REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
1/29/96

3. REPORT TYPE AND DATES COVERED
Final Report, 12/01/89 - 01/31/94

4. TITLE AND SUBTITLE
Detection and classifcation of signals and noise with long memory

5. FUNDING NUMBERS
ONR N00014-90-J-1287

6. AUTHOR(S)
Murad S. Taqqu and Gennady Samorodnitsky

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Boston University
Department of Mathematics
Boston, MA 02215

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Office of Naval Research
Boston Regional Office
495 Summer Street, Room 103
Boston, MA 02210-2109

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for Public Release;
Distribution is Unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

Long memory occurs when low frequencies have a fundamental impact on the dependence structure of the data. There may also be high variability which occurs when the data has fat distribution tails. Methods were developed for the detection and classification of signals with such characteristics.

Some of these techniques were applied to the analysis of computer traffic. The corresponding article, authored by Leland, Taqqu, Willinger and Wilson, was reprinted a number of times. Its extended version has received the 1995 William J. Bennett Award from the IEEE Communications Society and the 1996 IEEE W.R.G. Baker Prize Award. The Baker Prize Award recognizes "the most outstanding paper reporting original work" in all publications of the IEEE.

14. SUBJECT TERMS
Long-range dependence, 1/f noise, stable distributions, networks, self-similarity, infinite variance

15. NUMBER OF PAGES
13

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT

GENERAL INSTRUCTIONS FOR COMPLETING SF 298

The Report Documentation Page (RDP) is used in announcing and cataloging reports. It is important that this information be consistent with the rest of the report, particularly the cover and title page. Instructions for filling in each block of the form follow. It is important to ***stay within the lines*** to meet ***optical scanning requirements***.

Block 1. Agency Use Only *(Leave blank)*.

Block 2. Report Date. Full publication date including day, month, and year, if available (e.g. 1 Jan 88). Must cite at least the year.

Block 3. Type of Report and Dates Covered. State whether report is interim, final, etc. If applicable, enter inclusive report dates (e.g. 10 Jun 87 - 30 Jun 88).

Block 4. Title and Subtitle. A title is taken from the part of the report that provides the most meaningful and complete information. When a report is prepared in more than one volume, repeat the primary title, add volume number, and include subtitle for the specific volume. On classified documents enter the title classification in parentheses.

Block 5. Funding Numbers. To include contract and grant numbers; may include program element number(s), project number(s), task number(s), and work unit number(s). Use the following labels:

C -	Contract	**PR** -	Project
G -	Grant	**TA** -	Task
PE -	Program Element	**WU** -	Work Unit Accession No.

Block 6. Author(s). Name(s) of person(s) responsible for writing the report, performing the research, or credited with the content of the report. If editor or compiler, this should follow the name(s).

Block 7. Performing Organization Name(s) and Address(es). Self-explanatory.

Block 8. Performing Organization Report Number. Enter the unique alphanumeric report number(s) assigned by the organization performing the report.

Block 9. Sponsoring/Monitoring Agency Name(s) and Address(es). Self-explanatory.

Block 10. Sponsoring/Monitoring Agency Report Number. *(If known)*

Block 11. Supplementary Notes. Enter information not included elsewhere such as: Prepared in cooperation with...; Trans. of...; To be published in.... When a report is revised, include a statement whether the new report supersedes or supplements the older report.

Block 12a. Distribution/Availability Statement. Denotes public availability or limitations. Cite any availability to the public. Enter additional limitations or special markings in all capitals (e.g. NOFORN, REL, ITAR).

DOD - See DoDD 5230.24, "Distribution Statements on Technical Documents."
DOE - See authorities.
NASA - See Handbook NHB 2200.2.
NTIS - Leave blank.

Block 12b. Distribution Code.

DOD - Leave blank.
DOE - Enter DOE distribution categories from the Standard Distribution for Unclassified Scientific and Technical Reports.
NASA - Leave blank.
NTIS - Leave blank.

Block 13. Abstract. Include a brief *(Maximum 200 words)* factual summary of the most significant information contained in the report.

Block 14. Subject Terms. Keywords or phrases identifying major subjects in the report.

Block 15. Number of Pages. Enter the total number of pages.

Block 16. Price Code. Enter appropriate price code *(NTIS only)*.

Blocks 17. - 19. Security Classifications. Self-explanatory. Enter U.S. Security Classification in accordance with U.S. Security Regulations (i.e., UNCLASSIFIED). If form contains classified information, stamp classification on the top and bottom of the page.

