THE OSCILLATORY BEHAVIOR OF A

TWO-PHASE NATURAL CIRCULATION LOOP

Eugene H. Wissler, H. S. Isbin, and N. R. Amundson

University of Minnesota, Minneapolis, Minnesota

Contributed by

American Institute of Chemical Engineers

THE OSCILLATORY BEHAVIOR OF A
TWO-PHASE NATURAL CIRCULATION LOOP

by

Eugene H. Wissler, H. S. Isbin, and N. R. Amundson
Department of Chemical Engineering, University of
Minnesota, Minneapolis, Minnesota.

ABSTRACT

A natural circulation loop with water as the
circulating fluid was studied for a range of operation
covering two-phase flow. Periodic oscillations of the
flow rate and fluid temperature occur even with constant
heatinput and constant cooling water properties for the
heat exchangers. Several conclusions concerning the
stability of operation are given. Use is made of the
theoretical analysis of an open-ended system, and an
analogue computer. For use in a more detailed numerical
analysis, the equation of motion, the continuity equation,
and the energy equation are presented for a transient two-
phase flow model.

The Oscillatory Behavior of a
Two-Phase Natural Circulation Loop

by

Eugene H. Wissler, H. S. Isbin, and N. R. Amundson

Introduction

A program of study of the transient operation of
natural circulation loops has been underway at the University
of Minnesota (1), and this paper is concerned with the
oscillatory behavior of a two-phase natural circulation
loop. These studies are of interest for the emergency
cooling of nuclear reactors and in the design of boiling
water reactors. The literature survey pertaining to the
transient operation of a natural circulation loop is
given by Alstad, Isbin, Amundson and Silvers (1), and a
survey on two-phase flow is given by Isbin, Moen and Mosher (2).

Experimental Loop

Figure 1 is a schematic diagram of the natural
convection loop which was studied. The loop was constructed
primarily of 16 gage, 1 inch O.D. (0.872 inch I.D.) hard
drawn brass tubing. The major features of the loop are
described in reference (1). During a natural circulation
run, the flowrators were by-passed and only the electro-
magnetic flowmeter was used. Normally the surge tank,

MINNESOTA NATURAL
CIRCULATION LOOP NO.3

SCALE : 1" = ~40"

KEY :

A   HEATERS
B   PRESSURE DROP TEST SECTION
C   HEAT EXCHANGER
D   FLOW METERS
E   SURGE TANKS
F   PUMP
G   ELECTROMAGNETIC METER
H   COOLANT FLOW METER
J   COLD WATER
K   COOLANT OUTLET
O   OVERFLOW
P   PRESSURE TAPS
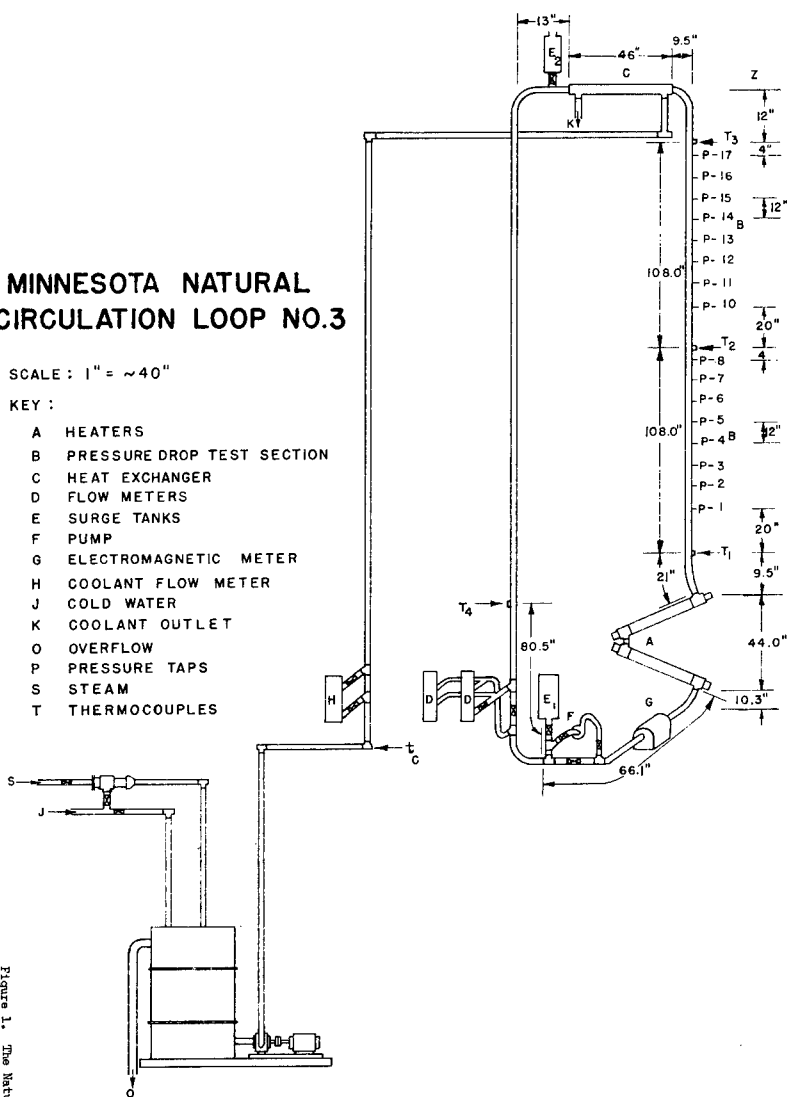S   STEAM
T   THERMOCOUPLES

