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John P. Best

Maritime Operations Division
Aeronautical and Maritime Research Laboratory

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ABSTRACT

In this report a simple model of the bubble formed by detonating a limpet mine in contact with a plane surface is developed. The key features are the assumption of a hemispherical bubble and the account taken of the outflow of detonation products from the bubble through the hole caused by the detonation. The outflow of the gas is assumed to be one dimensional. The equations describing the time variation of the bubble radius are solved for various values of the physical parameters describing the explosion. The model displays oscillatory behaviour which is damped due to the loss of energy via the outflowing gas. Under some circumstances, the outflow of gas is so large that only one oscillation of the bubble occurs.

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EXECUTIVE SUMMARY

When an explosion occurs underwater there are two principal physical phenomena that follow. A shock wave is transmitted into the water and an oscillating bubble is formed. When the explosion occurs some distance from the target the damage effects associated with these two phenomena are reasonably well understood. When the detonation occurs in very close proximity to the target the damage causing effects are not so well understood.

A particular underwater weapon that detonates in very close proximity to a target is a limpet mine. This is a small explosive charge that is detonated in contact with the hull of a naval target. In this case, not only does a shock wave form and propagate into the surrounding ocean and through the target but the explosion itself is expected to immediately rupture the hull. Subsequently a bubble may form and collapse against the hull. Recent studies have shown that when an explosion bubble collapses against the hull of a naval target a high speed liquid jet may form and be directed towards the hull. This effect bears a degree of similarity with armour defeating shaped charge warheads.

In order to enhance understanding of damage causing effects associated with limpet mines and investigate damage mitigation techniques, a series of simulated detonations were carried out against the decommissioned destroyer escort HMAS Derwent. This series formed part of the Ship Survivability Enhancement Program (SSEP) conducted jointly by DSTO and the RAN in November 1994.

Part of the instrumentation of the limpet mine trial series of detonations was high speed underwater photography, in order to gain some appreciation of the possible contribution to vessel damage by the collapse of the bubble formed by the detonation. Optimum deployment of the photographic equipment required an estimate of the maximum bubble radius and period of oscillation in order that a correct field of view and filming period were employed.

To provide these estimates an analytical model of the motion of the bubble formed upon detonation of a limpet mine was developed and is described in this report. The model takes account of the possibility that a hole may be formed in the hull upon detonation and that explosion gases may escape into the vessel. The calculations show that in this case the bubble oscillations are damped compared to the behaviour predicted if no hole is formed. These calculations demonstrate that upper estimates of the maximum bubble radius and bubble period may be made using the model and assuming that no hole is formed upon detonation.

Model predictions were shown to be in reasonable agreement with high speed photographic measurements of the bubble parameters. It is concluded that the model provides a useful capability to predict and analyse the bubble effect of limpet mines on the hull of naval platforms.

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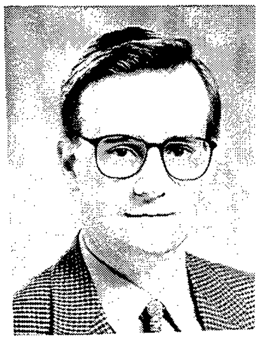

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Contents

1. Introduction

A limpet mine is a small explosive device that is deployed against naval targets. It is deployed in contact with the target and magnets are usually used to hold the mine in place. Limpet mines are typically deployed in a clandestine manner by divers. During the Ship Survivability Enhancement Program (SSEP) conducted by DSTO and the RAN during November 1994, a series of simulated limpet mine detonations against the hull of the decommissioned destroyer escort HMAS Derwent were conducted in order to enhance understanding of the damage mechanisms associated with these weapons and possible techniques for damage mitigation.

Associated with an underwater detonation are two principal phenomena; shock wave formation and bubble formation. Typically a great deal of emphasis has been placed upon the damage causing potential of the shock wave and the bubble oscillation has been considered in the context of hull whipping, which is primarily a far field bubble effect. In recent years bubble collapse in close proximity to targets has become much better understood, with the phenomenon of high speed liquid jet formation upon bubble collapse being investigated as a damage causing effect possibly comparable to the shock wave. In view of this progress the SSEP limpet mine trial series was instrumented in such a manner that the bubble motion event could be visualised using high speed underwater photography (Thornton *et al.* 1996). In order to determine the appropriate field of view and filming time requirements a simple model of limpet mine bubble oscillation was developed and is described here.

The model assumes that the bubble formed upon detonation of a limpet mine is hemispherical and attached to the vessel hull which is assumed to be plane. It is assumed that a fixed proportion of the explosion energy manifests itself in the bubble motion. The detonation of a limpet mine will almost certainly cause the formation of a hole in the vessel hull. The evidence from both internal and external high speed photography of limpet mine detonation events during the SSEP is that the hole is formed within an interval of the order of 10 μs after detonation whereas the lifetime of the bubble is of the order of 100 ms. Some account of the effect of this has been taken in the model. It is assumed that a hole is formed immediately upon detonation and the effect is that bubble energy is lost as explosion gases are transported through the hole.

