



SENSITIVITY OF AVAILABILITY ESTIMATES
TO INPUT DATA CHARACTERIZATION

THESIS

Darren P. Durkee, Major, USAF

AFIT/GOR/ENS/97M-06

DISTRIBUTION STATEMENT A

Approved for public release;
Distribution Unlimited

DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY

AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

DTIC QUALITY INSPECTED 1

19970429 223

AFIT/GOR/ENS/97M-06

SENSITIVITY OF AVAILABILITY ESTIMATES
TO INPUT DATA CHARACTERIZATION

THESIS

Darren P. Durkee, Major, USAF

AFIT/GOR/ENS/97M-06

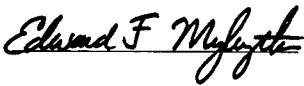
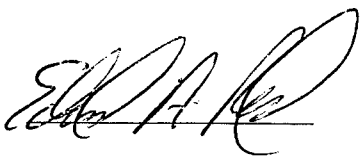
Approved for public release; distribution unlimited

THESIS APPROVAL

NAME: Darren P. Durkee, Major, USAF **CLASS:** GOR-97M

THESIS TITLE: Sensitivity of Availability Estimates to Input Data Characterization

DEFENSE DATE: 11 February 1997

COMMITTEE:	NAME/TITLE/DEPARTMENT	SIGNATURE
Co-Advisor	Edward F. Mykytka Associate Professor of Operations Research Department of Operational Sciences Air Force Institute of Technology	
Co-Advisor	Edward A. Pohl, Major, USAF Assistant Professor of Aerospace and Systems Engineering Department of Aeronautics and Astronautics Air Force Institute of Technology	
Reader	W. Paul Murdock, Jr., Major, USAF Assistant Professor of Operations Research Department of Operational Sciences Air Force Institute of Technology	

The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U. S. Government.

SENSITIVITY OF AVAILABILITY ESTIMATES
TO INPUT DATA CHARACTERIZATION

THESIS

Presented to the Faculty of the Graduate School of Engineering
Air Force Institute of Technology
Air University
In Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Operations Research

Darren P. Durkee, B. S., M. S. B. A.
Major, USAF

February 1997

Approved for public release; distribution unlimited

Acknowledgments

I am indebted to my thesis committee, Dr. Mykytko, Major Pohl, and Major Murdock for their guidance and assistance in completing this thesis. My research spanned several subject areas, including an area in which I had little prior knowledge (reliability theory), and their knowledge and experience were invaluable assets. I am also very appreciative of their even-handed and no nonsense approach in the oversight of my research efforts.

I would also like to thank my AFOTEC thesis sponsor, Major Chris Swider, who provided a clear research objective, was very responsive to my queries, and proved very flexible when I was 'fine tuning' the research objectives and scope.

I am also very grateful to the Air Force for providing me the opportunity to receive a degree and conduct research in the area of Operations Research in a full-time academic environment. The knowledge and experiences I have gained here will benefit me and the Air Force as I continue to serve.

Finally, I would like to express my gratitude to my family, Karin and Caroline, for their unwavering support throughout my AFIT and thesis experience. Their patience and encouragement helped keep me going when things seemed to be a little too overwhelming.

Darren P. Durkee

Table of Contents

	Page
Acknowledgments.....	ii
Table of Contents.....	iii
List of Figures.....	vii
List of Tables.....	viii
Abstract.....	ix
 I. Introduction.....	 1
Overview.....	1
Background.....	2
Research Objectives.....	5
Scope.....	6
Overview of Subsequent Chapters.....	7
 II. Literature Review.....	 9
Overview.....	9
Component Importance Measures.....	9
Terminology.....	10
Structural Importance Measures.....	10
Birnbaum Structural Measure.....	10
Barlow-Proschan Structural Measure.....	11
Time-Dependent Component Importance Measures.....	11
Birnbaum Reliability Importance Measure.....	11
Veseley-Fussell Importance Measure.....	12
Time-Independent Component Importance Measures.....	12
Barlow-Proschan Time-Independent Measure.....	13
Natvig Importance Measure.....	13
Numerical Example of Component Importance Measures.....	14
Birnbaum Structural Measure Example.....	15
Birnbaum Reliability (Time-Dependent) Example.....	16
Barlow-Proschan Time-Independent Example.....	17
Natvig Time-Independent Importance Measure Example.....	18
Simulation Experimental Design and Factor Screening Methods.....	19
Two-Level (2^k) Factorial Designed Experiments.....	20

	Page
Fractional Factorial Designs.....	20
Plackett-Burman Experimental Designs.....	21
Projection Properties of Plackett-Burman Designs.....	21
Lin and Draper's Plackett-Burman Projection Techniques.....	22
Plackett-Burman Projections.....	23
Benefits of Plackett-Burman Designs.....	25
Past Research.....	26
Sensitivity Analysis of Availability Estimates.....	26
Availability Analysis Using Simulation.....	27
System Complexity (or Size).....	27
Constant Failure Rate Assumption.....	28
Repair Distributions.....	28
Comparison of Screening Designs for Simulation Models.....	28
Box-Meyer Bayesian Method.....	28
Response Surface Methodology.....	29
Summary.....	29
 III. Methodology: Preliminary Experiment.....	 31
General Methodology Overview.....	31
Preliminary Experiment.....	32
Definitions.....	32
Assumptions.....	33
Software.....	34
Design of Preliminary Experiment.....	34
Simulation Runs.....	38
Analysis Methods and Software.....	39
 IV. Results: Preliminary Experiment.....	 41
Simulation Results.....	41
Statistical Analysis.....	42
Graphical Analysis.....	43
Additional Analysis.....	44
Summary.....	45

	Page
V. Methodology: Final Experiment.....	46
Insights Gained from Preliminary Experiment.....	46
Final Experiment.....	47
Assumptions.....	47
Structures.....	47
Design of Final Experiment.....	48
Distributional Fittings.....	50
Important Components.....	50
Simulation Runs.....	50
Analysis Methods.....	51
VI. Results: Final Experiment.....	52
Simulation Results.....	52
Statistical Analysis.....	53
Graphical Analysis.....	54
Significant Effect Model.....	55
Response Surface.....	56
Additional Analysis.....	57
Summary.....	57
VII. Summary and Conclusions.....	58
Research Objectives.....	58
Overview of Results.....	58
Preliminary Experiment.....	58
Final Experiment.....	59
Multivariate Analysis.....	59
Conclusions.....	60
Comparison with Past Research Results.....	60
Sensitivity to Component Failure Rate Characterization.....	60
Exponential Assumption.....	61
Suggestions for Future Research.....	61
Identifying Other Factors.....	61
Mean-Time-to-Failure / Mean-Repair Time (MTTF/MRT) Ratio.....	62
Response Surface Methodology (RSM).....	62

	Page
Appendix A: Statistical Analysis Output.....	63
Appendix B: Final Experiment Structures and True Component Distribution Functions.....	75
Appendix C: Data Fitting (Preliminary Experiment).....	77
Appendix D: Data Fitting (Final Experiment).....	83
Appendix E: Data Fitting Graphs.....	107
Appendix F: Birnbaum Structural Component Importance Measure Results for Final Experiment.....	110
Appendix G: Multivariate Analysis of Raptor Output.....	111
Analysis Techniques.....	111
Overview.....	111
Discriminant Analysis (DA).....	111
Neural Networks.....	112
Logistic Regression.....	112
Principal Component Analysis (PCA).....	113
Factor Analysis (FA).....	113
Database.....	114
Specific Output Measures.....	114
Analysis Objectives.....	118
Purpose of Investigation.....	118
Variables Used.....	118
Analysis Results.....	121
Special Problems Encountered.....	121
Discrimination Between Categories of Component Structures.....	121
Neural Net Results.....	123
Reduction in Dimensionality (PCA).....	124
Identification of Underlying Factors (FA).....	127
Insights.....	129
References.....	131
Vita.....	134

List of Figures

Figure	Page
1. Simple Series System	2
2. Simple Parallel System	3
3. Series-Parallel System	4
4. Bridge Structure	4
5. Example System	14
6. Plackett-Burman (P-B) Design ($n = 12$)	24
7. P-B Design Projection for $n = 12$ and $k = 3$	24
8. Experimental Structure for Preliminary Experiment	34
9. Component 5 Failure PDF Fitting Graph	37
10. Component 5 Repair PDF Fitting Graph	37
11. Preliminary Experiment Normal Probability Plot	43
12. Preliminary Experiment Pareto Plot of Scaled Estimates	43
13. Preliminary Experiment Box-Meyer Bayes Plot	44
14. Final Experiment Normal Probability Plot	54
15. Final Experiment Pareto Plot of Scaled Estimates.....	54
16. Final Experiment Box-Meyer Bayes Plot	55
17. Two-Factor Model Response Surface and Contour Plot	56

List of Tables

Table	Page
1. Component Failure Distributions and Reliability Functions for Example System	15
2. Importance Measure Rankings for Example System	18
3. Projection of 12-Run Plackett-Burman Design into k Dimensions	23
4. Selected Experimental Factors	31
5. Experimental Factors and Levels for Preliminary Experiment	35
6. System Failure True and Fitted Distributions (Replication 1)	36
7. System Repair True and Fitted Distributions (Replication 1)	36
8. Barlow-Proschan Time-Independent Importance Measure Values.....	39
9. Preliminary Experimental Design and Responses	41
10. Preliminary Experiment Model Statistical Results	42
11. Estimated Effects and Statistical Analysis	42
12. Significant Factors Assessing Alternative Responses and a Blocking Variable.....	44
13. Factors and Levels for Final Experiment	49
14. 12-Run Placket-Burman Design for Final Experiment	49
15. Top 20% Important Components for Final Experiment Structures.....	50
16. Numerical Results for Final Experimental Runs	52
17. Final Experiment Model Statistical Results	53
18. Estimated Effects and Statistical Analysis	53
19. Estimated Effects and Statistical Analysis for C, H, C*H Model	55

Abstract

Reliability analysts are often faced with the challenge of characterizing the behavior of system components based on limited data. Any insight into which model input data is most significant and how much data is necessary to achieve desired accuracy requirements will improve the efficiency and cost effectiveness of the data collection and data characterization processes. This thesis assesses potential significant factors in the probabilistic characterization of component failure and repair behavior with respect to the effect on system availability estimates. Potential factors were screened for significance utilizing fractional factorial and Plackett-Burman experimental designs for several system models developed using an AFOTEC simulation program entitled RAPTOR.

Two input data characterization factors were found to have a significant affect on availability estimation accuracy: the size of the structure and the number of data points used for component failure and repair distributional fitting. Estimation error was minimized when the structures analyzed were small and many data points (in this case, 25) were used for the distributional fittings. Assuming constant component failure rates and using empirical repair distributions were found to be equally effective component characterization methods (pertaining to model availability estimation error) compared to using automated software fitting tools (or 'wizards'). The results of this study also indicate that there is no apparent benefit in concentrating on 'important' components for the highest fidelity distributional fittings.

SENSITIVITY OF AVAILABILITY ESTIMATES TO INPUT DATA CHARACTERIZATION

I. INTRODUCTION

Overview

Reliability, maintainability, and availability (RM&A) analysis plays an integral part in the design and production of efficient, cost-effective systems. According to Kapur and Lamberson,

“The reliability of a system is the probability that, when operating under stated environmental conditions, the system will perform its intended function adequately for a specified time.” [1:1]

“Maintainability is defined as the probability that a failed system can be made operable in a specified interval of downtime.” [1:225]

“Availability is defined as the probability that a system is operating satisfactorily at any point in time...” and “is a measure of the ratio of the operating time of the system to the operating time plus the downtime.” [1:225]

The Department of Defense and the Air Force conduct numerous studies into the reliability and maintainability of current and future weapons systems in an effort to control RM&A costs of fielded systems and to verify RM&A characteristics of systems which are still in development. One key Air Force agency which conducts such studies is Headquarters Air Force Operational Test and Evaluation Center (HQ AFOTEC). AFOTEC manages a large portion of the Air Force's weapons system operational verification and validation testing.

In an effort to describe a system's RM&A characteristics, analysts frequently represent the system with an analytical and/or simulation model. Reliability analysts will base these models on observed component failure and repair data, historical performance of similar systems, contractor estimates, as well as on certain traditional theoretical assumptions which have been developed in the field of reliability. In an ideal circumstance, data from extensive testing will be available for accurate probabilistic characterization of the various system components. However, due to various constraints and limitations, the analyst is often faced with the challenge of characterizing the behavior of system components based on limited data. In this instance, the analyst will need to make judgments as to how best characterize the input data to obtain acceptable analytical results.

Background

Systems are frequently broken down into sub-structures of components for RM&A analysis. Several categories of component structures have been defined in the field of reliability. The more common classes of structures include series, parallel, series-parallel, and complex structures. A complex structure is one that cannot be defined as series, parallel, or series-parallel. The simplest example of a series system contains two components as shown in Figure 1.



Figure 1. Simple Series System

Given that p_1 and p_2 (ranging in value from 0 to 1.0) represent the reliability of components 1 and 2, respectively, and that all components operate independently of each other, then the system reliability function, $h(\mathbf{p})$, is

$$h(\mathbf{p}) = p_1 \cdot p_2.$$

A two component parallel system is shown in Figure 2.

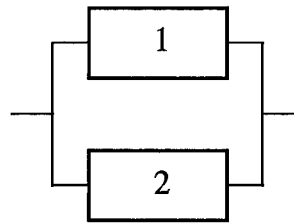


Figure 2. Simple Parallel System

In this case, the system reliability function is

$$h(\mathbf{p}) = 1 - [(1 - p_1) \cdot (1 - p_2)].$$

Series-parallel systems consist of combinations of series and parallel components in the system. An example is shown in Figure 3.

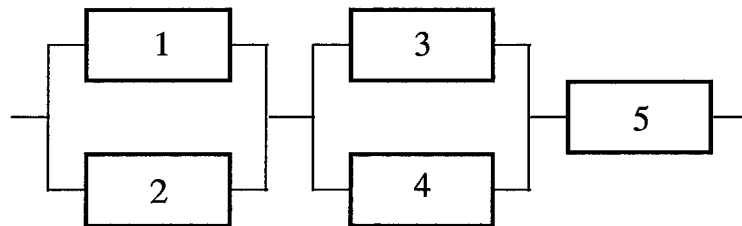


Figure 3. Series-Parallel System

The system reliability function for this series-parallel system is

$$h(\mathbf{p}) = [1 - (1 - p_1) \cdot (1 - p_2)] \cdot [1 - (1 - p_3) \cdot (1 - p_4)] \cdot p_5.$$

A typical complex structure can be illustrated by a bridge structure as shown in Figure 4.

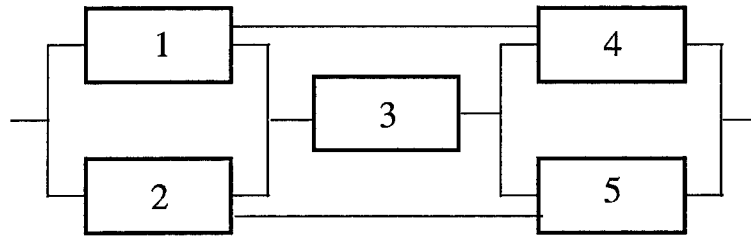


Figure 4. Bridge Structure

The system reliability function for a bridge structure is

$$h(\mathbf{p}) = 1 - [(1 - p_1 \cdot p_4) \cdot (1 - p_1 \cdot p_3 \cdot p_5) \cdot (1 - p_2 \cdot p_5) \cdot (1 - p_2 \cdot p_3 \cdot p_4)].$$

As can clearly be seen, the complexity of the system reliability function increases significantly as the size and complexity of the system structure increases.

Several analytical methods exist for determining steady-state properties of systems of components, including Markovian models, network theory, fault tree analysis, path and cut set analysis, Venn decomposition, non-homogenous Poisson processes (NHPP), and power law processes, to name a few. However, if the system under study is large and/or complicated, as is often the case, analytical methods can become cumbersome.

Furthermore, most analytical methods provide insight only into the system's steady-state properties, not its transient properties. The task is further complicated when estimating system availability, since component repair rates must be considered. In such situations where analytical methods are inadequate or overly cumbersome, simulation provides a viable (and often times preferable) alternative [2:112].

In developing a simulation model, analysts must collect component failure and repair rate data (and/or use existing data) and then characterize this data to accurately represent the true behavior of the components of interest. More often than not, this data collection

process is time consuming and expensive. Any insight into which model input data is most significant and how much data is necessary to achieve desired accuracy requirements should improve the efficiency and cost effectiveness of the data collection and data characterization processes.

Research Objectives

The general purpose of this study is to provide insight into input data characterization factors (such as volume of data utilized, data fitting methods, system size, type of system structure, and component importance) which may affect the accuracy of simulation model availability estimates. If we can identify the key factors which have a significant affect on model accuracy, the analyst can focus more attention on modeling these significant factors and less on the insignificant factors when soliciting and characterizing input data for an RM&A model.

Questions which need to be researched include:

- (1) How much failure rate and repair rate data are needed for each component to obtain a desired model accuracy?
- (2) Which data fitting techniques for characterizing component failure and repair probability distributions produce significant errors in model accuracy, and which do not?
- (3) Do all components need the same fidelity of characterization, or can increased efficiency be realized by focusing on only the 'important' components?
- (4) Are the answers to the above questions affected by system size, the underlying true component failure distributions, or other system characteristics?

Although the scope of this effort does not allow for a complete research of the above questions, much can be ascertained by conducting a controlled experiment. This research

is intended, using a design of experiment approach, to help identify the most critical pieces of data needed to ensure representative simulation results. Many efficiencies could be achieved if analysts were provided general input data characterization guidelines based on experimental results. Insights gained from this research may assist in the reduction of expensive live testing and unproductive data collection through the efficient use of simulation models.

The overall research objectives are to:

- (1) Identify potential factors which affect availability model output accuracy.
- (2) Screen these potential factors to determine which have a statistically significant effect (or interaction effect) on output accuracy.
- (3) Assess the magnitude of the significant effects.
- (4) Provide basic insight to analysts to aid in efficient input data characterization for availability models.

Scope

Although several model output measures may be of interest when analyzing a system, this study focused on the system availability output measure. A total of nine input data characterization factors (defined in Chapter 3), identified by several RM&A analysis experts and the author as factors with a potential affect on the accuracy of availability estimates, were analyzed. The probability density functions (pdf) used to define system component failure and repair rates were limited to 'common' functions encountered in reliability analysis, namely the Weibull and Lognormal pdf's. Component sparing was not considered in this research. To maintain economy of effort, the maximum size of any

analyzed system was limited to 20 total components and the structure types analyzed were series-parallel and complex.

Overview of Subsequent Chapters

Chapter 2 contains a review of existing literature covering several topics pertinent to this research. Major component importance measures, experimental designs for simulation (including screening designs), Plackett-Burman two-level experimental screening designs, and past research relating to this effort are all explored.

The research was conducted in two stages: a preliminary experiment to validate and refine the methodology, followed by a larger-scale experiment. Chapter 3 includes a description of the research methodology for the preliminary experiment which assessed five input data characterization factors. Chapter 3 also includes a discussion of the specific designed experimental screening methods used as well as specific analytical techniques used for data analysis for the preliminary experiment. The software used for availability model development, random variate generation, and data fitting are described.

Chapter 4 contains the results from the preliminary experiment. Statistical results are presented which identify the factors which proved significant in affecting availability model output accuracy.

Chapters 5 and 6 include descriptions of the methodology refinements and results of the final experiment. This experiment analyzed nine input data characterization factors.

Chapter 7 contains a summary of the thesis effort, including an overview and discussion of the impact of the results, how these results may benefit reliability analysts, and ideas for future research.

II. LITERATURE REVIEW

Overview

This chapter provides an overview of the current literature in areas pertaining to this thesis. This chapter begins by reviewing several major methods of defining component importance which are found in the literature. It then provides an overview of two-level designed experimental methods for factor screening in simulation experiments. One screening experimental technique, Plackett-Burman (P-B) experimental designs, was used in this research and is discussed in detail. Finally, past research which relate to this effort are reviewed.

Component Importance Measures

Systems are frequently broken down into sub-structures of components to aid in system design, analysis, and repair. Component importance measures provide a scientific, quantitative approach of identifying the most important components in a given structure of components. As an example of a common application, system designers can use component importance measure to identify which components are most critical in the proposed design structure. Furthermore, reliability analysts can use component important measures to determine which components are most crucial in defining the overall system reliability [3:195].

Several component importance measures have been developed in reliability theory since Birnbaum introduced the first mathematical component importance measures in 1969. Current component importance measures can be categorized into three areas: structural,

time dependent, and time independent. This section provides an outline of several of the major component importance measures which have been published in recent years and are common in use.

Terminology. All systems considered in this paper are coherent systems comprised of binary state components. A coherent system is one in which all components are relevant in maintaining a functional system. Binary state components have just two states: functioning or failed. The states are typically represented as

$$\begin{aligned} X(t) &= 1 \text{ if the component functions at time } t \\ &= 0 \text{ if the component is failed at time } t. \end{aligned}$$

A system's (as opposed to a component's) reliability function is depicted as $h(\mathbf{p})$, where $\mathbf{p}$ represents the component reliability vector. A component's reliability function is a function of time and is depicted as $p_{(i)}(t)$ for component i .

Structural Component Importance Measures. Structural importance measures are based solely upon the structural design of the system. They are used when the system structure function is known, but the individual component reliabilities are not known [4:583]. Two key structural methods have been developed by Birnbaum as well as Barlow and Proschan.

Birnbaum Structural Measure. The Birnbaum structural measure provides a measure of the criticality of a component in maintaining a system's functional state. Annotated as $I_{B,\phi}^{(i)}$ for component i , it represents the proportion of system state vectors which are critical for component i [5:456]. When the system components are independent, it can be calculated by the following equation [4:584]:

$$I_{B,\phi}^{(i)} = \left. \frac{\partial h(\mathbf{p})}{\partial p_i} \right|_{p_1=\dots=p_n=\frac{1}{2}} \quad (1)$$

This measure does not take into account the individual reliabilities of each system component.

Barlow-Proschan (B-P) Structural Measure. The Barlow-Proschan (B-P) structural measure assumes that component reliabilities are not known, but can be assumed to be the same for each component and assigned the value p . It is defined by the equation

$$I_{BP,\phi}^{(i)} = \int_0^1 [h(1_i, \mathbf{p}) - h(0_i, \mathbf{p})] dp \quad (2)$$

where $h(1_i, \mathbf{p})$ represents the system reliability function when component i is functioning and $h(0_i, \mathbf{p})$ represents the system reliability function when component i is not functioning [5:457].

Time-Dependent Component Importance Measures. While structural importance measures are only dependent upon the underlying system structure, time-dependent measures take into consideration the component reliabilities at some chosen time t . They are typically utilized when both the system structure and the component reliability functions are known. Two frequently used time-dependent measures include one developed by Birnbaum and another introduced by Veseley and Fussell.

Birnbaum Reliability Importance Measure. Birnbaum's reliability importance measure assesses a component's importance at time t . If a system is comprised of n components whose reliabilities at time t are $p_1, p_2, \dots, p_n$ and $h(p_1, p_2, p_3, \dots, p_n)$ represents

the system reliability at time t , then the Birnbaum reliability importance measure for component i is given by

$$I_B^{(i)}(t) = h(p_1, \dots, p_{i-1}, 1, p_{i+1}, \dots, p_n) - h(p_1, \dots, p_{i-1}, 0, p_{i+1}, \dots, p_n) \\ = \frac{\partial h(\mathbf{p})}{\partial p_i} \quad (3)$$

It represents the decrease in system reliability when component i fails [6:266]. The Birnbaum reliability importance measure is the most frequently used time-dependent measure because of relative ease in calculation and because it provides the 'fairest' basis of comparison between components [5:458].

Veseley-Fussell (V-F) Importance Measure. Another popular time-dependent component importance measure, introduced by Veseley and Fussell in 1972, utilizes cut-set theory to define component importance. The V-F importance measure, $I_{VF}^{(i)}(t)$, represents the conditional probability that a cut set containing component i has failed at time t , given that the system has failed at time t .

