

NONLINEAR INTERACTION BETWEEN ACOUSTICS AND PROPELLANT  
COMBUSTION

Final Technical Report of Phase 1

by  
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# Chapter 1

## Russian and American Approaches to the Theory of Nonsteady Solid Propellant Combustion

A concise summary of certain problems in the theory of solid propellant nonsteady burning is given with emphasis to the difference in the approaches used by the scientists in the USA and Russia. Major topics include: two methods in the theory of nonsteady propellant combustion outlined, namely, the flame modeling (FM) approach and the Zel'dovich- Novozhilov (ZN) theory; two forms of the ZN theory – differential and integral; the burning rate response function to oscillatory pressure; the burning temperature response function to oscillatory pressure; the acoustic admittance of the propellant burning surface.

### 1.1 Introduction

The modern theory of nonsteady combustion of homogeneous propellants is based on the assumption of quasi-steady gas phase and reaction zones in both gas and condensed phases. Two approaches have been formulated in the framework of this approximation.

The flame model (FM) method was elaborated by the USA scientists. They develop detailed models that describe the processes occurring in the quick response zones. A mechanism concerned with transformation of a condensed matter into gaseous intermediate products, transport processes in the gas phase, kinetics of chemical reactions, and values of heat releases in various burning zones are proposed.

Researchers in Russia and the former Soviet Union have adopted the Zel'dovich-Novozhilov (ZN) theory. The advantage of this theory is that it permits considering nonsteady burning without involving a steady state theory. Calculation of nonsteady processes by the ZN method has no fitting parameters but only the properties of the given system in the steady state burning regime.

The aim of the present paper is to present a draft of the review concerning certain problems in the theory of solid propellant nonsteady burning with emphasis to the difference in the approaches used by the scientists in the USA and Russia. The following questions were considered:

a) Two methods of the theory of nonsteady propellant combustion are outlined, namely, the flame modeling (FM) approach and the Zel'dovich- Novozhilov (ZN) theory.

b) Two forms of the ZN theory - differential and integral - are presented. In the first case an essential element of the theory is the temperature distribution in the condensed phase.

This function, however, is used very seldom in practice. It was found possible to build the theory in such a way that only the values of interest to the researcher, such as external parameters and the burning rate, are directly interrelated.

c) The burning rate response functions to oscillatory pressure in linear approximation is obtained in the framework of the ZN theory.

d) In solving certain problems of the nonsteady theory it is necessary to know the temperature of the gas formed. In nonsteady regime it will differ from the steady state burning temperature. The burning temperature response function to oscillatory pressure is considered.

e) Acoustic admittance of the burning surface of a propellant is related to the burning rate and burning temperature response functions.

## 1.2 ZN and FM Approaches

### 1.2.1 Quasi-Steady Approximation

Most problem of the theory of nonsteady combustion of homogeneous propellantss are solved under the assumption that only the preheat layer in the condensed phase has a thermal inertia. The theory includes the only quantity of time dimension, the relaxation time of the heated layer,  $t_c \sim \kappa/u^2$  where  $\kappa$  is the condensed phase thermal diffusivity and  $u$  is the propellant burning rate. The finite relaxation times of the chemical reaction zones in the gas and condensed phases and heating regions in the gas are ignored. This approach is named QSHOD (Quasi-Steady, Homogeneous, One-Dimensional) analysis or  $t_c$ -approximation.

There are two different methods in the framework of QSHOD analysis or  $t_c$ -approximation. They are the Flame Modeling (FM method) and the Zel'dovich–Novozhilov (or ZN) theory. To show the difference between the two methods, we wish to dwell on one of the specific features of the nonsteady burning theory for condensed substances.

Combustion always involves a number of chemical reactions, and in most cases the burning rate depends on chemical kinetics. Therefore, practically each combustion theory essentially incorporates the kinetic characteristics of the reactions. However, combustion kinetics are not sufficiently understood at present, with only a few exceptions. Little is known, for example, about the kinetics of reactions involved in the combustion of condensed substances. This fact leads to the need for the introduction certain reaction models into theoretical calculations which, more often than not, only slightly resemble the real chemical processes (it is common practice to adopt the Arrhenius dependence of the reaction rate on the temperature and the power dependence of the reaction rate on the reactant concentrations). It is obvious that such investigations are but of a qualitative nature and are hardly suitable for comparison with experiment. Thus, for instance, the steady-state burning theory for condensed substances has been developed exclusively for the simplest types of chemical reactions and, although it can supply a qualitative explanation of the dependence of the burning rate, say, on the pressure or the initial temperature of the propellant, it is practically impossible to compare its results with experiment simply because it is associated with a very idealized model. Real physicochemical processes are much more complicated than theoretical ones. Moreover, it is evidently altogether impossible to develop a quantitative steady-state burning theory which would hold good for a broad class of substances, because they differ so widely.

At first glance, a nonsteady burning theory claiming quantitative agreement with experiment should be more complicated than a steady-state theory. This is true when we deal with a theory incorporating real kinetics of chemical reactions. It is, however, possible to deduce, to a rather good approximation, a phenomenological nonsteady theory in which the kinetics

of chemical reactions and all the complex physical processes involved in combustion would be automatically included by introducing data obtained from steady-state experiments.

This approach to the nonsteady theory was proposed by Zel'dovich<sup>1</sup> in 1942. He showed that in considering sufficiently slow nonsteady processes one should take into account solely the thermal inertia of the condensed phase. With this approximation, all the complex physicochemical processes find their way into the theory through the dependences of the burning rate on the pressure and initial temperature known from steady-state experiments.

This theory has explained qualitatively certain phenomena inherent in nonsteady burning, but its quantitative comparison with experiment leads to a contradiction. The latter is manifested most vividly in the fact that according to this theory steady-state burning in real systems turns out to be unstable. This discrepancy is caused by the excessive simplification of the combustion theory, where the surface temperature of the condensed phase was considered constant.

Experimental data and theoretical consideration show that the surface temperature of a propellant is also a function of the initial temperature and pressure. A nonsteady theory with a variable surface temperature was worked out by the author in Refs.2-3 which showed that the surface temperature in a nonsteady regime, as well as the burning rate, is determined by instantaneous values of the pressure and the temperature gradient at the surface.

Thus, the possibility of quantitative explanation of nonsteady phenomena in the burning of a solid propellant is due to the fact that the ZN theory includes experimental data relating to steady-state burning. These data contain all information on the kinetics of chemical reactions and of various physical processes (heat conduction and diffusion in the gas phase, fuel dispersion, etc.).

Reviews of the ZN theory have been presented in Refs.4-6.

A great number of investigations show a different approach to the nonsteady theory (see, for example, Refs.7-17). It is so called the Flame Modeling (FM) method. First, a steady-state model of propellant burning is constructed which includes various assumptions as to the mechanism of the chemical reactions in the condensed and gas phases, the nature of the transfer processes and the values of heat release in the different combustion zones. As a result, the conclusions of such a theory prove valid only for the model considered by its authors, which only crudely reflects the real processes occurring in propellant burning. Naturally, such theories contain a considerable number of parameters (e.g. the activation energies and heat releases of chemical reactions, transfer coefficients, the heat capacities of the gas and propellant, and so on), which are largely unknown. This also hinders comparison of theory and experiment.

Let us now compare the two approaches in detail.

## 1.2.2 Basic Assumptions

Both the ZN theory and the FM method use the following basic assumption.

1. A one-dimensional problem is considered (all values depend on only one spatial variable,  $x$ ). It is assumed that the propellant is homogeneous and isotropic and that the boundaries between the zones are planes. These requirements are necessarily fulfilled for homogeneous propellants. For the heterogeneous compositions, such an approach is valid when the size of oxidizer and fuel particles is much less than the characteristic size of the thermal layer following the steady-state theory, i.e., the propellant thermal layer  $\kappa/u$ .

During combustion, chemical processes accompanied by heat transfer, reactants diffusion, and gas motion take place in the condensed phase and in gas region near the interface. Accordingly, the entire space can be divided into three regions.

*C* region, in which condensed phase heating occurs. There is no chemical transformation whatsoever.

*S* region where, the condensed phase is transformed into intermediate gaseous products.

*G* region; here, as a result of gas-phase reactions, the intermediate products are transformed into the end products of burning. This transformation is accompanied by heat transfer, mass diffusion, and gas motion.

2. The relaxation times of *S* and *G* regions (respectively,  $t_s$  and  $t_g$ ) are taken to be zero. In other words, these regions are considered to respond quickly to changing external conditions. Experimental investigations of the burning zones in ballistite propellants, indicate a good fulfillment of the inequalities  $t_s \ll t_c$  and  $t_g \ll t_c$ . This result can also be obtained from simple estimates of the processes occurring in these zones. The first inequality results from the fact that the chemical transformation zone of the condensed phase is narrow. The second one is connected with a small ratio of gas and propellant densities.

3. In order to derive basic relationships of nonsteady burning theory in the  $t_c$ -approximation, it is necessary to consider the *S* region as infinitely thin. Thus, instead of the *S* region we obtain an interface *S* plane ( $x = 0$ ) whose temperature will be called the "surface temperature", and will be denoted as  $T_s$ . The approximation of an infinitely thin *S* region does not permit us to consider physicochemical processes occurring in this region in detail. As a result, it is necessary to provide the *S* plane with some definite properties.

4. In the ZN theory or the FM method, one can consider only rather slow changes of an external parameter (for example, pressure). If the characteristic time of the changing external parameter is  $t_p$ , the inequalities  $t_s \ll t_p$  and  $t_g \ll t_p$  should be fulfilled. In other words, *S* and *G* regions should adjust themselves without delay to a changing external parameter.

5. The preceding mayor assumptions are essential for the given approach. In order to simplify the analysis, minor assumptions are introduced. We do not consider thermal losses, or the influence of external forces. Moreover, it is assumed that the density of the condensed phase, its specific heat, and the coefficient of thermal conductivity are temperature-independent.

In the framework of these assumptions the thermal inertia of the condensed phase is described by the heat conduction equation

$$-\infty > x \geq 0, \quad \frac{\partial T(x,t)}{\partial t} = \kappa \frac{\partial^2 T(x,t)}{\partial x^2} - u(t) \frac{\partial T(x,t)}{\partial x}. \quad (2.1)$$

The space coordinate system is used to move the unreacted propellant in the positive direction of the  $x$  axis with a velocity that coincides with the linear regression velocity  $u(t)$ , so that the interface surface remains fixed at any combustion regime.

