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13. ABSTRACT (Maximum 200 words) We propose a research program to investigate new techniques for computing image descriptions in early vision. The program will incorporate computational, physiological, and psychophysical aspects. Research will be focused on the certain non-linearities that we have discovered can play an important role in improving the robustness of edge, line, and curve detection. These non-linearities are totally different from those previously studied in early vision. We embody them in what we call logical/linear operators, because these operators exhibit both logical (i.e., they form an appropriate Boolean algebra) and linear properties. We submit that these operators can lead to both more robust computer vision, and to new insights into biological vision.

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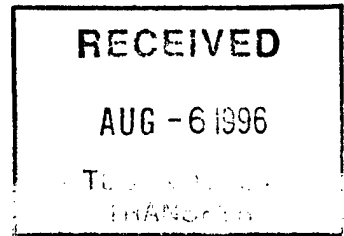
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1 Objectives

This document is the Final Report on the research grant: "Logical/Linear Operators in Early Vision." The overall objectives, as stated in the abstract from the original proposal, were as follows:

We propose a research program to investigate *new techniques for computing image descriptions in early vision*. The program will incorporate computational, physiological, and psychophysical aspects. Research will be focused on the certain non-linearities that we have discovered can play an important role in improving the robustness of edge, line, and curve detection. These non-linearities are totally different from those previously studied in early vision. We embody them in what we call logical/linear operators, because these operators exhibit both logical (i.e., they form an appropriate Boolean algebra) and linear properties. We submit that these operators can lead to both more robust computer vision, and to new insights into biological vision.

The research program has matured in a manner that exceeded our original expectations. The logical/linear operator abstraction has developed a particular tangent-field model that has developed into a basis for texture and shading analysis, as well as edge and curve detection, it has provided the basis for a new approach to segmentation and grouping, and it has provided the basis for a new approach to visual shape analysis. This latter study has created enormous interest in the computer vision community, in that it brought the mathematics of curve evolution to shape analysis for the first time. Finally, and perhaps most surprisingly, mathematical analysis of the tangent abstraction has led to a relationship between complexity theory and segmentation, the first well-founded approach to intermediate-level problems in perceptual grouping. Thus we have the theoretical foundations in place for a full theory of vision, from early measurements to abstract descriptions of generic shape. To our knowledge this broad covering of the vision problem from a coherent mathematical perspective is unique in the field.

The research program has matured professionally as well. The logical/linear operators have been embedded into a software package available over the Wide World Web, and hundreds of sites around the world have ftp'd a copy of the package. This site has been active for nearly a year, which is a short time for such material to diffuse into the research community, but we have already recieved very favorable feedback regarding it. Furthermore, Steven Zucker presented a public lecture series at the Isaac Newton Institute for Mathematical Sciences in Cambridge on "Computational Vision and Biological Perception", which covered much of the above material. These lectures will be published next year by Princeton University Press under the title: "Textures and Tangents", and it will be the second volume in their Newton lecture series. (The first volume was Hawking and Penrose's "The Nature of Space and Time.")

In the next section, we provide an overview of this tangent-map approach to vision, and a few example figures. Many additional examples and technical details are contained in the accompanying papers.

2 Status of Effort

It is difficult to express our satisfaction with the past three years of research without feeling some degree of embarrassment. The logical/linear operators have the potential to revitalize early vision, the complexity analysis approach to segmentation has the potential to provide the first foundation for intermediate-level vision problems that are consistent with both high- and low-level processing, and the curve-evolution approach to shape analysis has played a key catalytic role in introducing these mathematical techniques to the community. Although curve-evolution ideas had previously been applied to image processing by Osher, Sethian, and Rudin in California, their implications for vision tasks had not been previously realized. Connecting with the non-linear diffusion community, this has helped to create a movement in the U.S., Canada, and Europe to explore such techniques. Meetings have been held in Cambridge, UK, Paris, France, Utrecht, The Netherlands, Providence, RI, and Palo Alto, CA of this community, to say nothing of the specialized sessions at the annual conferences in our field. Thus our embarrassment is positive in the best possible sense.

3 Accomplishments/New Findings

In addition to completing the software system for logical/linear operators, we have continued research on several related topics; these are briefly discussed below.

3.1 Logical/Linear Operators

Edge detection is normally considered the first stage of image analysis, and there is little doubt that one of the major frustrations around current computer vision systems is the poor quality of initial edge detection and the subsequent segmentations they produce. It is here that our logical/linear operators apply ([7]). They provide more robust local estimates of edge and line structure by enforcing continuity conditions along the tangential extent and contrast variations along the normal extent of their spatial support. (Recall that a notion of orientation, or tangent, is essential to defining these conditions.) I believe these operators will become extremely important to those portions of the Air Force's mission that involve automatic or semi-automatic analysis of imagery, as well as to related civilian technology challenges.

A key aspect of the logical/linear operators is that they formally connect edge structure to the tangent of image curves. Thus they provide an abstract description of the (differential) geometric structure in the image in what we call a *tangent field*. A "portable" version of a system to support these applications has been developed

by Dr. Lee Iverson. He completed a first implementation as a Ph. D. student here to test the basic ideas, and we found that they worked extremely well on complex biomedical images (e.g., angiograms and retinograms), as well as the “standard test images” circulating in the field (e.g., “Paulina”); see Fig. 1.

