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A Double-Sum Technique for Performing a Fourier Transformation on an Integrand Composed of Aliased Factors

by

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Structural acoustics often utilizes Fourier transformation to either reveal a phenomenon that is more directly recognized in one domain than in its Fourier conjugate domain or to employ complementarity in order to decipher a phenomenon. In this paper, a novel computational technique is introduced. This technique advantageously performs this transformation in situations in which specific spatial scales govern aliased factors in the integrand of a Fourier transformation. The technique can factorially save computational steps over corresponding computations that employ discrete Fourier transform procedures.

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I. INTRODUCTION

Structural acoustics deals largely with the description of wave propagations in structures and in the fluids that are in contact with their internal and external surfaces. Models of the structure are constructed so that they facilitate the derivation of these descriptions. A description may be more readily derived when the model is idealized and adapted to accommodate a given Fourier domain. Yet a phenomenon in the description may be, in some cases, more advantageously interpreted in a domain that is a Fourier conjugate to that in which the description is originally derived. Moreover, a complementary description of this kind is itself of considerable benefit in the interpretation of a phenomenon in the description. In this sense, one is often called upon to perform Fourier transformations in structural acoustics. The purpose of this report is to propose a novel computational technique to accomplish a certain class of Fourier transformations. It transpires that a number of phenomena in structural acoustics are featured in this class; in particular, those phenomena that are related to aliasing and to pass and stop bands in regularly ribbed structures [1-11].

A typical Fourier transform in this report is expressed

$$f(x,\omega) = (2\pi)^{-1/2} \int dk \;\; exp(-ixk) \;\; F(k,\omega) \;, \tag{1a}$$

$$F(k,\omega) = (2\pi)^{-1/2} \int dx \;\; exp(ixk) \;\; f(x,\omega) \;, \tag{1b}$$

where the $k-$domain and the $x-$domain are Fourier conjugate domains; (k) and (x) are a Fourier variable pair; and (ω) is a silent variable in these transformations. For these equations to be valid, $F(k,\omega)$ and $f(x,\omega)$ need to be well behaved functions of (k) and (x), respectively, so that convergence can be guaranteed. In this report the concern is mainly focused on Eq. (1a). Using the self-aliased wavenumber operator $S_b(k)$, Eq. (1a) may be cast in the form

$$f(x,\omega) = (2\pi)^{-1/2} \int_o^{\kappa_1} dk \;\; S_b(k) \; \{exp(-ixk) \; F(k,\omega)\} \;;$$

$$\kappa_1 = (2\pi)/(b) \;. \tag{2a}$$

The self-aliased wavenumber operator $S_b(k)$ is defined

$$S_b(k) \; \{N(k,\omega)\} = \sum_q N(k+\kappa_q,\omega) = M(k,\omega) = M(k+\kappa_j,\omega) \;;$$

$$S_b(k) \; \{K(k,\omega) \;\; G(k,\omega)\} = K(k,\omega) \;\; S_b(k) \; \{G(k,\omega)\} \;, \quad \text{if}$$

$$K(k,\omega) = K(k+\kappa_j,\omega) \;, \tag{3}$$

where (b) is an appropriately selected spatial scale factor,

$$\kappa_q = q\kappa_1 \; ; \qquad \kappa_j = j\kappa_1 \; ; \qquad \kappa_1 = (2\pi/b) \; , \tag{4}$$