The boundary condition are obvious

$$x \rightarrow -\infty, \quad T = T_a; \quad x = 0, \quad T = T_s(t) \quad (2.2)$$

where  $T_a$  is the initial temperature.

To consider any nonsteady problem we must add to Eqs.(2.1-2) certain information on physical and chemical processes occurring in the quick-reponse zones *S* and *G*. The ways to use this information are quite different in the ZN theory and FM method.

### 1.2.3 ZN Theory

In order to build the ZN theory, it is necessary to know the steady-state dependencies of the combustion rate and surface temperature on the initial temperature and external parameters (pressure or tangential gas velocity),  $u^0(T_a, p)$  and  $T_s^0(T_a, p)$ . The correlations of this type will be called as "steady state burning laws" and the zero superscript correspond to steady

state values. In the following, we denote an external parameter by  $p$  and name it as pressure. In some cases, further information about the steady-state regime is also needed; for example, the dependence of the temperature of combustion products on the same parameters  $T_b^0(T_a, p)$ .

Under steady state conditions, the propellant burning rate, surface temperature, and any other properties depend on initial temperature and pressure. Thus, the variables  $T_a$  and  $p$  are suitable for studying steady state burning, and we can deliberately change them to examine the dependencies by systematic variation.

In general, it is impossible to use the steady-state relationships directly in the theory of nonsteady burning. In fact, the instantaneous state of  $S$  and  $G$  regions by no means depends on the temperature profile of the propellant far from these regions. At any given moment, the state of these zones can be determined only by the neighboring region of the condensed phase. Therefore, to consider the nonsteady processes, we should introduce, instead of  $T_a$ , some other parameter of the condensed phase that would directly affect the processes in  $S$  and  $G$  regions. Such a transition was realized by Zel'dovich<sup>1</sup> at the case of constant surface temperature. That value was demonstrated to be a temperature gradient at the surface on the condensed phase side

$$f = \left( \frac{\partial T}{\partial x} \right)_{x=0}.$$

The first attempt to prove this assumption for variable surface temperature was made by the author<sup>3</sup>. In that work, only the thermokinetic aspects of the phenomenon were considered. Without any essential changes, the results have been cited in some monograph<sup>4,5</sup>, a paper<sup>18</sup>, and a review<sup>16</sup>; in Ref.14 there was considerable simplification of the problem and erroneous references to the background material.

Later, two main conclusions were proved by the author within the framework of the foregoing assumptions (for details see Ref.6).

1. There exist the dependencies between the instantaneous values of the temperature gradient, pressure, and any properties of  $S$  and  $G$  regions

$$u = u(f, p), \quad T_s = T_s(f, p) \quad (2.3)$$

which are named as "nonsteady burning laws". In contrast to Ref.3 it was also considered the diffusion aspect of this problem, along with the kinetic and thermal aspects of the phenomena.

In the ZN theory, the dependencies given by Eqs.(2.3) are of great significance because the linear burning rate is included in the heat-conduction equation and the surface temperature in the boundary conditions.

Apart from the dependencies in Eqs.(2.3), other properties of the quick-response regions, i.e., temperature of the combustion products  $T_b(f, p)$  can be of interest in various problems of nonsteady burning theory. This function plays an important role in studying the acoustic admittance of the propellant surface.

2. The nonsteady laws of burning can be put into agreement with the steady-state dependencies  $u^0(T_a, p)$  and  $T_s^0(T_a, p)$ .

Let the steady-state combustion laws be

$$u^0 = F(T_a, p) \quad T_s^0 = \Phi(T_a, p). \quad (2.4)$$

From Eqs.(2.1,2) the steady state temperature distribution and the corresponding value of the gradient follow:

$$T^0(x) = T_a + (T_s^0 - T_a)e^{ux/\kappa}, \quad f^0 = \frac{u^0}{\kappa}(T_s^0 - T_a). \quad (2.5)$$

By expressing  $T_a$  through  $f^0$ ,  $T_s^0$ , and  $u^0$ , we get from Eqs.(2.4):

$$u^0 = F \left( T_s^0 - \frac{\kappa f^0}{u^0}, p \right), \quad T_s^0 = \Phi \left( T_s^0 - \frac{\kappa f^0}{u^0}, p \right). \quad (2.6)$$

As we stated that the instantaneous state of quick-response regions is defined by the pressure and temperature gradient in the same instant, we did not set limits on the type of burning regime (it could be either steady-state or nonsteady). Thus, the dependencies in Eq.(2.3) are true for any regime; they are the same both for steady state and nonsteady. But for the steady state regime they are known, and are represented in the form of Eqs.(2.6). Therefore, by omitting the index responsible for the steady state operation in Eqs.(2.6), we obtain the dependencies that are true in the nonsteady process as well:

$$u = F \left( T_s - \frac{\kappa f}{u}, p \right), \quad T_s = \Phi \left( T_s - \frac{\kappa f}{u}, p \right). \quad (2.7)$$

If necessary, these relationships can be resolved with respect to  $u$  and  $T_s$ , and represented in the form of Eqs.(2.3). The same is valid for any other function determining the state of  $S$  and  $G$  regions.

The possibility of transition from steady-state laws [(Eqs.(2.4)] to nonsteady [(Eqs.(2.7))] means that in the considered approximation, where the inertia of only the condensed phase is taken into account, any state of  $S$  and  $G$  regions in the nonsteady regime will coincide exactly with the definite steady regime. It can be characterized by some effective initial temperature:

$$T_e = T_s - \frac{\kappa f}{u}. \quad (2.8)$$

It should be noted that in some cases such a coincidence may turn out to be purely formal. In fact, stationary states can be realized when the initial temperatures are higher than absolute zero (if the system is at all capable of burning at sufficiently low initial temperatures). The values of the effective temperature can be low as we want (in particular, the temperature can be negative). The negative values of  $T_e$  do not lead to any physical contradiction since  $T_e$  is only the asymptotic value of the initial temperature of some stationary profile at a given value of burning rate, surface temperature, and temperature gradient. Such a steady-state regime can by no means be realized experimentally, but theoretically one may also consider the stationary regime at negative effective initial temperatures.

## 1.2.4 FM Method

In the FM method the boundary condition for the condensed phase energy Eq.(2.1) at the interface ( $x = 0$ ) is written through variation in the heat feedback from the gaseous flame zone to the solid propellant surface.

All models considered by the FM method assume that in burning process a solid propellant undergoes a two-stage transformation into combustion products. At the surface, the solid phase turns into combustible gases with a mass velocity  $m_s$ , and then gas-phase reactions with the formation of combustion products proceed at a mass velocity  $m_g$ . In the steady state approximation for the gas phase, naturally,  $m_s = m_g$ . The models considered in investigations on nonsteady burning have different dependences of  $m_s$  and  $m_g$  on the reaction zone temperatures and on the pressure.

American scientists often use a model proposed by Dennison and Baum<sup>8</sup>. It was assumed by these authors that burning in the condensed phase is described by the Arrhenius law

$$m_s = A_s \exp(-E_s/RT_s) \quad (2.9).$$

For the mass burning rate in the gas phase, an expression similar to the mass burning rate in premixed gas flames was used:

$$m_g = A_g p^{n/2} T_b^{(n/2+1)} \exp(-E_g/2RT_b) \quad (2.10).$$

In these relationships  $A_s$ ,  $A_g$ ,  $E_s$ ,  $E_g$ ,  $n$ , are constants, and  $T_b$ , the burning temperature.

A slightly simpler expression for the gas-phase reaction rate was used by Istratov and Librovich<sup>9</sup> in their investigation on the stability of steady state propellant burning at a constant pressure:

$$m_s = A_s \exp(-E_s/RT_s), \quad m_g = A_g \exp(-E_g/2RT_b).$$

Since in Ref.9 the pressure was assumed to be constant, the pressure dependence of pre-exponents was not specified. Heat release in the condensed and gas phases was taken to be constant in Refs.8,9.

The paper by Novikov and Ryazantsev<sup>10</sup> considered two combustion models which differed in the condition determining the termination of the reaction in the condensed phase. A model termed by the authors the  $Q$ -model is characterized by the fact that the chemical reactions in the condensed phase proceed until a definite (and constant even in a nonsteady regime) amount of heat is liberated; the surface temperature is variable in a nonsteady regime. In the other,  $T$ -model, the chemical reactions cease at certain given temperature  $T_s$  constant in a nonsteady regime, whereas heat release is variable. For both models the authors studied the stability of steady state propellant burning at a constant pressure; zero- and first-order reactions were investigated. For the rate of decomposition of the condensed phase use was made of expressions allowing for the heat flux from the gas phase. In calculating the mass rate of gas burning, approximations of the theory of steady state gas flame propagation were adopted.

In all the above-mentioned models the reaction rate in the condensed phase was described by the Arrhenius law, it being assumed that the chemical reactions in the gas proceed within a narrow temperature range near the combustion temperature. Krier et al.<sup>11</sup> proposed a model (the KTSS model) in which gasification obeys the power law

$$m_s = A_s (T_s - T_a)^n$$

where  $A_s$  and  $n$  are constants.

The model of the gas zone also differs substantially from the previously proposed ones in that the heat release in the gas begins directly at the propellant surface and terminates at the flame temperature; the heat release intensity in this zone is constant in space. This assumption enabled the authors of Ref.11 to find the expression for the heat flux from the gas to the condensed phase. The dependence of this value on the pressure was specified with an allowance for experimental data on steady state burning. The flux-pressure relationship obtained was used in the boundary condition for the heat conduction equation in the condensed phase describing its thermal inertia.

A more detailed survey of the models used in American investigations on nonsteady burning can be found in the reviews of Culick<sup>12</sup>, Kuo et al.<sup>16</sup>, and De Luca<sup>17</sup>.

### 1.2.5 Relationship between ZN and FM methods

Both approaches were developed for quasi-steady gas behavior. In this respect the ZN theory and FM method are equivalent. However, any theory requires some experimental data to be

compared with observed phenomena. The main difference between the two approaches is the way by which input experimental data are included in the theory.

Flame models involved a number of fitting parameters, most of which are unknown, and the authors of the models do not pointed out how these parameters might be measured.

