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THE APPLICATION OF A MAGNETIC LENS SPECTROMETER TO THE MEASUREMENT OF GAMMA RADIATION FROM Zn^{65} AND Co^{60}

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ABSTRACT

A thin magnetic lens spectrometer for the investigation of gamma-ray spectra is described. The effect of the thickness of the radiator used for the production of photoelectrons and the influence of the earth's magnetic field are reported. Based on a calibration of the instrument by means of annihilation radiation and the F line of ThB, energy values of 1.106 Mev for the gamma-ray of Zn^{65}, and 1.155 and 1.317 MeV for the two lines of Co^{60} are obtained. The probable error is estimated as 0.5 per cent.

INTRODUCTION

The use of a thin magnetic lens spectrometer for the study of beta and gamma radiations has been reported by several investigators.[1,2,3,4] The flexibility of such an instrument and, if iron-free, the convenience of its linearity have been previously indicated.[1] It is the purpose of this paper to describe briefly a magnetic lens spectrometer which we have constructed, to present the results of studies to determine the corrections which should be made to data obtained with it, and to give the energies found for the gamma radiations from Zn^{65} and Co^{60}.

DESCRIPTION OF SPECTROMETER

The spectrometer is shown in Figures 1 and 2. The design is similar to that employed by previous workers,[1,2,3] except that the instrument is mounted with its axis parallel to the magnetic field of the earth and, to minimize scattering, the chamber proper has been constructed of aluminum tubing. To preserve linearity, nonferromagnetic materials have been used throughout.

The spectrometer chamber is 7 inches in diameter and 40 inches long, evacuated by means of a two-stage oil diffusion pump backed by a mechanical pump. The baffles, shown in Figure 2, are of micarta, $\frac{1}{2}$-inch thick, except for the gamma-ray shields, which are of lead sheathed with aluminum. Baffle C, which is adjustable by means of a brass rod passing out of the chamber through a Wilson Seal, serves to delimit the electrons analyzed and so, for a given diameter of source and counter window, determines the intensity and resolution obtained. The lead shield surrounding the counter is primarily for the purpose of absorbing scattered gamma radiation and was designed to lie within the shadow of the lead shield in the center of the spectrometer. An indication of the small extent of electron scattering obtained with the arrangement described is seen from the fact that, with no current in the coil, the counting rates obtained with and without a 10 microcurie beta-ray source in the instrument were, respectively 21.1 ± 0.4 and 20.7 ± 0.2 cts/min.

Radioactive sources are mounted on lucite holders at the end of a brass tube which enters the upper end of the spectrometer through a Wilson seal and through a $2\frac{1}{2}$-inch gate valve modified to be suitable for vacuum service. The counter is mounted within a similar brass tube at the lower end of

