HEAT AND MASS TRANSFER FROM A ROTATING DISK

Preprint 20

F. Kreith, J. H. Taylor, and J. P. Chong
Lehigh University, Bethlehem, Pennsylvania

AIChE

Presented at the

Second National Heat Transfer Conference

A.I.Ch.E.-A.S.M.E.

Chicago, Illinois

August 18 to 21, 1958

Copy No.

W
L ① [illegible]
E

Preprinted for the conference by

AMERICAN INSTITUTE OF CHEMICAL ENGINEERS
25 West 45 Street, New York 36, New York

I. Introduction

Heat transfer by convection from a rotating body is of importance in the thermal analysis of rotating components of various types of machinery. The rotating disk is one of a number of geometrical configurations of interest because many practical systems can be idealized in terms of a disk rotating in an infinite environment or in a housing.

The hydrodynamic phenomena associated with a disk rotating in an infinite environment have been investigated theoretically by V. Kármán (1), Cochran (2), Goldstein (3), and Stuart (4), and experimentally by Theodorsen and Regier (5), Smith (6), and Gregory and Walker (4). Heat transfer by convection has been investigated theoretically by Wagner (7) and Millsaps and Pohlhausen (8) for laminar flow and by Kreith and Taylor (9) for turbulent flow, and experimental data for cooling in air have been obtained by Young (10) in laminar flow, and by Cobb and Saunders (11) in the laminar as well as in the turbulent flow regime.

For a disk rotating in an enclosure Pantell (12) and Jimbo (13) have measured skin friction and velocity profiles experimentally, and Soo (14) and Schultz-Grunow (15) have investigated the boundary layer and flow characteristics theoretically; neither experimental nor theoretical work on heat transfer are available to date.

This paper contains the results of one phase of a long range research program designed to investigate transfer phenomena in rotating systems. In this phase of the work mass transfer rates from a disk rotating at various speeds in an infinite environment in laminar and turbulent flow were measured and compared with the results of an analysis which is also applicable to heat transfer. The effect of placing a stationary surface at various distances parallel to the rotating disk surface was investigated experimentally in the laminar flow regime. Some unusual observations of flow patterns resulting from transitional "Goertler Vortices" and from some as yet unexplained turbulent vortex phenomena are also reported.

II. Experimental Procedure

During the initial stages of developing equipment suitable for studying transfer processes from a rotating disk, an electrically heated test section was considered. The design of this test section involved the use of a guard heater at the rim of the rotating disk and sliprings for thermocouple or thermistor leads as well as for the electrical heating element. The initial cost of constructing this type of test section was estimated to be quite high and the reliability of measured heat transfer coefficients appeared to be uncertain. In order to circumvent the experimental difficulties involved in taking heat transfer and surface temperature measurements, it was decided to use a mass transfer analogy, similar to one employed by Sogin (16). This decision shifted the experimental difficulties to finding a suitable material which could withstand rotative speeds up to 10,000 rpm, but eliminated the need for taking heat flux and temperature measurements in a rotating body, and also obviated the necessity of correcting for heat leakages.

The arrangement of the equipment used in the experimental part of this study is shown in Fig. 1. An 8 in. diameter aluminum disk was mounted horizontally on a Serval "Superspeed" centrifuge which could be adjusted to and maintained at speeds ranging from 400 to 10,000 rpm.

The speed was measured by means of a stroboscope. The temperature of the air above the disk was determined with a precision thermometer. The tests were carried out in a large closed room to eliminate stray air currents.

Before each test run crystal grade naphthalene ($C_{10}H_8$, molecular weight 128.2, melting point 79-81 C, residue after ignition less than 0.001%) was melted in a covered flask in an electrically heated oven at about 120F. The molten naphthalene was then cast into the undercut and recessed center of the aluminum disk on whose rim a split retaining ring was mounted. The disk with the molten naphthalene was then cooled in air at room temperature. After the naphthalene had solidified the retainer ring was removed and the surface of the naphthalene was machined on a lathe to obtain a smooth surface raised about 1/32 in. above the rim of the disk.

Prior to each test the surface of the naphthalene was carefully cleaned of loose particles and the aluminum disk with the cast naphthalene coating was weighed on a precision balance to within 0.002 gm. The disk was then mounted on the centrifuge and brought up to speed. The time was recorded at initial weighing, at centrifuge start, at attainment of the desired speed, at centrifuge power off, at centrifuge stop, and at reweighing.

Since the measured weight difference between initial and final weighings included losses between weighing and mounting, between dismounting and weighing, and during the start up and slow down periods, appropriate corrections had to be applied to obtain the difference in weight due to mass transfer at the test speed. The losses before mounting and after dismounting were due to natural convection transfer. Mass transfer rates during natural convection were therefore measured independently, multiplied by the total time elapsed between the initial weighing and mounting and between dismounting and the final weighing, and the resulting quantity subtracted from the measured weight difference. To correct similarly for losses during speeding up and slowing down of the disk, it was assumed that the mass transfer rate varied linearly with time and that the rates of mass transfer were independent of the acceleration. These assumptions were checked by repeating test runs at a given rotational speed, but varying their total duration. Since the results were reproducible within the estimated accuracy of the experimental data, the method of correction was considered satisfactory. All test runs were of sufficient duration that the total of the mass loss corrections necessary for any of them was less than 1.5 per cent of the measured weight difference.

The average mass transfer coefficient for the system $\bar{k}_c$ is defined by the equation

$$\bar{k}_c = m R_v T_s / p_{vs} A \qquad (1)*$$

where m is the total rate of mass transfer in lb/hr

R_v is the ideal gas constant of the vapor in ft #/lb R

T_s is the surface temperature of the solid naphthalene in R

p_{vs} is the vapor pressure of the naphthalene at T_s in #/sq.ft.

A is the surface area of the naphthalene in sq. ft. and

$\bar{k}_c$ is the average mass transfer coefficient in lb/hr sq. ft. (lb/ft^3)

* The concentration of naphthalene vapor in the air above the plate is so small that no correction need to be made to distinguish the process from equimolar counterdiffusion.

In Eq. 1 the vapor pressure p_{vs} is the saturation pressure corresponding to the temperature T_s which was not actually measured. However, as shown in Ref. 17, the temperature measured by the thermometer differs from the disk surface temperature only by two small corrective terms. One accounts for the surface temperature depression by cooling due to sublimation. Its numerical value is about 0.1 F. The other term corrects for the difference between the recovery temperature indicated by the thermometer and that of the cylinder surface. For the rotating disk this term is $[(\omega r)^2/2Jg_c c_p](1-\eta_r)$ where η_r is the recovery factor. Assuming that the recovery factor for a rotating disk is of the same order of magnitude as for a flat plate at the same velocity, this correction term never exceeds 1 degree F. The omission of these correction terms does therefore not introduce an appreciable error in the results. An analysis of the accuracy of the experimental results in terms of uncertainty intervals based on 20 to 1 odds (18), indicates that the value of any of the mass transfer coefficients is accurate to within ± 6 per cent without applying a surface temperature correction.