The ZN theory based on the use of experimental relationships  $u^0(T_a, p)$  and  $T_s^0(T_a, p)$  is free from this drawback. Its conclusions refer to real systems, since the indicated relationships are taken from experiments precisely with such systems. Besides, it is possible to show that all the results obtained for any specific model, are particular cases of the general ZN theory which operates with familiar relationships  $u^0(T_a, p)$  and  $T_s^0(T_a, p)$ . To do this, it is sufficient to find the above relationships in the particular model under consideration.

Let us compare the two methods. We consider the solution of the stability problem obtained by Denison and Baum (Ref.8) and by the ZN-method.

In 1965, the author<sup>2</sup> showed that the stability condition of a steady state propellant burning at constant pressure accounting for only the inertia of the heated layer of the condensed phase is expressed only through two parameters,  $k$  and  $r$

$$k = (T_s^0 - T_a) \left( \frac{\partial \ln u^0}{\partial T_a} \right)_p, \quad r = \left( \frac{\partial T_s^0}{\partial T_a} \right)_p, \quad (2.11)$$

which characterize the variation in burning rate and surface temperature with initial temperature.

Steady state burning is always stable when the parameter  $k < 1$ ; for  $k > 1$  steady-state burning is stable only when

$$r > \frac{(k-1)^2}{k+1} \quad (2.12).$$

Note that the criterion (2.12) was obtained without any assumptions as to the nature of the chemical reactions in the condensed and gas phases.

Consideration of any specific type of chemical reaction should lead to the above criterion, and the values  $k$  and  $r$  will be expressed through the kinetic characteristics and other parameters characterizing the model adopted. Therefore, in order to investigate the stability of steady-state burning of a propellant at a constant pressure in any specific model it is sufficient to calculate the parameters  $k$  and  $r$  and substitute them into Eq.(2.12). We will illustrate this statement on the model used by Denison and Baum in Ref.8.

First we must define the parameters  $k$  and  $r$  used in the ZN approach. In the steady-state regime  $m_s^0 = m_g^0 = m^0$ . Therefore, from Eqs.(2.9-10)

$$A_s \exp(-E_s/RT_s^0) = A_g p^{n/2} (T_b^0)^{(n/2+1)} \exp(-E_g/2RT_b^0).$$

If this equation is differentiated with respect to initial temperature  $T_a$ , we obtain a parameter  $r$ :

$$r = \epsilon \frac{cR(T_s^0)^2}{c_g E_s T_b^0}$$

where  $c$  and  $c_g$  are the specific heats of the condensed and gas phases respectively, and

$$\epsilon = \frac{n}{2} + \frac{E_g}{2RT_b^0}$$

is one of the parameters used in DB-model.

From the expression for stationary mass rate,

$$m^0 = A_g p^{n/2} (T_b^0)^{(n/2+1)} \exp(-E_g/2RT_b^0).$$

we have

$$k = \epsilon \frac{c(T_s^0 - T_a)}{c_g T_b^0}$$

Three other parameters are introduced in the DB-model :

$$\alpha = \frac{c_g T_b^0}{c(T_s^0 - T_a)}, \quad A = \frac{E_s(T_s^0 - T_a)}{R(T_s^0)^2}, \quad q = 1 + A(1 - \alpha).$$

It can be shown that

$$\alpha = \frac{1}{k}, \quad A = \frac{k}{r}, \quad q = \frac{k + r - 1}{r}.$$

Denison and Baum provided the next conditions of unstable burning:

$$q > 1, \quad q^2 - q - 2A > 0.$$

The first of these expressions means  $k > 1$ , and the second means  $r < (k - 1)^2 / (k + 1)$ .

There is an unbounded nonoscillatory increase in perturbations for  $q^2 > 4A$ . It is easy to see that this condition can be written as  $r < (\sqrt{k} - 1)^2$ , which has been also obtained in Ref.2

Thus the results given by the DB-model are identical to those obtained by the ZN-method.

Any specific model with a constant surface temperature should satisfy the stability criterion  $k < 1$ . It is also satisfied by the  $T$ -model considered by Novikov and Ryazantsev<sup>10</sup>. The stability criterion for the  $Q$ -model can also be expressed by Eq.(2.12); a great number of parameters, which have been introduced to characterize the chemical reactions in the condensed and gas phases, appear in the stability criterion in the form of a combinations representing the derivatives of the burning rate and surface temperature with respect to the initial temperature. This conclusion also holds for the combustion model adopted by Istratov and Librovich<sup>9</sup> and in KTSS model<sup>11</sup>.

## 1.3 Two Forms of the ZN Theory

Two aspects may be considered in the theory of nonsteady burning. The first is associated with determining the burning rate with given external conditions (pressure or tangential flow velocity). Let us call this an "internal" problem of the nonsteady burning theory. Solution of problems in nonsteady burning with given external conditions opens a way to study combustion with a variable burning rate in combustion chambers. In investigating the second class of problems, the pressure-time relationship should be replaced by equations relating the pressure and temperature in the combustion chamber to the nonsteady burning rate and the temperature of the gases formed. As a result, we can find both the burning rate and pressure, besides the temperature inside the combustion chamber. This problem (let us call it "external") can be solved only if the internal problem has been investigated. Solution of the internal problem, which is basic for all kinds of applied problems, should be considered the main problem in the nonsteady combustion theory. Therefore, we shall restrict the following to consideration of the internal problem only.

### 1.3.1 Differential Form

The problem of finding the nonsteady burning rate in the ZN theory is reduced to accounting for the thermal inertia of the condensed phase by solving the heat-conduction equation:

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2} - u \frac{\partial T}{\partial x} \quad (3.1)$$

with boundary conditions:

$$x \rightarrow -\infty, \quad T = T_a; \quad x = 0, \quad T = T_s \quad (3.2)$$

at the given initial state and pressure in time:

$$t = 0, \quad T(x, 0) = T(x); \quad p = p(t). \quad (3.3)$$

The laws of nonsteady combustion are known, i.e., the relationships between the linear burning rate, surface temperature, temperature gradient, and pressure:

$$u = u(f, p), \quad T_s = T_s(f, p) \quad (3.4)$$

where

$$f = \left( \frac{\partial T}{\partial x} \right)_{x=0}. \quad (3.5)$$

Dimensionless variables are sometimes used. In any problem, it is possible to determine a basic steady-state regime. Let  $u^0$  be the linear rate of the steady-state combustion at pressure  $p^0$ , and introduce a dimensionless space coordinate, time, pressure, and burning rate with the help of the following definitions:

$$\xi = \frac{u^0 x}{\kappa}, \quad \tau = \frac{(u^0)^2 t}{\kappa}, \quad \eta = \frac{p}{p^0}, \quad v = \frac{u}{u^0}.$$

The dimensionless space coordinate and time are expressed in terms of characteristic length and characteristic time of the condensed phase.

The temperature in the condensed phase, the gradient and the temperature at the surface can be conveniently expressed in the form of

$$\theta = \frac{T - T_a}{T_s^0 - T_a}, \quad \vartheta = \frac{T_s - T_a}{T_s^0 - T_a}, \quad \varphi = \frac{f}{f^0}.$$

The internal problem, in terms of these variables, is formulated in the following manner. Find the burning rate  $v(\tau)$  from the heat-conduction equation, which takes into account the thermal inertia of the propellant:

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \xi^2} - v \frac{\partial \theta}{\partial \xi} \quad (3.6)$$

with initial and boundary conditions:

$$\theta(\xi, 0) = \theta_i(\xi) \quad \theta(-\infty, \tau) = 0 \quad \theta(0, \tau) = \vartheta(\tau). \quad (3.7)$$

The following relationships are also given:

$$v = v(\varphi, \eta) \quad \vartheta = \vartheta(\varphi, \eta) \quad (3.8)$$

where

$$\varphi = \left( \frac{\partial \theta}{\partial \xi} \right)_{\xi=0}$$

and the pressure dependence on time  $\eta(\tau)$ .

At the steady-state regime for  $\eta = 1$  we have

$$\theta^0 = e^\xi, \quad \varphi^0 = 1, \quad v^0 = 1, \quad \vartheta^0 = 1. \quad (3.9)$$

In the foregoing text, the pressure can be replaced by any other external factor, for instance the tangential flow velocity. We assume that the steady-state laws of erosion ( $g$  is the erosive flow velocity) are known, namely:

$$u^0 = F_g(T_a, g) \quad T_s^0 = \Phi_g(T_a, g).$$

With the use of the gradient - initial temperature relationship these steady-state relations can be converted into the functions

$$u^0 = F_g\left(T_s^0 - \frac{\kappa f^0}{u^0}, g\right), \quad T_s^0 = \Phi_g\left(T_s^0 - \frac{\kappa f^0}{u^0}, g\right).$$

In the nonsteady case, when the flow velocity is variable, the last expressions hold good, since they represent relationships between values referring to the inertialess region. Therefore the superscript characterizing a steady-state regime can be omitted. Thus, in studying nonsteady phenomena associated with erosive burning, the relationships (2.7) must be replaced by the functions

$$u = F_g\left(T_s - \frac{\kappa f}{u}, g\right), \quad T_s = \Phi_g\left(T_s - \frac{\kappa f}{u}, g\right) \quad (3.10)$$

and Eqs.(3.4) should be replaced by

$$u = u(f, g), \quad T_s = T_s(f, g). \quad (3.11)$$

Within the framework of the theory formulated it is possible to allow for the effect of radiation reaching the surface of the propellant from the combustion products or from an external source.

### 1.3.2 Integral Equation for Nonsteady Burning Rate

When solving the problems of nonsteady burning theory in terms of Eqs.(3.7-8), along with the burning rate we also find the nonsteady temperature distribution in the condensed phase,  $\theta(\xi, \tau)$ . This function is a by-product of the theory, since it is not needed for solving problems of internal ballistics (with the exception of certain special problems). The principal objective of the nonsteady theory is to predict the behaviour of the burning rate  $v(\tau)$  with given functions  $\eta(\tau)$ . The theory can be represented without the previously mentioned function of the two variables, as it was shown in Ref.19. Since the condensed phase is thermal inertial, the relation between the burning rate and the pressure is integral; i.e. the value of the burning rate at the moment  $\tau$  is determined by both the initial conditions and the pressure behavior  $\eta(\tau')$  at  $0 \leq \tau' \leq \tau$ .