Block 20. Limitation of Abstract. This block must be completed to assign a limitation to the abstract. Enter either UL (unlimited) or SAR (same as report). An entry in this block is necessary if the abstract is to be limited. If blank, the abstract is assumed to be unlimited.

Boston University

College of Liberal Arts
111 Cummington Street
Boston, Massachusetts 02215
Tel: 617/353-2560
Fax: 617/353-8100

Department of Mathematics

Dr. Neil L. Gerr
Code 1111
Office of Naval Research
800 North Quiny Street
Arlington, Virginia 22217-5000

January 29, 1996

Dear Dr. Gerr:

I am enclosing my final technical report on the N00014-90-J-1287 grant *Detection and Classification of Signals and Noise with Long Memory* with Murad S. Taqqu and Gennady Samorodnitsky as Principal Investigators. It is only now that all submitted articles have been accepted for publication and/or appeared in print. Some of the results that we have obtained were included in our book *Stable Non-Gausian Random Processes.*

Let me also mention that my work in the article Leland, Taqqu, Willinger and Wilson *On the Self-Similar Nature of the Ethernet Traffic (Extended Version)*, which appeared in *IEEE/ACM Transactions on Networking*, has received the 1995 William J. Bennett Award from the IEEE Communications Society and, as I also just learned, the 1996 IEEE W.R.G. Baker Prize Award, as well. The Baker Prize Award "is presented for the most outstanding paper reporting original work in the TRANSACTIONS, JOURNALS and MAGAZINES of the Societies or in the PROCEEDINGS of the IEEE".

I would like to thank ONR for its generous support.

Sincerely yours,

Murad Taqqu
Professor

Cc.

Michelle Tritt, Administrative Grants Officer
Director, Naval Research Laboratory (Code 2627)
Defense Technical Information Center.

Enc.

FINAL TECHNICAL REPORT
ON ONR GRANT N00014-90-J-1287

R&T Number: 4119368—01
Grant Number: ONR N00014-90-J-1287
Grant Title: Detection and Classification of Signals and Noise with Long Memory
Principal Investigators: Murad S. Taqqu and Gennady Samorodnitsky
Mailing Address: Department of Mathematics, 111 Cummington Street,
Boston University, Boston, MA 02215-2411
Phone Number: 617-353-3022
Email address: murad@math.bu.edu
Date: January 29, 1996

This is a summary of the main results obtained under ONR grant N00014-90-J-1287 grant *Detection and Classification of Signals and Noise with Long Memory* with Murad S. Taqqu and Gennady Samorodnitsky as Principal Investigators. Some of the results obtained were included in our book Samorodnitsky and Taqqu [37] on Infinite Variance Stable Processes. The following 38 articles have now all appeared or are scheduled to appear in refereed publications. For ease of exposition the headings below may include the description of more than one article.

The article Leland, Taqqu, Willinger and Wilson [21] on the analysis of computer traffic, was reprinted a number of times. Its extended version [23] has received the 1995 William J. Bennett Award from the IEEE Communications Society and the 1996 IEEE W.R.G. Baker Prize Award. The Baker Prize Award recognizes "the most outstanding paper reporting original work" in all publications of the IEEE.

Analysis of computer network traffic

In the papers Leland, Taqqu, Willinger and Wilson [22], [21], [23], [39], we show how a careful statistical analysis of large sets of actual traffic measurements can reveal new features of network traffic that have gone unnoticed by the literature and yet, seem to have serious implications for predicted network performance. We use hundreds of millions of high-quality traffic measurements from an Ethernet local area network to demonstrate that Ethernet traffic is statistically self-similar, and that this property clearly distinguishes between currently used models for packet traffic and our measured data. We also indicate how such a unique data set (in terms of size and quality) (i) can be used to illustrate a number of different statistical inference methods for self-similar processes, (ii) gives rise to new and challenging problems in statistics, statistical computing and probabilistic modeling, and (iii) opens up new areas of mathematical research in queueing theory and performance analysis of future high-speed networks.

The statistical nature of variable-bit-rate video images

With the recent availability of large data sets of real variable-bit-rate video data (representing 1/2 to 2 hours of actual video), the following question arises naturally: what are the inherent features of variable-bit-rate video images; i.e., features which are independent of scene and compression technique? This question is of particular importance as one tries to move away from models that are highly scene and compression technique dependent toward a more universal description of variable-bit-rate video. In Beran, Sherman, Taqqu and Willinger [4], we analyze approximately 20 sets of actual video data, generated by a variety of different compression techniques and representing a wide range of different scenes. Performing extensive statistical and graphical tests, our main conclusion is that long-range dependence is a predominant characteristic of video images. The Hurst parameter H is one measure of the intensity of long-range dependence. We found that low activity scenes like video-conferencing or video phone have a lower value of H than high activity scenes where there is a lot of motion. These findings challenge the currently proposed models in the literature, all of which concentrate on short-range dependence and are not able to account for the presence of long-range dependence.

