FULL WAVE THEORY FOR ROUGH SURFACE SCATTERING
USING THE KIRCHHOFF SOLUTION FOR PRIMARY FIELD

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13. ABSTRACT (Maximum 200 words)

    This final report summarizes work done under Grant DAAH04-94-G-0035. The objective of the investigation was to develop a more rigorous full wave approach to rough surface scattering than that used by Bahar in his full wave method. Instead of relying on the use of telegraphists equations a local spectral expansion method was found that is applicable to general rough surface scattering, not just surfaces with roughness in one dimension only. The local spectral expansion method can be formulated so as to involve the tangential fields at the surface as the only unknowns. This is in contrast with Bahar's full wave method which involves coupling integrals between the modes used in expanding the field and is more difficult to use in an iterative procedure. The new method turns out, in the lowest approximation involving the Kirchhoff approximation for the surface tangential fields, to yield significantly more accurate results for most scattering angles. The new approach also covers a wider range of surface parameters that extend from the small perturbation regime to the standard Kirchhoff regime.

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Full Wave Theory for Rough surface Scattering Using the Kirchhoff Solution for Primary Field.

Forward

More than two decades ago Bahar formulated a full wave theory for rough surface scattering(1,2). This theory was developed along the lines used for tapered waveguides and based on the use of generalized telegraphists equations. As a first step Bahar introduced a spectral expansion of the fields in the transverse plane and in the direction perpendicular to the interface separating two different media. This formulation is essentially limited to treating rough surfaces that are rough in one direction only, although Bahar did extend the theory in a more or less ad hoc way to treat rough surfaces with two dimensional roughness governed by isotropic statistics (3). By expanding the fields in the vertical direction in terms of the spectral expansion and substituting into Maxwell's equations one obtains certain coupling integrals that corresponded to mode coupling in the analogous waveguide problem, along with some boundary terms. The resultant equations are too difficult to solve exactly so Bahar resorted to an iterative method. In order to obtain a first iterative solution Bahar approximates the fields in the mode coupling integrals by an approximate solution corresponding to a plane wave incident and reflected from the surface using the approximation that locally the field is reflected as though the rough surface was flat with a height equal to the local height. Later on Bahar modified his theory by referring the fields and scattering coefficients to the local normal. A detailed study of this theory has shown that it has serious limitations and is generally unreliable despite claims made by Bahar as to its wide applicability (4,5). Since the first order field that Bahar uses for the first iterative solution is in itself not a solution to Maxwell's equations Bahar's method is not a systematic iterative or perturbation solution. For this reason it is difficult to proceed to higher order solutions since the starting point already has errors that are difficult to quantify.

The above somewhat shaky foundation for the full wave theory led us to reconsider the basic theory from the beginning and to follow a more rigorous mathematical derivation as far as it could be pursued without approximations. The first

step was the recognition that a spectral expansion of the field components in the vertical direction above the rough surface could be carried out using basis functions that locally satisfy the boundary conditions for a surface that is flat but having the local height (6). By using this expansion in Maxwell's equations one can formulate a full wave theory for rough surfaces that have arbitrary roughness in two directions, without any need to introduce the telegraphists equations. We have also found that it is possible to carry the theory forward without approximations to a point where no mode coupling integrals are present, the only unknowns being boundary values for the unknown field. We call this method the local spectral expansion method to distinguish it from Bahar's method which is based on the telegraphists equations. Based on this approach it was of interest to examine what results could be obtained if the same tangent field approximation to the surface fields as used in the Kirchhoff method was also used in the local spectral expansion method. We found that this led to useful results that had a range of application that covered both the small perturbation regime as well as the Kirchhoff regime. A curious aspect of this more rigorous method was that the solutions obtained were intrinsically non-reciprocal, similar with what happens in the phase perturbation method (7). In spite of this lack of reciprocity the predicted scattering cross sections turn out to be more accurate than those found using either the small perturbation solution, the Kirchhoff solution, or Bahar's full wave solution. Although reciprocity is physically required,reciprocity or the lack of it in approximate solutions is not a criteria for the accuracy of the approximate solution.