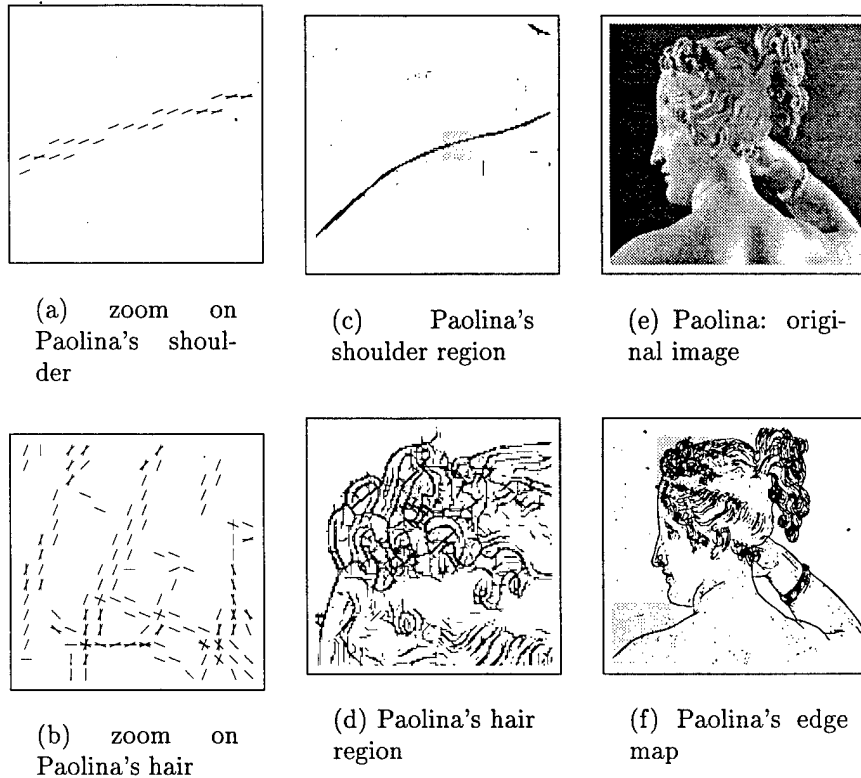


Figure 1: An illustration of the tangent map created by logical/linear operators and the subtlety of interpreting (“walking through”) it: (top row) curves, (bottom row) texture. The original image in (e) and the full edge map in (f). Notice how it is relatively easy to “walk” from one tangent to another when the tangents depict a curve, but that there is substantial confusion for the more complex texture (hair) pattern. For many of the tangents in the hair it is functionally impossible to determine which one is “next” along the same hair. Segmenting curves from hair is based on a complexity analysis.

This figure also illustrates the intimate connection between edge detection and segmentation issues, which is described next.

3.2 Complexity Analysis and Segmentation

We illustrate this segmentation problem with a specific example. Imagine standing on an edge element in an unknown image, as in the first Figure. Is this edge part of a curve, or perhaps part of a texture? If the former, which is the next element along

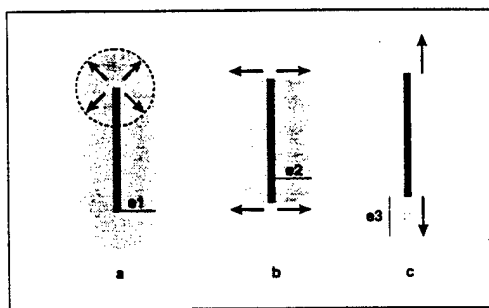
the curve? If the pattern is a texture, is it a hair pattern, in which nearby elements are oriented similarly, or a spaghetti pattern, in which they are not. The question is in part one of complexity, since curves are "simpler" than textures and in part one of dimensionality, since curves are 1-D and textures are 2-D.

Benoit Dubuc and I (paper appended) have proposed a measure of representational complexity that seeks to answer these questions, and it involves building a scale space over the tangent field. We sketch these ideas in this Section.

The mathematical foundation for our measure of complexity is geometric measure theory. This differs from classical differential geometry in that curves are not given as prespecified maps, but rather we use a parameterization-free characterization of curve-like sets as those with Besicovitch tangents; informally, these are the locations in the image at which contrast organizes into locally dense, oriented arrangements.

Viewing curves as curve-like sets of points, we now consider dilations over it. Minkowski dilations are routinely used in mathematical morphology and for the estimation of fractal dimension. The approach consists in creating a new set which is the Minkowski sum with a dilating (structuring) element. The dilation is done isotropically over the set. More formally, given a set $E \subset \mathbf{R}^2$ and a compact convex set $F \subset \mathbf{R}^2$, the *dilation* of E by F is given by $E \oplus F = \{a + b, a \in E, b \in F\}$, where $+$ here denotes the vector sum. Given $\epsilon > 0$, we can also scale the structuring element F and obtain $\epsilon F = \{\epsilon x : x \in F\}$. An example of the isotropic dilation of a line segment with a ball is shown in the next Figure.

Our complexity measure will be based on *oriented dilations* that will adapt to the local structure of the set. Reconsidering the line segment example, the same Figure illustrates the concepts of both *normal* and *tangential dilations*. Notice that it is the tangent interpretation that allows oriented dilations; without it, such an analysis would be impossible.