Conditional moments of stable random variables

No α-stable random variable has a finite second moment when $\alpha < 2$ and even the first moment does not exist when $\alpha \leq 1$. This makes it nontrivial to extend to the α-stable case $0 < \alpha < 2$, prediction and filtering techniques developed for the Gaussian case $\alpha = 2$. The existence of conditional moments is closely related to a certain integrability property of the spectral measure Γ of the α-stable vector $(X(t), X(s))$. The papers Cioczek-Georges and Taqqu [7], [9] provide detailed conditions for

$$E(|X_2|^p|X_1 = x) < \infty$$

to exist for values of p greater than α. In Cioczek-Georges and Taqqu [8], we study the conditional variance.

Numerical computation of non-linear stable regression functions

In Hardin, Samorodnitsky and Taqqu [11] we derive the explicit formulas for the regression function of one stable random variable upon another. Although the regression may sometimes be linear, it is in general not a linear function. It involves the quotient of two integrals which cannot be computed analytically and must therefore be approximated numerically. The general problem of computing the integrals is fraught with difficulties. In order to allow the practitioner to apply the formulas, we present in Hardin, Samorodnitsky and Taqqu [12], a self-contained exposition of the regression problem and a software package, written in the C language, which overcomes the numerical difficulties and allows the user control over the accuracy of the approximation. The package also allows the user to compute numerically the probability density function of a stable random variable.

Does asymptotic linearity of the regression extend to stable domains of attraction?

While the regression $E[Y|X = x]$ is in general not linear when (X, Y) is a stable random vector, it is asymptotically linear. What is the asymptotic behavior of the regression if the random vector is not stable but in the stable domain of attraction? This question is of importance in applications, where one encounters more often random vectors that belong to the stable domain of attraction than vectors whose distributions are precisely stable.

The paper Cioczek-Georges and Taqqu [6] gives an answer to the question in the special case where the stable random variable X is perturbed by an independent noise. More precisely, we suppose that the vector is $(X + \mathcal{E}, Y)$, where (X, Y) is α-stable and $\mathcal{E}$ is a random variable independent of (X, Y), such that $X + \mathcal{E}$ is in the domain of normal attracaation of X. We show that one cannot generally expect that the regression $E[Y|x + \mathcal{E} = x]$ is asymptotically linear even if (X, Y) is symmetric α-stable. The asymptotic linearity of $E[Y|X + \mathcal{E} = z]$, it turns out, is equivalent to the statement that the density of $X + \mathcal{E}$ is asymptotically the same as the density of X. We provide sufficient conditions for the latter and give a counterexample where the regression is not asymptotically linear.

Linear models with long-range dependence and with finite or infinite variance

The paper Samorodnitsky and Taqqu [34] discusses a number of linear models that display long-range dependence and either finite or infinite variance. Our goal is to show that these models, while different, share many common features. The models include the finite variance fractional ARIMA and fractional Gaussian noise, as well as processes with long-range dependence and infinite variance such as the linear fractional stable noise, the harmonizable stable noise, and the sub-Gaussian and sub-stable processes. All these processes are linear transformations of white noise.

Analyzing the asymptotic dependence structure of stable processes

The covariance function characterizes the dependence structure of a stationary Gaussian process. What can one use when the process $\{X(t), -\infty < t < \infty\}$ is a non-Gaussian infinite variance stable process? Such a process shares many common features with the Gaussian process, but because it has infinite variance, the covariance is not defined. It is possible to use the covariation, but the covariation is not a symmetric function and is not always defined.

We have used instead the function

$$r(t) = Ee^{i(\theta_1 X(t)+\theta_2 X(0))} - Ee^{i\theta_1 X(t)}Ee^{i\theta_2 X(0)},$$

where θ_1 and θ_2 are real constants. The function $r(t)$ is the difference between the joint characteristic function of the process at time 0 and at time t, and the product of the marginal characteristic functions. In the Gaussian case, $r(t)$ is asymptotically proportional to the covariance, if the covariance converges to zero. And, unlike the covariation, $r(t)$ is always defined in the stable case. Hence the function $r(t)$ is a natural replacement of the covariance when t is large and the covariance is not defined.

