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# OPTIMAL CORRELATION FILTERS FOR DETECTING A TARGET IN BACKGROUND NOISE

University of Connecticut

Prof. Bahram Javidi

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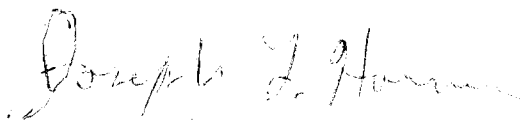
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APPROVED:



JOSEPH L. HORNER  
Project Engineer

FOR THE COMMANDER:



ROBERT V. MCGAHAN  
Acting Director  
Electromagnetics & Reliability Directorate

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13. ABSTRACT (Maximum 200 words) This report focuses on the development of optimal correlation filters for pattern recognition with spatially disjoint target and scene noise. In particular, it is shown that, for this class of problems, the matched filter expressions and optimum receivers derived under the overlapping input signal and scene noise assumption may not perform well in the presence of spatially disjoint input signal and scene noise.				
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## 1. Introduction

Since the introduction of the holographic matched filter,<sup>[1]</sup> many different types of filters for optical correlators have been proposed.<sup>[2-4]</sup> The matched filter is designed under the assumption that a wide sense stationary input noise is additive (e.g., spatially overlapping) with the target. The limitations of the matched filter in terms of broad correlation peak and sensitivity to distortion are known.<sup>[2,3]</sup> Different criteria have been proposed to characterize the filter performance in terms of light efficiency, sharpness of the correlation peak, and peak-to-sidelobe ratio<sup>[5,6]</sup>, and the importance of finding trade-offs between the different criteria by changing the filter characteristics has been established.<sup>[7-9]</sup>

The matched filter function is derived under the condition that the additive input noise is overlapping with the target and is at least wide sense stationary.<sup>[1]</sup> In many pattern recognition applications, however, the input scene noise does not overlap the target. The idea, in plain words, is that the target is in the foreground and blocks the scene noise.

In this report, we focus on the development of optimal correlation filters for pattern recognition with spatially disjoint target and scene noise. We analyze the consequences of this assumption which describes many realistic physical situations. In particular, we show that for this class of problems the matched filter expressions and the optimum receivers derived under the overlapping input signal and scene noise assumption may not perform well in the presence of spatially disjoint input signal and scene noise. The matched-filter functions are derived by maximizing the conventional correlation peak signal-to-noise ratio (SNR) metric. Under the spatially disjoint target and scene noise condition, the conventional definition of SNR metric needs to be modified and new optimum receivers and filters need to be determined. Owing to the spatially disjoint input scene noise and the target, the procedure for the maximization of the correlation peak SNR will not result in a useful optimum matched-filter function.<sup>[1]</sup>

In this report, we present four techniques to detect a signal in the presence of spatially disjoint

scene noise. The first approach uses multiple hypothesis testing to describe an optimum receiver to detect a signal in spatially disjoint scene noise. The second approach is based on minimum mean square error filtering; that is, a Wiener filter is designed to detect a signal in spatially disjoint scene noise. The third technique is based on maximizing a performance metric to design an optimum filter to detect signals in spatially disjoint scene noise. The new performance metric is defined as the ratio of the output signal at the target location to the output signal energy for the spatially disjoint target and scene noise case. The fourth approach is based on modifying the conventional definition of the correlation peak SNR to take into account the non-overlapping nature of the target and the scene noise. A generalized matched filter is designed to detect a signal in spatially disjoint scene noise by maximizing the modified SNR .

In section 2, the limitations of the conventional SNR as a measure of the matched filter performance for spatially disjoint target and scene noise are discussed. A limited number of computer simulation tests will be provided to show that the conventional matched filter obtained by maximizing the classic definition of SNR will not perform well for this case.

In section 3, the design of an optimum receiver for pattern recognition problems with input scene noise that is spatially disjoint with the target is described. The processor is designed based on multiple alternative hypothesis testing.

In section 4, a minimum mean square error filter for pattern recognition problems with input scene noise that is spatially disjoint with the target is described. The filter is designed to have an output which is a delta function located at the position of the target. The filter minimizes the mean square of the difference between the desired delta function and the filter output in response to a target in the presence of spatially disjoint scene noise. We show that the filter output has a well defined peak and small sidelobes in the presence of disjoint signal and scene noise.

In section 5, an optimum filter is designed to maximize the ratio of the correlation peak to the average output energy for a signal which is spatially disjoint with the input scene noise. This filter

takes into account the non-overlapping nature of the target and the scene noise, the effect of the non-whiteness of input scene noise, the limited spatial size of the input window, and the unknown illumination of the target. We will show that the filter produces a sharp output signal at the target location with a low output noise sidelobes. We show that this optimum filter is very similar to the minimum mean square error filter for input scene noise that is spatially disjoint with the target.

In section 6, the classic definition of the correlation peak SNR is modified to obtain a new metric for spatially disjoint target and scene noise. The modified SNR is defined as the ratio of the expected value squared of the correlation peak to the spatial average of the output variance and is denoted  $\widetilde{\text{SNR}}$ . By maximizing  $\widetilde{\text{SNR}}$ , we have designed a generalized matched filter function that can detect a target in the presence of spatially disjoint scene noise. It will be shown that the conventional matched filter function is a special case of this generalized matched filter under the condition that the input noise is overlapping with target, and has infinite size and zero mean.

## **2. Limitations of the Classic Definition of the Correlation SNR in Pattern Recognition with Spatially Disjoint Target and Scene Noise<sup>[10]</sup>**

In this section, we show that the matched filter expressions derived under the overlapping input signal and scene noise assumption may not perform well in the presence of spatially disjoint input signal and scene noise. The commonly used optimum matched-filter functions are derived by maximizing the conventional SNR. We discuss the limitation of the conventional SNR as a performance measure in matched filter based optical pattern recognition for input scene noise that is spatially disjoint (or non-overlapping) with the target. For matched filters the classic definition of the SNR is the ratio of the expected value squared of the correlation peak to the variance of the peak.<sup>[1]</sup> Under the assumption that the input scene noise is stationary and overlaps the target, the correlation peak variance is a good statistical measure of the variance at any other sample point of the correlation signal. We show that when the input noise is spatially disjoint with the target, the

correlation peak variance may not be a good measure of the noise fluctuations over the entire output plane. For clarity, one dimensional notation will be used in the following analysis.

We examine the correlation SNR generated by a matched filter function for an input signal  $s(t)$  that contains a target (reference signal)  $r(t)$  in the presence of spatially disjoint additive scene noise  $\tilde{n}(t)$ . The input signal and its Fourier transform are:

$$s(t)=r(t)+\tilde{n}(t); \quad (1)$$

$$|S(\omega)| \exp[j\phi_S(\omega)]= |R(\omega)| \exp[j\phi_R(\omega)] + |\tilde{N}(\omega)| \exp[j\phi_{\tilde{N}}(\omega)] \quad (2)$$

Here,  $|S(\omega)| \exp[j\phi_S(\omega)]$ ,  $|R(\omega)| \exp[j\phi_R(\omega)]$ , and  $|\tilde{N}(\omega)| \exp[j\phi_{\tilde{N}}(\omega)]$  are the Fourier transforms of the input signal, the reference signal, and the input scene noise, respectively. We assume that the noise  $\tilde{n}(t)$  is bounded in a finite window and that the Fourier transform of its realization exists.

We consider  $k^{\text{th}}$  law nonlinearly transformed matched filters.<sup>[9]</sup> The first order component of the  $k^{\text{th}}$  law nonlinearly transformed matched filter that generates the correlation signal is:<sup>[9]</sup>

$$H_{k1}(\omega) = |R(\omega)|^k \exp\{-j\phi_R(\omega)\}, \quad (3)$$

Here, the carrier frequency is dropped for simplicity, and  $k$  is the severity of the nonlinearity (a given constant). A family of filters is generated by changing  $k$ . A conventional matched filter<sup>[1]</sup> can be produced by setting  $k=1$ . A phase-only filter<sup>[4,9]</sup> is produced by using a binary nonlinearity ( $k=0$ ).

The light leaving the filter plane that generates the first order correlation term is the product of the input signal Fourier transform and the filter function in Eq.(3):

$$C_1(\omega) = |R(\omega)|^k \exp\{-j\phi_R(\omega)\} |R(\omega)| \exp[j\phi_R(\omega)] + |R(\omega)|^k \exp\{-j\phi_R(\omega)\} |\tilde{N}(\omega)| \exp[j\phi_{\tilde{N}}(\omega)] \quad (4)$$

The first term in Eq.(4) produces the correlation term and the second term produces the output noise term. It is assumed that carrier frequency is large enough to separate the higher order terms from the first order correlation term.

We denote the filter output by  $y(t)$ . The conventional correlation peak SNR is defined as the ratio of the expected value squared of the correlation peak amplitude to the variance of the correlation peak amplitude:

$$SNR = \frac{|E\{y(0)\}|^2}{\text{Var}\{y(0)\}} \quad (5)$$

Here,  $y(0)$  is the correlation peak amplitude:

$$y(0) = \frac{1}{2\pi} \int |R(\omega)|^{k+1} d\omega + \frac{1}{2\pi} \int |R(\omega)|^k |\tilde{N}(\omega)| \exp\{j[\phi_{\tilde{N}}(\omega) - \phi_R(\omega)]\} d\omega \quad (6)$$

In Eq. (6), the first term is the signal term and is deterministic. The variation in the correlation peak owing to the noise fluctuations is represented by the second term. Using Parseval's theorem, the contribution of the noise term to the correlation peak,  $n_o$ , is:

$$n_o = \int_{(t) \in S_1 \cup S_2} \tilde{n}(t) r_k(t) dt = \int_{(t) \in S_2} \tilde{n}(t) r_k(t) dt \quad (7)$$

where  $r_k(t)$  is the inverse Fourier transform of  $|R(\omega)|^k \exp\{j\phi_R(\omega)\}$ . Here,  $\cup$  is a union,  $S_1$  is the area

in the input plane that covers the target  $r(t)$ , and  $S_2$  is the remaining area in the input plane that covers the input noise. The second equality follows since  $\tilde{n}(t)$  is zero over  $S_1$ . The SNR is

$$\text{SNR} = \frac{\left| \frac{1}{2\pi} \int |\mathcal{R}(\omega)|^{k+1} d\omega + \int_{(t) \in S_2} E[\tilde{n}(t)]r_k(t) dt \right|^2}{E \left\{ \left[ \int_{(t) \in S_2} \tilde{n}(t)r_k(t) dt - \int_{(t) \in S_2} E[\tilde{n}(t)]r_k(t) dt \right]^2 \right\}} \quad (8)$$

For the conventional filter ( $k=1$ ), we have  $r_1(t)=r(t)$  which is spatially disjoint with the noise term. In this case, the contribution of the noise term to the correlation peak is zero; that is,  $n_0$  is zero. Thus the denominator of the expression for the SNR goes to zero. Since the noise does not affect the correlation peak, the correlation peak variance is zero, and the correlation SNR of the conventional filter is infinite. This is the problem with the classical definition of the SNR under the spatially disjoint condition for measuring the correlation performance: a large input noise could generate large output noise amplitude at points other than the correlation peak position such that false alarms are generated indicating an overall poor system performance. However, for these cases the conventional SNR could remain high, presenting a misleading measure of system performance.

