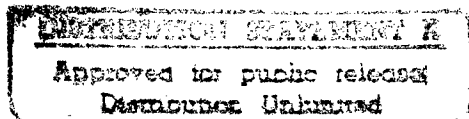AECD - 1850

UNITED STATES ATOMIC ENERGY COMMISSION

PRELIMINARY REPORT ON THE "THREE QUARTER WAVE" R.F. SYSTEM

FOR FREQUENCY MODULATED CYCLOTRONS

by

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PRELIMINARY REPORT ON THE "THREE QUARTER WAVE" R.F. SYSTEM

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ABSTRACT

This report covers the theory and model test results on a radio frequency system for a frequency modulated cyclotron as required for acceleration of 350 Mev protons. A rotating condenser is used at approximately one-third of the length of the dee line from the shorted end. It is shown to be possible to operate through the range of approximately 9 to 24 megacycles per second as required for the acceleration of either protons or deuterons in the 184-inch Berkeley cyclotron.

INTRODUCTION

The following report is primarily written as information for the people working on the 184-inch project. By the time it is available, the experimental results quoted in the report will be superseded, it is hoped, by much more recent and encouraging data. However, the theory should apply, and may help to clarify some of the improvements found by empirical methods.

This r.f. system, which, for lack of a better name will be referred to as the "three quarter wave system," seems to be capable of meeting the demands of any of the frequency modulated cyclotrons planned at the present time (excluding those designs in which both magnetic field and frequency are varied). It seems theoretically possible to build a unit with a frequency change in the ratio of about 3 to 1. The experimental attainment of such a continuous frequency change is limited, at the moment, by methods of coupling the oscillator tube to the system. A ratio of 2.5 to 1 has been achieved but not with the desired efficiency throughout the range.

The advantage of this system is that the rotating condenser, which affects the frequency change, may be located outside the magnetic field and therefore avoid eddy current losses. In addition, much more space is available, allowing a more rugged and serviceable unit. A further consideration which may turn out to be quite important, is that servicing and repairing the unit can be done without much danger of an overdose of radiation from a "hot" dee.

To offset this advantage, it should be stated that the system is much more bulky than designs which employ a rotating condenser on the dee directly. It uses more power and has many more possible modes of oscillation. Of course the possibility of numerous modes should not be considered a disadvantage after all but one of the modes have been eliminated by proper design. The power consumption of the oscillator is small compared to magnet power, so bulkiness is probably the only feature which might be objectionable.

PRINCIPLE OF OPERATION

The operation of the system may be easily understood by referring to Figure 1 which represents a uniform transmission line 3/4 wavelength long, shorted at one end and a series variable condenser

$$\text{————— } f$$
$$\text{— —— —— } \tfrac{2}{3} f$$
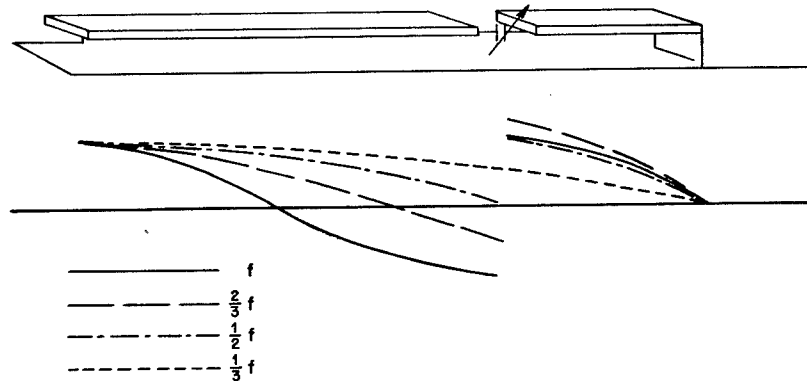$$\text{—·—·—·— } \tfrac{1}{2} f$$
$$\text{———————— } \tfrac{1}{3} f$$

Figure 1.

located 1/4 wavelength from the short. The voltage distribution is shown for 4 different settings of the variable condenser. Case 1 represents the practically unattainable condition of zero minimum capacity. Cases 2 and 3 represent intermediate settings of the condenser, and case 4 represents the case of infinite capacity or, in other words, a dead short, and the system behaves as a 1/4 wave system at 1/3 the original frequency.

Figure 2 represents the other fundamental mode of the system. Since there is no voltage across the condenser, its value is unimportant and the frequency of this mode is constant and coincides with the upper limit of the other mode. This mode will be referred to as the parallel or symmetric mode. Since we can never achieve zero minimum capacity, the frequency of the symmetric mode will always lie above the upper limit of the antisymmetric mode.

In Figure 1 the left-hand end of the line is supposed to represent the edge of the dee and in its simplest form the system would take the shape of a rectangular box, 3/4 wavelength long and as wide as the diameter of the magnet pole face. One-half wavelength from the dee edge we would insert a rotary condenser (Figure 3).

The oscillator tube is shown directly coupled to the system. The reason this is possible is that over about a 2 to 1 frequency shift, the voltage on the 1/4 wave-shorted line (which will be referred to as the "stub" line) is practically the same as the dee voltage. Such a system, of course, cannot distinguish between the 2 fundamental modes of oscillation so it is necessary to eliminate the wrong mode by means of a wavetrap. This is possible as the symmetric mode has a constant frequency.