We investigate the asymptotic behavior of $r(t)$ in a number of papers. In Levy and Taqqu [24], we obtain the asymptotic structures of the moving average, sub-Gaussian and real harmonizable processes. We find that the intensity of the dependence tends to zero in the moving average case but not in the sub-Gaussian and real harmonizable case.

In Kokoszka and Taqqu [14], we investigate the asymptotic dependence structure of a large class of self-similar stable random fields which are extensions of the linear fractional Lévy motion to the parameter space $\mathbf{R}^n$. We show that the intensity of the dependence decreases to zero like a power function as the lag tends to infinity and we obtain the exact expression for the exponent in the power function. The exponent depends on both the stability parameter and the self-similarity parameter.

In Kokoszka and Taqqu [13], we obtain the asymptotic dependence structure of Chentsov-type processes. These processes are derived from random fields. They can be defined geometrically, and as such they can be regarded as extensions of Lévy Brownian motion on $\mathbf{R}^n$.

Our study of $r(t)$ shows that the Chentsov-type processes are different from the fractional Lévy motion, log-fractional stable motion, harmonizable fractional stable process and sub-Gaussian process. These results generalize those of Y. Sato. Our methodology, moreover, is easier to apply. Sato's approach was based on the fact that the spectral measure of two-dimensional distributions of a Chentsov type process is discrete whereas the spectral measure of two-dimensional distributions of the fractional Lévy motion has an absolutely continuous component. Our method is simpler, because, instead of analyzing the whole spectral measure we concentrate on a particular functional of the measure.

Finally, the paper Lee, Rachev and Samorodnitsky [20] describes several additional dependence results for stable random variables, including "association". In the Gaussian case, random variables are associated if and only if their correlations are all non-negative. While correlations are not defined in the stable case, it is possible to exactly characterize the effect of "association".

Infinite variance stable ARMA time series

In most applications of the ARMA model

$$X_n - b_1 X_{n-1} - \ldots - b_p X_{n-p} = \epsilon_n + a_1 \epsilon_{n-1} + \ldots + a_q \epsilon_{n-q},$$

one assumes that the innovations ϵ_n are independent and identically distributed Gaussian random variables or at least have finite second moments. In this paper we study a more general ARMA model by considering the innovations ϵ_n with symmetrical stable distributions. Because the time series will have infinite variance one cannot use the covariances to describe the asymptotic dependence structure. In Kokoszka and Taqqu [15], we show by using the function $r(t)$ defined above, that while the stable ARMA time series have infinite variance, they have nevertheless the same type of asymptotic dependence structure as Gaussian ARMA time series. We show in Kokoszka and Taqqu [17] that these results extend to stable fractional ARIMA time series and in Kokoszka and Taqqu [19] to other infinite variance time series with long-range dependence.

New classes of stable self-similar random fields

The class of stable self-similar processes with stationary increments has been the focus of much study. Very little work exists, however, on stable self-similar random fields, where the time-parameter is multidimensional. The paper Kokoszka and Taqqu [16] focuses on random fields. Two new classes of symmetric stable self-similar random fields with stationary increments are investigated, one of the "moving-average" type, the other of the "harmonizable" type.

Self-similar processes

In Samorodnitsky and Taqqu [32], we first solve an outstanding problem posed by other authors and show that when $0 < \alpha < 1$ the only α-stable self-similar stationary process with index $H = 1/\alpha$ is the one that has independent increments, namely the α-stable Lévy process. Then, considering the cases $1 < \alpha < 2$ we construct *new* α-stable self-similar stationary increment processes with index $H = 1/\alpha$. These are related to random fields. To show that the new processes are different from each other we use a technique we developed, which is based on the conditional distributions.

Characterization of linear and harmonizable fractional stable motions

A self-similar can be transformed into a stationary process by applying a non-random transformation. The paper Cambanis, Maejima and Samorodnitsky [5] characterizes the linear and harmonizable fractional stable motions as the self-similar stable processes with stationary increments whose left-equivalent (or right-equivalent) stationary processes are moving averages and harmonizable respectively.