For nonlinearly transformed matched filters ( $k \neq 1$ ) including the phase-only filter,  $r_k(t)$  "leaks" into the input scene noise owing to the Fourier plane nonlinear transformation. (Leaks can also occur for non-conventional matched filters.) The spreading of  $r_k(t)$  "grabs" the noise and "pulls" it into the correlation peak. Let us consider the case of  $k$  varying from one to zero. As the severity of the nonlinearity increases ( $k$  decreases), the degree of the leakage increases. The most severe leakage is for  $k=0$  (phase-only filter). In the presence of leakage the noise contribution in Eq.(7) is the product of  $r_k(t)$  and  $\tilde{n}(t)$  integrated only over the area that  $r_k(t)$  spreads into  $\tilde{n}(t)$ . This means integration over the leakage area within  $S_2$ . However, most of the energy of  $r_k(t)$  may remain in

the  $S_1$  area which means that the portion of the signal in the leakage area may be small and that the contribution of the noise fluctuation to the correlation peak may be small. As a result, the variance of the correlation peak would remain small and the SNR would be large (although naturally smaller than the conventional case, which is infinity).

Computer simulations are performed to investigate the effects of the input scene noise on the correlation peak SNR. In the computer simulations, white Gaussian noise is added to a deterministic reference signal (tank image). For the overlapping case, the conventional matched filter is optimum for wide sense stationary white Gaussian noise. The additive input noise is considered for both the overlapping and non-overlapping target, as shown in Fig. 1. The expected value of the noise,  $m_n$ , is 0.45. The standard deviation of the noise,  $\sigma_n$ , is 0.2. Both the conventional filter ( $k=1$ ) and the phase only filter ( $k=0$ ) are considered [please see Eq.(4)]. The phase only filter results in the most severe leakage of the output signal into output noise. In the computer simulation, the target (a tank image) and the input image are sampled to reduced size with  $15 \times 26$  and  $50 \times 69$  pixels respectively. The size of the input array is  $128 \times 256$  pixels. The reference signal amplitude in the input plane is normalized to have a maximum value of unity. The SNR is measured for the first order correlation peak according to Eq.(8). The tests were repeated 100 times to obtain the expected values and to measure each output SNR.

Using the target in the spatially disjoint scene noise [please see Fig. 1(a)], we show an example of the output correlation signal intensity for the conventional filter in Fig. 2(a). In the tests, we measured the conventional SNR, and the failure frequency in the detection of the target. The failure frequency indicates the number of times that any pixel value of the output noise intensity becomes larger than the correlation peak intensity. It is assumed that target is detected successfully when the correlation peak is larger than every pixel value of the output noise.

For the noise parameters used here the tests performed for the conventional correlator ( $k=1$ ) produced very large numerical values for the conventional SNR (infinite in theory and larger than

$10^{30}$  in computer simulation) but failed to detect the tank image many times by producing a correlation peak to sidelobe ratio of less than unity. The failure frequency for the conventional filter was about 78 out of 100. On the other hand, the  $k$ th law nonlinear filter for  $k=0$  (that is, the phase-only filter) produced a much lower conventional correlation peak SNR (about 320), but it was more successful in detecting the target (the failure frequency was zero). It is evident that under the spatially disjoint condition a large correlation peak SNR may not be a good indication of the system performance.

We have performed the same correlation tests with the same target and the same scene noise that is overlapping with the target [please see Fig. 1(b)]. The correlation signal intensity for the conventional filter is shown in Fig. 2(b). In the tests, the conventional filter performed very well with no failure and produced a SNR of about  $2.2 \times 10^3$ . In this case, the input noise satisfies the conditions that the filter is designed under, and depending on the noise color, the conventional expressions for the matched filter result in optimum performance. The effect of spatially disjoint input noise mean on the correlation performance is discussed in Appendix A.

In summary, we have investigated the limitation of the classic definition of the correlation peak SNR as a measure of system performance for input scene noise that is spatially disjoint (non-overlapping) with the target. The correlation peak SNR is considered for the classic (conventional) matched filter and for the nonlinearly transformed filters, including the phase-only filter. The output of the filter contains a signal term and a noise term. For spatially disjoint noise, it is shown that the correlation peak SNR is infinity for the conventional filter ( $k=1$ ). For nonlinearly transformed filters, such as the phase-only filter, the output signal term "leaks" into the output noise term. The degree of the leakage depends on the filter type. However, even for a severe leakage (phase only filter), the peak SNR remains large since most of the signal energy is non-overlapping with the noise. Computer simulations that use spatially disjoint target and scene noise are provided. The filter can produce a large numerical correlation peak SNR and can fail to detect the target by generating output noise larger than the correlation peak.

### 3. Design of an Optimum Receiver for Pattern Recognition with Spatially Disjoint Target and Scene Noise<sup>[11]</sup>

In Section 2, it was shown that the conventional filters designed under the spatially overlapping target and input scene noise assumption may not perform well for input scene noise that is spatially disjoint (that is, non-overlapping) with the target.<sup>[10]</sup> We know from classic results in communication theory that using hypothesis testing optimum receivers designed to detect a known signal in the presence of additive stationary Gaussian noise can be implemented using matched filters. For pattern recognition, a theoretical design of an optimum processor under the spatially disjoint target and scene noise condition is needed. In this section, we describe the design of an optimum receiver for pattern recognition problems when the input signal contains a target in the presence of input scene noise such that the input scene noise is disjoint (non-overlapping) with the target in the input scene. We show that for such problems, the conventional correlator solution that results from the overlapping input signal and scene noise assumption will not be the optimum solution. We shall compare the optimum receiver obtained for the spatially disjoint target and scene noise case with the classic results to describe the additional operations that need to be performed on a matched filter to obtain the optimum receiver. Since in general detectors produce an additive noise on the input image, we will also assume that the target can be noisy. The detector noise is added both to the target and to the background noise. However, we will make the assumption that the noise on the target is statistically independent of the noise on the other parts of the image. This assumption is easily fulfilled if one considers that the detector noise is white and statistically independent.

We use multiple hypothesis testing with unknown parameters to design a pattern recognition system for the spatially disjoint target and scene noise problem. For each hypothesis, the received signal  $s(t)$  contains a target (reference signal)  $r(t-t_j)$  at location  $t_j$  in the presence of spatially disjoint

additive input scene noise  $n_j(t)$ . Let  $w_r(t)$  denote a unit magnitude window function that defines the support of the reference signal such that  $w_r(\cdot)$  is unity within the reference signal, and is zero elsewhere. It is evident that  $r(t-t_j)w_r(t-t_j) = r(t-t_j)$ . We denote the detector noise by  $n_d(t)$  and we assume that it is statistically independent of the spatially non-overlapping background noise  $n_B(t)$ . We have:

$$n_j(t) = [n_B(t) + n_d(t)][1 - w_r(t-t_j)]. \quad (9)$$

If the detector noise is negligible, we can let the variance of  $n_d(t)$  go to zero, and the target will become noise free. Since the amplitude of the signal  $r(t)$  can be unknown,  $r(t)$  is multiplied by an unknown parameter " $\tilde{a}$ " which could represent the illumination variation. Different hypotheses  $H(j, \tilde{a})$  correspond to a possible position  $t_j$  of the target and to a possible amplitude parameter " $\tilde{a}$ ":

$$H(j, \tilde{a}): s(t) = [\tilde{a}r(t-t_j) + n_d(t)]w_r(t-t_j) + n_j(t) \quad (10)$$

This is an estimation/detection problem. The unknown parameter " $\tilde{a}$ " needs to be determined followed by signal detection. We use the Bayes test for multiple alternative hypotheses testing to decide which hypothesis is more probable, and to determine both  $t_j$  and " $\tilde{a}$ ". Assuming all the hypotheses equally likely and equal costs for all errors, the situation reduces to a maximum likelihood test,<sup>[12]</sup> and the decision rule becomes: choose the hypothesis  $H(j, \tilde{a})$  for which the corresponding probability density function (likelihood function) is maximum. The assumption of equally likely hypotheses is reasonable since we do not have the probability of each hypothesis. We consider " $m$ " amplitude samples of the observation  $s(t)$ , where  $s(t_i)$  is the observation sample, and the vector  $\mathbf{s}$  represents the  $m$  samples. We denote  $\mathbf{s}_j$  the  $m_r$  samples taken over the target, that is, within the window  $w_r(t_i-t_j)$ . We denote  $\bar{\mathbf{s}}_j$  the remaining samples taken over the scene noise, that is, within the scene window  $[1-w_r(t_i-t_j)]$ . The joint probability density function of the  $m$  samples of  $s(t)$  given

that the hypothesis  $H(j, \tilde{a})$  is true is denoted by  $p[s/H(j, \tilde{a})]$ . We denote the joint probability density functions of the samples of  $n_j(t)$  and  $n_d(t)$ , as  $p_{\mathbf{n}_j}(\cdot)$  and  $p_{\mathbf{n}_d}(\cdot)$ , respectively. Since it is assumed that  $n_j(t)$  and  $n_d(t)$  over the target are statistically independent,  $p[s/H(j, \tilde{a})] = p_{\mathbf{n}_j}(\bar{\mathbf{s}}_j) p_{\mathbf{n}_d}(\mathbf{s}_j - \tilde{\mathbf{a}}\mathbf{r}_j)$ . The vector  $\mathbf{r}_j$  represents the samples of the target  $r(t_i - t_j)$ . To proceed further, the probability density function of the detector noise  $p_{\mathbf{n}_d}(\cdot)$  is needed. We shall assume that  $n_d(t)$  is white Gaussian with variance  $\sigma_d^2$ :

$$p_{\mathbf{n}_d}(\mathbf{n}_d) = \prod_{i=1}^m p_{n_d}(n_d(t_i)) = [(1/2\pi\sigma_d^2)^{0.5m}] \exp\{-\sum_{i=1}^m [n_d(t_i)]^2/2\sigma_d^2\}.$$

Under these assumptions, the log-likelihood function,  $\log\{p[s/H(j, \tilde{a})]\}$ , is:

$$\log\{p[s/H(j, \tilde{a})]\} = \log[p_{\mathbf{n}_j}(\bar{\mathbf{s}}_j)] - (1/2\sigma_d^2) \sum_{i=1}^m w_r(t_i - t_j) [s(t_i) - \tilde{\mathbf{a}}r(t_i - t_j)]^2 + 0.5m \log(1/2\pi\sigma_d^2). \quad (11)$$