The physical properties used in the reduction of the experimental data, including the diffusion coefficient of naphthalene vapor in air, the specific gravity of solid naphthalene, and the vapor pressure of solid naphthalene, were taken from Table 2 of Ref. 19 which is based on the best available data.

In the tests designed to study the effect of a shroud on the transfer process, the experimental procedure described above was followed, but after the disk was mounted a glass plate was placed parallel to and at a predetermined distance above the disk surface on an adjustable stand. The same data reduction method as for the free disk was used.

III. Evaluation of Average Mass and Heat Transfer Coefficients for a Disk Rotating in an Infinite Environment.

The experimental technique used in this investigation as well as that in the heat transfer measurements of Cobb and Saunders (11) yielded average coefficients of mass and heat transfer. To compare the measured values with those calculated analytically it is therefore necessary to evaluate average values of the coefficients.

As shown in Appendix I as well as in Refs. 7 and 8, the theory predicts that in the laminar flow regime mass and heat transfer coefficients are uniform over the entire surface of a rotating disk. For the system used in the experiments, i. e., for a Schmidt number of 2.4, the approximate analysis presented in Appendix I yields the equation

$$Sh = k_c r/D_v = 0.67\, Re^{0.5} \tag{2}$$

Equation 2 applies also to heat transfer for a fluid having a Prandtl number of 2.4 if the Sherwood number is replaced by the Nusselt number $h_c r/k$.

An exact solution of the complete Navier-Stokes and energy equations for the rotating disk was obtained by Millsaps and Pohlhausen (8). For cases where viscous dissipation can be neglected, the usual situation in laminar flow, their results can be reduced to the equation

$$Nu = Sh = c\, Re^{0.5} \tag{3}$$

where the constant C depends on the Schmidt or Prandtl number as shown in Table I

Table I

Pr or Sc	0.74	1.0	2.5	5.0	7.5	10
C in Eq. 3	0.33	0.39	0.60	0.80	1.0	1.1

As shown in Ref. 4, laminar flow exists at Reynolds numbers $\omega r^2/\nu$ below about 2×10^5. At about this value of Reynolds number disturbances in the laminar boundary layer on the disk are amplified and rapidly cause transition to turbulent flow. At a Reynold's number $\omega r_o^2/\nu$ larger than about 2.8×10^5 the flow is completely turbulent. On a rotating disk having a Reynolds number larger than 2.8×10^5 there exists therefore a laminar inner core surrounded by a ring shaped transition region and a ring shaped turbulent regime.

In the turbulent regime mass- and heat transfer coefficients increase with increasing radius. As shown in Appendix II, a modified mass, heat, and momentum transfer analogy predicts local values of Sherwood or Nusselt numbers given by the equations

$$Sh = \frac{Re\ Sc\ \beta_m \sqrt{C_{Dr}/2}}{5\beta_m Sc + 5\ln(5\beta_m Sc + 1) + \sqrt{2/C_{Dr}} - 14} \quad (4)$$

and

$$Nu = \frac{Re\ Pr\ \beta_H \sqrt{C_{Dr}/2}}{5\beta_H Pr + 5\ln(5\beta_H Pr + 1) + \sqrt{2/C_{Dr}} - 14} \quad (5)$$

respectively, where the symbols are defined in the nomenclature.

To evaluate average values of the mass coefficients the local values must be integrated over the entire disk surface according to the equation

$$\bar{k}_c = \frac{1}{\pi r_o^2}\left(k_{c_l}\pi r_c^2 + \int_{r_c}^{r_o} k_{c_t}\, 2\pi r\, dr\right) \qquad (6)$$

where r_c is the radius corresponding to the critical Reynolds number.

The same procedure must be followed to evaluate the average heat transfer coefficient. Since the expressions for k_{ct} and h_{ct} derived in Appendix II can not be integrated analytically, the averaging was done numerically according to

$$\bar{k}_c = k_{c_l}\left(\frac{r_c}{r_o}\right)^2 + 2\sum_{n=1}^{n=N} k_{c_t}\left(\frac{r_n}{r_o}\right)\frac{\Delta r}{r_o} \qquad (7)$$

where Δr was taken as *0.2 inches* and

$N = (r_o - r_c) / \Delta r$

In the averaging procedure the transition region was neglected and it was assumed that turbulent flow sets in at the radial distance corresponding to the critical Reynolds number (see Fig. 2). The average Sherwood number corresponding to the average value of the mass transfer coefficient was then evaluated according to

$$\overline{Sh} = \bar{k}_c r_o / D_v \qquad (8)$$

A similar procedure was followed in the evaluation of the average Nusselt numbers.

IV. Presentation and Discussion of Results

The experimental results for mass transfer from the disk rotating in an infinite environment are summarized in Table II. The speed was varied from about 500 to 10,000 rpm, corresponding to a rotational Reynolds number variation from about 3.5×10^4 to 7×10^5. At the highest Reynolds number about 70 percent of the total disk surface was covered by a turbulent boundary layer.

The test results are presented graphically in Fig. 3 where the average Sherwood number is plotted as a function of the rotational Reynolds number. Also shown in Fig. 3 are the results of theoretical analyses for the laminar flow regime and the results of the analogy calculations according to Eqs. 7 and 8. In addition to the mass transfer results, averaged heat transfer data obtained by Cobb and Saunders (11) with a disk rotating in air are compared with the results of the analogy calculations in Fig. 3.

In the laminar regime the experimental data fall slightly below the curve predicted by the analysis in Appendix I, but slightly above Millsaps and Pohlhausen's theory (8). When the Reynolds number exceeds the critical value and part of the disk surface is covered by a laminar boundary and part by a turbulent boundary layer, the experimental mass transfer data fall slightly below the results predicted by the analogy if a critical Reynolds number of 2×10^5 is used, but above the calculated curve if 2.5×10^5 is taken to be the critical Reynolds number. In view of the numerous assumptions made in the analysis the agreement between theory and experiment is quite satisfactory for a Schmidt number of 2.4.