Figure 1. The Natural Convection Loop.

E_1, was not used for two-phase natural circulation runs.
For those runs in which the pressure at one point in the
loop was held constant, the gate valve between the surge
tank, E_2, and the loop was opened; for the constant volume
run, the gate valve was closed. A heater and pump were
installed to maintain the cooling water supply at 5 gpm and
at 130°F.

Theoretical Analysis

General Equations

The continuity equation, the equation
of motion and the energy equation for
a viscous fluid flowing in a region of general geometry
were formulated for a phase having continuous properties.
A similar set of equations were derived for flow across a
surface of discontinuity. The combination of these two
sets of equations permits one to write the equations for
the two-phase flow. An annular flow model was selected
to illustrate some of the essential properties of two-phase
flow. The model is sufficiently simple to permit attempting
a numerical solution. Each phase is assumed to flow
through a well defined cross-sectional area with a uniform
velocity with the reservation that the liquid velocity at
the wall must equal zero. Further, the steam and water
phases are assumed to be in equilibrium; that is, p and
T correspond to the saturation pressure and temperature.

For these conditions, the continuity equation may be written as

$$A \frac{\partial \rho}{\partial t} = - \frac{\partial W}{\partial x} \tag{1}$$

The equation of motion for upward flow through a pipe of constant diameter becomes

$$\frac{1}{g_c} \frac{\partial W}{\partial t} + \frac{1}{g_c} \frac{\partial (W\phi_1 V)}{\partial x} = - A \frac{\partial \rho}{\partial x} - A \frac{g}{g_c} \rho \, z' - A \, F_f \tag{2}$$

Finally, the energy equation is

$$A \frac{\partial (\rho H)}{\partial t} + \frac{\partial (W\phi_2 H)}{\partial x} - \frac{A}{J} \frac{\partial \rho}{\partial t} = Q' + \frac{\partial}{\partial x} [(k_w A_w + k_s A_s) \frac{\partial T}{\partial x}] \tag{3}$$

where $\quad 2\pi r_p k_w \left(\frac{\partial T}{\partial r}\right)_{r_p} = Q' =$ rate at which heat is conducted through the pipe.

The terms ϕ_1 and ϕ_2 represent the ratio of the true rate of transport to the rate of transport of the mean flow for momentum and energy respectively. If both phases have the same linear velocity, ϕ_1 and ϕ_2 are unity.