Many other time-dependent measures, most of which are variations of those discussed previously, also exist. For the sake of brevity, these additional measures, including those developed by Butler and Aven arising from network theory, will not be discussed in this paper.

Time-Independent Component Importance Measures. Both structural measures and time-dependent measures have inherent characteristics which make them inappropriate for certain analyses. Structural measures do not consider component reliabilities, and time-dependent measures are only valid for one specific instance in time. As a result,

time-independent measures have been developed in an attempt to address these issues. Time-independent measures allow component importance rankings for a desired time interval. Several time-independent measures have been developed, most of which are some form of weighted average of the Birnbaum reliability measure [7:160]. Two of the most prominent time-independent measures are those developed by Barlow and Proschan and B. Natvig.

Barlow-Proschan Time-Independent Measure. The first time-independent component importance measure was introduced by Barlow and Proschan in 1975. The B-P measure represents the probability that component i causes system failure in the time period $(0, \tau)$. It is represented by

$$I_{BP}^{(i)} = \int_0^{\tau} I_B^{(i)}(t) \cdot f^{(i)}(t) dt \quad (4)$$

where $I_B^{(i)}(t)$ represents the Birnbaum reliability measure at time t and $f^{(i)}(t)$ is the failure probability density function for component i . $I_{BP}^{(i)}$ can also be interpreted as the probability that the system life equals the life of component i [8:158].

Natvig Importance Measure. In 1979, Natvig introduced another time-independent component importance measure. The Natvig measure is defined by

$$I_N^{(i)} = \int_0^{\tau} I_B^{(i)}(t) \cdot p_{(i)}(t) \cdot (-\ln p_{(i)}(t)) dt \quad (5)$$

where $p_{(i)}(t)$ represents the reliability function for component i . The Natvig measure represents the reduction in expected remaining system lifetime (up to time τ) due to the failure of the i^{th} component [9:280].

Other time-independent measures have been developed by Aven, Bergman, Narros, Boland, and Xie, most of which are extensions or advancements of the above listed measures. Furthermore, a significant amount of work has been done in the development of importance measures for multi-state and repairable components. Space does not allow discussion of these additional measures, but Boland and El-Newehi [5] is an excellent reference providing an overview of each method and a list of applicable references.

Numerical Example of Component Importance Measures. To further demonstrate the calculation of the various importance measures, a numerical example is offered. For the given structure shown in Figure 5, the Birnbaum structural measure, Birnbaum reliability time-dependent measure, and the Barlow-Proschan and Natvig time-independent measures will be calculated.

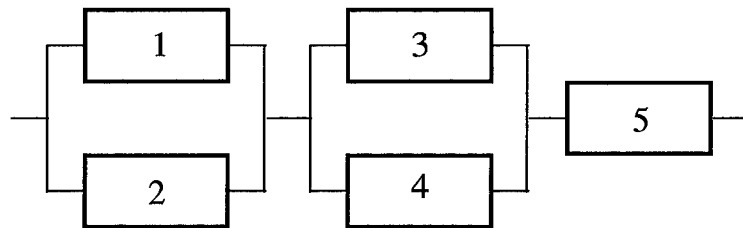


Figure 5. Example System

Table 1 defines the probability distribution and reliability functions for the various system components.

Table 1. Component Failure Distributions and Reliability Functions for Example System

Component (i)	Failure Distribution	$f^{(i)}(t)$	$p_{\phi}(t)$
1	Weibull: Shape = 1.1 (hrs) Scale = 3500 Location = 0	$\frac{1.1 \cdot t^{(1)}}{3500^{(1.1)}} e^{\left[-\left(\frac{t}{3500}\right)^{1.1}\right]}$	$e^{\left[-\left(\frac{t}{3500}\right)^{1.1}\right]}$
2	Weibull: Shape = 1.1 Scale = 3500 Location = 0	$\frac{1.1 \cdot t^{(1)}}{3500^{(1.1)}} e^{\left[-\left(\frac{t}{3500}\right)^{1.1}\right]}$	$e^{\left[-\left(\frac{t}{3500}\right)^{1.1}\right]}$
3	Weibull: Shape = 1.5 Scale = 2000 Location = 0	$\frac{1.5 \cdot t^{(5)}}{2000^{(1.5)}} e^{\left[-\left(\frac{t}{2000}\right)^{1.5}\right]}$	$e^{\left[-\left(\frac{t}{2000}\right)^{1.5}\right]}$
4	Weibull: Shape = 1.5 Scale = 2000 Location = 0	$\frac{1.5 \cdot t^{(5)}}{2000^{(1.5)}} e^{\left[-\left(\frac{t}{2000}\right)^{1.5}\right]}$	$e^{\left[-\left(\frac{t}{2000}\right)^{1.5}\right]}$
5	Weibull: Shape = 2.0 Scale = 2000 Location = 0	$\frac{2.0 \cdot t}{2000^{(2.0)}} e^{\left[-\left(\frac{t}{2000}\right)^{2.0}\right]}$	$e^{\left[-\left(\frac{t}{2000}\right)^{2.0}\right]}$

Based on the structure function, the system reliability function is

$$h(\mathbf{p}) = [1 - (1 - p_1) \cdot (1 - p_2)] \cdot [1 - (1 - p_3) \cdot (1 - p_4)] \cdot p_5 \quad (6)$$

Birnbaum Structural Measure Example. Since both components 1 and 2 as well as 3 and 4 are identical and in-parallel (and the structural importance measure does not consider component reliability), the structural importance measure values for components 1 through 4 will be the same.

Recall from equation (1) that

$$I_{B,\phi}^{(i)} = \frac{\partial h(\mathbf{p})}{\partial p_i} \bigg|_{p_1=\dots=p_n=\frac{1}{2}}.$$

For component 1,

$$\frac{\partial h(\mathbf{p})}{\partial p_1} = (1 - p_2) \cdot [1 - (1 - p_3) \cdot (1 - p_4)] \cdot p_5 \quad (7)$$

When $p_i = \frac{1}{2}$, from equation (7),

$$I_{B,\phi}^{(1)} = \frac{\partial h(\mathbf{p})}{\partial p_1} = .1875 = I_{B,\phi}^{(2)} \quad .$$

For component 3,

$$\frac{\partial h(\mathbf{p})}{\partial p_3} = [1 - (1 - p_1) \cdot (1 - p_2)] \cdot (1 - p_4) \cdot p_5 \quad (8)$$

Therefore, when $p_i = \frac{1}{2}$,

$$I_{B,\phi}^{(3)} = \frac{\partial h(\mathbf{p})}{\partial p_3} = .1875 = I_{B,\phi}^{(4)} \quad .$$

Using the same method to calculate the measure for component 5,

$$I_{B,\phi}^{(5)} = \frac{\partial h(\mathbf{p})}{\partial p_5} = .5625 \quad .$$

Therefore, the Birnbaum structural measure component ranking (in descending order) is 5, {1, 2, 3, 4}.

Birnbaum Reliability (Time-Dependent) Measure Example. Recall from

equation (3), $I_B^{(i)}(t) = \frac{\partial h(\mathbf{p})}{\partial p_i}$. Since this is a time-dependent measure, a specified time

value (t) must be selected. In this example, $t = 1000$ hours. Therefore,

for component 1,

$$I_B^{(1)}(t) = \frac{\partial h(\mathbf{p})}{\partial p_1} = (1 - p_2(t)) \cdot [1 - (1 - p_3(t)) \cdot (1 - p_4(t))] \cdot p_5(t) \quad (9)$$

$$= .158135$$

where $p_i(t)$ is given in Table 1. Since component 1 and 2 are identical and in-parallel, component 2's importance measure will also equal .158135.

Similarly, for components 3 and 4, $I_B^{(3)}(t) = \frac{\partial h(\mathbf{p})}{\partial p_3} = .220421 = I_B^{(4)}(t)$.

For component 5, $I_B^{(5)}(t) = \frac{\partial h(\mathbf{p})}{\partial p_5} = .866066$.

Therefore, the Birnbaum reliability (time-dependent) importance measure component ranking (in descending order) is 5, {3, 4}, {1, 2}.

Barlow-Proschan Time-Independent Measure Example. From equation (4),

$I_{BP}^{(i)} = \int_0^{\tau} I_B^{(i)}(t) \cdot f^{(i)}(t) dt$. A time period of interest (for the range of integration) must be specified to calculate time-independent measures. In this example, the time period will be 0 to 50,000 hours (i.e. $\tau = 50,000$). For components 1 and 2, where $I_B^{(1)}(t)$ is given in equation (9) and $p_1(t)$ and $f^{(1)}(t)$ are provided in Table 1,

$$I_{BP}^{(1)} = \int_0^{50,000} I_B^{(1)}(t) \cdot f^{(1)}(t) dt = .056671 = I_{BP}^{(2)}.$$

Similarly, for components 3 and 4,

$$I_{BP}^{(3)} = \int_0^{50,000} I_B^{(3)}(t) \cdot f^{(3)}(t) dt = .145126 = I_{BP}^{(4)}.$$

For component 5,

$$I_{BP}^{(5)} = \int_0^{50,000} I_B^{(5)}(t) \cdot f^{(5)}(t) dt = .596417.$$

Therefore, the Birnbaum time-independent importance measure component ranking (in descending order) is 5, {3, 4}, {1, 2}.

Natvig Time-Independent Importance Measure Example. From equation (5),

$I_N^{(i)} = \int_0^{\infty} I_B^{(i)}(t) \cdot p_{(i)}(t) \cdot (-\ln p_{(i)}(t)) dt$. For components 1 and 2, where $I_B^{(1)}(t)$ is given in equation (9) and $p_1(t)$ is provided in Table 1,

$$I_N^{(1)} = \int_0^{50,000} I_B^{(1)}(t) \cdot p_{(1)}(t) \cdot (-\ln p_{(1)}(t)) dt = 66.7423 = I_N^{(2)}.$$

For components 3 and 4,

$$I_N^{(3)} = \int_0^{50,000} I_B^{(3)}(t) \cdot p_{(3)}(t) \cdot (-\ln p_{(3)}(t)) dt = 142.9822 = I_N^{(4)}$$

and for component 5,

$$I_N^{(5)} = \int_0^{50,000} I_B^{(5)}(t) \cdot p_{(5)}(t) \cdot (-\ln p_{(5)}(t)) dt = 402.3612.$$

Therefore, the Natvig importance measure component ranking (in descending order) is 5, {3, 4}, {1, 2}.

In this particular example, the various demonstrated measures resulted in equivalent importance rankings for the system components (the Birnbaum structural method did not differentiate between components {1, 2} and {3, 4} because it considered only system structure and not component reliability) as summarized in Table 2.

Table 2. Importance Measure Rankings for Example System

Importance Measure	Ranking (highest to lowest)
Birnbaum Structural	5, {1, 2, 3, 4}
Birnbaum Reliability	5, {3, 4}, {1, 2}
Barlow-Proschan	5, {3, 4}, {1, 2}
Natvig	5, {3, 4}, {1, 2}

However, due to the different methods used in the calculation of component importance measures, there will not necessarily be agreement in component rankings between the various measures. Several instances were cited in the literature where one measure produced completely opposite ranking results from another measure. Therefore, analyst judgment is required for the selection of the most appropriate importance measure for any given situation [10:1431].

Simulation Experimental Design and Factor Screening Methods

The purpose of any experiment is to gain insight about a particular system [11:424]. Typically, changes are made to particular inputs (called *factors*), and the effects of these changes on some output parameter(s) (called *responses*) are analyzed and measured. Computer simulation models allow analysts the benefit of experimenting with a system model instead of the actual system. This usually saves time and money, and is frequently the only practical method of analyses.

Rather than randomly trying different combinations of input factor levels to ascertain their affect on the response, designed experiments provide an efficient and systematic method for conducting such analysis. Using a designed approach, the analyst can determine in advance the number of simulation runs and input configurations for each run to obtain the desired information about the system [12:657]. When more than just a few factors are under study, a logical first step is to determine or 'isolate' those factors which significantly affect the response measure. The literature commonly describes this as

factor screening. Several methods of factor screening are outlined in the literature including two-level factorial designed experiments, fractional factorial experiments, and Plackett-Burman (P-B) designs. Most factor screening methods consist of two-level designed experiments [13:50]. In fact, the most popular two-level experimental designs are fractional factorials and P-B designs [14:94]. Not until recently have designed factor screening experiments been used in the field of reliability to identify important factors which affect system performance [15:206].

A P-B designed experiment was used in this effort to identify the subset of active factors which affect availability estimation accuracy. This section provides a brief discussion of two-level factorial designed experiments, fractional factorial experiments, as well as an in-depth discussion of P-B designs and their projection properties.

Two-Level (2^k) Factorial Designed Experiments. A full two-level factorial experiment, where each factor is assigned a high and low level, will be used to estimate the effects of each of the k factors under study as well as their interaction effects. It requires simulation runs for each of the 2^k possible factor-level combinations (called design points) [12:660]. When a relatively small number of factors are under consideration, a full two-level factorial experiment is desirable for factor screening because it identifies all active effects without confounding. However, when k becomes moderate in size, which is most often the case, the amount of runs required can become unreasonably large.

Fractional Factorial Designs. To reduce the number of runs required, a fractional factorial experiment can be run using a subset (2^{k-p}) of the 2^k full-factorial design points. This will introduce confounding, thus reducing the amount of conclusive information

gained from the experiment. However, since we commonly assume higher-order interactions are negligible in factor screening experiments [16:17], fractional factorials can serve as excellent screening designs where only the main and two-factor interactions are of interest. The main disadvantage of fractional factorials is, like full factorials, they frequently require an impractical amount of simulation runs.

Plackett-Burman (P-B) Experimental Designs. P-B designs have traditionally been used in factor screening experiments to identify significant main effects [17:137], and they require significantly fewer runs than full and fractional factorials. P-B designs are designed experiments with two levels for estimating the effects of $n - 1$ factors at two levels in n runs. The number of runs (n) must be a multiple of four [18:423]. P-B designs are useful for screening experiments where several factors are of interest, but only a portion of these factors are suspected as being significant. They allow analysis of the main effects with a minimal number of experimental runs. The aliasing structure of P-B designs is complex, with the main effects being aliased with other interaction effects. Therefore, P-B designs are most effective when the experimenter has good reason to believe that the interaction effects are negligible. However, if some interaction effects are significant, they may be identified when using the P-B projection techniques outlined by Lin and Draper in [19].

Projection Properties of P-B Designs. When an experimental design is projected, analysis is conducted in a smaller dimension factor space to provide more detailed information concerning certain retained factors. For example, let's say an initial full factorial experiment was conducted assessing four factors with no replicates (i.e. 16 runs)

and only two factors proved significant. By ignoring the two insignificant factors, the design could be projected into a 2^4 full factorial experiment with four replicates. In this example, the projection produces replicates which allow for the calculation of pure error and the assessment of the appropriateness of the model fit.

Because of the saturated nature of Plackett-Burman designs, their projection properties are limited, but they can still be useful. Myers and Montgomery address this limitation by describing the projection properties of Plackett-Burman (P-B) experimental designs as unattractive [20:170]. However, with augmentation of additional runs to the original P-B design, some beneficial projection properties can be obtained. As Lin and Draper show, P-B designs can be quite useful in conducting screening experiments using a limited number of runs. Additionally, interaction effects can be analyzed by utilizing Lin and Draper's P-B projection techniques to obtain a higher resolution design in the significant factor space.

Lin and Draper's P-B Projection Techniques. An overview of Lin and Draper's P-B projection concepts can be summarized in a few concise steps:

- (1) Conduct a P-B designed experiment with the appropriate number of runs (n) for the factors which are to be screened and analyzed.
- (2) Using Yates algorithm [21:323-324], identify the k factors which exhibit significant main effects.
- (3) Use the associated P-B design columns for the k significant factors as the projected design in the k factor dimension.
- (4) If necessary, conduct supplemental experimental runs using specified levels for the k significant factors to achieve a desired resolution for the projected design.

P-B Projections. Table 3 delineates the projections identified for the 12-run Plackett-Burman design.

Table 3. Projection of a 12-run Plackett-Burman Design into k Dimensions [19]

k	Design Number	Description
2	2.1	2^2 design with 3 replicates
3	3.1	2^3 design plus 2^{3-1} design
4	4.1	Add one more run to obtain a 2^{4-1} design Add two more runs to obtain 3/4 replicate design Add five more runs to obtain a 2^4 design
5	5.1	Add two more runs to obtain a 2_{III}^{5-2} design Add six more runs to obtain a 2_V^{5-1} design
	5.2	Add two more runs to obtain a 2_{III}^{5-2} design Add eight more runs to obtain a 2_{IV}^{5-1} design Add ten more runs to obtain a 2_V^{5-1} design

A brief theoretical example may be the best method to demonstrate Lin and Draper's P-B projection techniques. The following is an example where $n = 12$ and $k = 3$. After conducting the 12 P-B runs, suppose only 3 of the 11 main effects prove to be significant (i.e. $k = 3$). By focusing only on the 3 columns that correspond to the k significant factors (in this example A, B, and C), the smaller design can be decomposed into a full 2^3 design and a 2^{3-1} design (where $I = \pm ABC$). Figure 6 shows a full 12-run P-B design. If, after conducting the 12 runs for the P-B design, only factors A, B, and C possess significant main effects, the design can be projected (with rows rearranged) into the arrangement shown in Figure 7.

Run	A	B	C	D	E	F	G	H	I	J	K
1	+	-	+	-	-	-	+	+	+	-	+
2	+	+	-	+	-	-	-	+	+	+	-
3	-	+	+	-	+	-	-	-	+	+	+
4	+	-	+	+	-	+	-	-	-	+	+
5	+	+	-	+	+	-	+	-	-	-	+
6	+	+	+	-	+	+	-	+	-	-	-
7	-	+	+	+	-	+	+	-	+	-	-
8	-	-	+	+	+	-	+	+	-	+	-
9	-	-	-	+	+	+	-	+	+	-	+
10	+	-	-	-	+	+	+	-	+	+	-
11	-	+	-	-	-	+	+	+	-	+	+
12	-	-	-	-	-	-	-	-	-	-	-

Figure 6. Plackett-Burman Design ($n = 12$)

Run	A	B	C
1	+	+	+
2	+	+	-
3	+	-	+
4	+	-	-
5	-	+	+
6	-	+	-
7	-	-	+
8	-	-	-
9	+	-	+
10	+	+	-
11	-	+	+
12	-	-	-

Figure 7. P-B Design Projection for $n = 12$ and $k = 3$ (A, B, C)

As can clearly be seen, runs 1 through 8 represent a full 2^3 design, and runs 9 through 12 represent a 2^{3-1} fractional design (where $I = -ABC$). These 12 runs will estimate all main effects of the 3 selected factors without aliasing and will also provide information to calculate pure error needed for lack of fit testing [19].

When $k = 4$ and $k = 5$ for the 12-run P-B design, no complete projection exists for the factors of interest. However, viable projections can be achieved by conducting

supplemental runs. When $k = 4$, one run can be added to obtain a 2_{IV}^{4-1} design, or five runs can be added to obtain a full 2^4 factorial design. An additional option is to supplement the runs to project the design into a three-quarter replicate. The three-quarter replicate consists of fewer runs than a full factorial design but more runs than a half fraction. The three-quarter replicate allows for estimation of the main effects and 2-factor interactions without aliasing with other 2-factor interactions [22]. For $k = 4$, two additional runs are needed to complete a three-quarter fraction design for the 4 factors of interest. When $k = 5$, two possible projection opportunities occur depending on the structure of the rows of the 5 selected columns from the original P-B design. If a repeat-run pair emerges, Lin and Draper call this a 5.1 design, where two more runs can be added to obtain a 2_{III}^{5-2} design, and six more runs can be added to obtain a 2_V^{5-1} design. If a mirror image pair emerges from the selected columns of the P-B design, this is a 5.2 design, where two additional runs gives a 2_{III}^{5-2} design, eight additional runs gives a 2_{IV}^{5-1} design, and ten additional runs achieves a 2_V^{5-1} design.

Benefits of P-B Designs. Utilizing Plackett-Burman designs and Lin and Draper's projection techniques offer an efficient way to conduct screening experiments when many factors are being considered, only a few are suspected of being significant, and higher order effects are assumed to be negligible. The projection techniques outlined allow analysis of the two-factor interactions in the k -dimensions of the projection while requiring less additional runs than a standard foldover.

Using a P-B experimental design for factor screening in this research provided the benefit of accomplishing the required objectives with maximum efficiency. In the final experiment, nine input data characterization factors were assessed for significance. A substantial amount of effort was required to set up each experimental run. The completion of a full two-level factorial experiment would have required 512 runs, while any viable fractional factorial design would also have required a large amount of runs. This was well beyond the scope of this research. On the other hand, the selected P-B design required only 12 experimental runs, while still providing analysis of the main effects and some two-factor interactions.

Past Research

The literature was reviewed for research in the areas of input data characterization and factor screening for system availability estimation. Numerous examples of factor screening experiments were found in the current literature. A few articles reviewed were closely related to this research and many facets of the final experimental design were extracted from these specific efforts. This section will briefly discuss six articles which closely paralleled and/or helped formulate the methodology for this thesis.

Sensitivity Analysis of Availability Estimates. Wolf [23] assessed the sensitivity of space system availability estimates to the underlying component reliability estimates. He utilized an iterative response surface methodology (RSM) to identify the system components whose component reliability significantly affected average system availability estimates. Individual component reliabilities were perturbed to high and low levels, and

fractional factorial experiments were used for factor screening. From this analysis, Wolf formulated a regression model predicting average system availability regressed against the estimated component reliabilities. Extensive regression analysis, involving several iterations, was necessary to identify the significant or 'important' components. Four of the initial one hundred components were retained in the final system availability regression model. Wolf found very little sensitivity of predicted system availability to individual component failure rate estimates. He surmised that this insensitivity may be due in part to the simplicity of the model [24:69].

Availability Analysis Using Simulation. Edgar and Bendell [24] tested the robustness of Markov models in estimating mean-time-to-failure (MTTF), mean-time-to-repair (MTTR), mean-time-to-first-failure (MTTFF), and availability for coherent systems of identical repairable components (up to 10) by use of simulation. Using Weibull distributions to define component failure and repair rates, the authors analyzed steady-state simulation versus Markov analytical results for both increasing failure rate (IFR) and decreasing failure rate (DFR) component failure and repair distributions. In general, the simulation steady-state and Markov model results were found to be consistent. The authors concluded that failure distributions (as opposed to repair distributions) were more critical in defining overall system behavior, and that decreasing failure rates were more critical than increasing failure rates [24:125].

System Complexity (or Size). Hwang, Tillman, and Lee [25] performed a literature review of works which evaluate reliability calculation methods for complex systems. Their definition of a complex system was one that could not be categorized as a series-parallel

structure. They categorized these complex systems as either small (1 - 6 components), moderate (7 - 9 components), or large (10 or more components). The article provided diagrams of the chosen example complex systems for the study with some small, some moderate, and some large. They applied various methods defined in the literature to evaluate the reliability of each example complex system. Hwang, Tillman, and Lee's definitions of complexity/size were utilized in this research effort.

Constant Failure Rate Assumption. A common practice in reliability analysis is to assume that time between failure follows an exponential distribution (i.e. a constant failure rate). Mortin, Krolewski, and Cushing [26] provided examples where this assumption produced erroneous results. They concluded that indiscriminate use of this simplifying assumption can introduce significant error in the analysis [26:54].

Repair Distributions. Kline [27], through in-depth analysis of several systems, verified that the lognormal is a good distribution for describing repair rates. He also concluded that use of the exponential distribution for repair rates resulted in negligible error when the true underlying repair distribution was lognormal [27:79].

Comparison of Screening Designs for Simulation Models. Webb and Bauer [28], using a large-scale computer simulation, compared three methods of analysis for a Plackett-Burman screening design: the Box and Meyer approach, the traditional response surface methodology (RSM) approach, and the Hamada and Wu approach. This thesis employed the RSM and Box-Meyer analysis methods.