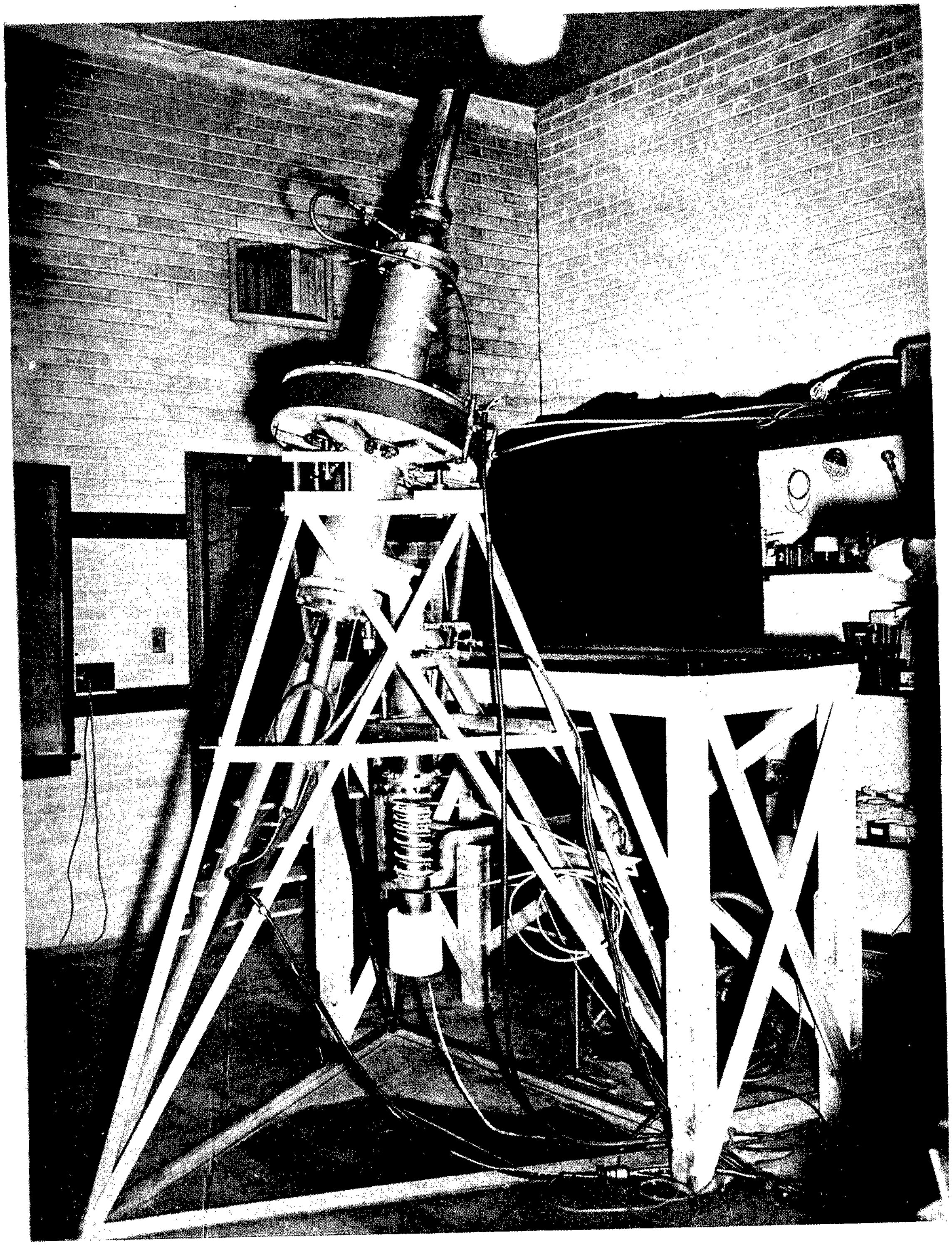

Figure 1. The magnetic lens spectrometer, aligned with its axis parallel to the magnetic field of the earth.

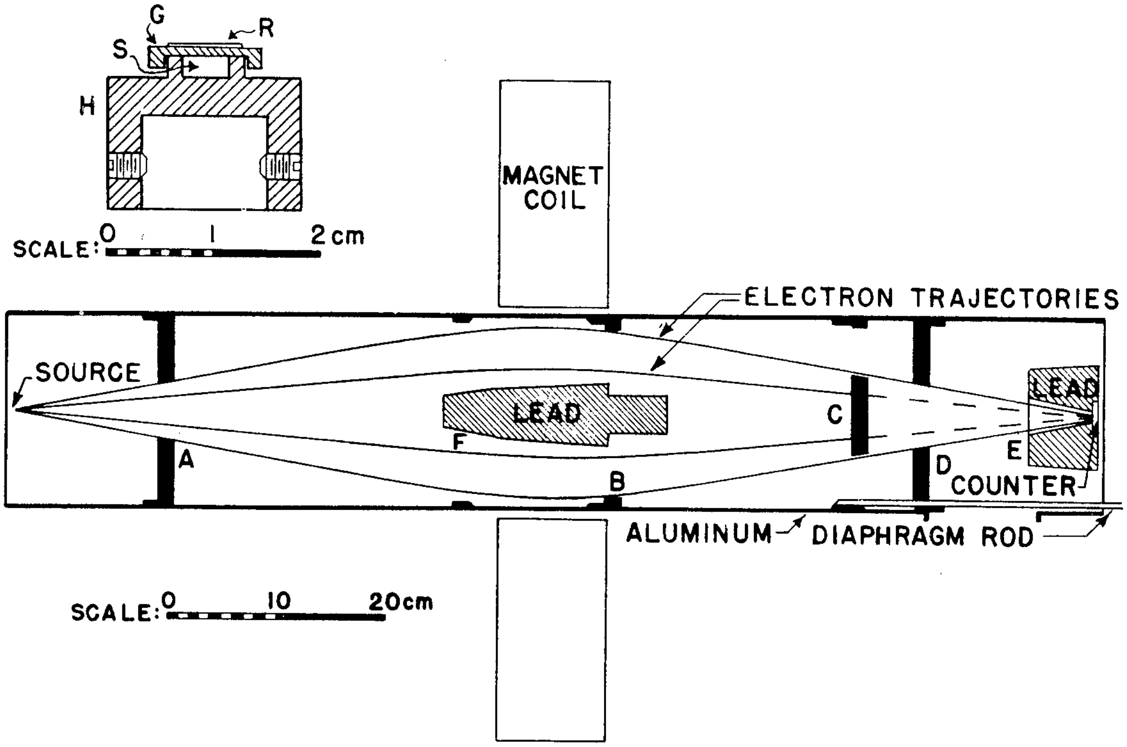

Figure 2. Diagram of spectrometer chamber. Insert: Source holder.

the instrument, where Wilson seals are again used to facilitate assembly and adjustment. The counter was originally used with a mica window of 4 mg/cm² surface density; for the ThB measurements a 1.1 mg/cm window was used and for the most recent work a thin formvar-polystyrene film (≅ 0.3 mg/cm²) was employed.

The coil for producing the magnetic field consists of 2799 turns of No. 12 single cotton-covered enameled copper wire, wound on a form consisting of a brass hub and two aluminum castings. Every fourth layer of wire is followed by a copper sheet, 0.030-inch thick, provided with 12 tabs which are soldered to water-cooled brass blocks mounted on the exterior surface of the castings. The completed coil has an inside radius of 9.9 cm, an outside radius of 28.3 cm, and an axial length of 10 cm. When the full number of turns is used with 220 volts across the coil, a focal length of 25 cm is obtained for electrons of approximately 3.4 Mev energy.

The focusing current for the coil is provided by a 2 kw motor-generator set. To stabilize the current, a portion of it is passed through a bridge circuit which has as one of its elements a 60-watt tungsten lamp bulb to serve as a non-linear resistance. Changes in the coil current affect the balance of the bridge and the resulting error-signal, when amplified, is used to correct the generator field. The magnetic field is thereby maintained constant within a probable error of 0.1 per cent. The coil current is measured by means of a series resistance and a potentiometer.

DETERMINATION OF GAMMA-RAY ENERGIES

General Method

In the work described in this paper, the gamma-ray energies were determined by a study of the spectra of photoelectrons produced in radiator foils. For calibration, use was made of photoelectrons produced by the annihilation radiation from Zn^{65} and of conversion electrons from ThB (F line). Each gamma-ray source S (Figure 2), a few mm thick, was mounted in a lucite holder H and covered by an aluminum cap G, which carried the radiator R.