The three functions ϕ_1, ϕ_2 and F_f were determined experimentally from steady state data. It was found that ϕ_1 and ϕ_2 could be correlated as functions of ρ alone, and that

$$F_f = a(\rho) W^{1.79} \tag{4}$$

where $a(\rho)$ is a function of ρ. Figures 2, 3 and 4 illustrate the variation of ϕ_1, ϕ_2 and $a(\rho)$ with ρ.
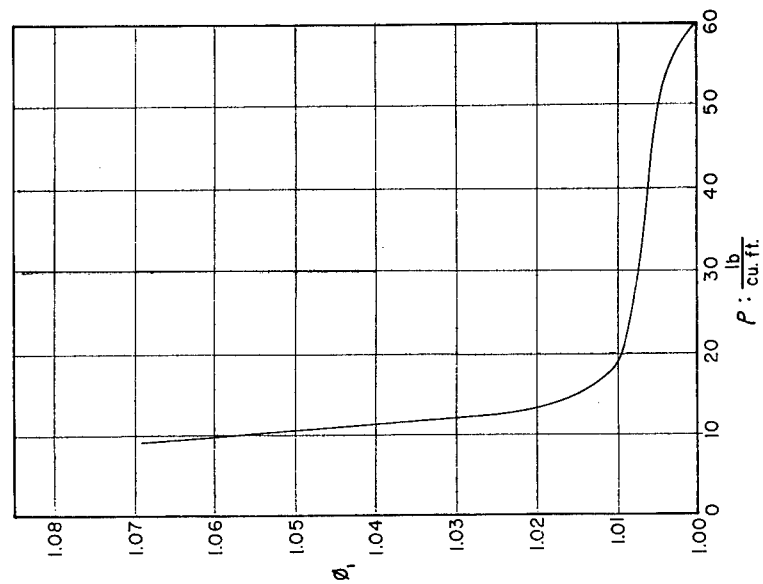
Figure 2. The Momentum Transport Coefficient, φ_1.
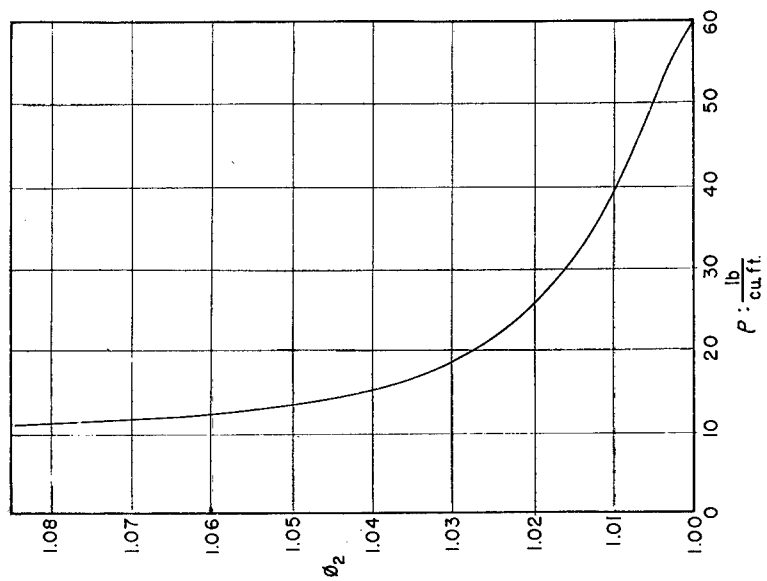

Figure 3. The Energy Transport Coefficient, φ_2.