Maxima and minima of limiting stable stochastic processes

The investigation of functional limit theorems for processes with paths with jumps was started by Skorohod, the Russian mathematician, in 1956. Skorohod, in his seminal paper, introduced four ways to evaluate distances now known respectively as J_1, J_2, M_1, and M_2 and proved that the convergence of sums of i.i.d. random variables to an α-stable Lévy motion, with $0 < \alpha < 2$, holds in the J_1 sense. J_1 is, today, the commonly used distance. We show in Avram and Taqqu [3] that for sums of moving averages with at least two nonzero coefficients, convergence in the J_1 sense *cannot* hold because adjacent jumps of the process may coalesce in the limit; however, if the moving average coefficients are positive, then the adjacent jumps are essentially monotone and one can have convergence in the M_1 sense. Our result is strong enough that is can be used to evaluate limits of maxima and minima of stable stochastic processes.

Slepian inequality

The Slepian inequality and its modifications compares the behavior of the suprema of two Gaussian processes. It is based on properties of the covariances. We have found the corresponding inequality in the stable case, where covariances do not exist. Our inequality, given in Samorodnitsky and Taqqu [35], involves the spectral measure which characterizes the distribution of stable processes. To obtain the inequality, we developed new relations involving stochastic dominance of infinitely divisible processes. In fact, our Slepian inequality is valid not only for stable processes but also for the so-called "G type" processes which extend the stable ones.

In Samorodnitsky and Taqqu [36], we study Slepian inequalities for general non-Gaussian infinitely divisible random vectors. Conditions for such inequalities are expressed in terms of the corresponding Lévy measures of these vectors. These conditions are shown to be nearly best possible, and for a large subfamily of infinitely divisible random vectors, these conditions are necessary and sufficient. As an application we consider symmetric α-stable Ornstein-Uhlenbeck processes and a family of infinitely divisible random vectors introduced by Brown and Rinott.

How linear combinations affect the distribution of stable vectors

We show in Samorodnitsky and Taqqu [33] that if $\mathbf{X} = (X_1, \ldots, X_d)$ is a vector in $\mathbf{R}^d$ and all linear combinations of its components $\sum_{i=1}^{d} C_i X_i$ are random variables with index $\alpha = 1$, then the vector $\mathbf{X}$ is itself stable with index $\alpha = 1$. This settles an outstanding problem of Dudley and Kanter.

Zero-one laws for outputs of non-linear filters

Zero-one laws are basic to the analysis of sample paths of stochastic processes.

The history of zero-one laws for Gaussian processes goes back to Cameron and Graces and to Shepp. They established in 1951 and 1956 respectively, zero-one laws for Wiener processes. Their results have been progressively extended to more general Gaussian processes by Kallianpur and Jain followed by Cambanis and Rajput and Badrikian and Chevet. The first work on zero-one laws for non-Gaussian infinitely divisible laws is that of Dudley and Kanter, who in 1974 have shown that α-stable processes with $0 < \alpha < 2$ satisfy zero-one laws. Their argument indicates that a basic reason for the zero-one dichotomy in the Gaussian and stable case is the linearity property, namely that a linear combination of two independent copies of the process is equivalent in law to an affine transformation of the process. However, Janssen has demonstrated, in 1984, that zero-one laws hold also for many infinitely divisible processes that do not possess necessarily the above linearity property and gave sufficient conditions in terms of Lévy measures that ensure the zero-one dichotomy. The motivation for our paper Rosinski, Samorodnitsky and Taqqu [30] stems from our interest in non-linear filters with Gaussian inputs. We obtained the zero-one laws for non-homogeneous chaos in Gaussian and certain other random variables. Examples include non-linear processes with long-range dependence and processes that arise as limits of U-statistics.

Distributions of subadditive functionals of sample paths of infinitely divisible processes

Subadditive functions include suprema, integrals of paths, oscillation on sets, and many others. The paper Rosinski and Samorodnitsky [29] gives an optimal condition which ensures that, when the process is infinitely divisible, the distribution of a subadditive functional is subexponential. An exact tail behavior for the distributions is also provided, which improves many recent results in this area.

Subexponentiality of the product of independent random variables

Suppose X and Y are independent nonnegative random variables. The asymptotic behavior of $P(XY > t)$, as $t \to \infty$ is investigated in Cline and Samorodnitsky [10], whenever X has a subexponential distribution. Particular attention is given to obtaining sufficient conditions on $P(Y > t)$ for XY to have a subexponential distribution.

Subexponential distributions have been used in branching processes, queuing theory, renewal theory and large deviations.

Stable distributions in high-dimensional spaces

Geometric stable laws and processes have become an object of attention in recent publications dealing with heavy tailed modeling. Many applications require understanding geometric stable laws on infinite dimensional spaces. Rachev and Samorodnitsky [27] study geometric stable laws on Banach spaces and Norvaiša and Samorodnitsky [25] study stable processes on Orlicz spaces.