The spectra obtained from Zn^{65} and Co^{60} sources are shown in Figures 3 and 4. In addition to the photoelectric conversion lines generated in the lead by gamma and annihilation radiation, a broad distribution of Compton electrons is also obtained.

To permit an accurate determination of the energies of the photoelectrons ejected from the radiator, attention must be given to the effect of radiator thickness and to the influence of the earth's magnetic field, which is in the direction of the spectrometer axis.

Effect of the Magnetic Field of the Earth.

For a focusing field of a given shape the momentum of the focused electrons will quite generally be proportional to the strength of the field and, if the field in question is proportional to the coil current, we may write for this momentum

$$P = I \cdot F \tag{1}$$

where I is the current in the coil and F is dependent upon the shape of the field. In the presence of an additional magnetic field H, superposed upon that produced by the coil current, F may be regarded as a function of the ratio H/I, since the shape of the field would remain unchanged if H and I were to vary in a mutually proportional manner. The relation between the current I_1 required to focus electrons of a given energy in the presence of the field H and the current I_0 required in its absence may therefore be written

$$I_1 \ F(H/I_1) = I_0 \ F(O). \tag{2}$$

One then finds, to a first approximation, that

$$I_1 - I_0 = -H \ F'(O)/F(O) \tag{3}$$

indicating that this difference is independent of the energy of the electrons. This is in agreement with the conclusions of Quade and Halliday,[4b] who have shown experimentally that for their spectrometer very little error is made by applying equation 3 to electron energies as low as 10 kev.

In the use of the spectrometer, it is the current necessary to focus electrons in the absence of an external field which is to be taken as proportional to the momentum, so the difference $I_1 - I_0$ must be determined and applied as a correction. This correction is most readily found by observing the change in the focusing current required when the current in the coil is reversed. It is, however, of interest to note that an approximate calculation, described in the Appendix, leads to a value for the correction which is independent of the energy and is in good numerical agreement with that found empirically. When all the turns on the focusing coil are employed, the current required to focus a particular conversion line is found to change by 0.012 amp when the current is reversed, so the correction then to be applied because of the presence of the magnetic field of the earth has been taken as $\pm$ 0.006 amp.

Effect of Radiator Thickness

The photoelectrons ejected from a radiator will, for a particular gamma-ray energy, have energies which depend upon the depth of the point from which they originate. The momentum distribution of the emergent electrons will, to a first approximation, be rectangular, with a width equal to the momentum loss associated with a full traversal of the radiator foil. To a higher order of approximation it might be supposed that, due to the change of the rate of momentum loss as the electrons lose energy in the foil, a trapezoidal distribution should be considered. In addition, the scattering of electrons in their passage through the foil would cause the distribution to drop and tail off on the low momentum side. An approximate analysis of these phenomena, as well as the experimental results reported here, indicates, however, that these effects are not of importance in the energy range with which we are

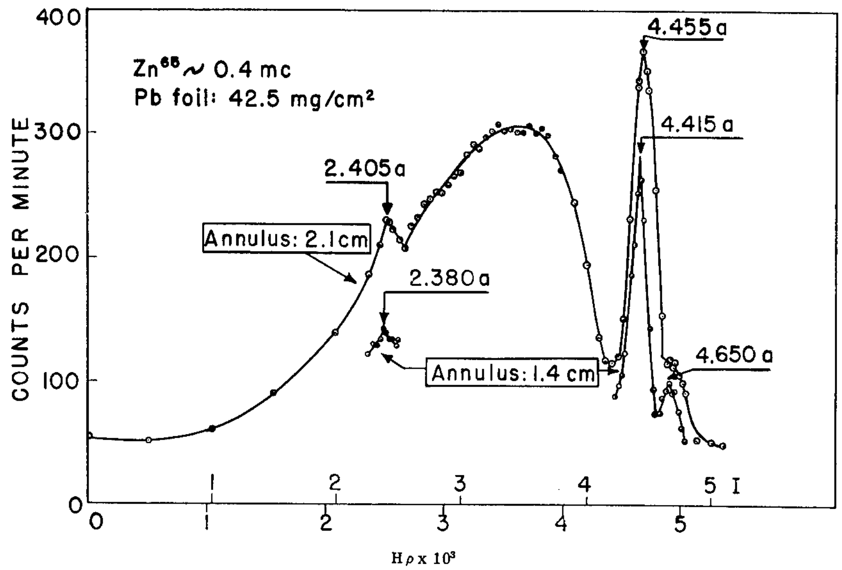

Figure 3. Spectrum of Zn^{65}, showing the photoelectric-conversion peaks produced in a lead radiator by annihilation radiation and the 1.11 Mev gamma ray, in addition to the broad distribution of Compton electrons. The sharper peaks shown separately were obtained with an adjustment which permitted the K and L lines to be resolved. The annulus is the width of the electron beam at the center of the spectrometer.