Other possible means of coupling are shown in Figure 4. Figure 4(a) was abandoned because of mechanical inconvenience and difficulty of making leads and coupling loop of sufficiently low inductance. Figure 4(b) was abandoned for reasons discussed later.
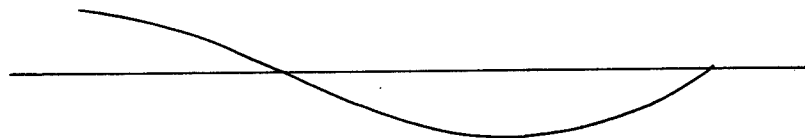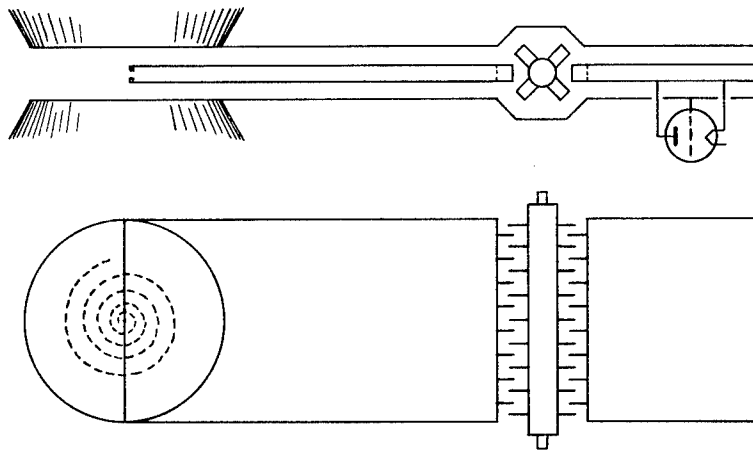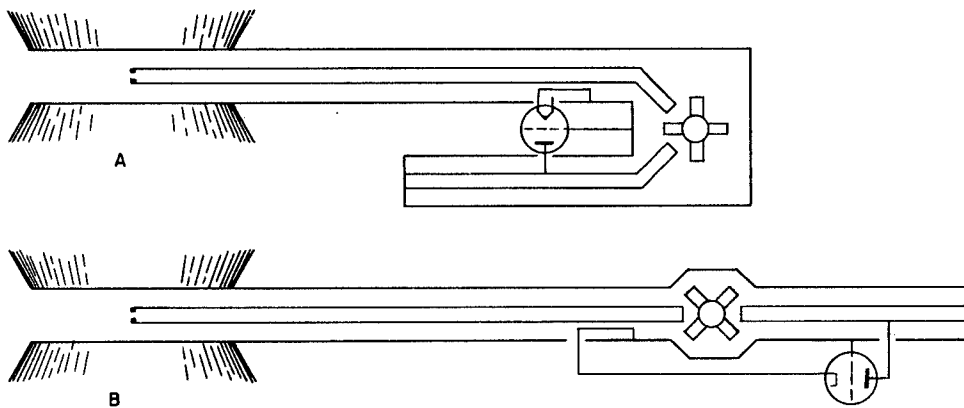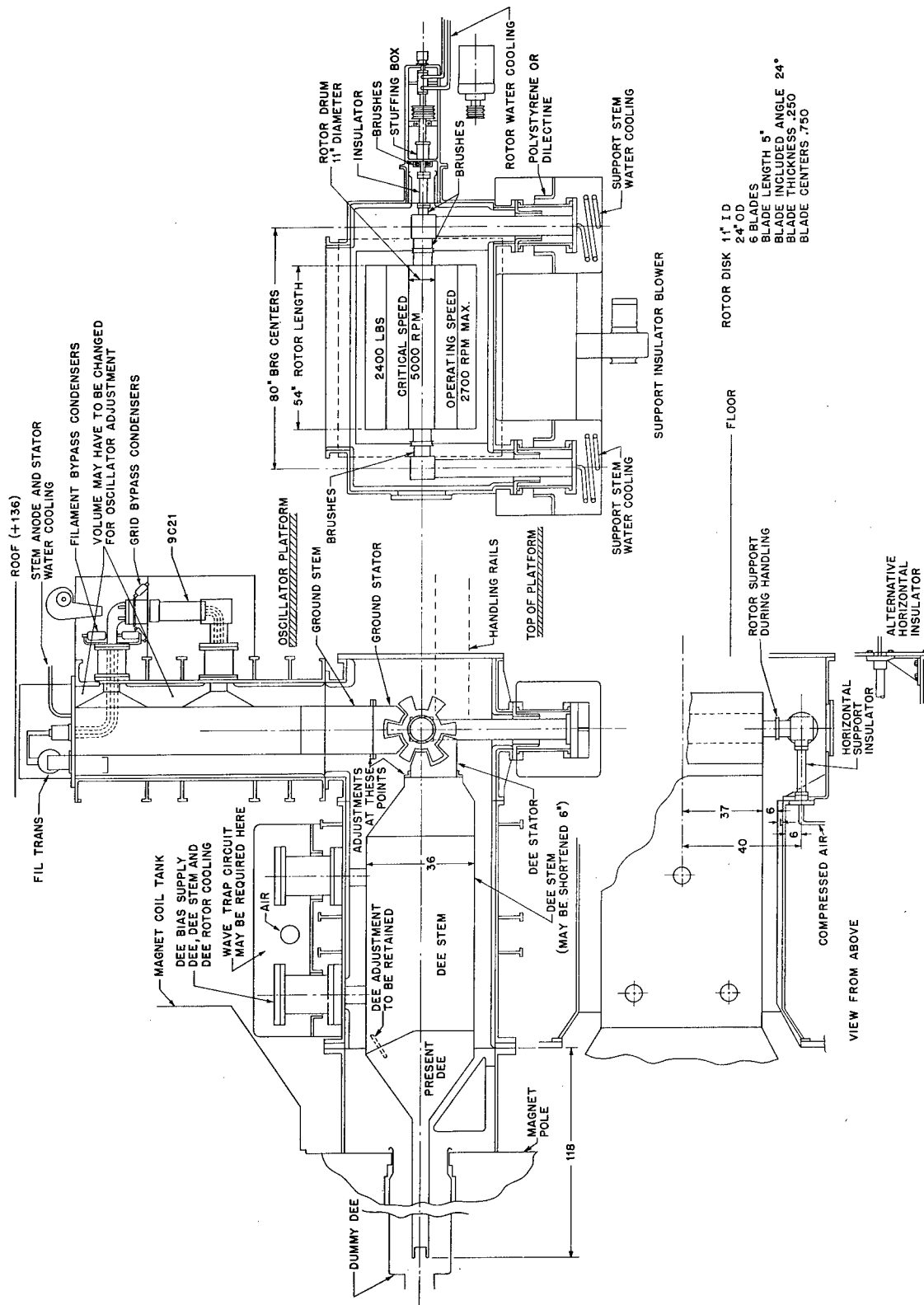

Figure 2.

Figure 3.

Figure 4.

Figure 5 – 184" cyclotron, proton conversion, RF system preliminary layout

MODEL TESTS

The foregoing discussion describes the thoughts that prevailed prior to the first model test. The first model was a 1/4 scale system using the present 184-inch deuteron dee which meant that the transmission lines were much narrower than the dee width. It was not excited satisfactorily due to the fact that at 100 megacycles the transit time effects were quite noticeable on the fundamental mode. With light house tubes such effects were negligible, but very high parasitic oscillations were also excited which normally would have been inhibited by transit time effects. So it was decided to forget the 1/4 scale model and build a 1/2 scale model, operating in the range from 50 to 20 megacycles where the Eimac 304TL tube serves as a very satisfactory scale model of a large power tube.

The 1/2 scale model was built as shown in Figure 5. It seemed desirable to use the present 184-inch deuteron dee in order to save the time and expense of making a new one. This of course meant that the transmission lines could not be very wide. However, there seemed no reason to suspect that this would cause any trouble. The only thing in favor of the system shown in Figure 3 is that dimensions can be calculated fairly exactly whereas in the system shown in Figure 5 one must depend on model tests (which are safer anyway).

In a 1/2 scale model the frequency will be up by a factor of 2 and the capacities and inductances down by a factor of 2. The Q of the model will be 0.7 times the Q of the actual installation and so will require 1.4 times as much power for a given dee voltage. This means that any coupling loops or taps in the model must pick up 1.4 times as much voltage, or else the tubes in the model must be capable of working into 0.7 the impedance seen by the full scale tubes. In the actual installation it is planned to use 9C21 tubes because they are specifically constructed for grounded grid operation. Another reason is that they are in use at present on the 184 inch so jackets and transformers are stocked as spare parts. It is planned to operate them at their handbook rating at 25 mc of 70 kw output or about 100 kw input. Typical maximum operation would be 12 kv at 8 amp per tube. It is more likely that operation most of the time would be around 12 kv at 6 amp as the tubes will operate much more efficiently than at the higher current figure. The Eimac 304TL will operate with reasonable efficiency at 1200 volts and 600 ma and so scales down very well. Because of the lower Q, the 304TL plate is tapped into the system in such a way as to pick up 40% more voltage than the corresponding tap for the 9C21. Also, the 304TL is a low mu tube (around 10 compared to 30 for the 9C21) so the excitation for the 304TL must be correspondingly higher.

The first thing to be settled in the model is the overall dimension so that the frequency limit will be correct. It was early recognized that this system can potentially cover both deuteron and proton ranges and so make the cyclotron doubly useful. The frequency limits for protons in the 15,000 gauss 184-inch field are 22.9 and 15.8 megacycles. The frequency limits for deuterons are 11.5 and 9.8 megacycles. When translated to the model these frequencies become 45.8 to 31.6 and 23.0 to 19.6 megacycles.

The question of how much extra range to allow in order that the ions can start when the frequency is changing uniformly probably cannot be answered in general. However, specific tests on the 184 inch showed that the upper frequency limit can be within 0.1 megacycles of the ion starting frequency without any loss in intensity. Similar tests at the low frequency end were not performed. At present the ions emerge at 9.8 megacycles and the lower limit of the frequency swing is 9.0 megacycles. It seems safe then to plan on a total frequency swing of 24 to 9 megacycles, which of course need not be continuous unless one wants protons and deuterons at the same time.

The model, as shown in Figure 5, will cover the range from 45 to 18 megacycles continuously when excited by a separate oscillator. By removing one set of stator blades, the upper limit is raised to 48 megacycles and the system will cover the proton range only. At present, the plan is to operate with one set of stator teeth only for protons. When deuterons are wanted, the condenser will have to be opened to air and the second set of stator teeth bolted in place. Probably later on a device will be designed to make the equivalent change under vacuum.

OSCILLATOR COUPLING

With the frequency limits established and realized, the remaining problem is the efficient coupling of the oscillator tube to the system. As mentioned earlier, the stub voltage is approximately equal to dee voltage over about a 2 to 1 range. Accordingly, the first oscillator, intended to cover the proton range, was coupled in as shown in Figure 6. The stub line was made vertical in order that leads to a water-cooled tube coupled in this fashion would be as short as possible. It also makes the rotary condenser more easily serviced.