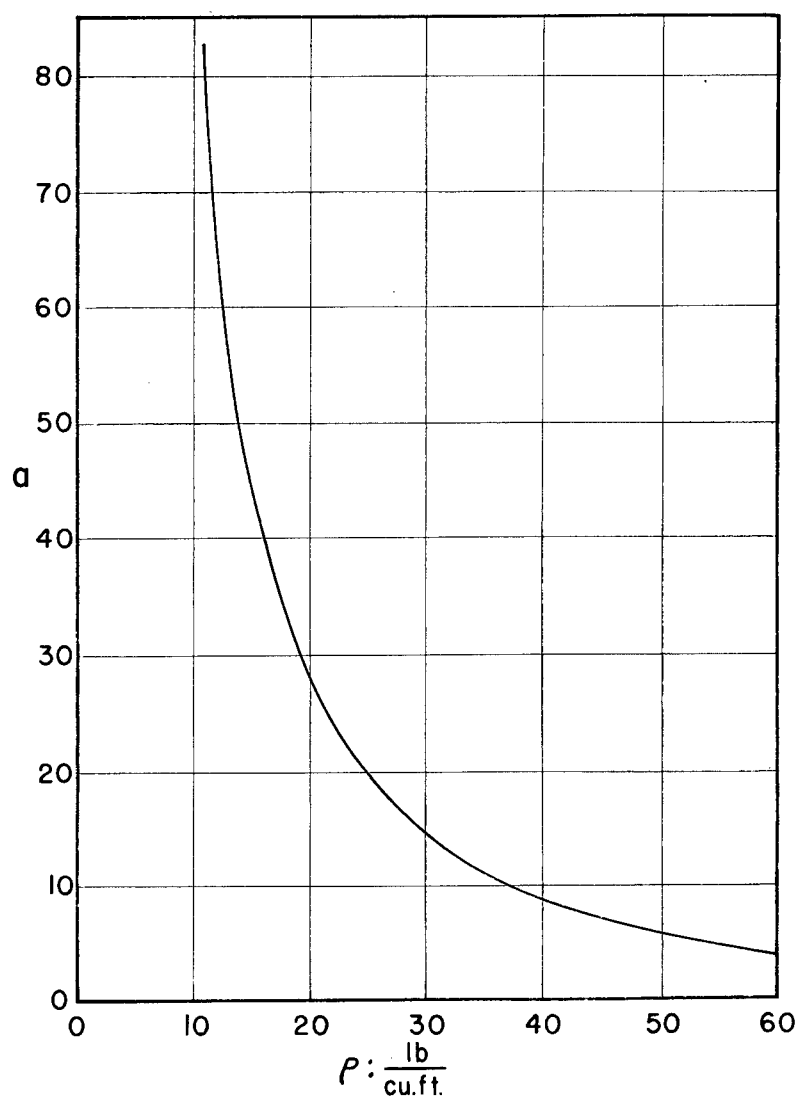Figure 4. The Friction Coefficient, a.

Equations (1) through (4) have been applied to a natural

circulation loop in the form of finite difference equations.

The discussion of these equations is not included in this

paper for the numerical calculations using the SEAC

(National Bureau of Standard's digital computer, Standard

Eastern Automatic Computer) have not been successfully

completed.

Stability Analyses

An insight on the factors which
determine the stability of a

natural convection system is gained through the analysis

of an open-ended loop, such as shown in Figure 5. The

fluid entering the heater always has the constant temperature

T_{o1}, and the velocity of the stream is fixed by the density

difference between the hot and cold leg. For any constant

heat input, one may define a state of equilibrium in which

the difference in weight of the two legs is just equal to

the frictional resistance to flow. Under certain conditions

this system may be unstable; that is a small deviation

from the equilibrium temperature distribution or the

equilibrium velocity may be propagated in space or time

with increasing amplitude. For example, if a velocity

perturbation of the form, $\epsilon_v \sin \omega t$, is present, under

what conditions could the temperature perturbation which is

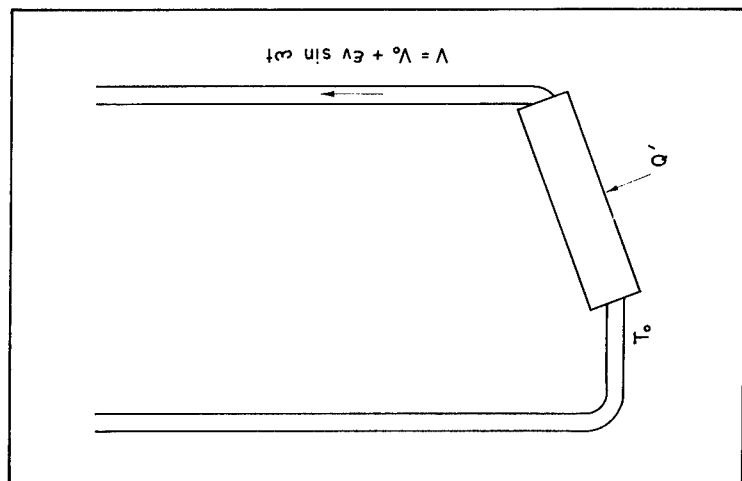generated cause the driving force that sustains the velocity

