

RL-TR-96-138  
Final Technical Report  
July 1996



# MICROWAVE IMPEDANCE MATCHING OF PHOTODIODES FOR HIGH SPEED OPTICAL COMMUNICATIONS SYSTEMS

David A. Sumberg, Consultant

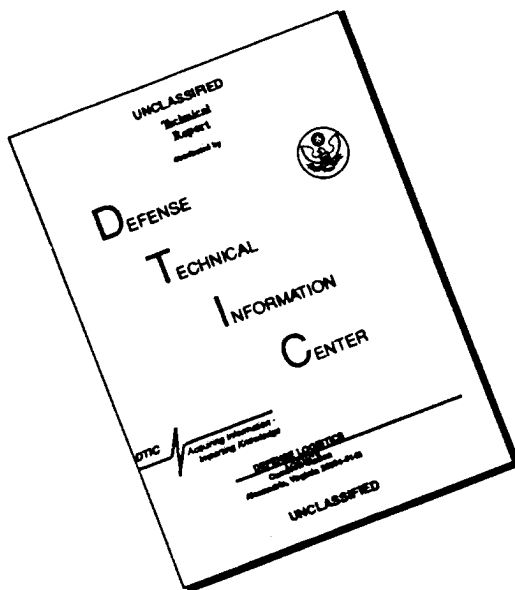
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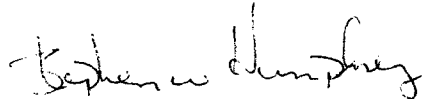
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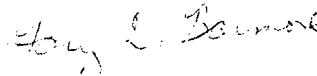
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REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.				
1. AGENCY USE ONLY (Leave Blank)	2. REPORT DATE July 1996		3. REPORT TYPE AND DATES COVERED Final Jun 94 - Sep 95	
4. TITLE AND SUBTITLE MICROWAVE IMPEDANCE MATCHING OF PHOTODIODES FOR HIGH SPEED OPTICAL COMMUNICATIONS SYSTEMS			5. FUNDING NUMBERS C - F30602-94-C-0099 PE - 62702F PR - 4600 TA - P2 WU - PR	
6. AUTHOR(S) David A. Sumberg				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) David A. Sumberg, Consultant 489 Claybourne Road Rochester NY 14618			8. PERFORMING ORGANIZATION REPORT NUMBER N/A	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Rome Laboratory/OCPC 25 Electronic Pky Rome NY 13441-4515			10. SPONSORING/MONITORING AGENCY REPORT NUMBER RL-TR-96-138	
11. SUPPLEMENTARY NOTES Rome Laboratory Project Engineer: Stephen W. Humphrey, 1Lt, USAF/OCPC/ (315) 330-1818				
12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution unlimited.			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words) The performance of optical communications systems can be influenced by the mismatch between the low impedance laser source and the high impedance generator. A procedure is described here for matching a laser diode to a 50 ohm generator by means of a lossless equalizer network. The equalizer network is realized in the form of a microstrip adaptation of a ladder network consisting of inductor and capacitor equivalents. The mathematical model predicts 50-70% improvement in transducer power gain. Experimental results confirm this.				
14. SUBJECT TERMS Transducer power, Impedance matching, Microstrip circuits			15. NUMBER OF PAGES 40	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT UL	

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## 1 EXECUTIVE SUMMARY

Many factors influence the performance of an optical communication system. This report addresses one of those factors, viz., the impedance mismatch between a low impedance laser diode and the generator. Based on a technique first described by Carlin<sup>1</sup>, a procedure is described for impedance matching the laser diode to a  $50 \Omega$  source over a prescribed frequency band by means of a lossless equalizer network. Carlin shows the form of the network to be that of a low-pass ladder circuit consisting of inductors and capacitors. The values for these unknown components are determined by maximizing, in a least-square sense, the transducer power gain over the prescribed frequency band. The procedure uses measured  $S_{11}$  data for the laser rather than deriving these values from an analytic model. The mathematical model predicts a 50% - 70% improvement in the transducer power gain and a flatter frequency response when compared with the laser diode alone.

In order to test the theory, a microstrip adaptation of the equalizer is designed based on stepped-impedance filter theory. The equalizer is fabricated and tested. Test results are found to be consistent with the results predicted by the model.

## 2 MATHEMATICAL MODEL

### 2.1 INTRODUCTION AND BACKGROUND

The problem of broad-band matching a load to a resistive generator, the so-called single matching problem, has been described by Carlin<sup>1</sup>. The block diagram of Figure 1 illustrates the major components, i.e., the generator, the equalizer, and the load. The problem, of course, is to determine the exact form of the equalizer that will broad-band match the load to the generator.

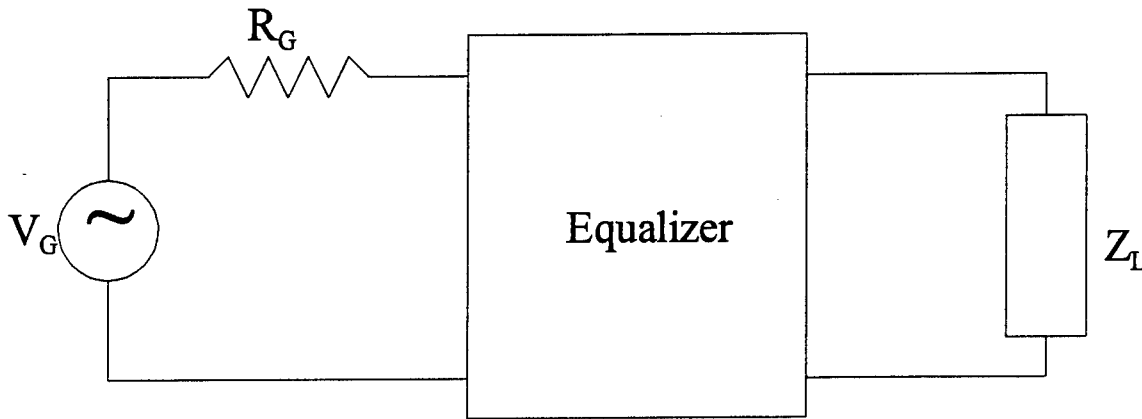


Figure 1. Block diagram of a circuit in which the equalizer matches the load to the generator.

Carlin approaches this problem by considering the transducer power gain  $T(\omega^2)$  of the load-equalizer system, where

$$\begin{aligned} T(\omega^2) &= \frac{\text{Power to the load}}{\text{Power available from the generator}} = 1 - |\Gamma|^2 \\ &= \frac{4 R_L(\omega) R_q(\omega)}{|Z_L(\omega) + Z_q(\omega)|^2} \end{aligned} \quad (1)$$

In the above equations  $\Gamma = (Z_L - Z_q)/(Z_L + Z_q)$  is the complex reflection coefficient at the load-equalizer interface,  $Z_q(\omega) = R_q(\omega) + jX_q(\omega)$  is the equalizer impedance, and  $Z_L(\omega) = R_L(\omega) + jX_L(\omega)$

is the known load impedance. Carlin solves the problem by representing  $R_q(\omega)$ , the real part of the equalizer impedance, as a number of straight line segments of unknown slopes between the frequency breakpoints  $0 < \omega_1 < \omega_2 \dots < \omega_n$  spanning the passband, and choosing the slopes such that the transducer power gain is a maximum. The reactance  $X_L(\omega)$  is then determined from  $R_q(\omega)$  by a Hilbert transform. Once the line segments describing  $R_q(\omega)$  are determined, this piecewise continuous function is approximated by a rational function of  $\omega$  denoted by  $\hat{R}_q(\omega)$ . A rational function for  $X_q(\omega)$  denoted by  $\hat{X}_q(\omega)$  is determined as above by a Hilbert transform. Using the rational form of the impedance ( $\hat{Z}_q(\omega) = \hat{R}_q(\omega) + j \hat{X}_q(\omega)$ ) the equalizer is realized as a Darlington reactance 2-port.

## 2.2 MATHEMATICAL FORMULATION

While Carlin's method provides a novel solution to the problem, it is not always the most straight-forward approach. In this report the problem of broad-band matching a laser diode to a 50 ohm generator is solved using a variation of the above technique. To begin, we note that a laser diode can be modeled as an LCR load, which Carlin shows is matched to the generator by a low-pass LC ladder network as in Figure 2. Thus, we postulate the form of the equalizer to be that of a low-

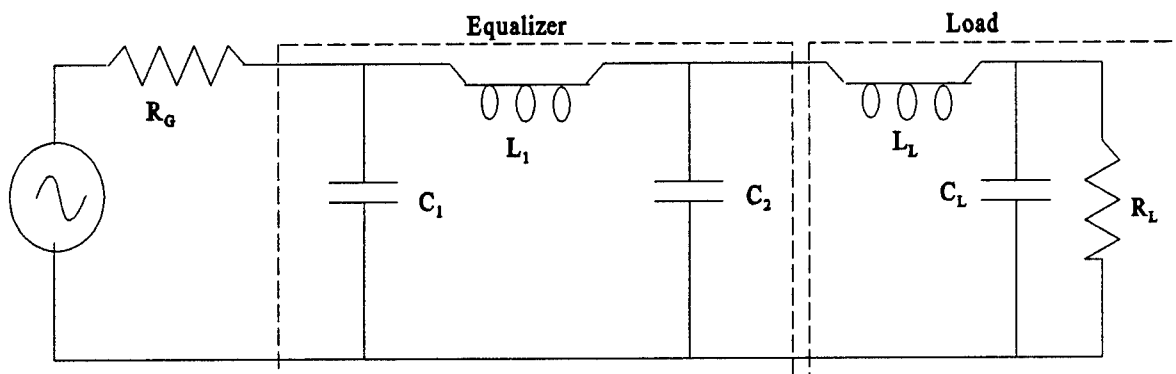


Figure 2. Three element low-pass equalizer for laser diode (modeled as an LCR load).

pass LC ladder network. We then write the transducer power gain in terms of the unknown parameters  $L$  and  $C$  and determine their values such that the transducer power gain is maximized.

