



# Feedback Control of Nonlinear Systems

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## Final Technical Report

Principal Investigator:

Dennis S. Bernstein  
Department of Aerospace Engineering  
The University of Michigan  
Ann Arbor, MI 48109-2118 (313) 764-3719  
(313) 763-0578 (fax)  
dsbaero@umich.edu

Grant Monitor:

Dr. Marc Jacobs  
Air Force Office of Scientific Research  
Bolling Air Force Base  
Washington, DC 20332

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# 1 Introduction

This AASERT grant was originally linked with the parent grant F49620-92-J-0127 entitled Robust Fixed-Structure Control. That grant ended in November 1994 and the ASSERT grant is now linked with the parent grant F49620-95-1-0019 entitled Robust, Nonlinear Feedback Control.

Funding under this AASERT grant was used primarily to support Mr. Robert Bupp. Mr. Bupp has been supported by this grant since January 1994. He is expected to receive the Ph. D. degree shortly after the close of this grant.

A brief description of the research conducted by Mr. Bupp under the direction of the Principal Investigator is given in the following section. Relevant publications are listed in Section 4.

## 2 Active Emulation of Passive Absorbers for Vibration Suppression

Mr. Bupp's research involved the problem of active vibration suppression by means of nonlinear feedback techniques. The specific objective of this work is to analyze and emulate the operation of passive absorbers to design reliable active controllers. The analysis of the active emulation algorithms requires modern techniques in nonlinear stability analysis and performance assessment.

The main focus of this research has been the control of translational vibration using a rotational actuator [1]. This problem is inherently nonlinear and provides the opportunity to compare the performance of classical absorbers with modern techniques such as integrator backstepping. An experimental testbed based upon this problem has been designed and fabricated and is described in the next section.

This nonlinear control problem, proposed by the Principal Investigator, became an invited session on a benchmark problem for nonlinear control at the 1995 American Control Conference in Seattle, WA [5, 10-14]. Since then, this problem has been accepted as a topic for a special issue of the *International Journal of Robust and Nonlinear Control*, which is due to be published in 1997 [2].

A major theoretical advance resulting from this work is the concept of resetting absorbers. The idea of a resetting absorber is to use an undamped emulated absorber to extract energy from the plant and then reset the absorber's (virtual) energy to zero. This technique efficiently removes the plant's energy without the use of dissipative components. The appropriate mathematical setting for this technique is the theory of impulsive differential equations [15, 16].

A specialized application of the resetting absorber to the double integrator (undamped rigid body) plant was considered in [3, 6]. It was shown that suitable choice of the reset time allows all of the plant's energy to be removed instantaneously as if it had passed through a trap door, hence the name, "virtual trap-door absorber." This approach yields an output-feedback, finite-settling-time controller. A more general development of the theory of virtual resetting absorbers is given in [4,9], where Lyapunov theory is used to guarantee the stability of the closed-loop system when resetting controllers are implemented.

### 3 RTAC Testbed for Experimental Investigation

We have also undertaken an experimental program in support of this research. To do this, we have designed and constructed a rotational actuator involving translational vibration, shown in Figure 1. A practical motivation for this experiment is to explore the feasibility of using conventional rotary motors as actuators for vibration suppression.

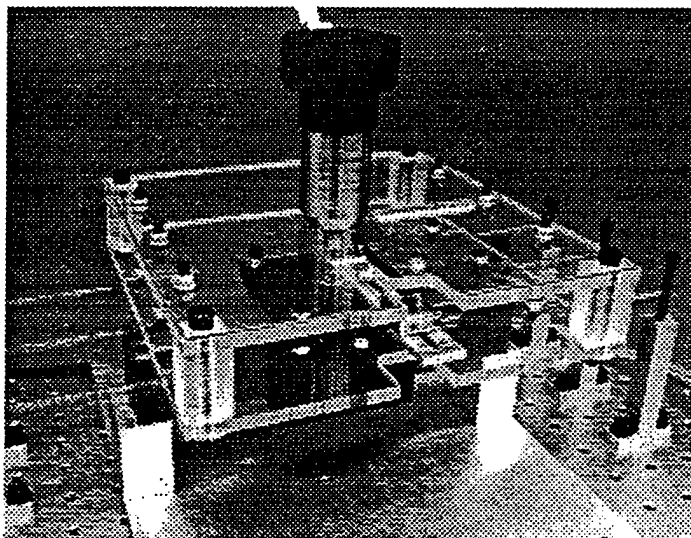


Figure 1: The RTAC Experiment

The experimental testbed involves a linear air slide with 4" of travel upon which is mounted a DC motor with eccentric proof mass. Translational motion is measured with a linear variable displacement transducer (LVDT), while rotational motion is measured with an optical encoder mounted on the motor shaft. Motor torque is commanded by means of a current-regulated amplifier, which in turn is commanded by the real-time control computer.

A variety of control laws have been studied in simulation for this experiment, including feedback linearization, integrator backstepping, and absorber emulation [7, 8]. In hardware experiments, we have implemented and evaluated many controllers, including simple passive compensators, integrator backstepping controllers, and resetting absorbers. These results, which appear in [2, 5], are summarized in Table 1. A typical result for the virtual resetting absorber implemented on the RTAC is given in Figure 2.