TABLE II

SUMMARY OF RESULTS FOR

MASS TRANSFER FROM DISK ROTATING

IN AN INFINITE ENVIRONMENT

Test Run No.	Speed (RPM)	Temperature of Air (°F)	Rate of Mass Transfer (GM/Min.)	Average Sherwood Number	Reynolds Number Based on Disk Radius
1	2,900	79.7	0.02546	293	1.98 $\times 10^5$
2	4,500	77	0.04074	545	3.10
3	5,500	77	0.05776	773	3.78
4	2,000	77.4	0.01762	231	1.37
5	7,000	77.4	0.08817	1,154	4.81
6	7,200	76.6	0.08520	1,216	4.96
7	1,000	75.9	0.01137	162	0.691
8	2,000	72.9	0.01421	240	1.40
9	3,000	72.9	0.01812	305	2.09
10	6,520	74.9	0.06238	937	4.52
11	500	69.8	0.00568	114	0.352
12	1,200	72.1	0.0101	179	0.839
13	3,600	79.6	0.03364	388	2.45
14	3,300	77.9	0.02931	373	2.26
15	8,300	78.0	0.10949	1,397	5.69
16*	10,300	77.2	0.15111	2,005	7.08
17	700	72.8	0.00852	144	0.488
18	1,500	70.8	0.01065	202	1.05
19	1,730	70.9	0.01169	220	1.21
20	2,250	76.7	0.01896	258	1.55
21	2,600	77.7	0.02144	275	1.78

*In this test a small piece of solid naphthalene chipped off near the rim and it was estimated that the actual mass transfer rate is about 6 per cent less than the measured value.

The experimental heat transfer data, that is the average Nusselt numbers for the disk, agree in the laminar regime within 5 percent with the results predicted theoretically in Refs. 7 and 8. At turbulent Reynolds numbers the heat transfer data fall as much as 10 per cent below the result predicted on the basis of the analogy calculations for a Prandtl number of 0.74 and a/β_H of unity. One possible reason for this deviation is that the analogy calculations take no account of variations in physical properties which are insignificant in mass transfer, but should be taken into account in heat transfer. Moreover, since the disk with which the heat transfer data were obtained was electrically heated, the surface temperature may not have been uniform. It is of course also possible that the assumptions made in the analogy calculations could be responsible for the deviation between theory and experiment.

Some observations incidental to the primary objective of the research are worth mentioning. At the end of several tests in the turbulent Reynolds number range the imprints of certain features of the flow could be observed on the naphthalene surface. These traces corroborate the observations made by Gregory and Walker (4) by means of the china-clay evaporation technique in the transition regime. According to these authors a vortex system is generated in the region of flow upstream of transition, and between two critical radii vortices occur which are stationary relative to the moving surface. Also the imprints left on the naphthalene surface indicate the presence of fairly large scale vortex patterns locked to the surface in the transition regime. However, whereas in the British experiments (4) no traces of the vortex pattern were visible in the turbulent regime, the imprints on the naphthalene suggest that also in the fully developed turbulent regime vortices remain attached to the disk surface and penetrate the so called laminar sublayer in a more or less regular manner. The scale of the vortices in the turbulent regime appears to be much smaller than in the transition regime. To verify the existence of stationary vortices in the turbulent zone the tests should be repeated with a disk whose surface is covered with china-clay, but instead of looking for traces merely after completion of a test, the streak pattern should be recorded photographically throughout the test. It is planned to perform such tests in the future.

The effect of a shroud on the mass transfer process from a rotating disk was studied experimentally in the laminar flow regime at rotational speeds of 1,000 and 2,000 rpm, corresponding to rotational Reynolds numbers of about 7×10^4 and 1.4×10^5. The test results are summarized in Table III and presented graphically in Fig. 4 where the ratio of the average Sherwood number for the shrouded disk to the Sherwood number for the free disk at the same rotational Reynolds number $\overline{Sh}_y/\overline{Sh}_\infty$ is plotted versus the ratio of the distance between the disk and the shroud surface and the disk diameter, y/d.

An inspection of the results shows that within the range of Reynolds number covered in the experiments the effect of the Reynolds number on the Sherwood number ratio is less than the scatter of the data. The presence of a shrouding surface decreases the rate of mass transfer. At a clearance to diameter ratio of 0.25 the mass transfer for the shrouded disk is only about 10 percent less than that for a comparable free disk

TABLE III

SUMMARY OF RESULTS FOR MASS TRANSFER

FROM A DISK ROTATING IN A SHROUD

Test Run No.	Speed (RPM)	Dimensionless Shrouding Distance (y/d)	Sherwood No. Ratio ($\overline{Sh}_y/\overline{Sh}_\infty$)
a	1,000	0.143	0.871
b	1,000	0.059	0.823
c	1,000	0.129	0.878
d	1,000	0.031	0.717
e	1,000	0.094	0.872
f	1,000	0.250	0.896
h	2,000	0.079	0.898
i	2,000	0.031	0.754
j	2,000	0.035	0.776
k	2,000	0.023	0.741
l	2,000	0.016	0.651
m	2,000	0.012	0.585
n	2,000	0.250	0.926

and drops to 80 percent of the free disk mass transfer rate at a clearance ratio of 0.05. As the clearance is further decreased the mass transfer rate decreases rapidly.

These observations are in qualitative agreement with analyses of the boundary characteristics in this sort of system (14, 15). When the fixed boundary is sufficiently far away from the surface of the rotating disk there will be two distinct boundary layers, one on the stationary, the other one on the rotating surface, leaving between them a "core" of finite width which rotates at approximately one half the velocity of the disk. In this regime the presence of the stationary surface would not be expected to affect the heat or mass transfer appreciably. However, as the clearance is decreased a point is reached where the two boundary layers meet. Any further decrease in clearance will impede the circulation appreciably and eventually cause choking of the central core. In this regime the rate of heat or mass transfer would be expected to drop off rapidly with decreasing clearance. The experimental results corroborate these phenomena qualitatively, but additional studies of the flow and boundary layer characteristics are necessary before the various flow regimes can be delineated quantitatively.

V. Conclusions

1. The analogy between heat, mass and momentum transfer can be applied to a disk rotating in an infinite environment to calculate heat and mass transfer coefficient from friction coefficients in the range of Prandtl and Schmidt numbers between 0.7 and 2.5.

2. In the laminar regime of rotational Reynolds numbers, a stationary surface placed parallel to the surface of a rotating disk reduces the rate of mass transfer, but the reduction is less than 20 percent of the transfer rate for a free disk at clearance to diameter ratios larger than 0.05.

3. Traces left by the air flowing over a disk rotating in an infinite environment suggest that stationary vortices occur in the turbulent as well as in the transition regime.