Isotropic and oriented dilations. (a) isotropic dilation with a ball of radius ϵ . (b) normal dilation (c) tangential dilation. Oriented dilations are possible because of the intermediate representation provided by the Besicovitch tangent sets.

The departure from the standard Minkowski dilation approach by using oriented dilations will be essential for our analysis since it will segregate the classification *curve*

vs. *texture* (using normal dilations) from the one of *dust* (or discontinuities) vs. *curve* (using tangential dilations). In particular, we shall use them to define what we call the *normal complexity* $C_N(\delta)$ and the *tangential complexity* $C_T(\delta)$ of a curve-like set at a given scale $\delta > 0$. The main idea is to look at the rate of growth of the measure of the dilated sets. In the case of normal complexity, it will be the area of $E_N(\epsilon)$. If in a neighborhood of δ the area grows like δ^α , then we say that the *normal complexity* $C_N(\delta)$ for E is $2 - \alpha$.

Returning to the Paolina image, we observe that, in the hair region, the rate of growth α is approximatively 0 at the chosen scale δ , thus the complexity is $2 - \alpha \approx 2$ (texture). For the shoulder, the rate of growth is linear ($\alpha \approx 1$), leading to a normal complexity of $2 - \alpha \approx 1$ (curve). This suggests the following principles for segmenting the tangent field:

dust: sets in which the tangent map is sparse, the object almost nowhere extends along its length locally, and might be thought of as being curve-free (low normal complexity - low tangential complexity);

curves: discrete tangent maps for which a curve representation is completely adequate. Objects extend along their length, like Paolina's shoulder, and the density of other tangents is low almost everywhere along it in a local neighborhood (low normal complexity - high tangential complexity);

turbulence: tangent maps that are characterized by objects that do not extend along their length but are dense in the normal direction; e.g., Paolina's uncombed hair (high normal complexity - low tangential complexity);

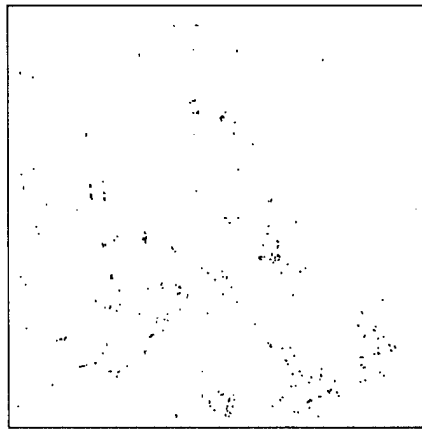
flow: tangent maps for which the objects extend along their length and are also dense in the normal direction (high normal complexity - high tangential complexity).

The result of this segmentation applied to the Paolina tangent field is in the next Figure.

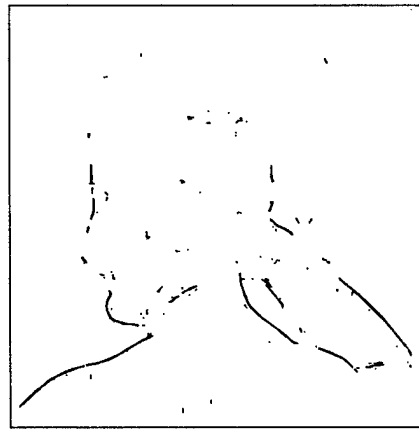
The analysis thus far has "focused" (*sic*) on sharp edges. However, vision on natural images must function for less than perfect images as well, a topic we turn to in the next subsection.

3.3 Blurry Edges

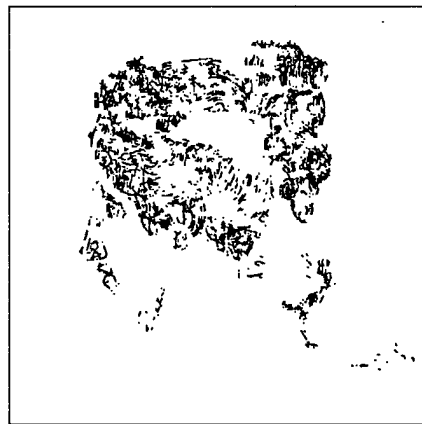
Not all edges are sharp, and there is significant information in the blur profile of an edge. James Elder and I have generalized the logical/linear framework to provide an estimate of these blur profiles, as dictated by the notion of "minimum reliable scale". In other words, with edges interpreted as tangents to image curves, they should be estimated at a scale which is sufficiently small so that the "straight-line" (or tangent) approximation is sensible from a numerical perspective on differential geometry, but at a scale which is sufficiently large that the estimates are reliable from a statistical perspective on signal detection theory. This extension is described in



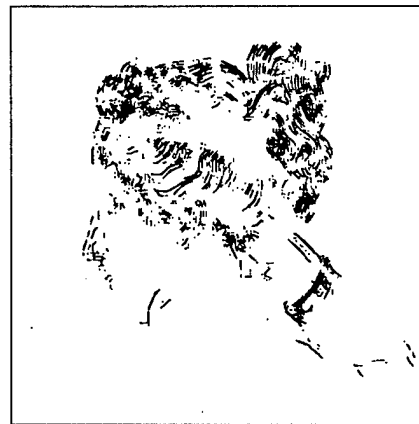
(a) dust



(b) curves



(c) turbulence



(d) flow

Figure 2: Segmentation can be pulled back to the level of the complexity map. This illustrates the four structural classes of image structure.