Let us apply the Fourier transform to the heat conduction equations (3.6).

$$F(k, \tau) = \int_{-\infty}^0 \theta(\xi, \tau) e^{-ik\xi} d\xi.$$

Since the function  $\theta(\xi, \tau)$  on the right-hand side of the surface is not given, it is convenient to take it equal to zero at ( $\xi > 0$ ).

The partial time derivatives of the temperature will then convert into a total derivative of the Fourier-transformed one.

The first term in the right-hand side will yield, after being integrated twice by parts with an allowance for the boundary conditions,

$$\int_{-\infty}^0 \frac{\partial^2 \theta}{\partial \xi^2} e^{-ik\xi} d\xi = \varphi + ik\vartheta - k^2 F.$$

Finally, the term corresponding to the convective heat transfer is transformed as follows

$$-v \int_{-\infty}^0 \frac{\partial \theta}{\partial \xi} e^{-ik\xi} d\xi = -v(\vartheta + ikF).$$

The equation for the transformed function is of the form

$$\frac{dF}{d\tau} + (k^2 + ikv)F = \varphi - v\vartheta + ik\vartheta \quad (3.12)$$

with the initial condition

$$F(k, 0) = \int_{-\infty}^0 \theta_i(\xi) e^{-ik\xi} d\xi. \quad (3.13)$$

The linear equation (3.12) has the following solution

$$F(k, \tau) = \int_0^{\tau} [\varphi(\tau') - v(\tau')\vartheta(\tau') + ik\vartheta(\tau')] e^{-k^2(\tau-\tau')-ikI} d\tau' + F(k, 0) e^{-k^2\tau-ikJ}, \quad (3.14)$$

where the notations

$$I = \int_{\tau'}^{\tau} v(\tau'') d\tau'', \quad J = \int_0^{\tau} v(\tau'') d\tau''.$$

are introduced.

We will now apply to Eq.(3.14) the reverse transformation

$$\theta(\xi, \tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(k, \tau) e^{ik\xi} dk$$

Here, we obtain the integrals

$$\int_{-\infty}^{\infty} e^{-q^2 k^2} \cos p k dk = \frac{\sqrt{\pi}}{q} e^{-\frac{p^2}{4q^2}}, \quad \int_{-\infty}^{\infty} k e^{-q^2 k^2} \sin p k dk = \frac{\sqrt{\pi} p}{2q^3} e^{-\frac{p^2}{4q^2}}.$$

For the temperature, we have the integral expression

$$\theta(\xi, \tau) = \frac{1}{2\sqrt{\pi}} \left[ \int_0^{\tau} \left( \varphi - v\vartheta + \frac{\vartheta(I - \xi)}{2(\tau - \tau')} \right) e^{-\frac{(\tau-\xi)^2}{2(\tau-\tau')}} \frac{d\tau'}{\sqrt{\tau - \tau'}} + \frac{1}{\sqrt{\tau}} \int_{-\infty}^0 \theta_i(z) e^{-\frac{(z+J-\xi)^2}{4\tau}} dz \right] \quad (3.15)$$

This expression contains three unknown functions of time: the burning rate, the gradient and temperature at the surface. The two relationships (3.8) between them are not sufficient for determining them and finding  $\theta(\xi, \tau)$ . It is, however, possible to obtain a third relationship between  $v$ ,  $\vartheta$  and  $\varphi$  if we use Eq.(3.15) at the point  $\xi = 0$ , i.e. on the propellant surface. It should be remembered that for  $\xi = 0$  the temperature suffers a discontinuity (it is equal to  $\vartheta$  on the left and to the zero on the right); putting  $\xi = 0$  in Eq.(3.15), we must simultaneously multiply the right-hand side by two. Then we obtain

$$\vartheta(\tau) = \frac{1}{\sqrt{\pi}} \int_0^{\tau} \left( \varphi(\tau') - v(\tau')\vartheta(\tau') + \frac{\vartheta(\tau')I(\tau, \tau')}{2(\tau - \tau')} \right) \exp \left[ -\frac{I(\tau, \tau')^2}{4(\tau - \tau')} \right] \frac{d\tau'}{\sqrt{\tau - \tau'}}$$

$$+ \frac{1}{\sqrt{\pi\tau}} \int_{-\infty}^0 \theta_i(z) \exp \left[ -\frac{[z + J(\tau)]^2}{4\tau} \right] dz. \quad (3.16)$$

Taking into account the instantaneous relationships

$$v = v(\varphi, \eta), \quad \vartheta = \vartheta(\varphi, \eta) \quad (3.17)$$

we obtain a closed system for determining any one of the functions  $v$ ,  $\vartheta$ , or  $\varphi$  from the given function  $\eta(\tau)$ .

It is now possible, if required, to find from Eq.(3.15) the temperature distribution in the propellant at any instant.

Of greatest interest is the burning rate. If the explicit form of the functions (3.17) is known, it is always possible to represent the system (3.16-17) as a single integral equation for  $v(\tau)$ , whose value at a given instant will depend on the entire history of change in the external parameter  $\eta(\tau)$ . In solving problems of internal ballistics Eq.(3.16) is preferable to the initial system (3.7-8) for the following reasons.

In the first place, one need not find the function of two variables  $\theta(\xi, \tau)$ , which is not used anywhere. This will evidently considerably simplify the numerical solution of those problems which have no analytical solution.

Further, a number of problems of the nonsteady theory can be solved by means of a series expansion in a small parameter, for instance the amplitude of a harmonically varying pressure. In this case the use of an integral equation will greatly simplify the calculations by eliminating all kind of corrections to the steady-state temperature distribution.

Finally, another feature of the equation obtained is that it closes up the system of internal ballistics equations which contain, among other things, the pressure and the burning rate. Where they are constant or vary only slightly (a quasi-steady regime), the system of internal ballistics equations is closed up by the steady-state relation  $u^0 = u^0(p, T_a)$ . In the nonsteady case this relation should be replaced by the integral expression (3.16) with the additional condition (3.17). Of course, the differential form of the theory can also be used to the same end, but in that case the system of internal ballistics equations is greatly complicated, because it contains an additional function of two variables, i.e. the temperature inside the propellant.

## 1.4 Burning Rate Response Function to Oscillatory Pressure

In the framework of ZN approach the solid-propellant burning rate response function has been found in Ref.20. Before that several investigation (see, for example, Refs.7,8,21) were performed in the framework of F-M method. Those studies were reviewed in Ref.12.

Denote by  $\omega$  the dimensionless frequency of pressure variation (angular frequency multiplied by the characteristic time of the condensed phase  $\kappa/(u^0)^2$ , and by  $p^0$ , the average pressure; then

$$p = p^0 + p_1 \cos(\omega\tau)$$

where  $p_1$  is the amplitude of oscillations ( $p_1 \ll p^0$ ). We have to find the burning rate

$$u = u^0 + u_1 \cos(\omega\tau + \psi).$$

Using the dimensionless variables and the complex amplitude method we have in the linear approximation

$$\eta = 1 + \eta_1 e_1 + \bar{\eta}_1 \bar{e}_1, \quad v = 1 + v_1 e_1 + \bar{v}_1 \bar{e}_1$$

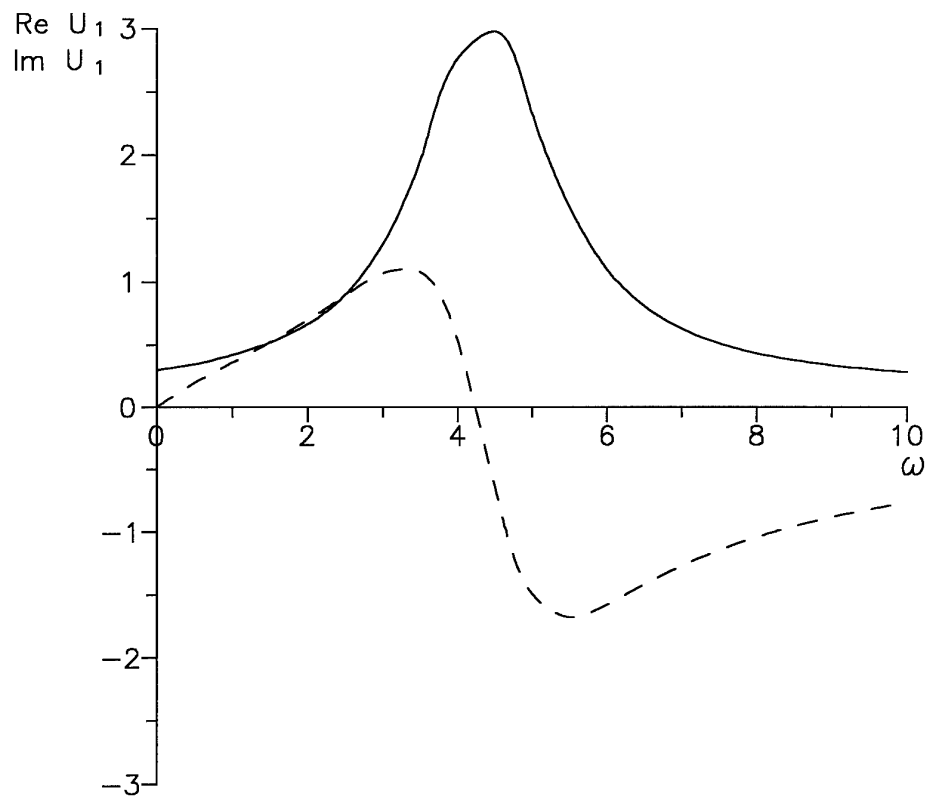


Figure 1.4.1 The real (solid) and imaginary (dashed) parts of  $U_1(\omega)$ .  $k = 1.8$ ,  $r = 0.3$ ,  $\nu = 0.3$ ,  $\delta = 0$ .

with the notations

$$\eta_1 = \frac{p}{p^0}, \quad v_1 = \frac{u}{2u^0} e^{i\psi}, \quad e_1 = e^{i\omega\tau}$$

and  $\bar{\eta}_1, \bar{v}_1$  and  $\bar{e}_1$  are the complex conjugations of  $\eta_1, v_1$  and  $e_1$

From the solution of the heat-conduction equation (3.6) we have the relation between the amplitudes of  $\vartheta_1, v_1$  and  $\varphi_1$

$$z_1 \vartheta_1 - \varphi_1 + \frac{v_1}{z_1} = 0 \quad (4.1)$$

where

$$z_1 = \frac{1}{2}[1 + \sqrt{1 + 4i\omega}] \quad (4.2)$$

The nonsteady burning laws in the same approximation are

$$v_1 = \frac{k}{D} \varphi_1 + \frac{\delta - \nu}{D} \eta_1, \quad \vartheta_1 = \frac{r}{D} \varphi_1 - \frac{\delta + \mu}{D} \eta_1. \quad (4.3)$$

where

$$k = \Delta \left( \frac{\partial \ln u^0}{\partial T_a} \right)_p, \quad r = \left( \frac{\partial T_s^0}{\partial T_a} \right)_p, \\ \nu = \left( \frac{\partial \ln u^0}{\partial \ln p^0} \right)_{T_a}, \quad \mu = \frac{1}{\Delta} \left( \frac{\partial T_s^0}{\partial \ln p^0} \right)_{T_a}, \quad (4.4)$$

$$\Delta = (T_s^0 - T_a), \quad \delta = \nu r - \mu k, \quad D = k + r - 1.$$

The three algebraic Eqs.(4.1,3) make it possible to find the amplitude of the burning rate, gradient and surface temperature from a given pressure amplitude.