The oscillator tube was tried on both sides of the stub line, functioning somewhat better on the side toward the magnet as more magnetic flux could be coupled at the tap due to the shorter path. It was necessary to suppress unwanted modes with wavetraps. Several modes are apparent. The most important unwanted mode is the parallel mode which changes frequency very little with condenser position, provided the voltage across stub and dee stem stator are equal at the high frequency limit. It was suppressed with 2 wavetraps slightly staggered in tuning to cover a small frequency band. Other wavetraps suppressed a mode which appeared at around 80 mc, presumably due to the high frequency circuits consisting of the plate and filament leads and tube element capacities. A total of 6 wavetraps sufficed to suppress all unwanted modes throughout the proton range. The parasitic modes were not completely eliminated, however, when the dee was shorted, as would occur in a discharge. It seems, therefore, that there is some risk of tube and insulator damage during discharges.

It was found that the parallel mode was discouraged by grounding the supports for the rotor. These supports have inductance and so introduce a third mode of oscillation. There are now 3 fundamental modes, but two of them require that large currents flow in the rotor supports, which can be made fairly high resistance. These modes are therefore not excited in favor of the much higher Q correct mode. The three modes and voltage distributions are shown in Figure 7. They were examined with a separate oscillator loosely coupled to the system but could not be excited with the oscillator shown in Figure 6.

Grounding the rotor supports is of course an advantage mechanically, but introduces the problem of keeping the r.f. current from flowing through the bearings. This will presumably be done with brushes, as in the present 184-inch condenser. Unfortunately, a long-life brush for carrying r.f. current which operates in a vacuum has not been developed to date, so means for easily servicing and replacing brushes must be worked out.
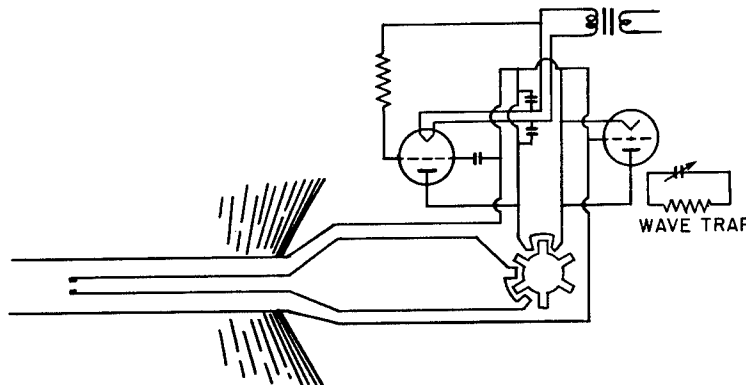

WAVE TRAP

Figure 6.

RIGHT MODE

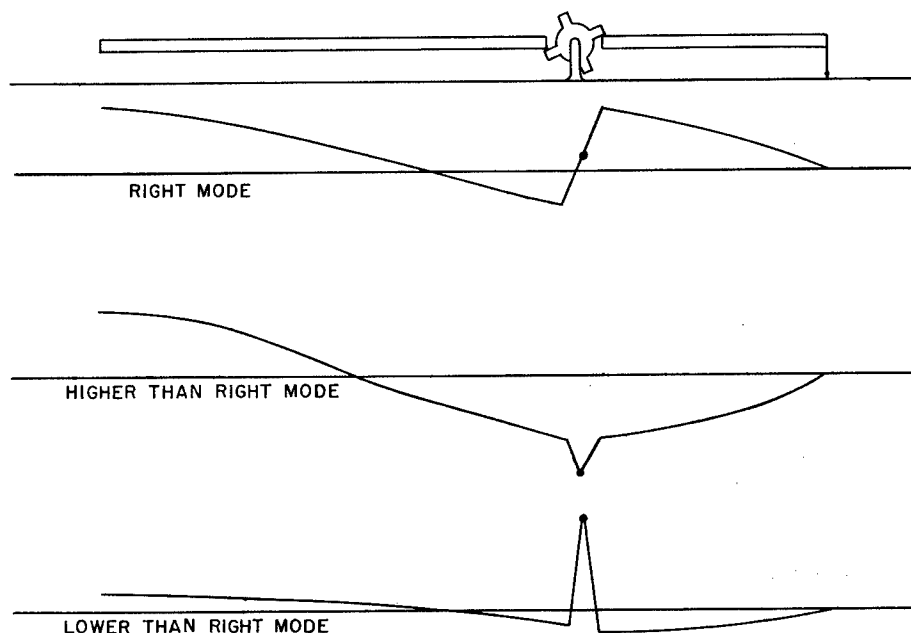HIGHER THAN RIGHT MODE

LOWER THAN RIGHT MODE

Figure 7.

Before leaving the question of grounding or ungrounding the rotor supports, it should be pointed out that if the supports, as shown in Figure 5, are supported on insulators, the insulators will be subjected to about 3 times the rotor voltage to ground. This is because the long metal supports act as open transmission lines and of course the voltage is a maximum at the open end. Currents, of course, must flow, and brushes are still necessary in addition to the excessive voltage on the insulators.

The only system which avoids high currents through the brushes is an all ceramic support structure. Mechanically, of course, this is undesirable.

THE DEUTERON RANGE

With constant dee voltage the stub voltage does not vary by more than 40% throughout the proton range. However, the stub voltage varies by 2 to 1 through the deuteron range, so the above oscillator coupling scheme does not seem practical. The variation of stub voltage with frequency for constant dee voltage is given in the Appendix.

Of course, it is not true that an oscillator tube will not work over the deuteron range if tapped onto the stub line. The result would be an amplitude modulation on the dee by 2 to 1 with about four times the power necessary at the low frequency end where the dee voltage would be highest. This type of amplitude modulation is theoretically desirable.

However, the maximum voltage will not be limited by plate dissipation but by filament emission, since the oscillator tubes will be pulsed. Therefore, if sufficient emission is available to produce a given dee voltage at the low frequency end, it does not seem quite right to use only 1/4 of the emission to produce 1/2 voltage at the high frequency end, when the circuit can be arranged so that the tubes can deliver full power at all times.

The tubes cannot be coupled to the dee stem magnetically because of insufficient space and prox-
imity to the magnetic field. More important, a loop cannot pick up enough voltage without being so
large that its self-inductance makes it resonant in the range to be covered. The solution seems to lie
in coupling the plate of the tube to the stub line through a transmission line. In order to eliminate the
unwanted fundamental modes, it seems desirable to get the excitation by means of a loop and line coupled
on the other side of the condenser. The two wrong modes have the wrong phase.

PLATE TRANSMISSION LINE

The following calculations apply to the model. The deuteron range (allowing a little to spare) is
taken as 25 to 18 mc (twice actual frequency). In order to supply the required power, two 9C21's will
be used, so two 304TL's are required in the model.

The plate to grid capacity of two 9C21's is 100 micromicrofarads. In addition, we must add the
capacity of the water jackets to ground which is of the order of 50 micromicrofarads apiece. If placed
close together, the capacity of the two jackets is somewhat less than twice 50 micromicrofarads and
might be as low as 60 micromicrofarads if the oscillator box is reasonably spacious. If the transmission
line comes into the oscillator box from the bottom, the water jacket can be made to appear as a con-
tinuation of the line and hence we can disregard its capacity to ground in our calculations. In any case,
it seems desirable to calculate line lengths for a total capacity of 100 micromicrofarads and also for
160 micromicrofarads.

The Case Where Capacity = 160 $\mu\mu f$

On model at twice frequency, the capacity equals 80 $\mu\mu f$. We will use a high impedance line to
reduce losses, say 100 ohms.

> At 25 mc, 80 $\mu\mu f$ = 80 ohms
> At 18 mc, 80 $\mu\mu f$ = 110 ohms
> The node occurs at $\theta = \tan^{-1} 80/100 = 39°$ at 25 mc
> The node occurs at $\theta = \tan^{-1} 110/100 = 48°$ at 18 mc

At an intermediate frequency of 21 mc, 80 $\mu\mu f$ = 94 ohms and the node occurs at $\theta = \tan^{-1} 94/100 = 43°$.