The equations derived for bubble radius and internal pressure are solved numerically to gain some understanding of the time variation of the limpet mine bubble radius. The radius of the explosion hole is varied and the results indicate that the loss of energy via gas transport through the hole will dampen the bubble oscillation and as the hole size is increased the damping increases to the extent the bubble does not rebound.

Finally, data is presented for the variation in time of the radius of the bubble formed following one of the limpet mine detonations. In this example the adjacent hull compartment was flooded and some comment is made as to the applicability of the simple model in this case. This single experimental data set is in acceptable agreement with model calculations.

2. The mathematical model

The geometry is as shown in figure 1. The centre of the hemispherical bubble is the origin and this point is also the centre of the hole in the plane boundary caused by the detonation. The radius of this hole is a and the radius of the hemispherical bubble is R, which is time dependent. The water is described as an ideal fluid and the flow irrotational and hence may be described by a velocity potential ϕ. Since gravity is neglected the flow may be considered as due to a source of strength $4\pi R^2\dot{R}$ located at the origin. Hence the potential is

$$\phi = -\frac{R^2\dot{R}}{|\boldsymbol{r}|}, \tag{1}$$

where $\boldsymbol{r}$ is the position vector of a point in the flow field. Note that this velocity potential satisfies the appropriate rigid boundary condition on the plane boundary.

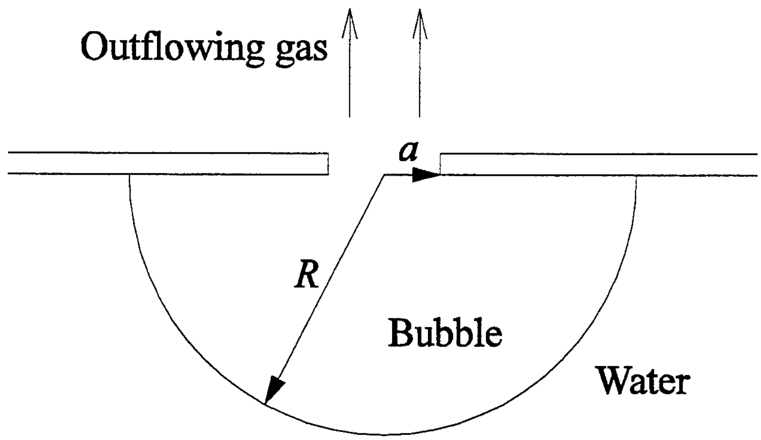

Figure 1: A schematic representation of the bubble formed upon detonation of a limpet mine. The detonation causes a hole of radius a to be formed in the boundary.

The Bernoulli equation in the water takes the form

$$\frac{\partial\phi}{\partial t} + \frac{1}{2}|\nabla\phi|^2 + \frac{p}{\rho} = \frac{p_\infty}{\rho}, \tag{2}$$

where p is the pressure in the fluid, p_∞ is the hydrostatic pressure at the point of detonation and ρ is the water density. If we denote by p_b the pressure inside the bubble which is assumed to be uniform, then evaluating (2) at the bubble boundary and exploiting (1) and the continuity of pressure across this interface yields the equation

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 + \frac{p_\infty - p_b}{\rho} = 0. \tag{3}$$

In the case where there is no hole created by the detonation, so that no explosion products escape, it is usually assumed that the products behave as an ideal gas undergoing adiabatic expansions. In this circumstance p_b is given by

$$p_b = p_0(V_0/V)^\gamma = p_0(R_0/R)^{3\gamma}, \tag{4}$$

where V denotes the volume, the subscript 0 denotes initial quantities and γ denotes the ratio of specific heats. Discussion of the determination of γ, p_0 and V_0 is postponed until later in the report. It is noted at this point that use of (4) in (3) gives the Rayleigh-Plesset equation.

In order to determine an equation for the time variation of the pressure within the bubble it is necessary to consider the thermodynamics of the bubble contents, taking account of the passage of gas through the explosion hole. It is supposed that the speed of the gas flowing through the hole is v and that it is uniform over the hole. v is considered to be positive when the gas is escaping from the bubble. The detonation products are considered to be ideal so the equation of state is

$$pV = nRT, \tag{5}$$

where n is the number of moles of gas, R is the universal gas constant and T is the temperature. Since heat exchange with the surrounds is neglected, the internal energy of the gas will change due to work done as the volume changes and due to changes in the number of moles of gas owing to outflow and inflow through the hole.