Note that in Eq. (11), the summation in the second term is over the entire scene, but only  $m_r$  samples are added due to the window  $w_r(t_i - t_j)$ . To choose a hypothesis  $H(j, \tilde{a})$ , we set to maximize its corresponding  $\log\{p[s/H(j, \tilde{a})]\}$ . The last term of Eq. (11) does not affect the maximization and will be ignored. The maximum likelihood estimate of " $\tilde{a}$ " can be determined by maximizing the likelihood function over " $\tilde{a}$ " given that  $t_j$  is fixed. To do this, we set to zero the gradient of  $\log\{p[s/H(j, \tilde{a})]\}$  with respect to " $\tilde{a}$ " and obtain:

$$\sum_{i=1}^m [s(t_i) - \hat{\mathbf{a}}r(t_i - t_j)]r(t_i - t_j) = 0. \quad \text{or} \quad \hat{\mathbf{a}} = \frac{\sum_{i=1}^m s(t_i)r(t_i - t_j)}{\sum_{i=1}^m r^2(t_i - t_j)}, \quad (12)$$

where  $\hat{\mathbf{a}}$  is the maximum likelihood estimate of " $\tilde{a}$ " given  $t_j$ . We substitute the maximum likelihood

estimate of " $\tilde{a}$ " in  $\log\{p[s/H(j,\tilde{a})]\}$  of Eq. (11), and then try to find the best estimate of the location of the target by maximizing  $\log\{p[s/H(j,\tilde{a})]\}$  over  $t_j$ . Thus, we need to find the maximum of:

$$\log\{p[s/H(j,\hat{a})]\} = \log[p_{\mathbf{n}_j}(\bar{s}_j)] - (1/2\sigma_d^2) \sum_{i=1}^m w_r(t_i-t_j) [s(t_i) - \hat{a}r(t_i-t_j)]^2, \quad (13)$$

where  $\log\{p[s/H(j,\hat{a})]\}$  is the log likelihood function using  $\hat{a}$ . In Eq.(13), we have ignored the last term of Eq. (11) for simplicity. Expanding the right hand side of Eq.(13), and substituting for the estimated value of " $\tilde{a}$ " given by Eq.(12) in Eq.(13), the optimal decision is to select the maximum of the following:

$$\log\{p[s/H(j,\hat{a})]\} = \log[p_{\mathbf{n}_j}(\bar{s}_j)] - (1/2\sigma_d^2)C_N(t_j) \quad (14)$$

where

$$C_N(t_j) = \sum_{i=1}^m s^2(t_i)w_r(t_i-t_j) - \frac{[\sum_{i=1}^m s(t_i)r(t_i-t_j)]^2}{\sum_{i=1}^m r^2(t_i-t_j)}.$$

Equation (14) provides the optimal decision rule to obtain  $t_j$  and needs to be evaluated for all  $t_j$ .

It is interesting to note that for a bounded  $p_{\mathbf{n}_j}(\bar{s}_j)$  [that is  $0 < p_{\mathbf{n}_j}(\bar{s}_j) < \infty$ ], if the noise on the target is sufficiently small ( $\sigma_d \ll 1$ ), then the actual  $p_{\mathbf{n}_j}(\bar{s}_j)$  becomes irrelevant to the detection process. In this case, the maximization of the likelihood function over  $t_j$  reduces to finding the maximum of  $-C_N(t_j)$  in Eq. (14). Thus, the optimal decision to obtain  $t_j$  is to find the minimum of the following:

$$C_N(t_j) = \sum_{i=1}^m s^2(t_i) w_r(t_i - t_j) - \frac{[\sum_{i=1}^m s(t_i) r(t_i - t_j)]^2}{\sum_{i=1}^m r^2(t_i - t_j)}. \quad (15)$$

When the input scene noise is spatially disjoint with the target and the target is noise free, the optimum receiver is independent of the scene noise probability density function. In Eq. (15), the first term computes the energy of the input scene over the window function for different  $t_j$ . The second term is the cross correlation between the reference function and the input scene normalized by the reference signal energy. This normalization is due to the presence of the unknown amplitude illumination " $\tilde{a}$ ". The minimum possible value of  $C_N(t_j)$  is zero, which value occurs only when  $t_j$  is the target's true location.

As an aside, since  $C_N(t_j) \geq 0$ , Eq. (15) may be rearranged as:

$$M_j(\mathbf{r}) = \frac{[\sum_{i=1}^m s(t_i) r(t_i - t_j)]^2}{\sum_{i=1}^m r^2(t_i - t_j) \sum_{i=1}^m s^2(t_i) w_r(t_i - t_j)} \leq 1 \quad (16)$$

where there is again equality only if  $t_j$  is the target's true location. In general, maximization of  $-C_N(t_j)$  and  $M_j(\mathbf{r})$  will not yield the same results; however, for the noise free target which is disjoint with scene noise, the two are equivalent. Equation (16) is the cross-correlation between the target and the input scene normalized by the energy of the input scene over the window function  $w_r(t_i - t_j)$  and the target energy.<sup>[13]</sup> It is evident that the normalized correlation used with heuristic arguments is optimal when the background noise is spatially disjoint with the target and when there is no noise on the target.

It is interesting to note that if the noiseless target in the input scene is known to have an amplitude scaling of one (that is,  $\tilde{a}=1$ ), using Eq.(11), the test is to find the maximum of:

$$- \sum_{i=1}^m w_r(t_i-t_j)[s(t_i)-r(t_i-t_j)]^2 \quad (17)$$

This test implies that if the target is noise free and is known exactly, the solution is to find  $t_j$  where

$\sum_{i=1}^m s(t_i)r(t_i-t_j) - 0.5 \sum_{i=1}^m w_r(t_i-t_j)s^2(t_i)$  is maximum. This occurs for  $t_j$  where the window is located on the target in the scene. It is evident that the conventional correlation is not optimal in this case, and a bias sum of  $-0.5 \sum_{i=1}^m w_r(t_i-t_j)s^2(t_i)$  should be added to the standard correlation receiver  $\sum_{i=1}^m s(t_i)r(t_i-t_j)$ .

If the noise on the target is not small, then the probability density function of the scene noise  $p_{\mathbf{n}_j}(\bar{\mathbf{s}}_j)$  is needed, and the normalized correlation will not be optimal. Let's assume that  $p_{\mathbf{n}_j}(\bar{\mathbf{s}}_j)$  is Gaussian, white, with mean  $m_n$  and variance  $\sigma_n^2$ , and that  $(m-m_r)$  statistically independent samples of the scene noise are considered. In this case, the decision rule is to find the maximum of:

$$\log\{p[s/H(j,\hat{a})]\} = -[1/2(\sigma_n^2 + \sigma_d^2)] \sum_{i=1}^{m-m_r} \{[1-w_r(t_i-t_j)]s(t_i)-m_n\}^2 - (1/2\sigma_d^2)C_N(t_j). \quad (18)$$

A limited number of computer simulation tests are presented to investigate the performance of the optimum receiver designed for the spatially disjoint input signal and scene noise. The tests are

performed to find the location of the target according to the optimal decision rule presented in Eq. (15). The maximum of  $[-C_N(t_j)]$  when equal to zero indicates the position of  $t_j$ . In the simulations, white Gaussian scene noise is used with an expected value of  $m_n = 0.5$  and a standard deviation of  $\sigma_n = 0.3$ . The reference signal is a tank image as shown in Fig. 3. The size of the target (tank image) is 15x26 pixels, the size of the input signal is 50x60 pixels, and the size of the input array is 128x128 pixels. The reference signal amplitude in the input plane is normalized to have a maximum value of unity. The reference signal energy is 28.3 and its mean is 0.14. An example of the noise free target with non-overlapping scene noise, is shown in Fig. 3(a). The target has an illumination of 0.5 which is unknown to the receiver. For this case, we show an example of  $[-C_N(t_j)]$  in Fig. 3(b). The maximum value is equal to zero and indicates the location of the target. The test was repeated many times and the target detection was always successful (as it should be). It is assumed that target is detected successfully when the position of the maximum of  $[-C_N(t_j)]$  when equal to zero coincides with the actual location of the target. In addition, in Fig. 3(c), we have shown the cross-correlation between the target and the input scene, that is,  $\sum_{i=1}^m s(t)r(t-t_j)dt$ , for the disjoint signal and scene noise of Fig. 3(a). Note that this is the optimum receiver solution under overlapping signal and scene noise assumption for additive white zero mean Gaussian noise. It is evident from this figure that the cross-correlation test by itself may not be a good test to determine the location and presence of the target for the disjoint signal and scene noise problem.

In summary, we have used hypothesis testing to design an optimum receiver for pattern recognition problems with input scene noise that is spatially disjoint (or non-overlapping) with a noisy target. We have used white Gaussian statistics for the detector noise to illustrate the optimum receiver. The cases of a noise free target which is spatially disjoint with the scene noise and of a target which has an unknown illumination scale are discussed. It is shown that for a noise free target, the optimum receiver is similar to a correlator normalized by the input scene energy within the target window. In

addition, when the detector noise is sufficiently small (noise free target), and given that the scene noise probability density function is bounded, then the actual scene noise statistics becomes irrelevant to the detection process. These results clearly indicate that the spatially non-overlapping target and scene noise case results in optimum solutions that are very different from the conventional solutions obtained by the additive spatially overlapping signal and noise case.

#### **4. Minimum Mean Square Error Filter for Pattern Recognition with Spatially Disjoint Target and Scene Noise<sup>[15,16]</sup>**

In this section, we describe a minimum mean square error filter for pattern recognition problems with input scene noise that is spatially disjoint (or non-overlapping) with the target. To provide a sharp output response and a quiet background, the filter is designed to have an output which is a delta function located at the position of the target in the input scene. The filter minimizes the mean square of the difference between the desired delta function and the filter output in response to a noisy input. Under the disjoint signal and scene noise condition, the minimum mean square error filter solution will be dependent on the window function that defines the target and it will be different from the conventional matched filter function and the Wiener filter. Computer simulation is provided to show the performance of the minimum mean square error filter in the presence of the spatially disjoint target and scene noise. We show that the filter output has a well defined peak and small sidelobes in the presence of disjoint signal and scene noise.