For the burning rate, which is of greatest interest to us, we have

$$U_1 = \frac{\nu + \delta(z_1 - 1)}{1 + (r - k/z_1)(z_1 - 1)}$$

At this point we introduce the definition of the burning rate response function to oscillatory pressure

$$U_1(\omega) = v_1/\eta_1 \quad (4.5)$$

for which the following expression holds

$$U_1(\omega) = \frac{\nu + \delta(z_1 - 1)}{1 + (r - k/z_1)(z_1 - 1)} \quad (4.6)$$

Figure 1.4.1 illustrates the real and imaginary parts of this function. One can see that the real part of may reach values much greater than its steady-state value at

$$U_1(\omega) = \nu$$

It is a consequence of the high quality of our oscillatory system. The maximum of is at frequency close to the natural frequency . In this frequency region the denominator of Eq.(4.7) is much less than unity. So any small perturbation of the system can change very strongly the response function. As an example of such a change we consider in the next section the burning rate response function of highly metallized propellants.

Fig.1.4.1 illustrates the real and imaginary parts of this function.

### 1.4.1 Relation between ZN and FM Pressure Driven Response Functions

The propellant burning rate response function to oscillatory pressure based on the FM approach was first reported by Denison and Baum<sup>8</sup>. In 1968 Culick<sup>12</sup> reviewed a variety of the other flame models and showed that all results in this area had the same form.

The next expression is commonly used for the FM burning rate response function

$$U = \frac{nAB + n_s(\lambda - 1)}{\lambda + A/\lambda - (1 + A) + AB} \quad (4.7)$$

where the complex function  $\lambda$  has the same form as  $z$ -function in the ZN theory

$$\lambda = z_1, \quad z_1 = \frac{1}{2}[1 + \sqrt{1 + 4i\omega}] \quad (4.8)$$

The parameters  $n$ ,  $n_s$ ,  $A$  and  $B$  are functions of propellant models.

Equation (4.7.) can be related to Eq.(4.6) of the ZN theory. It easy to show that these equations are equivalent if

$$A = \frac{k}{r}, \quad B = \frac{1}{k}, \quad n = \nu, \quad n_s = \frac{\delta}{r}. \quad (4.8)$$

In this connection see also the rview of Beckstead<sup>22</sup>.

## 1.5 Burning Temperature Response Function to Oscillatory Pressure

As indicated above, in solving certain problems of the nonsteady theory it is necessary to know the temperature of the gas formed. In nonsteady regime it will differ from the steady state burning temperature  $T_b^0$ . The burning temperature response function to oscillatory pressure is considered in this section.

In linear approximation, using the method of complex amplitudes we represent the burning temperature during a nonstesdy process in the form

$$T_b = T_b^0 + T_{b1}e^{i\omega\tau}.$$

where  $T_{b1}$  is the complex amplitude of burning temperature oscillations.

Let us introduce the burning temperature response function to oscillatory pressure

$$U_b(\omega) = \frac{(T_{b1}/T_b^0)}{p_1/p^0} \quad (5.1).$$

Since the state of the inertialess gas phase is completely determined by the pressure and the gradient at the solid phase surface, the burning temperature in a nonsteady regime can also be represented as a function of the gradiend and pressure. In order to find the combustion temperature in a nonsteady regime one should know the steady state function

$$T_b^0 = \Psi(T_a, p).$$

To solve our problem the nonsteady theory this function must be converted into the relationship  $T_b(f, p)$  in the usual way

$$T_b = \Psi\left(T_s - \frac{\kappa}{u}f\right) \quad (5.2).$$

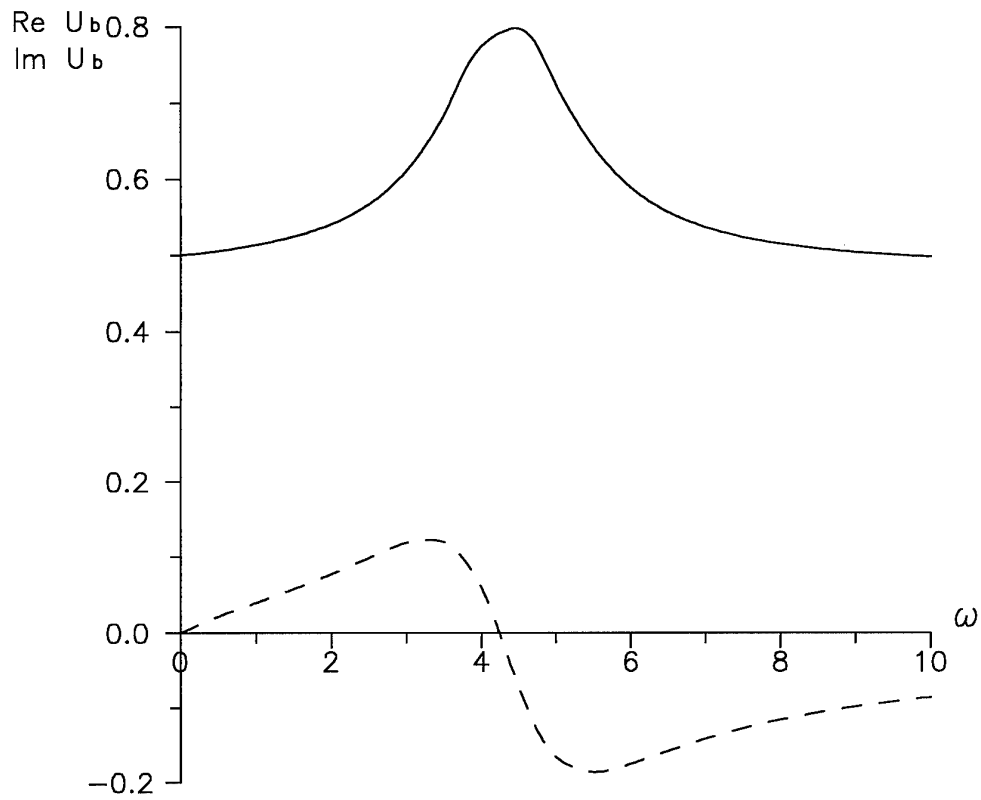


Figure 1.5.1 The real (solid) and imaginary (dashed) parts of  $U_b(\omega)$ .  $k = 1.8$ ,  $r = 0.3$ ,  $\nu = 0.3$ ,  $\delta = 0$ ,  $r_b = 0.1$ ,  $\mu_b = 0.5$ ,  $\gamma = 1.25$ .

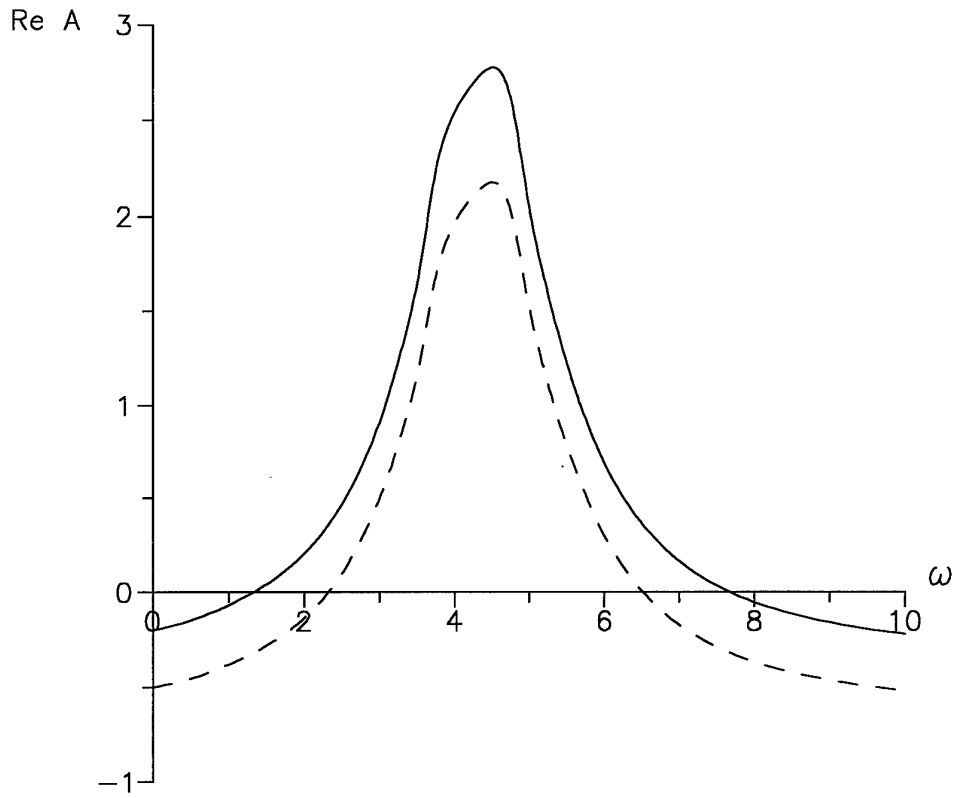


Figure 1.6.1 The real parts of  $A$  (solid) and  $A_0$  (dashed).  $k = 1.8$ ,  $r = 0.3$ ,  $\nu = 0.3$ ,  $\delta = 0$ ,  $r_b = 0.1$ ,  $\mu_b = 0.5$ ,  $\gamma = 1.25$ .