Under these circumstances of no heat exchange, the change in internal energy is

$$dU = -p\,dV + e\,dn, \tag{6}$$

with e the internal energy per mole of gas. The internal energy per mole of gas within the bubble is

$$e = \frac{RT}{\gamma - 1}, \tag{7}$$

which is the same as the internal energy per mole of gas escaping or entering, since it is at the same temperature. The volume of gas escaping per unit time is

$$\pi v a^2,$$

so the number of moles of gas escaping per unit time is

$$\frac{dn}{dt} = -\frac{\pi v n a^2}{V}. \tag{8}$$

The internal energy of the gas within the bubble is

$$U = \frac{nRT}{\gamma - 1} = \frac{pV}{\gamma - 1}, \tag{9}$$

where use has been made of (5). Hence (6) becomes

$$\frac{1}{\gamma - 1}(V\,dp + p\,dV) = -p\,dV + e\,dn. \tag{10}$$

Using (6), (7) and the chain rule in (9) yields

$$V\frac{dp}{dV} + \gamma p = -\frac{\pi v p a^2}{\dot{V}}. \tag{11}$$

Noting that $V = 2\pi R^3/3$ and replacing p in (11) by p_b yields the equation for the time variation of the pressure within the bubble as

$$\frac{dp_b}{dt} + 3\gamma p_b\frac{\dot{R}}{R} = -\frac{3a^2 v}{2R^3}p_b. \tag{12}$$

Notice that if $v = 0$ this equation may be immediately integrated to give (4).

The model is completed with specification of an equation giving the speed with which the gas escapes from the hole. An idealised situation is considered in order to expedite analysis. It is supposed that the gas outflow is one dimensional, as illustrated in figure 1. This may be thought of as assuming flow out a pipe attached to the hole. The outflow is assumed isentropic and the possibility of shock wave formation is neglected. The assumption of isentropic flow permits writing the pressure as a function of the density as

$$p\rho^{-\gamma} = \kappa, \tag{13}$$

where κ is a constant, and equal to that value that describes the detonation products at $t = 0$. Since the flow is into uniform gas at atmospheric pressure, p_a, the flow is a simple wave with a constant Reimann invariant. For outflowing gas this condition is expressed as

$$\frac{2}{\gamma - 1}c - u = \frac{2}{\gamma - 1}c_a, \tag{14}$$

where c is the sound speed and is given by

$$c^2 = \gamma p/\rho, \tag{15}$$

and c_a is the sound speed in the quiescent fluid in the far field where the gas pressure is equal to p_a. In (14) u is the fluid speed in the outgoing simple wave.

The conditions in the outflowing gas need to be matched to the conditions within the bubble at the hole. In bubble dynamics, the pressure within the bubble is assumed uniform and the flow speed of the gas within the bubble is neglected. Hence it is assumed that the only effect that outflow through the hole has is to reduce the pressure within the bubble in accord with (12). Thus the matching of conditions is achieved by assuming continuity of pressure at the hole. Then c in (14) may be evaluated in terms of p_b and the outflow velocity is then given by (14) as

$$v = \frac{2}{\gamma - 1}(\gamma\kappa^{1/\gamma})^{1/2}(p_b^{(\gamma-1)/(2\gamma)} - p_a^{(\gamma-1)/(2\gamma)}), \tag{16}$$

where use has been made of (13) to eliminate the density from (15).

To proceed to numerical solution of equations (3) and (12) it is helpful to introduce appropriate scalings. Distances are scaled with respect to R_m, which is of the order of the maximum bubble radius. Pressure is scaled with respect to p_∞, the hydrostatic pressure at the detonation point. The time scale is $R_m(\rho/p_\infty)^{1/2}$ and the velocity scale is $(p_\infty/\rho)^{1/2}$. Scaling with respect to these quantities yields the equations describing the bubble dynamics as

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 + 1 - p_b = 0, \tag{17}$$

$$\frac{dp_b}{dt} + 3\gamma p_b \frac{\dot{R}}{R} = -\frac{3a^2 v}{2R^3}p_b, \tag{18}$$

and

$$v = \frac{2}{\gamma - 1}(\gamma\kappa^{1/\gamma})^{1/2}(p_b^{(\gamma-1)/(2\gamma)} - p_a^{(\gamma-1)/(2\gamma)}), \tag{19}$$

where all quantities here are non-dimensional. It should be noted that the constant κ appearing in (19) is here non-dimensionalised in accord with the scaling given above.

3. Initial conditions

The initial conditions utilised for studies of the bubble dynamics are that the bubble initially has a non-dimensional radius R_0 and that, in the absence of gas loss through an explosion hole, would expand to a maximum non-dimensional radius of one. Given the initial bubble radius the initial bubble pressure may also be determined in a manner described in what follows. The initial radial velocity of the bubble surface is taken to be zero. With such assumptions, at both maximum and minimum radius the kinetic energy of the water is equal to zero. Hence the total energy is equal to the sum of potential and internal energy. The potential energy is the work done against the hydrostatic pressure in forming a bubble of radius R and is equal to

$$\frac{2}{3}\pi R^3 p_\infty,$$

and the internal energy is the work done in compressing the bubble contents adiabatically from infinite volume to the bubble V and is

$$\frac{2}{3}\frac{\pi p_0}{\gamma - 1} R_0^{3\gamma} R^{-3(\gamma-1)}.$$

The statement of energy conservation at maximum and minimum volume is then

$$\frac{2}{3}\pi R^3 p_\infty + \frac{2}{3}\frac{\pi p_0}{\gamma - 1} R_0^{3\gamma} R^{-3(\gamma-1)} = E_0, \tag{20}$$

where E_0 is the energy in the bubble motion. In this expression, the quantity $p_0 R_0^{3\gamma}$ may be eliminated in favour of κ given in (13). In order to perform calculations for practical problems, this is the strategy followed, with κ determined from experimental observations.