Box-Meyer Bayesian Method. The Box-Meyer method entails deriving the marginal posterior probability that a factor is active (i.e. statistically significant) using

Bayesian techniques. This method determines which model best fits the data by examining all possible hypotheses and is analogous to all-subsets regression [28:307]. Box and Meyer explain their method as follows:

“The Bayesian approach to model identification is as follows. We consider the set of all possible models labeled $M_0, \dots, M_m$. Each model M_i has an associated vector of parameters θ_i , so that the sampling distribution of data y , given the model M_i , is described by the probability density $f(y|M_i, \theta_i)$. The prior probability of the model M_i is $p(M_i)$, and the prior probability density of θ_i is $f(\theta_i|M_i)$. The predictive density of y , given model M_i , is written $f(y|M_i)$, and is given by the expression

$$f(y|M_i) = \int_{R_i} f(y|M_i, \theta_i) d\theta_i$$

where R_i is the set of possible values of θ_i . The posterior probability of the model M_i , given the data y , is then

$$p(M_i|y) = \frac{p(M_i)f(y|M_i)}{\sum_{h=0}^m p(M_h)f(y|M_h)}.$$

The posterior probabilities $p(M_i|y)$ provide a basis for model identification. Tentatively plausible models are identified by their large posterior probability” [14:95].

Since it considers the possibility of interactions, the Box-Meyer method increases the likelihood of identifying active factors. This is “particularly true of Plackett-Burman designs where the number of runs is not a power of two” [14:94].

Response Surface Methodology (RSM). The RSM approach consists of examining the magnitude of the main effects, using analysis of variance (ANOVA), and examining normal probability and/or Pareto plots. A Pareto plot is a bar chart where the length of the bars is proportional to the absolute value of the estimated effects [28:309].

Summary

A key objective of this research was to ascertain whether there is utility in focusing on ‘important’ components when characterizing input data for availability models. This

chapter provided a detailed review of current methods for computing component importance. Additionally, a general overview of two-level screening designs as well as a thorough review of Plackett-Burman (P-B) designs was provided. A P-B screening experimental design was used in this thesis to determine which selected characterization factors were significant. Finally, pertinent literature which shaped the methodology for this effort was discussed.

Many factors contribute to the accuracy of availability models. In an effort to supplement the analyst interviews, the literature review helped identify input data characterization factor candidates for analysis: component importance, underlying component failure and repair distribution characteristics (IFR versus DFR), system structure type, and system complexity level (or size). The literature review also provided insight into appropriate factor levels for the two-level screening experiments and applicable analysis methods.

III. METHODOLOGY: PRELIMINARY EXPERIMENT

General Methodology Overview

The general methodology for this research entailed a designed screening experiment to identify significant input data characterization factors affecting availability estimate accuracy. The RSM and Box-Meyer methods discussed previously were used for analysis of the experimental output data. The research was done in two steps: a simplified preliminary experiment analyzing five factors to validate and refine the methodology, and a final experiment analyzing nine factors.

Component input data characterization factors of interest were identified using reliability analyst interviews, ideas derived from the literature review, as well as personal judgment. The nine factors identified for analysis are listed in Table 4.

Table 4. Selected Experimental Factors

Input Data Characterization Factors
True Failure probability density function (pdf) of <i>important</i> components
True Failure probability density function (pdf) of <i>non-important</i> components
Number of data points (assumed to be same for all components)
Fitting technique for Failure pdf of <i>important</i> components
Fitting technique for Repair pdf of <i>important</i> components
Fitting technique for Failure pdf of <i>non-important</i> components
Fitting technique for Repair pdf of <i>non-important</i> components
System Complexity Level (Size)
System Structure Type

For the conduct of the two-level screening experiments, two levels for each factor were selected, labeled high and low for simplicity. Availability models for various generic systems of components were created using a PC-based RM&A software program developed by the Headquarters Air Force Operational Test and Evaluation Center (HQ AFOTEC). Each system of components was designed by the researcher for complete experimental control and do not represent actual existing systems. In accordance with the experimental design, factors were set to the appropriate levels for each design point. The response measure for each simulation run was system availability absolute estimation error. Following the simulation runs, the responses were analyzed to screen the active factors via traditional RSM as well as Box-Meyer statistical analysis techniques.

Preliminary Experiment

To validate the general methodology and to expose potential problem areas, an initial smaller scale screening experiment was performed on a subset of the factors listed above. A 2^{5-1}_v factorial designed experiment was conducted to determine which of 5 input data characterization factors (for a simple series-parallel structure) might significantly affect availability model accuracy.

Definitions. The system considered in the preliminary experiment was a coherent system comprised of binary state components. As defined previously, a coherent system is one in which all components are relevant in maintaining a functional system. Binary state components have just two states: functioning or failed. The states are represented as

$X(t) = 1$ if the component functions at time t and

$X(t) = 0$ if the component is failed at time t .

A system's (as opposed to component) reliability function is depicted as $h(\mathbf{p})$, where $\mathbf{p}$ represents the component reliability vector. System availability (A_o) is defined as the percentage of time the system will perform its specified function (i.e. in operational condition) in a given period of time [29:253].

Assumptions. The following assumptions were applied to the preliminary experiment:

- (1) The structure is coherent consisting of binary state components.
- (2) All component failure and repair distribution means are bounded by the following limits:
 - (a) Weibull failure distributions: $1000 < \text{mean} < 5000$ (hours)
 - (b) Lognormal repair distributions: $10 < \text{mean} < 200$ (hours).
- (3) Only these specific distributions (Weibull and Lognormal) are used to represent the true component failure and repair distributions.
- (4) All parallel components are identical.
- (5) No negative location parameters are allowed in distribution data fitting.
- (6) Distributional fitting results obtained for identical parallel components require only one set of input data sampled from one component.
- (7) Maximum Likelihood Estimation (MLE) methods are used to calculate fitted distribution parameters.
- (8) The response function, defined as the absolute error of the system availability measure from each simulation run, is approximately linear with respect to the input variables.
- (9) Higher order interaction effects are negligible.
- (10) The component with the highest ranking Barlow-Proschan time-independent importance measure represents the most important system component.

Software. The software used to create the availability simulation model is a PC-based program entitled Rapid Availability Prototyping Tool for Testing Operational Readiness (RAPTOR), written by the Headquarters Air Force Operational Test and Evaluation Center (HQ AFOTEC). RAPTOR can be used to create availability, reliability, maintainability, and sparing models for various systems undergoing operational testing and evaluation (OT&E). The program was written in MODSIM II, an object-oriented simulation language, and requires the user to graphically define the system Reliability Block Diagram (RBD). Component failure and repair rates are simulated over time to determine overall system R & M characteristics [30]. Weibull++ Version 4.0 was the software used to generate and fit component failure and repair data sets. Weibull++ Version 4.0 is a reliability software program created by ReliaSoft, Inc. which has robust data generation and fitting routines for common reliability distributions [31].

Design of Preliminary Experiment. The structure studied was a simple series-parallel structure consisting of five components depicted in Figure 8.

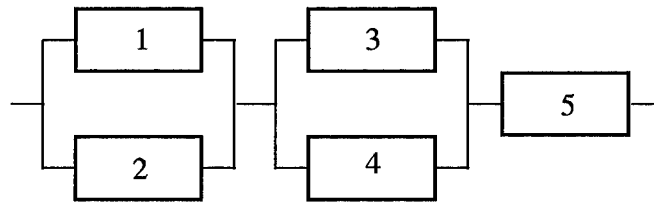


Figure 8. Experimental Structure for Preliminary Experiment

The experiment consisted of a 2^{5-1}_V factorial design (with three replicates) on the five component series-parallel system shown in Figure 8. Since this is a resolution V design, the main effects and two-factor effects can be estimated without aliasing with each other. However, two-factor

interactions are confounded with three-factor interactions [32:163]. The associated experimental factors and levels are depicted in Table 5.

Table 5. Experimental Factors and Levels for Preliminary Experiment

Factors		Levels	
A	Number of data points (assumed to be same for all components)	50	+
		10	-
B	Fitting technique for Failure pdf of <i>important</i> components	Weibull++ Top MLE Ranking	+
		Weibull++ MLE: Exponential	-
C	Fitting technique for Repair pdf of <i>important</i> components	Weibull++ Top MLE Ranking	+
		Empirical	-
D	Fitting technique for Failure pdf of <i>non- important</i> components	Weibull++ Top MLE Ranking	+
		Weibull++ MLE: Exponential	-
E	Fitting technique for Repair pdf of <i>non- important</i> components	Weibull++ Top MLE Ranking	+
		Empirical	-

The Weibull++ Monte Carlo data generation module was used to generate simulated failure and repair times from the defined component distributions. The Weibull++ distribution wizard was used to fit theoretical distributions to the generated data set and to calculate distribution parameters using the maximum likelihood estimation (MLE) method. A ‘forced-fit’ exponential distribution was used for the low level for component failure data fitting due to the frequent use of the exponential assumption in component failure analysis. Separate data sets were generated and fitted for each of the three replications.

The defined system failure and repair distributions as well as the (replication 1) fitted distributions for each component are listed in Tables 6 and 7.

Table 6. System Failure True and Fitted Distributions (Replication 1)

Component	True Failure Distribution	10 Data Points	50 Data Points
		Wizard Fit	Exponential Fit
1/2	Weibull: (hrs) Shape = 1.1 Scale = 3500 Location = 0	Weibull: Shape = 1.142 Scale = 3677 Location = 0	Exponential: Mean = 3333 Location = 0
3/4	Weibull: Shape = 1.5 Scale = 2000 Location = 0	Normal: Mean = 1284 St Dev = 771	Exponential: Mean = 1250 Location = 14.2
5	Weibull: Shape = 2.0 Scale = 2000 Location = 0	Weibull: Shape = 1.872 Scale = 2014 Location = 0	Exponential: Mean = 1428 Location = 384.5

Table 7. System Repair True and Fitted Distributions (Replication 1)

Component	True Repair Distribution	10 Data Points	50 Data Points
		Wizard Fit	Low Level Fit
1/2	Lognormal: Mean = 40 (hrs) St Dev = 10	Lognormal: Mean = 43.4 St Dev = 6.5	Empirical
3/4	Lognormal: Mean = 70 St Dev = 15	Weibull: Shape = 10.73 Scale = 65.2 Location = 0	Empirical
5	Lognormal: Mean = 60 St Dev = 8	Weibull: Shape = 1.582 Scale = 20.0 Location = 38.9	Empirical

Since components 1 and 2 as well as 3 and 4 were identical, the same data fit was used for each identical pair. Graphical examples of the results for failure and repair pdf data

fittings for component 5 are shown in Figures 9 and 10. The generated data sets for the preliminary experiment data fittings are available in Appendix C.

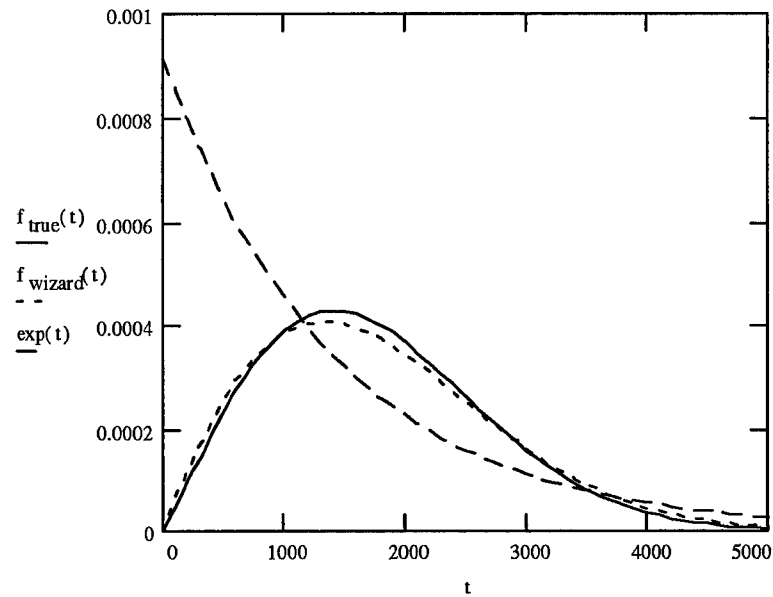


Figure 9. Component 5 True Failure pdf versus Weibull++ wizard and exponential fits (Replication 1 using 10 data points)

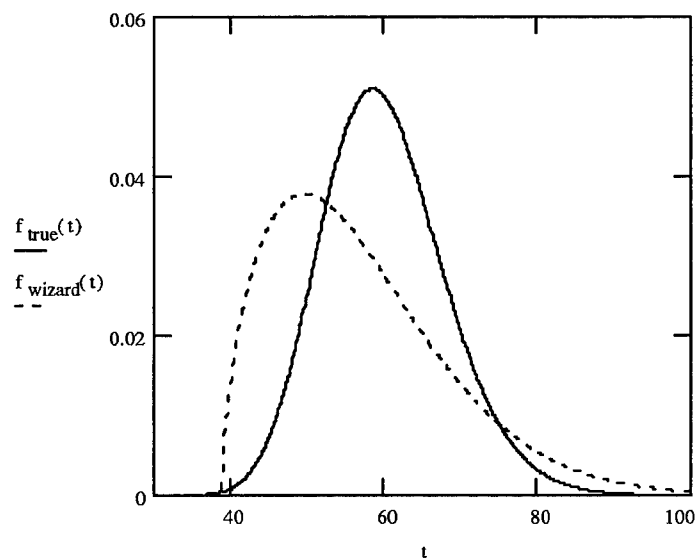


Figure 10. Component 5 True Repair pdf versus Weibull++ wizard fit (Replication 1 using 10 data points)

Simulation Runs. Run duration for each replication was 50,000 hours in simulated time. Three replications were conducted at each of the 2^4 design points, resulting in 48 total runs. The response variable was defined as the absolute error of the system availability measure from each simulation run. The value representing true availability ($A_0 = 96.6355\%$) used for calculation of absolute error was obtained by conducting 2000 runs using the defined component failure and repair distributions. Banks, Carson, and Nelson's [33:449] formula was used to calculate the initial estimate of the number of runs needed to obtain a 95% confidence limit and a $\pm .015\%$ tolerance for the 'true' system availability measure:

$$R \geq \left(\frac{z_{\alpha/2} S_0}{\epsilon} \right)^2 \quad (10)$$

where R is the estimated number of runs needed, S_0 is the standard deviation of the initial sample, and ϵ is the desired tolerance.

Since each run represents independent and identically distributed random variables, traditional statistical methods apply. One hundred initial runs of 50,000 hours duration were completed resulting in an S_0 for A_0 of .3168%. From equation (10), $R \geq 1713.56$. Therefore, 1714 or more runs were necessary to obtain a baseline availability measure which would meet the specified tolerance of $\pm .015\%$ at a 95% confidence level. A total of 2000 runs were completed which resulted in an average availability value (A_0) of 96.6355%. This point estimate of system availability for time 0 to 50,000 hours was the benchmark of comparison to calculate the absolute error of the system availability measure for each design point in the experiment.

Components were rank-ordered by their Barlow-Proschan time-independent importance measure for 0 to 50,000 hours, where component 5 was deemed the most important component. Table 8 shows the calculated B-P importance measure values.

Table 8. Barlow-Proschan Time-Independent Importance Measure Values

Component(s)	Calculated B-P Importance Measure
1, 2	.056671
3, 4	.145126
5	.596417

Analysis Methods and Software. The analyzed multiple regression main-effects model can be described in the following format:

$$Y_{ij} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \epsilon_{ij} \quad (11)$$

where

Y_{ij} is the response value for run number i and replication j ;

β_0 represents the intercept (or response mean);

β_k represents the regression coefficients for factors $k = 1, \dots, 5$;

X_k represents the factor level (either +1 or -1) for factor k ; and

ϵ_{ij} represents the residual error for run number i and replication j .

Yates algorithm [21:323-324] and least squares methods were used to calculate the main and interaction effects. The correlation coefficient (R^2), ANOVA, and lack of fit statistics were calculated to assess model adequacy. To identify significant factors, normal probability plots, Pareto plots, Box-Meyer Bayes plots, and linear regression coefficient t-test statistics were used. The primary analysis software was JMP version 3.1,

a PC-based statistical analysis program developed by the SAS Institute. JMP possesses data graphing, experimental design, and statistical analysis routines [34:319-341] which proved very useful in this research.

IV. RESULTS: PRELIMINARY EXPERIMENT

Simulation Results

The 2_{v}^{5-1} experimental design and resulting responses for the preliminary experiment are shown in Table 9.

Table 9. Experimental Design and Responses

Design Point	Factors					Observed Availability*			Absolute Error (Y)		
	A	B	C	D	E	Replication 1	Replication 2	Replication 3	Replication 1	Replication 2	Replication 3
1	-1	-1	-1	-1	1	96.8046%	96.9903%	95.8230%	0.1691%	0.3548%	0.8125%
2	-1	-1	-1	1	-1	96.7202%	97.0266%	95.8372%	0.0847%	0.3911%	0.7983%
3	-1	-1	1	-1	-1	96.6918%	96.9858%	95.7640%	0.0563%	0.3503%	0.8715%
4	-1	-1	1	1	1	96.6324%	96.8985%	95.7639%	0.0031%	0.2630%	0.8716%
5	-1	1	-1	-1	-1	96.7904%	96.8941%	95.9042%	0.1549%	0.2586%	0.7313%
6	-1	1	-1	1	1	96.7261%	96.8518%	95.9385%	0.0906%	0.2163%	0.6970%
7	-1	1	1	-1	1	96.6137%	96.8172%	95.8354%	0.0218%	0.1817%	0.8001%
8	-1	1	1	1	-1	96.5398%	96.7937%	95.9377%	0.0957%	0.1582%	0.6978%
9	1	-1	-1	-1	-1	96.7274%	96.0124%	96.3905%	0.0919%	0.6231%	0.2450%
10	1	-1	-1	1	1	96.7276%	96.0496%	96.3695%	0.0921%	0.5859%	0.2660%
11	1	-1	1	-1	1	96.6290%	95.7957%	96.2251%	0.0065%	0.8398%	0.4104%
12	1	-1	1	1	-1	96.5982%	95.8427%	96.2454%	0.0373%	0.7928%	0.3901%
13	1	1	-1	-1	1	96.7642%	95.9374%	96.2951%	0.1287%	0.6981%	0.3404%
14	1	1	-1	1	-1	96.7929%	95.9976%	96.4092%	0.1574%	0.6379%	0.2263%
15	1	1	1	-1	-1	96.6571%	95.8386%	96.2923%	0.0216%	0.7969%	0.3432%
16	1	1	1	1	1	96.8079%	95.8528%	96.2844%	0.1724%	0.7827%	0.3511%

* 2000 Run 'Truth' Availability = 96.6355%

Note that all system availability estimates from each run were within $\pm .88\%$ of the defined true system availability.

Statistical Analysis

A summary of the key model statistics is provided in Table 10.

Table 10. Preliminary Experiment Model Statistical Results

Statistic	Value	Interpretation
R^2	.004537	Model explains virtually none of output variability
Whole Model F-test p-value	.9991	Model as a whole is not significant
Lack of Fit F-test p-value	1.0	Linear model is appropriate (no curvature)

The model statistics show that the defined main-effects model explains very little of the response variation and that a linear model is appropriate for the experimental region. A summary of the calculated factor effects and statistics is shown in Table 11.

Table 11. Estimated Effects and Statistical Analysis

Factor	Effect Estimate	t-test p-value	Interpretation
Intercept	.37850%	<.0001	Significant (mean response)
A	-.00386%	.9654	Not significant
B	-.02694%	.7624	Not significant
C	.01933%	.8282	Not significant
D	-.01871%	.8336	Not significant
E	.00598%	.9465	Not significant

The t-test for each effect estimate indicates that only the mean response (regression model intercept term) is significant. A supplemental listing of statistical analysis outputs for the preliminary experiment is provided in Appendix A.

Graphical Analysis

Figures 11, 12, and 13 show the normal probability, the Pareto, and the Box-Meyer Bayes plots.

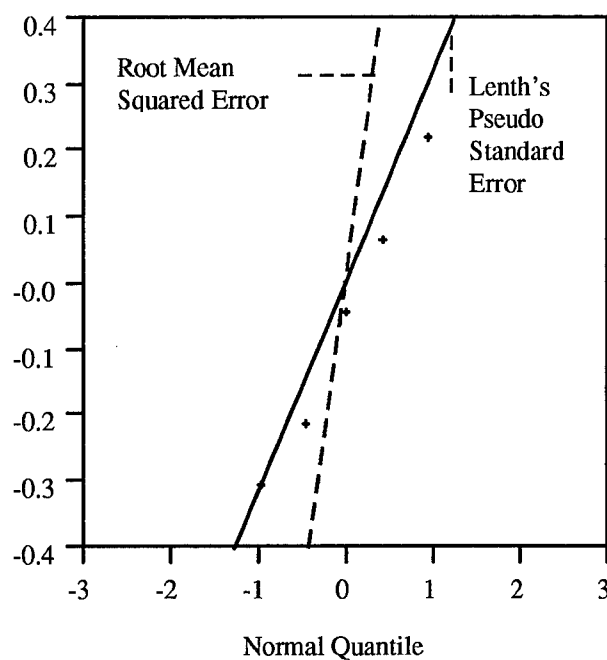


Figure 11. Normal Probability Plot

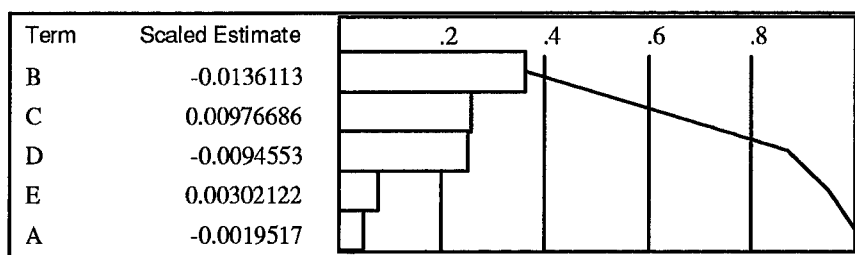


Figure 12. Pareto Plot of Scaled Estimates

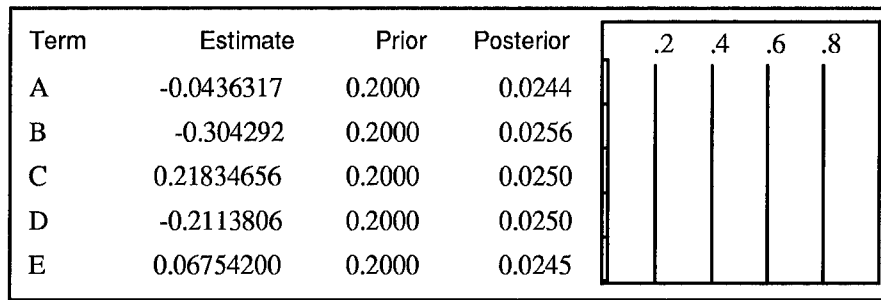


Figure 13. Box-Meyer Bayes Plot

The normal probability and Bayes plot results are consistent and indicate that no effects are significant. The Pareto plot indicates that factors B, C, and D explain the most variation, but since the amount of explained variation by the model is negligible this result has little significance.

Additional Analysis

Upon closer inspection of the absolute error responses shown in Table 9, it was discovered that a possible blocking effect may be present between replications. For example, notice (in Table 9) that the absolute errors in replication 1 are the smallest values in all cases. To address this, additional data analysis was conducted on models which included a blocking variable as well as other response measures: error and squared error. Table 12 contains a summary of the possible significant factors resulting from all analyses on the preliminary experimental data.

Table 12. Significant Factors Assessing Alternative Responses and a Blocking Variable

Blocking Variable	Response		
	Absolute Error	Error	Squared Error
No	None	Possibly C	Possibly A & C
Yes	None	None	A and possibly C

Statistical analysis showed that the blocking variable was strongly significant with all three response measures.

With the additional responses (error and squared error), factors A (number of data points) and C (fitting technique for repair pdf of important component: component 5), presented themselves as possible significant factors. However, these conclusions are not definitive and thus were addressed again in the final experiment.

Summary

The statistical analysis, using absolute error as a response measure, supports the hypothesis that there are no significant effects. With the absolute error response, no effects were shown to be significant in the t-tests, and the normal probability, Pareto, and Bayes plots revealed no clear significant factor effects. This means that using fewer data points (i.e. 10 versus 50) and less aggressive fitting techniques (i.e. exponential assumption for failure rates and use of empirical repair distributions) on important as well as non-important components did not significantly degrade model accuracy for this particular structure.