In linear approximation we obtain

$$T_{b1} = \left( \frac{d\Psi}{dT_a} \right)_p \left( T_{s1} - \frac{\kappa}{u^0} f_1 + \frac{\kappa f^0}{(u^0)^2} u_1 \right) + \left( \frac{d\Psi}{dp} \right)_{T_a} p_1$$

or

$$\frac{T_{b1}}{T_s^0 - T_a} r_b \left( \frac{T_{s1}}{T_s^0 - T_a} - \frac{f_1}{f^0} + \frac{u_1}{u^0} \right) + \mu_b + \frac{p_1}{p^0}$$

where

$$r_b = \frac{\Delta}{T_b^0} \left( \frac{\partial T_b^0}{\partial T_a} \right)_p, \quad \mu_b = \frac{1}{T_b^0} \left( \frac{\partial T_b^0}{\partial \ln p^0} \right)_{T_a} \quad (5.3).$$

To obtain the final expression for the burning temperature response function to oscillatory pressure we should use Eqs.(.). The result is

$$U_b = \frac{T_s^0 - T_a}{T_b^0} \left[ \frac{r_b}{k} (U - \nu) + \mu_b \right] \quad (5.4).$$

To characterize burning temperature oscillations it is useful to introduce the value

$$I_b = \Gamma U_b, \quad \Gamma = \frac{\gamma}{\gamma - 1} \quad (5.5)$$

where  $\gamma$  is the ratio of specific heats. We shall call this quantity as the isentropicity of the flame front. If the adiabatic relation between the temperature and pressure holds  $\text{Re}I_b = 1$  and  $\text{Im}I_b = 0$ . The quantity

$$N_b = \Gamma U_b - 1 \quad (5.6)$$

can be referred to as flame nonisentropicity (in a flame with  $\text{Re}N_b = 0$  the and  $\text{Im}N_b = 0$  adiabatic relation between the pressure and temperature perturbations holds).

## 1.6 Acoustic Admittance of Burning Propellant Surface

One of the important problems in the theory of nonsteady burning of condensed propellants is the possibility of amplification of pressure waves when they are reflected from the surface of a solid propellant. The solution of this problem is closely connected with the practically important problem of suppression of pressure oscillations in rocket motor chambers. If the propellant surface is capable of amplifying waves whose frequency coincides with one of the acoustic frequencies of the chamber, propellant burning in the chamber may be unstable: the pressure and burning rate will vary with time. Since the frequency of their variation depends on the acoustical properties of the chamber, this instability is termed acoustic, or high-frequency, instability.

Acoustical instability, as well as low-frequency instability, results from the interaction of two objects - the chamber void and the burning propellant surface. Since the acoustical properties of voids of different shapes have been studied very thoroughly, the principal difficulty in solving the problem of high-frequency instability is in establishing the properties of the propellant surface, i.e. its response to the varying pressure.

A property of a surface to enhance or attenuate an incident acoustic wave is characterized in acoustics by a dimensionless quantity which is called acoustic admittance of the surface.

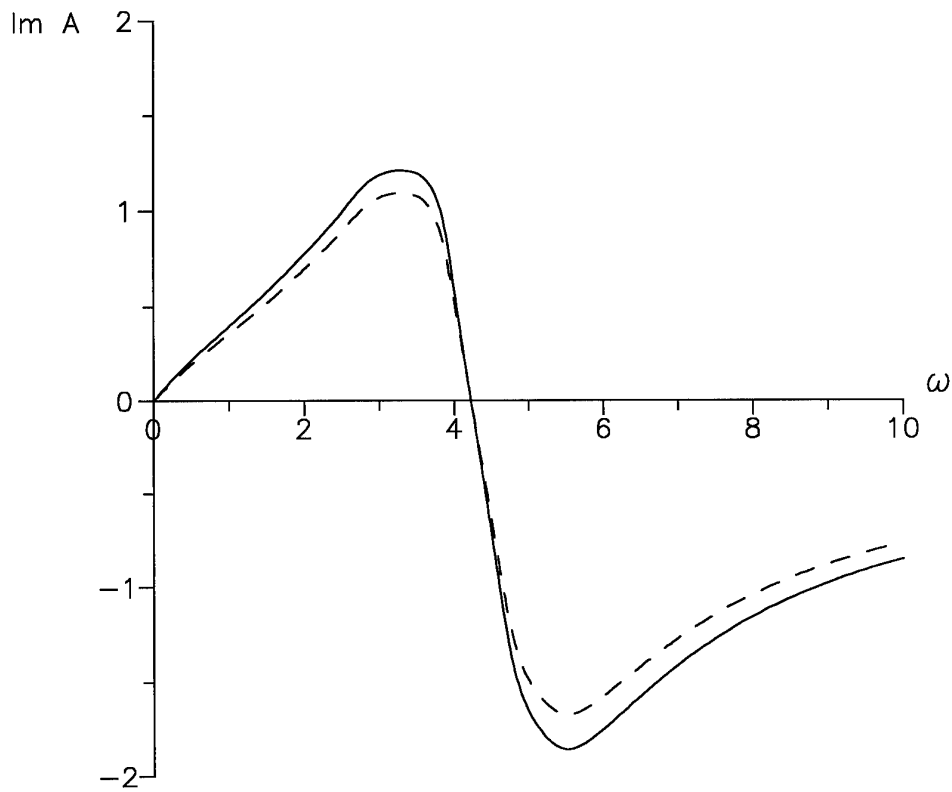


Figure 1.6.2 The imaginary parts of  $A$  (solid) and  $A_0$  (dashed).  $k = 1.8$ ,  $r = 0.3$ ,  $\nu = 0.3$ ,  $\delta = 0$ ,  $r_b = 0.1$ ,  $\mu_b = 0.5$ ,  $\gamma = 1.25$ . Figure 1.5.1 The real (solid) and imaginary (dashed) parts of  $U_b(\omega)$ .  $k = 1.8$ ,  $r = 0.3$ ,  $\nu = 0.3$ ,  $\delta = 0$ ,  $r_b = 0.1$ ,  $\mu_b = 0.5$

If the acoustic wave propagates in a moving medium (e.g., in burning gases or in propellant combustion) the acoustic admittance may be written as

$$\zeta = \gamma M \frac{(v_{g1})}{\eta_1}, \quad (6.1)$$

rather than the whole perturbation. Here  $M = u_g^0/a$  and  $a$  are the Mach number and sound velocity in the unperturbed gas, and  $v_{g1}$  is a dimensionless complex amplitude of gas velocity response to a harmonically varying pressure.

If the thermal lag of the gas phase is neglected, the gas velocity response in Eq.(6.1) can be simply expressed through the burning rate response. Indeed, the burning rate  $\rho_g u_g$  in the  $t_c$  approximation is constant over the space and is equal to the instantaneous burning rate  $m$ . Therefore, taking into account that the gas is perfect we have

$$\frac{\zeta}{\gamma M} = U_1 + U_b - 1. \quad (6.2)$$

Fig.1.6.1 and 1.6.2 show the real and imaginary parts of  $A = \frac{\zeta}{\gamma M}$ . In the same figures the value  $A_0 = U_1 - 1/\gamma$  is pictured that is usually used in the USA.

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## Chapter 2

# Solid Propellant Burning Rate Response Functions of the Second Order

In solid-propellant motors such conditions during which pressure oscillations arise are frequently realized. To find the amplitude of a self-excited oscillation and to analyze the possibility of triggering an oscillation by finite size disturbances, one should go out of the framework of the linear analysis. Some recent results for nonlinear acoustics in combustion chambers are presented by Culick and Yang in Ref.1.

Most analytical nonlinear calculations performed relate the nonlinearity, as a rule, to gasdynamic behavior since a propellant is assumed to be linear, although there are some papers<sup>2,3</sup> considering nonlinear combustion modeling in numerical analyses.

It is evident, nevertheless, that combustion instability is highly dependent on propellant characteristics. To produce the nonlinear phenomena observed in experiments such as limiting amplitudes, triggering, and a mean pressure shift, the nonlinear burning rate response has to be considered. A propellant itself is a nonlinear oscillatory system with a definite natural frequency and damping decrement. Some nonlinear effects of solid propellant combustion near the steady-state stability boundary were considered before in Refs.4-6.

A new concept of solid propellant burning rate response function of the higher order is introduced in Ref.7. In this Chapter a few examples of those functions of the second order are considered.

### 2.1 Response Functions of the Second Order

Suppose that the acoustic field contains two first harmonics so the pressure can be represented as

$$p = p^0 + p_1 \cos \omega \tau + p_2 \cos(2\omega \tau + \psi_2). \quad (1.1)$$

The burning rate in this case should be written as

$$u = u^0 + u_0 + u_1 \cos(\omega \tau + \psi_{u1}) + u_2 \cos(2\omega \tau + \psi_{u2}) \quad (1.2)$$

Let us introduce nondimensional complex amplitude of the pressure and the burning rate.

$$\eta_k = \frac{p_k}{2p^0} e^{i\psi_k}, \quad v_k = \frac{u_k}{2u^0} e^{i\psi_{u_k}}$$

The Eqs.(1.1-2) have the following forms

$$\eta = 1 + (\eta_1 e_1 + \eta_2 e_2 + c.c)$$

$$v = 1 + v_0 + (v_1 e_1 + v_2 e_2 + c.c)$$

where and denotes the complex conjugation.

To distinguish the linear and nonlinear response function we shall supply the former to one subscript and the latter to several subscripts.