We will now assume a constant voltage at the plate of the tube and progress down the line at all
three frequencies until we find a line length where the voltage variation fits the voltage variation on the
stub line. To fit a 2 to 1 variation on the stub line, we choose a line length, X = 119° at 25 mc.
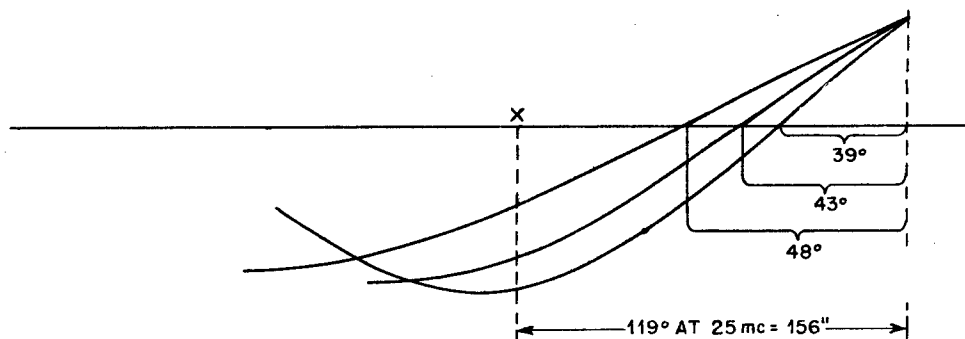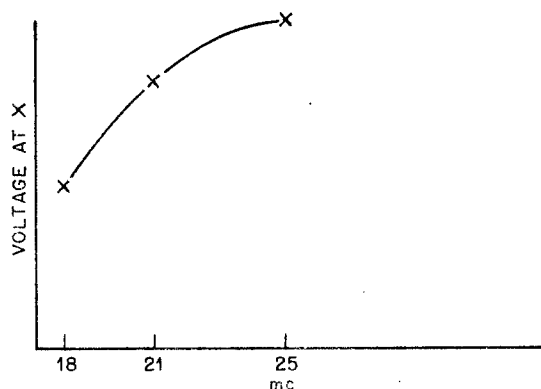

Figure 8.

Figure 9.

If the variation is more than 2 to 1, we choose a shorter line to match. The voltage at X is shown in Figure 9 and is seen to be reasonably close to the measured stub voltage (Figure 25). The proposed circuit is shown in Figure 10.

The correct line length compensates for variation with frequency, while the actual tube voltage, for a given dee voltage, is determined by the tap position and also by the length and Z_0 of the stub line (as shown in the Appendix). The empirical procedure in locating the correct tap position is to start with the tap at the end of the stub line and then move toward the shorted end until the tube draws rated plate current at rated voltage.

The Case Where the Capacity of Two 9C21's Is Considered to Be 100 $\mu\mu$f, Which Corresponds to 50 $\mu\mu$f on the Model

We arbitrarily chose a 100-ohm line. If, for this case, we choose a line of Z_0 = 80/50 x 100 = 160-ohm line, then Figure 8 applies exactly, and we use a line Z_0 = 160 ohms and 156 inches long.

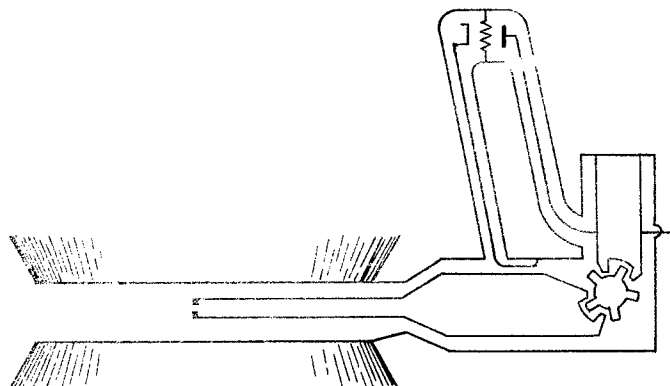Or we can recalculate, using a 100-ohm line just to get the "feel" of what is going on.

Figure 10.

At 25 mc, 50 $\mu\mu$f = 127 ohms
At 18 mc, 50 $\mu\mu$f = 176 ohms
At 25 mc, distance to node = $\tan^{-1}$ 127/100 = 52°
At 18 mc, distance to node = $\tan^{-1}$ 176/100 = 60°

As seen from Figure 8 and Figure 11, the difference in line length using 50 $\mu\mu$f and 80 $\mu\mu$f plate to grid and ground capacity is only 15 inches. End effects and bends in the line can cause this much variation. The conclusion is that there should be some possibility of varying the line length by a small amount so that an exact adjustment may be made empirically.

Two 304TL's have a plate to grid capacity of about 25 $\mu\mu$f. The actual apparent capacity at the terminal is somewhat higher due to lead inductances. In order to know exactly how much capacity to add, it would be desirable to construct a crude scale model of a 9C21 and then pad the 304TL's until the line resonated at the same frequency in both cases. This, however, seems hardly worth while as the line length is such a noncritical function of the plate to grid capacity.

THE FILAMENT LINE

As mentioned earlier, the filament excitation is obtained from the dee side of the rotary condenser. In the range from 25 to 18 mc it is possible to obtain a very suitable voltage by tying the filament line to one of the large insulator stems which hold up the dee structure. The voltage at this point varies by about 30% thru the deuteron range. Of course, it becomes a voltage node near the top of the proton range. Referring to Figure 8, it is seen that a line about 30% longer than 156 inches will take care of a 30% voltage change. Figure 8 does not apply exactly, of course, and reference to it was made only to show the direction in which the line should be changed. No detailed analysis will be given as on trial an unsuspected mode appeared which rendered the system useless over half the range. This mode is shown in Figure 12.

It was therefore necessary to consider loop coupling as shown in Figure 10. So far, no easy way of calculating the quantities involved in loop coupling has been found. To pick up sufficient voltage, the loop is large enough so that it cannot be considered as a lumped inductance only. To enclose the maximum flux with the smallest possible loop, the loop is designed to project into a slot in the inner dee stem
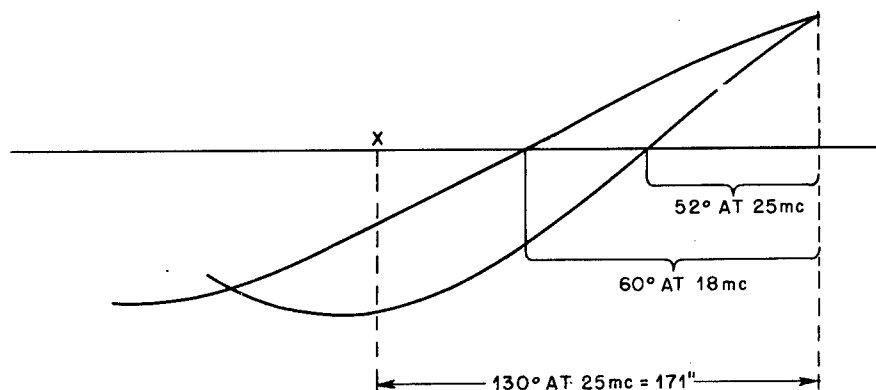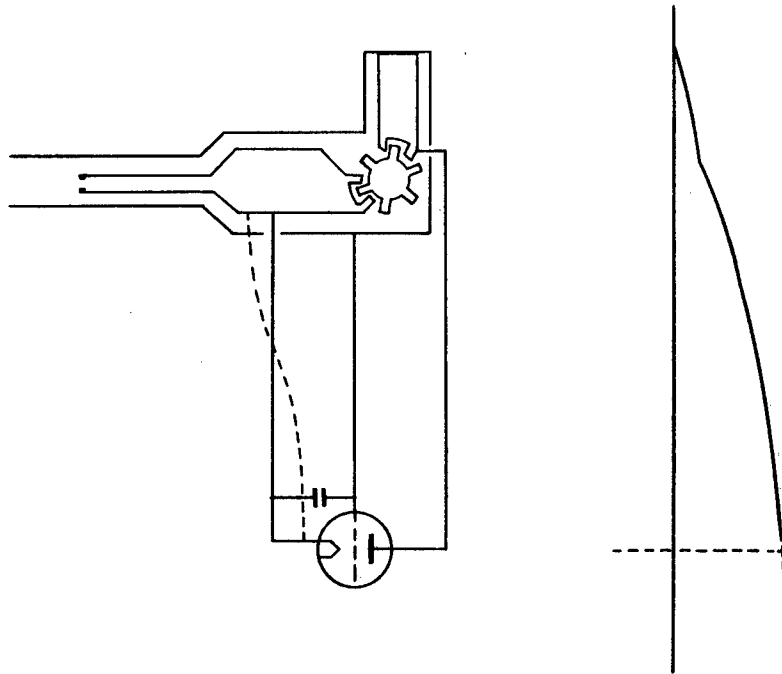

Figure 11.