A. Statement of the Problem Studied.

The problem investigated was the use of the local spectral expansion method along with the Kirchhoff tangent field approximation to the surface fields to evaluate the scattering cross sections of rough surfaces governed by Gaussian statistics, and to compare the results with numerical solutions based on the Monte Carlo method as well as those predicted by other theories. Perfectly conducting surfaces with roughness along one direction as well as along two directions were studied. In addition scattering from rough dielectric surfaces were also investigated, but limited to surfaces with one dimensional roughness. This work was to be used to develop higher order iterative solutions. The

foundations for higher order iterative solutions were developed in a mathematically sound manner but no results beyond using the first order Kirchhoff tangent field approximation was obtained during the course of the present investigation.

B. Summary of the Most Important Results.

Consider a perfectly conducting rough surface with one dimensional roughness described by the height profile h(x). The y axis is perpendicular to the mean surface which coincides with the y=0 plane. For TE and TM waves the following local basis functions are used

$$\psi_o(x,y,k_y) = \frac{1}{\sqrt{2\pi}}[e^{-jk_y(y-h(x))} - e^{jk_y(y-h(x))}]$$

$$\psi_e(x,y,k_y) = \frac{1}{\sqrt{2\pi}}[e^{-jk_y(y-h(x))} + e^{jk_y(y-h(x))}]$$

It is convenient to define the following transform of the scattered electric field for TE waves

$$A^s(x,k_y) \equiv \frac{1}{\sqrt{2\pi}} \int_{h(x)}^{\infty} E^s(x,y)e^{jk_y y}dy$$

The transform of the scattered electric field in terms of the odd local basis function can be expressed in terms of the above transform. The equation satisfied by the A^s transform was found to be

$$\frac{\partial^2 A^s}{\partial x^2} + (k_o^2 - k_y^2)A^s = -\frac{jk_y}{\sqrt{2\pi}}e^{jk_y h}E^s\Big|_h - \frac{1}{\sqrt{2\pi}}\frac{\partial}{\partial x}(h_x E^s\Big|_h e^{jk_y h}) +$$

$$+\frac{2}{\sqrt{2\pi}}\sqrt{1+h_x^2}e^{jk_y h}\frac{\partial E^s}{\partial n}.$$

Note that the only unknown in this equation is the normal derivative of the scattered electric field. This normal derivative is related to the unknown current density on the surface, i.e. the tangential magnetic field at the surface. In the tangent field approximation this unknown normal derivative is approximated by the normal derivative of the incident plane wave electric field. The resultant equation can now be formally solved using a

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Fourier transform with respect to the x coordinate. From this solution the scattered electric field in the far zone can be obtained by using the method of stationary phase. This solution is

$$E^s(x,y) = -\frac{E_0 e^{-jk_s \cdot r + j\frac{\pi}{4}}}{2\sqrt{2\pi k_o r}} F_1(\theta_i, \theta_s) \int_{-L}^{L} e^{-jv_x x'} (e^{jp_y h(x')} - 1) dx' +$$

$$+ \frac{E_0 e^{-jk_s \cdot r + j\frac{\pi}{4}}}{2\sqrt{2\pi k_o r}} F_2(\theta_i, \theta_s) \int_{-L}^{L} e^{-jv_x x'} (e^{jq_y h(x')} - 1) dx' -$$

$$- \frac{E_0 e^{-jk_s \cdot r + j\frac{\pi}{4}}}{2\sqrt{2\pi k_o r}} (p_y - q_y) \int_{-L}^{L} e^{-jv_x x'} dx',$$

where $k_s = \hat{a}_x k_x^s + \hat{a}_y k_y^s = k_o(\hat{a}_x \sin\theta_s + \hat{a}_y \cos\theta_s)$, $v_x = k_x^i - k_x^s = k_o(\sin\theta_i - \sin\theta_s)$,

$p_y = k_y^i + k_y^s = k_o(\cos\theta_i + \cos\theta_s)$, $q_y = k_y^i - k_y^s = k_o(\cos\theta_i - \cos\theta_s)$, $F_1 = (v_x^2 + p_y^2)/p_y = $

$2k_o(1 + \cos(\theta_i + \theta_s))/(\cos(\theta_i) + \cos(\theta_s))$, $F_2 = (v_x^2 + q_y^2)/q_y = $

$2k_o(1 - \cos(\theta_i - \theta_s))/(\cos(\theta_i) - \cos(\theta_s))$

The scattering cross sections can be evaluated using standard procedures when it is assumed that the surface roughness is governed by Gaussian statistics.