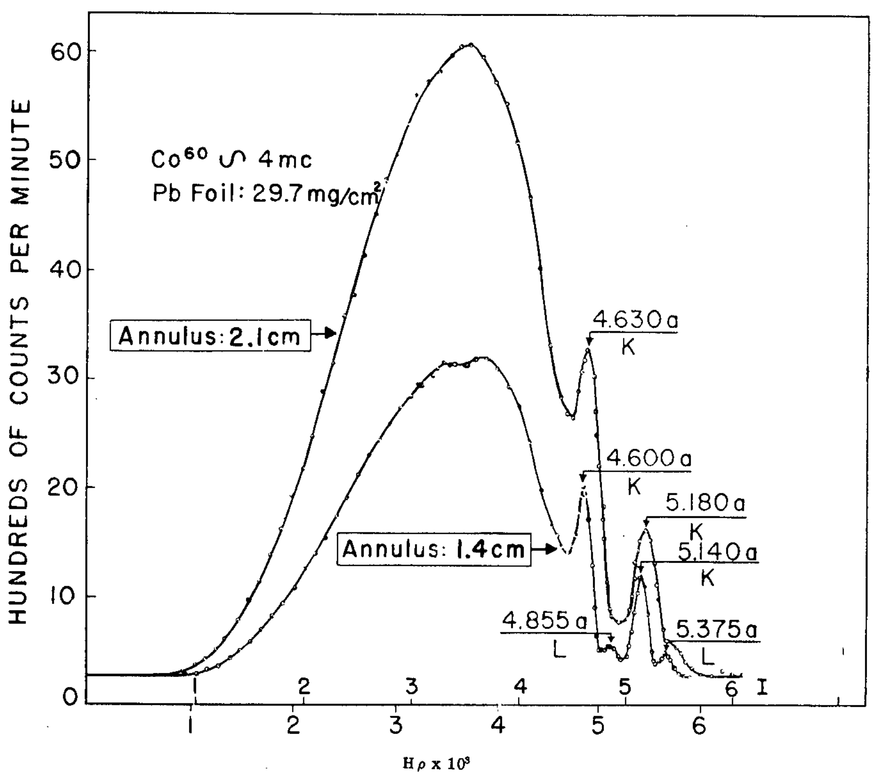

Figure 4. Spectrum of Co^{60}, showing the photoelectric-conversion peaks produced in a lead radiator by the two gamma rays present.

concerned in this paper. At lower energies scattering will certainly play a prominent role [cf. Bethe, Rose, and Smith, Proc. Amer. Phil. Soc. 78:573 (1938)]. Figure 5(A) shows a momentum distribution of this type, which extends from a momentum P_a to the maximum momentum P_m. The result of the combination of this distribution function with the transmission curve of the spectrometer must be considered and will indicate the manner by which the experimental data may be corrected in cases for which the effect of radiator thickness is not completely negligible. The result of an analysis of this character will be applicable with equal validity to internal conversion lines which arise from a source of non-vanishing thickness.

The transmission curve of a magnetic lens spectrometer has been investigated by Deutsch, et al[1], and has approximately the shape of an isosceles triangle for the case in which the image and counter windows have the same size. As the current is changed in the coil of the spectrometer, the width of the transmission curve will vary in direct proportion to the momentum of the electrons which it passes. For a triangular transmission curve, we therefore take the half-width b as equal to a constant K multiplied by the momentum P_0 corresponding to the point of maximum transmission. This is illustrated by Figure 5(B). The constant K evidently serves as a measure of the resolution of the instrument. When, in order to obtain the expected line shape we pass such a transmission curve

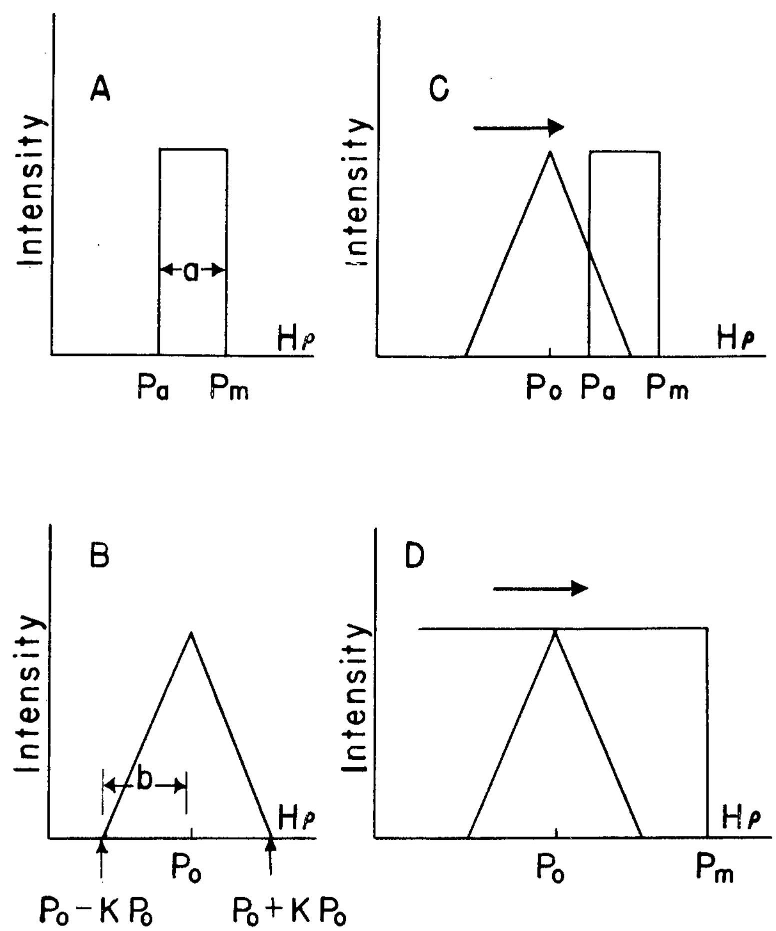

Figure 5. Momentum distribution and transmission curve of spectrometer, as assumed for the purposes of the analysis given in the text.