Pozar<sup>2</sup> expresses the transducer power gain in terms of the scattering parameters  $S_{ij}$  of the equalizer as follows:

$$T(\omega^2) = \frac{|S_{21}(\omega)|^2 \cdot (1 - |S_G|^2) \cdot (1 - |S_L(\omega)|^2)}{|1 - S_{11}(\omega) \cdot S_G|^2 \cdot |1 - S_2(\omega) \cdot S_L(\omega)|^2} \quad (2)$$

where  $S_L(\omega)$  is the experimentally determined reflection coefficient ( $S_{11}$ ) at the load and is presented in Figure 3. The reflection coefficient at the generator is  $S_G = (R_G - R_0)/(R_G + R_0)$ , where  $R_0$  is the characteristic line impedance and  $R_G$  is the generator resistance, and

$$S_2(\omega) = S_{22}(\omega) + \frac{S_{21}(\omega)^2 \cdot S_G}{1 - S_{22}(\omega) \cdot S_G} \quad (3)$$

For the case of a generator matched to the line ( $R_G = R_0$ ),  $S_G = 0$ , and the following simplifications result:

$$S_2(\omega) = S_{22}(\omega) \quad (4)$$

and

$$T(\omega^2) = \frac{|S_{21}(\omega)|^2 \cdot (1 - |S_L(\omega)|^2)}{|1 - S_{22}(\omega) \cdot S_L(\omega)|^2} \quad (5)$$

The scattering parameters  $S_{ij}$  depend on the L and C components that comprise the equalizer. Expressions for the scattering parameters in terms of L and C are reproduced in Appendix A for a three element low-pass ladder network and in Appendix B for a five element low-pass ladder network.

Optimization is carried out for the load of Figure 3. The data, which represents  $|S_L(\omega)|^2$  in Eq (5), is obtained using a network analyzer to measure  $S_{11}$  with the laser biased at its operating point. The transducer power gain is then optimized in a least-square sense by minimizing the following function:

$$SSE = \sum_{i=1}^n [T(\omega_i) - T_I]^2 \quad (6)$$

where  $T_I$  is the idealized value for  $T(\omega)$  and must be in the range (0, 1). SSE is the sum of the square of the errors. Several currently available software packages, Mathcad and Matlab to name but two, have built-in optimization capability. The specific results presented in this report were obtained using Mathcad's "minerr" function.

The resulting values for C and L for the three and five element networks are summarized in Table 1. The transducer power gains with and without the equalizer network are plotted in Figure 4 for the three element network and in Figure 5 for the five element network. Detailed information for the three and five element networks appear in Appendix A and Appendix B, respectively.

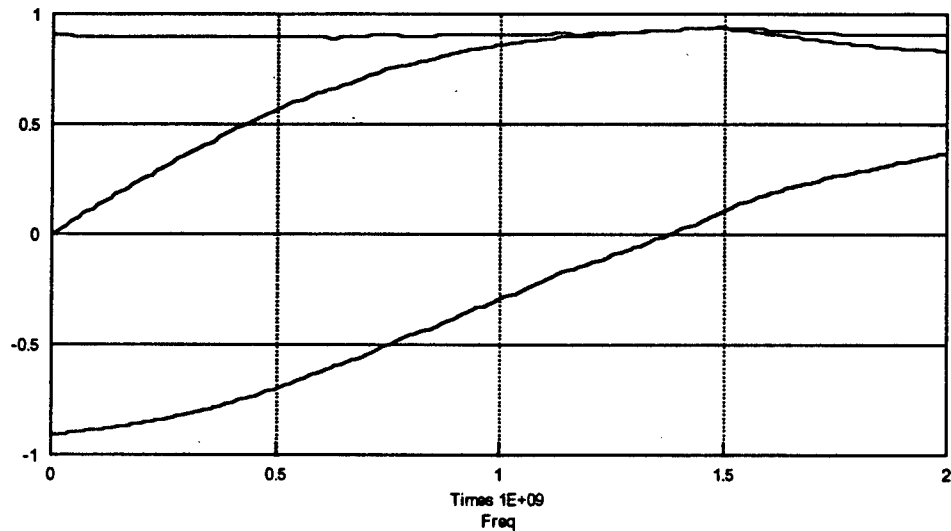


Figure 3. Experimentally measured  $S_{11}$  parameter for a Sharp laser diode, type LTO22. Lower curve:  $\text{Re}\{S_{11}\}$ , middle curve:  $\text{Im}\{S_{11}\}$ , upper curve:  $|S_{11}|$ .

	3 elements	5 elements
$C_1$ (pF)	1.90	1.44
$C_2$ (pF)	3.80	1.30
$C_3$ (pF)	N.A.	3.84
$L_1$ (nH)	6.51	2.20
$L_2$ (nH)	N.A.	5.41

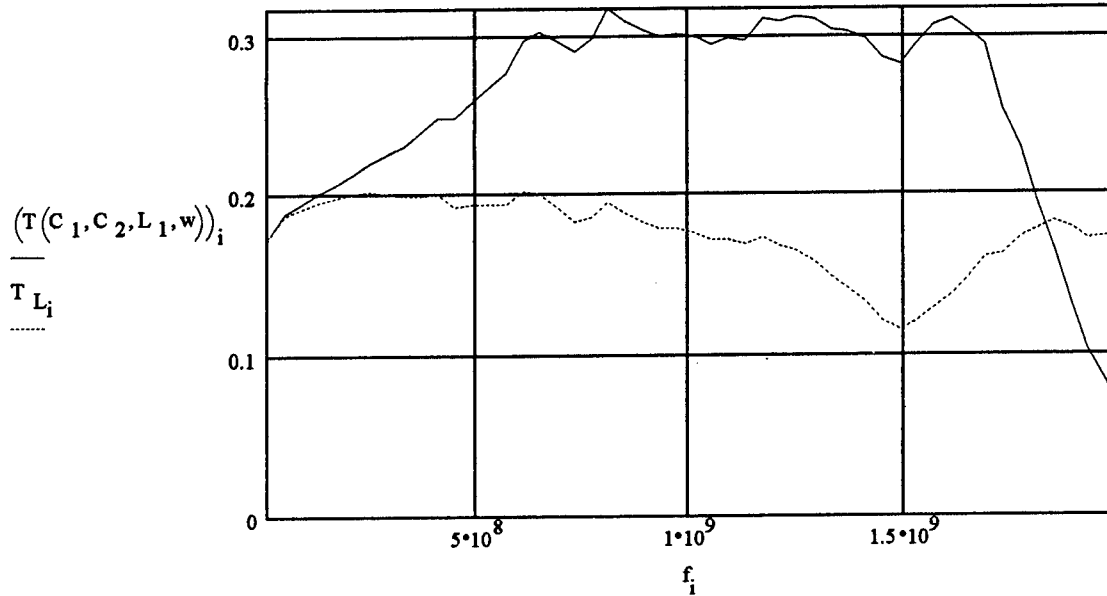


Figure 4. Transducer power gain. **Solid curve:** Three-element equalizer terminated with laser diode. Values for  $C_1$ ,  $C_2$ , and  $L_1$  are listed in Table 1. **Dashed curve:** Transducer power gain for the laser diode without equalizer.

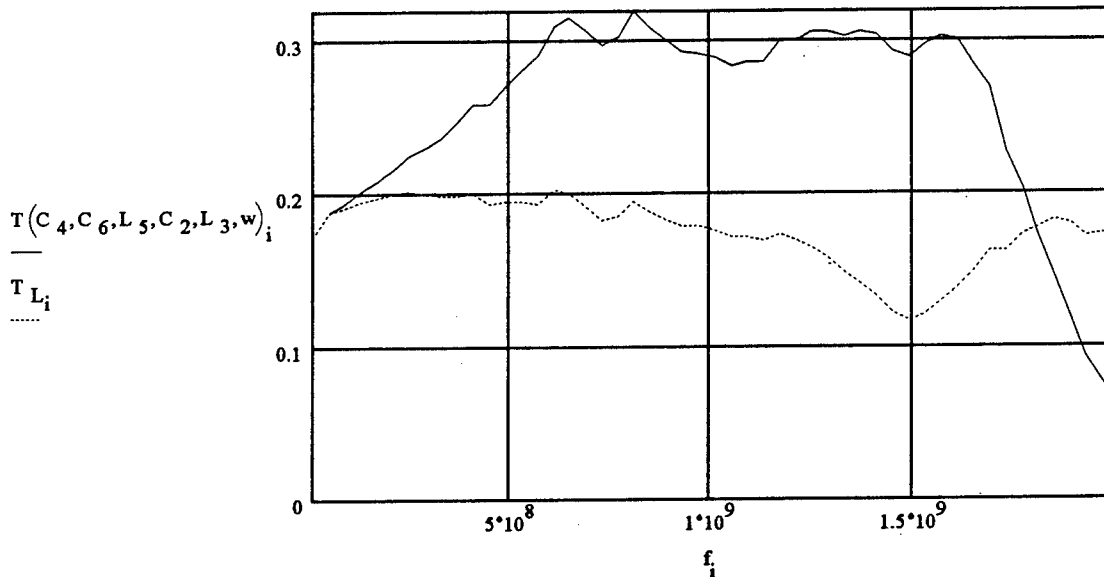


Figure 5. Transducer power gain. **Solid curve:** Five-element equalizer terminated with laser diode. Values for  $C_1$ ,  $C_2$ ,  $C_3$ ,  $L_1$ , and  $L_2$  are listed in Table 1. **Dashed curve:** Transducer power gain for the laser diode without equalizer.

### 3 EXPERIMENTAL DESIGN AND RESULTS

#### 3.1 EQUALIZER DESIGN

The transition from the mathematical model to an actual physical realization of the equalizer is hampered by the difficulty in obtaining discrete components (capacitors and inductors) of the exact values required. The problem is further exacerbated by the sensitivity of the equalizer circuit to the values of the components. The results of a sensitivity analysis in which the values of L and C are varied by  $\pm 10\%$  in the mathematical model is shown in Figure 6. It should be noted that small variations in the values of the components tend to destroy much of the potential benefits of the equalizer.