However, introducing a blocking variable in conjunction with two alternative responses, error and squared error, revealed that factors A and C *may* be significant. Therefore, the results from this experiment are inconclusive. Further analysis is required to determine conclusively if the number of data points (factor A) and the fitting technique for repair pdf of important component (factor C) are significant.

V. METHODOLOGY: FINAL EXPERIMENT

Insights Gained from Preliminary Experiment

While the preliminary experiment assessed five input data characterization factors, the final experiment assessed nine factors listed in Table 4. Several insights were gained from the preliminary experiment which helped refine the methodology for the final experiment. After reviewing the methodology and results of the preliminary experiment, AFOTEC analysts recommended low and high levels of 5 and 25 for the 'number of data points' factor levels. They felt that levels of 10 and 50 data points were too generous based upon their experience in past operational availability analyses. They also pointed out that the mean-time-to-failure (MTTF) / mean-time-to-repair (MRT) ratios were relatively large for all five components of the experimental structure, and that a wider range of ratios may be more appropriate for future experimental designs. It was also pointed out that frequently the analyst will not have a priori knowledge of component failure behavior. This information is normally required for the calculation of component importance measures, with the exception of structural importance measures. An additional suggestion was to analyze the variability of several availability model outputs for individual runs. This was addressed in a separate study conducted using multivariate techniques on several RAPTOR model output measures. A summary of the study is provided in Appendix G. Finally, it was discovered that a significant amount of time and effort was required to set-up the experimental runs, which included component failure and repair data point generation and fitting, construction of RAPTOR models, and completion of 'truth' data

runs. Since the required effort would increase dramatically with the addition of 4 more factors, any subsequent experimental screening design would need to economize on the number of simulation runs.

Final Experiment

Assumptions. To produce diversity in the MTTF/MRT ratios for the system components, wider bounds were allowed for the means of the component failure and repair distributions. They were bounded by the following limits:

(1) Weibull failure distributions: $1000 < \text{mean} < 6500$ (hours)

(2) Lognormal repair distributions: $50 < \text{mean} < 3000$ (hours).

The most important components in a structure were deemed as the ones which fell in the top 20% of component importance measure rankings based upon component failure distributions. To allow for the calculation of the importance measures without knowledge of the underlying failure distributions, the Birnbaum structural importance measure was used. This measure is based solely upon system structure. All other assumptions outlined in the preliminary experiment also applied to the final experiment.

Structures. 20 components were designed which were used for the building of system structures for the RAPTOR models. Each component was designed to have true Weibull failure and lognormal repair distributions randomly set within the established bounds for the distribution means. Increasing failure rate (IFR) and decreasing failure rate (DFR) configurations were created for each component while maintaining the same distribution mean. To accomplish this, randomly selected Weibull shape and scale parameters were

utilized to create the IFR failure distributions. Using a randomly generated DFR shape parameter for each component, the same *average* failure rate was maintained by adjusting the Weibull scale parameter to achieve an identical mean failure rate as in the IFR configuration. This procedure was used to ensure that the results were not biased by producing a different average failure rate when reconfiguring a component from IFR to DFR. The shape parameters ranged from 1.1 to 4.0 for IFR configurations and from .4 to .95 for DFR configurations. A complete listing of component failure and repair distribution parameters (for both configurations) is shown in Appendix B.

Four basic structures were created from the set of 20 components described above: a small/series-parallel structure, a small/complex structure, a large/series-parallel structure, and a large/complex structure. The small structures used components 1 through 5, while the large structures were comprised of all 20 components. Appendix B provides reliability block diagrams for each structure.

Design of Final Experiment. The factors and levels for the final experiment are shown in Table 13. Since each run demanded a large set-up effort, a design which minimized the number of runs was preferable. Replications were still desired to increase the confidence in the results and to estimate pure error for lack of fit testing. A full factorial experiment would require 1536 runs (i.e. $512 * 3$ replications), and a 2_{III}^{9-5} fractional factorial design would require 48 runs (i.e. $16 * 3$ replications). A Plackett-Burman (P-B) design was chosen because it required only 36 (i.e. $12 * 3$ replications) total simulation runs to assess the nine factors.

Table 13. Factors and Levels for Final Experiment

Factors		Levels	
A	True Failure probability density function (pdf) of <i>important</i> components	Weibull IFR	+
		Weibull DFR	-
B	True Failure probability density function (pdf) of <i>non-important</i> components	Weibull IFR	+
		Weibull DFR	-
C	Number of data points (assumed to be same for all components)	25	+
		5	-
D	Fitting technique for Failure pdf of <i>important</i> components	Weibull++ Top MLE Ranking	+
		Weibull++ MLE: Exponential	-
E	Fitting technique for Repair pdf of <i>important</i> components	Weibull++ Top MLE Ranking	+
		Empirical	-
F	Fitting technique for Failure pdf of <i>non-important</i> components	Weibull++ Top MLE Ranking	+
		Weibull++ MLE: Exponential	-
G	Fitting technique for Repair pdf of <i>non-important</i> components	Weibull++ Top MLE Ranking	+
		Empirical	-
H	System Complexity Level (Size)	Large (20 components)	+
		Small (5 components)	-
I	System Structure Type	Series-Parallel	+
		Complex	-

The 12-run 9-factor P-B design used for the final experiment is shown in Table 14.

Table 14. 12-run Plackett-Burman Design for Final Experiment

Design Point	Factors								
	A	B	C	D	E	F	G	H	I
1	+	+	+	+	+	+	+	+	+
2	-	+	-	+	+	+	-	-	-
3	-	-	+	-	+	+	+	-	-
4	+	-	-	+	-	+	+	+	-
5	-	+	-	-	+	-	+	+	+
6	-	-	+	-	-	+	-	+	+
7	-	-	-	+	-	-	+	-	+
8	+	-	-	-	+	-	-	+	-
9	+	+	-	-	-	+	-	-	+
10	+	+	+	-	-	-	+	-	-
11	-	+	+	+	-	-	-	+	-
12	+	-	+	+	+	-	-	-	+

Distributional Fittings. As in the preliminary experiment, Weibull++ was used to generate and fit the component failure and repair data sets for each configuration. Separate generations and fits were conducted for each replication. Components 14, 15, and 16 as well as 18, 19, and 20 were identical components, therefore only one generation and fitting was conducted for each triplicate set per replication. Final experiment fitting data is contained in Appendix D and graphical examples for the fitted distributions for some of the components are provided in Appendix E.

Important Components. A complete listing of the Birnbaum structural component importance measures calculated for each component in each of the four experimental structures is provided in Appendix F, with a summary provided in Table 15.

Table 15. Top 20% Important Components

Structure	Top 20% Important Components
Small / Series-Parallel	Component 3
Small / Complex	Component 1
Large / Series-Parallel	Components 4, 5, 13, 17
Large / Complex	Components 1, 4, 7, 8

Simulation Runs. 16 truth runs were required due to the four additional factors. For each of the four structures, 'truth' runs were done with the following configurations:

- (1) All components with IFR failure distributions
- (2) All components with DFR failure distributions
- (3) Important components with IFR failure distributions and non-important components with DFR failure distributions
- (4) Important components with DFR failure distributions and non-important components with IFR failure distributions.

As before, each simulation run duration was for 50,000 hours simulation time.

Two thousand replications were run to establish 'truth' availability values for each configuration. For the P-B experimental runs, the response measure was again the absolute error of the system availability measure from each simulation run as compared to the 'truth' measure.

Analysis Methods. The analysis methods were identical to those used for the preliminary experiment. Traditional statistical measures were used to assess model adequacy, and normal probability plots, Pareto plots, Bayes plots, and linear regression coefficient t-test statistics were used to identify the significant factor effects. A response surface was formed to graphically portray the combined affect of the active factors on model availability estimation error.

VI. RESULTS: FINAL EXPERIMENT

Simulation Results

The results from the truth and Plackett-Burman experimental RAPTOR runs for the final experiment are shown in Table 16.

Table 16. Numerical Results for Final Experimental Runs

Design Point	Structure	Component Failure PDF	Truth Runs	Observed Availability			Absolute Error (Y)		
		Important / Non-important		Replication 1	Replication 2	Replication 3	Replication 1	Replication 2	Replication 3
1	Large / S-P	IFR / IFR	83.1373%	81.0810%	78.166%	82.5297%	2.0563%	4.9713%	0.6076%
2	Small / Complex	IFR / DFR	77.3638%	81.2591%	78.2235%	80.9242%	3.8953%	0.8597%	3.5604%
3	Small / Complex	DFR / DFR	76.4428%	77.3795%	79.5758%	74.8265%	0.9367%	3.1330%	1.6163%
4	Large / Complex	DFR / IFR	60.4257%	38.6057%	61.4589%	55.8074%	21.820%	1.0332%	4.6183%
5	Large / S-P	IFR / DFR	82.7799%	80.6977%	77.8648%	71.5604%	2.0822%	4.9151%	11.219%
6	Large / S-P	DFR / DFR	81.6366%	82.0842%	76.8906%	82.4661%	0.4476%	4.7460%	0.8295%
7	Small / S-P	DFR / DFR	64.6009%	63.2109%	65.2901%	55.2021%	1.3900%	0.6892%	9.3988%
8	Large / Complex	DFR / IFR	60.4257%	41.0340%	61.8580%	54.8932%	19.391%	1.4323%	5.5325%
9	Small / S-P	IFR / IFR	65.9448%	64.6130%	65.4925%	64.7097%	1.3318%	0.4523%	1.2351%
10	Small / Complex	IFR / IFR	78.2001%	76.4971%	80.2750%	78.4965%	1.7030%	2.0749%	0.2964%
11	Large / Complex	IFR / DFR	60.7345%	60.7147%	56.1168%	60.1398%	0.0198%	4.6177%	0.5947%
12	Small / S-P	DFR / IFR	65.0705%	63.5087%	65.8172%	65.2438%	1.5618%	0.7467%	0.1733%

A much larger variability in the response was observed compared to the preliminary experiment. The observed absolute errors in availability estimates ranged from .0198% to 21.82%.

Statistical Analysis

A summary of the key model statistics is provided in Table 17.

Table 17. Final Experiment Model Statistical Results

Statistic	Value	Interpretation
R^2	.333092	Model explains one-third of output variability
Whole Model F-test p-value	.2241	Model as a whole is not significant
Lack of Fit F-test p-value	.9680	Linear model is appropriate (no curvature)

The model statistics show that the defined main-effects model explains approximately one-third of the response variation and that a linear model is appropriate for the experimental region. A summary of the calculated factor effects and statistics is shown in Table 18.

Table 18. Estimated Effects and Statistical Analysis

Factor	Effect Estimate	t-test p-value	Interpretation
Intercept	3.4997%	.0001	<i>Significant</i> (mean response)
A	.89372%	.5688	Not significant
B	-1.8335%	.2471	Not significant
C	-3.5403%	.0306	<i>Significant</i>
D	-.04232%	.9784	Not significant
E	.63297%	.6861	Not significant
F	-.53829%	.7310	Not significant
G	1.2852%	.4142	Not significant
H	3.1045%	.0555	<i>Significant</i>
I	-1.5712%	.3197	Not significant

The mean absolute error of availability estimates for all the P-B simulation runs is 3.4997%. The t-test for each effect estimate indicates that the mean response (regression model intercept term), factor C (number of data points) effect, and

factor H (system complexity/size) effect are significant. A supplemental listing of statistical analysis outputs for the final experiment is provided in Appendix A.

Graphical Analysis

Figures 14, 15, and 16 show the normal probability, the Pareto, and the Box-Meyer Bayes plots.

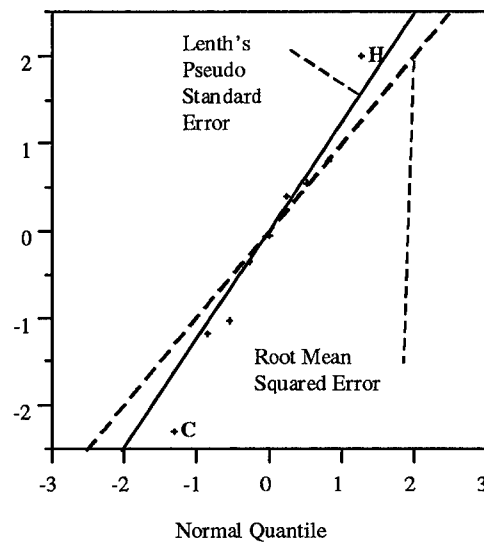


Figure 14. Normal Probability Plot

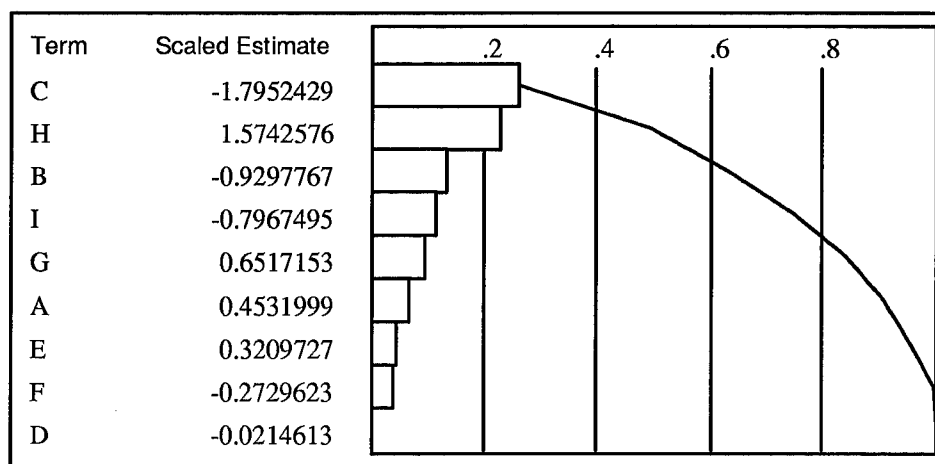


Figure 15. Pareto Plot of Scaled Estimates

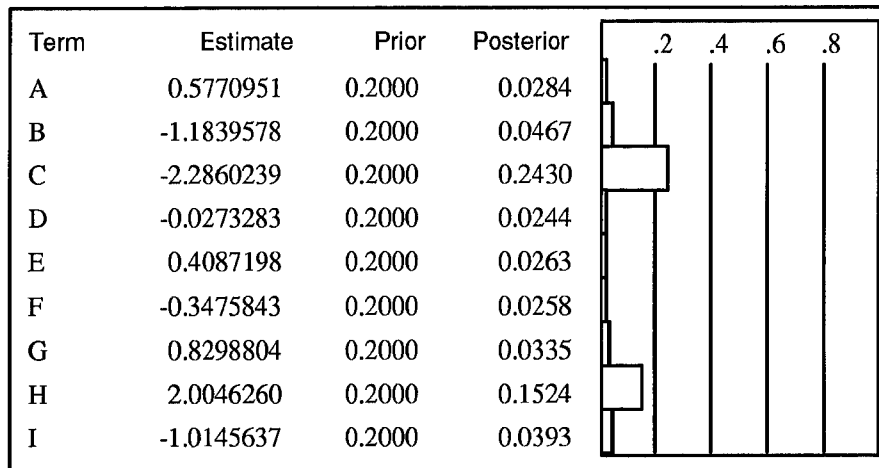


Figure 16. Box-Meyer Bayes Plot

The normal probability, Pareto, and Bayes plot results are consistent and suggest that factor C (number of data points) and factor H (system complexity/size) are significant, while all other factors are not significant.

Significant Effect Model

A subsequent regression model containing only factors C, H, and their interaction term was analyzed to determine if the C*H interaction term was significant. The results are shown in Table 19.

Table 19. Estimated Effects and Statistical Analysis for C, H, C*H Model

Factor	Effect Estimate	t-test p-value	Interpretation
Intercept	3.4997%	<.0001	Significant (mean response)
C	-3.5403%	.019	Significant
H	3.1045%	.0379	Significant
C*H	-2.365766	.1086	Not significant
Statistic	Value		Interpretation
R ²	.296981		Model explains approximately one-third of output variability
Whole Model F-test p-value	.0095		Model as a whole is significant

In this case, the model explained approximately 30% of the response variability, and the model as a whole was significant. As before, the main effects for factors C and H were significant. The C*H interaction effect was not significant at a 10% level of significance.

Response Surface

A response surface was developed for the resulting C and H main-effects model:

$$Y = 3.4997 - 1.770133C + 1.5522389H \quad (12)$$

where Y is the estimated absolute error in the availability estimate; and

C and H represent the factor level (either +1 or -1) for each factor.

The resulting response surface and contour plot are shown in Figure 17.

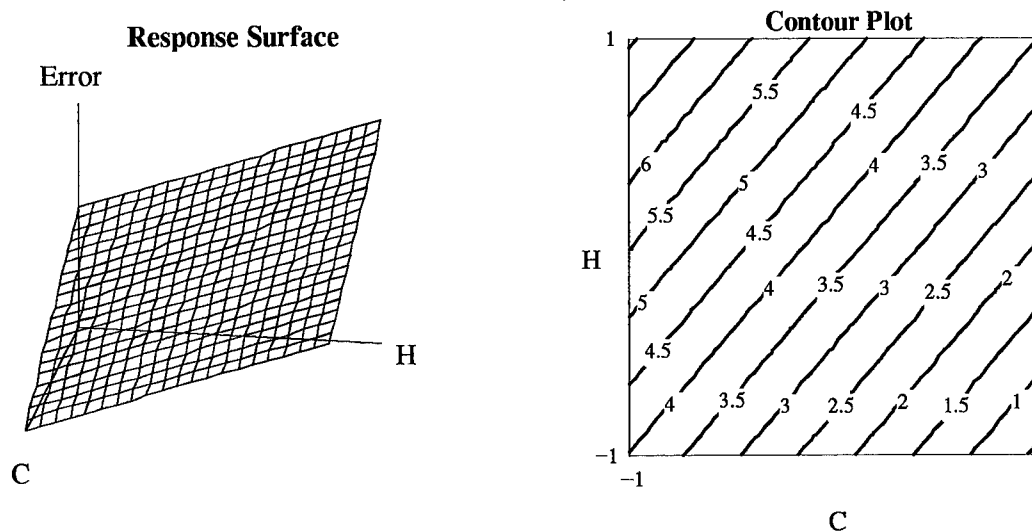


Figure 17. Two-Factor Model Response Surface and Contour Plot

As the plots in Figure 17 demonstrate, a high level for factor C (number of data points) and a low level for factor H (system size) result in the smallest availability estimation error.

Additional Analysis

As with the preliminary experiment, subsequent analysis was performed using error and squared error as response measures as well as introducing a blocking variable for the replications. In all cases, the blocking variable was insignificant. Furthermore, the results in all cases were consistent with those achieved using absolute error as the response, showing factors C and H as significant.

Summary

The statistical analysis tests and the normal probability, Pareto, and Bayes plots support the hypothesis that factors C and H are significant. Subsequent analysis indicates that the C*H interaction effect is not significant. The blocking effect between replications was insignificant, and using error and squared error as response variables resulted in identical conclusions to those achieved using the absolute error response. Analysis of the resulting two-factor model reveals that availability error is reduced when operating at a high level for factor C (number of data points) and a low level for factor H (system size).

VII. SUMMARY AND CONCLUSIONS

Research Objectives

The general purpose of this study was to provide insight into the input data characterization factors which may affect the accuracy of availability model output. The potential benefits of identifying the key factors would be the reduction of unproductive data collection and more efficient RM&A modeling.

. The overall research objectives were to:

- (1) Identify potential factors which affect availability model output accuracy.
- (2) Screen the potential factors to determine which have a statistically significant effect (or interaction effect) on output accuracy.
- (3) Assess the magnitude of the significant effects.
- (4) Provide basic insights to aid in efficient component input data characterization for availability models.

Overview of Results

Component input data characterization factors thought to possibly affect system availability estimates were identified and analyzed. Using a design of experiment approach with the absolute error of system availability estimates serving as the response, a two-stage experimental screening process was conducted to identify the active factors.

Preliminary Experiment. The results from the preliminary experiment were inconclusive, identifying number of data points and fitting method for the important components as possible significant factors. Using absolute error as the response, all

factors proved insignificant. The average system availability estimate absolute error was .3785%.

Final Experiment. The final experiment, analyzing four basic structures, revealed that system size (5-component versus 20-component) and the number of data points (5 versus 25) *do* affect estimate accuracy. It also showed that fitting technique, underlying component failure distribution (IFR versus DFR), and system structure type (series-parallel versus complex) *do not* have a significant effect. The interaction effect between the two active factors was not statistically significant. Using error and squared error as response variables resulted in the same conclusions achieved using the absolute error response. The average system availability estimate absolute error was 3.4997%, and the effect estimates were -3.504% for the 'number of data points' factor and 3.1045% for the 'system size' factor. The response surface from the two-factor model derived from the final experiment showed that estimation error is minimized when the number of data points is at a high level and the system size is small.

Multivariate Analysis. The supplemental multivariate analysis of RAPTOR output (Appendix G) revealed that multivariate techniques can be used to discriminate between various structures based on model outputs. It was also discovered that structures with predominantly DFR components produce higher variability in RAPTOR output measures than structures with predominantly IFR components.

Conclusions

Several insights were gained from this research:

- (1) More availability estimation error is to be expected when analyzing larger system structures;
- (2) Availability estimation error can be reduced by increasing the number of failure and repair data points collected for each system component;
- (3) There is no measurable significant difference in estimation error when analyzing systems with IFR component failure characteristics versus systems with DFR component failure characteristics;
- (4) There is no apparent benefit in focusing on 'important' versus 'non-important' components when characterizing component failure and repair probability distributions;
- (5) There is no apparent difference in estimation error when analyzing series-parallel structures versus complex structures; and
- (6) No single fitting technique utilized in this research provided any distinct advantage over any other method for availability estimate error reduction.

To summarize, the availability measure appears to be robust to fitting method, component failure characteristics, and system structure type, and sensitive to the number of data points used in data fitting and the system size.

Comparison with Past Research Results

Sensitivity to Component Failure Rate Characterization. In analyzing a large space system, Wolf found very little sensitivity of the predicted system availability to individual component failure rate estimates [23:69]. The preliminary experimental results showed that the number of data points *may* affect availability estimation accuracy. The final experiment showed conclusively that the number of data points used in the

characterization of component failure and repair behavior *can* have a statistically significant affect on availability estimation accuracy.

Edgar and Bendell concluded that failure distributions were more critical than repair distributions in defining overall system behavior and that decreasing failure rates (DFR) were more critical than increasing failure rates (IFR) [24:125]. This study revealed that, at least when measuring system availability estimation error, the fitting fidelity of the failure and repair distributions and the underlying component failure rate (IFR versus DFR) were not significant. System availability appears to be a highly robust system characteristic and may be less sensitive than other system characteristics to changes in certain factors. The multivariate study showed that DFR component structures have higher output variability than IFR component structures.

Exponential Assumption. Mortin, Krolewski, and Cushing provided examples where the indiscriminate use of the exponential distribution for component failure characterization can produce erroneous results [26:54]. In this study, the use of the exponential distribution for component failure characterization (when the true underlying failure distribution was Weibull) did not significantly alter system availability estimation accuracy. Again, this may indicate that the availability measure is robust to component distributional assumptions.

Suggestions for Further Research

Identifying Other Factors. The final regression model (using the absolute error response) explained only a portion of the overall response measure variability with an R^2

of .297, suggesting that other significant explanatory variables may exist. More formal methods could be conducted to identify other possible critical input data characterization factors not addressed in this study, such as a formal survey of several Air Force reliability analysts. A screening design could then be accomplished to identify other significant factors.

Mean-Time-to-Failure / Mean-Repair-Time (MTTF/MRT) Ratio. After reviewing the results of the preliminary experiment, AFOTEC analysts felt one important factor to analyze would be the component MTTF/MRT ratio. They suspected that system availability estimates might be more sensitive to some of the factors analyzed in this study when several components possessed a low MTTF/MRT ratio. Time did not allow for the inclusion of the MTTF/MRT factor in this study; in fact, it was randomized in the experimental design to mitigate ('spread around') its effect. Follow-on experiments which incorporate this factor may be insightful.