In the analysis, samples of the signals are represented in discrete form and are used in the analysis. Samples of the input signal sequence is  $s(t_i)$  where  $i = \dots, -1, 0, 1, \dots$  is an integer. It is assumed that the input scene has a infinite size, that is, the input signal sequence  $s(i)$  is infinitely long. The reference signal sequence  $r(t_i)$  in the scene has a finite size and has a window  $w_r(t_i)$ . The window sequence  $w_r(t_i)$  of the target  $r(t_i)$  is defined to be one wherever  $r(t_i)$  exists and to be zero otherwise. The non-overlapping scene noise is defined as  $n(t_i)[1-w_r(t_i)]$ , and the noise  $n(t_i)$  is assumed to be samples of a wide sense stationary random process. The target is assumed to be

located at the origin in the input scene. The input scene  $s(t_i)$  is:

$$s(t_i) = r(t_i) + n(t_i) [1 - w_r(t_i)] = r(t_i) + n(t_i) w(t_i) \quad (19)$$

where  $w(t_i) = [1 - w_r(t_i)]$

Let us denote the impulse response of the minimum mean square error filter by the sequence  $h(t_i)$ , the actual filter output response by  $\hat{y}(t_i) = s(t_i) * h(t_i)$ , and the ideal output response by  $y(t_i) = \delta(i)$ . Here, "\*" is the convolution operation and  $\delta(i)$  is the Kronecker delta sequence. We use the minimum mean square error criterion to optimize  $h(t_i)$ . The mean square error (MSE) is the error squared between the ideal output and the actual filter output:

$$\text{MSE} = \sum_i \text{MSE}(i) = \sum_i E\{|\hat{y}(t_i) - y(t_i)|^2\} \quad (20)$$

where  $\text{MSE}(i) = |\hat{y}(t_i) - y(t_i)|^2$ . In this section, we consider that the signal is periodic (cyclostationary<sup>[16]</sup>) and we sum over one period. This allows to avoid divergence problems in the MSE determination. Here  $|\cdot|$  denotes absolute value, and  $E\{\cdot\}$  the expected value. From Eq. (20), it can be shown that the mean square error MSE is:

$$\text{MSE} = 1 + \sum_i \sum_j h(t_i) b(t_i, t_j) h(t_j) - 2 \sum_i h(t_i) E\{s(t_i)\} \quad (21)$$

where

$$\begin{aligned} b(t_i, t_j) &= \sum_p E\{s(t_i + t_p) s(t_j + t_p)\} \\ &= R_{rr}^c(t_i - t_j) + m_n R_{wr}^c(t_i - t_j) + m_n R_{rw}^c(t_i - t_j) + R_{ww}^c(t_i - t_j) R_{nn}^c(t_i - t_j) = b(t_i - t_j) \end{aligned} \quad (22)$$

Here,  $R^c(\cdot)$  is the correlation function. The subscripts  $rr$ ,  $wr$  and  $ww$  denote autocorrelation of  $r(\cdot)$ , cross-correlations between  $w(\cdot)$  and  $r(\cdot)$ , and autocorrelation of  $w(\cdot)$  respectively.

Using Eq. (21), the mean square error MSE can be written as:

$$\text{MSE} = 1 + \sum_{\nu} H(\nu)B(\nu)H^*(\nu) - 2 \sum_{\nu} H(\nu)E\{S(\nu)\}, \quad (23)$$

where,

$$B(\nu) = |R(\nu)|^2 + m_n W(\nu)R^*(\nu) + m_n W^*(\nu)R(\nu) + |W(\nu)|^2 S_n(\nu) \quad (24)$$

where  $W(\nu) = [\delta(\nu) - m_n W_r(\nu)]$  is the discrete Fourier transform of  $w(t_i)$ . Here,  $H(\nu)$ ,  $B(\nu)$ ,  $S(\nu)$ ,  $R(\nu)$  and  $W_r(\nu)$  are discrete Fourier transforms of  $h(t_i)$ ,  $b(t_i)$ ,  $s(t_i)$ ,  $r(t_i)$  and  $w_r(t_i)$ , respectively.  $S_n(\nu)$  is the power spectral density of the input noise sequence  $n(t_i)$ .

The MMSE filter is obtained by minimizing the MSE [please see Eq. (23)]:

$$\begin{aligned} H^*(\nu) &= \frac{E\{S(\nu)\}}{B(\nu)} \\ &= \frac{R(\nu) + m_n W(\nu)}{|R(\nu)|^2 + m_n W(\nu)R^*(\nu) + m_n W^*(\nu)R(\nu) + |W(\nu)|^2 S_n(\nu)} \end{aligned} \quad (25)$$

The input noise sequence  $n(t_i)$  can be considered as a sum of a zero mean noise sequence  $n_0(t_i)$  and its mean value  $m_n$ , that is,  $n(t_i) = m_n + n_0(t_i)$ . The non-overlapping input scene noise  $n(t_i)w(t_i)$  can thus be decomposed into two parts,  $m_n w(t_i)$  and  $n_0(t_i)w(t_i)$ . The first component  $m_n w(t_i)$  is deterministic when the mean of the noise is known and the uncertainty comes from the second component  $n_0(t_i)w(t_i)$ . The input data can be written as:

$$s(t_i) = \{r(t_i)+m_n w(t_i)\} + \{n_0(t_i)w(t_i)\} \quad (26)$$

The first term,  $\{r(t_i)+m_n w(t_i)\}$ , can be considered as the signal to be detected while the second term  $\{n_0(t_i)w(t_i)\}$  is the noise term to be rejected. It is evident from Eq.(25) that the numerator of the filter transfer function,  $R(v) + m_n W(v)$ , is matched to detect  $\{r(t_i)+m_n w(t_i)\}$ . When the scene noise is spatially disjoint with the signal, the window of the reference signal becomes as important as the reference signal itself. This is extremely useful when the mean of the noise background is strong. In this case, the disjoint window  $\{w(t_i)=[1-w_r(t_i)]\}$  may contribute significantly to the output signal.

In correlation tests using spatially disjoint target and scene noise, we have observed that when the scene noise has large mean, most of the widely used filters designed for overlapping signal and scene noise may fail to detect the target. These filters do not utilize the disjoint window information of the reference signal and scene noise. On the other hand, our tests indicate that the minimum mean square error filter presented here works very well for spatially disjoint target and scene noise.

The image in Figure 4(a) is used as the input data in the computer simulation. The reference signal is a tank image of 15x26 pixels and normalized to a maximum of unity as shown in Fig. 4(a). The target (tank image) is placed at pixel coordinates (44,35). The input array is 64x64 pixels. The scene noise is Gaussian, white, with mean  $m_n=0.5$  and standard deviation  $\sigma= 0.3$  and is spatially disjoint with the target. The 3-D plot of the output of the minimum mean square error filter is shown in Fig.4(b). It is evident that the minimum mean square error filter provides a very good result as evidenced by a well defined correlation peak and a small background output noise. This is because the desired output of the minimum mean square error filter is a delta function which sharpens the correlation peak while it minimizes the output background noise.

In summary, we have described a minimum mean square error filter for pattern recognition problems with input scene noise that is spatially disjoint with the target. The filter is designed to have an output which is a delta function located at the position of the target. The window of the reference signal strongly influences the filter function and the filter is matched to detect both the reference signal and the spatially disjoint window function of the reference signal. The importance of the window information increases when the mean of the scene noise background is strong.

### 5. An Optimal Filter Function for Spatially Disjoint Signal and Input Scene Noise Using a Peak to Noise Ratio Metric

In this section, an optimum filter function is designed for an input signal  $r(t)$  which is spatially disjoint from the input scene noise  $\tilde{n}(t)$ . The filter is designed to detect a target  $r(t)$  located in an unknown random position  $\tau$ .

We assume that  $w_r(t)$  is the window function that defines the input target. The window is unity where the function is defined and zero otherwise, that is,  $r(t)w_r(t)=r(t)$ .  $w_o(t)$  is the window function which defines the limited size input scene, and  $w(t,\tau)$  is a window function that defines the scene noise, that is,

$$w(t,\tau)=w_o(t)-w_r(t-\tau). \quad (27)$$

The Fourier transforms of the target  $r(t)$  and its window  $w_r(t)$  are denoted by  $R(\omega)$  and  $W_r(\omega)$ , respectively. The Fourier transform of the input scene window  $w_o(t)$  is denoted by  $W_o(\omega)$ . The Fourier transform of the input scene noise window  $w(t,\tau)=w_o(t)-w_r(t-\tau)$  is:

$$W(\omega,\tau)=W_o(\omega)-W_r(\omega)\exp(-j\omega\tau). \quad (28)$$

In deriving the filter function, the following assumptions are made:

1) The scene noise  $\tilde{n}(t)$  is defined as:

$$\tilde{n}(t) = n(t)w(t, \tau) = n(t)[w_o(t) - w_r(t - \tau)], \quad (29)$$

where  $n(t)$  is a wide sense stationary random noise with expected value  $m_n$ . The fact that the scene noise is spatially disjoint with the target is indicated by the product of the wide-sense stationary noise  $n(t)$  and the window function  $w(t, \tau)$  in Eq. (29). The scene noise  $\tilde{n}(t)$  is not stationary due to the limited size of the input scene.

2) It is assumed that the target's random location " $\tau$ " has a uniform probability density function  $f(\tau)$ :

$$f(\tau) = \begin{cases} 1/d & \tau \in \text{scene area} \\ 0 & \text{elsewhere} \end{cases}, \quad (30)$$

where  $d = W_o(0)$ .

3) The unknown illumination coefficient  $\tilde{a}$  is a random variable with mean  $\bar{a}$  and variance  $\sigma_a^2$ .

4) It is assumed that the wide-sense stationary noise  $n(t)$ , the illumination coefficient  $\tilde{a}$ , and the target location  $\tau$  are statistically independent of one another.

The filter impulse response and transfer function are denoted by  $h(t)$  and  $H(\omega)$ , respectively.

The filter output  $y(t, \tau)$  when the target is located at position  $t = \tau$  in the input plane is:

$$y(t, \tau) = \int h(t - t') [\tilde{a}r(t' - \tau) + n(t')w(t', \tau)] dt' \quad (31)$$

The optimal filter function  $H_{\text{opt}}$  is designed to maximize the ratio of the expected value squared of the correlation peak to the average expected value of the output signal energy:

$$\text{PNR} = \frac{\{E[y(\tau, \tau)]\}^2}{E\{[y(t, \tau)]^2\}} \quad (32)$$

Here, PNR is the peak-to-noise ratio metric, the symbol in the denominator " $\{\cdot\}$ " denotes spatial averaging (integration) over  $t$ , and  $E\{\cdot\}$  denotes the statistical expected value.