Figure 12.(a) Voltage distribution on filament line. The whole dee structure + support acts as a lumped capacity. Frequency of mode is about 10 mc on model. (b) Voltage distribution on stub line and plate line.

conductor. To keep the impedance of the loop low, it should be of large conductor cross section (4-inch diameter is planned). This increases the distributed capacity to the dee stem which introduces an induced voltage which is hard to take into account. However, it can be made small compared to the voltage induced by the magnetic field.

The line from the filament of the tube to the loop terminal is calculated in exactly the same way as the plate line. The starting point in calculating the plate line was finding the impedance of the grid to plate capacity. Some previous experience (in particular the calculations for the Harvard model oscillator tests) showed that any attempt to construct a filament line using only the tube capacity usually results in rather large phase shifts between filament and plate r.f. voltages. In a fixed frequency oscillator this shift can be reduced to practically zero by suitable choice of line lengths. This is not very effective in a variable frequency oscillator, so the shift is reduced by making the filament grid capacity large, so that the out of phase r.f. current will be large compared with the in phase electron emission current. When we calculated the plate line constants we neglected the phase shift as the r.f. current (out of phase) through the capacity of the tube elements is about 6 times the emission current, due to the high r.f. plate voltage. However, the shift produced is not negligible and should be calculated. It is of the order of 10°. The excitation voltage is much lower (about 1/6 in the case of the 9C21) and the emission current higher (grid current added) so the grid filament capacity must be artificially raised to increase the out of phase current.

After the capacity (grid to filament) reaches a certain size, it does very little phase correcting and serves almost wholly as a means of adjusting the excitation voltage. This will become apparent as soon as an example is worked out.

We will try 800 $\mu\mu$f from filament to grid on the pair of 9C21's. On the model this would be 400 $\mu\mu$f.

At 25 mc, 400 $\mu\mu$f = 16 ohms
At 18 mc, 400 $\mu\mu$f = 22 ohms

Again using a 100-ohm line:

At 25 mc, the distance to the node = $\tan^{-1}$ 16/100 = 9°.
At 18 mc, the distance to the node = $\tan^{-1}$ 22/100 = 12.5°.

The question now arises as to what the length of the line should be. To answer this, we must ask what voltages will appear at the loop terminal. The voltage picked up by the loop will be Mωi. At the high frequency end (25 mc), we get a factor of 25/18 or 1.4 times the voltage at the low frequency end from the factor "ω" alone. The current "i" also increases, but not by such a large factor. In a transmission line (no lumped constants), "i" (at the node) would not increase with frequency for a given voltage at the open end. However, here there is some lumped effect and also the loop is closer to the node at 25 mc. We would probably not be far off if we said the total voltage increased by about 1.6 at 25 mc. This would require that the line be cut off at "X". However, another effect appears.

At 25 mc, very little current is flowing in the line at point X because the voltage is about maximum. However, at 18 mc the current may be appreciable and the back emf (Lωi) in the loop may considerably reduce the actual voltage impressed on the line at X. For this reason, the line should probably be somewhat shorter than 137 inches.

The actual length does not seem to be very critical. As can be seen in Figure 13, the point Y, 94 inches from the tube, represents a 2 to 1 variation in voltage at the end of the line. The line can therefore vary in length from 137 to 94 inches and only introduce a variation in excitation voltage in the ratio 1.6 to 2, or 25%.

The exact length has not been calculated, but with the general manner in which voltages vary shown in Figure 13, it should be possible to adjust in the line length empirically.

One trouble with using such a high impedance line is that the loop voltage must be about 6 times the filament grid voltage. The case will be recalculated using a 60-ohm line.

60-ohm Filament Line

At 25 mc, θ to node = $\tan^{-1}$ 16/60 = 15°

At 18 mc, θ to node = $\tan^{-1}$ 22/60 = 20°

Figure 13.

X

15°

20°

75° AT 18 mc

105° AT 25 mc = 137"
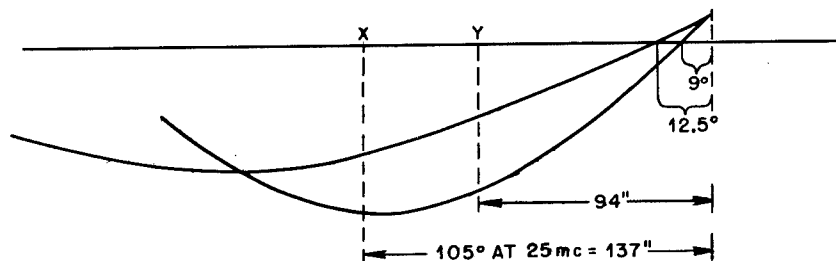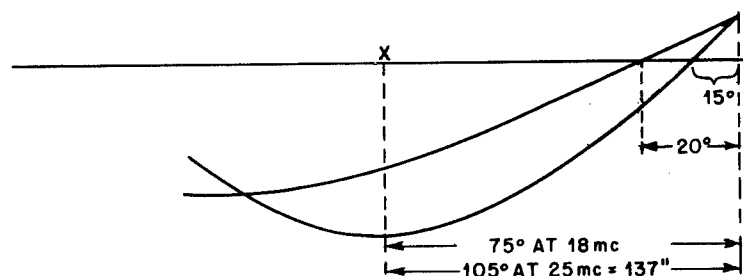
Figure 14.

The point X, corresponding to a voltage ratio of 1.6 to 1, at 25 and 18 mc, is still 137 inches from the tube within the accuracy of the sketch (Figure 14). Such accuracy is adequate for design purposes provided some adjustment of length is provided. Here the loop need pick up a maximum of 4 times the filament grid voltage. Self-inductance effects in the loop are somewhat more pronounced because the line current is greater due to the lower impedance. The line must therefore be shorter than 137 inches. A guess which would probably not be far off would be 100 inches.

FILAMENT LINE PHASE SHIFT

It is usually desirable to check the phase shift in the filament circuit. The quickest way to do this is by the graphical method by J. P. Shanklin (Electronics, 1945). In brief, anything that happens on a line is the result of an incident and reflected wave. The ratio of voltage to current in the incident and in the reflected wave is Z_0. Hence we will plot voltages and currents on the same diagram by simply multiplying all current vectors by Z_0.

We will calculate the phase shift for two 9C21's with 800 micromicrofarads from filament to grid, 2500 volts peak r.f. driving voltage and 16 amp plate current which corresponds to 30 amperes (peak) of fundamental component. Including grid current, we have about 35 amp of fundamental component.

The out of phase r.f. current due to 2500 volts across 800 micromicrofarads is:

$$i = \frac{2500}{16} = 156 \text{ amp at } 12.5 \text{ megacycles}$$

$$= \frac{2500}{22} = 114 \text{ amp at } 9 \text{ megacycles}$$

These out of phase and in phase currents are shown in Figure 15 for a $Z_0 = 60$ ohms.

The phase shift is seen to be quite appreciable. The shift due to the plate line is usually around 6° to 10° and in the opposite direction. Hence we might expect total phase shifts around 25°. The efficiency of the tubes will be reduced by such a shift, but it can be tolerated. Transit time will reduce the shift a little. However, this shift of 25° can not be increased very much without seriously impairing efficiency.

It is probably worth while discussing for a moment the factors which influence this shift. (1) If the grid filament capacity is made lower than 800 micromicrofarads (full scale) and the in phase emission current of 16 amp not correspondingly reduced, the shift will rapidly get worse. (2) Reducing the plate current for given plate and grid r.f. voltages by operating the tube into a higher impedance (tapping nearer the end of the stub line) will reduce the phase shift and also the maximum possible dee voltage. (3) Making the loop inductance as low as possible so that the line can be longer will reduce the shift.

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Figure 15. OB, OB , is voltage and Z_0X current in incident at 18 and 25 mc.

        BA, B'A are voltages in reflected wave at 18 and 25 mc.

        BC, B'C are currents in reflected wave at 18 and 25 mc.

        OB_1' is incident voltage advanced by 105° at 25 mc.

        $B_1'A_1'$ is reflected voltage retarded by 105° at 25 mc.

        OB_1 is incident voltage advanced by 75° at 18 mc.

        B_1A_1''' is reflected voltage retarded by 75° at 18 mc.

        OA_1'' and OA_1 are voltages which must appear at end of a line 200 inches long to give 2500

volts between filament and grid at 18 and 25 mc.

    In general, the phase shift problem can be looked at as follows when one has a model to experiment
with.

        1) Initially make the loop as large as convenient so that the induced voltage will be large.