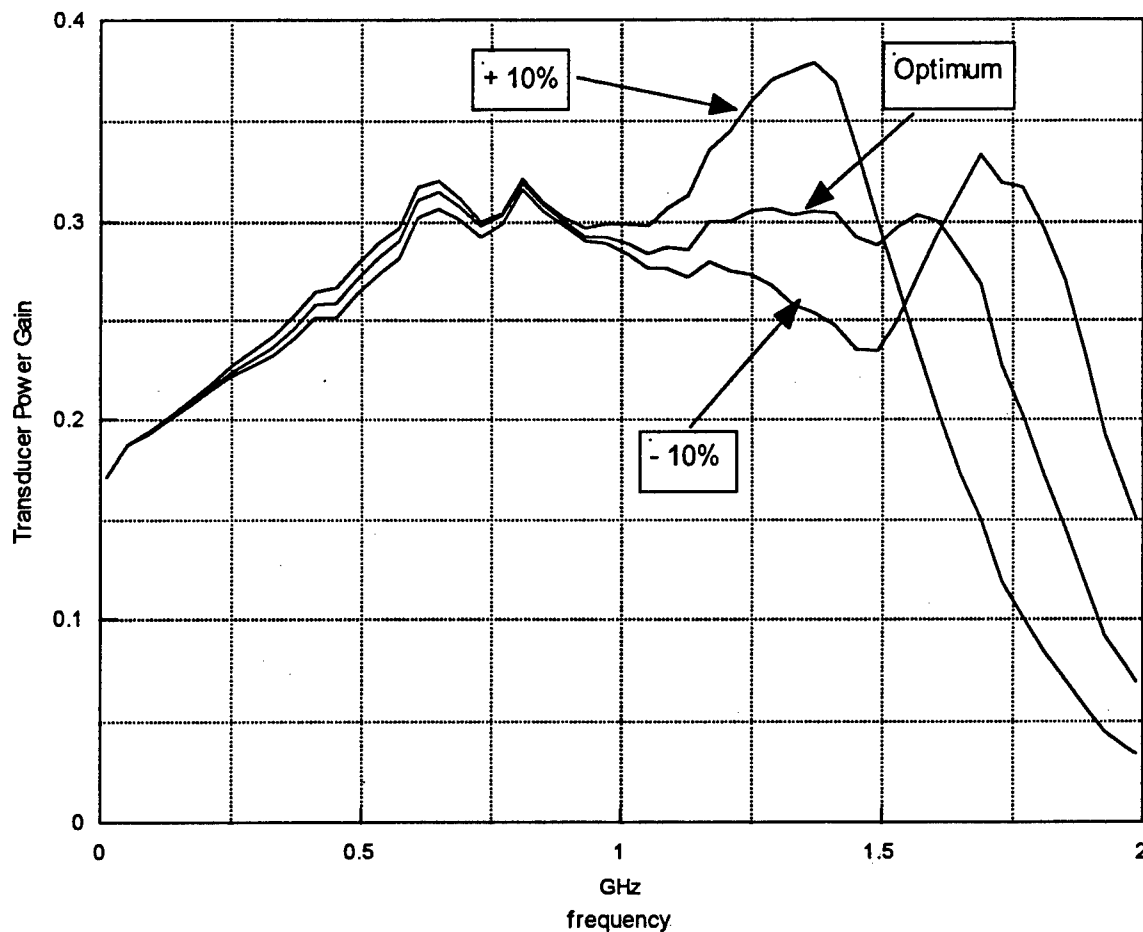


Figure 6. Results of sensitivity analysis in which the values of L and C are varied by  $\pm 10\%$  from their optimum values.

Based on these observations, a practical solution is to employ microstrip design techniques to fabricate the required equalizer network. Of the several choices available, a stepped-impedance filter design is selected, because of its ease of fabrication. As is well known, the stepped impedance design substitutes a series combination of high and low impedance transmission lines for the components of the ladder network; high impedance lines replace inductors while low impedance lines replace capacitors. A summary of the microstrip design equations used in this report may be found in Appendix C. An example of a stepped impedance design fabricated on microstrip is shown in Appendix D.

The parameters to be determined for the stepped impedance filter are the length and width of each line segment. As a practical matter, line widths are limited to values between (approximately) 0.1 mm and 35 mm. According to the theory of microstrip lines summarized in Appendix C, the stated range of line widths confine the characteristic impedance of the transmission line to values between (approximately) 20  $\Omega$  and 200  $\Omega$ . See the figure of Appendix C for a plot of characteristic impedance vs. line width for typical Duroid material.

When designing a stepped-impedance filter, standard approximations<sup>4</sup> are used to calculate the length of each line segment based on the value of the inductor or capacitor that it replaces and by the characteristic impedance of the transmission line. However, rather than simply use the approximations to translate the previously determined values of L and C to transmission line lengths, the lengths of the transmission lines are themselves treated as variables to be determined by the optimization process. This is achieved by first writing the scattering parameters in terms of the lengths of the line segments by means of the ABCD matrices (see below), and then by writing the transducer power gain in terms of the scattering parameters and optimizing the response, as was done previously. This calculation is more precise, since it uses the exact transmission line equations rather than the approximations referred to above. The relevant equations are stated below.

The ABCD matrix for the  $i^{\text{th}}$  transmission line segment<sup>5</sup> of length  $\ell_i$  is

$$M_i = \begin{bmatrix} \cos(\beta_i \ell_i) & jZ_i \sin(\beta_i \ell_i) \\ jY_i \sin(\beta_i \ell_i) & \cos(\beta_i \ell_i) \end{bmatrix} \quad (7)$$

where  $Z_i$  is the characteristic impedance of the line segment,  $Y_i = 1/Z_i$ ,  $\beta_i = 2\pi f/u_i$ , with the phase velocity defined as  $u_i = c/(\epsilon_e)^{1/2}$ . The effective dielectric constant  $\epsilon_e$  is defined in Appendix C. The overall ABCD matrix for n line segments is then the product of the individual matrices written as

$$M = M_1 \cdot M_2 \cdots M_n = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad (8)$$

where  $M_1$  is the segment nearest the generator. Finally, the scattering parameters are obtained according to the following transformations<sup>5</sup>:

$$S_{11} = \frac{A + \frac{B}{Z_0} - C \cdot Z_0 - D}{K}$$

$$S_{22} = \frac{-A + \frac{B}{Z_0} - C \cdot Z_0 + D}{K} \quad (9)$$

$$S_{12} = S_{21} = \frac{2}{K}$$

where

$$K = A + \frac{B}{Z_0} + C \cdot Z_0 + D$$

Appendix D contains detailed calculations in which the above procedure is applied to a five element equalizer network. Table 2 summarizes the results of those calculations and includes the length, width, and characteristic impedance of each line segment as it appears in the final design. Lengths  $\ell_1 - \ell_5$  represent the five elements, while length  $\ell_0$  of the 50  $\Omega$  line joining the generator and the equalizer is arbitrary. The transducer power gain with and without the equalizer is plotted in Figure 7.

Table 2. Segment Lengths			
	length in mm.	width in mm.	characteristic impedance ( $\Omega$ )
$\ell_0$	-	9.43	50
$\ell_1$	6.75	31.6	200
$\ell_2$	2.72	0.308	20
$\ell_3$	6.19	31.6	200
$\ell_4$	5.23	0.308	20
$\ell_5$	15.83	31.6	200

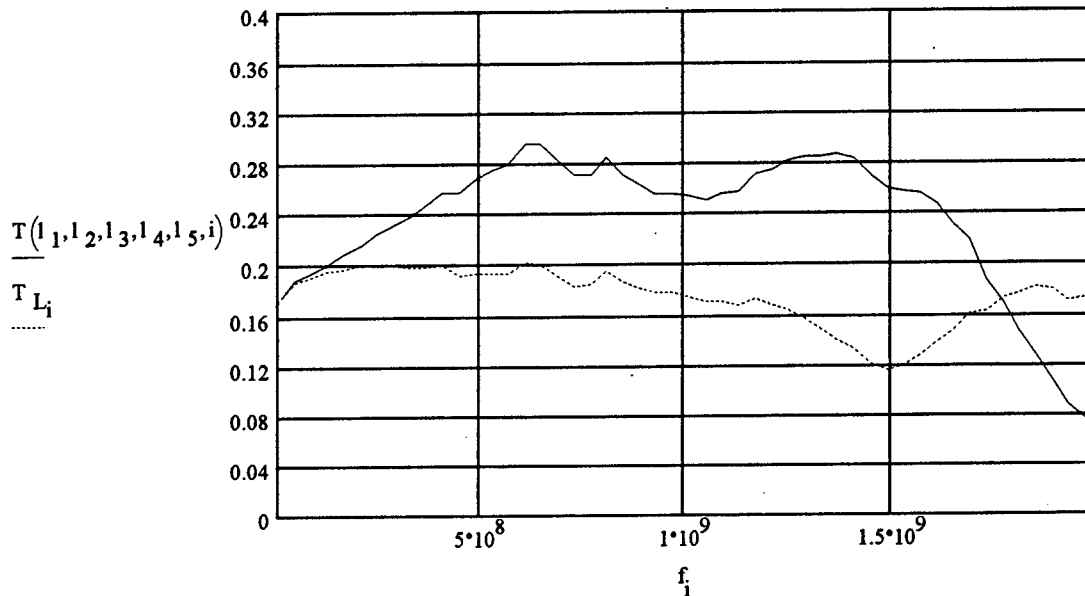


Figure 7. Transducer power gain. **Solid curve:** Five-segment stepped-impedance equalizer terminated with laser diode. Values for  $\ell_1 - \ell_5$  are listed in Table 2. **Dashed curve:** Transducer power gain for the laser diode without equalize

### 3.2 EXPERIMENTAL RESULTS AND DISCUSSION

The experimental results consist of two sets of data: (1) the transducer power gain and (2)  $S_{21}$ . Measured values of transducer power gain are presented in Figure 8 and Figure 9. The data of Figure 8 represents  $T(\omega)$  for the load with and without the equalizer. The data of Figure 9 is  $T(\omega)$  when a small amount of stray capacitance is introduced by placing a finger near one of the line segments. Figure 9 is included, because it suggests the possibility that different architectures (perhaps a capacitor in parallel with one of the inductors) may result in greater values of  $T(\omega)$ . It also illustrates