Given the target location  $\tau$ , the conditional expected value of the output signal is:

$$E[y(t, \tau)|\tau] = \int h(t-t') [\bar{a}r(t'-\tau) + m_n w(t', \tau)] dt' = \frac{1}{2\pi} \int H(\omega) [\bar{a}R(\omega) e^{-j\omega\tau} + m_n W(\omega, \tau)] e^{j\omega t} d\omega, \quad (33)$$

where  $E[\cdot]$  denotes expected value. The expected value of the correlation peak at  $t=\tau$  is:

$$E[y(\tau, \tau)] = E\{E[y(\tau, \tau)|\tau]\} = \frac{1}{2\pi} \int H(\omega) [\bar{a}R(\omega) + m_n \tilde{W}_1(\omega)] d\omega, \quad (34a)$$

where

$$\tilde{W}_1(\omega) = E\{W(\omega, \tau) e^{j\omega\tau}\} = |W_o(\omega)|^2 / d - W_i(\omega), \text{ and } d = W_o(0); \quad (34b)$$

The conditional variance of the output signal is:<sup>[14]</sup>

$$\text{Var}[y(t, \tau)|\tau] = \iint h(t-t') h(t-t'') [\sigma_a^2 r(t'-\tau) r(t''-\tau) + w(t', \tau) w(t'', \tau) C_n(t'-t'')] dt' dt''$$

$$\begin{aligned}
&= \left(\frac{1}{2\pi}\right)^3 \int_{\omega_3} \int_{\omega_2} \int_{\omega_1} H(\omega_1)H(\omega_2) \left\{ 2\pi\sigma_a^2 R(\omega_3)R(\omega_1+\omega_2-\omega_3)\delta(\omega_1-\omega_3)e^{-j(\omega_1+\omega_2)\tau} \right. \\
&\quad \left. + W(\omega_3,\tau)W(\omega_1+\omega_2-\omega_3,\tau)[S_n(\omega_1-\omega_3)-2\pi m_n^2\delta(\omega_1-\omega_3)] \right\} e^{j(\omega_1+\omega_2)t} d\omega_1 d\omega_2 d\omega_3, \quad (35)
\end{aligned}$$

where  $\text{Var}[\cdot]$  denotes variance, and  $C_n(t)$  and  $S_n(\omega)$  are the covariance and the power spectrum of the scene noise  $n(t)$ , respectively.

Let us denote the spatial average of  $x(t)$  as  $\bar{x} = \int x(t)dt$ . We compute  $\overline{\{[y(t,\tau)]^2|\tau\}} = \overline{\text{Var}[y(t,\tau)|\tau]} + \{\overline{E[y(t,\tau)|\tau]}\}^2$ . Provided that the filter function is real and the spatial extent of the output is  $L$ , we have:

$$\overline{\text{Var}[y(t,\tau)|\tau]} = \frac{1}{L} \int \text{Var}[y(t,\tau)|\tau] dt = \frac{1}{(2\pi)^2 L} \int [2\pi\sigma_a^2 |R(\omega)|^2 + |W(\omega,\tau)|^{2*} [S_n(\omega) - 2\pi m_n^2 \delta(\omega)]] |H(\omega)|^2 d\omega \quad (36)$$

$$\overline{\{E[y(t,\tau)|\tau]\}^2} = \frac{1}{L} \int \{E[y(t,\tau)|\tau]\}^2 dt = \frac{1}{2\pi L} \int |\bar{a}R(\omega)e^{-j\omega\tau} + m_n W(\omega,\tau)|^2 |H(\omega)|^2 d\omega, \quad (37)$$

where  $*$  denotes convolution. If the spatial extent of the output is equal to that of the input, then  $d=L$ .

Using Eqs. (36)-(37), we have:

$$\begin{aligned}
\overline{\{[y(t,\tau)]^2|\tau\}} &= \overline{\text{Var}[y(t,\tau)|\tau]} + \{\overline{E[y(t,\tau)|\tau]}\}^2 \\
&= \frac{1}{2\pi L} \int \left[ \sigma_a^2 |R(\omega)|^2 + \frac{1}{2\pi} |W(\omega,\tau)|^{2*} [S_n(\omega) - 2\pi m_n^2 \delta(\omega)] + |\bar{a}R(\omega)e^{-j\omega\tau} + m_n W(\omega,\tau)|^2 \right] |H(\omega)|^2 d\omega \quad (38)
\end{aligned}$$

Given that  $\tau$  is uniformly distributed, the expected value of  $\overline{E\{[y(t,\tau)]^2|\tau\}}$  is:

$$\overline{E\{[y(t,\tau)]^2\}} = \frac{1}{2\pi L} \int \left\{ \sigma_a^2 |R(\omega)|^2 + \frac{1}{2\pi} \widetilde{W}_2(\omega) * S_n(\omega) + |\bar{a}R(\omega) + m_n \widetilde{W}_1(\omega)|^2 - m_n^2 |\widetilde{W}_1(\omega)|^2 \right\} |H(\omega)|^2 d\omega \quad (39)$$

where  $\widetilde{W}_1(\omega)$  is defined in Eq. (34b), and:

$$\widetilde{W}_2(\omega) = E\{|W(\omega, \tau)|^2\} = |W_o(\omega)|^2 + |W_r(\omega)|^2 - 2|W_o(\omega)|^2 \text{Real}[W_r(\omega)]/d \quad (40)$$

Therefore, the output PNR of the filter is:

$$\text{PNR} = \frac{\left| \int H(\omega) [\bar{a}R(\omega) + m_n \widetilde{W}_1(\omega)] d\omega \right|^2}{\frac{2\pi}{L} \int \left\{ \sigma_a^2 |R(\omega)|^2 + \frac{1}{2\pi} \widetilde{W}_2(\omega) * S_n(\omega) + |\bar{a}R(\omega) + m_n \widetilde{W}_1(\omega)|^2 - m_n^2 |\widetilde{W}_1(\omega)|^2 \right\} |H(\omega)|^2 d\omega} \quad (41)$$

Applying the Schwarz inequality, we have:

$$\text{PNR} \leq \frac{1}{\frac{2\pi}{L}} \int \left| \frac{\bar{a}R(\omega) + m_n \widetilde{W}_1(\omega)}{\sqrt{\left\{ \sigma_a^2 |R(\omega)|^2 + \frac{1}{2\pi} \widetilde{W}_2(\omega) * S_n(\omega) + |\bar{a}R(\omega) + m_n \widetilde{W}_1(\omega)|^2 - m_n^2 |\widetilde{W}_1(\omega)|^2 \right\}}} \right|^2 d\omega \quad (42)$$

The equality holds when

$$H_{\text{opt}}^*(\omega) = \frac{\bar{a}R(\omega) + m_n \widetilde{W}_1(\omega)}{\sigma_a^2 |R(\omega)|^2 + \frac{1}{2\pi} \widetilde{W}_2(\omega) * S_n(\omega) + |\bar{a}R(\omega) + m_n \widetilde{W}_1(\omega)|^2 - m_n^2 |\widetilde{W}_1(\omega)|^2}, \quad (43)$$

where  $H_{\text{opt}}^*(\omega)$  is the complex conjugate of the filter transfer function.

Equation (43) describes the optimal filter function which detects a signal in the presence of spatially disjoint scene noise. The filter optimizes the correlation performance by maximizing the ratio of the correlation peak intensity to the average output energy. The filter function can be determined in terms of the reference signal, the spatial size of the input scene, the first and second order statistics of the input scene noise, and the input window functions. The filter takes into account the non-whiteness of the input scene noise, the non-stationarity of the input signal due to the limited spatial size of the scene, the spatial disjointness of the target and the scene noise, and the possible variation of the target illumination.

For an infinite size input scene,  $w(t,\tau)=1-w_r(t-\tau)$ , and we can write:

$$\tilde{W}_1(\omega)=\delta(\omega)-W_r(\omega)=W_1(\omega) \quad (44)$$

$$\tilde{W}_2(\omega)=|W_1(\omega)|^2 \quad (45)$$

In this case the optimal filter is:

$$H^*(\omega)=H_{opt}^*(\omega)=\frac{\bar{a}R(\omega)+m_n W_1(\omega)}{\sigma_n^2 |R(\omega)|^2 + \frac{1}{2\pi} |W_1(\omega)|^2 * [S_n(\omega) - 2\pi m_n^2 \delta(\omega)] + |\bar{a}R(\omega) + m_n W_1(\omega)|^2} \quad (46)$$

In this case, if the scene noise is a constant [ $n(t)=m_n$ ], and the illumination is known [ $\tilde{a} = \bar{a}$ ], then using Eq. (46), the optimum filter is:

$$H(\omega)=H_{opt}(\omega)=\frac{1}{\bar{a}R(\omega)+m_n W_1(\omega)} \quad (47)$$

Equation (47) is an inverse filter and its output is a delta function.

It can be seen from Eqs. (43)-(46) that for the case of disjoint signal and scene, the optimum filter

is very different from the conventional matched filter expression. The optimum filter is matched to the expected value of the input scene and the window of the input scene noise plays an important role in the filter function. For large scene noise, the scene noise window function can be as important as the reference function.

If we ignore the amplitude variation on the target, Eq.(46) can be simplified to the form which is exactly the same as the MMSE filter in Eq.(25). Both filters attempt to maximize the output signal at the target location while minimizing the output signal at other locations. The MMSE filter is designed to reduce the difference between ideal delta function at the target location and the filter output. Thus, a very well defined output is produced at the target location. The optimum filter presented in Eq. (43) achieves this by maximizing the output at the target location while minimizing the output signal energy at other locations. Therefore, both approaches result in similar filter solutions. The optimum filter is derived in continuous form and takes into account the effect of the limited size of the input scene and the amplitude variation of the target. The MMSE filter is derived in discrete form.

Computer simulation of the optimum filter of Eq.(43) is provided for the input scene shown in Fig. 5(a). The tank is used as the reference signal. The target is placed in a scene noise which is non-stationary due to the limited spatial size of the input[see Fig. 5(a)]. The input scene noise  $n(t)$  is white Gaussian noise with mean of 0.5, and standard deviation of 0.3. The amplitude coefficient  $\tilde{a}$  is assumed to be one. The outputs of the optimum filter [please see Eq.(43)], the conventional matched filter, and the phase-only filter for the same input scene are plotted in Figures (5b), (5c), and (5d), respectively.

In this section, an optimum filter function is described to detect a target in the presence of spatially disjoint input scene noise. The filter function is designed to optimize the ratio of the expected value squared of the output correlation peak to the average expected value of the output signal energy. This metric maximizes the correlation peak and minimizes the output energy. The filter function takes into account the scene noise color, the non-stationarity of the input scene due to the limited spatial

size of the input, the spatially disjoint target and scene noise, and the unknown variation of the target illumination. The computer simulation results demonstrate that this filter function yields good performance with well defined output correlation peak and low output sidelobe.

## 6. Generalization of the Matched Filter Function for Spatially Disjoint Target and Input Scene Noise

Optimum filters such as matched filters are designed by maximizing a metric such as the correlation SNR. It was shown in section 2 that the classic definition of the correlation SNR may not be a good criterion for designing a filter or evaluating the performance of a filter when the scene noise is spatially disjoint with the target in the scene. In this case, the scene noise is non-stationary since it does not overlap the target. The other reason for the non-stationarity of the input scene noise could be the limited size of the input scene. Thus, a modified version of the correlation SNR is necessary to design the filter function.