        2) The larger the induced voltage the larger one can make the filament grid capacity.

        3) Make the loop inductance as low as possible.

        4) This will allow the use of a reasonably low impedance line without excessive back emf in the
loop due to heavy currents flowing in the line.

        5) The lower the line impedance, the larger the filament grid capacity can be made for a given
voltage at the loop end of the line.

With the present model there is probably some difficulty in getting a single loop with low enough self-inductance to allow the use of much less than a 30-ohm line (running). Several loops in parallel have been suggested, and by such means the shift may be reduced almost without limit. There is no point, however, in going so far that an appreciable fraction of the power is circulating in the filament line. It should be noted that several loops in parallel have different characteristics from a single "sheet" loop of rectangular conductor cross section and equivalent volume of metal. The difference lies in capacity to the inner dee stem and consequent distortion of the lines of current flow. As one increases the width of such a "sheet" loop, one can have the seemingly paradoxical condition of current flowing in opposite directions on the inside and outside of the loop.

EXPERIMENTAL TESTS ON DEUTERON OSCILLATOR

To date, the problem of keeping efficiency high and dee voltage constant over the deuteron range has not been examined very closely. It can be said that from 18 to 24 megacycles (on the model) the oscillator operates quite satisfactorily. Efficiency is apparently nowhere less than 50%, and dee voltage does not vary by more than 30% throughout the range. The reason that more definite results cannot be quoted is that progress was held up for several days while looking for a peculiar trouble. At about 20 megacycles the dee voltage would dip to zero just as though some resonant circuit, coupled to the system, was absorbing power. The trouble turned out to be a transverse mode along the rotor of the variable condenser. It was overlooked for a while because grounding or ungrounding the rotor supports had very little effect, and it was argued that such a procedure should greatly affect the frequency of a transverse mode. The oscillating circuit actually is a long folded transmission line consisting of the two rows of meshed teeth. When the distance the teeth overlapped was multiplied by the number of meshed teeth, the length so obtained comes out to be a half wavelength around 20 megacycles.

The solution to this problem was to remove some of the teeth and so raise the frequency of this circuit. To regain the lost capacity, an extra row of stator teeth was added to both the stub line and dee support. Although designed for two rows of teeth initially, the model had been used up till now with only one row, because one row had sufficient capacity and the maximum to minimum capacity ratio is larger with one row than two rows. The frequency of the transverse mode is not affected by adding the second row. The frequency of this transverse mode will always cross the fundamental frequency if the row of teeth is long enough. This is easily seen by remembering that the fundamental frequency approaches $1/3$ f for infinite capacity. If the row of teeth is made long enough it will resonate at $1/3$ f or lower. In the unmeshed condition the transverse mode approaches the physical length of the rotor, which is much higher than the fundamental frequency. It therefore seems safe to assume that if the frequency of the transverse mode is still higher than the fundamental when the teeth are fully meshed, then the transverse frequency will always lie above the fundamental for any partially meshed position.

POWER CONSUMPTION

Only rough figures are available at the moment. The 1/2 scale model uses about 400 watts input to the oscillator at 48 megacycles and 600 watts input at 18 megacycles to produce 1500 volts on the dee. The oscillators referred to are the direct coupled proton oscillator and transmission line oscillator for deuterons. This means 28 kw and 42 kw, respectively, to produce 15,000 volts on the dee, after taking into account $\sqrt{2}$ difference in Q. To produce 30 kv on the dee, we will require 112 kw and 168 kw input at 24 megacycles and 9 megacycles. It is probable that the power required will be somewhat less, as the model suffers from numerous bad joints and brass surfaces where copper will be used in the actual installation. Nevertheless, the power consumption will be much higher than the consumption of the present 184-inch oscillator for the same dee voltage.

A new power supply will probably not be necessary, however. It is planned to pulse the oscillators over only their useful range. The average power will therefore be only about a quarter to one-third of the maximum, and should be easily handled by the present supply.

THE FREQUENCY-TIME CURVE

A curve of frequency against rotor angle (time) is shown in Figure 16. The curve was taken with a single row of stator teeth on stub and dee stem and does not apply any more. It is shown to give a general idea of the probable shape. The proton acceleration is all over in 5 degrees of rotation or 1/2 of a cycle. The deuteron acceleration could not be accomplished without adding a little capacity to the dee support stem (effectively adding capacity to the dee). On the lower curve the deuteron acceleration is then over in 8 degrees or 1/7 cycle.

The shape and slope of the frequency-time curve will be considerably altered under the present plan in which the deuteron range will be covered with two rows of teeth. The proton range will be covered by only one row of teeth. Such an arrangement will probably allow greater rotor speeds for a given dee voltage and consequently more output.
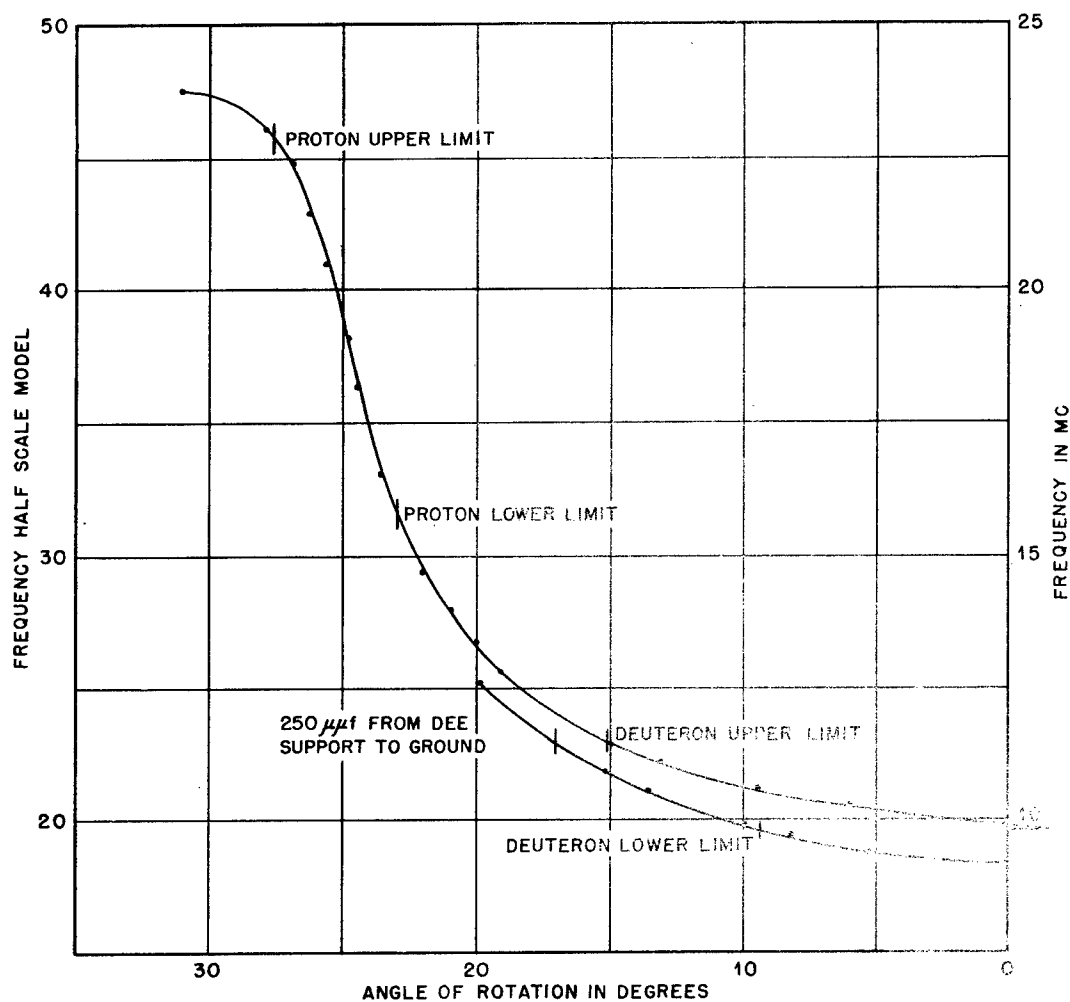

Figure 16. Frequency vs angle of rotation of rotary condenser.

The Transmission Line Oscillations in the Proton Range

Consider that the frequency limits are 48 mc and 30 mc. Let two 9C21's have 100 $\mu\mu$f plate-to-ground capacity. On the model this is 50 $\mu\mu$f. Use 100-ohm line.