So the linear response function for the first mode is where is given in Chapter 1 by Eq.(4.6).

$$U_1(\omega) = \frac{v_1}{\eta_1}$$

The linear response function of the second mode is

$$U_2(\omega) = \frac{v_2}{\eta_2}, \text{ skip } 1.5 \text{ cm } U_2(\omega) = U_1(2\omega)$$

or

$$U_2 = \frac{\nu + \delta(z_2 - 1)}{1 + (r(z_2 - k/z_2)(z_2 - 1))}$$

where

$$z_2 = \frac{1}{2}[1 + \sqrt{1 + 8i\omega}]$$

We introduce here the definition of response function of the order. Nonlinearity of combustion process can give the oscillation of the burning rate with frequency  $\omega_k \pm \omega_i$  by interaction of two the modes with frequencies  $\omega_k$  and  $\omega_i$ . The simplest examples are:

1) self-interaction of the first harmonic may give the second order correction to the second or zeroth modes of the burning rate

$$v_{1,1} = U_{1,1}|\eta_1|^2, \quad v_{1,-1} = U_{1,-1}\eta_1\bar{\eta}_1$$

2) interaction of the first and second modes give the second order correction to the linear response function for the first mode

$$v_{2,-1} = U_{2,-1}\eta_2\bar{\eta}_1$$

3) self-interaction of the second harmonic may give the second order correction to zeroth mode of the burning rate

$$v_{2,-2} = U_{2,-2}|\eta_2|^2$$

The burning rate response functions  $U_{1,1}$ ,  $U_{1,-1}$ ,  $U_{2,-2}$ , and  $U_{2,-1}$ . will be found in the next sections of this chapter.

## 2.2 Relations from Integral Equation

We will now obtained from the integral Eq.(3.16) of chapter 1 the relationships between the second order corrections to the burning rate, gradient, and temperature at the propellant surface.

In a stationary oscillatory  $\tau \rightarrow \infty$ , so the effect of the initial conditions disappears. The term of the integral equation which contains the initial temperature distribution vanishes because of the factor  $1/\sqrt{\tau}$ :

$$\vartheta(\tau) = \frac{1}{\sqrt{\pi}} \int_0^\tau \left( \varphi(\tau') - v(\tau')\vartheta(\tau') + \frac{\vartheta(\tau')I(\tau, \tau')}{2(\tau - \tau')} \right) \exp \left[ -\frac{I(\tau, \tau')^2}{4(\tau - \tau')} \right] \frac{d\tau'}{\sqrt{\tau - \tau'}}$$

Let us introduce a new integration variable

$$y^2 = \frac{\tau - \tau'}{4}$$

As the result we have

$$\vartheta(\tau) = \frac{4}{\sqrt{\pi}} \int_0^{\frac{\sqrt{\tau}}{2}} \left( \varphi(\tau - 4y^2) - v(\tau - 4y^2)\vartheta(\tau - 4y^2) + \frac{\vartheta(\tau - 4y^2)I(\tau, y)}{8y^2} \right) e^{-y^2} dy$$

The terms of the zeroth order give

$$1 = \frac{2}{\sqrt{\pi}} \int_0^{\frac{\sqrt{\tau}}{2}} e^{-y^2} dy$$

The terms of the first order give

$$z_1 \vartheta_1 = \varphi_1 + \frac{v_1}{z_1}$$

In the second order we have

$$\vartheta_{1,-1} = \varphi_{1,-1} - v_{1,-1} + Z_{1,-1}|\eta_1|^2, \quad \vartheta_{2,-2} = \varphi_{2,-2} - v_{2,-2} + Z_{2,-2}|\eta_2|^2 \quad (2.1)$$

$$z_2 \vartheta_{1,1} = \varphi_{1,1} - \frac{v_{1,1}}{z_2} + Z_{1,1}\eta_1^2, \quad z_1 \vartheta_{2,-1} = \varphi_{2,-1} - \frac{v_{2,-1}}{z_1} + Z_{2,-1}\eta_2 \bar{\eta}_1 \quad (2.1)$$

where

$$Z_{1,-1} = -\frac{1}{|\eta_1|^2}(v_1 \bar{\vartheta}_1 + \bar{v}_1 \vartheta_1) \quad Z_{2,-2} = -\frac{1}{|\eta_2|^2}(v_2 \bar{\vartheta}_2 + \bar{v}_2 \vartheta_2) \quad (2.3)$$

$$Z_{1,1} = \frac{1}{\eta_1^2} \left[ \frac{v_1^2}{2\omega^2} (1 - z^2 + 2z_1(z_2 - z_1)) + \frac{v_1 \vartheta_1}{i\omega} z_1 (z_1 - z^2) \right] \quad (2.4)$$

$$Z_{2,-1} = \frac{1}{\eta_2} \left[ \frac{\bar{v}_1}{i\omega} \left( \vartheta_2 + \frac{v_2}{2i\omega} \right) z_2 (z_1 - z_2) - \frac{v_2}{2i\omega} \left( \bar{\vartheta}_1 + \frac{\bar{v}_1}{i\omega} \right) \bar{z}_1 (z_1 - \bar{z}_1) + \frac{\bar{v}_1 v_2}{2\omega^2} (z_1 - 1) \right] \quad (2.5)$$

## 2.3 Relations from Nonsteady Burning Laws

The relations between complex amplitude of burning rate, temperature gradient, and pressure are obtained to second order terms in this section. Expanding the nonsteady combustion laws  $v(\varphi, \eta)$  and  $\vartheta(\varphi, \eta)$  in a Taylor series up to second order terms we have

$$\begin{aligned} v_e &= v_\varphi \varphi_e + v_\eta \eta_e + \frac{1}{2} v_{\varphi\varphi} \varphi_e^2 + v_{\varphi\eta} \varphi_e \eta_e + \frac{1}{2} v_{\eta\eta} \eta_e^2 \\ \vartheta_e &= \vartheta_\varphi \varphi_e + \vartheta_\eta \eta_e + \frac{1}{2} \vartheta_{\varphi\varphi} \varphi_e^2 + \vartheta_{\varphi\eta} \varphi_e \eta_e + \frac{1}{2} \vartheta_{\eta\eta} \eta_e^2 \end{aligned} \quad (3.1)$$

where subscript denotes extra-terms to the steady-state conditions:

$$\begin{aligned} \eta &= 1 + \eta_e, \quad \varphi = 1 + \varphi_e \\ v &= 1 + v_e \quad \vartheta = \vartheta^0 + \vartheta_e \end{aligned}$$

Subscripts  $\varphi$  and  $\eta$  relate to derivatives with respect to temperature gradient and pressure. For any function  $Y$

$$\begin{aligned} \left(\frac{\partial Y}{\partial \varphi}\right)_\eta &= Y_\varphi, & \left(\frac{\partial Y}{\partial \eta}\right)_\varphi &= Y_\eta \\ \left(\frac{\partial^2 Y}{\partial \varphi^2}\right)_\eta &= Y_{\varphi\varphi}, & \frac{\partial^2 Y}{\partial \varphi \partial \eta} &= Y_{\varphi\eta}, & \left(\frac{\partial^2 Y}{\partial \eta^2}\right)_\varphi &= Y_{\eta\eta}. \end{aligned}$$

Here the list of these derivatives:

$$v_\varphi = \frac{k}{D}, \vartheta_\varphi = \frac{r}{D}, v_\eta = \frac{\delta - \nu}{D}, \vartheta_\eta = -\frac{\delta + \mu}{D}$$

$$v_{\varphi\varphi} = \frac{\Delta}{D} \left(\frac{\partial v_\varphi}{\partial T_a}\right)_p + v_\varphi^2 - v_\varphi, \vartheta_{\varphi\varphi} = \frac{\Delta}{D} \left(\frac{\partial \vartheta_\varphi}{\partial T_a}\right)_p - \vartheta_\varphi v_\varphi, \quad (3.2)$$

$$v_{\varphi\eta} = \frac{\Delta}{D} \left(\frac{\partial v_\eta}{\partial T_a}\right)_p + v_\varphi v_\eta, \vartheta_{\varphi\eta} = \frac{1}{D} \left[ \Delta \left(\frac{\partial \vartheta_\eta}{\partial T_a}\right)_p - \vartheta_\eta \right] + \vartheta_\eta v_\varphi,$$

$$v_{\eta\eta} = \left(\frac{\partial v_\eta}{\partial \ln p}\right)_{T_a} + \Delta(v_\eta + \vartheta_\eta) \left(\frac{\partial v_\eta}{\partial T_a}\right)_p + v_\eta^2 - v_\eta, \vartheta_{\eta\eta} = \left(\frac{\partial \vartheta_\eta}{\partial \ln p}\right)_{T_a} + \Delta(v_\eta + \vartheta_\eta) \left(\frac{\partial \vartheta_\eta}{\partial T_a}\right)_p + \vartheta_\eta^2 - \vartheta_\eta,$$

If the steady-state burning laws are

$$u^0 = A(p^0)^n \exp(\beta T_a), \quad u^0 = B(p^0)^{n_s} \exp(\beta_s T_s^0) \quad (3.3)$$

then

$$\nu = n, \quad k = \beta \Delta, \quad \mu = \frac{r(\nu - n_s)}{k}, \quad r = \frac{\beta}{\beta_s}, \quad \delta = r n_s, \quad (3.4)$$

and the second derivatives have the following forms

$$v_{\varphi\varphi} = -\frac{k^2(r-1)}{D^3}, \quad \vartheta_{\varphi\varphi} = -\frac{kr}{D^3} [k + 2(r-1)],$$

$$v_{\varphi\eta} = \frac{k^2(\delta - \nu)}{D^3}, \quad [k + 2(r-1)], \vartheta_{\varphi\eta} = \frac{r-1}{D^3} [\mu - \delta(r-1)], \quad (3.5)$$

$$v_{\eta\eta} = \frac{\delta - \nu}{D^3} [(\delta - \nu)(2k + r - 1) - D^2], \quad \vartheta_{\eta\eta} = \frac{\delta - \nu}{D^3} [\delta k + \mu(2k + r - 1)] + \frac{(\delta + \mu)^2}{D^2} + \frac{\delta + \mu}{D^2}$$

## 2.4 The Zeroth Mode Corrections

From the relations obtained in the previous sections it easy to obtain the zeroth mode burning rate response functions of the second order:

$$U_{1,-1} = D[v_\varphi Z_{1,-1} + L_{\varphi\varphi} |\varphi_1/\eta_1|^2 + L_{\varphi\eta} (\varphi_1/\eta_1 + (\bar{\varphi}_1/\bar{\eta}_1) + L_{\eta\eta}] \quad (4.1)$$

$$U_{2,-2} = D[v_\varphi Z_{2,-2} + L_{\varphi\varphi} |\varphi_2/\eta_2|^2 + L_{\varphi\eta} (\varphi_2/\eta_2 + (\bar{\varphi}_2/\bar{\eta}_2) + L_{\eta\eta}] \quad (4.2)$$

where

$$L_{\varphi\varphi} = (\vartheta_\varphi - 1)v_{\varphi\varphi} - v_\varphi \vartheta_{\varphi\varphi},$$

$$L_{\varphi\eta} = (\vartheta_\varphi - 1)v_{\varphi\eta} - \vartheta_\varphi v_{\varphi\eta},$$