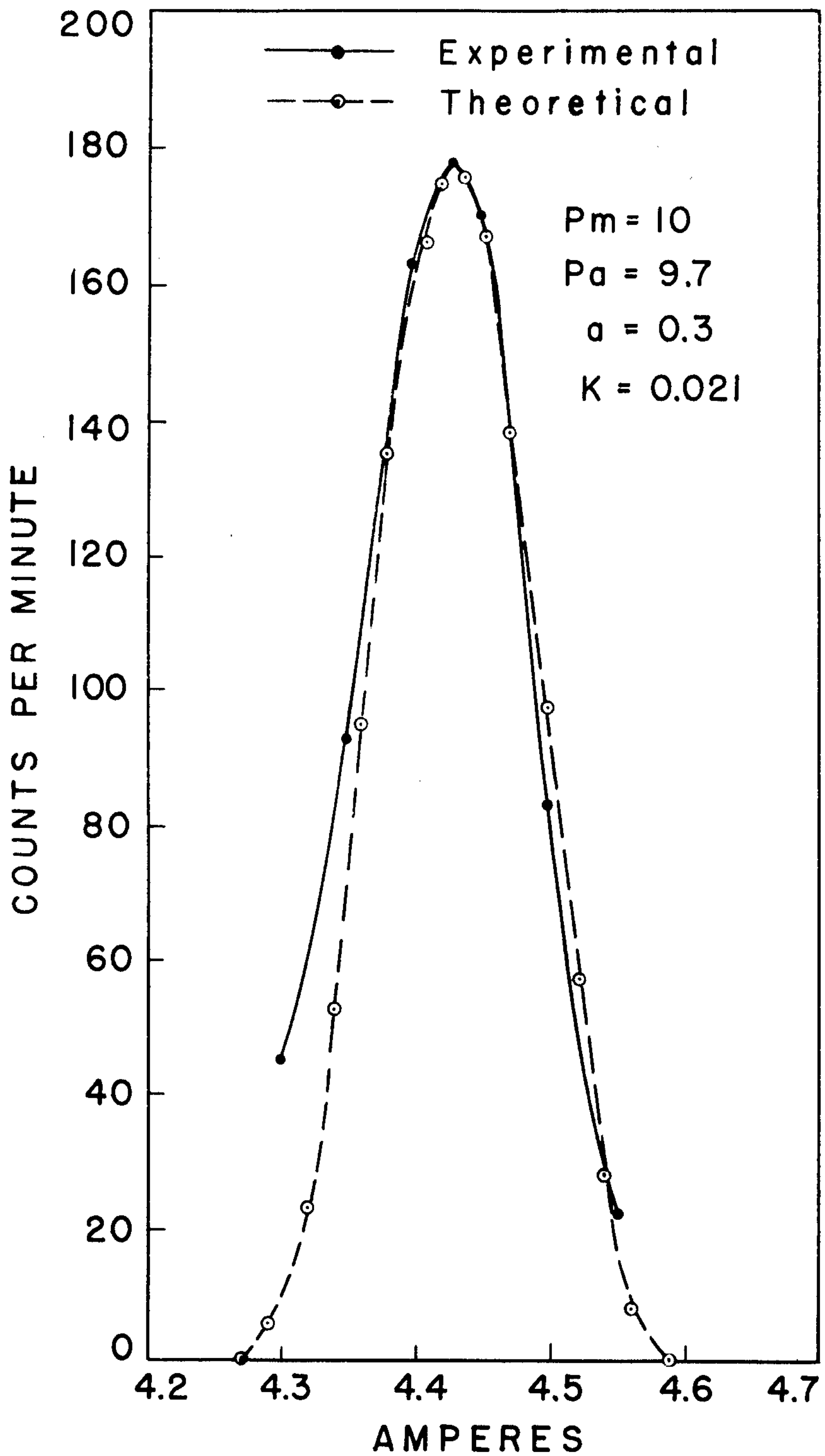

Figure 6. Resultant line shape obtained with electrons for which $P_m \simeq 4800$ gauss-cm. The dotted curve represents the line shape calculated for $a/P_m = 0.03$ and $K = 0.021$; the solid curve represents the shape obtained experimentally under comparable conditions.

across the momentum distribution for the electrons, there are two cases to consider. The first of these is that for which the momentum spread of the electrons is less than the full width of the transmission curve, as illustrated by Figure 5(C); the other is that for which the momentum spread is greater than the width of the transmission curve and is shown in Figure 5(D).

In the case of a thin radiator, specifically one for which the momentum spread $P_m - P_a$ is less than 2b, the maximum transmission is found to occur when

$$P_o \cong (P_a + P_m)/2 \tag{4}$$

neglecting terms small compared to $P_m - P_a$.

Thus

$$P_m \cong P_o + a/2 \tag{5}$$

where

$$a \equiv P_m - P_a \tag{6}$$

The effect of radiator thickness is, therefore, to give maximum transmission at a momentum which is less than the maximum momentum of the electrons by an amount which is equal in a first approximation to one-half the momentum loss experienced by electrons which traverse the full thickness of the radiator.

For a thick radiator, for which $P_m - P_a > 2b$ maximum transmission is to be expected when the transmission curve lies just inside the momentum distribution, if terms in K^2 are neglected. Thus

$$P_m \cong P_o (1 + K) \tag{7}$$

where

$$K = b/P_o \tag{8}$$

In determining, from the current corresponding to maximum transmission, the upper limit of the momentum distribution of electrons generated by an unknown gamma ray, the factor $(1 + K)$ may be absorbed into the calibration constant of the spectrometer provided the radiator thickness is such that Equation 7 is applicable. It should be noted that, due to the variation of the rate of momentum loss, a radiator which can be correctly regarded as a thin foil for high energies may, on the other hand, be effectively a thick foil at lower energies. We shall, therefore, apply the correction indicated by Equation 7 in an explicit fashion in those cases to which it applies. In analyzing the data reported in this paper, we have based the energy determinations on the positions of the maxima of the curves obtained, subject to the corrections indicated, since the maximum appears to be the point most accurately located for every line.