$V = V_0 + Ev \sin \omega t$

Q'

T_o

Figure 5.   The Stability Model
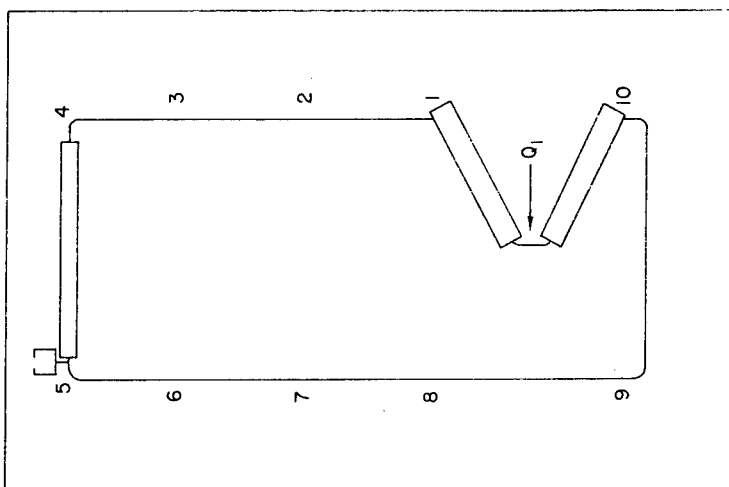
4 3 2 1 10

Q_1

5 6 7 8 9

Figure 6.   Subdivisions for the Analogue Computer Problem.

disturbance? The problem has been treated analytically
for a one-phase fluid (3), and the following conclusions
were obtained:

a) If an oscillatory flow rate is to be possible,
the force cannot be generated in the heater; it must be
generated in the vertical riser.

b) The product of the coefficient of expansion
of the fluid and the vertical height of the riser must
exceed a certain value (defined by an analytical expression)
if the velocity perturbation is to be sustained.

c) The period of oscillation will be approxi-
mately equal to the residence time of the fluid in the
heater and the vertical riser.

In order to predict the period for a closed
loop, a problem was solved on a Reeves Electronic
Analogue Computer. The number of non-linear terms was
limited to 16, and the model used was necessarily a
simple one. For such a problem, the period of the oscilla-
tions should be meaningful even though the wave shape is
not correct, if the period is a function of only the
geometry and the mean velocity. The computer could handle
only ten subdivisions, with the driving force expressed as
a linear function of all ten enthalpies, and the frictional
force as a quadratic function of the velocity. The number

of available summing amplifiers limited the problem to
the case in which boiling occurs only at the top sub-
division of the vertical riser. The equations solved
were of the following form:

$$\frac{dH_n}{dt} = -\frac{V}{(A\Delta x)_n}(H_n - H_{n-1}) + \frac{Q_n}{(A\Delta x)_n \rho C_p} \tag{5}$$

$$\frac{dV}{dt} = \frac{-\Sigma \rho_n z'_n \Delta x_n - F}{\frac{\bar{\rho}_n}{g_c} \Sigma \frac{\Delta x_n}{A_n}} \tag{6}$$

The loop subdivisions are given in Figure 6.
For section 1, $Q_n = Q_1$, for section 5, $Q_5 = -bV(\frac{H_5 + H_4}{2} - h_c)$
where the cooler heat transfer coefficient is taken to be
a linear function of velocity, and all remaining Q's are
taken equal to zero. The density at point n is written as

$$\rho_n = \rho_{n_o} + \alpha_n (H_n - H_{n_o}) \tag{7}$$

and the mean density of the nth subinterval is set equal
to the average of the densities at n and $n-1$.