To fix ideas, consider that the explosive is TNT. Experimental data recorded in Taylor (1950) indicates that the ratio of specific heats may be taken as

$$\gamma = 5/4 \tag{21}$$

and that

$$p\rho^{-\gamma} = \kappa = 1.386 \times 10^5, \tag{22}$$

where p and ρ are respectively measured in $\mathrm{kg\,m^{-1}\,s^{-2}}$ and $\mathrm{kg\,m^{-3}}$. The energy yield per kilogram of TNT is denoted by $\mathcal{E}$ and is equal to $3.683 \times 10^6\,\mathrm{J\,kg^{-1}}$ so supposing that a proportion μ of the total energy yield of the explosion manifests itself in the bubble motion, the energy balance at maximum and minimum radius is

$$\frac{2}{3}\pi R^3 p_\infty + (\frac{2\pi}{3})^{1-\gamma}\frac{\kappa w^\gamma}{\gamma - 1} R^{-3(\gamma-1)} = \mu w \mathcal{E}, \tag{23}$$

where w is the charge mass in kilograms. In the calculations presented later μ is taken to be equal to 1/2.

It is convenient at this stage to introduce a distance scale

$$\beta = \left(\frac{3\mu w \mathcal{E}}{2\pi p_\infty}\right)^{1/3}, \tag{24}$$

and if $R = \beta R^*$ then (23) becomes

$$R^{*3} + \epsilon R^{*-3(\gamma-1)} = 1, \tag{25}$$

with

$$\epsilon = \left(\frac{\kappa}{\gamma - 1}\right)(\mu\mathcal{E})^{-\gamma} p_\infty^{\gamma-1}, \tag{26}$$

which is a small number. The maximum and minimum radii of the bubble are then given by

$$\begin{aligned} R_{\max} &= R_1^* \beta, \\ R_{\min} &= R_2^* \beta, \end{aligned} \tag{27}$$

where R_1^* and R_2^* are the roots of (25) which are given with good accuracy as

$$R_1^* = 1 - \epsilon/3 - 7\epsilon^2/36, \qquad R_2^* = \epsilon^{4/3}. \tag{28}$$

Having determined the maximum and minimum radii, it remains to determine the non-dimensional value of p_0, the initial pressure within the bubble. Using (22) and (28), the dimensional value of p_0 is

$$p_0 = \kappa p_\infty^\gamma (\mu \mathcal{E})^{-\gamma} R_2^{*-3\gamma}, \tag{29}$$

and the non-dimensional value of this, also known as the strength parameter α (Best & Kucera 1992), is

$$\alpha = \kappa p_\infty^{\gamma-1} (\mu \mathcal{E})^{-\gamma} R_2^{*-3\gamma}. \tag{30}$$

The initial non-dimensional bubble radius is

$$R_0 = R_{\min}/R_{\max} = R_2^*/R_1^*. \tag{31}$$

4. Example calculations

In order to integrate equations (17)-(19) a fourth order Runge-Kutta method has been employed using a variable time step. The time step is given by

$$\Delta t = \frac{\Delta\phi}{\dot{R}^2/2 + p_b + 1}, \tag{32}$$

where $\Delta\phi$ is a constant here taken to be equal to 0.01. This choice of time step has been introduced by Best & Kucera (1992) in order to perform boundary integral method calculations of non-spherical bubble motion. This choice of time step ensures that small steps are taken when the speed of the bubble boundary is large or when the bubble contents are highly compressed near to rebound. At other times larger steps are taken to minimise the computational resources utilised.

To illustrate the features of the model a number of calculations are presented for various charge masses and various explosion hole diameters. This model provides no estimate of the size of the explosion hole expected to be created by the detonation and this quantity is simply input to the model. The results presented here are thus only indicative of the behaviour that might be expected in a real detonation scenario. Figures 2-4 illustrate the variation of bubble radius with time for charges of mass 10 kg, 5 kg and 1 kg, these being values that were considered for the SSEP limpet mine trial. The depth of detonation is 3.5 m which corresponds to deployment of the mine near the keel of a typical target warship.

Given the strong assumptions made in derivation of the model equations, particularly concerning the flow of the explosion gases through the hole, these results are best used to determine qualitatively the trends in bubble behaviour with varying charge mass and hole radius. The trend with respect to increasing hole size is illustrated by all examples and figure 2 is as good as any. For the case where no explosion hole is formed there is no mechanism in the model whereby energy is lost from the bubble motion. Thus an oscillatory motion is predicted as illustrated in this figure. In reality any non-uniformity in the flow field surrounding the bubble will cause the bubble to deform from hemispherical shape and a liquid jet will most likely form upon the collapse. Such sources of flow asymmetry include buoyancy and the flow field geometry itself. It should be noted that the expansion phase of a bubble motion is stable and any deviations from hemispherical shape are expected to occur during the later stages of the bubble collapse. Thus in the event that a hole is not formed by the detonation this model is expected to well predict the motion of the bubble for most of its first oscillation and particularly its maximum radius and period.