Some typical results that have been obtained are shown in Fig.'s 1-3 and are compared with results obtained using the small perturbation method, the Kirchhoff method, Bahar's regular full wave method, and numerical solutions provided to us by Dr. E.I.Thorsos. The results obtained with the local spectral expansion method (LSEM) and shown in Fig.'s 1 and 2 show good agreement with the numerical evaluations and are clearly superior to those obtained using the regular full wave method (RFWM) or the Kirchhoff method (KA). In Fig. 3 the angle of incidence is 80° and for this near grazing

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angle of incidence none of the approximate theories are accurate in the back scatter direction.

Similar results were also obtained for TM polarization but there was less in the way of numerical solutions with which to compare the results. For TM polarization the local spectral expansion method appears to be closer to the Kirchhoff results.

Computations of the scattering cross sections for rough dielectric surfaces were also carried out and compared with the numerical results of Sanchez-Gil and Nieto-Vesperinas (8) for the case of an angle of incidence of 40° and a relative permittivity of 2.04. The agreement with the numerical simulations are better than those obtained using the Kirchhoff approximation or the regular full wave theory, as can be seen from Fig. 4. Again there is a scarcity of numerical results to compare with.

Overall it was found that by using the tangent field approximation with the local spectral expansion method a more accurate prediction of the scattering cross sections could be made than that provided by either the small perturbation method or the Kirchhoff methods alone. It is generally believed by the current investigators that the local spectral expansion method is a useful contribution to the collection of approximate theories for rough surface scattering.

C. List of Publications.

1. A. Garcia-Valenzuela, A New Theory for Electromagnetic Scattering from Rough Surfaces: The Local Spectral Expansion Method, Ph.D. Dissertation, Case Western Reserve University, Nov. 1995.

2. A. Garcia-Valenzuela and R.E.Collin, The Local Spectral Expansion Method for Scattering from Perfectly Conducting Surfaces Rough in One Dimension, Submitted for publication.

3. A. Garcia-Valenzuela, The Local Spectral Expansion Method for Two-Dimensional Perfectly Conducting Rough Surfaces, Submitted for publication.

4. A. Garcia-Valenzuela, An Heuristic Correction to the First Order Local Spectral Expansion Method Approximation for Random Rough Surface Scattering, Submitted for publication.

5. A. Garcia-Valenzuela, The Local Spectral Expansion Method for Scattering from One-Dimensional Dielectric-Dielectric Rough Surfaces, Submitted for publication.

D. Participating Scientific Personnel.

1. R.E.Collin, Principal investigator.

2. Augusto Garcia-Valenzuela, Graduate student. Mr. Garcia-Valenzuela obtained his Ph.D. degree based on work supported by this grant.

3. Chung-Jen Hsu, Mr. Hsu participated in this investigation by assisting with some of the computer programming and numerical evaluation of scattering cross sections. He obtained his Ph.D. degree based on other work.

Inventions.