The complete line shape which results from a combination of a rectangular momentum distribution and a triangular transmission curve has been calculated for the case that a/P_m, the relative momentum spread from the radiator, is 0.03 and the resolution of the spectrometer is such that $K = 0.021$. The calculated curve is represented by the broken line in Figure 6 and may be compared with the solid line, which gives the results experimentally obtained under these conditions with photoelectrons produced in lead by Zn^{65} radiation (P_m = 4800 gauss-cm). The two curves were made to fit at their peaks and it is thought that their shapes are in satisfactory agreement. The somewhat larger counting rate obtained experimentally on the low momentum side of the line may be ascribable to straggling and scattering phenomena, the importance of which is indicated by, for example, the work of White and Millington.[5]

An experimental study was made of the positions of the points of maximum intensity when various radiator thickness are used. For this purpose the 1.1 Mev gamma ray of Zn^{65} was again used, with the results shown in Figures 7 and 8. It is seen that, in agreement with our previous discussion, the shift of the peaks obtained with thin foils is proportional to the thickness of the radiator, but becomes constant when the foil thickness exceeds a value of approximately 65 mg/cm^2. The slope of the initial part

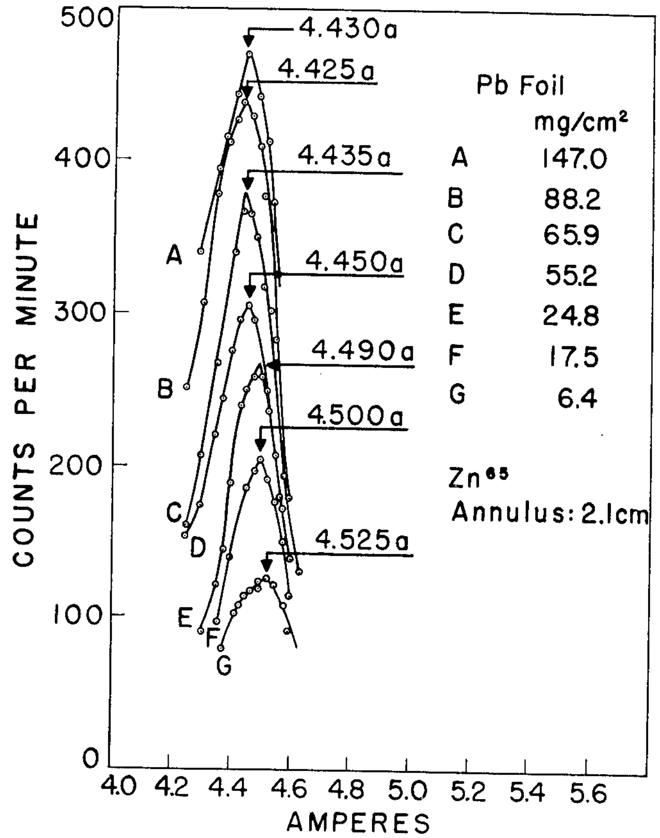

Figure 7. Photoelectric lines obtained from the 1.11 Mev gamma ray of Zn^{65} with various thicknesses of the lead radiator.

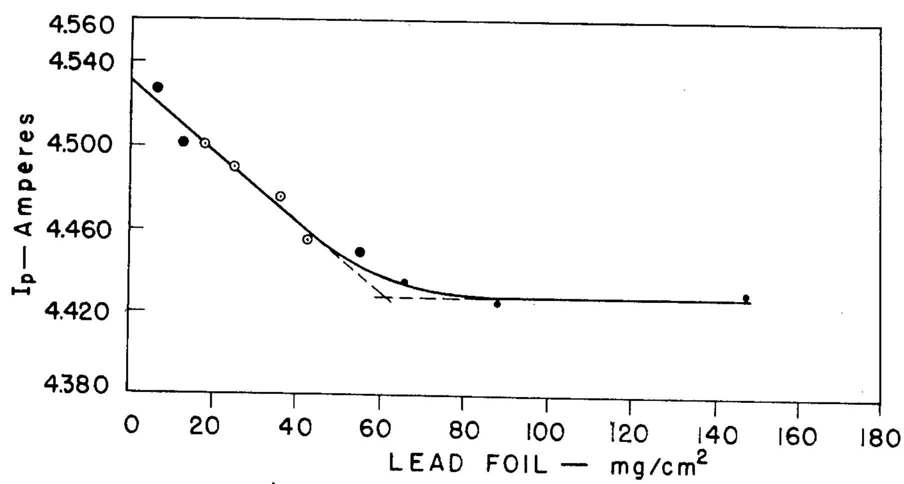

Figure 8. Current values corresponding to the peaks of the lines of Figure 7, as a function of radiator thickness. The results of additional data, not shown in Figure 7, are included. In determining the slope of the line, the points designated by the heavy solid circles were given half the weight of those marked by open circles.

of the curve in Figure 8 corresponds to 1.75 gauss-cm/mg-cm^{-2}. The theoretical rate of energy loss in lead, as obtained from a formula given by Heitler,[6] is 1.0 Mev/gm-cm^{-2} for electrons of the energy with which we are concerned here. This theoretical energy loss corresponds to a momentum loss of 3.5 gauss-cm/mg-cm^{-2} and, when compared with the slope of the experimental curve, affords confirmation of the statement that the peaks should be shifted by an amount which is half the momentum loss associated with a full traversal of the radiator foil.

The horizontal portion of the curve of Figure 8 occurs at a current value which is 2.3 per cent below the extrapolated value for zero foil thickness. This implies that K = 0.023, which is consistent with the expected resolution for the spectrometer at the time the data were obtained. The break in the curve of Figure 8 occurs, as expected, at a radiator thickness for which a = 2b. Similar data obtained with a lower energy gamma ray, for which the photoelectrons have an energy of 0.177 Mev, indicate that the break occurs for a foil thickness between 6.6 and 11.3 mg/cm^2. In this case the condition a = 2b would imply a thickness of 10 mg/cm^2.