For matched filters the classic definition of the correlation SNR is the ratio of the expected value squared of the correlation peak to the variance of the peak.<sup>[1]</sup> If the input noise is stationary (input noise overlaps the target), the correlation peak variance is a good statistical measure of the variance at any other sample point of the output signal. However, statistical measurements of a non-stationary random process, such as mean and variance, depend on the sample position. Therefore, when the target and scene noise are spatially disjoint, the variance of the correlation peak is not a good measure of the output noise fluctuations. Thus, to obtain a measure of the output noise fluctuations, we could consider the spatial average of the output noise variance over all the output pixel positions. The classic definition of the correlation SNR is thus modified to obtain a new metric  $\widetilde{\text{SNR}}$ , defined as the ratio of the expected value squared of the correlation peak to the spatial average of the output variance:

$$\widetilde{\text{SNR}} = \frac{|E[y(\tau, \tau)]|^2}{\text{Var}[y(t, \tau)]} \quad (48)$$

The input signal model and the notations used in this section are the same as those established in section 5. Here,  $y(t, \tau)$  is the filter output when the target is located at position  $\tau$  in the input plane, the symbol " $\overline{\{\cdot\}}$ " denotes spatial averaging (integration) over  $t$ , and  $E\{\cdot\}$  denotes the statistical expected value. We design a generalized matched filter function  $H_g(\omega)$  which maximizes the  $\overline{SNR}$  in Eq. (48). It will be shown that the conventional matched filter designed for overlapping signal and noise is a special case of  $H_g(\omega)$  when the input scene noise is a wide sense stationary noise with zero mean and is overlapping with the target.

Using Eq. (36), the spatial average of the output variance is:

$$\overline{\text{Var}[y(t, \tau)]} = E\{\overline{\text{Var}[y(t, \tau)|\tau]}\} = \frac{1}{(2\pi)^2 L} \int [2\pi\sigma_a^2 |R(\omega)|^2 + \widetilde{W}_2(\omega) * [S_n(\omega) - 2\pi m_n^2 \delta(\omega)]] |H(\omega)|^2 d\omega \quad (49)$$

where  $\widetilde{W}_2(\omega)$  is defined in Eq.(40). Therefore, using Eqs. (34a), (48) and (49), we have:

$$\overline{SNR} = \frac{\left| \int H(\omega) [\overline{aR}(\omega) + m_n \widetilde{W}_1(\omega)] d\omega \right|^2}{\frac{2\pi}{L} \int \left\{ \sigma_a^2 |R(\omega)|^2 + \frac{1}{2\pi} \widetilde{W}_2(\omega) * [S_n(\omega) - 2\pi m_n^2 \delta(\omega)] \right\} |H(\omega)|^2 d\omega} \quad (50)$$

$$\leq \frac{1}{\frac{2\pi}{L}} \int \left| \frac{\overline{aR}(\omega) + m_n \widetilde{W}_1(\omega)}{\sqrt{\left\{ \sigma_a^2 |R(\omega)|^2 + \frac{1}{2\pi} \widetilde{W}_2(\omega) * [S_n(\omega) - 2\pi m_n^2 \delta(\omega)] \right\}}} \right|^2 d\omega \quad (51)$$

where  $\widetilde{W}_1(\omega)$  is defined in Eq.(34b). The equality yields the new generalized matched filter:

$$H^*(\omega) = H_g^*(\omega) = \frac{\overline{aR}(\omega) + m_n \widetilde{W}_1(\omega)}{\sigma_a^2 |R(\omega)|^2 + \frac{1}{2\pi} \widetilde{W}_2(\omega) * [S_n(\omega) - 2\pi m_n^2 \delta(\omega)]} \quad (52)$$

This is the generalization of the matched filter when the input noise does not overlap the target. For an input scene with infinite window size, the generalized matched filter is [please see Eqs.(44) and (45)]:

$$H_g^*(\omega) = \frac{\bar{a}R(\omega) + m_n W_1(\omega)}{\sigma_a^2 |R(\omega)|^2 + \frac{1}{2\pi} |W_1(\omega)|^2 * [S_n(\omega) - 2\pi m_n^2 \delta(\omega)]} \quad (53)$$

If the input scene noise is zero mean, and the illumination is known ( $\tilde{a} = \bar{a} = 1$ ), we obtain:

$$H_g^*(\omega) = \frac{R(\omega)}{\frac{1}{2\pi} |W_1(\omega)|^2 * S_n(\omega)}, \quad (54)$$

where  $W_1(\cdot)$  is defined in Eq. (44). Furthermore, if the input noise is overlapping with the target, then  $W_1(\omega) = \delta(\omega)$  and the generalized matched filter function is simplified to:

$$H_g^*(\omega) = \frac{R(\omega)}{\frac{1}{2\pi} S_n(\omega)} \quad (55)$$

Equation (55) is the same as the conventional matched filter function obtained by maximization of the classic definition of SNR under the condition that the target to be detected is in the presence of zero mean overlapping stationary noise. The conventional matched filter function in Eq. (55) is a special case of the generalized matched filter function in Eq. (52). When the stationary input noise is overlapping the target, the variance of the correlation peak is a good statistical measure of the variance at any other point of the output signal. Thus, the spatial average of the output variance is the same as the variance of the correlation peak. For the overlapping target and input noise, the metric  $\widetilde{SNR}$  is equivalent to the classic definition of SNR, and the conventional matched filter solution is obtained

by maximizing these metrics.

## 7. Summary

In many pattern recognition problems, the input scene noise is spatially disjoint with the target in the scene. We have shown that for this class of problems, the matched filter expressions and the optimum receivers derived under the overlapping input signal and scene noise assumption may not perform well in the presence of spatially disjoint input signal and scene noise. The matched filter function is derived by maximizing the conventional correlation SNR metric. The limitations of the conventional SNR metric for spatially disjoint target and scene noise are discussed.

In this report, we have presented four possible solutions to detect targets in spatially disjoint scene noise. The first solution is an optimum receiver designed based on multiple alternative hypothesis testing to detect a signal in spatially disjoint scene noise. Both noisy targets (detector noise present) and noise free targets which are spatially disjoint with the scene noise and have an unknown illumination scale are discussed. It is shown that for a noise free target, the actual scene noise statistics becomes irrelevant to the detection process. In this case, the optimum receiver is similar to a correlator normalized by the input scene energy within the target window.

The second approach is based on Wiener filtering. A minimum mean square error filter is presented for pattern recognition problems with input scene noise that is spatially disjoint with the target. The filter is designed to have an output which is a delta function located at the position of the target. It is shown that the filter performs very well. The window of the reference signal strongly influences the filter function and the filter is matched to detect both the reference signal and the spatially disjoint window function of the reference signal.

The third technique is based on maximizing a performance metric to design an optimum filter to detect signals in spatially disjoint scene noise. An optimum filter function is designed to maximize the ratio of the correlation peak to the average output energy for a signal which is spatially disjoint with

the input scene noise. It is shown that the filter produces a sharp output signal at the target location with a low output noise. Other types of optimal filters can be designed by maximizing variations of similar performance metrics which are defined for the spatially disjoint target and scene noise problem.

The fourth approach is based on modifying the conventional definition of the correlation peak SNR to take into account the non-overlapping nature of the target and the scene noise. For spatially disjoint target and scene noise, the classic definition of the correlation SNR is modified to obtain a new metric  $\widetilde{\text{SNR}}$  defined as the ratio of the expected value squared of the correlation peak to the spatial average of the output variance. By maximizing the  $\widetilde{\text{SNR}}$ , we have designed a generalized matched filter function that can detect a target in the presence of spatially disjoint scene noise. It is shown that the widely used conventional matched filter functions designed for additive signal and noise are a special case of this generalized matched filter when the input noise is overlapping with the target and has zero mean.

We wish to thank Prof. Bahaa E. A. Saleh and V. Laude for many rewarding discussions, and A. Fazlolahi, G. Zhang, and F. Parchekani for performing the computer simulation tests.

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## Figure Captions

Figure 1 Input images

(a) The target (tank) in the spatially disjoint (non-overlapping) additive input scene noise used in the correlation tests.

(b) The target (tank) in the overlapping additive input scene noise used in the correlation tests. The input noise statistics are the same as in Fig. 1(a).

Figure 2 Output correlation intensity.

(a) The correlation output obtained by a conventional matched filter for the reference signal in the presence of the spatially disjoint noise as shown in Fig. 1(a).

(b) The correlation output obtained by a conventional matched filter for the reference signal in the presence of the additive overlapping noise as shown in Fig. 1(b).

Figure 3 Computer simulation results of an optimum receiver designed for spatially disjoint target and scene noise.

(a) Target (tank) in spatially disjoint (non-overlapping) background noise used in the tests. The background noise is Gaussian with mean of 0.5 and standard deviation of 0.3. The target has an illumination of 0.5 which is unknown to the receiver.

(b) Example of the test function  $[-C_N(t_j)]$  for the reference signal in the presence of spatially disjoint background noise shown in Fig. 1(a).  $C_N(t_j)$  is given by Eq. (15).

(c) Classic correlation receiver solution  $\sum_{i=1}^m r(t_i)s(t_i-t_j)$  for the reference signal in the presence of spatially disjoint background noise shown in Fig. 1(a). The figure illustrates the pixels around the correlation peak. The correlation peak cannot be seen because it is below the output noise floor.

Figure 4 Computer simulation results of a minimum mean square error filter designed for spatially disjoint target and scene noise.

(a) The input scene with a target (tank) and spatially disjoint scene noise.

(b) minimum mean square error filter output in response to the data in (a).

Figure 5 Computer simulation results of an optimum filter [please see Eq. (43)] designed for spatially disjoint target and scene noise.

(a) Input image used in the computer simulations. The target is the tank. The input scene noise is white Gaussian with mean 0.5, standard deviation of 0.3.

(b) Output of the optimal matched filter

(c) Correlation output of the conventional matched filter

(d) Output of the phase only filter

## 9. Appendix A

### The Effect of Input Scene Noise Mean on the Correlation Performance of the Conventional Matched filter

We examine the effect of input scene noise mean on the correlation performance of the conventional matched filters for both the overlapping and non-overlapping (spatially disjoint) input scene. One dimensional notation is used for clarity and simplicity.

It is assumed that the input signal contains a target  $r(t)$  and scene noise  $\tilde{n}(t)$ . For overlapping input scene and signal, we have  $\tilde{n}(t)=n(t)$ , where  $n(t)$  is wide sense stationary. For non-overlapping input scene and signal, we have  $\tilde{n}(t)=[1-w_r(t)]n(t)$ . Here,  $w_r(t)$  is a uniform window function, that is,  $w_r(t)$  is unity within the area where the target is defined, and is zero elsewhere. To maintain stationarity, it is assumed that the input scene is sufficiently large along the coordinate axis.

The filter impulse response is denoted by  $h(t)$ . The filter output is:

$$y(t)=\int h(t-t')r(t')dt'+\int h(t-t')\tilde{n}(t')dt' \quad (a-1)$$

The noisy output correlation peak is

$$y(0)=\int h(-t')r(t')dt'+\int h(-t')\tilde{n}(t')dt' \quad (a-2)$$

The difference between the correlation peak and the output signal at any position "t" is:

$$\Delta s=\int [h(-t')-h(t-t')]r(t')dt'+\int [h(-t')-h(t-t')]\tilde{n}(t')dt' \quad (a-3)$$

We consider the difference  $\Delta s$  to investigate the filter performance for detecting the target  $r(t)$  in the presence of the scene noise  $\tilde{n}(t)$ . Obviously, a large  $\Delta s$  indicates that the peak is much higher than the output sidelobe.