None

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8. J.A. Sanchez-Gil and M. Nieto-Vesperinas, Light scattering from random rough dielectric surfaces, *J. Opt. Soc. Am.*, vol. 8(8), pp. 1270-1286, 1991.
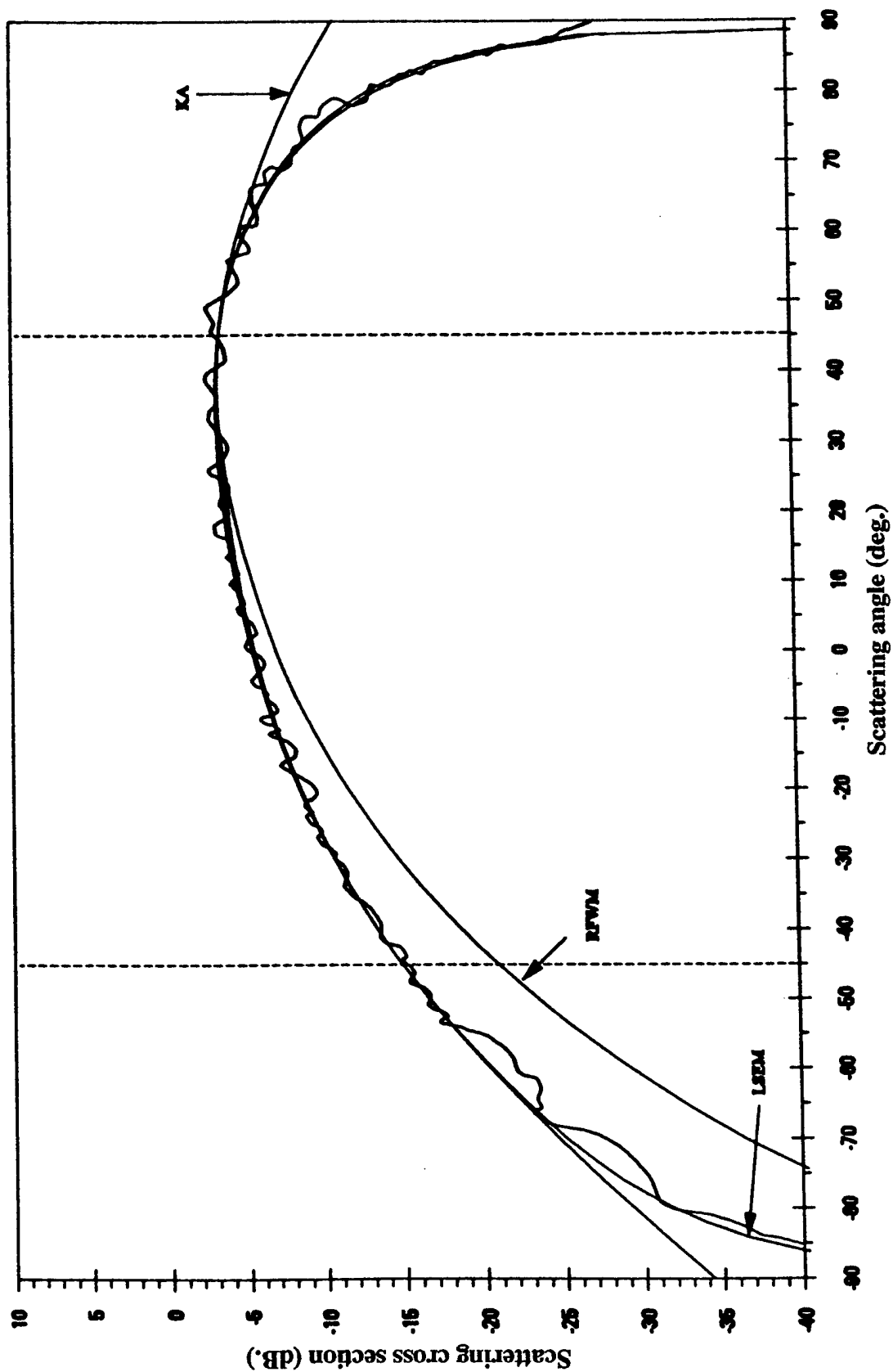
Fig. 1. Scattering cross section for horizontal polarization (TE waves) for a perfectly conducting rough surface. Angle of incidence = 45°, rms surface height 0.212 