Response Surface Methodology (RSM). This research addressed qualitative as well as quantitative factors. In all cases, the qualitative factors proved insignificant. However, two quantitative factors (number of data points and system size) were significant. A simple linear response surface was developed for the resultant two-factor model for the defined experimental region. The factor levels used for the experiment (number of data points: 5 and 25; system size: 5 components and 20 components) represents a limited experimental region. Using RSM, the experimental region could be expanded and explored in more detail.

Appendix A: Statistical Analysis Output

Preliminary Experiment: JMP Output (Without Blocking Variable)

Screening Fit ABS Error Summary of Fit

RSquare	0.004537
RSquare Adj	-0.11397
Root Mean Square Error	0.30666
Mean of Response	0.378498
Observations (or Sum Wgts)	48

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	5	0.0180009	0.003600	0.0383
Error	42	3.9496998	0.094040	Prob>F
C Total	47	3.9677006		0.9991

Lack of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack of Fit	10	0.0758773	0.007588	0.0627
Pure Error	32	3.8738225	0.121057	Prob>F
Total Error	42	3.9496998		1.0000
Max RSq				0.0237

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	0.3784979	0.044263	8.55	<.0001
A	-0.001931	0.044263	-0.04	0.9654
B	-0.013469	0.044263	-0.30	0.7624
C	0.0096646	0.044263	0.22	0.8282
D	-0.009356	0.044263	-0.21	0.8336
E	0.0029896	0.044263	0.07	0.9465

Effect Test

Source	Nparm	DF	Sum of Squares	F Ratio	Prob>F
A	1	1	0.00017903	0.0019	0.9654
B	1	1	0.00870755	0.0926	0.7624
C	1	1	0.00448340	0.0477	0.8282
D	1	1	0.00420189	0.0447	0.8336
E	1	1	0.00042901	0.0046	0.9465

Error Summary of Fit

RSquare	0.057138
RSquare Adj	-0.05511
Root Mean Square Error	0.427632
Mean of Response	0.237094
Observations (or Sum Wgts)	48

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	5	0.4654480	0.093090	0.5090
Error	42	7.6805195	0.182870	Prob>F
C Total	47	8.1459675		0.7678

Lack of Fit				
Source	DF	Sum of Squares	Mean Square	F Ratio
Lack of Fit	10	0.0345872	0.003459	0.0145
Pure Error	32	7.6459323	0.238935	Prob>F
Total Error	42	7.6805195		1.0000
Max RSq				
0.0614				

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	0.2370937	0.061723	3.84	0.0004
A	0.0841313	0.061723	1.36	0.1801
B	-0.000431	0.061723	-0.01	0.9945
C	0.0507771	0.061723	0.82	0.4154
D	-0.003435	0.061723	-0.06	0.9559
E	0.0053354	0.061723	0.09	0.9315

Effect Test					
Source	Nparm	DF	Sum of Squares	F Ratio	Prob>F
A	1	1	0.33974723	1.8579	0.1801
B	1	1	0.00000893	0.0000	0.9945
C	1	1	0.12375899	0.6768	0.4154
D	1	1	0.00056650	0.0031	0.9559
E	1	1	0.00136640	0.0075	0.9315

SQ Error	
Summary of Fit	
RSquare	0.020457
RSquare Adj	-0.09616
Root Mean Square Error	0.274629
Mean of Response	0.225921
Observations (or Sum Wgts)	48

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	5	0.0661547	0.013231	0.1754
Error	42	3.1676947	0.075421	Prob>F
C Total	47	3.2338494		0.9703

Lack of Fit				
Source	DF	Sum of Squares	Mean Square	F Ratio
Lack of Fit	10	0.0548711	0.005487	0.0564
Pure Error	32	3.1128236	0.097276	Prob>F
Total Error	42	3.1676947		1.0000
Max RSq				
0.0374				

Term	Parameter Estimates			
	Estimate	Std Error	t Ratio	Prob> t
Intercept	0.2259211	0.039639	5.70	<.0001
A	-0.01106	0.039639	-0.28	0.7816
B	-0.018303	0.039639	-0.46	0.6466
C	0.0275692	0.039639	0.70	0.4906
D	-0.011699	0.039639	-0.30	0.7693
E	0.0048954	0.039639	0.12	0.9023

Source	Nparm	Effect Test			
		DF	Sum of Squares	F Ratio	Prob>F
A	1	1	0.00587120	0.0778	0.7816
B	1	1	0.01608043	0.2132	0.6466
C	1	1	0.03648302	0.4837	0.4906
D	1	1	0.00656969	0.0871	0.7693
E	1	1	0.00115031	0.0153	0.9023

Preliminary Experiment: JMP Output (With Blocking Variable)

Screening Fit ABS Error Summary of Fit	
RSquare	0.526943
RSquare Adj	0.444158
Root Mean Square Error	0.216619
Mean of Response	0.378498
Observations (or Sum Wgts)	48

Source	Analysis of Variance			
	DF	Sum of Squares	Mean Square	F Ratio
Model	7	2.0907506	0.298679	6.3652
Error	40	1.8769500	0.046924	Prob>F
C Total	47	3.9677006		<.0001

Term	Parameter Estimates			
	Estimate	Std Error	t Ratio	Prob> t
Intercept	0.3784979	0.031266	12.11	<.0001
A	-0.001931	0.031266	-0.06	0.9511
B	-0.013469	0.031266	-0.43	0.6689
C	0.0096646	0.031266	0.31	0.7588
D	-0.009356	0.031266	-0.30	0.7663
E	0.0029896	0.031266	0.10	0.9243
Block[1-3]	-0.291992	0.044217	-6.60	<.0001
Block[2-3]	0.1172021	0.044217	2.65	0.0115

Effect Test					
Source	Nparm	DF	Sum of Squares	F Ratio	Prob>F
A	1	1	0.0001790	0.0038	0.9511
B	1	1	0.0087075	0.1856	0.6689
C	1	1	0.0044834	0.0955	0.7588
D	1	1	0.0042019	0.0895	0.7663
E	1	1	0.0004290	0.0091	0.9243
Block	2	2	2.0727498	22.0864	<.0001

Error
Summary of Fit

RSquare	0.434239
RSquare Adj	0.335231
Root Mean Square Error	0.339436
Mean of Response	0.237094
Observations (or Sum Wgts)	48

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	7	3.5372968	0.505328	4.3859
Error	40	4.6086707	0.115217	Prob>F
C Total	47	8.1459675		0.0011

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	0.2370937	0.048993	4.84	<.0001
A	0.0841313	0.048993	1.72	0.0937
B	-0.000431	0.048993	-0.01	0.9930
C	0.0507771	0.048993	1.04	0.3062
D	-0.003435	0.048993	-0.07	0.9444
E	0.0053354	0.048993	0.11	0.9138
Block[1-3]	-0.30305	0.069287	-4.37	<.0001
Block[2-3]	-0.013144	0.069287	-0.19	0.8505

Effect Test					
Source	Nparm	DF	Sum of Squares	F Ratio	Prob>F
A	1	1	0.3397472	2.9488	0.0937
B	1	1	0.0000089	0.0001	0.9930
C	1	1	0.1237590	1.0741	0.3062
D	1	1	0.0005665	0.0049	0.9444
E	1	1	0.0013664	0.0119	0.9138
Block	2	2	3.0718488	13.3307	<.0001

SQ Error
Summary of Fit

RSquare	0.373498
RSquare Adj	0.26386
Root Mean Square Error	0.225056
Mean of Response	0.225921
Observations (or Sum Wgts)	48

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	7	1.2078352	0.172548	3.4066
Error	40	2.0260142	0.050650	Prob>F
C Total	47	3.2338494		0.0060

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	0.2259211	0.032484	6.95	<.0001
A	-0.01106	0.032484	-0.34	0.7353
B	-0.018303	0.032484	-0.56	0.5763
C	0.0275692	0.032484	0.85	0.4011
D	-0.011699	0.032484	-0.36	0.7206
E	0.0048954	0.032484	0.15	0.8810
Block[1-3]	-0.215254	0.045939	-4.69	<.0001
Block[2-3]	0.0771839	0.045939	1.68	0.1007

Effect Test					
Source	Nparm	DF	Sum of Squares	F Ratio	Prob>F
A	1	1	0.0058712	0.1159	0.7353
B	1	1	0.0160804	0.3175	0.5763
C	1	1	0.0364830	0.7203	0.4011
D	1	1	0.0065697	0.1297	0.7206
E	1	1	0.0011503	0.0227	0.8810
Block	2	2	1.1416805	11.2702	0.0001

Final Experiment - Full Main Effect Model: JMP Output (Without Blocking Variable)

Screening Fit	
Abs Error	
Summary of Fit	
RSquare	0.333092
RSquare Adj	0.102239
Root Mean Square Error	4.645971
Mean of Response	3.499722
Observations (or Sum Wgts)	36

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	9	280.30078	31.1445	1.4429
Error	26	561.21112	21.5850	Prob>F
C Total	35	841.51190		0.2214

Lack of Fit				
Source	DF	Sum of Squares	Mean Square	F Ratio
Lack of Fit	2	1.51785	0.7589	0.0325
Pure Error	24	559.69327	23.3206	Prob>F
Total Error	26	561.21112		0.9680
Max RSq				
0.3349				

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	3.4997222	0.774328	4.52	0.0001
A	0.4468611	0.774328	0.58	0.5688
B	-0.916772	0.774328	-1.18	0.2471
C	-1.770133	0.774328	-2.29	0.0306
D	-0.021161	0.774328	-0.03	0.9784
E	0.3164833	0.774328	0.41	0.6861
F	-0.269144	0.774328	-0.35	0.7310
G	0.6426	0.774328	0.83	0.4142
H	1.5522389	0.774328	2.00	0.0555
I	-0.785606	0.774328	-1.01	0.3197

Effect Test					
Source	Nparm	DF	Sum of Squares	F Ratio	Prob>F
A	1	1	7.18865	0.3330	0.5688
B	1	1	30.25697	1.4018	0.2471
C	1	1	112.80139	5.2259	0.0306
D	1	1	0.01612	0.0007	0.9784
E	1	1	3.60582	0.1671	0.6861
F	1	1	2.60779	0.1208	0.7310
G	1	1	14.86565	0.6887	0.4142
H	1	1	86.74004	4.0185	0.0555
I	1	1	22.21834	1.0293	0.3197

Error	
Summary of Fit	
RSquare	0.364237
RSquare Adj	0.144165
Root Mean Square Error	5.134356
Mean of Response	2.3826
Observations (or Sum Wgts)	36

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	9	392.6757	43.6306	1.6551
Error	26	685.4020	26.3616	Prob>F
C Total	35	1078.0777		0.1515

Lack of Fit				
Source	DF	Sum of Squares	Mean Square	F Ratio
Lack of Fit	2	7.62238	3.8112	0.1350
Pure Error	24	677.77964	28.2408	Prob>F
Total Error	26	685.40201		0.8744
Max RSq				
0.3713				

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	2.3826	0.855726	2.78	0.0099
A	0.9243389	0.855726	1.08	0.2900
B	-0.987061	0.855726	-1.15	0.2592
C	-1.6128	0.855726	-1.88	0.0707
D	-0.121572	0.855726	-0.14	0.8881
E	-0.203883	0.855726	-0.24	0.8135
F	-0.784844	0.855726	-0.92	0.3675
G	0.8526778	0.855726	1.00	0.3282
H	2.2535167	0.855726	2.63	0.0140
I	0.0108167	0.855726	0.01	0.9900

Effect Test					
Source	Nparm	DF	Sum of Squares	F Ratio	Prob>F
A	1	1	30.75849	1.1668	0.2900
B	1	1	35.07443	1.3305	0.2592
C	1	1	93.64046	3.5522	0.0707
D	1	1	0.53207	0.0202	0.8881
E	1	1	1.49646	0.0568	0.8135
F	1	1	22.17531	0.8412	0.3675
G	1	1	26.17414	0.9929	0.3282
H	1	1	182.82015	6.9351	0.0140
I	1	1	0.00421	0.0002	0.9900

SQ Error	
Summary of Fit	
RSquare	0.295785
RSquare Adj	0.052019
Root Mean Square Error	97.38942
Mean of Response	35.62339
Observations (or Sum Wgts)	36

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	9	103578.18	11508.7	1.2134
Error	26	246602.20	9484.7	Prob>F
C Total	35	350180.38		0.3290

Lack of Fit				
Source	DF	Sum of Squares	Mean Square	F Ratio
Lack of Fit	2	922.26	461.1	0.0450
Pure Error	24	245679.94	10236.7	Prob>F
Total Error	26	246602.20		0.9560
Max RSq				
0.2984				

Term	Parameter Estimates			
	Estimate	Std Error	t Ratio	Prob> t
Intercept	35.623386	16.23157	2.19	0.0373
A	17.173384	16.23157	1.06	0.2998
B	-22.02642	16.23157	-1.36	0.1864
C	-30.17743	16.23157	-1.86	0.0744
D	1.7020264	16.23157	0.10	0.9173
E	-0.217741	16.23157	-0.01	0.9894
F	-2.477217	16.23157	-0.15	0.8799
G	8.4618496	16.23157	0.52	0.6066
H	27.486565	16.23157	1.69	0.1023
I	-18.71392	16.23157	-1.15	0.2594

Source	Nparm	DF	Effect Test		
			Sum of Squares	F Ratio	Prob>F
A	1	1	10617.304	1.1194	0.2998
B	1	1	17465.871	1.8415	0.1864
C	1	1	32784.393	3.4566	0.0744
D	1	1	104.288	0.0110	0.9173
E	1	1	1.707	0.0002	0.9894
F	1	1	220.918	0.0233	0.8799
G	1	1	2577.704	0.2718	0.6066
H	1	1	27198.406	2.8676	0.1023
I	1	1	12607.593	1.3293	0.2594

Final Experiment - Full Main Effect Model: JMP Output (With Blocking Variable)

Screening Fit
Abs Error
Summary of Fit

RSquare	0.369889
RSquare Adj	0.081088
Root Mean Square Error	4.70038
Mean of Response	3.499722
Observations (or Sum Wgts)	36

Source	DF	Analysis of Variance		
		Sum of Squares	Mean Square	F Ratio
Model	11	311.26611	28.2969	1.2808
Error	24	530.24579	22.0936	Prob>F
C Total	35	841.51190		0.2931

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	3.4997222	0.783397	4.47	0.0002
A	0.4468611	0.783397	0.57	0.5737
B	-0.916772	0.783397	-1.17	0.2534
C	-1.770133	0.783397	-2.26	0.0332
D	-0.021161	0.783397	-0.03	0.9787
E	0.3164833	0.783397	0.40	0.6898
F	-0.269144	0.783397	-0.34	0.7342
G	0.6426	0.783397	0.82	0.4201
H	1.5522389	0.783397	1.98	0.0591
I	-0.785606	0.783397	-1.00	0.3260
Block[1-3]	1.2199611	1.10789	1.10	0.2817
Block[2-3]	-1.027106	1.10789	-0.93	0.3631

Effect Test					
Source	Nparm	DF	Sum of Squares	F Ratio	Prob>F
A	1	1	7.18865	0.3254	0.5737
B	1	1	30.25697	1.3695	0.2534
C	1	1	112.80139	5.1056	0.0332
D	1	1	0.01612	0.0007	0.9787
E	1	1	3.60582	0.1632	0.6898
F	1	1	2.60779	0.1180	0.7342
G	1	1	14.86565	0.6728	0.4201
H	1	1	86.74004	3.9260	0.0591
I	1	1	22.21834	1.0056	0.3260
Block	2	2	30.96533	0.7008	0.5061

Error Summary of Fit	
RSquare	0.415505
RSquare Adj	0.147612
Root Mean Square Error	5.124008
Mean of Response	2.3826
Observations (or Sum Wgts)	36

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	11	447.9467	40.7224	1.5510
Error	24	630.1310	26.2555	Prob>F
C Total	35	1078.0777		0.1778

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	2.3826	0.854001	2.79	0.0102
A	0.9243389	0.854001	1.08	0.2898
B	-0.987061	0.854001	-1.16	0.2591
C	-1.6128	0.854001	-1.89	0.0711
D	-0.121572	0.854001	-0.14	0.8880
E	-0.203883	0.854001	-0.24	0.8133
F	-0.784844	0.854001	-0.92	0.3672
G	0.8526778	0.854001	1.00	0.3280
H	2.2535167	0.854001	2.64	0.0144
I	0.0108167	0.854001	0.01	0.9900
Block[1-3]	1.45715	1.20774	1.21	0.2394
Block[2-3]	-1.571483	1.20774	-1.30	0.2056

Effect Test					
Source	Nparm	DF	Sum of Squares	F Ratio	Prob>F
A	1	1	30.75849	1.1715	0.2898
B	1	1	35.07443	1.3359	0.2591
C	1	1	93.64046	3.5665	0.0711
D	1	1	0.53207	0.0203	0.8880
E	1	1	1.49646	0.0570	0.8133
F	1	1	22.17531	0.8446	0.3672
G	1	1	26.17414	0.9969	0.3280
H	1	1	182.82015	6.9631	0.0144
I	1	1	0.00421	0.0002	0.9900
Block	2	2	55.27102	1.0526	0.3646

SQ Error	
Summary of Fit	
RSquare	0.374379
RSquare Adj	0.087636
Root Mean Square Error	95.54236
Mean of Response	35.62339
Observations (or Sum Wgts)	36

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	11	131100.15	11918.2	1.3056
Error	24	219080.23	9128.3	Prob>F
C Total	35	350180.38		0.2802

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	35.623386	15.92373	2.24	0.0348
A	17.173384	15.92373	1.08	0.2915
B	-22.02642	15.92373	-1.38	0.1793
C	-30.17743	15.92373	-1.90	0.0702
D	1.7020264	15.92373	0.11	0.9158
E	-0.217741	15.92373	-0.01	0.9892
F	-2.477217	15.92373	-0.16	0.8777
G	8.4618496	15.92373	0.53	0.6000
H	27.486565	15.92373	1.73	0.0972
I	-18.71392	15.92373	-1.18	0.2514
Block[1-3]	38.210871	22.51955	1.70	0.1027
Block[2-3]	-26.2954	22.51955	-1.17	0.2544

Effect Test					
Source	Nparm	DF	Sum of Squares	F Ratio	Prob>F
A	1	1	10617.304	1.1631	0.2915
B	1	1	17465.871	1.9134	0.1793
C	1	1	32784.393	3.5915	0.0702
D	1	1	104.288	0.0114	0.9158
E	1	1	1.707	0.0002	0.9892
F	1	1	220.918	0.0242	0.8777
G	1	1	2577.704	0.2824	0.6000
H	1	1	27198.406	2.9796	0.0972
I	1	1	12607.593	1.3811	0.2514
Block	2	2	27521.966	1.5075	0.2417

Final Experiment - C, H, C*H Model: JMP Output

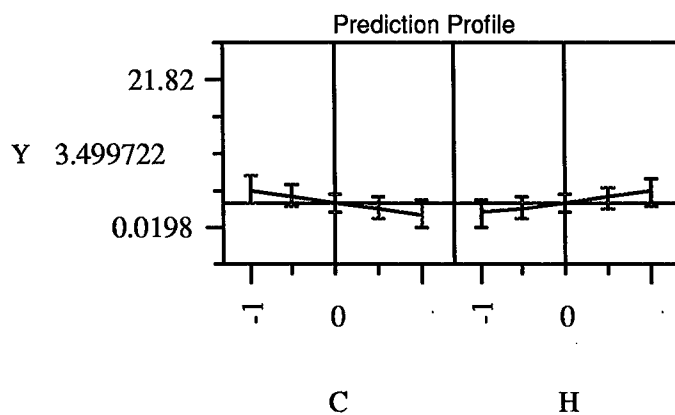
Response Variable: Absolute Error

Screening Fit	
Y	
Summary of Fit	
RSquare	0.296981
RSquare Adj	0.231073
Root Mean Square Error	4.299705
Mean of Response	3.499722
Observations (or Sum Wgts)	36

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	3	249.91310	83.3044	4.5060
Error	32	591.59880	18.4875	Prob>F
C Total	35	841.51190		0.0095

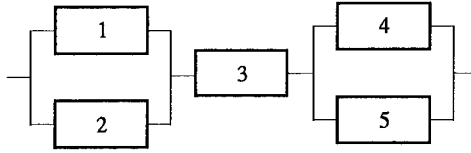
Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	3.4997222	0.716617	4.88	<.0001
C*H	-1.182883	0.716617	-1.65	0.1086
C	-1.770133	0.716617	-2.47	0.0190
H	1.5522389	0.716617	2.17	0.0379

Effect Test					
Source	Nparm	DF	Sum of Squares	F Ratio	Prob>F
C*H	1	1	50.37167	2.7246	0.1086
C	1	1	112.80139	6.1015	0.0190
H	1	1	86.74004	4.6918	0.0379

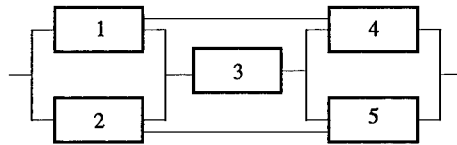


Appendix B: Final Experiment Structures and True Component Distribution Functions

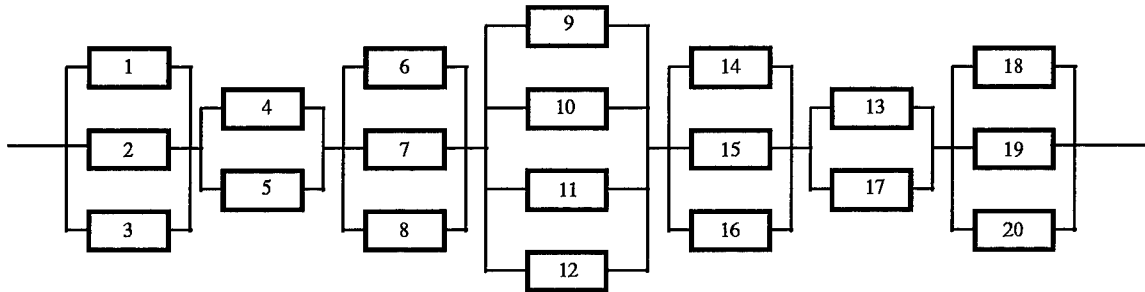
Small / Series-Parallel:



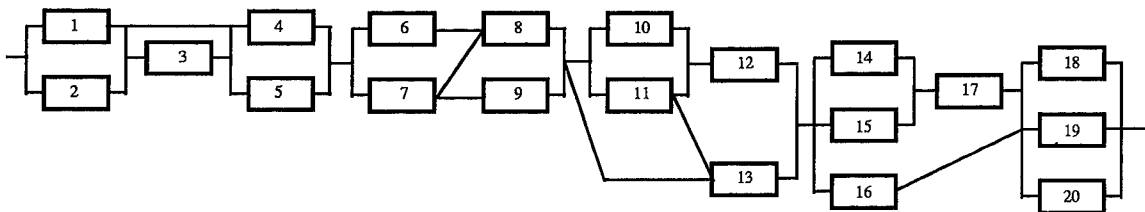
Small / Complex (Bridge Structure):



Large / Series-Parallel:



Large / Complex:



Component True Failure and Repair Distributions (Final Experiment)