and $N(k,\omega)$, $K(k,\omega)$, and $G(k,\omega)$ are arbitrary but well behaved functions of the wavenumber (k). The quantities $K(k,\omega)$, by definition, and $M(k,\omega)$, by construction, are aliased; i.e., once $K(k,\omega)$ and $M(k,\omega)$ are specified in a segment that spans the width (κ_1) in the $k-$domain, they are specified, by repetitive extrapolation, throughout that domain. In particular, if $K(k,\omega)$ and $M(k,\omega)$ are known within the segment $0 \leq k < \kappa_1$, they are known throughout the $k-$domain. Equation (1a) is transformed into Eq. (2a) by constructing an integrand that is "aliased in (k) with respect to (κ_1)" and, therefore, the integral over a single segment (κ_1) wide suffices; all the information that is in $\{exp(-ixk)\ F(k,\omega)\}$ over the entire $k-$domain is concentrated in $S_b(k)\ \{exp(-ixk)\ F(k,\omega)\}$ over the segment $0 \leq k < \kappa_1$ of that domain. After the aliased integrand is sampled in this segment, the integration over this segment may be evaluated adaptively or otherwise. Alternatively, the segmental span (κ_1) may be chosen to be (κ_1') which is small enough so that a "mean value theorem" can be relied upon to evaluate the integral in Eq. (2a). At this stage, the alternative procedure is selected for the evaluation of Eq. (2a); namely

$$f(x,\omega) = (2\pi)^{-1/2}\ (\kappa_1')\ S_{b'}(k_o)\ \{exp(-ixk_o)\ F(k_o,\omega)\} \; ;$$

$$\kappa_1' = (2\pi/b') \; ; \qquad k_o = (\alpha\kappa_1') \; ;$$

$$S_{b'}(k)\ \{N(k,\omega)\} = \sum_q (N+\kappa_q',\omega) \; ; \qquad \kappa_q' = q\kappa_1' \; , \tag{2b}$$

where (α) is a positive constant less than unity. [cf. Eqs. (3) and (4).] To define "small enough," the details in the integrand in Eq. (1a) may be conveniently typified by a wavenumber (k_γ) and a loss factor (η_γ). The wavenumber (k_γ) locates the position of the $(\gamma)th$ detail and (η_γ) estimates the extent of that detail in the k-domain. In order to capture all relevant details in this integrand, when Eq. (1a) is converted into Eq. (2b), it is necessary to impose

$$\kappa_1' << (\eta_\gamma k_\gamma) , \tag{5}$$

for all relevant γ's. Indeed, under the condition stated in Eq. (5), and after carrying out the operation dictated by $S_{b'}(k_o)$ in Eq. (2b), (k_o) can be safely set equal to zero by choosing (α) to be zero. In this limit, Eq. (2b) becomes a Fourier series expansion for $f(x,\omega)$. Clearly, the infinite summation that is dictated in Eq. (2b) by the operator $S_{b'}(k_o)$ cannot be met in practice; the summation needs to be truncated. To ensure that the truncated terms can be discarded, a maximum wavenumber (k_M) is defined so that the convergence of $F(k,\omega)$ is substantially completed when this wavenumber is reached. A number (J') is then defined and it is required that

$$f(x,\omega) \simeq (2\pi)^{-1/2} \ (\kappa_1') \ S_{b'}^{J'}(k_o) \ \{exp(-ixk_o) \ F(k_o,\omega)\} ;$$

$$k_M = (J'\kappa_1') , \tag{2c}$$

constitutes an adequate approximation to Eq. (2b), where

$$S_{b'}^{J'}(k) \ \{N(k,\omega)\} \sum_{j=-J'}^{J'} N(k+\kappa_j',\omega); \qquad \kappa_j' = (j\kappa_1') . \tag{6}$$

[cf. Eqs. (2b) and (3).] How can an adequate approximation be defined in the context of Eq. (2c)? For this purpose one needs to specify the expected "overall spatial span" (x_M) and the "spatial sampling span" (x_1') for the Fourier transform $f(x,\omega)$. The specification of these spatial parameters defines a number (J_x') given by

$$x_M = (J_x' \, x_1') \ . \tag{7a}$$

The uncertainty principle and sampling theory relate $\{k_M, \kappa_1'\}$ to $\{x_M, x_1'\}$ in the form

$$(x_1' \, k_M) = (x_M \, \kappa_1') \geq 2\pi \ ; \qquad J_x' = J' \geq 2\pi \ (x_1' \, \kappa_1')^{-1} \ . \tag{7b}$$