Acknowledgment

The work reported here was accomplished as a part of a research program at Lehigh University supported by a grant from the National Science Foundation.

NOMENCLATURE

The following nomenclature is used in this paper:

A = heat or mass transfer area, sq. ft.

C_{Dr} = local drag coefficient at r

$\bar{C}_D$ = average drag coefficient, $\frac{1}{\pi r_o^2} \int_o^{r_o} C_{Dr} \; 2\pi \; rdr$

c_p = specific heat at constant pressure, Btu/lb F

d = diameter of disk, ft.

D_v = diffusivity of vapor in mass transfer, sq.-ft./hr.

g_c = conversion factor, 32.2 x $(3600)^2$ ft. lb/# hr.^{2}

h_c = local convection heat transfer coefficient at r, Btu/hr. sq.-ft. F

$\bar{h}_c$ = average convection heat transfer coefficient,

$\frac{1}{\pi r_o^2} \int_o^{r_o} h_c \; 2\pi \; rdr$, Btu/hr. sq.-ft. F

k = thermal conductivity, Btu/hr. ft. F

k_c = local mass transfer coefficient at r, lb/hr. sq. -ft. (lb/cu.-ft.)

$\bar{k}_c$ = average mass transfer coefficient

$= \frac{1}{\pi r_o^2} \int_o^{r_o} k_c \; 2\pi \; rdr$, lb/hr. sq.-ft. (lb/cu.-ft.)

M = mass-flux parameter $(m/A)_s$ lb/hr. sq.-ft.

m = rate of mass transfer, lb/hr.

p_v = partial pressure of vapor component A, #/sq.-ft.

P = total pressure, #/sq.-ft.

q_c = rate of heat transfer by convection, Btu/hr.

Q = heat flux parameter $(q/A)_s/c_p\rho$, ft. F/hr.

r = radial distance, ft.

r_c = critical radius at which transition from laminar to turbulent flow begins, ft.

r_o = outside radius of disk, ft.

R_v = ideal gas constant for naphthalene vapor, ft. #/lb. R

Nomenclature (continued)

S = $\sqrt{g_c \tau_s / \rho}$, shear velocity, ft./hr.

T = temperature, F

V (r) = net velocity at r relative to disk velocity at r, ft./hr.

u_r = radial velocity component, ft./hr.

u = peripheral velocity component, ft./hr.

y = distance from disk surface, ft.

α = thermal diffusivity, $k/\rho c_p$, sq.-ft./hr.

ϵ_M = eddy diffusivity for momentum, sq.-ft./hr.

ϵ_H = eddy diffusivity for heat, sq.-ft./hr.

ϵ_m = eddy diffusivity for mass, sq.-ft./hr.

δ = hydrodynamic boundary layer thickness, ft.

δ_H = thermal boundary layer thickness, ft.

δ_m = mass transfer boundary layer thickness, ft.

ν = kinematic viscosity, $g_c \mu / \rho$, sq.-ft./hr.

ω = rotational speed, rad./hr.

ρ = fluid density, lb/cu. ft.

τ = fluid shear stress, #/sq.-ft.

μ = fluid viscosity, # hr./sq.-ft.

V^+ = nondimensional velocity in the boundary layer, V/S

y = distance from disk surface, ft.

Y = clearance between disk surface and stationary shroud surface, ft.

y^+ = nondimensional distance from the surface, yS/ν

Nu = local Nusselt number, $h_c\ r/k$

$\overline{Nu}$ = average Nusselt number, $\bar{h}_c\ r_o/k$

Sc = Schmidt number, ν / D_v

β_M = ratio of eddy diffusivities of mass and momentum ϵ_m / ϵ_M

β_H = ratio of eddy diffusivities of heat and momentum ϵ_H / ϵ_M

Nomenclature (continued)

Sh = local Sherwood number, $k_c r/D_v$

$\overline{Sh}$ = average Sherwood number, $\overline{k}_c r_o/D_v$

$\overline{Sh}_Y$ = average Sherwood number for a shrouded disk with clearance y

St = local Stanton number, $h_c/\omega r \rho c_p$

$\overline{St}$ = average Stanton number $\overline{h}_c/\omega r_o \rho c_p$

Re = local rotational Reynolds number, $\omega r^2/\nu$

Re_o = rotational Reynolds number, $\omega r_o^2/\nu$

Pr = Prandtl number, ν/α

Subscripts

H - heat transfer

l - laminar

M - momentum transfer

m - mass transfer

o - outside radius of disk

s - surface of disk

t - turbulent

∞ - far away from disk

b - buffer layer

BIBLIOGRAPHY

1. "Über Laminare und Turbulente Reibung" by Th. von Karman, Z. ang. Math. Mech., Vol. 1, 1921, p. 233.

2. "The Flow Due to a Rotating Disk" by W. G. Cochran, Proc. Camb. Phil. Soc., Vol. 30, 1934, p. 365.

3. "On the Resistance of a Disk Immersed in a Fluid" by S. Goldstein, Proc. Camb. Phil. Soc., Vol. 31, 1935, p. 232.

4. "On the Stability of Three-Dimensional Boundary Layers with Application to Flow due to a Rotating Disk" by N. Gregory, J. T. Stuart, and W. S. Walker, Phil. Trans. Roy. Soc. London, Math. Phys. Sci., A, Vol. 248, 1956, p. 155.

5. "Experiments on Drag of Revolving Disks, Cylinders, and Streamlined Rods at High Speeds" by T. Theodorsen and A. Regier, NACA Rep. No. 793, 1944.

6. "Exploratory Investigation of Laminar Boundary Layer Oscillations on a Rotating Disk" by N. S. Smith, NACA TN No. 1227, 1947.

7. "Heat Transfer from a Rotating Disk to Ambient Air" by C. Wagner, J. of Appl. Physics, Vol. 19, 1948, p. 837.

8. "Heat Transfer by Laminar Flow from a Rotating Plate" by K. Millsaps and K. Pohlhausen, J. of Aero. Sci., Vol. 19, 1952, p. 120.

9. "Heat Transfer from a Rotating Disk in Turbulent Flow" by F. Kreith and J. H. Taylor, ASME paper No. 56-A-146, 1956.

10. "Heat Transfer from a Rotating Plate" by R. L. Young, ASME Trans., Vol. 78, 1956, p. 1163.

11. "Heat Transfer from a Rotating Disk", by E. C. Cobb and O. A. Saunders, Proc. Roy. Soc., Vol. 236, 1956, p. 343.