As the hole caused by the detonation increases in radius the oscillation of the bubble becomes damped. In figure 2 the hole of radius 0.05 m causes the first maximum to be less than in the case of no hole. The bubble rebounds in this case with the rate of collapse near the minimum so great that despite the outflow of gas through the hole the pressure builds to an extent that it cannot be relieved by gas outflow and the bubble rebounds. The second maximum is damped with respect to the first as is the third maximum with respect to the second. As the bubble collapses a third time the remaining gas within the bubble flows out the hole and the bubble collapses completely.

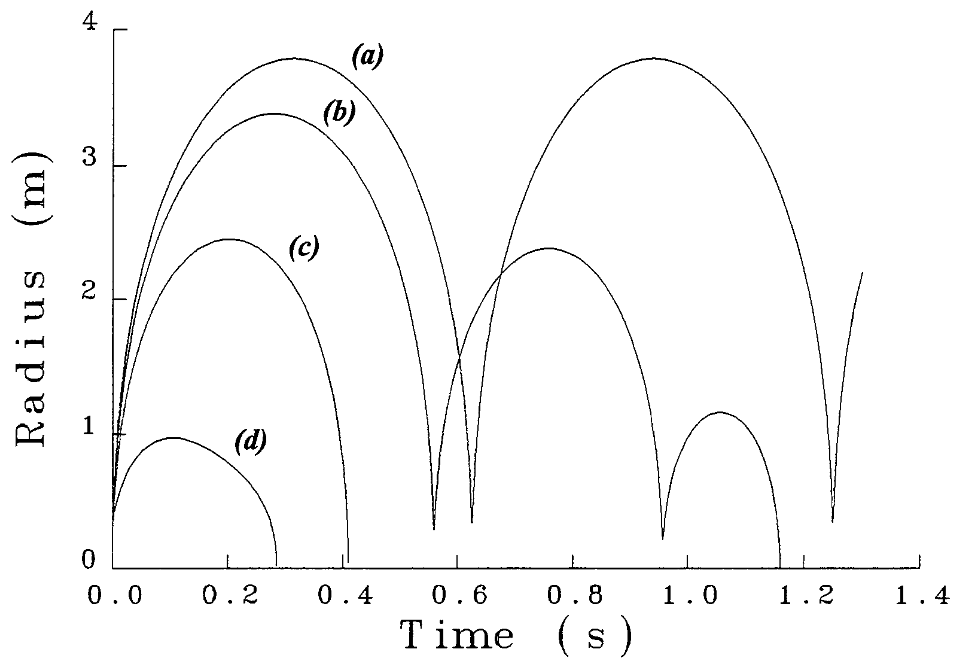

Figure 2: The bubble radius as a function of time following detonation of 10 kg TNT at a depth of 3.5 m. The curves correspond to an explosion hole of radius: (a) 0.0 m, (b) 0.05 m, (c) 0.1 m, (d) 0.25 m.

The final two examples illustrate the case where the hole is so large that all bubble gases flow out during the first collapse and the bubble does not rebound. It is interesting to note the shape of the radius-time curve for the case $a = 0.25$ m. All other oscillation periods of the bubble are qualitatively symmetrical about the maximum but in this case the curve is noticeably skewed.

The examples for smaller charge masses illustrated in figures 3 and 4 show similar features. As the hole size increases the bubble motion is damped to the extent that the bubble eventually collapses completely. Notice that in the case where $w = 1$ kg and $a = 0.025$ m the third and final bubble period demonstrates significant asymmetry in the radius-time curve.