Summarizing Eqs. (2) through (7) yields

$$f(x,\omega) \simeq (2\pi)^{-1/2} \ (\kappa_1') \ S_{b'}^{J'}(k_o) \ \{exp(-ixk_o) \ F(k_o,\omega)\} \ ;$$

$$k_o = (\alpha \kappa_1') \ , \tag{8}$$

and the approximation is adequate, provided

$$(\eta_\gamma)^{-1} << (k_\gamma \, / \, \kappa_1') << J'; \qquad J' = (k_M \, / \, \kappa_1') = (x_M \, / \, x_1') \ , \tag{9}$$

is satisfied, where (k_γ) and (η_γ) define all details in $F(k,\omega)$ in the $k-$domain that need to be captured to render $f(x,\omega)$ a reliable Fourier transform. Usually $(x_1' \, \kappa_1')$ is selected as small compared with unity and (α) is selected as zero. Choosing the equality sign in Eq. (7b) and, as such, substituting it in Eq. (8), the resulting equation can be readily manipulated to conform with the discrete Fourier transform. The discrete Fourier

transform is the basis for the formulation of the Fast Fourier Transform (FFT) computational procedures [12]. In this report, however, a different computational path is selected.

II. FORMALISM OF A DOUBLE-SUM COMPUTATIONAL PROCEDURE

The approximation that underlies Eq. (8) requires Eq. (9) to be satisfied. The latter equation implies that some knowledge of $F(k,\omega)$ is on hand in that the various vectors $\{k_\gamma, \eta_\gamma\}$ that typify details in this function can be reasonably estimated. These estimations are then used to determine the vector $\{\kappa_1', J'\}$ that needs to be imposed on Eq. (8). This vector, in turn, determines the "wavenumber sampling span" (κ_1') and the "overall wavenumber span" $k_M = (J'\kappa_1')$. The first determines the separation between adjacent sampling positions and the second ensures convergence. Situations may arise in which the wavenumber sampling span is selected, for some reason, to be (κ_1) instead of (κ_1') and $(\kappa_1) > (\kappa_1')$. Were this selection of (κ_1) for (κ_1') to be made, what modification needs to be instituted in Eq. (1a) to bring in a finite summation, while ensuring that an adequate approximation is installed? Indeed, the approximation is required to match the accuracy that is enjoyed by Eq. (8) when this equation is covered by the imposition of Eq. (9).

Using the self-aliased operator $S_b(k)$, Eq. (1a) may be recast in the form

$$f(x,\omega) = (2\pi)^{-1/2} \int_o^{\kappa_1} dk \;\; S_b(k) \;\; \{exp(-ixk) \;\; K(k,\omega) \;\; G(k,\omega)\} \; ;$$

$$\kappa_1 = (2\pi / b) \; , \tag{10a}$$

where, for the sake of flexibility, $F(k,\omega)$ is expressed in the factorial form

$$F(k,\omega) \;=\; K(k,\omega)\;\; G(k,\omega)\;\; . \qquad (11)$$

[cf. Eq. (2a).] Ensuring the convergence of $F(k,\omega)$ by an overall wavenumber span $k_M \;=\; J\kappa_1$, the infinite summation implied by $S_b(k)$ is then replaced by a finite summation; namely, Eq. (10a) is approximated in the form

$$f(x,\omega) \;\simeq (2\pi)^{-1/2} \int_o^{\kappa_1} dk \;\; S_b^J(k) \;\; \{exp(-ixk) \;\; K(k,\omega) \;\; G(k,\omega)\} \;;$$

$$k_M \;=\; (J\kappa_1)\;, \qquad (10b)$$

where

$$S_b^J(k)\;\{N(k,\omega)\} \;=\; \sum_{j\,=\,-J}^{J} N(k+\kappa_j,\omega); \qquad \kappa_j \;=\; j\kappa_1 \;;$$