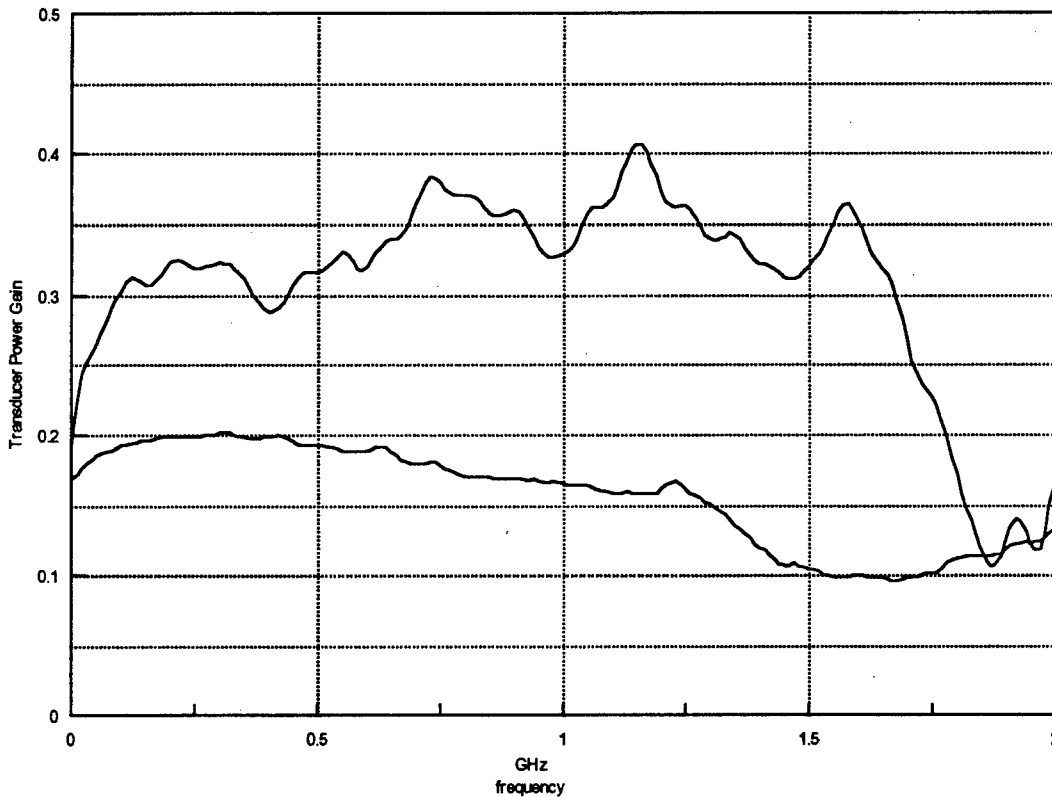


Figure 8. Measured values of  $T(\omega)$  for the five segment stepped-impedance equalizer terminated with the laser diode (upper curve) and the laser diode without the equalizer (lower curve).

how the optimization process may be dependent on the guessed values that are used to initiate the process, i.e., different initial values may produce a different outcome. Comparison of Figure 8 with Figure 7 shows a somewhat better gain than anticipated, perhaps caused by the influence of stray capacitance.

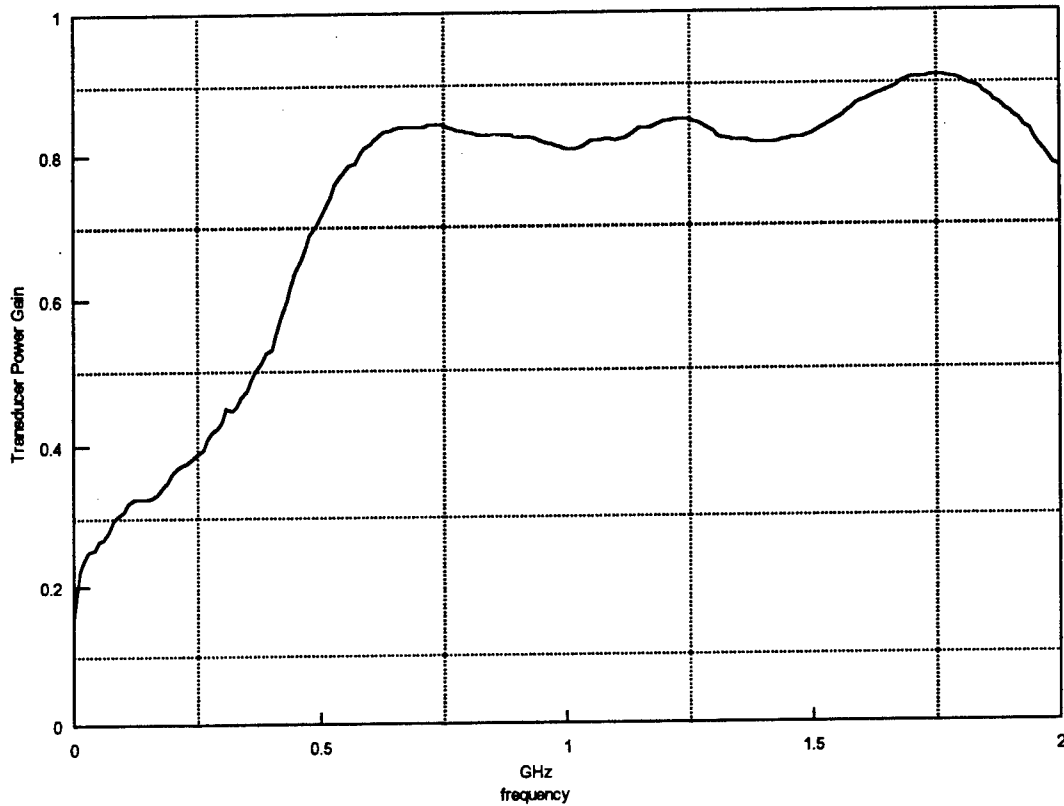


Figure 9. Transducer power gain. Equalizer with stray capacitance.

The second set of data is a measurement of  $S_{21}$ . The apparatus used to obtain these data is shown in Figure 10. The data itself is plotted in Figure 11, which shows  $S_{21}$  for the laser diode with and without the equalizer. A comparison of the two curves of Figure 11 shows a significant improvement in frequency response over the range 0 - 1.25 GHz. The equalizer provides a frequency response that is flat to within 3 dB, whereas without the equalizer the response fluctuates by more than 10 dB.

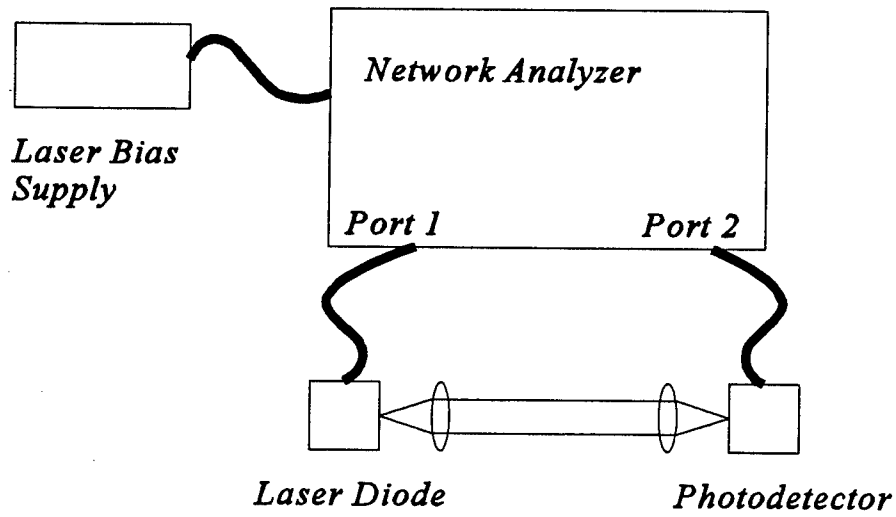


Figure 10. Experimental apparatus used to measure  $S_{21}$ .

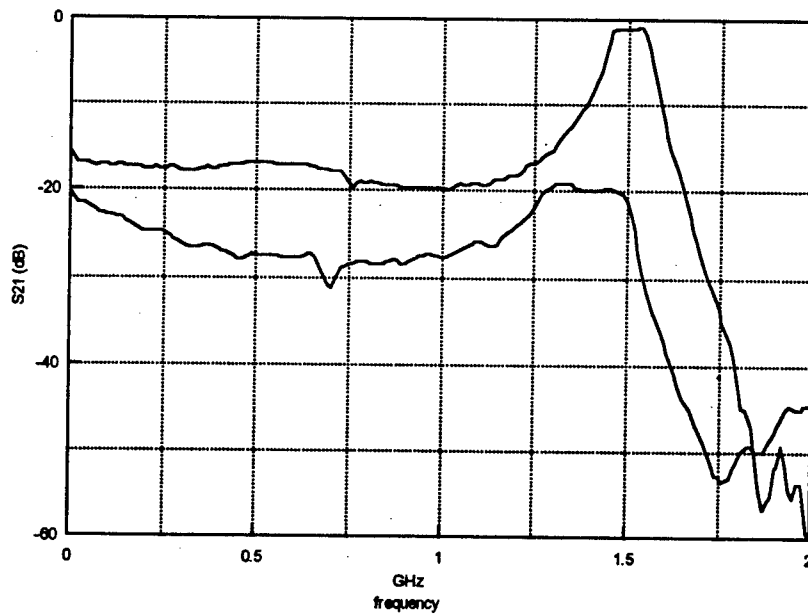


Figure 11.  $S_{21}$  for the load with equalizer (upper curve) and without equalizer (lower curve).

#### 4 CONCLUSION

This report describes a procedure for impedance matching a low impedance laser diode to a  $50\ \Omega$  source over a prescribed frequency band by means of a lossless equalizer network. The form of the network is that of a low-pass ladder circuit consisting of inductors and capacitors. The values for these unknown components are determined from measured  $S_{11}$  data for the laser diode by maximizing, in a least-square sense, the transducer power gain over the prescribed frequency band. Based on the derived component values, the theory predicts a 50% - 70% improvement in the transducer power gain and a flatter frequency response when compared with the laser diode alone.