At 48 mc, 50 $\mu\mu$f = 66 ohms
At 30 mc, 50 $\mu\mu$f = 105 ohms
At 40 mc, 50 $\mu\mu$f = 79 ohms

θ to node = tan^{-1} 66/100 = 33° at 48 mc
θ to node = tan^{-1} 79/100 = 38° at 40 mc
θ to node = tan^{-1} 105/100 = 46° at 30 mc
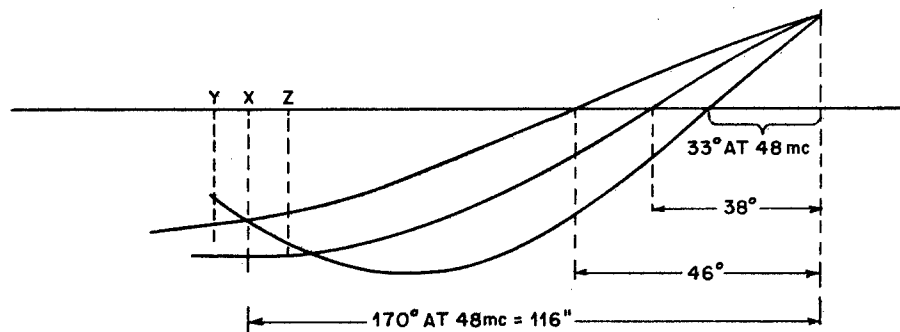
Figure 17.

Now we can cut the line off at X, Y, or Z, or in the vicinity. The voltage at the three places as function of frequency and constant plate voltage is shown in Figure 18. Reference to the Appendix and Figure 25 shows that the stub voltage behaves in just this manner. Variation in length of just a few inches will make the line fit any stub voltage variation.
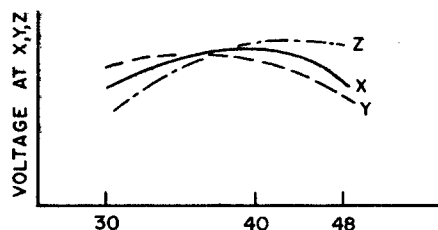

Figure 18.

Filament Line in Proton Range

Use same 60-ohm line as for line in deuteron range

400 $\mu\mu$f = 8.3 ohms at 48 mc
400 $\mu\mu$f = 13.2 ohms at 30 mc

θ = tan^{-1} 8.3/60 = 8° at 48 mc
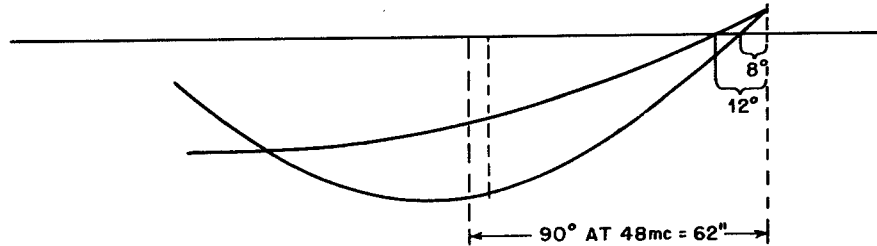θ = tan^{-1} 8.3/60 = 12° at 30 mc

Figure 19.

The loop voltage ($M\omega$ i) increases by (48/30 = 1.6) and also by any amount the current "i" increases. Assuming a total loop voltage increase of 2 to 1 (including $L\omega$ i drop at 30 mc), we would choose a point X around 60 inches from tube. A question arises —will the same loop do?

Here the loop voltage at 48 mc is about 7 times the filament grid voltage. For the deuteron case it was 4 times. The ratio 48/25 = 1.9. The ratio 7/4 = 1.75, so it seems that the same loop should work and any line adjustment of "drive" could be obtained by varying filament grid capacity. In conclusion, it seems that if set up correctly for deuterons, the system will work all right for protons by shortening lines.

APPENDIX

Voltage Variation on Stub Line

As mentioned earlier, the voltage on the stub line does not vary widely over most of the frequency range. For this reason it is a good place to couple an oscillator into the system. Since there are a number of variables that affect this voltage, it seems worth while to carry out a certain degree of investigation.

If the characteristic impedance of the system is fairly constant throughout, this means that the node will be about midway between the dee edge and condenser teeth at the high frequency end of the range. In order to have a starting point, we will represent the system by a transmission line of low Z_0, connecting a sort of "effective" lumped dee capacity to the capacity of the rotating condenser. At some point in the frequency range the effective dee capacity and condenser capacity will be equal (provided the condenser has low enough minimum capacity) and we will have equal voltages on dee and condenser. We will call this frequency f_0.

Referring to Figure 20, we will define the following quantities:

Z_0 is the characteristic impedance of the 1/2 wave section.

θ_0 is the electrical length of the 1/2 wave section at f_0.

Z_{0_S} is the characteristic impedance of the 1/4 wave section.

θ_{S_0} is the electrical length of the 1/4 wave section at f_0.

θ_d is the electrical distance from dee to node.

θ_c is the electrical distance from node to condenser.

Z_d is the impedance of the dee.

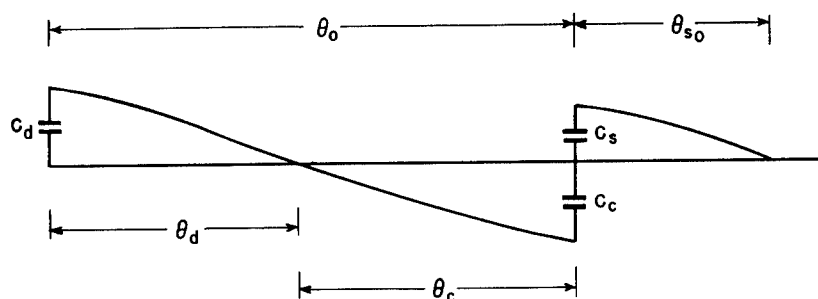Z_{d_0} is the impedance of the dee at f_0.

Figure 20.

C_c and C_s are two hypothetical series condensers with values such that the midpoint is always at ground potential.

$$\frac{1}{C_c} - \frac{1}{C_s} = \frac{1}{C_r}$$

where C_r is the value of the rotary condenser at any time.

In order that the system be resonant at any frequency, the impedances of the condensers C_d, C_c, and C_s must be conjugate to the impedances of their respective transmission line section.

Z_c is impedance of the shorted line θ_c and conjugate to impedance of C_c.
Z_s is impedance of the shorted line θ_s and conjugate to impedance of C_s.
V_d is the voltage on the dee.
V_c is the voltage across C_c.
V_s is the voltage across C_s.
The following equations can now be developed.

At any frequency f:

$$\theta_s = \frac{f}{f_0}\,\theta_s$$

$$\theta_d = \tan^{-1}\frac{Z_d}{Z_0}$$

$$\theta_c = \frac{f}{f_0}\,\theta_0 - \tan^{-1}\frac{Z_d}{Z_0}$$

$$Z_c = Z_0 \tan\left[\frac{f}{f_0}\,\theta_0 - \tan^{-1}\frac{Z_d}{Z_0}\right] \text{ but } Z_d = \frac{f_0}{f}\,Z_{d_0}$$

$$Z_c = Z_0 \tan\left[\frac{f}{f_0}\,\theta_0 - \tan^{-1}\frac{Z_{d_0}f_0}{Z_0 f}\right]$$

$$Z_s = Z_{0s} \tan\frac{f}{f_0}\,\theta_{s_0}$$

The ratio of voltages at the rotary condenser is the ratio of the impedances of the condensers C_c and C_s, or the conjugate impedances of their associated lines.

$$\frac{V_s}{V_c} = \frac{Z_{0s} \tan\frac{f}{f_0}\,\theta_{s_0}}{Z_0 \tan\left[\frac{f}{f_0}\,\theta_0 - \tan^{-1}\frac{Z_{d_0}f_0}{Z_0 f}\right]}$$

$$\frac{V_d}{V_c} = \frac{\sin \theta_d}{\sin \theta_c} = \frac{\sin \left[\tan^{-1} \frac{Z_{do} \, f_0}{Z_0 \, f}\right]}{\sin \left[\frac{f}{f_0} \, \theta_0 - \tan^{-1} \frac{Z_{do} \, f_0}{Z_0 \, f}\right]}$$

$$\frac{V_d}{V_s} = \frac{\sin \left[\tan^{-1} \frac{Z_{do} \, f_0}{Z_0 \, f}\right] \times Z_0 \, \tan \left[\frac{f}{f_0} \, \theta_0 - \tan^{-1} \frac{Z_{do} \, f_0}{Z_0 \, f}\right]}{\sin \left[\frac{f}{f_0} \, \theta_0 - \tan^{-1} \frac{Z_{do} \, f_0}{Z_0 \, f}\right] \times Z_{0s} \, \tan \frac{f}{f_0} \, \theta_{s0}}$$

$$= \frac{\sin \left[\tan^{-1} \frac{Z_{do} \, f_0}{Z_0 \, f}\right] \times Z_0}{\cos \left[\frac{f}{f_0} \, \theta_0 - \tan^{-1} \frac{Z_{do} \, f_0}{Z_0 \, f}\right] \times Z_{0s} \, \tan \frac{f}{f_0} \, \theta_{s0}}$$

This can be handled graphically by plotting the expression in parentheses and the tangent function separately. The quantity in parentheses can be plotted quickly by the construction shown in Figure 21.