$$\kappa_1 \;=\; (2\pi/b)\;, \qquad (12a)$$

[cf. Eqs. (2c) and (6).] With the help of Eq. (7) it emerges that the approximation in Eq. (10b) matches that in Eq. (2c) provided the equalities and the inequality

$$x_1 \;=\; x_1' \;; \qquad x_M \;=\; J'x_1 \;; \qquad k_M \;=\; (J'\kappa_1') \;=\; (J\kappa_1);$$

$$J \;\geq\; 2\pi(x_1\kappa_1)^{-1}\;, \qquad (13)$$

are maintained. Since the new wavenumber sampling span (κ_1) is larger than the old wavenumber sampling span (κ_1'), the integral cannot be evaluated by a single segment in the manner of Eq. (2c). Each segment in the summation needs to be spanned more finely. In this vein, the integral is carried out over $(1+R)$ regular intervals, notwithstanding that an adaptive procedure may, under certain circumstances, be preferred. Executing the integral in the manner just specified, one derives from Eq. (10b) the Fourier transform in the form

$$f(x,\omega) \simeq (2\pi)^{-1/2} \ (1+R)^{-1} \ (\kappa_1) \sum_{r=o}^{R} S_b^J(k_r) \ \{exp(-ixk_r) \ K(k_r,\omega) \ G(k_r,\omega)\} \ ;$$

$$k_M = (J\kappa_1) \ ; \qquad k_r = (1+R)^{-1} \ \kappa_r \ ; \qquad \kappa_r = (r\kappa_1) \ ; \qquad \kappa_1 = (2\pi/b) \ . \tag{14}$$

The "effective" wavenumber sampling span in Eq. (14) is then given by $(1+R)^{-1} \ \kappa_1$. Using Eqs. (9) and (12), the approximation in Eq. (14) adequately matches that of Eq. (8) provided

$$\{\eta_\gamma(1+R)\}^{-1} << (k_\gamma/\kappa_1) << J \ ; \qquad J(1+R) \simeq J' \ , \tag{15}$$

notwithstanding that the number of terms in the double-sum in Eq. (14) is $\{(2J+1) + (1+R)\}$ versus $(2J'+1)$ in the single-sum in Eq. (8). However, the number of computational steps is $\{(2J+1) \ (1+R)\}$ in Eq. (14) versus $(2J'+1)$ in Eq. (8), which is in agreement with the second of Eq. (15) when J and $J' >> 1$. There is then no factorial saving in computational steps between Eq. (14), as it stands, and Eq. (8); using

one or the other equation is merely a matter of convenience. The crux of the report is revealed when one or the other factors composing $F(k,\omega)$, $K(k,\omega)$, etc. is aliased in (k) with respect to (κ_1), and (κ_1) matches the (κ_1) in Eq. (10a). Under this condition, a factorial saving of computational steps may be accrued by an appropriate manipulation of the integrand. A few examples in this regard are considered next.

III. INTEGRAND COMPOSED OF ALIASED FACTORS

The expression depicted in Eq. (11) is specialized in that the factor $K(k,\omega)$ is assumed to be aliased in (k) with respect to (κ_1). [cf. Eq. (3).] Using the properties of the self-aliased operator $S_b(k)$ as defined in Eq. (3), the integrand in Eq. (10a) can be equivalently expressed in form

$$S_b(k)\ \{exp(-ixk)\ \mathrm{K}(k,\omega)\ G(k,\omega)\} \equiv K(k,\omega)\ S_b(k)\ \{exp(-ixk)\ G(k,\omega)\}\ ,\ \text{if}$$

$$K(k,\omega) = K(k+\kappa_j,\omega)\ . \tag{16a}$$

In terms of the approximation stated in Eq. (14), the "R-summand" equivalence assumes the form