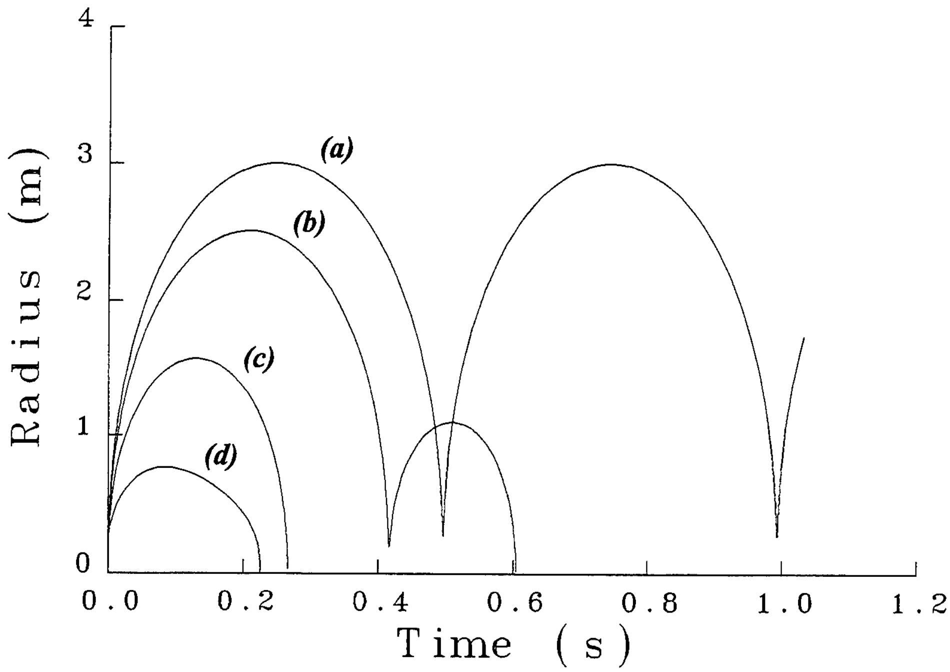

Figure 3: The bubble radius as a function of time following detonation of 5 kg TNT at a depth of 3.5 m. The curves correspond to an explosion hole of radius: (a) 0.0 m, (b) 0.05 m, (c) 0.1 m, (d) 0.2 m.

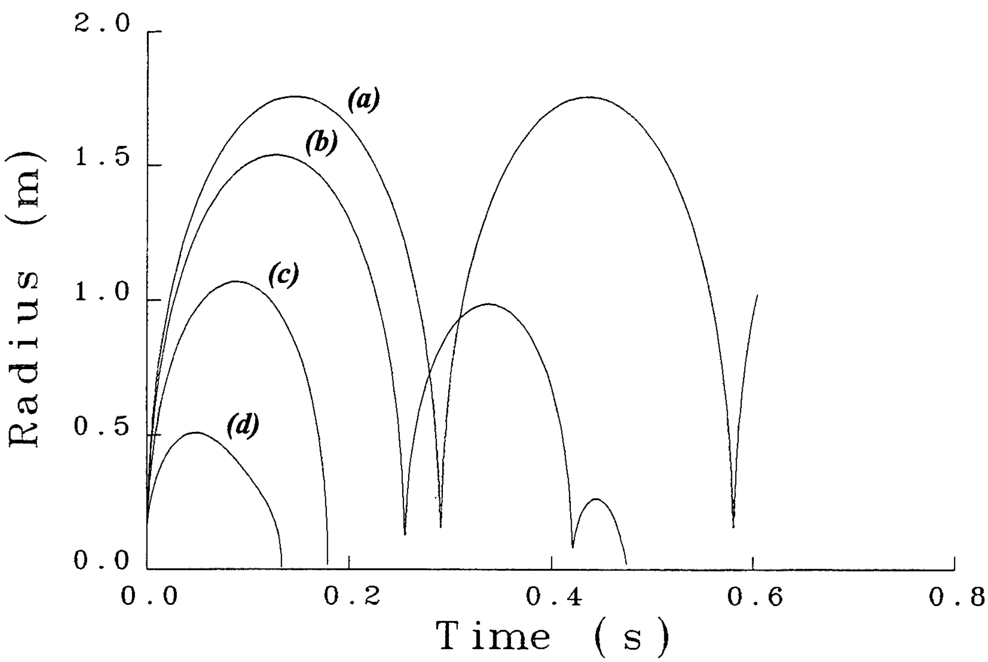

Figure 4: The bubble radius as a function of time following detonation of 1 kg TNT at a depth of 3.5 m. The curves correspond to an explosion hole of radius: (a) 0.0 m, (b) 0.025 m, (c) 0.05 m, (d) 0.1 m.

This model has been formulated making some strong assumptions as to the flow of gas through the explosion bubble hole. Indeed, if the variation of escaping gas velocity with time is plotted as in figure 5, for the case where $w = 10$ kg and $a = 0.05$ m, then the high values near bubble minimum suggest that shock waves may be formed. Note also that when the bubble is near its maximum radius the pressure within the bubble is so low that the model predicts gas flow into it. The possibility of shock formation in the outflowing gas could be incorporated into the model, but given the lack of information regarding key aspects of the limpet mine detonation event, such as the formation of the hole and the proportion of the explosion energy yield that manifests itself in the bubble motion, such effort seems unwarranted. To proceed detailed hydrocode modelling would appear to be the most appropriate course.