Integrability of stable processes

Let $\{X(t),\ t \in T\}$ be an α-stable stochastic process with $0 < \alpha < 2$. Samorodnitsky [31] investigates its integrability with respect to some measure ν and obtains necessary and sufficient conditions for $\int_T |X(t)|^p \nu(dt) < \infty$ a.s. where $p > 0$. It is shown that the distribution of the above integral has a power tail behavior, and a Fubini-type theorem is proved which justifies a change of order of integration and stochastic integration with respect to a stable ordinary random measure.

The result has important implications for the computation of Fourier and inverse Fourier transforms of stable processes.

Random recursions

The paper Rachev and Samorodnitsky [28] investigates certain processes defined by random recursions, e.g.,

$$S_n^* = S_{n-1}^* Z_n + Y_n Z_n, \quad n = 1, 2, \ldots$$

where $\{(Y_n, Z_n),\ n \geq 1\}$ is an independent and identically distributed sequence of random vectors. Application of the model include the steady of spoilage, decay, etc. A particular subclass of such random recursions are the so-called ARCH processes, which have become recently popular in time series analysis.

Fractional super Brownian motion

Adler and Samorodnitskky [1] study limits of systems of noninteracting particles undergoing critical branching which follow a self similar spatial motion with stationary increments. The limit processes are of the super and historical process type. In the case in which the underlying motion is that of a fractional Brownian motion, a characterisation of the limit process is obtained as a kind of stochastic integral against the historical process of a Brownian motion defined on the full real line.

Level crossings for stable processes

Let $\{X(t), t \geq 0\}$ be a harmonisable, symmetric, α-stable stochastic process, and $C_u(T)$ the number of times that X crosses the level u during the time interval $[0, T]$. Adler, Samorodnitsky annd Gadrich [2] obtain the precise numerical value of $C := \lim_{u \to \infty} u^\alpha EC_u(T)$. By way of examples, including an explicit evaluation of EC_u for a stationary process and a combination of analytic and Monte Carlo techniques for some others, we show that the asymptotic approximation $EC_u \sim Cu^{-\alpha}$ is remarkably accurate, even for quite low values of the level u. This formula therefore serves, for all practical purposes, as a "Rice formuala" for harmonisable stable processes, and should be as important in the applications of harmonisable stable processes as the original Rice formula was for their Gaussian counterparts.

A characterization of mixing processes of type G

Mixing is a form of asymptotic independence. A stationary Gaussian process is mixing if and only if its covariance function tends to zero as the lag increases to infinity. In Kokoszka and Taqqu [18], we give an analogous characterization for a large class of symmetric infinitely divisible processes, known as processes of type G, whose marginal distributions are variance mixtures of the normal distribution.

Book on infinite variance stable processes

Stable processes, which have attracted growing interest in recent years, have previously not been the single subject of any monograph or comprehensive overview. In the book Samorodnitsky and Taqqu [37] , we hope to make this important branch of probability widely accessible and provide both an introduction and a basic reference text. We include this book in our report because it presents in a systematic fashion some of the results of the articles we mentioned above.

The central limit theorem which offers the fundamental justification for approximate normality points to the importance of the stable distributions: they are the only limiting distributions of normalized sums of independent, identically distributed random variables, and perforce include the Gaussian as distinguished elements. Gaussian distributions and processes have long been well understood and their utility as both stochastic modeling constructs and analytical tools is well-accepted. However, they do not allow for large fluctuations and are thus often inadequate for modeling high variability. Non-Gaussian stable models, on the other hand, do not share such limitations. In general, the upper and lower tails of their marginal distributions decrease like a power function. The rate of decay depends on a number α, which takes a value between 0 and 2. The smaller α, the slower the decay and the heavier the tails. The distributions always have infinite variance and when $\alpha \leq 1$, they have an infinite mean as well.

In the last two or three decades, data with "heavy tails" have been collected in fields as diverse as economics, telecommunications, hydrology and physics of condensed matter, which suggests using non-Gaussian stable processes as possible models. Such models offer the additional merit of flexibility and variety when compared to Gaussian processes. The latter are completely specified by their mean and autocovariance functions, whereas non-Gaussian stable processes command a much richer parameterization. Gaussian distributions, moreover, are always symmetric around their mean; the non-Gaussian stable ones can have an arbitrary degree of skewness.