12. "Versuche über Scheibenreibung" by K. Pantell, Forsch. Geb. Ing. wes, Vol. 16, 1949, p. 97.

13. "Investigation of the Interaction of Windage and Leakage Phenomena in a Centrifugal Compressor" by H. Jimbo, ASME paper No. 56-A-47, 1956.

14. "Laminar Flow over an Enclosed Rotating Disk" by S. L. Soo, ASME Trans., Vol. 80, 1958, p. 287.

15. "Der Reibungswiderstand rotierender Scheiben in Gehäusen" by F. Schultz-Grunow, Z. f. ang. Math. u. Mech., Vol. 15, 1935, p. 191.

16. "Sublimation from Disks to Air Streams Flowing Normal to their Surface" by H. H. Sogin, Trans. ASME, Vol. 80, 1958, p. 593.

17. "Laminar Mass and Heat Transfer from Elliposidal Surfaces of Fineness Ratio 4 in Axisymmetric Flow" by Shao-Yen Ko and H. H. Sogin, ASME paper No. 57-SA-44, 1957.

18. "Describing Uncertainties in Single-Sample Experiments" by S. J. Klein and F. A. McClintock, Mechanical Engineering, Vol. 75, 1953, p. 3.

19. "Experimental Investigation of Mass Transfer by Sublimation from Sharp-Edged Cylinders in Axisymmetric Flow with Laminar Boundary Layer" by W. J. Christian and S. P. Kezios, 1957 Heat Transfer and Fluid Mechanics Institute, Calif. Inst. of Tech., Pasadena, Calif., June, 1957.

20. "The Analogy between Fluid Friction and Heat Transfer" by Th. von Karman, Trans. ASME, Vol. 61, 1939, p. 705.

21. Introduction to the Transfer of Heat and Mass by E. R. G. Eckert, McGraw Hill Book Company, Inc., New York, N. Y., 1950, pp. 120-125.

22. "Analysis of Turbulent Heat Transfer, Mass Transfer, and Friction in Smooth Tubes at High Prandtl and Schmidt Numbers" by R. G. Deissler, NACA TN 3145, 1954.

23. "Heat Transfer, Mass Transfer, and Fluid Friction - Relationships in Turbulent Flow" by T. K. Sherwood, Ind. and Eng. Chem., Vol. 42, 1950, p. 2077.

24. "Fluid Dynamics and Heat Transfer" by J. G. Knudsen and D. L. Katz, Eng. Res. Bul. No. 37, Eng. Res. Inst. Univ. of Mich., Ann Arbor, 1953.

25. "Heat Transfer from a Rotating Cylinder With and Without Cross Flow" by W. M. Kays and I. S. Bjorklund, Trans. ASME, Vol. 80, 1958, p.

26. "Momentum and Mass Transfer by Eddy Diffusion in a Wetted-Wall Channel" by A. M. Dhanak, paper No. 57-HT-32, Heat Transfer Conference, University Park, Pa., 1957.

27. "Turbulence in Thermal and Material Transport" by J. P. Opfell and B. H. Sage, Advances in Chemical Engineering, Vol. I. T. B. Drew and J. W. Hoopes, Jr., ed., Academic Press Inc., New York, 1956.

28. Die Grundegesetze der Waermeuebertragung by U. Grigull, Springer Verlag, Berlin 1955.

29. "The Transfer of Heat and Momentum in a Turbulent Stream of Mercury" by H. E. Brown, B.H. Amstead, and B.E. Short, ASME paper No. 55-A-106.

30. "Variation of the Eddy Conductivity with Prandtl Modulus and its Prediction of Turbulent Heat Transfer Coefficients" by R. Jenkins, Heat Transfer and Fluid Mechanics Institute, Stanford University, 1951, p. 147.

Appendix I

Approximate Calculations of Rates of Heat and Mass Transfer from a Disk Rotating in an Infinite Environment in Laminar Flow.

In cylindrical coordinates the steady state equation for mass transfer of component A by molecular diffusion through an inert gas in laminar motion is

$$D_v\left(\frac{\partial^2 c}{\partial r^2}+\frac{1}{r}\frac{\partial c}{\partial r}+\frac{\partial^2 c}{\partial y^2}\right)-u_r\frac{\partial c}{\partial r}-u_y\frac{\partial c}{\partial y}=0 \qquad 1$$

where D_v is the diffusivity of component A in ft^2/hr and c is the concentration of component A in the inert gas in lb/cu. ft.

The corresponding equation for heat transfer to or from a rotating plate is

$$\alpha\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial y^2}\right)-u_r\frac{\partial T}{\partial r}-u_y\frac{\partial T}{\partial y}=0 \qquad 2$$

if viscous dissipation is neglected and the fluid properties are uniform. Assuming that the temperature or the concentration at the surface of the rotating plate is uniform and that the temperature or the concentration in the fluid far away from the plate is zero, the boundary conditions pertaining to the system are

at $y = 0$: $T = T_s$ or $c = c_s$

at $y = \infty$ $T = 0$ or $c = 0$

Von Kármán (1) and Millsaps and Pohlhausen (8) have shown that the boundary layer thickness δ is constant over the entire plate, provided the small influence of flow toward the plate is neglected; and, that the radial flow velocity u_r at a given distance y from the disk surface is proportional to r. Consequently the total mass rate of fluid flowing outward in the radial direction at any distance r is proportional to the area πr^2. Thus a concentration or temperature distribution dependent on y, but independent of r may be assumed (7). Equations 1 and 2 reduce then to the ordinary differential equations

$$(D_v)\left(\frac{d^2c}{dy^2}\right)-(u_y)\left(\frac{dc}{dy}\right)=0 \qquad 3$$

and

$$(\alpha)\left(\frac{d^2T}{dy^2}\right)-(u_y)\left(\frac{dT}{dy}\right)=0 \qquad 4$$

respectively. Equations 3 and 4 yield upon integration,

$$\frac{dc}{dy}=-exp\left(\frac{1}{D_v}\int_0^y u_y\,dy+C\right) \qquad 5$$

and

$$\frac{dT}{dy}=-exp\left(\frac{1}{\alpha}\int_0^y u_y\,dy+C\right) \qquad 6$$

Another integration gives

$$c=\int_y^\infty exp\left(\frac{1}{D_v}\int_0^y u_y\,dy+C\right)\cdot dy \qquad 7$$

and

$$T=\int_y^\infty exp\left(\frac{1}{\alpha}\int_0^y u_y\,dy+C\right)\cdot dy \qquad 8$$