Let us assume that the input noise  $n(t)$  is white with mean  $m_n$ . We examine the effect of the noise mean on the correlation performance of the conventional matched filter, that is  $h(t)=r(-t)$ , for both the overlapping and the non-overlapping input signal and scene noise cases. The expected value of the output correlation peak using Eq. (a-2) is:

$$E[y^{ov}(0)] = R_{rr}^c(0) + m_n \int r(t') dt' \quad (a-4)$$

$$E[y^{nov}(0)] = R_{rr}^c(0) \quad (a-5)$$

The superscripts "ov" and "nov" denote the output measurements corresponding to the overlapping and non-overlapping input cases, respectively. The expected value of the difference  $\Delta s$  between the output correlation peak and the output correlation is:

$$E[\Delta s^{ov}] = R_{rr}^c(0) - R_{rr}^c(t) + m_n \int [r(t') - r(-t+t')] dt' = R_{rr}^c(0) - R_{rr}^c(t) \quad (a-6)$$

$$E[\Delta s^{nov}] = R_{rr}^c(0) - R_{rr}^c(t) - m_n \int r(-t+t') [1 - w_r(t)] dt' \quad (a-7)$$

The second equality in Eq. (a-6) holds under the assumption that the bounds of the integral are sufficiently large.

For large values of  $t$  in Eqs. (a-6)-(a-7),  $r(-t+t')$  is shifted out of the window  $w_r(t')$ , and we have:

$$E[\Delta s^{ov}] = R_{rr}^c(0) \quad (a-8)$$

$$E[\Delta s^{nov}] = R_{rr}^c(0) - m_n \int r(t') dt' \leq E[\Delta s^{ov}] \quad (a-9)$$

Here, it is assumed that  $R_{rr}(t)$  approaches zero for large  $t$ .

It can be seen from Eqs. (a-4), (a-6), and (a-8) that for the overlapping signal and scene noise case, the output correlation peak is affected by the noise mean, and the difference between the output correlation peak and the noisy output sidelobe,  $\Delta s$ , is independent of the noise mean.

In contrast, it can be seen from Eq. (a-5) that for the non-overlapping (spatially disjoint) signal and scene noise case, the correlation peak of the conventional matched filter is independent of the noise mean. However, it can be seen from Eqs. (a-7) and (a-9) that the difference between the correlation peak and the noisy output sidelobe,  $\Delta s$ , is affected by the noise mean. Inequality (a-9) shows that the correlation performance of the conventional matched filter for the spatially disjoint input signal and scene noise is inferior to that of the overlapping input case. In addition, the filter performance for the spatially disjoint input signal and scene noise deteriorates as the input scene noise mean value increases.

The correlation performance of the conventional matched filter for the spatially disjoint input signal and scene noise case is improved when the mean of the scene noise is small. For instance, when the scene noise mean is zero,  $E[\Delta s^{nov}]$  is maximized. However, real images have a non-zero mean. In a filter based correlation system, using a DC block in the Fourier plane can remove the mean of the input signal. Under this condition, each output signal pixel value is shifted down by the same amount. This amount is determined by the mean of the input signal and the mean of the reference signal. However, the expected value of the difference between the correlation peak and the output,  $\Delta s$ , is not affected. Furthermore, due to the spatial size of DC block, this method will remove the low frequency information of the input scene. If the target has significant energy in the low spatial frequencies, a DC block can result in loss of light efficiency, and loss of information about the target.

## 10. Appendix B

### List of Symbols

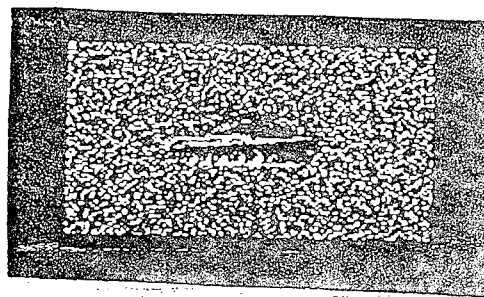
$\tilde{a}$	unknown illumination coefficient of the target in the input scene
$\bar{a}$	mean of the random variable $\tilde{a}$
$\hat{a}$	maximum likelihood estimate of $\tilde{a}$
$C_1(\omega)$	Fourier component which generates the first order correlation output
$C_n(t)$	covariance of the input noise $n(t)$
$E\{\cdot\}$	expected value
$f(t)$	probability density function of the target location in the input scene
$h(t)$	impulse response of the filter
$H(j, \tilde{a})$	hypothesis $H(j, \tilde{a})$ correspond to the target located at position $t_j$ and with amplitude $\tilde{a}$
$H(\omega)$	Fourier transform of $h(t)$ , filter transfer function
$H_g(\omega)$	transfer function of the generalized matched filter
$H_{k1}(\omega)$	first order component of the $k$ th law nonlinearly transformed matched filter function
$H_{opt}(\omega)$	optimal filter transfer function
$k$	degree of the $k$ th law nonlinearity
$m$	number of the samples taken over the input scene $s(t)$
$m_n$	expected value of the input noise $n(t)$
$m_r$	number of the samples taken over the target $r(t)$
MSE	mean square error
$n(t_i)$	sample of the noise $n(t)$
$n(t)$	wide sense stationary noise in the input plane

$n_B(t)$	spatial disjoint background noise in the input plane
$n_d(t)$	detector noise
$n_j(t)$	spatially disjoint input scene noise in section 3 which is the sum of the background noise and the detector noise
$n_o(t)$	input noise minus by its mean value
$\tilde{n}(t)$	disjoint input scene noise in section 5,6 and appendix A
$n_o$	output noise contribution to the output correlation peak
$N(\omega)$	Fourier transform of the noise $n(t)$
$\tilde{N}(\omega)$	Fourier transform of the spatially disjoint input scene noise $\tilde{n}(t)$
$P_{n_j}(\cdot)$	joint probability density function of samples of $n_j(t)$
$P_{n_d}(\cdot)$	joint probability density function of samples of $n_d(t)$
$p(\cdot)$	likelihood function
PNR	the peak to noise ratio defined as the ratio of the expected value squared of the correlation peak to the average expected value of the output signal energy
$r_k(t)$	inverse Fourier transform of $ R(\omega) ^k \exp\{j\phi_R(\omega)\}$
$r(t)$	target function in the input scene
$r(t_i)$	sample of the target function
$R(\omega)$	Fourier transform of the target function $r(t)$
$R(v)$	discrete Fourier transform of the target sample sequence $[r(t_i)]$
$R_{rr}^c(t_i)$	correlation of the sample sequence of the target $[r(t_i)]$
$R_{wr}^c(t_i)$ or $R_{rw}^c(t_i)$	correlation between $[r(t_i)]$ and $[w(t_i)]$
$R_{nn}^c(t_i)$	correlation of the sample sequence of the noise $[n(t_i)]$
$s(t_i)$	sample of the input scene
$s(t)$	input scene function
$s_j$	$m_t$ samples taken over the target in the input scene

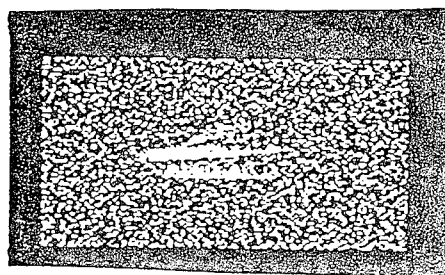
$\overline{s_j}$	(m-m <sub>r</sub> ) samples taken over the input scene noise
$S(\omega)$	Fourier transform of the input scene function s(t)
$S(v)$	discrete Fourier transform of the input scene sample sequence [s(t <sub>i</sub> )]
$S_n(\omega)$	power spectrum of the noise n(t)
SNR	conventional definition of the signal to noise ratio defined as the ratio of the expected value squared of the correlation peak to the variance of the peak
$\widetilde{SNR}$	modified signal to noise ratio for spatially disjoint target and scene noise defined as the ratio of the expected value squared of the correlation peak to the spatial average of the output variance
$w_r(t)$	window function which defines the target in the input scene
$w_o(t)$	window function which defines the input scene
$w(t)$	window function which defines the spatially disjoint noise
$w(t_i)$	sample of the window function which defines the spatially disjoint input noise
$W(\omega)$	Fourier transform of w(t)
$W_r(\omega)$	Fourier transform of w <sub>r</sub> (t)
$W_o(\omega)$	Fourier transform of w <sub>o</sub> (t)
$y(t_i)$	discrete form of ideal filter output in response to an input with spatially disjoint signal and scene noise
$\hat{y}(t_i)$	discrete form of filter output in response to an input with spatially disjoint signal and scene noise
$y(t)$	filter output in response to an input with spatially disjoint target and scene noise
$\Delta s$	difference between the correlation peak and the output signal at any position
$\phi_{\tilde{N}}(\omega)$	Fourier phase of the spatially disjoint input noise $\tilde{n}(t)$
$\phi_R(\omega)$	Fourier phase of the target r(t)
$\phi_S(\omega)$	Fourier phase of the input scene s(t)
$\sigma_n$	standard deviation of the noise n(t)

$\sigma_d$  standard deviation of the detector noise  $n_d(t)$

$\overline{\{ \cdot \}}$  spatial averaging operation



(a)

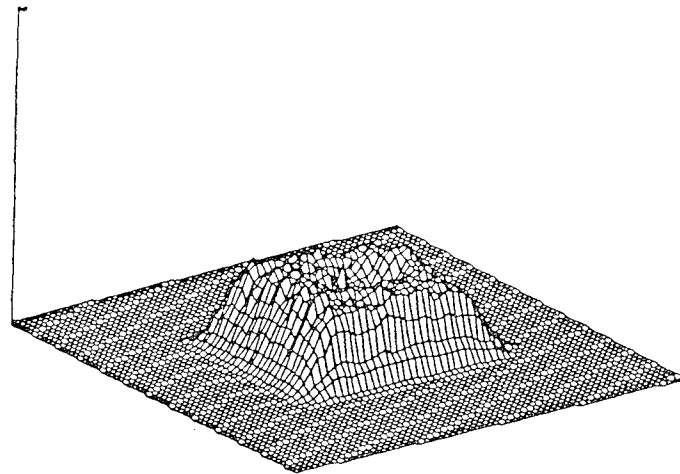


(b)

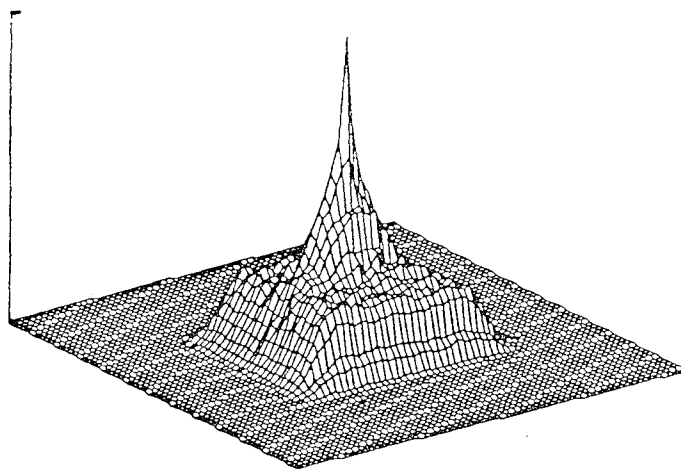
Figure 1 Input images.