A microstrip adaptation of the equalizer is designed based on stepped-impedance filter theory. The equalizer is fabricated and tested. Test results confirm the 50% - 70% performance improvement predicted by the model.  $S_{21}$  measurements show a 3 dB fluctuation in the frequency response compared with 10 dB for the laser diode without the equalizer.

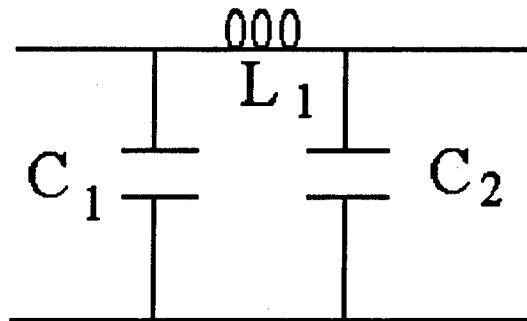
Finally, during system testing it was shown that stray capacitance resulted in transducer power gains of nearly 80%. This observation suggests that future activity should center on alternative architectures for the equalizer, particularly those which place capacitors in parallel with the inductors.

## 5 REFERENCES

1. H. J. Carlin, "A new approach to gain-bandwidth problems," *IEEE Trans. Circuits Syst.*, vol. CAS-24, pp. 170-175, Apr. 1977
2. Ibid.
3. D. M. Pozar, *Microwave Engineering*, Reading, MA, Addison-Wesley, 1990, p 243
4. Ibid. See, for example, equations 9.86a and 9.86b on p. 503.
5. Ibid. See Table 5.1, p233.
6. Ibid. See Table 5.2, p235.

## APPENDIX A

Optimization for a 3-element low-pass equalizer (pi-section consisting of  $C_1$ ,  $C_2$ ,  $L_1$ ).  
The following calculation was written using Mathcad 5.0.



Characteristic line impedance;      Impedance of the generator

$$R_0 := 50$$

$$R_g := 50$$

Read from disk the data file for the laser diode.

```
A := READPRN(comb_sht)
```

```
f := A<1>
```

```
w := 2·π·f
```

```
n := length(w)
```

```
n = 11
```

```
i := 1..n
```

Define reactance of the elements

$$x_{C1}(C_1) := \frac{1}{j \cdot C_1 \cdot w}$$

$$x_{C2}(C_2) := \frac{1}{j \cdot C_2 \cdot w}$$

$$x_{L1}(L_1) := j \cdot w \cdot L_1$$

Input Admittance and Impedance looking into port 1 when the output is terminated with  $R_0$

$$Y_{in1}(C_1, C_2, L_1) := \overbrace{\left[ \frac{1}{x_{C1}(C_1)} + \frac{1}{x_{L1}(L_1) + \frac{1}{\frac{1}{x_{C2}(C_2)} + \frac{1}{R_0}}} \right]}$$

$$Z_{in1}(C_1, C_2, L_1) := \overbrace{\frac{1}{Y_{in1}(C_1, C_2, L_1)}}$$

Input Admittance and Impedance looking into port 2 when the input is terminated with  $R_0$

$$Y_{in2}(C_1, C_2, L_1) := \overbrace{\left[ \frac{1}{x_{C2}(C_2)} + \frac{1}{x_{L1}(L_1) + \frac{1}{\frac{1}{x_{C1}(C_1)} + \frac{1}{R_0}}} \right]}$$

$$Z_{in2}(C_1, C_2, L_1) := \overbrace{\frac{1}{Y_{in2}(C_1, C_2, L_1)}}$$

Scattering matrix elements for the equalizer

$$S_{11}(C_1, C_2, L_1) := \overbrace{\frac{Z_{in1}(C_1, C_2, L_1) - R_0}{Z_{in1}(C_1, C_2, L_1) + R_0}}$$

$$S_{22}(C_1, C_2, L_1) := \overbrace{\frac{Z_{in2}(C_1, C_2, L_1) - R_0}{Z_{in2}(C_1, C_2, L_1) + R_0}}$$

$$S_{12}(C_1, C_2, L_1) := \overbrace{\frac{1 + S_{22}(C_1, C_2, L_1)}{1 + x_{L1}(L_1) \cdot \left( \frac{1}{R_0} + \frac{1}{x_{C1}(C_1)} \right)}}$$

$$S_{21}(C_1, C_2, L_1) := \overbrace{\frac{1 + S_{11}(C_1, C_2, L_1)}{1 + x_{L1}(L_1) \cdot \left( \frac{1}{R_0} + \frac{1}{x_{C2}(C_2)} \right)}}$$

Read (from disk) the scattering matrix (reflection coef) for the laser diode at frequencies  $f_i$

$$S_L := A^{\langle 2 \rangle} + j \cdot A^{\langle 3 \rangle}$$

$$m := \text{length}(S_L)$$

$$m = 11$$

Scattering matrix (reflection coef) for the generator at frequencies  $f_i$

$$S_g := \frac{R_g - R_0}{R_g + R_0} \quad S_g = 0$$

Transducer power gain

$$S_2(C_1, C_2, L_1) := \overrightarrow{\left( S_{22}(C_1, C_2, L_1) + \frac{S_{21}(C_1, C_2, L_1)^2 \cdot S_g}{1 - S_{22}(C_1, C_2, L_1) \cdot S_g} \right)}$$

$$T(C_1, C_2, L_1, w) := \frac{\overrightarrow{\left( |S_{21}(C_1, C_2, L_1)|^2 \cdot [1 - (|S_g|)^2] \cdot [1 - (|S_L|)^2] \right)}}{\overrightarrow{\left( |1 - S_{11}(C_1, C_2, L_1) \cdot S_g|^2 \cdot |1 - S_2(C_1, C_2, L_1) \cdot S_L|^2 \right)}}$$

**Calculate the component values ( $C_1$ , etc.) that maximize the transducer power gain  $T$  at the frequencies  $f_i$  (in a least-squares sense).**

Set the idealized value for  $T$  (this is arbitrary)

$$T_I := 0.30$$

Compute the sum of the squares, which is to be minimized

$$\text{SSE}(C_1, C_2, L_1) := \sum_{i=1}^n \left( T(C_1, C_2, L_1, w)_i - T_I \right)^2$$

Initial guess for parameters

$$C_1 := 1.904314 \cdot 10^{-12}$$

$$C_2 := 3.8038 \cdot 10^{-12}$$

$$L_1 := 6.50727 \cdot 10^{-9}$$

Set up a solve block

Given

$$\text{SSE}(C_1, C_2, L_1) = 0$$

Dummy equations: Mathcad needs 3 equations for 3 unknowns

$$1 = 1 \quad 2 = 2$$

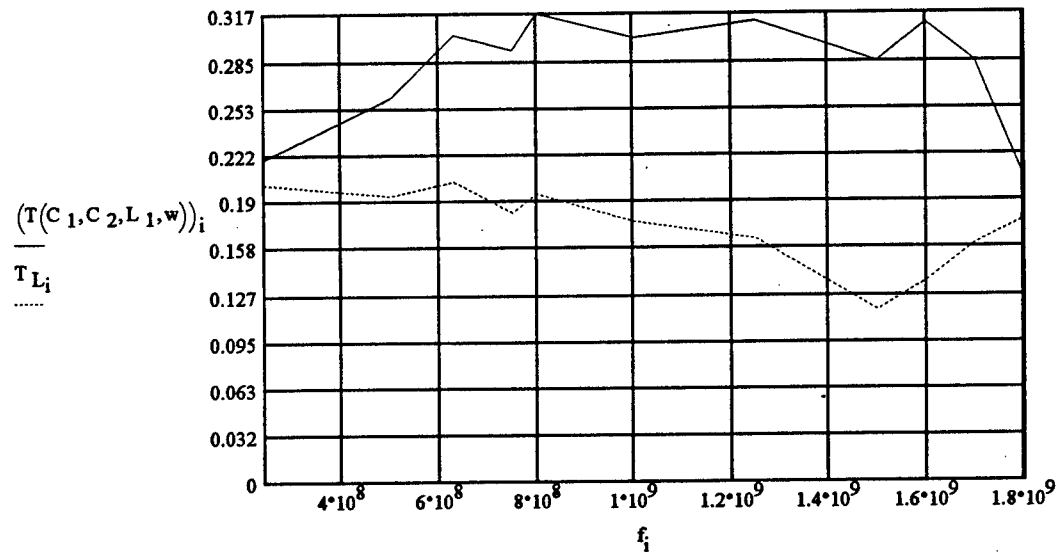
Minerr function gives best fit to data

$$\begin{bmatrix} C_1 \\ C_2 \\ L_1 \end{bmatrix} := \text{Minerr}(C_1, C_2, L_1) \quad \begin{bmatrix} C_1 \\ C_2 \\ L_1 \end{bmatrix} = \begin{bmatrix} 1.90431 \cdot 10^{-12} \\ 3.8038 \cdot 10^{-12} \\ 6.50727 \cdot 10^{-9} \end{bmatrix}$$

Calculate the transducer power gain for the laser diode without the equalizer (assume  $S_g = 0$ ) and calculate the mean-square error.

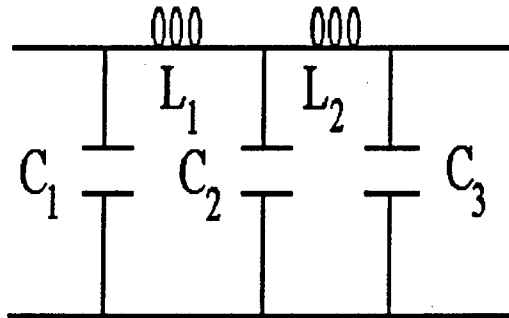
$$T_L := \sqrt{1 - (|S_L|)^2} \quad \frac{\text{SSE}(C_1, C_2, L_1)}{n - 2} = 0.002$$

Plot the transducer power gain with and without the equalizer



## APPENDIX B

Optimization for a 5-element low-pass equalizer (pi-section consisting of  $C_1, C_2, C_3, L_1, L_2$ ).  
The following calculation was written using Mathcad 5.0



Characteristic line impedance;      Impedance of the generator

$$R_0 := 50$$

$$R_g := 50$$

Read from disk the data file for the laser diode. Specify the frequency  $f_i$  as the 1<sup>st</sup> column.