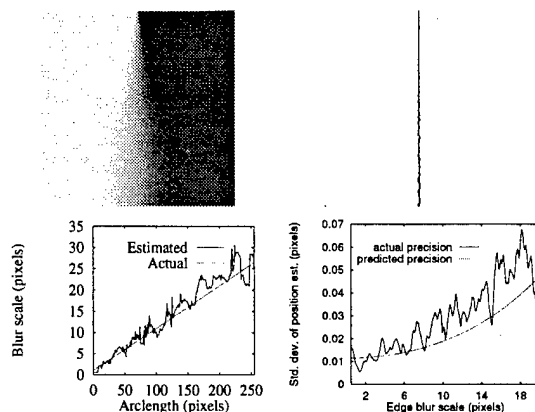


Figure 3: **Top left:** The blur grade is linear, ranging from $\sigma_b = 1$ pixel to $\sigma_b = 26.6$ pixels. **Top right:** Edges detected by local scale control at the minimum reliable scale. **Bottom left:** Estimated blur scale. **Bottom right:** Predicted and measured localization precision.

the accompanying papers. Note that it is a very different interpretation of the role of scale than is common in computer vision, as practiced by Witkin, Koenderink, Lindeberg, etc. Its reliability against an edge with increasing blur is shown in the next Figure, followed by an illustration of how it can be used to segment a “sharp” tree branch from a field of defocussed ones. Clearly such problems are of interest to primates swinging through a forest, and to robotic vehicles traversing a terrain with imaging sensors of finite depth-of-field. Note, in particular, how such ideas could be used for quick depth estimation in such vehicles. The full paper is appended.

A surprising consequence of the minimum reliable scale extension is that it opens an entirely new class of algorithms for image reconstruction from contour codes. An example is shown in the next figure; details are in the paper “Precision, Blur, and the Perceptual Content of the Contour Code. We mention this here because the question of image communication is becoming extremely prominent for military and civilian applications, and several companies in the U.S. have begun to investigate this new approach; see *Transitions*. The key for us was understanding the deblurring inherent in traditional reconstruction algorithms: note how shadows, shading and defocused contours appear quite unnatural. It appears that obtaining perceptually accurate reconstructions from the contour code depend upon accurate *re*blurring of the reconstructed luminance function. The extent of this reblurring is dictated by the minimum reliable scale.

Of course, some blur in the image is not due to optical effects, but is due to shading effects. Recalling the complexity analysis of texture flows, we now introduce a notion of shading flow.

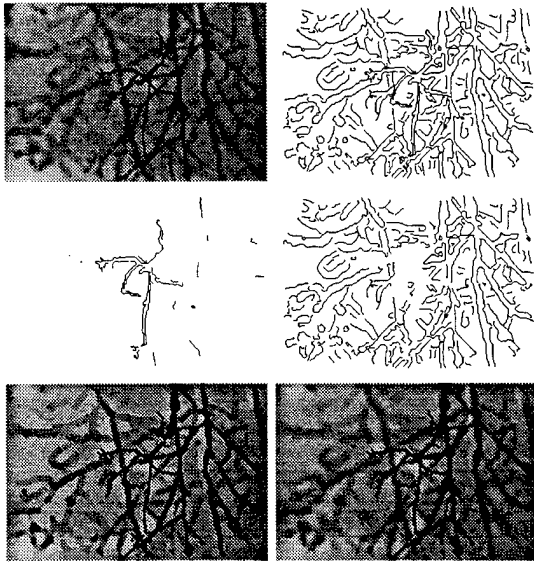


Figure 4: **Top left:** A photograph of tree branches with small depth of field ($f/3.5$) and near focus. **Top right:** Edge map computed by local scale control. **Middle left:** Foreground structure (focused contours). **Middle right:** Background structure (defocused contours). **Bottom left:** Reconstructed luminance terrain by standard techniques (Carlsson). **Bottom right:** Reconstructed, reblurred result. Note how much more natural this reconstruction appears.



Figure 5: **Top left:** Original image. **Top right:** Contours. **Bottom left:** Reconstructed luminance function. **Bottom right:** Reblurred result.

3.4 Shading Flow Fields

In addition to the applications-driven activities, we are pursuing more theoretical domains as well. The logical/linear “structural” constraints can be applied at a larger scale than can arise from contour blurring; for example, in the extended shading distributions that are common in natural images. The analysis of such surface coverings can also be done from a tangent-map-like structure, but this time as a full two-dimensional distribution. Dr. Pierre Breton has completed his Ph. D. thesis in this area, in which we define a notion of “shading flow field” as the basis for such inferences. It derives from the mathematical observation that the dual flow is a gradient field. Image gradients are of course important to shading analysis, and provide the basis for shape-from-shading inferences. An illustration follows, indicating both how the shading flow field can be used to support surface inferences, and how it provides the key to parsing shadows and other light-source changes from images. Details are in the attached paper: “Shadows and Shading Flow Fields”.