$$S_b^J(k_r)\ \{exp(-ixk_r)\ K(k_r,\omega)\ G(k_r,\omega)\} \Rightarrow K(k_r,\omega)\ S_b^{J_1}(k_r)\ \{exp(-ixk_r)\ G(k_r,\omega)\}\ ;$$

$$k_{M1} = J_1\kappa_1\ , \tag{16b}$$

where k_{M1} is the overall wavenumber span with respect to the convergence of the operand under $S_b^{J_1}(k_r)$ in this equation and in general

$$S_b^{J_\beta}(k_r)\ \{N(k_r,\omega)\} \ = \ \sum_{j=-J_\beta}^{J_\beta} N(k_r+\kappa_j,\omega)\ . \tag{12b}$$

It is noted that if the right side of Eq. (16b) is used in Eq. (14) instead of the left, a factorial saving equal to the smaller of $(2J_1+1)$ and $(2J+1)$ of computational steps is accrued. Usually, however, $J_1 = J$ and, therefore, the two factors are equal. Moreover, if the aliasing in $K(k,\omega)$ can be cast in the form

$$K(k,\omega) \ = \ S_b(k)\ \{K_A(k,\omega)\ \ K_B(k,\omega)\}\ , \tag{17a}$$

then in terms of the approximation stated in Eq.(14), the R-summand equivalence becomes

$$S_b^J(k_r)\ \{exp(-ixk_r)\ \ K(k_r,\omega)\ \ G(k_r,\omega)\}$$
$$\Rightarrow\ S_b^{J_2}\ \{K_A(k_r,\omega)\ \ K_B(k_r,\omega)\}\ \ S_b^{J_1}(k_r)\ \{exp(-ixk_r)\ \ G(k_r,\omega)\}\ ;$$

$$k_{M2} \ = (J_2\kappa_1)\ , \tag{17b}$$

where k_{M2} is the overall wavenumber span with respect to the convergence of the operand $\{K_A(k,\omega)\ \ K_B(k,\omega)\}$ under $S_b^{J_2}(k)$. It is noted that the factorial saving of computational steps pertaining to Eq. (17b) remains unchanged from that derived from Eq. (16b). If, in addition to the form of $K(k,\omega)$, as stated in Eq. (17a), it is assumed that

$K_A(k,\omega)$ is aliased in (k) with respect to (κ_1), then the factor $K(k,\omega)$ can be factorized further in the form

$$K(k,\omega) \;=\; K_A(k,\omega)\;\; S_b(k)\;\; \{K_B(k,\omega)\}\;, \quad \text{if}$$

$$K_A(k,\omega) \;=\; K_A(k+\kappa_j,\omega)\;. \tag{18a}$$

In terms of the approximation stated in Eq. (14), the R-summand equivalence assumes the additional factorization, over Eq. (17b), of the form

$$S_b^J(k_r)\;\{exp(-ixk_r)\;\; K(k_r,\omega)\;\; G(k_r,\omega)\}$$
$$\Rightarrow\; K_A(k_r,\omega)\;\; S_b^{J_3}(k_r)\;\; K_B(k_r,\omega)\}\;\; S_b^{J_1}(k_r)\;\{exp(-ixk_r)\;\; G(k_r,\omega)\}\;;$$

$$k_{M3} \;=\; (J_3\kappa_1)\;, \tag{18b}$$

where k_{M3} is the overall wavenumber span with respect to the convergence of the quantity $K_B(k,\omega)$ in this equation. It is noted that if in Eq. (14) the right side of Eq. (18b) is used instead of that of Eq. (16b), a factorial saving equal to the smaller of $(2J_2+1)$ and $(2J_3+1)$ of computational steps is accrued. Usually, however, $J_\beta \;=\; J$ and, therefore, the two factors are equal. Under this equality, it is noted that if the right side of Eq. (18b) is used in Eq. (14) instead of the left, a factorial saving of $(2J+1)^2$ of computational steps is accrued. In the event that the aliasing in the factor $K_A(k,\omega)$, as stated in Eq. (18a), stems from its functional dependence on an aliased quantity; e.g.