Component	IFR Failure Distribution	DFR Failure Distribution	Repair Distribution
1	Weibull: Shape = 1.5 (hrs) Scale = 3000 Location = 0	Weibull: Shape = .50 (hrs) Scale = 1354 Location = 0	Lognormal: (hrs) Mean = 2800 S.D. = 200
2	Weibull: Shape = 4.0 Scale = 2500 Location = 0	Weibull: Shape = .85 Scale = 2082 Location = 0	Lognormal: Mean = 1500 S.D. = 100
3	Weibull: Shape = 2.5 Scale = 4000 Location = 0	Weibull: Shape = .95 Scale = 3468 Location = 0	Lognormal: Mean = 1000 S.D. = 150
4	Weibull: Shape = 1.7 Scale = 1700 Location = 0	Weibull: Shape = .60 Scale = 1008 Location = 0	Lognormal: Mean = 150 S.D. = 25
5	Weibull: Shape = 2.8 Scale = 3500 Location = 0	Weibull: Shape = .40 Scale = 938 Location = 0	Lognormal: Mean = 850 S.D. = 90
6	Weibull: Shape = 1.9 Scale = 3333 Location = 0	Weibull: Shape = .70 Scale = 2336 Location = 0	Lognormal: Mean = 3000 S.D. = 125
7	Weibull: Shape = 1.2 Scale = 2575 Location = 0	Weibull: Shape = .55 Scale = 1423 Location = 0	Lognormal: Mean = 190 S.D. = 20
8	Weibull: Shape = 2.7 Scale = 1500 Location = 0	Weibull: Shape = .78 Scale = 1156 Location = 0	Lognormal: Mean = 1200 S.D. = 75
9	Weibull: Shape = 1.6 Scale = 6000 Location = 0	Weibull: Shape = .91 Scale = 5143 Location = 0	Lognormal: Mean = 1000 S.D. = 30
10	Weibull: Shape = 2.3 Scale = 4700 Location = 0	Weibull: Shape = .46 Scale = 1763 Location = 0	Lognormal: Mean = 2300 S.D. = 133
11	Weibull: Shape = 1.4 Scale = 2700 Location = 0	Weibull: Shape = .82 Scale = 2210 Location = 0	Lognormal: Mean = 500 S.D. = 60
12	Weibull: Shape = 1.9 Scale = 2700 Location = 0	Weibull: Shape = .67 Scale = 1812 Location = 0	Lognormal: Mean = 1000 S.D. = 100
13	Weibull: Shape = 1.3 Scale = 4200 Location = 0	Weibull: Shape = .86 Scale = 3591 Location = 0	Lognormal: Mean = 90 S.D. = 15
14/15/16	Weibull: Shape = 1.5 Scale = 2600 Location = 0	Weibull: Shape = .62 Scale = 1626 Location = 0	Lognormal: Mean = 2200 S.D. = 200
17	Weibull: Shape = 1.1 Scale = 3100 Location = 0	Weibull: Shape = .75 Scale = 2513 Location = 0	Lognormal: Mean = 750 S.D. = 60
18/19/20	Weibull: Shape = 1.6 Scale = 2000 Location = 0	Weibull: Shape = .48 Scale = 829 Location = 0	Lognormal: Mean = 280 S.D. = 50

Appendix C: Fitting Data (Preliminary Experiment)

Components 1 and 2:

Component 1 and 2		Failure Data									
Failure PDF		(Top Weibull++ Selection)									
10 Data Points		High Level Fitting Parameters					Low Level Fitting Parameters				
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3	
3400.223	290.2722	189.0183	Shape:	1.1424	0.8048	1.1105	Lambda	0.0003	0.0003	0.0004	
6623.047	5161.413	2434.877	Scale:	3676.78	2606.92	2763.633	mean	3333.333	3333.333	2500	
1599.312	2421.625	416.6079	Location:	0	161.866	0	Location	0	0	145.6065	
4116.536	4305.172	6656.384									
10858.39	935.8477	2309.267									
459.6741	3462.054	5599.139									
230.2787	1673.137	2719.866									
3307.555	634.8128	2945.3									
1954.587	245.1001	3211.058									
2521.536	11798.13	210.6919									
Failure PDF		(Top Weibull++ Selection)									
50 Data Points		High Level Fitting Parameters					Low Level Fitting Parameters				
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3	
8.3745	4565.173	925.3368	Shape:	1.3037	1.0481	1.2134	Lambda	0.0003	0.0002	0.0004	
695.5815	6988.86	2083.236	Scale:	4018.15	4179.29	2914.042	mean	3333.333	5000	2500	
721.8845	867.6317	5474.835	Location:	0	0	0	Location	8.3745	0	8.5878	
737.5764	2271.222	2352.89									
742.0527	4498.481	1261.039									
744.2721	3508.063	6648.759									
1169.983	14271.66	5350.916									
1170.862	782.0633	6206.759									
1322.077	2763.669	1753.056									
1708.436	3590.396	910.5857									
1742.418	8846.494	778.6821									
1802.004	3753.02	5657.847									
1850.785	1655.012	1970.843									
2004.512	3384.537	2468.072									
2014.792	11180.08	1525.289									
2016.208	8001.486	2610.481									
2052.169	4769.927	298.2684									
2182.257	737.3073	2800.817									
2189.692	1611.978	2004.737									
2252.016	4.9562	5362.177									
2295.502	571.789	5859.77									
2342.247	2022.819	3807.239									
2437.57	4195.4	245.7073									
2805.726	5324.474	596.207									
2923.359	1835.266	1682.965									
3095.633	9747.239	4229.763									
3210.884	1952.818	1186.952									
3330.358	117.2965	4641.868									
3425.986	8534.688	242.8431									
3465.598	7398.04	1407.097									
3474.296	542.9199	4102.534									
3505.905	2608.717	8.5878									
3766.494	5840.204	6416.082									
3830.914	1590.474	1962.96									
3975.112	3452.596	332.806									
3977.555	4376	274.9632									
4158.176	1302.869	1755.551									
4496.154	12241.56	202.5712									
4727.568	8041.356	7054.521									
5381.795	2394.354	4707.271									
5993.572	1635.696	747.6762									
6399.864	14488.04	1450.601									
6705.746	2623.179	1651.895									
6793.979	193.0907	4390.494									
7673.189	6415.875	1637.359									
7974.926	823.0479	4152.95									
8068.264	274.0156	3508.264									
9717.07	1407.548	5992.359									
12809.35	3493.224	2000.326									
13761.84	1838.203	2782.958									

TRUE PARAMETERS
Weibull
Shape: 1.1
Scale: 3500
Location: 0

Component 1 and 2			Repair Data									True Lognormal Mean:			40
Repair PDF						(Top Weibull++ Selection)			(Empirical)			True Lognormal St Dev:			10
10 Data Points						High Level Fitting Parameters			Low Level Fitting Parameters			True Lognormal Variance:			100
Set1	Set2	Set3				Rep1	Rep2	Rep3	Rep1	Rep2	Rep3				
33.9979	28.4262	27.4382	N Mean			3.7728	3.7099					Mean for Normal variates:			3.658567143
39.4116	33.7581	28.9223	N S.D.			0.147	0.2001				(Empirical)	Var for Normal variates:			0.060624622
39.6336	35.2395	30.0245	LogN Mean			43.97426	41.67577	1				St Dev for Normal Variates:			0.246220677
40.3567	37.5516	31.9426	LogN S.D.			6.499295	8.4235	0							
43.0280	37.6499	33.5554													
43.6418	43.1749	34.1744	Weibull Shape					0.9565							
45.1152	43.2111	35.3690	Scale					11.9569							
46.1252	51.8209	49.1101	Location					26.9208							
47.0883	51.9354	56.1393													
61.5145	53.8807	64.5566													
Repair PDF						(Top Weibull++ Selection)			(Empirical)						
50 Data Points						High Level Fitting Parameters			Low Level Fitting Parameters						
Set1	Set2	Set3				Rep1	Rep2	Rep3	Rep1	Rep2	Rep3				
24.1991	25.4645	18.0659	N Mean			3.6407		3.6131							
24.6871	25.9885	23.8515	N S.D.			0.2218		0.2944			(Empirical)				
27.6193	26.4718	24.0047	LogN Mean			39.06776	1	38.72308							
28.0627	26.7059	24.3652	LogN S.D.			8.772902	0	11.65161							
28.9823	26.8455	26.3355													
29.1609	27.5487	26.6495	Weibull Shape					3.0977							
29.7943	28.2030	26.7887	Scale					24.5055							
30.1743	29.6874	27.0128	Location					17.1612							
30.3590	31.8349	27.1011													
30.9141	32.9141	27.8885													
31.3476	33.7304	29.5760													
31.6416	33.9327	29.8710													
32.0434	34.5756	30.3222													
33.6861	34.6541	31.8326													
34.1794	35.1944	32.2594													
34.2628	35.2887	33.4084													
34.2756	35.6300	33.6012													
34.4363	36.0861	33.6211													
36.0875	36.5806	33.7551													
36.1988	37.3148	33.8202													
36.4695	37.3771	34.6326													
37.8517	37.4830	35.3132													
38.0252	38.1706	35.6575													
38.2895	38.2115	35.7772													
38.3758	38.5760	36.5698													
38.4396	39.0699	36.9051													
38.4502	39.4282	36.9805													
38.6779	39.5461	37.9663													
38.7602	39.9599	38.7156													
38.7681	40.3300	38.5875													
39.5696	40.9358	39.8971													
40.1211	41.1033	40.2322													
40.6761	41.2334	40.3086													
40.8159	41.5192	40.3624													
41.1818	41.5250	41.2963													
41.4396	42.2468	42.5026													
42.5334	43.1045	43.4949													
42.8620	44.4447	44.0890													
43.0596	44.4763	44.5110													
44.3323	45.9010	45.5003													
44.5534	46.8976	46.4135													
47.2101	47.3260	47.3535													
47.3783	47.6214	48.9973													
48.6732	48.4473	50.0807													
49.1984	48.4638	51.5531													
49.4929	48.7800	54.8195													
57.2485	48.9502	55.6666													
63.1859	52.6280	55.8298													
63.3980	57.2091	61.3957													
63.4086	57.3055	104.6072													

Components 3 and 4:

Component 3 and 4			Failure Data									
Failure PDF												
10 Data Points						(Top Weibull++ Selection)				(Weibull++ Exponential)		
Set1	Set2	Set3				High Level Fitting Parameters				Low Level Fitting Parameters		
						Rep1	Rep2	Rep3		Rep1	Rep2	Rep3
1419.354	2052.765	2207.199	Shape			1.842	1.701			0.0008	0.0008	0.0006
963.0771	1421.759	1785.776	Scale			1808.529	2257.799			1250	1250	1666.667
14.1518	1909.412	2657.587	Location			317.4287	43.7521			14.1518	664.5628	459.4963
544.4623	3221.591	5230.137										
382.2532	1794.981	1682.83	Normal									
2046.543	3727.739	1131.662	Mean		1284.115							
1318.988	2250.122	2304.446	S.D.		771.0283							
2070.357	1122.486	2220.419										
1539.051	664.5628	818.5274										
2542.909	1025.499	459.4963										
Failure PDF						(Top Weibull++ Selection)				(Weibull++ Exponential)		
50 Data Points						High Level Fitting Parameters				Low Level Fitting Parameters		
Set1	Set2	Set3				Rep1	Rep2	Rep3		Rep1	Rep2	Rep3
136.4218	2006.373	1696.307	Shape			1.2122	2.0219	1.6765		0.0007	0.0005	0.0005
171.3912	948.7334	1365.369	Scale			1663.22	2204.209	2125.105		1428.571	2200	2000
224.7112	2500.185	2687.736	Location			99.6879	0	0		136.4218	0	0
277.1905	1696.223	1805.908										
345.4204	809.518	2860.507										
350.7182	4187.265	2341.057										
389.1629	1980.925	2884.308										
479.4368	1541.282	240.2723										
500.3845	3635.917	4878.578										
527.7269	1674.282	2950.525										
533.4867	722.2443	4238.523										
545.9969	385.2486	1467.667										
586.504	1605.537	1540.711										
634.7171	3350.027	649.1847										
641.1439	4166.519	1783.356										
814.9092	2043.477	80.483										
817.0835	1839.547	4346.261										
817.7518	1992.607	2434.073										
1013.223	364.0332	701.3264										
1023.585	1415.027	1327.958										
1041.254	3404.917	695.0499										
1119.601	2785.675	4072.324										
1162.011	1613.91	641.4696										
1323.632	1456.764	3960.691										
1356.765	403.8959	1580.863										
1452.204	2513.195	1864.181										
1517.397	1049.062	2072.33										
1610.408	1199.236	1395.664										
1650.178	2189.122	1179.098										
1747.276	894.8619	616.6241										
1783.921	712.8388	2660.072										
1787.338	2992.213	2868.949										
1910.095	851.2721	589.046										
2061.335	3859.053	994.874										
2073.654	1165.431	595.9472										
2149.89	195.2311	1987.552										
2202.226	3017.145	890.1436										
2661.079	3160.684	1949.515										
2663.375	1935.184	1520.993										
2824.277	1190.268	4662.404										
2938.453	2587.891	1383.616										
3044.948	1584.65	2257										
3108.324	2100.979	1979.941										
3111.957	1745.846	2055.053										
3218.49	1494.447	1163.525										
3297.171	2859.441	1122.916										
3818.062	2195.199	2345.829										
3881.404	3036.767	1677.936										
4296.429	2630.787	1388.224										
5500.918	2086.617	465.4539										

TRUE PARAMETERS

Weibull	
Shape	1.5
Scale	2000
Location	0

Component 3 and 4			Repair Data									True Lognormal Mean:	70
Repair PDF						(Top Weibull++ Selection)		(Empirical)				True Lognormal St Dev:	15
10 Data Points						High Level Fitting Parameters		Low Level Fitting Parameters				True Lognormal Variance:	225
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3			
50.5429	50.1709	58.4619	N Mean		4.2835							Mean for Normal variates:	4.226047582
54.5919	59.9039	63.5271	N S.D.		0.1966			(Empirical)				Var for Normal variates:	0.04489532
54.8351	64.0371	69.9191	LogN Mean	1	73.90835	1						St Dev for Normal Variates:	0.211885157
61.5746	67.9009	70.1893	LogN S.D.	0	14.67192	0							
62.2426	70.2945	72.1974											
63.4417	70.9324	83.9836	Weibull Shape	10.7264		2.0011							
63.6161	80.1769	85.9047	Scale	65.159		31.3301							
68.8356	82.1574	88.4354	Location	0		51.6926							
69.2073	93.6345	91.2170											
72.6966	99.7863	110.0655											
Repair PDF						(Top Weibull++ Selection)		(Empirical)					
50 Data Points						High Level Fitting Parameters		Low Level Fitting Parameters					
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3			
34.1787	38.7913	41.6494	N Mean	4.2304	4.2268	4.2206							
48.3627	46.0611	43.2507	N S.D.	0.2308	0.1961	0.266		(Empirical)					
49.9477	53.5753	43.4841	LogN Mean	70.60029	69.82748	70.52576							
51.5149	53.7212	44.2777	LogN S.D.	16.51397	13.82587	19.09664							
54.3864	55.0942	47.1829											
54.4412	57.0988	49.7388											
54.5729	57.5047	49.7418											
56.2162	57.6721	51.2158											
56.7004	58.4367	52.5310											
56.9730	59.2594	54.6539											
57.3835	59.9447	54.7453											
58.8944	61.2913	56.0434											
59.0369	61.3342	57.1667											
61.1619	61.8150	60.0750											
61.4112	62.3685	60.3486											
61.8558	62.5415	61.1234											
62.0726	62.9040	61.8934											
62.0887	62.9070	61.9786											
63.3913	63.0202	62.6219											
63.4997	63.2976	63.4711											
63.7484	63.3424	64.3084											
64.2637	63.3544	64.3633											
64.9753	66.6072	65.3554											
65.8449	67.0459	65.5944											
67.5707	67.1534	67.4849											
68.3410	67.3088	67.9115											
68.5082	69.4443	68.4511											
69.2928	69.4803	68.7440											
69.5706	69.8782	69.7515											
70.0106	69.9384	70.2290											
70.2269	70.0798	70.4140											
70.7711	70.3432	71.1318											
71.6171	72.0448	71.4524											
75.8268	72.7633	73.2540											
75.8544	74.5976	73.2624											
76.7763	77.2004	74.5592											
76.8853	77.9867	76.8966											
80.2664	81.3173	77.4953											
81.9055	81.4892	78.2504											
83.1295	81.5827	78.5668											
84.8814	81.6052	82.8026											
85.2087	82.0497	87.8336											
85.4425	82.3824	88.8182											
88.9759	82.7042	89.0255											
91.0799	83.2923	102.6272											
91.6729	83.6743	108.3565											
94.6421	84.1560	114.8737											
105.4016	91.3150	115.7703											
108.3813	100.5538	122.6361											
132.2814	129.3339	122.9925											

Component 5:

Component 5		Failure Data									
Failure PDF				(Top Weibull++ Selection)				(Weibull++ Exponential)			
10 Data Points				High Level Fitting Parameters				Low Level Fitting Parameters			
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3	
384.4927	641.1133	244.2498	Shape	1.8723	1.1411	1.8549	Lambda	0.0007	0.0006	0.0009	
545.0513	836.1144	473.7554	Scale	2014.397	1492.544	1545.62	Mean	1428.571	1666.667	1111.111	
1014.724	1063.66	774.6268	Location	0	530.6213	0	Location	384.4927	440.0948	244.2498	
1467.796	1225.419	1200.055									
1528.46	1386.741	1259.333									
1796.037	1677.181	1431.017									
1805.159	1710.791	1463.062									
2685.534	2509.8	1628.846									
3171.514	4063.236	2380.459									
3470.297	4405.597	2878.333									
Failure PDF				(Top Weibull++ Selection)				(Weibull++ Exponential)			
50 Data Points				High Level Fitting Parameters				Low Level Fitting Parameters			
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3	
478.862	139.3333	244.5103	Shape	2.2201	1.7751	1.8758	Lambda	0.0007	0.0007	0.0007	
559.5032	216.6253	317.6606	Scale	2154.565	1712.647	1853.406	Mean	1428.571	1428.571	1428.571	
606.1648	481.2112	378.0646	Location	0	0	0	Location	478.862	139.3333	244.5103	
666.5704	481.4101	444.4303									
747.9887	575.2048	563.182									
874.4738	617.7887	606.8696									
898.0651	649.797	690.6264									
942.651	674.0816	697.5089					TRUE PARAMETERS				
956.5013	698.4854	728.1186					Weibull				
982.7919	713.8397	728.617					Shape	2			
1002.692	713.9761	739.3244					Scale	2000			
1229.721	723.6584	750.4836					Location	0			
1271.843	779.9904	944.7299									
1296.71	809.8509	985.4585									
1318.26	1014.29	994.7749									
1395.802	1023.576	1040.816									
1407.178	1053.333	1086.567									
1424.359	1101.47	1120.212									
1452.061	1115.344	1200.526									
1525.454	1172.539	1214.506									
1551.733	1205.425	1254.517									
1567.216	1285.991	1271.458									
1606.131	1297.485	1285.855									
1858.728	1374.334	1346.401									
1909.199	1406.672	1455.681									
1927.392	1414.709	1519.508									
1929.022	1518.297	1571.408									
1952.617	1541.507	1587.083									
1967.877	1592.483	1679.344									
1994.657	1647.969	1773.561									
2029.51	1663.765	1814.845									
2038.27	1691.25	2049.751									
2122.888	1723.812	2058.304									
2235.849	1737.343	2083.694									
2375.611	1740.056	2154.53									
2421.885	1749.907	2197.575									
2436.823	1755.844	2267.668									
2460.293	1764.924	2268.36									
2526.509	1910.202	2393.67									
2549.512	2171.504	2416.656									
2637.84	2201.866	2493.545									
2680.403	2210.847	2504.284									
2842.487	2315.208	2645.74									
2864.291	2401.005	2753.988									
2932.771	2402.354	2827.33									
3146.245	2633.574	2969.749									
3370.188	2672.251	3015.439									
3651.116	3801.817	3045.548									
3730.11	3867.55	3479.832									
4825.686	4566.505	4417.831									

Component 5			Repair Data									True Lognormal Mean:			60		
Repair PDF												True Lognormal St Dev:			8		
10 Data Points						(Top Weibull++ Selection)			(Empirical)			Desired Lognormal Variance:			64		
Set1			Set2			Set3			High Level Fitting Parameters			Low Level Fitting Parameters					
									Rep1			Rep2			Rep3		
42.3867			47.6347			51.4372			N Mean								
43.2308			48.1388			52.2682			N S.D.								
48.2416			52.5022			52.7358			LogN Mean			1			1		
51.4218			56.1148			54.5995			LogN S.D.			0			0		
53.5588			57.9414			55.6594											
53.7508			60.2239			56.2238			Weibull Shape			1.5821			2.9888		
61.3120			65.4354			56.9997			Scale			20.0399			24.3756		
62.2097			66.2377			59.3828			Location			38.8838			37.8505		
70.7762			68.6597			59.7857											
81.3838			72.4364			63.5537											
Repair PDF									(Top Weibull++ Selection)			(Empirical)					
50 Data Points									High Level Fitting Parameters			Low Level Fitting Parameters					
Set1			Set2			Set3			Rep1			Rep2			Rep3		
43.3633			40.7621			43.0694			N Mean			4.1117			4.0748		
44.4692			48.5947			46.7846			N S.D.			0.1551			0.1146		
45.4751			49.4850			50.4923			LogN Mean			1			61.78916		
48.6513			50.2172			51.6487			LogN S.D.			0			9.641424		
50.0403			51.6521			51.9374											
50.2316			52.1425			52.1729			Weibull Shape			2.7445					
51.7557			52.2455			52.3547			Scale			25.1628					
52.2082			52.5248			52.4985			Location			38.2597					
52.3060			53.1706			52.5289											
52.5053			53.1965			52.8929											
52.9333			55.0626			52.9659											
53.7309			55.3737			53.8248											
53.9276			55.7426			53.8827											
54.3129			55.8370			53.9105											
54.9343			56.1746			55.2268											
55.3858			56.2229			55.2281											
56.2124			56.3467			55.9278											
56.7725			56.5094			56.1931											
56.7782			56.5145			56.6390											
57.6110			56.6211			56.6643											
57.7294			58.0804			57.5460											
57.7883			58.1192			57.9742											
58.9255			58.6514			58.1499											
59.2438			59.3046			58.2710											
59.5306			59.5374			58.5631											
60.7142			59.7304			58.6642											
61.3204			60.5192			59.2110											
61.3279			60.5982			59.6570											
61.9357			60.7126			59.6694											
62.3702			61.4066			60.5763											
63.0976			62.7784			60.9542											
64.0290			63.1173			61.0330											
64.2367			63.4444			61.6104											
64.3788			65.2501			61.6800											
64.4379			65.6371			61.9636											
65.6116			66.0719			62.1527											
65.9567			66.3744			62.9827											
66.8411			66.3844			63.0855											
67.9192			67.3746			63.3517											
68.0796			67.6069			63.9280											
68.8105			67.6528			64.0222											
68.8541			68.6839			64.8892											
70.5540			68.7946			64.9155											
70.7772			75.6322			66.8994											
70.9301			78.0914			68.7119											
71.4910			79.7626			68.8166											
71.5870			80.2364			70.8727											
75.8337			81.8113			73.5066											
80.9494			87.1557			75.3719											
83.3002			87.2371			75.4592											

Appendix D: Fitting Data (Final Experiment)

Component 1:

Component 1: IFR Failure												TRUE IFR PARAMETERS		
Failure PDF				(Top Weibull++ Selection)			(Weibull++ Exponential)			Weibull			Shape	1.5
5 Data Points				High Level Fitting Parameters			Low Level Fitting Parameters			Scale			Location	3000
Set1	Set2	Set3		Rep1	Rep2	Rep3	Rep1	Rep2	Rep3					
181.1979	2223.7486	287.5772	Shape		0.6094	1.1231		0.0004	0.0008	0.0004				
982.1146	2379.3399	937.6879	Scale		383.7708	2363.258	Lambda	2500	1250	2500				
2226.3576	2383.1759	1497.2583	Location		2218.289	18.7925	mean	0	1547.451	0				
2388.7594	2472.4279	2778.1681					Location							
5399.2774	4496.5138	5903.7465	Exp. Lambda	0.0004										
			mean	2500										
			Location	0										
Failure PDF				(Top Weibull++ Selection)			(Weibull++ Exponential)							
25 Data Points				High Level Fitting Parameters			Low Level Fitting Parameters							
Set1	Set2	Set3		Rep1	Rep2	Rep3	Rep1	Rep2	Rep3					
68.4728	423.2879	319.593	Shape	1.6539	1.6359	1.4402	Lambda	0.0005	0.0003	0.0004				
331.6175	852.5509	651.7494	Scale	2430.633	3188.455	2918.504	mean	2000	3333.333	2500				
416.3119	872.1424	712.6503	Location	0	76.2918	126.3392	Location	68.4728	0	319.593				
765.5116	885.9164	756.8507												
985.3227	1339.2671	945.8708												
1091.8591	1384.6532	956.1866												
1209.8468	1873.6413	1104.1897												
1442.4947	2102.4384	1498.2734												
1595.0677	2249.4598	1754.579												
1651.5536	2297.4357	1915.9636												
1712.5829	2419.0937	1961.8824												
1829.5611	2440.497	2283.4476												
1970.983	2480.1319	2561.7131												
2395.9298	2690.5506	2579.4042												
2535.9024	2764.3675	2604.1896												
2590.8444	2930.4622	3034.4819												
2856.3818	2971.7748	3627.4104												
2939.8024	2976.9494	3725.7776												
2977.372	3598.5336	4224.4737												
3046.7539	3959.8607	4353.1658												
3702.9801	4424.5982	4731.5963												
3778.5931	4635.2492	4792.5074												
3787.6137	5039.1509	4808.6287												
3996.1117	7344.0245	5785.7093												
5170.1898	8029.4939	7695.7031												