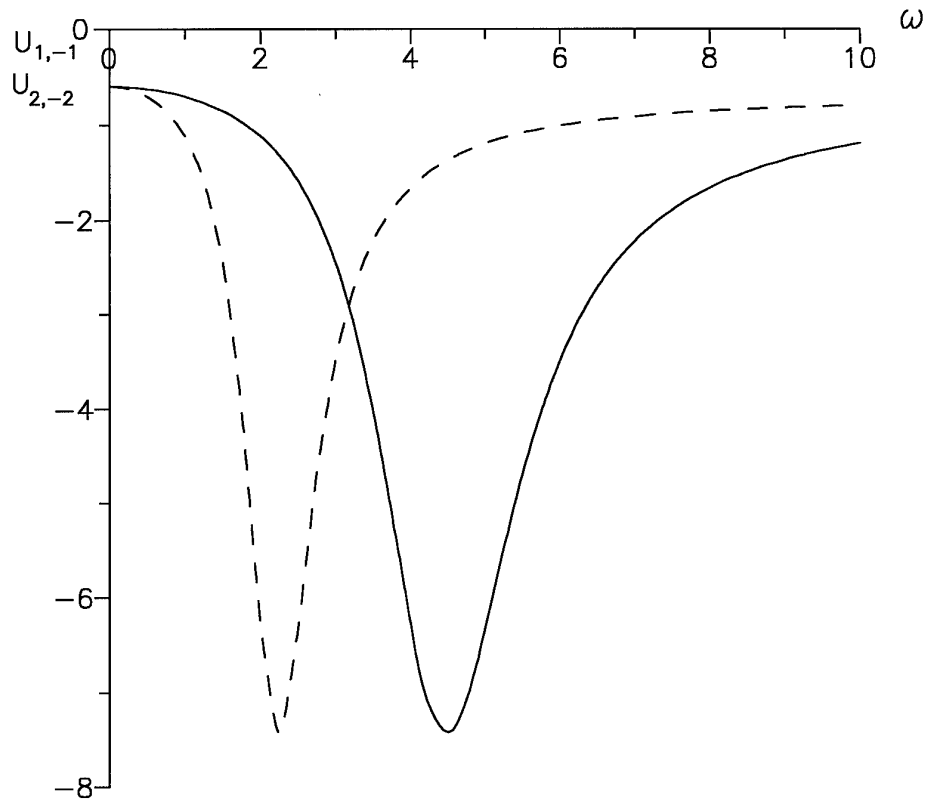


Figure 2.4.1 The zeroth mode burning rate response functions of the second order  $U_{1,-1}$ —solid,  $U_{2,-2}$ —dashed)  $k = 1.8$ ,  $r = 0.3$ ,  $\nu = 0.3$ ,  $\delta = 0$ .

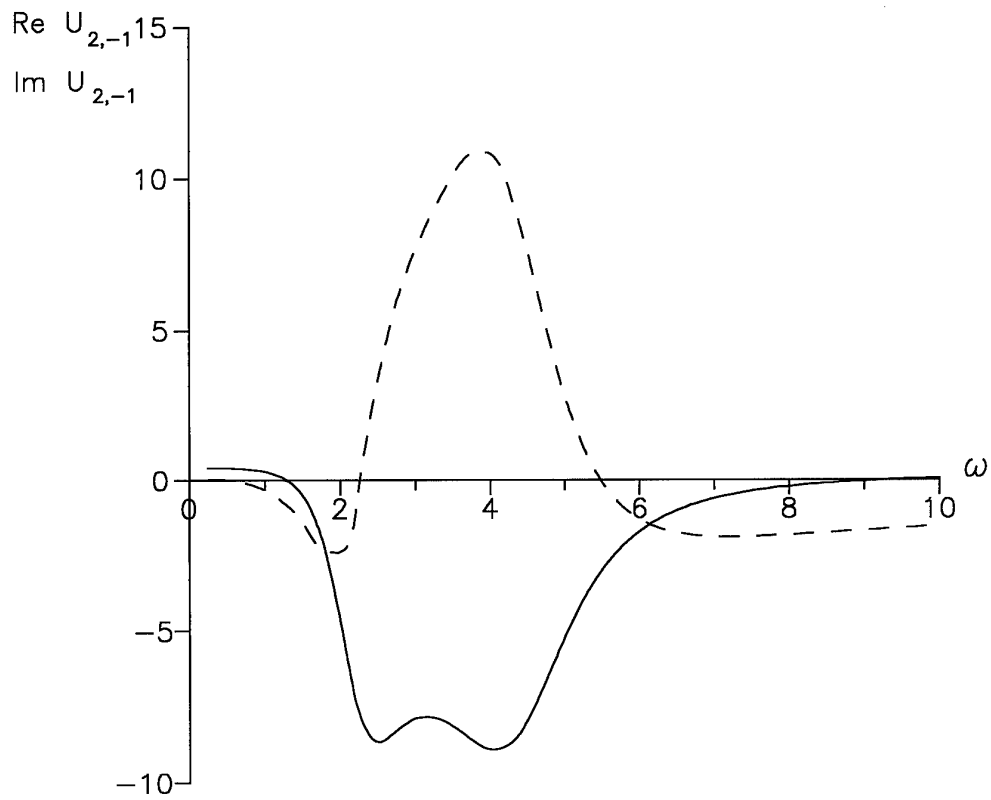


Figure 2.5.1 The real (solid) and imaginary (dashed) parts of  $U_{2,-1}$   $k = 1.8$ ,  $r = 0.3$ ,  $\nu = 0.3$ ,  $\delta = 0$ .

$$L_{\eta\eta} = (\vartheta_\varphi - 1)v_{\eta\eta} - v_\varphi\vartheta_{\eta\eta},$$

To end the calculation we should remember

$$\frac{\varphi_1}{\eta_1} = \frac{D}{k}U_1 - \frac{\delta - \nu}{k}, \quad \frac{\vartheta_1}{\eta_1} = \frac{r}{k}U_1 - \frac{\delta}{k},$$

$$\frac{\varphi_2}{\eta_2} = \frac{D}{k}U_2 - \frac{\delta - \nu}{k}, \quad \frac{\vartheta_2}{\eta_2} = \frac{r}{k}U_2 - \frac{\delta}{k},$$

Figure 2.4.1 shows the zeroth mode burning rate response functions of the second order.

## 2.5 The Second Order Interaction of the First and Second Harmonics

For the first mode in the second order we have

$$U_{2,-1} = \frac{v_\varphi Z_{2,-1} + L_{\varphi\varphi}\bar{\varphi}_1\varphi_2/\bar{\eta}_1\eta_2 + L_{\varphi\eta}(\bar{\varphi}_1/\eta_1 + \varphi_2/\eta) + L_{\eta_1\eta}}{z_1\vartheta_\varphi - 1 + v_\varphi/z_1} \quad (5.1)$$

Figure 2.5.1 shows the real and imaginary parts of  $U_{2,-1}$

## 2.6 The Second Mode From Self-Interaction of the First Harmonic

For this case we have

$$U_{1,1} = \frac{v_\varphi Z_{1,1} + L_{\varphi\varphi}\varphi_1/\eta_1^2 + L_{\varphi\eta}\varphi_1/\eta_1 + L_{\eta_1\eta}}{z_2\vartheta_\varphi - 1 + v_\varphi/z_2} \quad (5.1)$$

Figure 2.6.1 shows the real and imaginary parts of  $U_{1,1}$

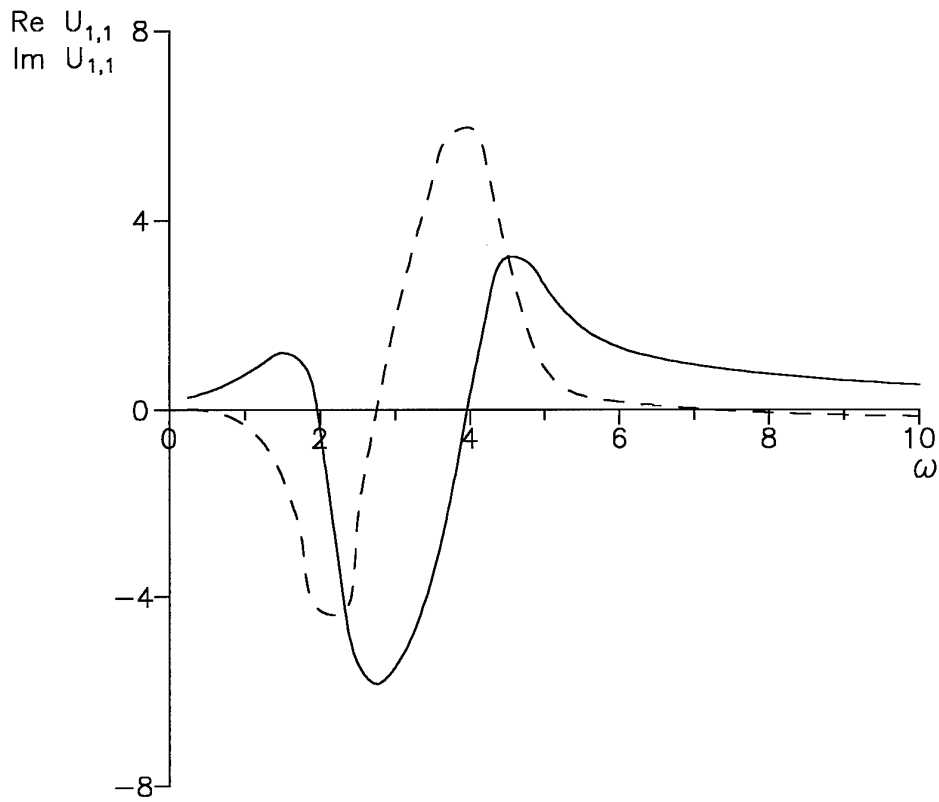


Figure 2.6.1 The real (solid) and imaginary (dashed) parts of  $U_{1,1}$   $k = 1.8$ ,  $r = 0.3$ ,  $\nu = 0.3$ ,  $\delta = 0$ .

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