$A := \text{READPRN}(\text{comb\_sht})$

$f := A^{<1>}$

$w := 2 \cdot \pi \cdot f$

$n := \text{length}(w)$

$n = 11$

$i := 1..n$

Input Impedance looking into port 1 when the output is terminated with  $R_0$

$$Z_{\text{inl}}(C_1, C_2, C_3, L_1, L_2) := \frac{1}{j \cdot w \cdot C_1 + \frac{1}{j \cdot w \cdot L_1 + \frac{1}{j \cdot w \cdot C_2 + \frac{1}{j \cdot w \cdot L_2 + \frac{1}{j \cdot w \cdot C_3 + \frac{1}{R_0}}}}}}$$

Input Impedance looking into port 2 when the input is terminated with  $R_0$

$$Z_{in2}(C_1, C_2, C_3, L_1, L_2) := \frac{1}{j \cdot \omega \cdot C_3 + \frac{1}{j \cdot \omega \cdot L_2 + \frac{1}{j \cdot \omega \cdot C_2 + \frac{1}{j \cdot \omega \cdot L_1 + \frac{1}{j \cdot \omega \cdot C_1 + \frac{1}{R_0}}}}}}$$

Scattering matrix elements for the equalizer

$$S_{11}(C_1, C_2, C_3, L_1, L_2) := \frac{Z_{in1}(C_1, C_2, C_3, L_1, L_2) - R_0}{Z_{in1}(C_1, C_2, C_3, L_1, L_2) + R_0}$$

$$S_{22}(C_1, C_2, C_3, L_1, L_2) := \frac{Z_{in2}(C_1, C_2, C_3, L_1, L_2) - R_0}{Z_{in2}(C_1, C_2, C_3, L_1, L_2) + R_0}$$

$$S_{21}(C_1, C_2, C_3, L_1, L_2) := \frac{1 + S_{11}(C_1, C_2, C_3, L_1, L_2)}{1 + j \cdot \omega \cdot L_2 \cdot \left( \frac{1}{R_0} + j \cdot \omega \cdot C_3 \right)} \cdot \frac{1}{1 + j \cdot \omega \cdot L_1 \cdot \left[ j \cdot \omega \cdot C_2 + \frac{1}{j \cdot \omega \cdot L_2 + \frac{1}{j \cdot \omega \cdot C_3 + \frac{1}{R_0}}} \right]}$$

$$S_{12}(C_1, C_2, C_3, L_1, L_2) := S_{21}(C_1, C_2, C_3, L_1, L_2)$$

Read (from disk) scattering matrix (reflection coef) for the laser diode at frequencies  $f_1$

$$S_L := A^{<2>} + j \cdot A^{<3>}$$

$$m := \text{length}(S_L)$$

$$m = 11$$

Scattering matrix (reflection coef) for the generator at frequencies  $f_i$

$$S_g := \frac{R_g - R_0}{R_g + R_0} \quad S_g = 0$$

Transducer power gain

$$S_2(C_1, C_2, C_3, L_1, L_2) := \overbrace{\left( S_{22}(C_1, C_2, C_3, L_1, L_2) + \frac{S_{21}(C_1, C_2, C_3, L_1, L_2)^2 \cdot S_g}{1 - S_{22}(C_1, C_2, C_3, L_1, L_2) \cdot S_g} \right)}$$

$$T(C_1, C_2, C_3, L_1, L_2, w) := \frac{\overbrace{\left( |S_{21}(C_1, C_2, C_3, L_1, L_2)| \right)^2 \cdot \left[ 1 - (|S_g|)^2 \right] \cdot \left[ 1 - (|S_L|)^2 \right]}}{\overbrace{\left( |1 - S_{11}(C_1, C_2, C_3, L_1, L_2) \cdot S_g| \right)^2 \cdot \left( |1 - S_{22}(C_1, C_2, C_3, L_1, L_2) \cdot S_L| \right)^2}}$$

**Calculate the component values ( $C_1$ , etc.) that maximize the transducer power gain  $T$  at the frequencies  $f_i$  (in a least-squares sense).**

Set the idealized value for  $T$  (this is arbitrary)

$$T_I := 0.29$$

Compute the sum of the squares, which is to be minimized

$$SSE(C_1, C_2, C_3, L_1, L_2) := \sum_{i=1}^n \left( T(C_1, C_2, C_3, L_1, L_2, w)_i - T_I \right)^2$$

Initial guess for parameters

$$C_1 := 1.435 \cdot 10^{-12} \quad C_2 := 1.3 \cdot 10^{-12} \quad C_3 := 3.835 \cdot 10^{-12} \quad L_1 := 2.2 \cdot 10^{-9} \quad L_2 := 5.4065 \cdot 10^{-9}$$

Set up a solve block

Given

$$SSE(C_1, C_2, C_3, L_1, L_2) = 0$$

Dummy equations: Mathcad needs 5 equations for 5 unknowns

1=1      2=2      3=3      4=4

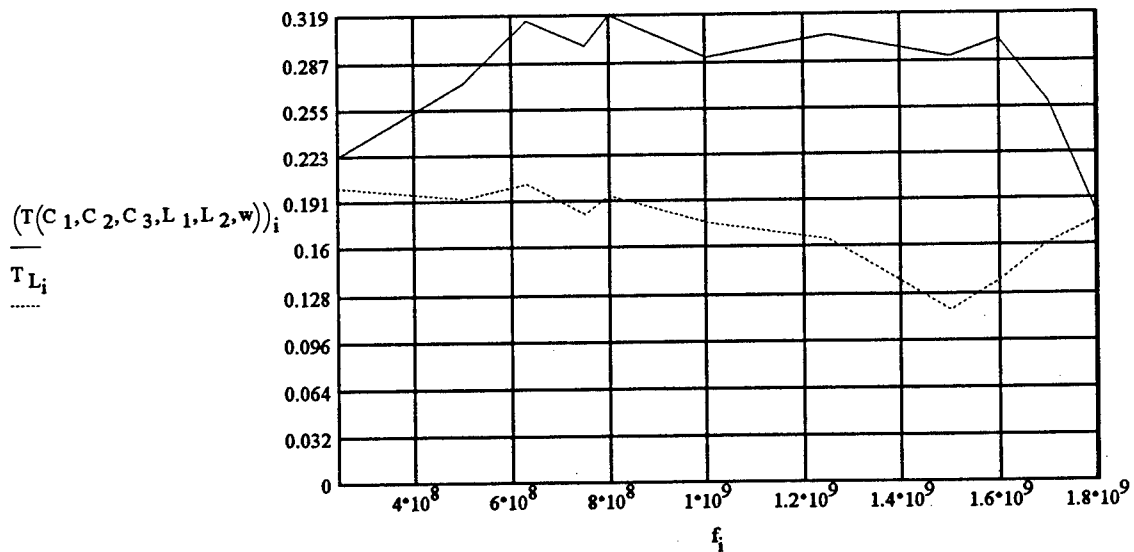
Minerr function gives best fit to data

$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ L_1 \\ L_2 \end{bmatrix} := \text{Minerr}(C_1, C_2, C_3, L_1, L_2) \quad \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ L_1 \\ L_2 \end{bmatrix} = \begin{bmatrix} 1.435 \cdot 10^{-12} \\ 1.3 \cdot 10^{-12} \\ 3.835 \cdot 10^{-12} \\ 2.2 \cdot 10^{-9} \\ 5.407 \cdot 10^{-9} \end{bmatrix}$$

Calculate the transducer power gain for the laser diode without the equalizer (assume  $S_g = 0$ )  
the mean square error

$$T_L := \overline{1 - (|S_L|)^2} \quad \frac{\text{SSE}(C_1, C_2, C_3, L_1, L_2)}{n - 2} = 0.002$$

Plot the transducer power gain with and without the equalizer.



## APPENDIX C

### A SUMMARY OF MICROSTRIP EQUATIONS

#### 1. Impedance of a microstrip line given the "width" to "dielectric thickness" ratio ( $r=W/d$ ).

$W$  = width;  $d$  = thickness of dielectric ;  $r = W/d$

Define  $a(r)$  such that:

$$a(r) = 1 \quad \text{if } 0 \leq r \leq 1$$

$$a(r) = 0 \quad \text{otherwise}$$

$$a(r) := \text{if}(r \leq 1, 1, 0)$$

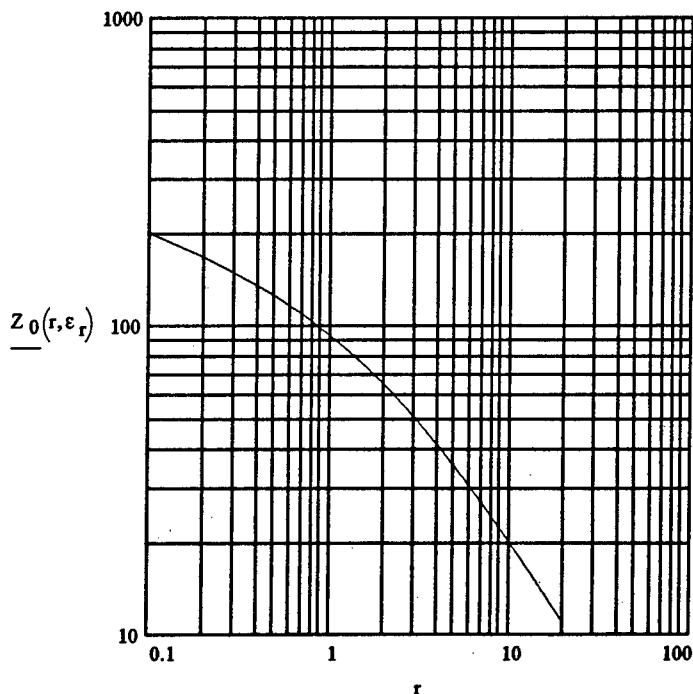
Effective dielectric constant:

$$\epsilon_e(r, \epsilon_r) := \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \cdot \left[ \frac{1}{\sqrt{1 + \frac{12}{r}}} + a(r) \cdot 0.041 \cdot (1 - r)^2 \right]$$