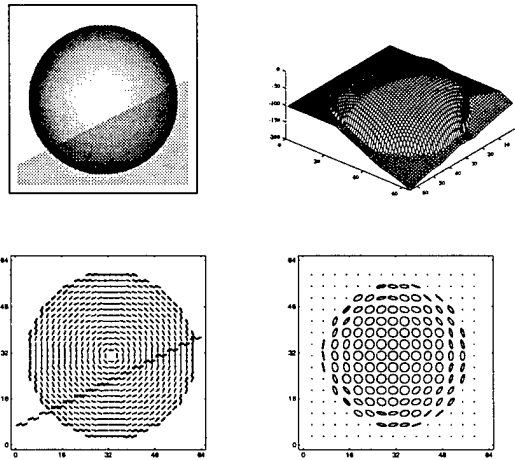


Figure 6: Top-left: an image for which assumptions of standard shape-from-shading (Bischel and Pentland, 1992) are violated. Top-right: neither of the two most plausible scenes (the surface albedo changes abruptly or a semi-transparent object lies somewhere in the scene) can be implied from the depth map obtained. Bottom-left: the shading flow field is depicted by thin arrows and the tangent field, by thick arrows. Bottom-right: the accurate shape recovery illustrated by the surface normals is obtained from the shading flow field.

Dr. M. Langer has completed a Ph. D. thesis on a very different aspect of how light interaction with surfaces. The classical model is derived by modeling how image contrast changes as a function of surface orientation. Our new perspective is obtained by modeling how light propagates through a scene; i.e., by modeling light rays instead of surface patches. This has given rise to an entirely new class of shape from shading algorithms, which are naturally parallel (for SIMD computers). Quite surprisingly, these methods are analogous in a certain technical sense to what is required for modeling (free)space for robot navigation and to do computer graphics on parallel (SIMD) machines.

3.5 Contour Completion

An outline drawing often serves as an excellent depiction of a visual scene, and somehow our visual systems can form two- and three-dimensional percepts solely from one-dimensional contour information. The complexity analysis showed how, for the first time, those tangents belonging to image contours could be segmented from those belonging to textures and other images structures. Thus we are now in a position to consider how to link these together.

In mathematics, contour closure plays a key role in bridging this gap, however in perception the link between closure and shape is unclear. James Elder and I have been attempting to better understand this relationship. We devised a set of visual search experiments in which subjects discriminate outline figures by means of their two-dimensional shape. By modulating the degree of closure of the outlines, we were able to show that two-dimensional shape processing is rapid for closed stimuli but slow for open stimuli. We further show that search speed can be characterized as a smooth, monotonic function of the degree of closure, supporting the notion of a perceptual closure continuum. We have just begun to translate these ideas into computer algorithms, the first paper for which is appended ("Computing Contour Closure").

From such algorithms we should be able to find the bounding contours of objects and major components within those objects, which leads us to the final topic, visual shape analysis.

3.6 Descriptions of Visual Shape

The analysis of visual shape naturally decomposes into a two-stage process, the first of which provides a general shape description to serve as a base key into categorical possibilities, and the second of which identifies specific members of the category. For example, an organization of a small blob on a large blob with several elongated structures emanating from the large blob might be categorized as a person, and then that person might be identified as "Richard Nixon". We shall concentrate on the first, categorical stage, since this is the stage that has been most frustrating for researchers in computer vision. In fact, we believe this to be the first mathematically-founded approach to generic shape analysis.

The analysis builds directly on that of the previous sections. Assume a tangent field has been inferred, and those tangents comprising a 1-dimensional curve have been selected. It is then necessary to infer a curve through this tangent field, and the technique in the Elder and Zucker paper, or in [3] suffice.

How can the generic 2-dimensional shape bounded by this closed curve be described? What are the natural parts, and are there other components to such a description? Biederman's "geons" ([1]), Hoffman and Richards' "codons" ([6]), Blum's medial axis transform ([2]), and Marr and Nishihara's "stick figures" ([11]) are possibilities, but they all differ. Some concentrate on boundary information, and others on interior (region) information. Each has a regime of figures on which they work

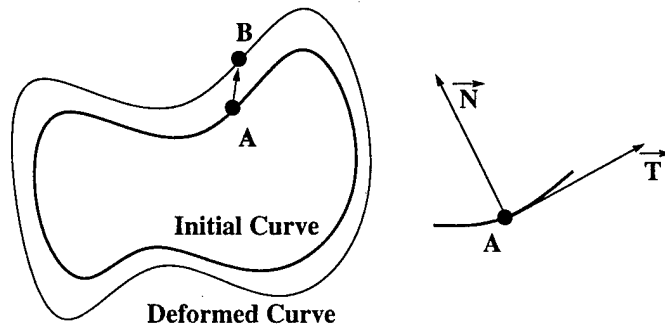


Figure 7: The points on the initial curve A move to B to generate a new curve. The direction and magnitude of this motion is arbitrary in order to capture general deformations. However, with mild restrictions appropriate to shape, one can classify this deformation to be a sum of *constant deformation* and *curvature deformation* along the normal.

well, e.g., machined industrial components for geons and leaves for Blum, but none provides a unifying methodology. Moreover, none provide a natural scale-space for significance of the components, in the sense that one can derive why the head is less significant than the torso, etc. This significance ordering requires a compositional component, as was earlier attempted in picture grammars ([13]; [10]).

Our approach to categorical shape description derives from the theory of curve evolution. This is a well-studied area of mathematics, for which a beautiful theory has been developed over the past half century ([5], [14, 12], [9], [4]). What will be of primary significance for us is that such evolutions develop singularities—or *shocks*—and these singularities define the natural shape descriptors. They also imply a natural significance hierarchy over shapes, which is the structural scale space that we seek. It is this specific connection between the singularities of the curve evolution process and the visual analysis of shape that is turning out to be so seminal for these applications (cf. the opening comments in the Sec. “Status of Effort”).