wavelength, correlation length 0.9 wavelength, rms slope=18.42°.
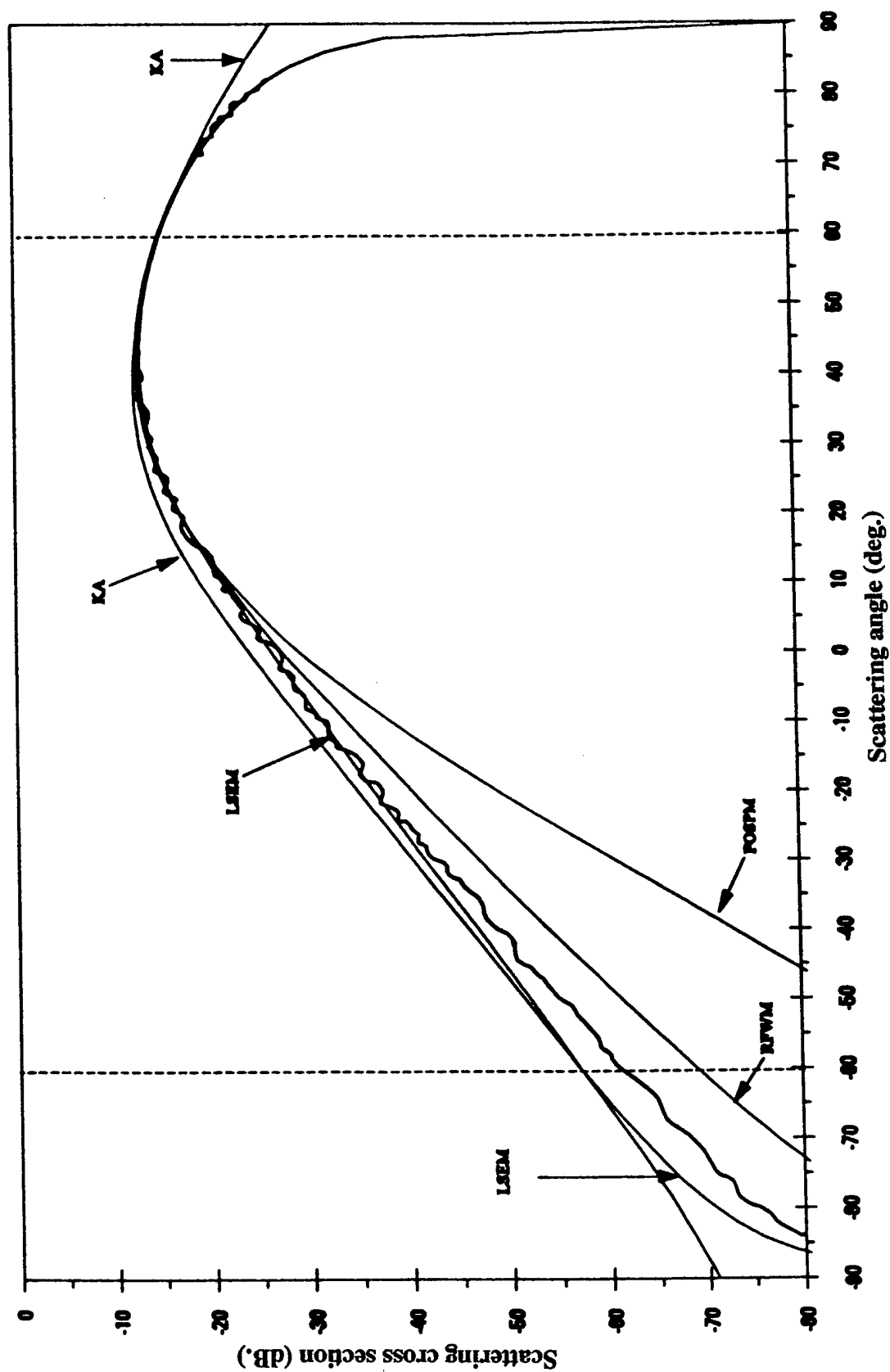
Fig. 2. Scattering cross section for horizontal polarization (TE waves) for a perfectly conducting rough surface. Angle of incidence = 60°, rms surface height 0.048 wavelength, correlation length 0.796 wavelength, rms slope=4.84°.
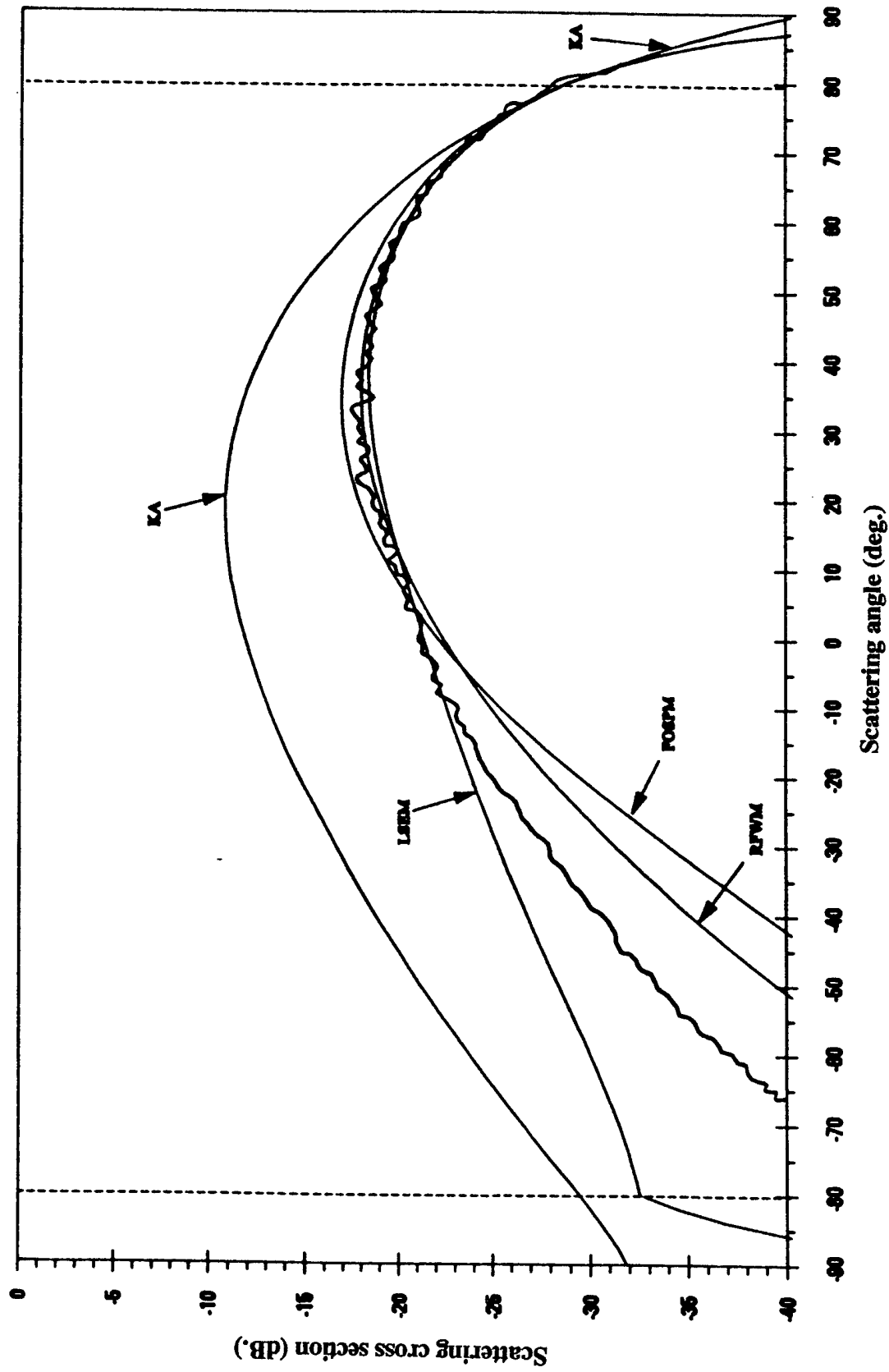