**Impedance** [formulas from Pozar<sup>3</sup> (p. 183)]

$$Z_0(r, \epsilon_r) := \frac{60}{\sqrt{\epsilon_e(r, \epsilon_r)}} \cdot \left[ a(r) \cdot \ln\left(\frac{8}{r} + \frac{r}{4}\right) + (1 - a(r)) \cdot \frac{2 \cdot \pi}{(r + 1.393 + 0.667 \cdot \ln(r + 1.444))} \right]$$

Dielectric constant :  $\epsilon_r := 2.33$



## 2. Width (W) of the Microstrip line

Calculate "r" (=W/d) given  $\epsilon_r$  and  $Z_0$  (Pozaar p. 185).

$$A(Z_0) := \frac{Z_0}{60} \cdot \sqrt{\frac{\epsilon_r + 1}{2}} + \frac{\epsilon_r - 1}{\epsilon_r + 1} \cdot \left(0.23 + \frac{0.11}{\epsilon_r}\right)$$

$$B(Z_0) := \frac{377 \cdot \pi}{2 \cdot Z_0 \cdot \sqrt{\epsilon_r}}$$

Pozaar gives the following two equations for r depending on whether  $r > 2$  or  $r < 2$ .

$$r_1(Z_0) := \frac{8 \cdot e^{A(Z_0)}}{e^{2 \cdot A(Z_0)} - 2} \quad \text{for } r < 2.$$

$$r_2(Z_0) := \frac{2}{\pi} \left[ B(Z_0) - 1 - \ln(2 \cdot B(Z_0) - 1) + \frac{\epsilon_r - 1}{2 \cdot \epsilon_r} \cdot \left( \ln(B(Z_0) - 1) + 0.39 - \frac{0.61}{\epsilon_r} \right) \right] \quad \text{for } r > 2.$$

The following equation for  $r = W/d$  selects either  $r_1$  or  $r_2$ , depending on whether  $r_1 < 2$  or  $r_1 > 2$

$$r(Z_0) := \text{if}(r_1(Z_0) \leq 2, r_1(Z_0), r_2(Z_0))$$

Linewidths for extreme values of  $Z_0$

$$Z_0 := 200$$

$$r(Z_0) = 0.09709$$

$$\text{Let } d := 0.125 \text{ inch}$$

$$d := d \cdot 25.4 \text{ mm}$$

$$W := r(Z_0) \cdot d \quad W = 0.30825 \text{ mm}$$

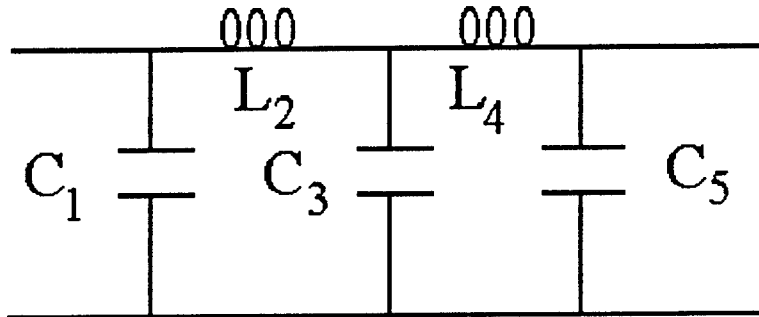
$$Z_0 := 20$$

$$r(Z_0) = 9.95253$$

$$W := r(Z_0) \cdot d \quad W = 31.599 \text{ mm}$$

## APPENDIX D

Optimization for a 5-element low-pass equalizer (pi-section consisting of  $C_1, C_2, C_3, L_1, L_2$ ).  
The following calculation was written using Mathcad 5.0.



### Width (W) of the Microstrip line given the impedance $Z_0$

Dielectric constant:  $\epsilon_r := 2.33$

Thickness of the board (in inches)  $d_{\text{inch}} := 0.125$

Thickness of the board (in mm)  $d := d_{\text{inch}} \cdot 25.4 \quad d = 3.175$

Speed of light:  $c := 3 \cdot 10^{10} \quad \text{cm/sec}$

$W$  = line width(mm);  $d$  = thickness of dielectric (mm);  $r = W/d$

Calculate "r" ( $=W/d$ ) as a function of  $Z_0$  given  $\epsilon_r$  (Poazar p. 185).

$$A(Z_0) := \frac{Z_0}{60} \cdot \sqrt{\frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{\epsilon_r + 1} \left( 0.23 + \frac{0.11}{\epsilon_r} \right)}$$

$$B(Z_0) := \frac{377 \cdot \pi}{2 \cdot Z_0 \cdot \sqrt{\epsilon_r}}$$

Poazar gives the following two equations for  $r$  depending on whether  $r > 2$  or  $r < 2$ .

$$r_1(Z_0) := \frac{8 \cdot e^{A(Z_0)}}{e^{2 \cdot A(Z_0)} - 2} \quad \text{for } r < 2.$$

$$r_2(Z_0) := \frac{2}{\pi} \left[ B(Z_0) - 1 - \ln(2 \cdot B(Z_0) - 1) + \frac{\epsilon_r - 1}{2 \cdot \epsilon_r} \left( \ln(B(Z_0) - 1) + 0.39 - \frac{0.61}{\epsilon_r} \right) \right] \text{ for } r > 2.$$

The following equation for  $r = W/d$  selects either  $r_1$  or  $r_2$ , depending on whether  $r_1 < 2$  or  $r_1 > 2$

$$r(Z_0) := \text{if}(r_1(Z_0) \leq 2, r_1(Z_0), r_2(Z_0))$$

Calculate width of high impedance, low impedance, and 50  $\Omega$  line

High impedance:  $Z_H := 200$  (inductors)

W/d ratio:  $r(Z_H) = 0.09709$

Define  $W_H$   $W_H := r(Z_H) \cdot d$

Line width for  $Z_H$   $W_H = 0.30825$  mm

Low impedance:  $Z_L := 20$  (capacitors)

W/d ratio:  $r(Z_L) = 9.95253$

Define  $W_L$   $W_L := r(Z_L) \cdot d$

Line width for  $Z_L$   $W_L = 31.599$  mm

Characteristic line impedance:  $Z_0 := 50$

W/d ratio:  $r(Z_0) = 2.97028$

Define  $W_0$   $W_0 := r(Z_0) \cdot d$

Line width for  $Z_0$   $W_0 = 9.431$  mm

Effective dielectric constant and  $\beta$  for  $Z_H$  and  $Z_L$ .

Define  $a(r)$  such that:

$a(r) = 1$  if  $0 \leq r \leq 1$

$a(r) = 0$  otherwise

$a(r) := \text{if}(r \leq 1, 1, 0)$

Effective dielectric constant:

$$\epsilon_e(r) := \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \cdot \left[ \frac{1}{\sqrt{1 + \frac{12}{r}}} + a(r) \cdot 0.041 \cdot (1 - r)^2 \right]$$

High impedance:

$\epsilon_H := \epsilon_e(r(Z_H))$

$\epsilon_H = 1.747$

Phase velocity:

$u_{pH} := \frac{c}{\sqrt{\epsilon_H}}$

$u_{pH} = 2.27 \cdot 10^{10}$

Low impedance:  $\epsilon_L := \epsilon_e(r(Z_L))$

$$\epsilon_L = 2.113$$

Phase velocity:  $u_{pL} := \frac{c}{\sqrt{\epsilon_L}} \quad u_{pL} = 2.064 \cdot 10^{10}$

Read from disk the data file for the laser diode. Specify the frequency  $f_i$  as the 1<sup>st</sup> column.

P := READPRN(comb\_sht)

f := P<1>

w := 2 · π · f

n := length(w)

n = 11

i := 1..n

Impedances:  $Z_0 = Z_6 = I/O$  sections;  $Z_1 = Z_3 = Z_5 =$  capacitors;  $Z_2 = Z_4 =$  inductor

$$Z_1 := Z_L \quad Z_2 := Z_H \quad Z_3 := Z_L \quad Z_4 := Z_H \quad Z_5 := Z_L \quad Z_6 := Z_0$$

Propagation constants in the High and Low impedance materials ( $\beta$ )

$$\beta_H := \frac{2 \cdot \pi}{u_{pH}} \cdot f \quad (\text{Inductors})$$

$$\beta_L := \frac{2 \cdot \pi}{u_{pL}} \cdot f \quad (\text{Capacitors})$$

ABCD elements for the capacitors (1,3 & 5), and the inductors (2, 4)

$$\begin{aligned}
 A_1(l_1) &:= \overrightarrow{\cos(\beta_L \cdot l_1)} & A_3(l_3) &:= \overrightarrow{\cos(\beta_L \cdot l_3)} & A_5(l_5) &:= \overrightarrow{\cos(\beta_L \cdot l_5)} \\
 B_1(l_1) &:= j \cdot Z_1 \cdot \overrightarrow{\sin(\beta_L \cdot l_1)} & B_3(l_3) &:= j \cdot Z_3 \cdot \overrightarrow{\sin(\beta_L \cdot l_3)} & B_5(l_5) &:= j \cdot Z_5 \cdot \overrightarrow{\sin(\beta_L \cdot l_5)} \\
 C_1(l_1) &:= \frac{j \cdot \overrightarrow{\sin(\beta_L \cdot l_1)}}{Z_1} & C_3(l_3) &:= \frac{j \cdot \overrightarrow{\sin(\beta_L \cdot l_3)}}{Z_3} & C_5(l_5) &:= \frac{j \cdot \overrightarrow{\sin(\beta_L \cdot l_5)}}{Z_5} \\
 D_1(l_1) &:= \overrightarrow{\cos(\beta_L \cdot l_1)} & D_3(l_3) &:= \overrightarrow{\cos(\beta_L \cdot l_3)} & D_5(l_5) &:= \overrightarrow{\cos(\beta_L \cdot l_5)}
 \end{aligned}$$

$$\begin{aligned}
 A_2(l_2) &:= \overrightarrow{\cos(\beta_H \cdot l_2)} & A_4(l_4) &:= \overrightarrow{\cos(\beta_H \cdot l_4)} \\
 B_2(l_2) &:= j \cdot Z_2 \cdot \overrightarrow{\sin(\beta_H \cdot l_2)} & B_4(l_4) &:= j \cdot Z_4 \cdot \overrightarrow{\sin(\beta_H \cdot l_4)} \\
 C_2(l_2) &:= \frac{j \cdot \overrightarrow{\sin(\beta_H \cdot l_2)}}{Z_2} & C_4(l_4) &:= \frac{j \cdot \overrightarrow{\sin(\beta_H \cdot l_4)}}{Z_4} \\
 D_2(l_2) &:= \overrightarrow{\cos(\beta_H \cdot l_2)} & D_4(l_4) &:= \overrightarrow{\cos(\beta_H \cdot l_4)}
 \end{aligned}$$