Starting at freq. f_0 the triangle with sides $Z_{do} f_0$ (= const) and $Z_0 f_0$ is constructed. The angle $\tan^{-1} \frac{Z_{do} \, f_0}{Z_0 \, f_0}$ is the distance to the node from the dee edge at f_0 and is $\frac{\theta_0}{2}$ and in the figure is arbitrarily chosen as $57°$. The line OA at angle θ_0 is made equal to OB. Simply OA' and OA''are made equal to OB' + OB'', e.g. the triangle OCB'' corresponds to a freq. $3/4 \, f_0$ and the angle θ_d is $64°$ by measurement. The lengths of lines with the arrows at the ends represent $\cos \left[\frac{f}{f_0} \, \theta_0 - \tan^{-1} \frac{Z_{do} \, f_0}{Z_0 \, f}\right]$ at the frequency intervals shown.
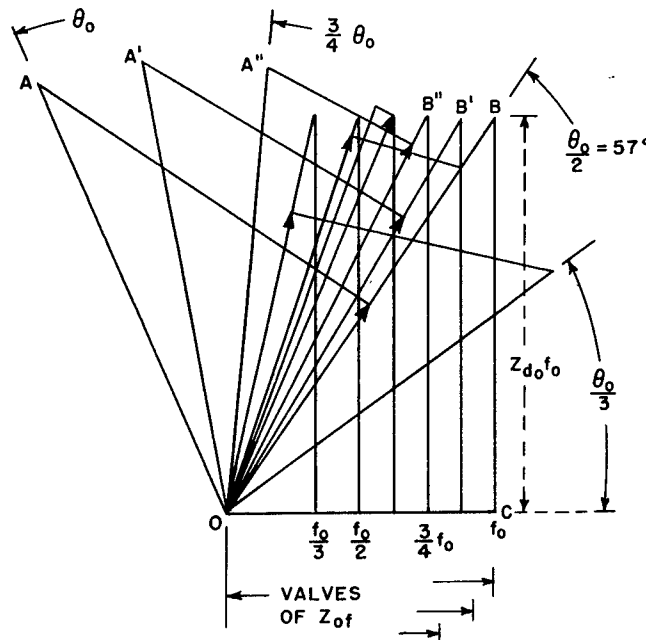

Figure 21.

The quantity $\dfrac{\sin\left[\tan^{-1}\dfrac{Z_{d_0}f_0}{Z_0 f_0}\right]}{\cos\left[\dfrac{f}{f_0}\,\theta_{0_1}-\tan^{-1}\dfrac{Z_{d_0}f_0}{Z_0 f}\right]}$ is then $=\dfrac{\text{length of vertical line } Z_{d_0}f_0}{\text{length of line with arrow}}$

The above ratio is plotted as the heavy line. Various values of the curve $Z_{0S}\tan\dfrac{f}{f_0}\,\theta_{S_0}$ are plotted depending on values of Z_{0S} and θ_{S_0}. It is evident that over quite a range the stub voltage can bear a one to one correspondence to the dee voltage by suitably choosing the length and characteristic impedance of the stub line. However, at the lower end of the frequency range, the stub and dee voltage vary widely from each other. It should be noted that it may not be possible to reach $\dfrac{f_0}{3}$ as it might require more than infinite capacity. However, this does not affect the analysis.

The preceding case was calculated assuming 57° from dee to node at the upper frequency (arbitrary choice). The actual model corresponds to about 73° from dee to node. The diagram for this case is shown in Figure 23. Nonessential lines are omitted.

The stub line can be adjusted to give approximately dee voltage or some fraction of dee voltage over the proton range but over the deuteron range, 18 to 24 mc, there is practically a 2 to 1 variation in the stub to dee voltage ratio.

An experimental curve taken with a separately excited oscillator is shown in Figure 25. Dee voltage is kept constant and the stub voltages are V_S and V_S'.
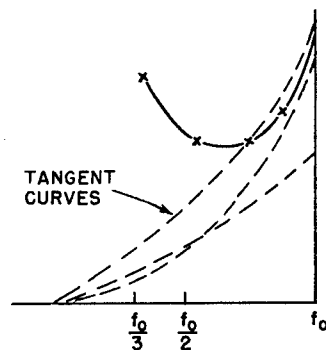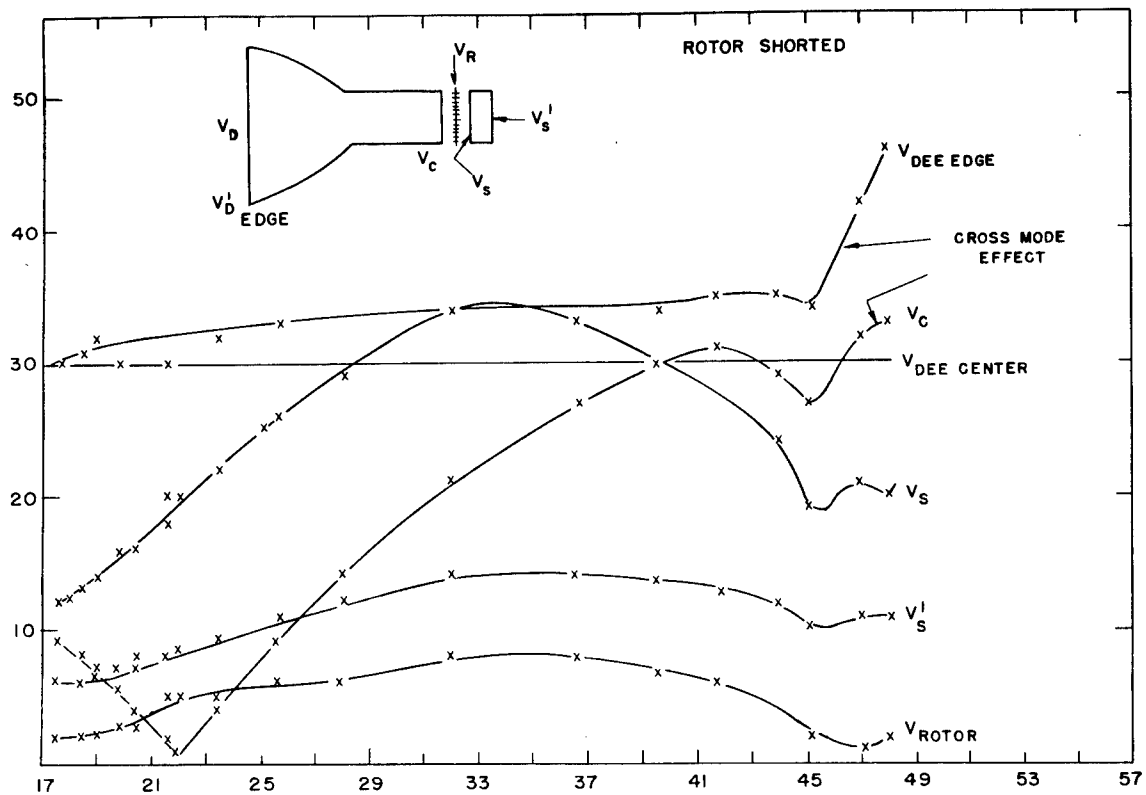

Figure 22.

Figure 23.

Figure 24.

Figure 25.

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