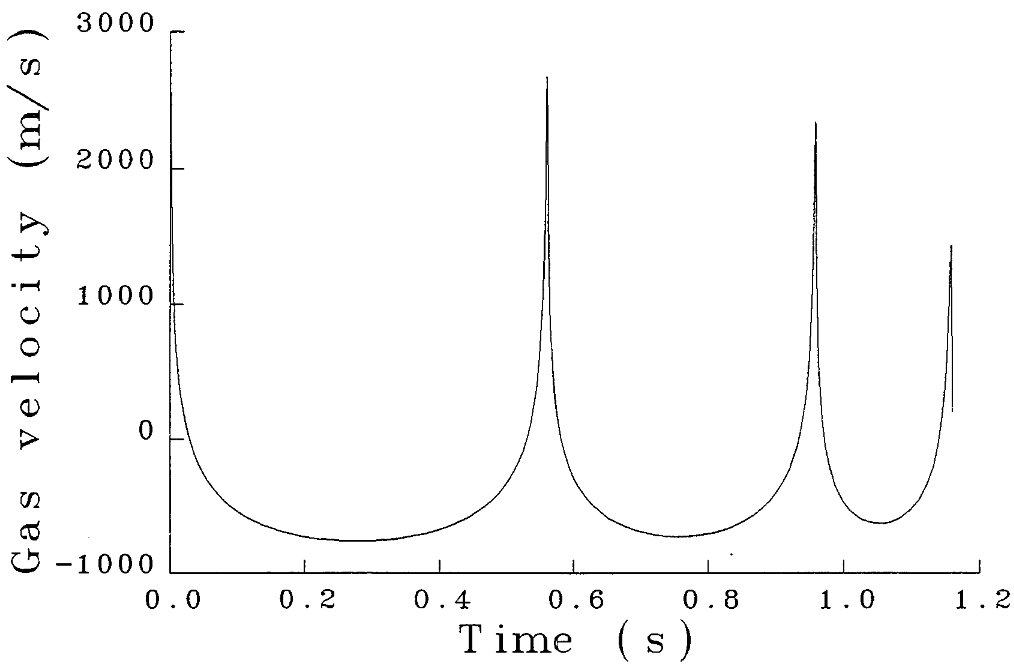

Figure 5: The escaping gas velocity as a function of time for the example of detonation of 10 kg TNT at a depth of 3.5 m. The explosion hole is of radius 0.05 m.

Table 1: The predicted upper bounds on the maximum bubble radius and first oscillation period for a range of charge masses. All detonations are assumed to take place at a depth of 3.5 m.

Charge mass (kg)	Maximum radius (m)	Oscillation period (s)
10	3.79	0.63
5	3.01	0.50
1	1.76	0.29

The primary aim of the calculations undertaken here was to make estimates of the period and maximum radius of the bubble formed upon detonation of a limpet mine.

In this context the model case where no hole is formed by the detonation is expected to provide upper bounds on these quantities, with the calculations performed here indicating that the effect of hole formation is to dampen the bubble oscillation. Hence for the charge masses considered here and detonated at a depth of 3.5 m table 1 shows the upper estimates of the period and maximum radius.

During the SSEP limpet mine trial high speed underwater photography was performed for a number of detonations (Thornton *et al.* 1996). The photographic record of a single shot was amenable to detailed analysis and yielded the radius-time history shown in figure 6. This example is for 2 kg of PE4 detonated at a depth of 2 m. It is emphasised that in this case the internal ship compartment adjacent to the detonation point was completely flooded with sea water. Measurements made after the event showed a hole of approximate diameter 275 mm compared with the charge diameter of 170 mm. Further, the hole showed a light secondary indentation, indicating that the hole size was not significantly increased by a flow of water or gas through it during the bubble motion. Hence it may be assumed that the hole was formed essentially upon detonation.

This scenario is not considered to be appropriately described by the model developed here. However, if the case is considered where a limpet mine is detonated adjacent to a plane boundary surrounded on both sides by water then it may be argued that the bubble oscillation parameters may be predicted assuming motion of a bubble in the free field. If it is considered that the shock formed upon detonation propagates only into the half space on which the charge is located and that the energy that would otherwise manifest itself in a shock propagating into the other half space is essentially expended in creating the explosion hole, then it may be deduced that the remaining energy manifests itself in the bubble. In accordance with the evidence supporting the proposition that the hole is formed immediately upon detonation, there is no preferential direction in which the explosion gases will expand. Thus the gases will expand equally on either side of the boundary and the bubble so formed will be equivalent to that formed if the detonation occurred in the free field. In this case the statement of energy conservation is slightly different owing to the spherical shape of the bubble with the potential energy given by

$$\frac{4}{3}\pi R^3 p_\infty,$$

and the internal energy is

$$\frac{4}{3}\frac{\pi p_0}{\gamma - 1} R_0^{3\gamma} R^{-3(\gamma-1)}.$$

The analysis follows through as for the hemispherical bubble with the exception that the length scale β is given by

$$\beta = \left(\frac{3\mu w \mathcal{E}}{4\pi p_\infty}\right)^{1/3}. \tag{33}$$

The non-dimensional maximum and minimum radii are determined as the roots of (25) and the expression for the non-dimensional parameter ϵ is exactly (26). The initial bubble pressure and strength parameter are given by (29) and (30) and the initial non-dimensional bubble radius by (31).