In this book, we emphasize the *probabilistic* approach over the analytic one. We talk of tails, moments and dependence structures and focus on multivariate properties and sample paths. The book will be useful to a wide spectrum of researchers in probability, applied probability and statistics. Our goal has been to write a very readable text and to keep the wider context in clear perspective.

References

[1] R. J. Adler and G. Samorodnitsky. Super Fractional Brownian motion and related self-similar (super) processes. *The Annals of Probability*, 23:743–766, 1995.

[2] R. J. Adler, G. Samorodnitsky, and T. Gadrich. The expected number of level crossings for stationary, harmonizable, symmetric, stable processes. *The Annals of Applied Probability*, 3:553–575, 1993.

[3] F. Avram and M. S. Taqqu. Weak convergence of sums of moving averages in the α -stable domain of attraction. *The Annals of Probability*, 20:483–503, 1992.

[4] J. Beran, R. Sherman, M. S. Taqqu, and W. Willinger. Long-range dependence in variable-bit-rate video traffic. *IEEE Transactions on Communications*, 43:1566–1579, 1995.

[5] S. Cambanis, M. Maejima, and G. Samorodnitsky. Characterization of linear and harmonizable fractional stable motions. *Stochastic Processes and their Applications*, 42:91–110, 1992.

[6] R. Cioczek-Georges and M. S. Taqqu. Does asymptotic linearity of the regression extend to stable domains of attraction? *Journal of Multivariate Analysis*, 48:70–86, 1994.

[7] R. Cioczek-Georges and M. S. Taqqu. How do conditional moments of stable vectors depend on the spectral measure? *Stochastic Processes and their Applications*, 54:95–111, 1994.

[8] R. Cioczek-Georges and M. S. Taqqu. Form of the conditional variance for symmetric stable random variables. *Statistica Sinica*, 5:351–361, 1995.

[9] R. Cioczek-Georges and M. S. Taqqu. Necessary conditions for the existence of conditional moments of stable random variables. *Stochastic Processes and their Applications*, 56:233–246, 1995.

[10] D. B. H. Cline and G. Samorodnitsky. Subexponentiality of the product of independent random variables. *Stochastic Processes and Their Applications*, 49:75–98, 1994.

[11] C. D. Hardin Jr., G. Samorodnitsky, and M. S. Taqqu. Non-linear regression of stable random variables. *The Annals of Applied Probability*, 1:582–612, 1991.

[12] C. D. Hardin Jr., G. Samorodnitsky, and M. S. Taqqu. Numerical computations of non-linear stable regression functions. In S. Cambanis, G. Samorodnitsky, and M. S. Taqqu, editors, *Stable Processes and Related Topics*, volume 25 of *Progress in Probability*, pages 143–180, Boston, 1991. Birkhäuser.

[13] P. S. Kokoszka and M. S. Taqqu. Asymptotic dependence of stable self-similar processes of Chentsov type. In R. M. Dudley, M. G. Hahn, and J. Kuelbs, editors, *Probability in Banach Spaces, 8: Proceedings of the Eighth International Conference*, pages 152–165, Boston, 1992. Birkhäuser.

[14] P. S. Kokoszka and M. S. Taqqu. Asymptotic dependence of moving average type self-similar stable random fields. *Nagoya Mathematical Journal*, 130:85–100, 1993.

[15] P. S. Kokoszka and M. S. Taqqu. Infinite variance stable ARMA processes. *Journal of Time Series Analysis*, 15:203–220, 1994.

[16] P. S. Kokoszka and M. S. Taqqu. New classes of self-similar symmetric stable random fields. *Journal of Theoretical Probability*, 7:527–549, 1994.

[17] P. S. Kokoszka and M. S. Taqqu. Fractional ARIMA with stable innovations. *Stochastic Processes and their Applications*, 60:19–47, 1995.

[18] P. S. Kokoszka and M. S. Taqqu. A characterization of mixing processes of type G. *The Journal of Theoretical Probability*, 9, 1996. To appear.

[19] P. S. Kokoszka and M. S. Taqqu. Infinite variance stable moving averages with long memory. *Journal of Econometrics*, 73, 1996. To appear.

[20] M. L. T. Lee, S. T. Rachev, and G. Samorodnitsky. Dependence of stable random variables. In M. Shaked and Y.L. Tong, editors, *Proceedings of the Conference on Stochastic Inequalities*, pages 219–234, Hayward, CA, 1993. Institute of Mathematical Statistics.