A stable solution was found if the coefficient
of expansion of water is used for all values of α_n. An
oscillatory solution was obtained, Figure 7, if vaporization
to a few per cent quality were assumed in the top section
of the vertical riser ($\alpha_4 = -9.500$, all other α's =
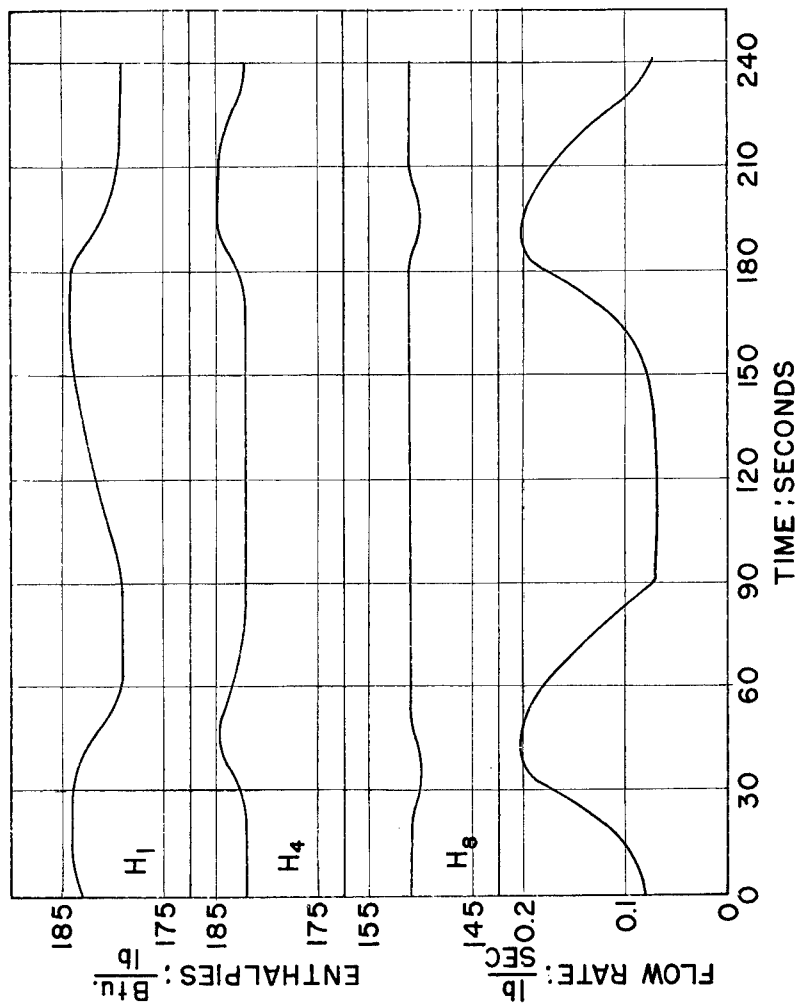-0.024 lb/cu.ft.°F).

Figure 7. Computer of
the Analog Computer Problem.

Results

Two steady equilibrium modes of operation were possible when the pressure at one point in the loop is held constant (surge tank E_2 open to the atmosphere). For a very low heat input the water temperature in the riser never exceeds the boiling point and a state of stable equilibrium may be defined. For a very high heat input, the entire riser contains both steam and water, and a maximum flow rate is obtained. Oscillatory modes of operation result for the intermediate heat input[c]. An illustration of the manner in which the period and amplitude depend on the heat input is given in Figures 8 to 12. The period and amplitude of the oscillations are determined by the mean temperature level of the fluid in the vertical riser.

The period was inversely proportional to the mean velocity providing some steam is in the riser at all times. When the system does not contain steam during most of the cycle, the period is considerably longer than that predicted by the extrapolation of the higher flow rate periods.

In the analogue computer problem, $Q_1 = 3.16$ Btu/sec, and the period was 149 seconds, which is about 21 seconds

less than the experimentally observed value for the lowest

flow rate. The computed period would be expected to be

less than the observed value, since the density function

used in the top subinterval in the riser was a two-phase

density function. Further, the density function used

for the boiling subinterval was linear and it was not

possible to exclude densities greater than the density of

saturated water. A special non-linear element is required

to generate a density function with the correct properties.