Fig. 3. Scattering cross section for horizontal polarization (TE waves) for a perfectly conducting rough surface. Angle of incidence = 80°, rms surface height 0.107 wavelength, correlation length 0.45 wavelength, rms slope=18.59°.
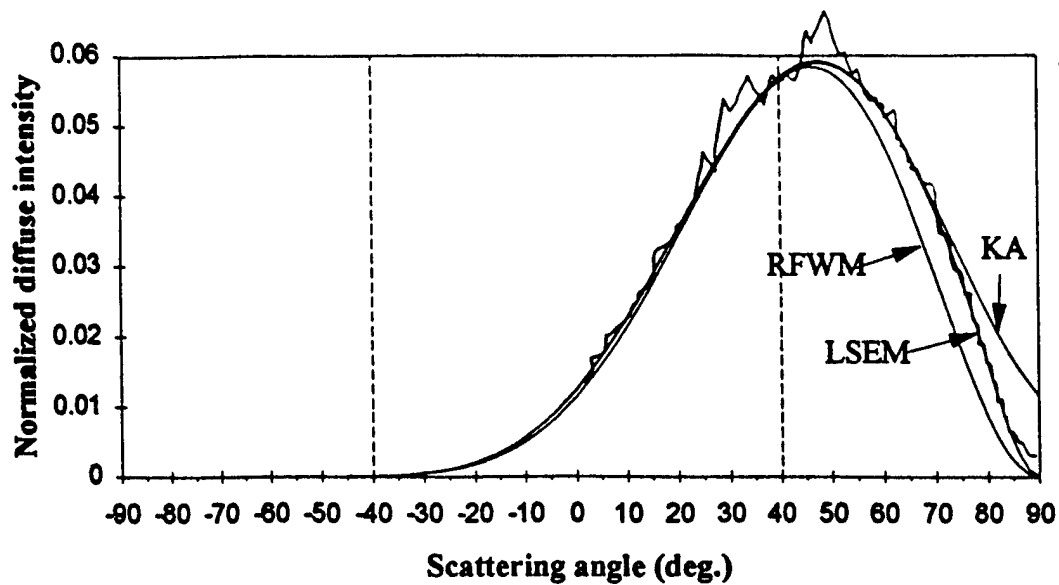
Fig. 4. Scattering cross section for a rough dielectric surface. Angle of incidence = 40°, relative permittivity = 2.04, rms surface height 0.5 wavelength, correlation length 3.16 wavelength.