In the absence of better data it is assumed that the performance of PE4 underwater may be described by the same parameters as TNT. Evaluating the maximum radius and bubble period using this model yields $R_{\max} = 1.83$ m and $T = 0.32$ s. These values agree favourably with those observed as evident from inspection of figure 6. Note, however, that the correspondence is particularly good during the early phase of the expansion but that the experimental data fall away from the theoretical curve as the bubble collapses. It was observed that the bubble in this case did not rebound so this trend in the bubble radius is likely to be due to the outflow of explosion bubble gases into the target. Note that the asymmetry of the radius time curve bears qualitative similarity to some of the calculated radius time curves for hemispherical bubbles for those oscillation periods where the bubble collapses completely. This single example provides some evidence that significant limpet mine bubble parameters may be approximately determined using the elementary analysis presented here.

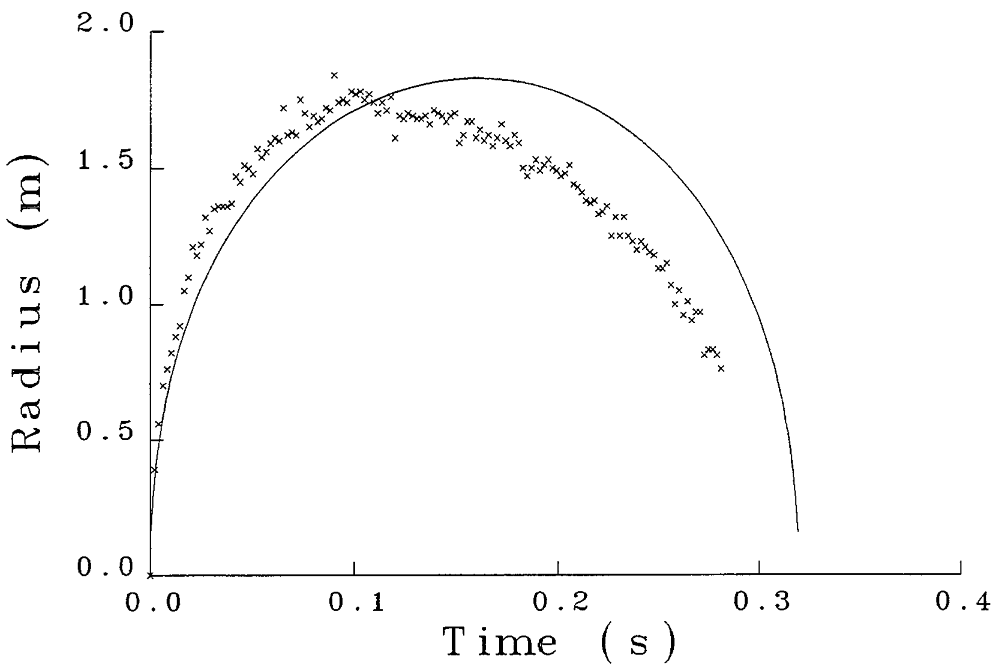

Figure 6: The bubble radius as a function of time as determined from a high speed photographic record of the detonation of 2 kg of PE4 at a depth of 2 m. In this example the internal ship compartment adjacent to the detonation point was flooded. The experimental data are denoted by a cross and the solid curve is calculated assuming motion of a spherical bubble.

5. Conclusions

In this report an analytical model has been developed that describes the motion of the bubble formed upon detonation of a limpet mine adjacent to a plane boundary. The model allows predictions to be made of the maximum bubble radius and period. A significant factor that affects the behaviour of the bubble is the radius of the hole presumed formed upon detonation. The hole allows explosion bubble contents to vent into the vessel hull and thus dampens the bubble oscillation. The results of calculations indicate that for a small hole a number of bubble oscillations may occur, with the maximum radius decreasing for each subsequent oscillation period. As the hole radius increases a stage is reached where all the bubble contents are vented on the first collapse and the bubble does not rebound.

A second significant factor affecting bubble behaviour is the manner in which the energy yield of the explosion manifests itself in propagating a shock wave into the water, cutting a hole in the vessel hull and in bubble motion. It has been assumed here that half the available energy manifests itself in bubble motion. In the absence of any data this may be a reasonable first approximation but is an issue requiring further investigation if more detailed calculations are required.

Finally, the primary aim of this modelling exercise was to provide upper bounds on the maximum bubble radius and period for detonation of a limpet mine against the hull of a naval target. It is expected that use of equations (17)-(19) with the hole radius set equal to zero will provide appropriate upper bounds. The effect of a non-zero hole radius will be to dampen the motion compared to these values.

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19. ABSTRACT

In this report a simple model of the bubble formed by detonating a limpet mine in contact with a plane surface is developed. The key features are the assumption of a hemispherical bubble and the account taken of the outflow of detonation products from the bubble through the hole caused by the detonation. The outflow of the gas is assumed to be one dimensional. The equations describing the time variation of the bubble radius are solved for various values of the physical parameters describing the explosion. The model displays oscillatory behaviour which is damped due to the loss of energy via the outflowing gas. Under some circumstances, the outflow of gas is so large that only one oscillation of the bubble occurs.

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