The integration constant C can be evaluated from the boundary condition at y = 0 while the boundary condition at y = oo is automatically satisfied by the upper limit of the first integral in Eqs. 7 and 8. The concentration or the temperature at the plate surface are according to Eqs. 7 and 8 respectively

$$c_s=\int_0^\infty exp\left(\frac{1}{D_v}\int_0^y u_y\,dy+C\right)\cdot dy \qquad 9$$

and

$$T_s=\int_0^\infty exp\left(\frac{1}{\alpha}\int_0^y u_y\,dy+C\right)\cdot dy \qquad 10$$

On division of Eq. 5 by Eq. 9 or Eq. 6 by Eq. 10 the integration constant C drops out and

$$\frac{dc}{dy} = -C_s \frac{\exp\left(\frac{1}{D_v}\int_0^y u_y\,dy\right)}{\int_0^\infty \exp\left(\frac{1}{D_v}\int_0^y u_y\,dy\right)\cdot dy} \qquad 11$$

or

$$\frac{dT}{dy} = -T_s \frac{\exp\left(\frac{1}{\alpha}\int_0^y u_y\,dy\right)}{\int_0^\infty \exp\left(\frac{1}{\alpha}\int_0^y u_y\,dy\right)\cdot dy} \qquad 12$$

For naphthalene vapor diffusing from the solid naphthalene - air interface at the disk surface into the surrounding air the concentration in the gas phase is related to the partial pressure p_v by the equation

$$c = p_v / R_v T \qquad 13$$

where p_v is the partial pressure of the naphthalene vapor in the air in psf, R_v is the ideal gas constant in ft #/lb R, and T is the absolute temperature in deg. R. The mass transfer coefficient k_c and the heat transfer coefficient h_c are given by

$$k_c = \frac{1}{c_s}\left[-D_v\left(\frac{\partial c}{\partial y}\right)_{y=0}\right] = \frac{1}{p_{v,s}}\left[-D_v\left(\frac{\partial p_v}{\partial y}\right)_{y=0}\right] \qquad 14$$
$$= D_v \Big/ \int_0^\infty \exp\left(\frac{1}{D_v}\int_0^y u_y\,dy\right)\cdot dy$$

and

$$h_c = \frac{1}{T_s}\left[-k\left(\frac{\partial T}{\partial y}\right)_{y=0}\right] = k \Big/ \int_0^\infty \exp\left(\frac{1}{\alpha}\int_0^y u_y\,dy\right)\cdot dy \qquad 15$$

The corresponding dimensionless numbers, the Sherwood number and the Nusselt number are

$$Sh = \frac{k_c r}{D_v} = r \Big/ \int_0^\infty \exp\left(\frac{1}{D_v}\int_0^y u_y\,dy\right)\cdot dy \qquad 16$$

and

$$Nu = \frac{h_c r}{k} = r \Big/ \int_0^{\infty} \exp\left(\frac{1}{\alpha}\int_0^{y} u_y\,dy\right)\cdot dy \qquad 17$$

The integrals of Eqs. 16 and 17 can be evaluated graphically with the aid of Von Kármán's solution (1) of the hydrodynamical problem according to which

$$\text{for } y \leqq \delta\,,\ u_y = -2\omega\int_0^{y}\left[1.026\left(\frac{y}{\delta}\right)\left(1-\frac{y}{\delta}\right)^2\left(1+2\frac{y}{\delta}\right)-\frac{1}{2}\left(\frac{y}{\delta}\right)^2\left(1-\frac{y}{\delta}\right)^2\right]dy$$

$$= -2\omega y\left[0.31\left(\frac{y}{\delta}\right)^4-0.52\left(\frac{y}{\delta}\right)^3-0.17\left(\frac{y}{\delta}\right)^2+0.51\left(\frac{y}{\delta}\right)\right] \qquad 18$$

$$\text{and for } y > \delta,\ u_y = -0.708(\nu\omega)^{\frac{1}{2}} \qquad 19$$

$$\text{where } \delta = 2.58\left(\frac{\nu}{\omega}\right)^{\frac{1}{2}} \qquad 20$$

The mean temperature during the experiments was 74 F. At atmospheric pressure and 74 F the kinematic viscosity of air ν is 0.60 ft^2/hr (19) and the vapor diffusivity D_v is 0.25 ft^2/hr (19), so that Sc = 2.4. With this value for D_v Eq. 16 becomes upon evaluation of the integral in the denominator

$$Sh = 0.67\left(\frac{\omega r^2}{\nu}\right)^{\frac{1}{2}} = 0.67\,Re^{\frac{1}{2}} \qquad 21$$

Similarly, for Pr = 2.4 the Nusselt number is

$$Nu = 0.67\left(\frac{\omega r^2}{\nu}\right)^{\frac{1}{2}} = 0.67\,Re^{\frac{1}{2}} \qquad 22$$

Wagner (7) evaluated the heat transfer coefficient in air (Pr = 0.74) and found

$$Nu = 0.34\left(\frac{\omega r^2}{\nu}\right)^{\frac{1}{2}} = 0.34\,Re^{\frac{1}{2}} \qquad 23$$

It should be noted that both heat and mass transfer coefficients are uniform over the entire surface in laminar flow.

Appendix II

Approximate Calculation of Rates of Heat and Mass Transfer from a Disk Rotating in an Infinite Environment in Turbulent Flow.

The analogy between heat-, mass-, and momentum- transfer has been used successfully to predict heat- and mass- transfer coefficients from experimentally measured friction coefficients. Typical analogy solutions for flow inside of tubes or ducts and for flow over plane surfaces are described in Refs. 20, 21, 22, 23, and 24. Kays and Bjorklund (25) obtained recently an analogy solution for a circular cylinder rotating in an infinite environment and found that the solution agreed satisfactorily with experimental data for air.