We begin with the principle that: *slight changes in the boundary of an object cause only slight changes to its shape*, which suggests that we consider slight boundary deformations. Let the shape be represented by the curve $C_0(s) = (x_0(s), y_0(s))$, where s is the parameter along the curve (not necessarily arclength), x_0 and y_0 are the Cartesian coordinates and the subscript 0 denotes the initial curve prior to deformation. We consider the specific deformation (see Kimia et al. ([8]) for derivation, background, and related references):

$$\begin{aligned} \frac{\partial C}{\partial t} &= (1 + \epsilon\kappa)\vec{N} \\ C(s, 0) &= C_0(s) \end{aligned}$$

An illustration of a curve evolving according to this equation is shown in the next Figure. The ϵ parameter runs along the x-axis, and the time of evolution, or scale, along the vertical axis. The result is a scale space built upon the shape, which we call a *reaction/diffusion space*.

This space is so named because of the parameter ϵ , which controls the amount of diffusion during the evolution. For large amounts of ϵ it dominates, and the result is a *geometric heat equation*.

At the opposite extreme, when $\epsilon = 0$, there is no smoothing and shocks are created. Four such shocks are illustrated in the Figure; they are:

1. *First-Order Shocks* are discontinuities in orientation of the boundary of a shape. They denote *protrusions* and *intrusions*, such as the nose on a face.
2. *Second-Order Shocks* result when, during the process of deformation, two distinct non-neighboring boundary points join and not all the other neighboring boundary points have collapsed together. Second-order shocks define the *parts* of a shape.
3. *Third-Order Shocks* result when, during the process of deformation, two distinct non-neighboring boundary points join in a manner such that neighboring boundaries of each point also collapse together. Third-order shocks indicate an extended axis, as in a tail or a *bend*.
4. *Fourth-Order Shocks* occur when, during the process of deformation, a closed boundary collapses to a point. Thus fourth-order shocks are the *seeds* of a shape, in that they indicate the locations where mass is to be attached.

Second-order shocks persist for only an instant in time, and then give rise to a pair of first-order shocks traversing in opposite directions. The locus of points traced out by the shocks is the Blum skeleton.

Because the shocks occur during an evolutionary process, the time of formation is related to significance. In particular, the least significant shocks form first, and the most significant ones last. The result is a hierarchical description that is suitable for categorical object description; see Figure.

The most recent activity has related this class of shape description to the fragment grouping problem via a principle of perceptual occlusion. Again, the connection is formal, and is described in the accompanying paper by August and Zucker.

3.7 Summary of Contributions

In this Section we have attempted to outline the contributions made during the tenure of this grant. They range from initial image measurement via logical/linear operators, and their extension to blurry edges, to an intermediate-level theory of complexity-based segmentation to, finally, a high-level theory of shape description. A notion of orientation, or tangent, is central to each of these levels.