Component 1: DFR Failure												TRUE DFR PARAMETERS		
Failure PDF				(Top Weibull++ Selection)			(Weibull++ Exponential)			Weibull			Shape	0.5
5 Data Points				High Level Fitting Parameters			Low Level Fitting Parameters			Scale			Location	1354
Set1	Set2	Set3		Rep1	Rep2	Rep3	Rep1	Rep2	Rep3					
139.4629	0.0637	0.6333	Shape	0.5196		0.2708	Lambda	0.0002	0.0004	0.0004				
469.7426	1038.2233	3.8248	Scale	2709.083		509.9078	mean	5000	2500	2500				
836.6793	1703.1888	425.0307	Location	120.3868		0.6248	Location	0	0	0				
7502.4567	2524.7519	3214.1432												
14660.2643	6112.2116	7872.6754	Exp. Lambda		0.0004									
			mean		2500									
			Location		0									
Failure PDF				(Top Weibull++ Selection)			(Weibull++ Exponential)							
25 Data Points				High Level Fitting Parameters			Low Level Fitting Parameters							
Set1	Set2	Set3		Rep1	Rep2	Rep3	Rep1	Rep2	Rep3					
8.5763	0.001	0.23	Shape	0.5068	0.4887	0.4491	Lambda	0.0003	0.0004	0.0006				
12.033	0.1838	3.3597	Scale	1517.284	1589.497	675.7722	mean	3333.333	2500	1666.667				
35.734	6.1297	8.0134	Location	7.7972	0	0.0953	Location	0	0	0				
38.9043	13.0001	9.1801												
43.0139	80.2815	14.0094												
73.9562	156.3244	22.8833												
117.0724	199.4423	38.816												
233.2414	285.532	65.5375												
298.2957	326.8303	90.886												
359.7071	765.5564	129.6294												
543.8688	865.7244	169.409												
545.6467	1046.201	199.2903												
920.3555	1061.9274	283.987												
973.9422	1665.1605	346.233												
1081.318	2491.2969	379.0188												
1525.3416	2855.2928	466.5133												
1646.0548	2916.4294	575.8503												
2183.3176	2917.5233	675.9627												
2858.4604	3149.028	1191.9816												
3234.4206	3167.2538	1654.3719												
4179.5228	5480.5496	3587.6506												
4360.3714	5517.3566	4577.0942												
10713.9065	6790.4468	5097.7195												
14261.0287	9955.8328	7507.7753												
22240.9338	12086.6422	14256.0549												

Component 3:

IFR Failure												TRUE IFR PARAMS	
Failure PDF			(Top Weibull++ Selection)						(Weibull++ Exponential)			Weibull	
5 Data Points			High Level Fitting Parameters						Low Level Fitting Parameters			Shape	2.5
Set1	Set2	Set3		Rep1	Rep2	Rep3			Rep1	Rep2	Rep3	Scale	4000
2883.5319	2998.5673	2434.8346	Shape	1.0003		3.4133	Lambda		0.0009	0.0011	0.0006	Location	0
3060.1663	3038.2884	2947.7713	Scale	925.7452		4586.741	mean		1111.111	909.0909	1666.667		
3514.6735	3636.2092	4363.5149	Location	2811.544		0	Location		2634.388	2788.847	2313.926		
3998.5685	3832.8183	4568.3683											
5228.9477	4806.3682	6214.0369	Exp. Lambda		0.0011								
			mean		909.0909								
			Location		2788.847								
Failure PDF			(Top Weibull++ Selection)						(Weibull++ Exponential)				
25 Data Points			High Level Fitting Parameters						Low Level Fitting Parameters				
Set1	Set2	Set3		Rep1	Rep2	Rep3			Rep1	Rep2	Rep3		
1711.4399	1278.0886	1557.278	Shape	1.2574	2.9605	2.8125	Lambda		0.0006	0.0004	0.0004		
1794.8646	1808.9081	1611.9093	Scale	1927.631	4258.217	4263.149	mean		1666.667	2500	2500		
2052.9023	2465.6908	1982.5562	Location	1617.411	166.2515	0	Location		1711.44	1278.089	1557.278		
2058.9274	2494.8888	2069.6192											
2073.6771	2546.2591	2151.6954											
2098.9349	2767.6346	2251.1667											
2404.9142	3152.5575	2314.9334											
2624.2049	3162.102	2443.4072											
2772.2025	3172.1098	2877.3355											
2833.8453	3592.8889	3028.6866											
2915.2481	3693.995	3596.0093											
2985.502	3757.219	3917.9257											
2987.2932	3801.8114	4019.9073											
3109.4728	3948.7351	4087.4007											
3131.6845	4141.2514	4146.7606											
3198.8659	4206.1661	4192.7926											
3283.6123	4225.2252	4734.3948											
3796.4672	4233.9697	4745.9309											
3801.7873	5123.1271	4787.0335											
4162.7148	5166.0151	4815.4448											
4228.3248	5574.046	4872.8703											
5341.9776	5658.4477	5028.4013											
6312.7596	5746.6372	6369.2437											
6389.2901	6280.8288	6387.1472											
7126.3486	7081.6374	6617.8441											

DFR Failure												TRUE DFR PARAMS	
Failure PDF			(Top Weibull++ Selection)						(Weibull++ Exponential)			Weibull	
5 Data Points			High Level Fitting Parameters						Low Level Fitting Parameters			Shape	0.95
Set1	Set2	Set3		Rep1	Rep2	Rep3			Rep1	Rep2	Rep3	Scale	3468
868.5596	698.6931	169.6434	Shape	2.3023	0.6745	0.8935	Lambda		0.0007	0.0003	0.0004	Location	0
1516.3796	1366.5098	401.407	Scale	2553.192	2135.705	2627.934	mean		1428.571	3333.333	2500		
2218.6249	1700.3733	2722.0258	Location	0	658.6182	0	Location		852.0483	0	0		
2697.295	2792.1665	3234.4815											
3966.098	10702.2462	7299.204											
Failure PDF			(Top Weibull++ Selection)						(Weibull++ Exponential)				
25 Data Points			High Level Fitting Parameters						Low Level Fitting Parameters				
Set1	Set2	Set3		Rep1	Rep2	Rep3			Rep1	Rep2	Rep3		
207.8036	42.6759	26.6541	Shape	0.9439	0.7877	0.8449	Lambda		0.0003	0.0003	0.0003		
266.2007	110.6362	73.5251	Scale	3714.093	2702.41	3310.37	mean		3333.333	3333.333	3333.333		
564.8699	125.2635	202.0481	Location	126.86	21.438	0	Location		0	0	0		
571.728	367.1112	481.4955											
575.5735	412.2275	578.7774											
956.9291	417.0253	911.8075											
1372.7997	514.4869	958.3334											
1458.9632	646.2646	1024.3937											
1807.3052	905.7708	1215.1514											
2216.5446	937.2157	1340.3025											
2250.8431	1167.433	1446.7933											
2279.4159	1336.8557	1703.5091											
2631.9957	1660.6554	1726.7708											
3200.9254	1813.3288	2022.7823											
3607.7149	2070.6515	2047.3684											
3808.4165	2174.3106	2517.398											
4135.8181	2720.8299	2566.4763											
4219.2527	3012.5395	5004.063											
4349.9903	4941.1598	5040.78											
4892.6842	5374.6816	6443.4536											
5591.2831	6328.194	6489.4017											
6274.9309	8346.4754	10469.55											
9276.5345	8829.5878	10626.242											
15164.2679	10020.3126	11879.798											
16880.6085	13523.2832	13441.007											

DFR Failure										TRUE DFR PARAMETERS			
Failure PDF				(Top Weibull++ Selection)				(Weibull++ Exponential)				Weibull	
5 Data Points				High Level Fitting Parameters				Low Level Fitting Parameters				Shape	0.6
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3		Scale	1008
11.0497	480.894	70.1082	Shape	0.4993		0.9992	Lambda	0.0002	0.0017	0.0026	Location		0
242.6425	559.0181	150.3176	Scale	2642.701		337.1318	mean	5000	588.2353	384.6154			
1947.525	728.0175	232.7354	Location	0		43.9176	Location	0	349.5782	0			
3472.224	1285.811	569.4263											
18283.15	1555.789	883.2029	p. Lambda		0.0017								
			mean		588.2353								
			Location		349.5782								
Failure PDF				(Top Weibull++ Selection)				(Weibull++ Exponential)					
25 Data Points				High Level Fitting Parameters				Low Level Fitting Parameters					
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3			
5.5541	16.9831	1.5864	Shape	0.6606	0.4855	0.6164	Lambda	0.0008	0.0009	0.0005			
19.0226	22.131	5.3811	Scale	1006.345	540.1314	1474.721	mean	1250	1111.111	2000			
54.6918	22.4463	11.718	Location	3.3075	16.7859	0	Location	0	0	0			
60.0507	23.8545	44.7068											
61.9173	27.0709	111.0731											
68.6155	29.8473	117.8029											
82.5532	37.277	184.8074											
114.3219	90.4036	400.416											
131.4785	106.9176	490.2925											
216.6277	149.1267	635.9928											
439.2135	155.9802	727.1333											
448.182	312.0662	746.0528											
851.5921	359.9542	902.3072											
859.8882	375.4828	990.0742											
1061.002	622.0638	1258.296											
1417.39	633.5568	1342.977											
1651.562	661.405	1478.737											
1744.063	754.3752	1785.762											
1967.511	758.3816	2833.623											
2031.855	1264.043	3223.336											
2600.598	1366.809	4326.368											
3238.654	1629.048	4453.8											
3680.447	3007.185	7217.69											
3779.719	5056.075	8415.314											
6110.383	11335.71	9351.056											

Repair										True Lognormal Mean: 150			
Repair PDF				(Top Weibull++ Selection)				(Empirical)				True Lognormal St Dev: 25	
5 Data Points				High Level Fitting Parameters				Low Level Fitting Parameters				True Lognormal Variance: 625	
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3			
107.9361	112.7261	110.8108	N Mean	4.9334									
126.2814	124.8062	148.5015	N S.D.	0.16									
141.0837	136.2086	156.1516	logN Mean	140.6395	1	1						Mean for Normal variates: 4.996936	
161.0093	141.4520	158.4165	LogN S.D.	22.64711	0	0						Var for Normal variates: 0.027399	
166.6536	158.3264	179.0876										St Dev for Normal Variates: 0.165526	
			Weibull Shape		3.7243	8.6173							
			Scale		56.8317	159.7675							
			Location		83.5173	0							
Repair PDF				(Top Weibull++ Selection)				(Empirical)					
25 Data Points				High Level Fitting Parameters				Low Level Fitting Parameters					
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3			
112.1810	109.6944	123.0544	N Mean										
115.7299	113.2002	128.6623	N S.D.										
116.9827	119.7126	130.3761	logN Mean	1	1	1							
122.1091	120.6516	132.2960	LogN S.D.	0	0	0							
123.3441	120.9405	137.8381											
135.6326	121.1742	139.3057	ull Shape	3.6758	6.7394	1.8917							
136.4452	134.9671	141.9222	Scale	82.7082	160.6862	43.151							
136.6714	137.0815	143.9501	Location	76.3831	0	116.0788							
137.6559	137.4096	145.6638											
143.8245	139.9656	146.0071											
147.6691	142.7575	146.1355											
151.1929	151.9322	146.617											
151.7625	153.15	147.6246											
157.357	155.0988	150.5233											
158.1261	159.2667	151.3792											
158.4126	159.6028	154.678											
158.8591	162.5519	158.9568											
159.9018	165.5747	160.7683											
170.0568	166.6327	164.3028											
171.5999	167.1319	165.501											
172.6964	169.7564	167.8884											
173.419	175.133	173.1062											
175.2563	176.0617	180.6055											
183.8417	198.3452	200.29											
201.9854	198.8682	218.1923											

Component 5:

IFR Failure										TRUE IFR PARAM			
Failure PDF				(Top Weibull++ Selection)				(Weibull++ Exponential)				Weibull	
5 Data Points				High Level Fitting Parameters				Low Level Fitting Parameters				Shape	2.8
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3		Scale	3500
1519.116	2113.463	1351.0364	Shape		3.3664	2.9259		Lambda	0.0006	0.001	0.0005	Location	0
1766.454	2682.808	2244.2143	Scale		2476.582	3631.724		mean	1666.667	1000	2000		
2884.758	3010.518	3833.9542	Location		919.46	0		Location	1262.577	2113.463	1351.036		
4180.014	3646.33	3848.6219											
4389.93	4226.435	4858.5665	Exp. Lambda	0.0006	Normal								
			mean	1666.667	s.d.								
			Location	1262.577									
Failure PDF				(Top Weibull++ Selection)				(Weibull++ Exponential)					
25 Data Points				High Level Fitting Parameters				Low Level Fitting Parameters					
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3			
900.3416	1123.459	1418.0763	Shape		2.4586	3.8567		Lambda	0.0005	0.0005	0.0005		
1428.436	1170.309	1520.0759	Scale		2852.708	3717.19		mean	2000	2000	2000		
1455.073	1770.679	2050.2135	Location		452.2922	0		Location	900.3416	1123.459	1418.076		
1484.823	1829.859	2147.0789											
1710.945	1835.78	2211.8287	Normal										
2134.966	1859.992	2589.9989	Mean	2954.431									
2138.175	2241.777	2819.1221	SD	1083.48									
2287.051	2250.085	2852.6587											
2573.535	2434.664	2996.6938											
2716.043	2599.351	3183.7396											
2876.521	2735.001	3205.9543											
2953.765	2854.768	3248.6523											
3030.953	2888.969	3412.3151											
3138.188	3017.953	3446.7971											
3194.012	3076.167	3481.3514											
3269.62	3108.421	3939.5758											
3300.383	3270.869	4048.7442											
3547.358	3326.3	4054.2415											
3653.454	3510.003	4102.369											
3676.913	3867.605	4237.113											
3751.156	4291.402	4267.0322											
3936.657	4704.565	4354.9116											
4682.204	4760.473	4406.399											
4768.117	4794.134	4441.4252											
5272.08	5119.947	5530.892											

										TRUE DFR PARAM			
Failure PDF				(Top Weibull++ Selection)				(Weibull++ Exponential)				Weibull	
5 Data Points				High Level Fitting Parameters				Low Level Fitting Parameters				Shape	0.4
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3		Scale	938
0.3887	237.7436	102.8577	Shape	0.3551	0.4961	0.3509		Lambda	0.0005	9.55E-05	0.0018	Location	0
44.7175	1030.879	103.8032	Scale	455.3751	5685.642	133.5759		mean	2000	10471.27	555.5556		
197.7363	1372.084	235.8229	Location	0	207.53	102.7		Location	0	0	0		
237.6625	17016.14	308.7439											
9511.298	32699.48	2043.3231	Exp. Lambda										
			mean		#DIV/0!								
			Location										
Failure PDF				(Top Weibull++ Selection)				(Weibull++ Exponential)					
25 Data Points				High Level Fitting Parameters				Low Level Fitting Parameters					
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3			
0.0044	0.0011	0.0001	Shape	0.4128	0.3741	0.3391		Lambda	0.0006	0.0004	0.0003		
0.1409	0.153	2.1644	Scale	889.8248	980.7368	862.057		mean	1666.667	2500	3333.333		
0.2395	2.0381	2.5861	Location	0	0	0		Location	0	0	0		
2.3157	2.8481	2.5863											
3.4037	3.3508	8.3013											
15.5762	9.2546	12.2399											
41.4359	69.4667	14.4416											
66.006	72.4795	21.25											
169.6024	98.3835	23.2016											
310.8927	188.9514	25.7567											
675.6514	228.1191	57.6914											
747.3958	472.309	161.199											
950.9716	542.8395	213.563											
1421.053	595.3248	1145.2239											
1425.232	829.2207	1530.8733											
1491.159	1195.228	1631.1368											
1533.18	1921.193	1795.8049											
1657.854	2272.61	2049.4273											
1816.85	2776.404	2686.2033											
2661.743	3540.973	2855.1948											
3469.24	4220.8	3805.9691											
3482.437	9999.785	10099.231											
5354.848	10923.76	12312.46											
7718.713	12973.89	16991.308											
10400.45	15384.06	24172.201											

Repair									True Lognormal Mean: 850		
Repair PDF			(Top Weibull++ Selection)						True Lognormal St Dev: 90		
5 Data Points			High Level Fitting Parameters			(Empirical)			True Lognormal Variance: 8100		
Set1	Set2	Set3	Rep1	Rep2	Rep3	Low Level Fitting Parameters					
747.1571	850.0651	772.8667	N Mean			Rep1	Rep2	Rep3			
828.7058	935.5551	827.0410	N S.D.						Mean for Normal variates: 6.739662		
830.6967	957.0606	937.0020	LogN Mean			(Empirical)			Var for Normal variates: 0.011149		
833.6765	981.7003	978.3519	LogN S.D.						St Dev for Normal Variates: 0.105587		
902.3605	988.9605	1027.9719									
			Weibull Shape			Normal					
			Scale			Mean					
			Location			SD					
Repair PDF			(Top Weibull++ Selection)			(Empirical)					
25 Data Points			High Level Fitting Parameters			Low Level Fitting Parameters					
Set1	Set2	Set3	Rep1	Rep2	Rep3	Rep1	Rep2	Rep3			
642.7415	691.2975	674.5797	N Mean								
696.8263	723.6644	683.9227	N S.D.			(Empirical)					
728.8017	735.9082	711.2389	LogN Mean								
745.0789	738.2158	752.3358	LogN S.D.								
773.4927	739.9673	779.8016									
774.3035	763.4049	786.2454	Weibull Shape								
792.5570	771.2505	787.2781	Scale								
792.6556	817.6816	795.3255	Location								
811.2078	822.2252	798.9932									
811.2933	823.0084	802.8723									
814.0875	827.8375	805.9737									
815.4539	831.2502	817.539									
830.9855	834.2116	838.9178									
834.2839	843.8531	843.2974									
858.9568	851.4224	843.6612									
862.9996	855.4944	844.3625									
867.1553	862.0037	870.6218									
872.307	862.5475	878.4302									
873.9636	876.9206	885.2466									
883.4647	936.0016	886.0297									
884.7831	941.7188	917.3511									
906.1892	943.284	940.4368									
940.6469	953.0136	958.6495									
943.1421	959.3408	1000.5771									
1061.907	964.0956	1022.3195									

Component 6:

IFR Failure												TRUE IFR PARAMETER		
Failure PDF			(Top Weibull++ Selection)			(Weibull++ Exponential)						Weibull		
5 Data Points			High Level Fitting Parameters			Low Level Fitting Parameters						Shape	1.9	
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3		Scale	3333	
943.0678	1272.8057	1373.1654	Shape	2.0214	1.0014	4.5581		0.0005	0.0003	0.0009	Location		0	
2219.2323	1816.7934	2175.7739	Scale	3343.678	2549.782	2673.545	Lambda	2000	3333.333	1111.111				
2853.9978	2781.1238	2552.2985	Location	0	1065.907	0	mean	864.6482	519.5057	1373.165				
3069.4	4368.796	2779.7657					Location							
5680.3975	7831.8613	3285.1362												
Failure PDF			(Top Weibull++ Selection)			(Weibull++ Exponential)								
25 Data Points			High Level Fitting Parameters			Low Level Fitting Parameters								
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3				
556.924	478.1575	338.3958	Shape	1.7668	1.8077	1.9447	Lambda	0.0004	0.0004	0.0003				
888.1491	548.2909	923.6208	Scale	3576.66	3112.881	3793.36	mean	2500	2500	3333.333				
980.9987	835.3915	938.8093	Location	151.86	0	0	Location	556.924	478.1575	338.3958				
1524.9478	1028.4228	1144.9473												
1551.9424	1252.9216	1383.9967												
1554.7179	1318.5908	2032.6482												
2150.2115	1326.7294	2056.6935												
2196.241	1403.6103	2558.1259												
2333.6979	1488.9602	2704.5543												
2639.6742	1779.9499	3017.2549												
2657.9665	1971.7594	3194.8691												
2838.8861	3073.1181	3298.5318												
2902.5407	3188.9704	3363.2244												
2911.8353	3240.6705	3371.6199												
3154.5195	3451.9756	3503.761												
3245.5848	3453.6347	3767.7098												
3798.1278	3474.3802	3811.3404												
4401.8804	3533.9596	3829.0021												
4454.7244	3609.0219	3895.2994												
4692.1923	3828.5382	4917.9287												
5156.627	3974.7389	5125.8579												
6195.3269	4201.9076	5385.3977												
6572.5149	4593.8342	5832.9327												
6595.2222	5073.4553	5849.498												
7321.4379	6986.9939	8078.036												

DFR Failure												TRUE DFR PARAMS		
Failure PDF							(Top Weibull++ Selection)			(Weibull++ Exponential)			Weibull	
5 Data Points							High Level Fitting Parameters			Low Level Fitting Parameters			Shape	0.7
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3		Scale	2336	0
126.6007	132.5105	468.6132	Shape	0.8337	0.7236		Lambda	0.0001	0.0003	0.0012	Location			
3871.6142	151.9939	542.8767	Scale	7308.818	2441.373		mean	10000	3333.333	833.3333				
4941.3851	1810.4893	1450.5999	Location	0	0		Location	0	0	347.763				
8492.4261	5181.3916	1450.9201												
22172.966	7327.3941	1977.9812					Normal							
						1178.198	Mean							
						582.3018	SD							
Failure PDF							(Top Weibull++ Selection)			(Weibull++ Exponential)				
25 Data Points							High Level Fitting Parameters			Low Level Fitting Parameters				
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3				
3.9292	50.9178	2.0586	Shape	0.745	0.682	0.6467	Lambda	0.0003	0.0004	0.0003				
49.8137	62.3909	4.4384	Scale	2679.482	2139.152	2139.572	mean	3333.333	2500	3333.333				
61.4687	116.6026	62.1906	Location	0	44.271	0	Location	0	0	0				
302.8609	137.147	170.9058												
318.0808	173.5285	173.6596												
448.9909	417.1683	174.1197												
562.4808	541.5138	348.7671												
609.2363	582.8722	638.6683												
754.8118	833.464	641.4655												
967.5236	1030.7575	697.9511												
1125.4453	1209.2986	1150.9406												
1273.4832	1575.5976	1324.3103												
1542.3704	1681.0466	1527.1351												
1607.1968	1915.0946	1677.1838												
2056.0146	1927.3436	1803.4187												
3319.1916	1932.9038	1816.459												
3580.5006	2428.3783	2521.6041												
3865.8573	2650.1936	2704.6759												
4930.0673	2672.4901	3679.97												
5678.6557	2715.7387	4895.6748												
7145.3277	2867.5279	5713.7405												
7246.6288	3093.7406	5798.1658												
8642.1941	8740.5562	6360.7816												
9803.4282	12507.125	6552.6711												
12761.655	19032.824	21372.6813												