[21] W. E. Leland, M. S. Taqqu, W. Willinger, and D. V. Wilson. On the self-similar nature of Ethernet traffic. *Computer Communications Review*, 23:183–193, 1993. Proceedings of the ACM/SIGCOMM'93, San Francisco, September 1993. Reprinted in *Trends in Networking – Internet*, the conference book of the Spring 1995 Conference of the National Unix User Group of the Netherlands (NLUUG). Also reprinted in *Computer Communication Review*, **25**, Nb. 1 (1995), 202-212, a special anniversary issue devoted to "Highlights from 25 years of the Computer Communications Review".

[22] W. E. Leland, M. S. Taqqu, W. Willinger, and D. V. Wilson. Statistical analysis of high time-resolution Ethernet LAN traffic measurements. In M. E. Tarter and M. D. Lock, editors, *Statistical Applications of Expanding Computer Facilities*, volume 25, pages 146–155. Interface Foundation of North America, 1993. *Computing Science and Statistics.*

[23] W. E. Leland, M. S. Taqqu, W. Willinger, and D. V. Wilson. On the self-similar nature of Ethernet traffic (Extended version). *IEEE/ACM Transactions on Networking*, 2:1–15, 1994.

[24] J. B. Levy and M. S. Taqqu. A characterization of the asymptotic behavior of stationary stable processes. In S. Cambanis, G. Samorodnitsky, and M. S. Taqqu, editors, *Stable Processes and Related Topics*, volume 25 of *Progress in Probability*, pages 181–198, Boston, 1991. Birkhäuser.

[25] R. Norvaiša and G. Samorodnitsky. Stable processes with sample paths in Orlicz spaces. *Annals of Probability*, 22:1904–1929, 1995.

[26] S. T. Rachev and G. Samorodnitsky. Option pricing formulae for speculative prices modelled by subordinated stochastic processes. *Serdica*, 19:175–190, 1993.

[27] S. T. Rachev and G. Samorodnitsky. Geometric stable distributions in Banach spaces. *Journal of Theoretical Probability*, 7:351–374, 1994.

[28] S. T. Rachev and G. Samorodnitsky. Limit laws for a stochastic process and random recursion arising in probabilistic modelling. *Advances in Applied Probability*, 27:195–202, 1995.

[29] J. Rosiński and G. Samorodnitsky. Distributions of subadditive functionals of sample paths of infinitely divisible processes. *The Annals of Probability*, 21:996–1014, 1993.

[30] J. Rosiński, G. Samorodnitsky, and M. S. Taqqu. Zero-one laws for multilinear forms in Gaussian and other infinitely divisible random variables. *Journal of Multivariate Analysis*, 46(1):61–82, 1993.

[31] G. Samorodnitsky. Integrability of stable processes. *Probability and Mathematical Statistics*, 13:191–204, 1992.

[32] G. Samorodnitsky and M. S. Taqqu. $(1/\alpha)$-self-similar processes with stationary increments. *Journal of Multivariate Analysis*, 35:308–313, 1990.

[33] G. Samorodnitsky and M. S. Taqqu. Probability laws with 1-stable marginals are 1-stable. *The Annals of Probability*, 19:1777–1780, 1991.

[34] G. Samorodnitsky and M. S. Taqqu. Linear models with long-range dependence and finite or infinite variance. In D. Brillinger, P. Caines, J. Geweke, E. Parzen, M. Rosenblatt, and M. S. Taqqu, editors, *New Directions in Time Series Analysis, Part II*, pages 325–340, New York, 1992. IMA Volumes in Mathematics and its Applications, Volume 46, Springer-Verlag.

[35] G. Samorodnitsky and M. S. Taqqu. Stochastic monotonicity and Slepian-type inequalities for infinitely divisible and stable random vectors. *The Annals of Probability*, 21:143–160, 1992.

[36] G. Samorodnitsky and M. S. Taqqu. Lévy measures of infinitely divisible random vectors and Slepian inequalities. *The Annals of Probability*, 22:1930–1956, 1994.

[37] G. Samorodnitsky and M. S. Taqqu. *Stable Non-Gaussian Processes: Stochastic Models with Infinite Variance.* Chapman and Hall, New York, London, 1994.

[38] W. Willinger and M. S. Taqqu. Toward a convergence theory for continuous stochastic securities market models. *Mathematical Finance*, 1:55–99, 1991.

[39] W. Willinger, M. S. Taqqu, W. E. Leland, and V. Wilson. Self-similarity in high-speed packet traffic: analysis and modeling of Ethernet traffic measurements. *Statistical Science*, 10:67–85, 1995.