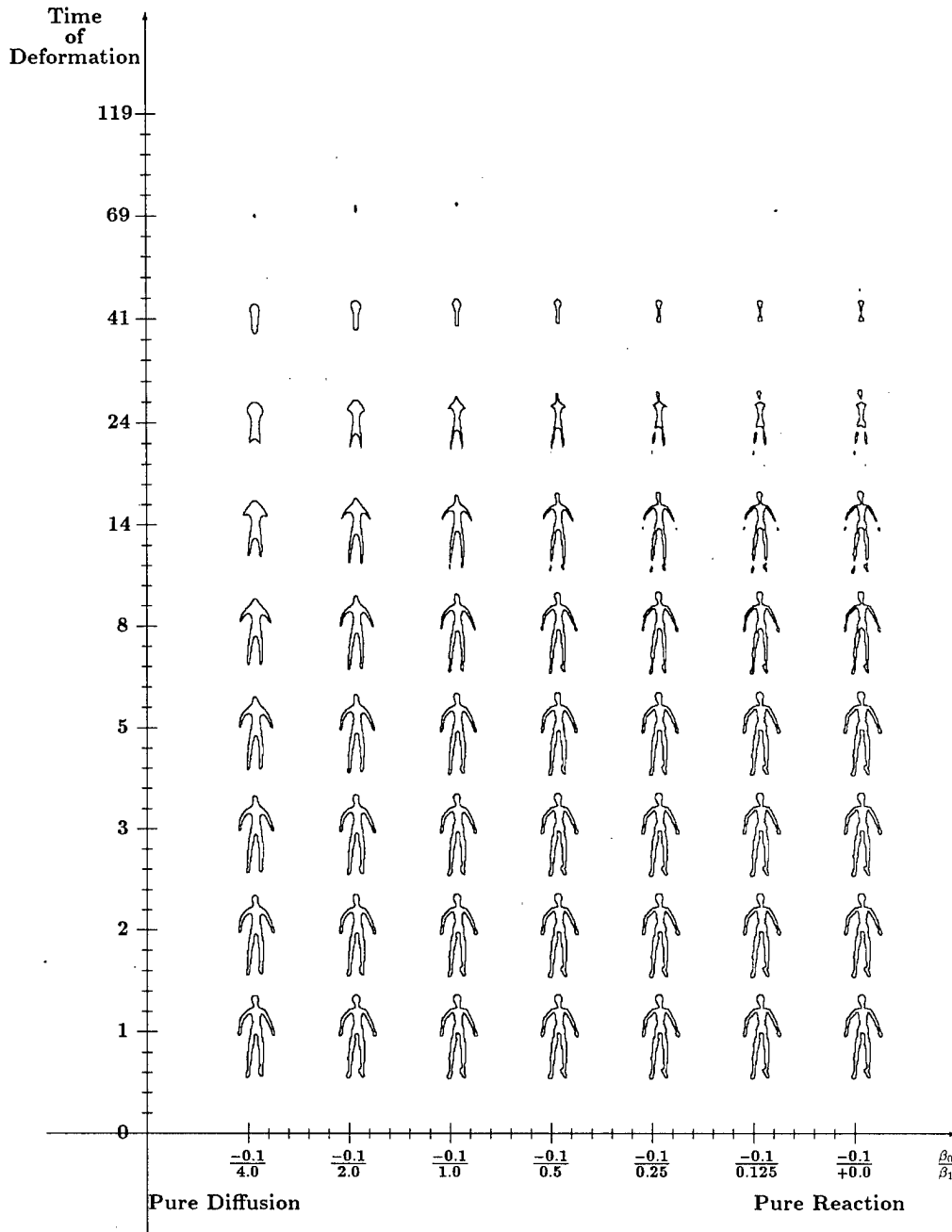


Figure 8: An illustration of a portion of the reaction-diffusion space for a DOLL image (from the National Research Council of Canada's Laser Range Image Library CNRC9077 Cat No 422). The image was thresholded and stored as a 128x128 image. The numbers on the x -axis are indicative of the ratio of reaction to diffusion. Note how the reaction creates shocks, and the diffusion "melts" the boundary of the shape. After [8].

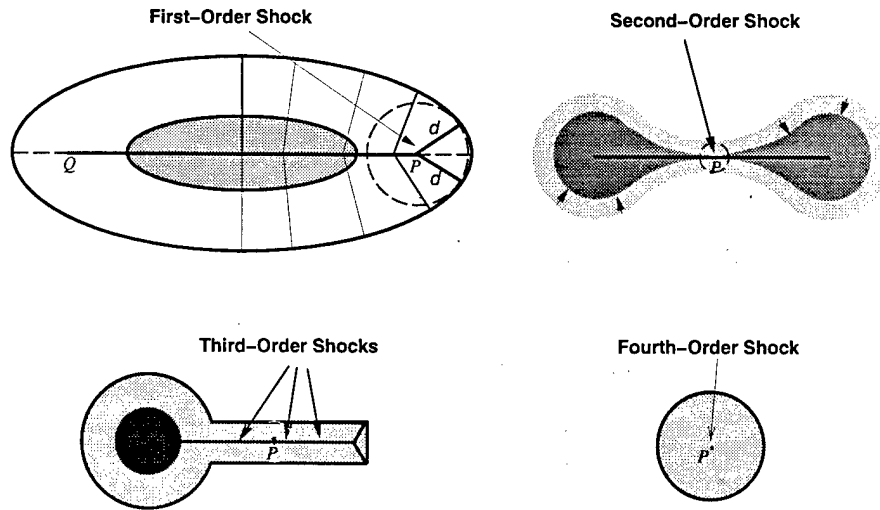


Figure 9: Illustration of the four types of shocks formed in the course of the reactive deformation process; each is correlated with a generic shape category, *i.e.*, protrusion, part, bend and seed. For each shock type, the maximal inscribed disc is overlaid. After [8].

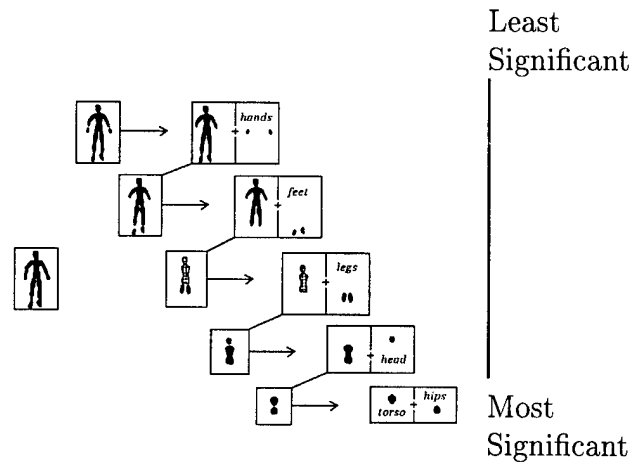
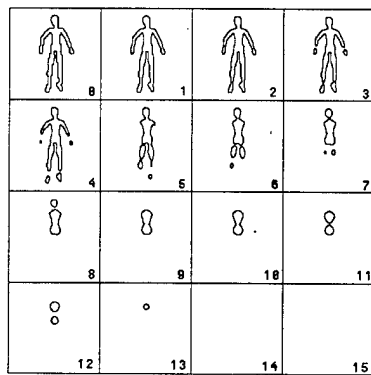


Figure 10: a) The evolution of shocks leads to parts, protrusions, and bends. Again the DOLL image is shown. The contour in box N corresponds to increasing boundary evolution (time) steps. Observe that the “feet” partition from the “legs” (via second-order shocks) between frames 3 and 4, and the “hands” from the “arms” between frames 2 and 3. Following these second-order shocks, first-order shocks develop as the “arms” are “absorbed” into the chest. Running this process in the other direction would illustrate how the arms “protrude” from the chest.

b) The hierarchical decomposition of a doll into parts. Selected frames were organized into a hierarchy according to the principle that the significance of a part is directly proportional to its survival duration. After [8]

4 Personnel Supported

The following graduate students and Post-Doctoral Fellows have been supported in whole or in part by this grant:

- Pierre Breton
- Benoit Dubuc
- Douglas Miller
- Michael Langer
- Jonas August
- James Elder
- Michael Langer

In addition, Dr. Lee Iverson has remained in a consultative capacity.