$$K_A(k,\omega) \;=\; K_A[S_b(k)\;\{K_{AA}(k,\omega)\}]\;, \tag{19a}$$

the approximation stated in Eq. (14) supports the R-summand equivalence of the form

$$S_b^J(k_r)\ \{exp(-ixk_r)\ \ K(k_r,\omega)\ \ G(k_r,\omega)\}$$
$$\Rightarrow\ K_A^{J_4}(k_r,\omega)\ \ S_b^{J_3}(k_r)\ \{K_B(k_r,\omega)\}\ \ S_b^{J_1}(k_r)\ \{exp(-ixk_r)\ \ G(k_r,\omega)\}\ ;$$

$$K_A^{J_4}(k_r,\omega)\ =\ K_A[S_b^{J_4}(k_r)\ \{K_{AA}(k_r,\omega)\}]\ ;\qquad k_{M4}\ =\ (J_4\kappa_1)\ ,\qquad (19b)$$

where k_{M4} is the overall wavenumber span with respect to the convergence of the quantity $K_{AA}(k,\omega)$ in this equation. It is noted that there is no factorial saving of computational steps between the use of Eqs. (18b) and (19b). If further factorization exists, either in $K_A(k,\omega)$, in $K_B(k,\omega)$ or in both, in the format just prescribed, one may gain more and more factorial savings. For the purpose of this report, however, the factorization and dependence considered in Eqs. (18a) and (19a) suffice.

A special case of Eq. (11) is that in which, in addition to the aliasing of $K(k,\omega)$ in (k) with respect to (κ_1), the function $G(k,\omega)$, in this equation, is independent of the wavenumber (k); namely

$$F(k,\omega)\ =\ G_o(\omega)\ \ K(k,\omega)\ ;\qquad K(k,\omega)\ =\ K(k+\kappa_j,\omega)\ .\qquad (20)$$

Substituting Eq. (20) in Eq. (10a) derives

$$f(x,\omega)\ =\ s_b(x)\ \{\delta(x)\ \ f_s(x,\omega)\}\ ;$$

$$f_s(x,\omega)\ =\ [G_o(\omega)/\kappa_1]\ (2\pi)^{-1/2}\ \int_o^{\kappa_1}\ dk\ \ exp(\text{-}ixk)\ K(k,\omega)\ ,\qquad (21)$$

where $s_b(x)$ is a self-aliased spatial (x) operator with respect to the fundamental separation distance (b); $b = (2\pi/\kappa_1)$. This operator is defined

$$s_b(x)\ \{n(x,\omega)\} = \sum_j n(x+x_j,\omega)\ ; \qquad x_j = (jb)\ ; \qquad x_o = 0\ . \quad (22)$$

[cf. Eq. (3).] Executing the integral in Eq. (21) in a manner analogous to that resulting in Eq. (14), one obtains

$$f_s(x,\omega) = (2\pi)^{-1/2}\ [G_o(\omega)/(1+R)] \sum_{r=o}^{R} \{exp(-ixk_r)\ K(k_r,\omega)\}\ , \quad (23)$$

and the evaluation of $f_s(x,\omega)$ is considered, in view of Eq. (21), to be synonymous with the evaluation of $f(x,\omega)$, notwithstanding the acknowledgment that, unlike $f(x,\omega)$, the evaluation of $f_s(x,\omega)$ is meaningful only for values of $x = x_j$; $x_j = (jb)$. In this view, the recognition of the aliasing in $K(k,\omega)$, as stated in Eq. (20), is worth a factorial saving of $(2J+1)$ of computational steps. This is similar to the factorial saving that is accrued in Eq. (16b), utilizing the same recognition. Indeed, the factorization of $K(k,\omega)$, as depicted in Eqs. (16) - (19) and beyond, are just as beneficial with respect to Eq. (23) as they are with respect to Eq. (14).

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