The analogy method is based on the assumption that the transfer mechanisms for heat, mass, and momentum are physically similar, or can at least be described by similar equations of the form

$$g_c \tau_s / \rho = (\epsilon_M + \nu)\left(\frac{dV}{dy}\right) \qquad 1$$

for the momentum transfer and resulting shearing stress, by

$$\left(\frac{q}{A}\right)_s / \rho c_p = (\epsilon_H + \alpha)\left(\frac{dT}{dy}\right) \qquad 2$$

for the rate of heat transfer, and by

$$\left(\frac{m}{A}\right)_s = (\epsilon_m + D_v)\left(\frac{dc}{dy}\right) \qquad 3$$

for the rate of mass transfer, where ϵ_M, ϵ_H, and ϵ_m are dependent on the amount and kind of turbulent mixing at a point. To complete the analogy one assumes that $\epsilon_H = \beta_H \epsilon_M$ and $\epsilon_m = \beta_m \epsilon_M$ where β_H and β_m must be determined theoretically or experimentally inasmuch as the equation of motions for a fluid contains terms which are not present in the diffusion equation. In the vicinity of the surface the shear stress, the heat flux, and the mass flux are nearly constant and if the fluid properties are uniform, Eqs. 1, 2, and 3 can be written with reference to conditions at the surface as

$$\nu = (\epsilon_M + \nu)\left(\frac{dV^+}{dy^+}\right) \qquad 4$$

$$Q = \left[\beta_H \epsilon_M + \frac{\nu}{Pr}\right]\frac{S}{\nu}\cdot\frac{dT}{dy^+} \qquad 5$$

and

$$M = \left[\beta_m \epsilon_M + \frac{\nu}{Sc}\right]\frac{S}{\nu}\cdot\frac{dc}{dy^+} \qquad 6$$

The application of the analogy method to heat and mass transfer from a rotating disk is greatly simplified if one assumes that the flow in the immediate vicinity of the disk surface is similar to the flow near the wall of a pipe. This assumption can be partly justified on the basis of available data on the drag coefficients in the turbulent regime (5) which are well correlated by the equation

$$\frac{1}{\sqrt{C_{D_r}}} = constant + 4.07 \log_{10}\left(Re_r\sqrt{C_{D_r}}\right) \qquad 7$$

Equation 7 is of the same form as the well known Kármán-Nikuradse resistance equation for pipe flow and the coefficient 4.07 corresponds to a magnitude of 0.4 for the universal non-dimensional profile constant for turbulent flow near a surface. Moreover, measurements by Gregory and Walker (4) and Theodorsen and Regier (5) indicate that the velocity profile near a disk is logarithmic outside the so called laminar sub-layer and that the radial velocity component is of a smaller order of magnitude than the tangential velocity component.

Although boundary layers on rotating disks are three dimensional and the differential equations for energy and momentum as well as the boundary conditions are dissimilar, for the purpose of the following approximate calculations a conventional analogy approximation was made. The flow field was divided into a socalled laminar sublayer extending from $y^+ = 0$ to $y^+ = 5$, a bufferlayer from $y^+ = 5$ to $y^+ = 30$, and a turbulent region beyond $y^+ = 30$. The upper limit of the buffer layer thickness was estimated by comparing the temperature distribution predicted on the basis of strict analogy calculations (25) with the temperature profile measured by Cobb and Saunders (11). The solid line in Fig. 5 represents the calculated temperature profile while the dashed line was faired through experimental data taken from Ref. 11. The experimental data follow the predicted curve quite well up to a value of y^+ of about 30, but beyond this point the experimental temperature gradient becomes less than the calculated value. On the basis of similar observations in experiments on heat transfer from a rotating cylinder Kays and Bjorklund (25) suggested that in the region beyond $y^+ = 20$ the transfer mechanism could be approximated by the simple Reynolds analogy. Although the two systems are dissimilar in many respects, the Reynolds analogy was found to be a satisfactory approximation for heat- and mass- transfer from a rotating disk beyond $y^+ = 30$. Although the following derivation is for mass transfer, it applies equally well to heat transfer since Eqs. 2 and 3 are analogous.

In the laminar sublayer $V^+ = y^+$, $\epsilon_M = 0$ and integration of Eq. 6 gives therefore

$$\Delta c_{\ell} = 5 M\, Sc / S \qquad (8)$$

In the buffer layer between $y^+ = 5$ and $y^+ = 30$ the von Kármán-Nikuradse profile (21) is

$$V^+ = -3.05 + 5.0 \ln y^+ \qquad 9$$

which gives a dimensionless velocity gradient $dV^+/dy^+ = 5/y^+$ substituting $5/y^+$ for the velocity gradient in Eq. 4 and solving for ϵ_M yields

$$\epsilon_M = \left(\frac{\nu y^+}{5} - \nu\right)$$

so that

$$\epsilon_m = \beta_m \nu \left(\frac{y^+}{5} - 1\right)$$

Eq. 6 can then be written as

$$M = \left(\beta_m \frac{y^+}{5} - \beta_m + \frac{1}{Sc}\right) S \cdot \frac{dc}{dy^+}$$

so that

$$dC_b = \frac{5M}{\beta_m S} \cdot \frac{dy^+}{y^+ - 5\left(1 - \frac{1}{\beta_m Sc}\right)}$$

Integration between $y^+ = 5$ and 30 yields the concentration potential drop in the buffer layer ΔC_b, or

$$\Delta C_b = \frac{5M}{\beta_m S} \cdot \ln(5\beta_m Sc + 1) \qquad 10$$

In the flow regime outside of $y^+ = 30$ it will be assumed that the intensity of transfer processes by eddy diffusion is much greater than by molecular diffusion and that the shear stress and mass flux are the same function of y so that

$$\frac{\nu}{M} = \frac{1}{\beta_m} \cdot \frac{\nu}{S} \cdot \frac{dV^+}{dc}$$

Integration between $y^+ = 30$ and ∞ yields ΔC_t, the concentration potential at the outer edge of the buffer layer, or

$$\Delta C_t = \frac{M}{\beta_m S} \int_{V^+=14}^{V^+=\frac{\omega r}{S}} dV^+ = \frac{M}{\beta_m S}\left(\frac{\omega r}{S} - 14\right) \qquad 11$$

Adding the concentration potentials and noting that $\omega r/s = \sqrt{2/C_{Dr}}$ gives

$$\Delta C = C_S - 0 = \frac{M}{\beta_m S}\left[5\beta_m Sc + 5\ln(5\beta_m Sc + 1) + \sqrt{2/C_{Dr}} - 14\right] \qquad 12$$

so that

$$\frac{Sh}{ReSc} = \frac{M}{\Delta c\,\omega r} = \frac{\beta_m\sqrt{C_{Dr}/2}}{5\beta_m Sc + 5\ln(5\beta_m Sc + 1) + \sqrt{2/C_{Dr}} - 14} \qquad 13$$

For the calculations in this paper, the experimental results of Dhanak (26) were used to evaluate β_m and the local drag coefficients were taken from Ref. 5.