(a) The target (tank) in the spatially disjoint (nonoverlapping) additive input scene noise used in the correlation tests.

(b) The target (tank) in the overlapping additive input scene noise used in the correlation tests. The input noise statistics are the same as in Fig. 1(a).



(a)

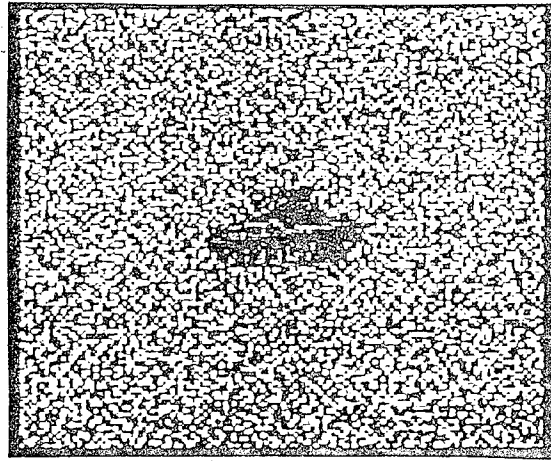


(b)

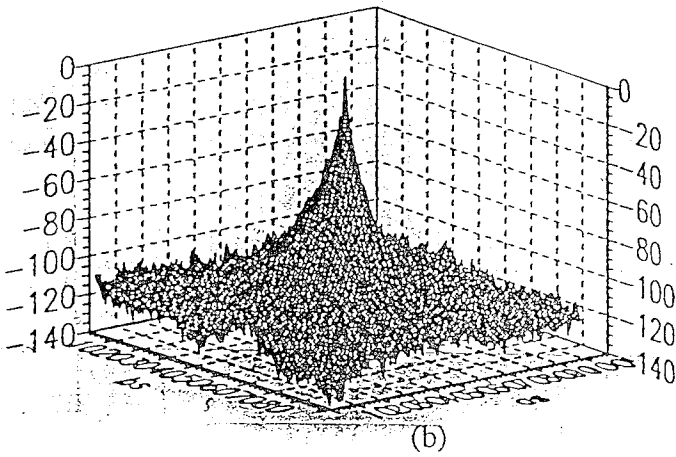
Figure 2 Output correlation intensity

(a) The correlation output obtained by a conventional matched filter for the reference signal in the presence of the additive spatially disjoint noise as shown in Fig. 1(a)

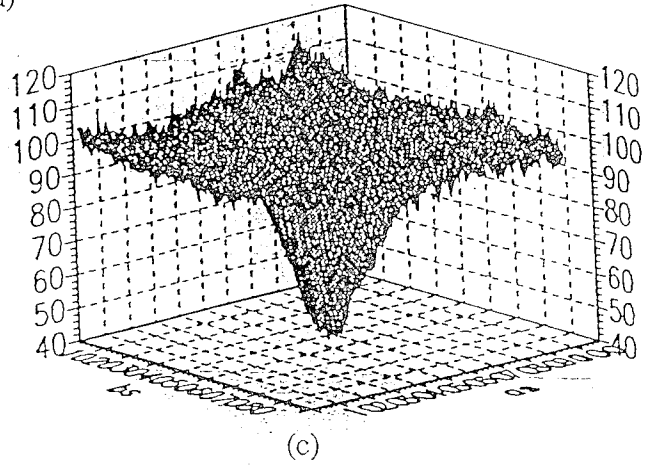
(b) The correlation output obtained by a conventional matched filter for the reference signal in the presence of the additive overlapping noise as shown in Fig. 1(b).



(a)



(b)



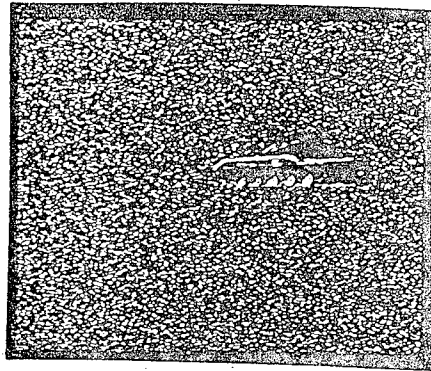
(c)

Figure 3 Computer simulation results of an optimum receiver designed for spatially disjoint target and scene noise.

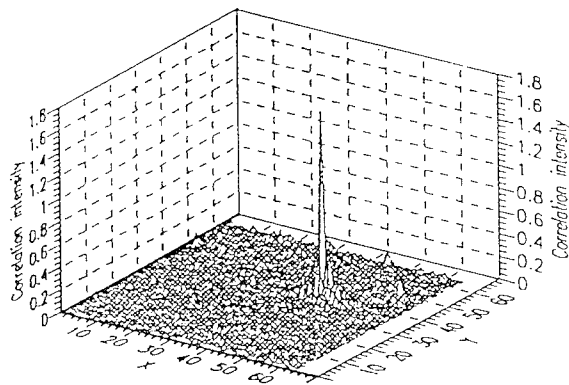
(a) Target (tank) in spatially disjoint (non-overlapping) background noise used in the tests. The background noise is Gaussian with mean of 0.5 and standard deviation of 0.3. The target has an illumination of 0.5 which is unknown to the receiver.

(b) Example of the test function  $[-C_N(t_j)]$  for the reference signal in the presence of spatially disjoint background noise shown in Fig. 1(a).  $C_N(t_j)$  is given by Eq. (15).

(c) Classic correlation receiver solution  $\sum_{i=1}^m r(t_i)s(t_i-t_j)$  for the reference signal in the presence of spatially disjoint background noise shown in Fig. 1(a). The figure illustrates the pixels around the correlation peak. The correlation peak cannot be seen because it is below the output noise floor.



(a)

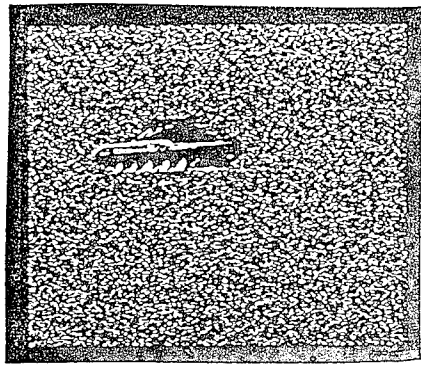


(b)

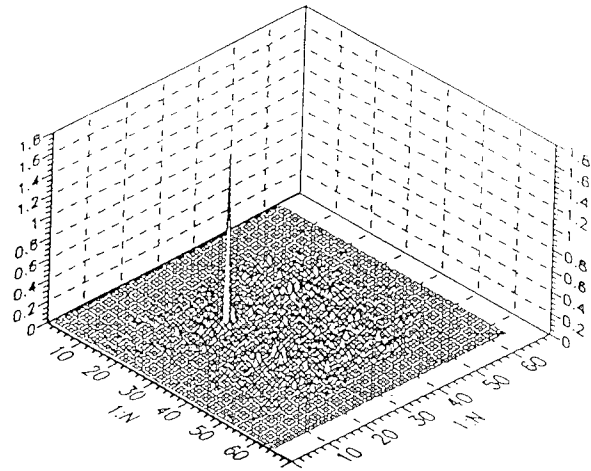
Figure 4 Computer simulation results of a minimum mean square error filter designed for spatially disjoint target and scene noise.

(a) The input scene with a target (tank) and spatially disjoint scene noise.

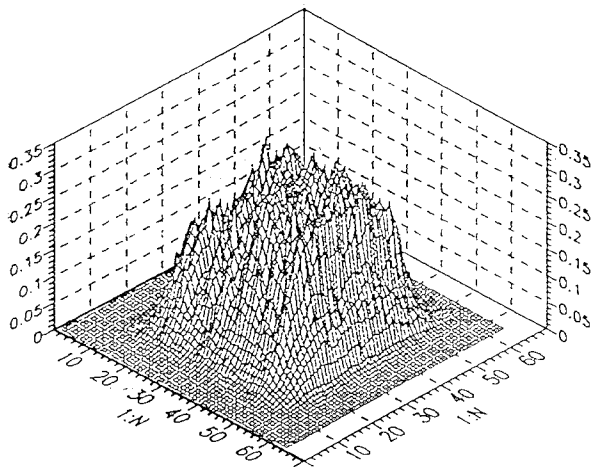
(b) minimum mean square error filter output in response to the data in (a).



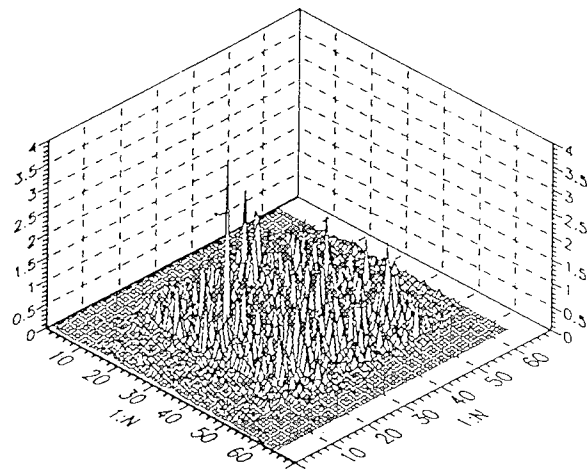
(a)



(b)



(c)



(d)

Figure 5 Computer simulation results of an optimum filter [please see Eq. (43)] designed for spatially disjoint target and scene noise.

(a) Input image used in the computer simulations. The target is the tank. The input scene noise is white Gaussian with mean 0.5, standard deviation of 0.3.

(b) Output of the optimal matched filter

(c) Correlation output of the conventional matched filter

(d) Output of the phase only filter

*MISSION  
OF  
ROME LABORATORY*

Mission. The mission of Rome Laboratory is to advance the science and technologies of command, control, communications and intelligence and to transition them into systems to meet customer needs. To achieve this, Rome Lab:

- a. Conducts vigorous research, development and test programs in all applicable technologies;
- b. Transitions technology to current and future systems to improve operational capability, readiness, and supportability;
- c. Provides a full range of technical support to Air Force Materiel Command product centers and other Air Force organizations;
- d. Promotes transfer of technology to the private sector;
- e. Maintains leading edge technological expertise in the areas of surveillance, communications, command and control, intelligence, reliability science, electro-magnetic technology, photonics, signal processing, and computational science.

The thrust areas of technical competence include: Surveillance, Communications, Command and Control, Intelligence, Signal Processing, Computer Science and Technology, Electromagnetic Technology, Photonics and Reliability Sciences.