5 Publications

The following papers were published during the previous 36 months, or the tenure of the grant.

1. Kimia, B., and Zucker, S.W., Symbolic inverse to Gaussian Blur, *Optical Engineering*, 1993, **32**(1), 166 - 176.
2. Elder, J., and Zucker, S.W., Contour closure and the perception of shape, *Vision Research*, 1993, **33**(7), 981 - 991.
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6 Interactions/Transitions

6.1 Participations/presentations

The following *Invited Lectures and Presentations* were given during the duration of the current Grant.

1. Distinguished AI Colloquium, Ohio State University, Columbus, May, 1996.

2. Opening Lecturer (invited), Workshop on Mathematical Methods in Computer Vision, The Geometry Center, University of Minnesota, Minneapolis, Sept., 1995.
3. Invited Lecturer, *Workshop on Computational Neuroscience*, Marine Biological Laboratories, Woods Hole, Ma, August, 1995.
4. Invited Lecturer, *Methods in Computational Neuroscience*, Marine Biological Laboratories, Woods Hole, August, Ma, 1995.
5. Invited Lecturer, *Stockholm Workshop on Computer Vision*, Rosenon, Sweden, August, 1995.
6. Invited Lecturer, *NECI Vision Workshop*, Princeton, N.J., March, 1995.
7. Invited Lecturer, *30-th Anniversary, Computer Vision Laboratory*, University of Maryland, College Park, 1994.
8. Invited Lecturer, *Second Workshop on Visual Form*, International Association for Pattern Recognition, Capri, May, 1994.
9. Invited Lecturer, *Workshop on Computational Neuroscience*, Marine Biological Laboratories, Woods Hole, Ma, 1994.
10. Invited Lecturer, *The Hebb Symposium on Neurons and Biological Dynamics*, The Fields Institute for Mathematical Sciences, University of Toronto, May, 1994.
11. Invited Lecturer, *U.S. - Israel Binational Workshop on Cerebral Cortex and Object Perception*, Jerusalem, March, 1994.
12. Newton Institute Public Lecture Series, *Computational Vision and Biological Perception*, University of Cambridge, October - November, 1993.
13. Invited Lecturer, *Fourth Annual British Machine Vision Association Meeting on Natural and Machine Vision*, British Institute of Radiology, London, October, 1993.
14. Invited Lecturer, *Workshop on Computational Neuroscience*, Marine Biological Laboratories, Woods Hole, Ma, August, 1993.
15. Invited Lecturer, *Workshop on Mechanisms of Visual Object Recognition*, Santa Fe Institute, Santa Fe, NM, August, 1993.

6.2 Transitions

The logical/linear operator system for local edge, bright line, and dark line detection has reached a sufficiently stable status that the software has been made available for the community. Access is free, and is distributed via anonymous ftp over the internet from a server at McGill's Center for Intelligent Machines. The distributed software runs on Sun Microsystems and Silicon Graphics platforms. Previous versions were tested here, at SRI International, and at Brown University. Dr. Iverson is now a Research Scientist at SRI International, in Palo Alto; working as a consultant on this grant, he has re-implemented the entire system to be more efficient and portable.

The response to this system has been fantastic. A giant log file of the different sites that have accessed the system is available, should it be desired.

7 New Discoveries, Inventions, Patent Disclosures

None.

8 Honors and Awards

- Nominated, *Best Paper Prize* (with J. August, K. Siddiqi) for the paper: Fragment grouping via the principle of perceptual occlusion, *International Conference on Pattern Recognition*, Vienna, August, 1996.
- *Best Paper Prize* (with J. Elder) for the paper: Scale-space surfaces and blur estimation, at *Vision Interface 95*, Quebec City, Canada, May, 1995.
- *Best Paper Prize* (with B. Dubuc) for the paper: La carte complexité et son impact sur le choix de représentation, *Vision Interface 95*, Quebec City, Canada, May, 1995.
- Visiting Professor (invited), Sloan Center for Theoretical Neurobiology, University of California at San Francisco, 1995 - 1998.
- *D.W. Ambridge Award*, 1993, for Lee Iverson's Ph. D. thesis.
- Elected (By)Fellow, Churchill College, Cambridge, 1993.
- SERC Fellow, Newton Institute for Mathematical Sciences, University of Cambridge, 1993.
- Invited Short-Term Visitor, Institute for Research in Cognitive Science, University of Pennsylvania, Philadelphia, March, - April, 1993.
- Invited Visiting Professor, School of Mathematical Sciences, Tel Aviv University, December, 1992 - January, 1993.

- Fellow, Canadian Institute for Advanced Research.
- Fellow, Institute of Electrical and Electronic Engineers (IEEE).