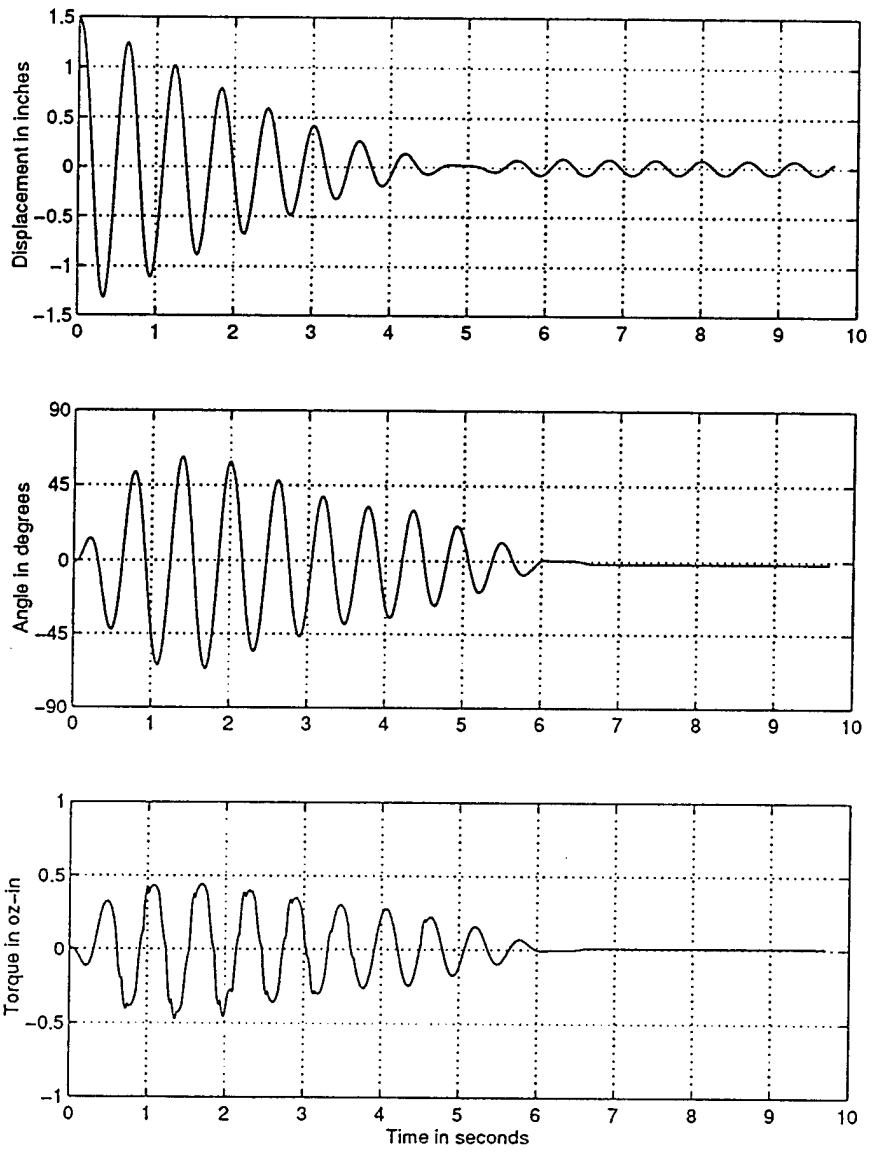


Figure 2: Virtual One-Way Absorber Controller: Response of the RTAC to a 1.5-inch initial displacement.

Controller	$\zeta_{\text{approx}}$	$u_{\text{max}}$
Open Loop	0.44%	—
Coupled Pendula Absorber	3.2%	0.52 oz-in
Integrator Backstepping	3.7%	12.2 oz-in (saturated)
Damped Pendulum Absorber	5.1%	0.47 oz-in
Virtual Resetting Absorber	5.2%	0.44 oz-in

Table 1: Summary of controller performance

## 4 References

### 4.1 Journal Papers

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- [2] R. T. Bupp, D. S. Bernstein, and V. T. Coppola, "Experimental Implementation of Integrator Backstepping and Passive Nonlinear Controllers on the RTAC Testbed," submitted to *Int. J. Robust and Nonlin. Contr.*
- [3] R. T. Bupp, D. S. Bernstein, V.-S. Chellaboina, and W. M. Haddad, "Finite Settling Time Control of the Double Integrator Using a Virtual Trap-Door Absorber," submitted to *Proc. IEEE Trans. Aut. Contr.*
- [4] R. T. Bupp, D. S. Bernstein, V.-S. Chellaboina, and W. M. Haddad. Resetting virtual absorbers for control. In preparation.

### 4.2 Conference Papers

- [5] R. T. Bupp, D. S. Bernstein, and V. T. Coppola, "A Benchmark Problem for Nonlinear Control Design: Problem Statement, Experimental Testbed, and Passive Nonlinear Compensation," *Proc. Amer. Contr. Conf.*, pp. 4363–4367, Seattle, WA, June 1995.
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- [7] R. T. Bupp, D. S. Bernstein, V.-S. Chellaboina, and W. M. Haddad, "Finite Settling Time Control of the Double Integrator Using a Virtual Trap-Door Absorber," *Proc. IEEE Conf. Contr. Appl.*, Dearborn, MI, September 1996.
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### 4.3 Other References

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