Similarly, the analogy gives for heat transfer

$$St = \frac{Nu}{RePr} = \frac{\beta_H\sqrt{C_{Dr}/2}}{5\beta_H Pr + 5\ln(5\beta_H Pr + 1) + \sqrt{2/C_{Dr}} - 14} \qquad 14$$

The selection of an appropriate value of β_H for the numerical evaluation of Eq. 14 was difficult because even for a pipe or a duct available experimental results are inconclusive (27, 28, 29, 30). Experimental measurements of β_H taken with air flowing in rectangular channels are summarized in Ref. 27. They indicate that the space average value of β_H increases with Reynolds number from about 1.1 at a Reynolds number of 2×10^7 to 1.4 at a Reynolds number of 9×10^7 but varies also with the distance from the surface at any given Reynolds number (27, 28). Data obtained with liquid metals (29) give values of β_H in the range from 0.7 to 0.9.

In the past a value of unity for β_H (20,22,25) has been used with good success for heat transfer analogy calculations. Also in this study a value of β_H equal to unity was found to yield satisfactory results, giving agreement within 10 per cent between the Nusselt numbers calculated from Eq. 14 and the experimental results of Ref. 11. For comparison the calculations were repeated with β_H equal to 1.4, the other extreme. This caused the calculated

Nusselt numbers to deviate about 27 per cent from the measured values.

In passing it is interesting to note that semi-empirical methods deduced originally from experimental data on the basis of a thought model implying a strictly two-dimensional structure of the turbulent boundary layer have in this work and in Ref. 25 been applied to systems in which the boundary layer structure must be three-dimensional in nature. Since these methods are apparently applicable to cases in which the structure of the boundary layer can by no stretch of the imagination be considered two-dimensional, it seems appropriate to question the hypothesis of a two-dimensional turbulent boundary layer in the other cases. The penetration of the so called laminar sublayer by vortices should supply additional food for thought.

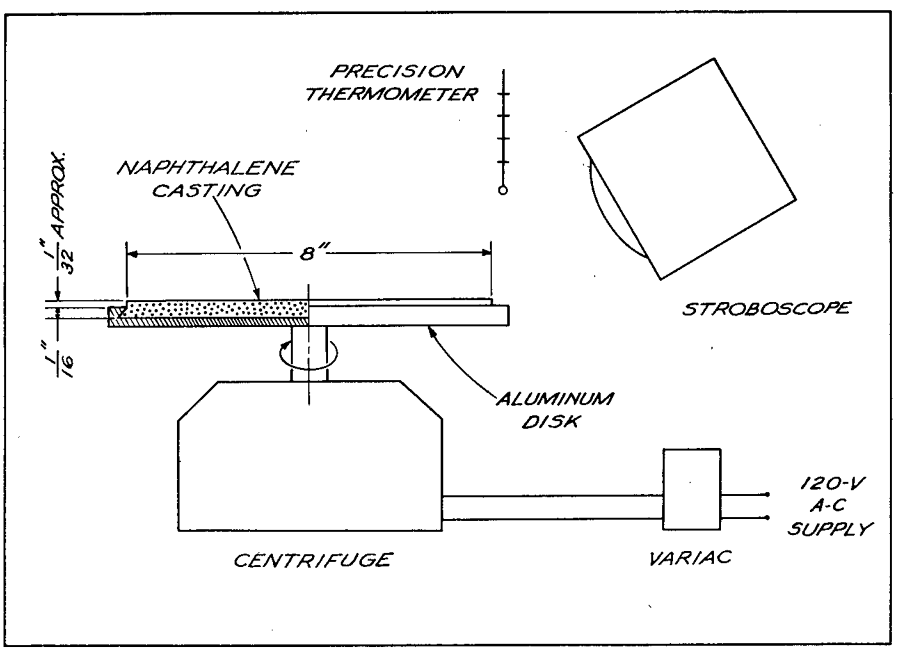

FIGURE-1. SCHEMATIC SKETCH OF TEST EQUIPMENT

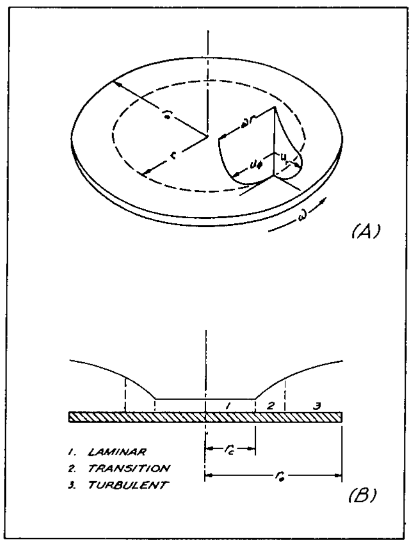

FIGURE-2. (A) VELOCITY PROFILE WITH OBSERVER RIDING ON DISK
(B) BOUNDARY LAYER PROFILE

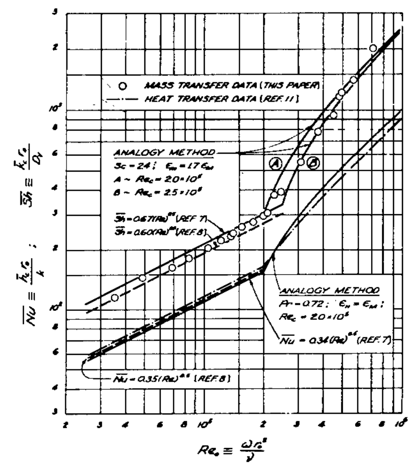

FIGURE-3 HEAT AND MASS TRANSFER FROM A ROTATING DISK

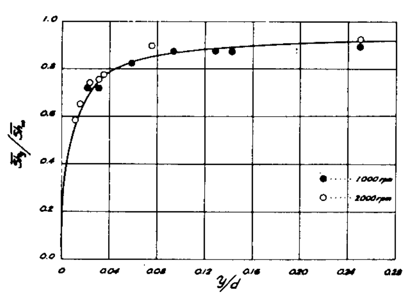

FIGURE-4. SHROUDING EFFECT ON MASS TRANSFER

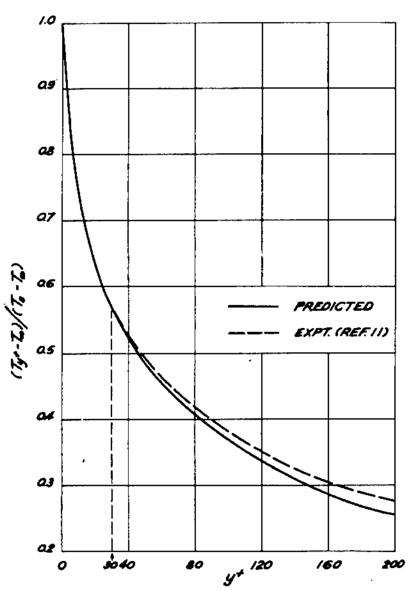

FIGURE-5 TEMPERATURE PROFILE

20-1abcd