sensing systems. These sensors usually include some combination of visual, radar or laser range, and thermal infrared modalities. Among the hard problems encountered in ATD/R are the high false-alarm rates frequently encountered due to nonrepeatability of the target signatures and possible obscuration of the targets from camouflage, environmental and sensor variations. In [6] a biologically motivated neural network system is presented. It is biologically motivated and based on the rattlesnake that integrates multichannel sensory inputs for ATD/R. The system demonstrates a probability of detection greater than 90% with a false-alarm rate less than 10^{-5} false-alarms/km² for very small fixed targets using two-channel infrared input. In addition, temporal properties of the thermal neurons in the rattlesnake are demonstrated to be of possible use for segmentation of mobile targets from background clutter. Also presented are the results of some experimental studies on real-world multichannel infrared images sampled throughout a day.

Ref. 20 Mathematical morphology, as originally described by Matheron and Serra, consists of the application of set theoretical operations between the image set X and a structure element set B . Image skeletons

are very efficient representations of shape, and can be directly derived using the morphological operations of erode and open. If done at full image resolution, derivation of a skeleton can be very time consuming without dedicated signal processing hardware.

In [7] we present an alternative to the standard approach that relies on morphological operations within the wavelet coefficient space. In particular, the skeleton transformation can be done very efficiently at the reduced resolution of the coarse wavelet coefficient levels. We investigate the relationship between wavelet image compression and morphological transforms for the derivation of skeletons. We also report the results of some experimental studies on binary and gray scale images.

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Ref. 22 A trend is emerging towards the use of networks of smaller distributed robots for complicated tasks. A number of areas need to be addressed before such systems can be put into practical environments. Among these are the transfer and sharing of information between robots, control strategies for sensing and movement, interfaces for teleoperator assistance to the multirobot systems, and degree of autonomy.

In [2] we present a cooperative multirobot system framework that has a flexible degree of autonomy, depending on the complexity of the task that is to be performed. The system uses a wavelet-based method to

address the pose and orientation calculations for robot positioning. Our previous work in this area demonstrated that reasonable sensor integration can be done within the wavelet domain at the coarse level. Augmented finite state machines are used under a subsumption architecture for control and integration of local and global maps for the multirobot system. This allows us to explicitly include the teleoperator interface in the system design.

We also present the results of an experimental simulation study of a spinning satellite retrieval by three cooperating robots. This simulation includes full orbital dynamics effects such as atmospheric drag and non-spherical gravitation field perturbations.

- Ref. 23 In [25] we present a method that allows us to edit a given surface to obtain a new one with desired, user-defined properties. The approach is based on a version of the Nash-Moser techniques combined with wavelets.
- Ref. 24 Functional MRI (fMRI) has recently become an effective method for studying brain activities. In this report [26] we use wavelet methods for denoising and enhancing the readability of fMRI images. Several experimental are presented and discussed.
- Ref. 25 In the note [27] we show how wavelets on Lipschitz curves are efficient tools for studying Laplace's equation on nonsmooth domains in the plane. Numerical results are included.
- Ref. 26 In [28] we construct compactly supported wavelets by the average-interpolation scheme with an integral decimation parameter $p \geq 2$ and Haar systems on the set of p -adic numbers. The wavelets have arbitrarily high regularity and they are symmetric if the functions in a Haar system used in the average-interpolation are reflectible and shift-invariant. We prove that an average-interpolation scaling function generated by a general Haar system with certain smoothness behaves asymptotically as a polynomial average-interpolation scaling function. Hence, it has at least the same regularity as a polynomial average-interpolation scaling function. The (closure of) the linear span of the translates of the scaling function reproduces the functions used in the Haar system for the average interpolation. These average-interpolation schemes provide many useful choices of wavelets that have several ad-

vantages over the more standard ones, which only have vanishing moments, in many applications.

Ref. 27 In [29] we develop a method of editing normal vector fields of surfaces that has the potential to considerably increase the efficiency of 3D system in generating of surfaces. The method is based on the variational approach and the standard implicit function theory in proper Banach spaces.

Ref. 28,29 In [31], [32] we present the lifting scheme, a new idea of constructing compactly supported wavelets with compactly supported duals. The lifting scheme provides a simple relationship between all multiresolution analyses with the same scaling function. It isolates the degrees of freedom remaining after fixing the biorthogonality relations. Then one has full control over these degrees of freedom to custom-design the wavelet for a particular application. It also leads to a faster implementation of the fast wavelet transform. We illustrate the use of the lifting scheme in the construction of wavelets with interpolating scaling functions.

Ref. 30 IMI Report 1994:9

The implicit sampling theorem of Bar-David gives a representation of band limited functions using their crossings with a cosine function. This cosine function is chosen such that its difference with the original function has sufficient zero crossings for a unique representation. In [1] we show how, on an interval, this leads to a multiplicative representation involving a Riesz product. This provides an alternative to the classic additive Fourier series. We discuss stability and implementation issues. Since we have an explicit reconstruction formula, there is no need for an iterative algorithm.

Ref. 31 Wavelets have proven to be powerful bases for use in numerical analysis and signal processing. Their power lies in the fact that they only require a small number of coefficients to represent general functions and large data sets accurately. This allows compression and efficient computations. Traditional constructions have been limited to simple domains such as intervals and rectangles. In the paper [33] we present a wavelet construction for scalar functions defined on surfaces and more particularly the sphere. Treating these bases in the fully biorthogonal case we show how bases with custom properties

can be constructed with the lifting scheme. The bases are extremely easy to implement and allow fully adaptive subdivisions. We give examples of functions defined on the sphere, such as topographic data, bi-directional reflection distribution functions, and illumination, and show how they can be efficiently represented with spherical wavelets.

Ref. 32 In [5], given a complete separable σ -finite measure space (X, Σ, μ) and nested partitions of X , we construct unbalanced Haar-like wavelets on X that form an unconditional basis for $L^p(X, \Sigma, \mu)$ where $1 < p < \infty$. Our construction and proofs build upon ideas of Burkholder and Mitrea. We show that if (X, Σ, μ) is not purely atomic, then the unconditional basis constant of our basis is $(\max(p, q) - 1)$. We derive a fast algorithm to compute the coefficients.

Ref. 33 Wavelets are a powerful tool for planar image processing. The resulting algorithms are straightforward, fast, and efficient. With the recently developed spherical wavelets this framework can be transposed to spherical textures. In [34] we describe a class of processing operators which are diagonal in the wavelet basis and which can be used for smoothing and enhancement. Since the wavelets (filters) are local in space and frequency, complex localized constraints and spatially varying characteristics can be incorporated easily. Examples from environment mapping and the manipulation of topography/bathymetry data are given.

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