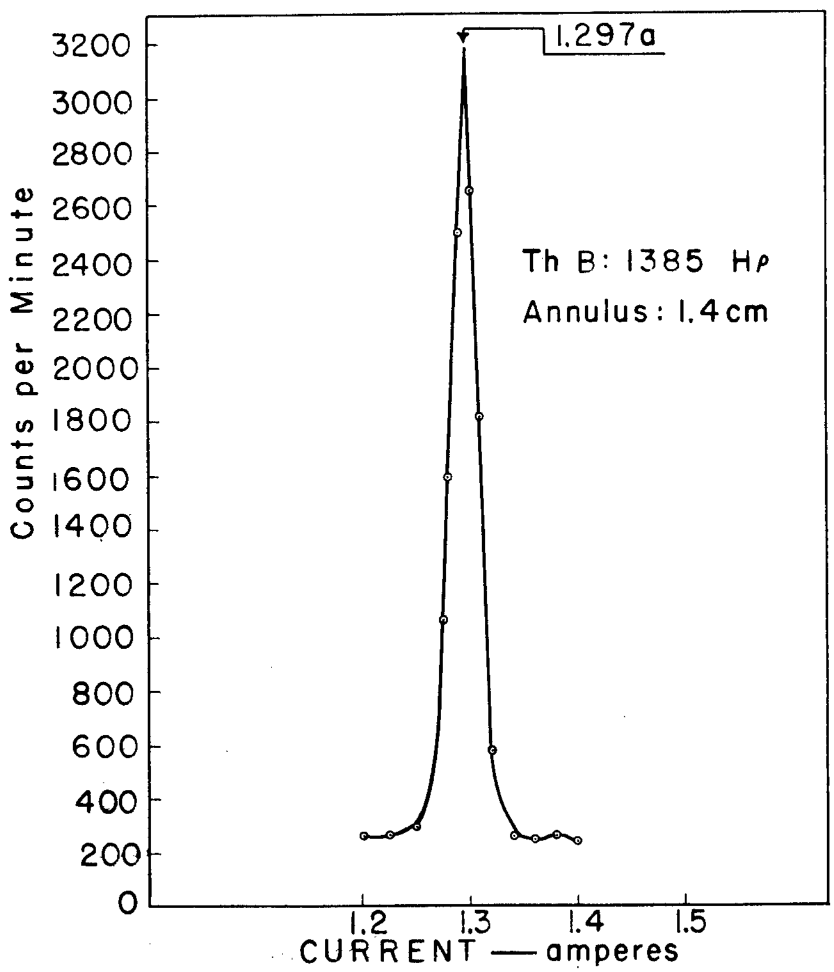

Figure 9. The F-line of ThB.

RESULTS

The photoelectric conversion lines obtained with lead radiators were measured for the Zn^{65} and Co^{60} radiations at each of two settings of the adjustable baffle. For these baffle positions, the radial width of the effective aperture at the center of the spectrometer assumed the values 2.1 and 1.4 cm. The resolution of the spectrometer was characterized by K = 0.023 and K = 0.021 in these two cases. As may be seen from Figures 3 and 4, lines were obtained from both the K and L shells of the lead in the second series of measurements.

For calibration, the F line of ThB and the photoelectric line produced by the Zn^{65} annihilation radiation were measured at each of the two adjustments of the instrument. For the two adjustments the calibrations from the annihilation radiation and the F line of ThB agree to 0.1% and 0.3% respectively. The ThB sample was deposited on an aluminum foil 0.00025-inch thick and mounted on the lucite source holder by means of a thin formvar-polystyrene film. The line obtained with this source is shown in Figure 9.

The results of the measurements are summarized in Table 1. Lines which are similar in character and for which the intensity measurements are made with equal precision can, presumably, be located with the same relative accuracy, although the lines are of different momenta and occur at different momenta and occur at different current values. In the work reported here, however, the data obtained were such that the location of the various lines could not be determined in all cases with the same relative accuracy; accordingly the estimated weighting factors indicated in Table 1 were applied to the current / momentum ratios.

In calculating, from the data of Table 1, the momenta of the photoelectrons generated in the lead radiator by the Zn^{65} gamma ray, a correction of 74 gauss-cm was taken as appropriate to the foil thickness employed. For the Co^{60} determinations, the correction was assigned the values 51 and 64 gauss-cm for the thinner and thicker Pb radiators, respectively. The correction made for the Th radiator was 48 gauss-cm and that for the U foil was 70 gauss-cm. Upon converting from the resulting momenta to the corresponding energy values and adding the binding energy appropriate to the photoelectric process involved, the gamma-ray energies shown in the final column of Table 1 resulted. Averaging for each line the energy values so found, taking into account the weights assigned to the individual determinations and to the calibration measurements the following gamma-ray energies are obtained:

Zn^{65},	1.106 Mev;
Co^{60}, I,	1.155 Mev; and
Co^{60}, II,	1.317 Mev.

A conservative estimate of the probable error for the values of the gamma-ray energies is ± 0.5 per cent. The constant of the spectrometer has the values 1063 and 1074 gauss-cm/amp for the two adjustments used.

It is seen that the value found for the energy of the Zn^{65} radiation is below the energy for either of the Co^{60} gamma-rays. Because of the possible interest[7] in the use of these radiations as standards, a direct comparison of the energies was thought to be desirable. To this end a source with both activities was put into the spectrometer. As reported[8] previously, the individual peaks in the composite spectrum were readily identified and indicated that the gamma-ray from Zn^{65} is of lower energy than either of the Co^{60} lines.