ABCD matrices for the capacitors (1,3 & 5), and the inductors (2, 4)

$$\begin{aligned}
 M_1(l_1, i) &:= \begin{pmatrix} A_1(l_1)_i & B_1(l_1)_i \\ C_1(l_1)_i & D_1(l_1)_i \end{pmatrix} & M_2(l_2, i) &:= \begin{pmatrix} A_2(l_2)_i & B_2(l_2)_i \\ C_2(l_2)_i & D_2(l_2)_i \end{pmatrix} & M_3(l_3, i) &:= \begin{pmatrix} A_3(l_3)_i & B_3(l_3)_i \\ C_3(l_3)_i & D_3(l_3)_i \end{pmatrix} \\
 M_4(l_4, i) &:= \begin{pmatrix} A_4(l_4)_i & B_4(l_4)_i \\ C_4(l_4)_i & D_4(l_4)_i \end{pmatrix} & M_5(l_5, i) &:= \begin{pmatrix} A_5(l_5)_i & B_5(l_5)_i \\ C_5(l_5)_i & D_5(l_5)_i \end{pmatrix}
 \end{aligned}$$

### ABCD matrix for the system

$$M(1_1, 1_2, 1_3, 1_4, 1_5, i) := M_1(1_1, i) \cdot M_2(1_2, i) \cdot M_3(1_3, i) \cdot M_4(1_4, i) \cdot M_5(1_5, i)$$

$$A(1_1, 1_2, 1_3, 1_4, 1_5, i) := M(1_1, 1_2, 1_3, 1_4, 1_5, i)_{1,1} \quad B(1_1, 1_2, 1_3, 1_4, 1_5, i) := M(1_1, 1_2, 1_3, 1_4, 1_5, i)_{1,2}$$

$$C(1_1, 1_2, 1_3, 1_4, 1_5, i) := M(1_1, 1_2, 1_3, 1_4, 1_5, i)_{2,1} \quad D(1_1, 1_2, 1_3, 1_4, 1_5, i) := M(1_1, 1_2, 1_3, 1_4, 1_5, i)_{2,2}$$

### Scattering matrix elements for the equalizer from the system ABCD matrix

$$K(1_1, 1_2, 1_3, 1_4, 1_5, i) := A(1_1, 1_2, 1_3, 1_4, 1_5, i) + \frac{B(1_1, 1_2, 1_3, 1_4, 1_5, i)}{Z_0} \dots \\ + C(1_1, 1_2, 1_3, 1_4, 1_5, i) \cdot Z_0 + D(1_1, 1_2, 1_3, 1_4, 1_5, i)$$

$$S_{11}(1_1, 1_2, 1_3, 1_4, 1_5, i) := \frac{A(1_1, 1_2, 1_3, 1_4, 1_5, i) + \frac{B(1_1, 1_2, 1_3, 1_4, 1_5, i)}{Z_0}}{K(1_1, 1_2, 1_3, 1_4, 1_5, i)} \dots \\ + \frac{-C(1_1, 1_2, 1_3, 1_4, 1_5, i) \cdot Z_0 - D(1_1, 1_2, 1_3, 1_4, 1_5, i)}{K(1_1, 1_2, 1_3, 1_4, 1_5, i)}$$

$$S_{12}(1_1, 1_2, 1_3, 1_4, 1_5, i) := \frac{2}{K(1_1, 1_2, 1_3, 1_4, 1_5, i)}$$

$$S_{21}(1_1, 1_2, 1_3, 1_4, 1_5, i) := \frac{2}{K(1_1, 1_2, 1_3, 1_4, 1_5, i)}$$

$$S_{22}(1_1, 1_2, 1_3, 1_4, 1_5, i) := \frac{-A(1_1, 1_2, 1_3, 1_4, 1_5, i) + \frac{B(1_1, 1_2, 1_3, 1_4, 1_5, i)}{Z_0}}{K(1_1, 1_2, 1_3, 1_4, 1_5, i)} \dots \\ + \frac{-C(1_1, 1_2, 1_3, 1_4, 1_5, i) \cdot Z_0 + D(1_1, 1_2, 1_3, 1_4, 1_5, i)}{K(1_1, 1_2, 1_3, 1_4, 1_5, i)}$$

Read in the scattering matrix (reflection coef) for the laser diode at frequencies  $f_i$

$$S_L := P^{<2>} + j \cdot P^{<3>}$$

$$m := \text{length}(S_L)$$

$$m = 11$$

Scattering matrix (reflection coef) for the generator at frequencies  $f_i$

Impedance of the generator.  $R_g := 50$

$$S_g := \frac{R_g - Z_0}{R_g + Z_0} \quad S_g = 0$$

Transducer power gain

$$S_2(1_1, 1_2, 1_3, 1_4, 1_5, i) := \left( S_{22}(1_1, 1_2, 1_3, 1_4, 1_5, i) + \frac{S_{21}(1_1, 1_2, 1_3, 1_4, 1_5, i)^2 \cdot S_g}{1 - S_{22}(1_1, 1_2, 1_3, 1_4, 1_5, i) \cdot S_g} \right)$$

$$T(1_1, 1_2, 1_3, 1_4, 1_5, i) := \frac{(|S_{21}(1_1, 1_2, 1_3, 1_4, 1_5, i)|)^2 \cdot [1 - (|S_g|)^2] \cdot [1 - (|S_{L_i}|)^2]}{(|1 - S_{11}(1_1, 1_2, 1_3, 1_4, 1_5, i) \cdot S_g|)^2 \cdot (|1 - S_2(1_1, 1_2, 1_3, 1_4, 1_5, i) \cdot S_{L_i}|)^2}$$

**Calculate the component values ( $C_1$ , etc.) that maximize the transducer power gain  $T$  at the frequencies  $f_i$  (in a least-squares sense).**

Set the idealized value for  $T$  (this is arbitrary)

$$T_I := 0.30$$

Compute the sum of the squares, which is to be minimized

$$\text{SSE}(1_1, 1_2, 1_3, 1_4, 1_5) := \sum_{i=1}^n (T(1_1, 1_2, 1_3, 1_4, 1_5, i) - T_I)^2$$

Initial guess for parameters

$$l_1 = 0.675 \quad l_2 = 0.272 \quad l_3 = 0.619 \quad l_4 = 0.523 \quad l_5 = 1.583$$

Set up a solve block

Given

$$\text{SSE}(l_1, l_2, l_3, l_4, l_5) = 0$$

Dummy equations: (Mathcad needs 5 equations for 5 unknowns)

$$l_1 = 1 \quad l_2 = 2 \quad l_3 = 3 \quad l_4 = 4$$

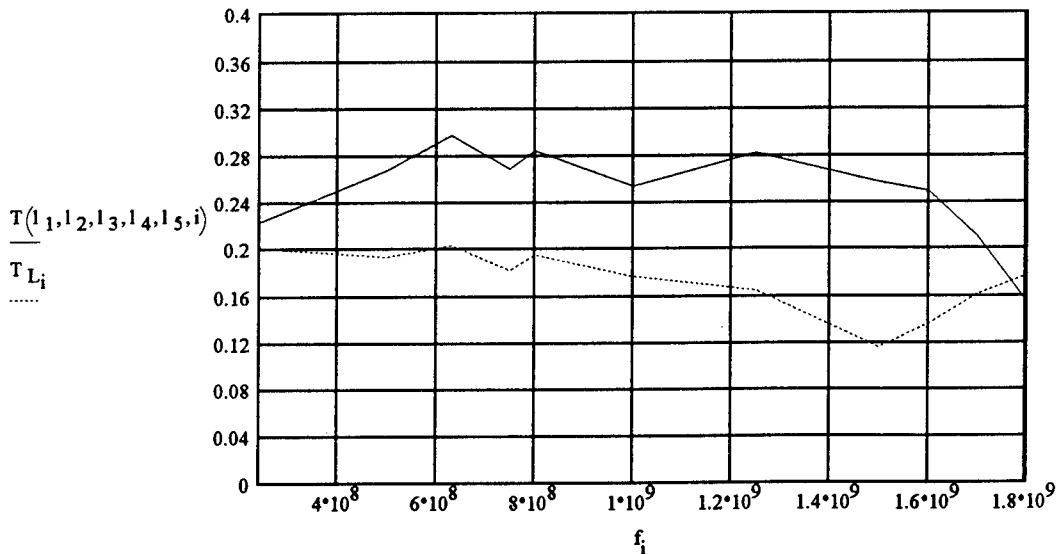
Minerr function gives best fit to data

$$\begin{bmatrix} l_1 \\ l_2 \\ l_3 \\ l_4 \\ l_5 \end{bmatrix} := \text{Minerr}(l_1, l_2, l_3, l_4, l_5) \quad \begin{bmatrix} l_1 \\ l_2 \\ l_3 \\ l_4 \\ l_5 \end{bmatrix} = \begin{bmatrix} 0.67491 \\ 0.27238 \\ 0.61918 \\ 0.52281 \\ 1.58304 \end{bmatrix}$$

Calculate the transducer power gain for the laser diode without the equalizer and the mean square error.

$$T_L := \overline{1 - (|S_L|)^2} \quad \frac{\text{SSE}(l_1, l_2, l_3, l_4, l_5)}{n - 2} = 0.00499$$

Plot the transducer power gain with and without the equalizer.



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