As a result, the computed mean flow rate is less than that

which could actually exist; however, the flow rate curve

and enthalpy curves have essentially the same shape as

the experimentally determined ones. A larger computer

is required for solving the stability problem at the higher

heat fluxes.

Conclusions

A natural circulation loop can be made unstable

in the sense that a small displacement from the equilibrium

state leads to undamped oscillations. Stable operations

result when the fluid temperature in the riser is restricted

to values less than the boiling point, and when the heat

input reaches a value such that the frictional force

changes more rapidly than the driving force. The theoreti-
cal treatment of an open-ended natural circulation system
and the solution of a simple problem using an analogue
computer have lent support to the general conclusions
of the stability analyses. A detailed numerical analyses
is in progress on a large digital computer.

Acknowledgments

NOTATION

Equations 1 through 4

A = cross-section area for flow

W = total mass flow rate

x = distance along streamline

t = time

g = local acceleration of gravity; g_c = conversion factor in Newton's law of motion

z' = slope of pipe at x

F_f = wall of frictional force

ρ = fluid density. $\rho \equiv \frac{1}{A}(A_w \rho_w + A_s \rho_s)$

V = volumetric flow rate. $V \cong \frac{A_w \rho_w V_w + A_s \rho_s V_s}{A\rho} = \frac{W}{A\rho}$

H = specific fluid enthalpy. $H \cong \frac{1}{A\rho}[A_w \rho_w H_w + A_s \rho_s H_s]$

$\phi_1 \cong \frac{A_w \rho_w V_w^2 + A_s \rho_s V_s^2}{WV}$; $\phi_2 \cong \frac{A_w \rho_w V_w H_w + A_s \rho_s V_s H_s}{WH}$

k = thermal conductivity

T = fluid temperature

p = fluid pressure

r = radius, r_p = pipe radius

Q' = rate at which heat is conducted through pipe.

$a(\rho)$ = an empirically defined function for Eq.4 for two-phase pressure drop.

Subscripts

s = steam phase; w = liquid water phase

Equations 5 through 7

The energy balance is reduced to a heat balance

H = specific enthalpy

Q = rate at which heat is added to fluid

V = volumetric flow rate

F = frictional force.

$$F = AV^{1.8} \cong F_o + \frac{1.8F_o}{V_o} (V-V_o) + \frac{0.72F_o}{V_o^2} (V-V_o)^2$$

F_o is the frictional force corresponding to a volumetric flow rate V_o.

h_c = cooler heat transfer coefficient

Δx = length of subinterval; z_n = vertical height of subinterval

ρ = density, $\bar{\rho}$ = mean density in a subinterval

C_p = heat capacity

α_n = a constant related to the coefficient of expansion of water and defined by Eq. 7

For the computer problem, for $n = 1, 2,$ and $3,$ H_{n_o} = 182 Btu/lb, ρ_{n_o} = 59.781 lb/cu.ft., α_n = - 0.024 lb/cu.ft.°F; $n = 4,$

$H_{n_o} = 182$, $\rho_{n_o} = 37.500$, $\alpha_n = -9.500$; $n = 5$ through 10,

$H_{n_o} = 153$, $\rho_{n_o} = 60.477$, $\alpha_n = -0.024$. $V_o = 0.0025$ cu.ft./sec,

$Q_1 = 3.16$ Btu/sec, and $Q_5 = -25.0V(\dfrac{H_5 + H_4}{2} - 89.6)$ Btu/sec.

References

1. Alstad, C. D., Isbin, H. S., Amundson, N. R., and Silvers, J.P. ANL - 5409 (in preparation for distribution to AEC depository libraries); A.I.Ch.E.Jr., (accepted for publication).

2. Isbin, H. S., Moen, R. H. and Mosher, D. R., AECU-2994, Nov. 1954.

3. Wissler, Eugene H.: The Transient Behavior of a Two-Phase Natural Circulation Loop, Ph.D. Thesis, Dept. Chem. Eng., Univ. of Minnesota, Minneapolis, Minn. (June, 1955).