The energies found for the Co^{60} gamma-rays are in good agreement with those given by Miller and Curtiss[3], although somewhat higher than the values of Deutsch, et al.[7] The energy found for the Zn^{65} gamma-ray is lower than the value given in an early report by Deutsch, Roberts, and Elliott[9] and that obtained by Mandeville and Fulbright[10] through a study of Compton electrons.

Table 1. Positions of conversion lines measured in magnetic-lens spectrometer

Line	Momentum (gauss-cm)	Aperture width (cm)	Radiator thickness (mg/cm^2)		Coil current (amp)*	Relative weight (of curr./momentum ratio)	Gamma-ray energy (Mev)
Annih.	2608/1.023†	2.1	42.5	Pb	2.401	5	
ThB	1385 ‡		Negligible		1.303	10	
Zn^{65} (K)			42.5	Pb	4.454 §	20	1.106
Co^{60}, I (K)			29.7	Pb	4.624	10	1.150
Co^{60}, II (K)			29.7	Pb	5.174	10	1.317
Annih.	2608/1.021 †	1.4	42.5	Pb	2.374	3	
ThB	1385 ‡		Negligible		1.291	10	
Zn^{65} (K)			42.5	Pb	4.409	10	1.106
(L)			42.5	Pb	4.644	5	1.106
Co^{60}, I (K)			42.5	U	4.501	10	1.160
(K)			28.5	Th	4.524	10	1.155
(K)			37.0	Pb	4.583	10	1.156
(K)			29.7	Pb	4.594	10	1.156
(L)			29.7	Pb	4.849	5	1.162
Co^{60}, II (K)			42.0	U	5.011	10	1.316
(K)			28.5	Th	5.045	10	1.314
(K)			37.0	Pb	5.104	10	1.315
(K)			29.7	Pb	5.134	10	1.321
(L)			29.7	Pb	5.369	5	1.321

*0.006 amp has been subtracted from the observed current values to correct for the magnetic field of the earth.

† Since the radiator is thick, in the sense $a > 2b$, for electrons of the energy with which we are concerned here, the momentum value of 2608 gauss-cm corresponding to 0.5108 Mev must be divided by $1 + K$ to correct for radiator thickness.

‡ C. D. Ellis, Proc. Roy. Soc. (London) A138:318 (1932).

§ Obtained from the sloping portion of the curve of Figure 8, so that data obtained with several foil thicknesses are in effect included.

ACKNOWLEDGMENTS

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APPENDIX

APPROXIMATE CALCULATION OF THE CORRECTION FOR THE MAGNETIC FIELD OF THE EARTH:

The solution of the differential equations for the trajectories of paraxial electrons in an axial magnetic field $H_Z = H_O/[1 + (z/a)^2]$ has been given by Glaser.[11] When the object distance and image distance are equal $(u = v = 2f)$, the focal length f may be written

$$f = \Lambda\ [H\rho]^2 \Big/ \int_{-u}^{u} H_Z^2\ dz \qquad \text{(i)}$$

where Λ is a numerical coefficient, calculable in terms of f/a, which takes on values extending from $\Lambda = 4$ for f/a large (thin lens) to π^2 for f/a small (solenoid). Here $[H\rho]$ serves as a measure of the momenta of the electrons in question in terms of their radius of curvature in a uniform magnetic field.

Assuming that to a field of the shape mentioned in the preceding paragraph, there is added a small constant axial field H, one can attempt to fit the resultant field in an approximate way to an equation of the original form and so obtain new values, H_O' and a' for the parameters. In this way we find that $H_O' - H_O \cong 8H/7$ and $a' - a \cong (12a/7)(H/H_O)$.

Introducing a constant A which connects the current in the coil with the magnetic field produced, so that

$$\int_{-u}^{u} H_{coil}^2\ dz = A\ I^2 \qquad \text{(ii)}$$

we then write the approximate relation for the total field as

$$\int_{-u}^{u} H_Z^2 \, dz \cong AI^2 + 2H \int_{-u}^{u} H_{coil} \, dz \qquad \text{(iii)}$$

The currents I_1 and I_0, which are respectively required to focus electrons of a given momentum in the presence and absence of the external field, are then, by Equation i, connected by the relation

$$\frac{AI_0^2}{\Lambda(f/a)} = \left[A\, I_1^2 + 2H \int_{-u}^{u} H_{coil} \, dz \right] \frac{1}{\Lambda(f/a')} \qquad \text{(iv)}$$

From this it follows that the difference $I_0 - I_1$ is approximately

$$I_0 - I_1 = H \left[\left(\frac{1}{A}\right) \int_{-u}^{u} (H_{coil}/I) \, dz + (6/7)\,(I/H_0) \frac{d \ln \Lambda}{d \ln (f/a)} \right] \text{(v)}$$

Through the use of Equations i and ii an approximate value of A is readily estimated experimentally by focusing electrons of known energy, while H_0/I and $\int_{-u}^{u} (H_{coil}/I)\, dz$ may be calculated from the geometry of the coil, the latter quantity being given closely by $4\pi/10$ times the number of turns on the coil.

For the spectrometer described in this paper, the following values apply when all the turns on the coil are employed:

$f = 25$ cm, $a \cong 13.6$ cm, $f/a = 1.84$

$\Lambda = 5.1$, $d\Lambda/d(f/a) = -0.65$, $\dfrac{d \ln \Lambda}{d \ln (f/a)} = -0.23$

$A = 2.27 \times 10^5$ gauss^2-cm/amp^2, $H_0/I = 93.5$ gauss/amp

$\int_{-u}^{u} (H_{coil}/I)\, dz = 3230$ gauss-cm/amp, and $H = 0.56$ gauss

With the substitution of these values in Equation v we find

$I_0 - I_1 = 0.007$ amp, in close agreement with the correction found experimentally.

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