Repair										True Lognormal Mean:	
Repair PDF										True Lognormal St Dev:	
5 Data Points										True Lognormal Variance:	
Set1	Set2	Set3		High Level Fitting Parameters				(Empirical)	Low Level Fitting Parameters		
				Rep1	Rep2	Rep3		Rep1	Rep2	Rep3	
2928.2077	2937.2639	2727.4405	N Mean								
2955.5721	3008.5301	2758.0500	N S.D.								
3118.5662	3023.0372	3009.5544	LogN Mean	1	1	1					
3192.1467	3067.5622	3059.0098	LogN S.D.	0	0	0					
3339.1724	3123.3886	3116.5662									
			Weibull Shape	2.9516			Normal				
			Scale	456.2813	3031.956	2934.124	Mean				
			Location	2701.37	66.0042	160.18	SD				
Repair PDF										True Lognormal Mean:	
25 Data Points										True Lognormal St Dev:	
Set1	Set2	Set3		High Level Fitting Parameters				(Empirical)	Low Level Fitting Parameters		
				Rep1	Rep2	Rep3		Rep1	Rep2	Rep3	
2725.9674	2758.7299	2741.9088	N Mean								
2729.9413	2770.4859	2747.9049	N S.D.								
2787.6873	2780.2585	2794.2454	LogN Mean	1	1	1					
2801.4099	2816.0094	2891.3966	LogN S.D.	0	0	0					
2833.9725	2866.0869	2930.3131									
2884.5562	2896.7268	2946.8886	Weibull Shape	5.9116	4.4958		Normal				
2920.6486	2927.8413	2965.6203	Scale	856.388	630.048	3032.937	Mean				
2930.9072	2934.3030	2976.0747	Location	2214.95	2420.89	145.72	SD				
2935.5857	2944.4503	2985.9873									
2941.8176	2948.8804	2997.5822									
2970.6641	2960.577	3001.2277									
3004.238	2966.7004	3011.5071									
3022.9127	2980.7188	3024.4465									
3034.4321	3026.4141	3044.4719									
3089.3571	3030.2903	3059.7489									
3103.579	3035.5214	3071.8224									
3107.0562	3076.3435	3080.8281									
3110.1218	3089.8191	3119.0444									
3116.661	3089.8886	3122.4055									
3122.7704	3097.7068	3165.1266									
3135.9401	3117.2651	3171.3008									
3170.1508	3126.9659	3187.1087									
3201.7674	3153.7086	3197.7748									
3202.9599	3179.3316	3213.9189									
3321.2597	3345.2504	3374.7772									

Component 7:

IFR Failure												TRUE IFR PARAM				
Failure PDF			(Top Weibull++ Selection)			(Weibull++ Exponential)			Weibull							
5 Data Points			High Level Fitting Parameters			Low Level Fitting Parameters			Shape			1.2				
Set1	Set2	Set3	Rep1	Rep2	Rep3	Rep1	Rep2	Rep3	Scale			2575				
1233.402	755.0101	507.7995	Shape	0.7525	0.871	1.2663	Lambda	0.0011	0.0002	0.0005	Location			0		
1333.494	1517.403	1140.251	Scale	587.5031	3216.492	1953.159	mean	909.0909	5000	2000						
1502.009	3151.586	1678.789	Location	1215	562.58	212.1063	Location	960.43	0	27.5108						
2398.809	3215.274	1955.761														
3042.558	11455.09	4809.848														
Failure PDF			(Top Weibull++ Selection)			(Weibull++ Exponential)										
25 Data Points			High Level Fitting Parameters			Low Level Fitting Parameters										
Set1	Set2	Set3	Rep1	Rep2	Rep3	Rep1	Rep2	Rep3								
75.305	11.1745	28.3818	Shape	1.2894	1.2411	1.2875	Lambda	0.0004	0.0004	0.0004						
163.5098	82.8646	351.9838	Scale	2989.621	2750.429	2639.941	mean	2500	2500	2500						
234.3425	689.9868	419.6121	Location	0	0	0	Location	75.305	11.1745	28.3818						
273.5559	799.5951	752.1258														
703.7099	1089.153	780.3233														
862.866	1099.429	888.8444														
999.166	1194.189	1020.826														
1593.052	1282.23	1250.599														
1876.896	1515.5	1383.95														
2199.876	1647.912	1635.23														
2224.738	1678.276	1971.803														
2360.094	2097.373	2005.742														
2548.09	2126.218	2273.373														
2998.434	2409.684	2282.09														
3619.917	2631.809	2284.143														
3627.051	2892.163	2568.483														
3737.317	3194.973	2640.37														
4502.703	3206.071	2681.885														
4507.146	3224.334	3046.235														
4518.089	3732.183	3501.405														
4562.136	3975.867	3775.582														
4720.62	5169.694	4871.772														
4807.826	5926.245	5547.4														
6007.031	6115.789	6538.232														
6192.146	7042.011	6864.931														
DFR Failure												TRUE DFR PARAM				
Failure PDF			(Top Weibull++ Selection)			(Weibull++ Exponential)			Weibull							
5 Data Points			High Level Fitting Parameters			Low Level Fitting Parameters			Shape			0.55				
Set1	Set2	Set3	Rep1	Rep2	Rep3	Rep1	Rep2	Rep3	Scale			1423				
492.9745	6.0805	33.8777	Shape	0.3744	0.3562	0.5816	Lambda	0.0002	0.0014	0.0007	Location			0		
526.3141	16.1423	340.1097	Scale	1592.354	181.6411	928.9105	mean	5000	714.2857	1428.571						
1281.924	37.7049	370.6554	Location	491.84	5.91	24.41	Location	0	0							
9009.097	385.8074	897.4695														
11366.33	3016.537	5712.437														
Failure PDF			(Top Weibull++ Selection)			(Weibull++ Exponential)										
25 Data Points			High Level Fitting Parameters			Low Level Fitting Parameters										
Set1	Set2	Set3	Rep1	Rep2	Rep3	Rep1	Rep2	Rep3								
0.2394	1.6826	1.4205	Shape	0.5956	0.5474	0.6307	Lambda	0.0004	0.0004	0.0006						
34.385	2.2468	4.2473	Scale	1671.235	1783.465	1159.606	mean	2500	2500	1666.667						
49.2658	8.1568	23.3267	Location	0	0	0	Location	0	0	0						
55.1196	9.7845	59.4488														
147.6313	57.1513	105.133														
215.7498	92.2753	180.356														
271.5501	188.8081	202.0613														
301.8768	457.0314	205.8487														
352.9605	483.8325	260.4402														
359.6454	655.3901	310.1456														
399.0531	888.7466	440.4745														
507.7724	1068.396	553.3837														
756.2171	1217.062	698.8552														
1014.603	1854.382	782.0061														
1450.057	1965.781	845.8762														
1476.586	2178.458	1603.001														
1505.085	2284.18	1717.025														
2169.176	2674.79	1776.987														
4144.399	2723.829	1810.474														
4645.451	4181.331	1889.635														
4864.779	4659.631	2164.609														
7281.768	6027.841	3077.744														
8678.169	7419.469	6075.051														
9202.056	8297.573	6330.696														
11153.06	20684.81	8855.911														

Component 8:

93

Component 9:

IFR Failure												TRUE IFR PARAM	
Failure PDF			(Top Weibull++ Selection)			(Weibull++ Exponential)			Weibull				
5 Data Points			High Level Fitting Parameters			Low Level Fitting Parameters			Shape			1.6	
Set1	Set2	Set3		Rep1	Rep2	Rep3			Rep1	Rep2	Rep3	Scale	6000
59.5546	1654.2175	613.4815	Shape	1.0849	3.3957	1.0719	Lambda	0.0006	0.0007	0.0002	Location		0
1158.651	2271.7078	1717.0383	Scale	1849.276	3368.241	6574.887	mean	1666.667	1428.571	5000			
1296.9513	3279.2189	6196.8696	Location	0	0	0	Location	0	1654.218	0			
3188.8792	3331.5225	7426.1392											
3320.8389	4547.5771	16070.173											
Failure PDF			(Top Weibull++ Selection)			(Weibull++ Exponential)			Weibull				
25 Data Points			High Level Fitting Parameters			Low Level Fitting Parameters			Shape				
Set1	Set2	Set3		Rep1	Rep2	Rep3			Rep1	Rep2	Rep3	Scale	
710.1353	315.042	677.8381	Shape	1.5749	1.3227	1.4201	Lambda	0.0002	0.0003	0.0002			
1404.275	319.0603	812.6982	Scale	5451.465	4452.107	4945.843	mean	5000	3333.333	5000			
1697.4788	865.1356	1029.7398	Location	136.8	0	188.2	Location	710.1353	315.042	677.8381			
1712.0311	964.765	1269.5125											
2399.8517	1555.3756	1283.0806											
2453.028	1788.3454	1556.249											
3059.8623	2018.1662	1768.0291											
3205.4976	2216.0917	2246.1425											
3348.8915	2299.0348	2708.4523											
3473.4032	2308.5815	3038.4733											
3590.5025	2723.8316	3711.0196											
3980.0035	2833.595	4279.3182											
4376.2988	2897.0913	4651.4644											
4563.0706	3439.8605	4745.8146											
4687.6538	3915.9635	5293.0437											
5026.6837	4195.1227	5386.0989											
5386.7755	4859.5895	5530.2301											
5665.2326	4934.6351	5907.2441											
6071.2068	5281.6802	6812.7298											
6290.1388	5447.2768	6856.5882											
7676.628	7422.3666	8055.3173											
8374.2954	7939.4749	8449.6528											
9111.7105	9350.079	8843.0188											
11337.734	9705.5429	9270.6163											
15570.734	12771.24	13027.447											

DFR Failure												TRUE DFR PARAM	
Failure PDF			(Top Weibull++ Selection)			(Weibull++ Exponential)			Weibull				
5 Data Points			High Level Fitting Parameters			Low Level Fitting Parameters			Shape			0.91	
Set1	Set2	Set3		Rep1	Rep2	Rep3			Rep1	Rep2	Rep3	Scale	5143
681.5702	596.0467	119.6449	Shape			0.8243	Lambda	0.0001	0.0002	0.0002	Location		0
2458.446	712.2351	936.5725	Scale			5389.666	mean	10000	5000	5000			
5510.2312	3365.8145	6880.5072	Location			0	Location	0	0	0			
10184.901	6731.4725	9813.6578											
24128.051	8823.3123	11431.131	Exp. Lambda	0.0001	0.0002								
			mean	10000	5000								
			Location	0	0								
Failure PDF			(Top Weibull++ Selection)			(Weibull++ Exponential)			Weibull				
25 Data Points			High Level Fitting Parameters			Low Level Fitting Parameters			Shape				
Set1	Set2	Set3		Rep1	Rep2	Rep3			Rep1	Rep2	Rep3	Scale	
288.5507	242.1327	41.8282	Shape	0.886	1.0473	0.8983	Lambda	0.0002	0.0002	0.0002			
454.0527	494.4891	167.0204	Scale	4238.59	5726.453	4840.68	mean	5000	5000	5000			
700.2991	677.5071	403.0769	Location	235.27	62.45	0	Location	0	0	0			
701.9181	1235.1553	619.0767											
1006.5467	1267.9978	879.9276											
1021.5041	1763.3643	930.1831											
1249.7236	2731.0845	1029.6775											
1519.4028	3326.5544	1328.8312											
1523.0142	3337.5112	1474.8382											
1751.3391	3365.3735	1684.7109											
2166.3748	3429.2985	2502.2854											
2540.1512	3515.0382	2693.31											
3018.4221	3761.8152	2807.075											
3857.1797	3920.6132	3551.0629											
4542.7513	4452.365	4450.4991											
4934.3277	4718.1228	5411.8571											
5396.2548	4722.0321	5604.2964											
5451.2574	4969.9776	6749.6905											
6577.9466	5292.6751	7225.6098											
6602.0741	8146.6918	9127.3135											
6605.3121	8953.005	9613.4038											
7288.3094	9086.6108	13044.791											
11396.714	15407.676	14046.558											
12643.835	17600.539	15157.287											
25368.238	25513.732	16563.952											

Repair										True Lognormal Mean:	1000
Repair PDF					(Top Weibull++ Selection)					True Lognormal St Dev:	30
5 Data Points					High Level Fitting Parameters			(Empirical)		True Lognormal Variance:	900
Set1	Set2	Set3		Rep1	Rep2	Rep3		Low Level Fitting Parameters			
957.3293	976.9499	980.0563	N Mean			6.9174		Rep1	Rep2	Rep3	
1004.1691	986.0260	993.8689	N S.D.			0.0225					Mean for Normal variates: 6.907305
1019.7928	1008.2677	1005.0293	LogN Mean	1	1	1009.947	(Empirical)				Var for Normal variates: 0.0009
1021.4267	1009.6679	1027.3645	LogN S.D.	0	0	22.72668					St Dev for Normal Variates: 0.029993
1063.3764	1033.7083	1043.4408									
			Weibull Shape	1013.219	1002.924		Normal				
			Scale	34.1523	19.9093		SD				
			Location								
Repair PDF					(Top Weibull++ Selection)			(Empirical)			
25 Data Points					High Level Fitting Parameters			Low Level Fitting Parameters			
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3	
969.4499	929.7908	906.8867	N Mean	6.9207							
987.9844	943.7861	965.8418	N S.D.	0.0214			(Empirical)				
988.6763	960.9697	978.3340	LogN Mean	1013.261	1	1					
991.4813	979.3975	979.6999	LogN S.D.	21.68627	0	0					
991.5687	983.9974	981.6159									
992.9107	990.1479	984.8527	Weibull Shape		1005.308	1002.698	Normal				
1001.3602	990.5475	986.9105	Scale		29.702	30.7377	SD				
1002.5114	995.3010	987.5617	Location								
1003.7589	1001.762	988.0836									
1005.574	1002.1008	995.0697									
1008.0845	1002.689	1003.6294									
1008.701	1004.0883	1004.238									
1009.1278	1008.1903	1004.3948									
1010.3378	1008.6868	1006.5613									
1015.067	1009.8558	1006.9701									
1015.6193	1017.8083	1008.422									
1019.8246	1020.2354	1009.5552									
1021.0363	1020.745	1010.9104									
1022.2379	1021.1124	1011.4882									
1028.7856	1024.58	1021.9361									
1030.7385	1032.176	1030.1529									
1044.5651	1037.4256	1032.2784									
1046.3787	1041.2503	1043.9363									
1055.5485	1049.9777	1050.9691									
1060.5552	1056.0762	1067.1425									

Component 10:

IFR Failure											TRUE IFR PARAMETERS	
Failure PDF					(Top Weibull++ Selection)			(Weibull++ Exponential)		Weibull		
5 Data Points					High Level Fitting Parameters			Low Level Fitting Parameters		Shape	2.3	
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3	Scale	4700
2460.3635	1391.2082	3104.3889	Shape	0.9031				0.0004	0.0002	0.0008	Location	0
2985.2778	3824.9763	3239.3708	Scale	1682.075			Lambda	2500	5000	1250		
3428.4506	4646.9928	4151.6844	Location	2349.75			Location	1454.85	939.0042	2888.68		
3829.4519	5099.4023	5067.2464										
7899.659	10868.3625	5136.9909	Exp. Lambda		0.0002	0.0008						
			mean		5000	1250						
			Location		939.0042	2888.68						
Failure PDF					(Top Weibull++ Selection)			(Weibull++ Exponential)				
25 Data Points					High Level Fitting Parameters			Low Level Fitting Parameters				
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3		
894.4406	331.2133	2131.6465	Shape	2.1419	2.0096	1.4895	Lambda	0.0003	0.0002	0.0004		
1023.1727	1715.7957	2340.2868	Scale	4366.964	4955.363	3212.777	mean	3333.333	5000	2500		
1310.8678	2173.6379	2502.6929	Location	38.11	0	1897.27	Location	894.4406	331.2133	2131.647		
2000.8766	2547.4421	2821.5088										
2112.3535	2581.314	2917.4431										
2350.6133	2846.8133	3105.3902										
2447.3256	2874.1741	3417.3013										
2531.0481	2918.4731	3425.1682										
2756.9077	3162.4144	3635.6986										
2899.8222	3171.9591	3778.0157										
2938.7555	3428.8095	3824.1597										
3549.4744	3508.1976	3860.8158										
3725.4581	3838.6692	4335.5436										
3818.6762	4515.6644	4792.2902										
4007.7806	4651.8767	5010.0428										
4367.2804	4715.7704	5324.771										
4622.0827	4729.5918	5377.7319										
5262.9528	4923.6739	5787.9227										
5510.4431	4983.704	6245.8785										
5649.2894	5094.7207	6666.7204										
6078.4149	6895.3167	6890.9982										
6198.2355	7500.7015	7054.6766										
6370.6497	8166.9252	7089.9942										
6873.8549	8813.8198	7181.9843										
8151.8431	9904.6635	10476.828										

Repair									True Lognormal Mean:	2300
Repair PDF					(Top Weibull++ Selection)				True Lognormal St Dev:	133
5 Data Points					High Level Fitting Parameters			(Empirical)	True Lognormal Variance:	17689
Set1	Set2	Set3		Rep1	Rep2	Rep3		Low Level Fitting Parameters		
2244.4884	2141.4443	2276.3188	N Mean		7.7629			Rep1	Rep2	Rep3
2260.0039	2328.3686	2482.6217	N S.D.		0.0551					
2354.3167	2335.8325	2514.6316	LogN Mean	1	2355.287	1		(Empirical)		Mean for Normal variates: 7.738995263
2362.9009	2463.2168	2525.4142	LogN S.D.	0	129.8749	0				Var for Normal variates: 0.003338278
2414.3795	2507.9680	2627.1859								St Dev for Normal Variates: 0.057777834
			Normal	2327.218		28.4045	Weibull Shape			
			SD	64.7559		2535.908	Scale			
						0	Location			
Repair PDF					(Top Weibull++ Selection)			(Empirical)		
25 Data Points					High Level Fitting Parameters			Low Level Fitting Parameters		
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3
2041.8124	1988.1396	2098.0090	N Mean							
2077.4957	2142.8664	2142.8664	N S.D.					(Empirical)		
2129.4412	2202.4947	2169.5474	LogN Mean	1	1	1				
2135.9223	2202.7754	2169.6936	LogN S.D.	0	0	0				
2177.7875	2241.5206	2197.0539								
2201.6623	2264.6235	2223.3397	Weibull Shape	2315.309	12.2186	2.5076	Normal			
2227.5785	2265.4355	2230.1468	Scale	141.1561	1213.158	337.9804	SD			
2247.9478	2265.6194	2234.5096	Location		1138.31	2024.68				
2278.6822	2269.1553	2258.3334								
2285.9912	2277.6492	2267.9941								
2288.4621	2287.4753	2273.7032								
2291.1901	2303.3847	2276.2228								
2325.2501	2303.3995	2282.6423								
2348.7154	2304.2275	2325.19								
2348.8709	2304.6355	2353.8731								
2351.348	2318.4564	2368.0734								
2372.4001	2332.3781	2368.1366								
2374.8429	2345.6113	2369.3179								
2383.5781	2368.9782	2453.5628								
2445.5536	2372.1516	2460.3674								
2453.2725	2375.4344	2464.775								
2453.3264	2404.7957	2475.733								
2473.8595	2469.9964	2530.6703								
2503.4369	2483.9576	2543.4697								
2664.3045	2511.8641	2558.3395								

Component 11:

IFR Failure																TRUE IFR PARAMETER
Failure PDF					(Top Weibull++ Selection)					(Weibull++ Exponential)						Weibull
5 Data Points					High Level Fitting Parameters					Low Level Fitting Parameters						Shape
Set1	Set2	Set3		Rep1	Rep2	Rep3			Rep1	Rep2	Rep3					Scale
399.3785	348.758	1171.1815	Shape	1.717	1.5995			Lambda	0.0008	0.0005	0.0004	Location				2700
830.8968	1017.428	1221.9844	Scale	1776.997	2057.368			mean	1250	2000	2500					0
1580.886	1719.111	3127.0986	Location	0	0			Location	321.4348	0	657.75					
1953.553	2494.546	4832.3086														
3139.557	3653.888	5037.1987				0.0004	Ex. lambda									
						2500	mean									
						657.75	location									
Failure PDF					(Top Weibull++ Selection)					(Weibull++ Exponential)						
25 Data Points					High Level Fitting Parameters					Low Level Fitting Parameters						
Set1	Set2	Set3		Rep1	Rep2	Rep3			Rep1	Rep2	Rep3					
135.1654	205.6775	125.7678	Shape	1.4844	1.4247	1.1671		Lambda	0.0005	0.0004	0.0005					
611.4846	394.5768	252.0802	Scale	2507.105	2599.055	2230.025		mean	2000	2500	2000					
651.0371	509.5215	291.4994	Location	0	45.86	49.46		Location	135.1654	0	125.7678					
660.8156	737.8067	355.3873														
1021.222	752.9524	716.5149														
1022.097	840.2453	781.2346														
1263.096	1089.669	869.563														
1281.615	1356.857	909.8222														
1330.08	1362.603	924.6418														
1384.751	1391.372	1208.1672														
1385.471	1643.994	1245.9077														
1576.088	1929.807	1306.397														
1871.908	1942.723	1530.4185														
1993.42	2024.086	1958.8213														
2015.435	2285.275	2204.4944														
2241.681	2586.417	2349.3161														
2455.512	2881.621	2484.0128														
2700.641	3504.116	2976.4458														
2942.081	4173.105	3206.0688														
3105.378	4200.117	3282.3602														
3995.412	4339.191	3711.8226														
4304.415	4461.944	4345.3492														
4901.151	4711.247	5114.9785														
5523.447	5461.185	5357.2256														
6151.231	5482.621	6614.3085														

DFR Failure												TRUE DFR PARAMS		
Failure PDF			(Top Weibull++ Selection)			(Weibull++ Exponential)			Weibull					
5 Data Points			High Level Fitting Parameters			Low Level Fitting Parameters			Shape			0.82		
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3		Scale	2210	
8.6383	257.351	652.8088	Shape	0.3717	0.8764	0.5997	Lambda	0.0016	0.0005	0.0003	Location	0	0	
25.3535	674.1752	871.4467	Scale	216.4645	1894.652	1694.464	mean	625	2000	3333.333				
34.4213	1307.778	1860.3552	Location	8.48	140.36	625.16	Location	0	0	0				
1408.127	2304.159	2150.5854												
1577.924	6288.88	10252.167												
Failure PDF			(Top Weibull++ Selection)			(Weibull++ Exponential)								
25 Data Points			High Level Fitting Parameters			Low Level Fitting Parameters								
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3				
62.2017	0.7289	102.2177	Shape	0.612	0.806	0.9371	Lambda	0.0005	0.0005	0.0004				
83.1745	39.6174	120.7591	Scale	1477.638	1885.069	2157.154	mean	2000	2000	2500				
98.2315	48.879	124.0102	Location	59.81	0	0	Location	0	0	0				
109.0633	118.1574	178.337												
132.6368	401.0883	320.1617												
141.8487	422.3874	496.5205												
223.3982	428.7619	568.8352												
285.5178	463.5295	570.9538												
291.1456	527.5761	951.019												
402.0495	891.8545	955.9224												
494.164	1148.423	1020.5197												
582.361	1270.221	1101.879												
931.2186	1693.32	1577.8085												
1277.169	1811.43	1969.9017												
1360.007	2027.364	2016.8713												
1848.355	2364.114	2172.5559												
2216.056	2416.477	2262.6457												
2803.269	2602.398	2444.5492												
3128.196	2804.394	2861.5444												
3542.853	3433.075	2865.5286												
3571.851	3981.664	3978.9659												
4313.159	4518.86	4345.9862												
7229.554	4981.317	5207.9364												
8298.052	5998.264	8198.7492												
10447.32	7468.848	9183.7914												

Component 12:

99

DFR Failure												TRUE DFR PARAM	
Failure PDF			(Top Weibull++ Selection)						(Weibull++ Exponential)			Weibull	
5 Data Points			High Level Fitting Parameters						Low Level Fitting Parameters			Shape	0.67
Set1	Set2	Set3		Rep1	Rep2	Rep3			Rep1	Rep2	Rep3	Scale	1812
7.6546	96.0899	21.1995	Shape	0.7685	0.63692	0.6072		Lambda	0.0019	0.0005	0.0003	Location	0
197.1153	113.9879	755.0449	Scale	471.1644	1448.012	2426.685		mean	526.3158	2000	3333.333		
325.2801	1001.18	781.0092	Location	0	0	0		Location	0	0	0		
513.3453	1377.4546	4275.7639											
1654.598	7131.7167	11255.07											
Failure PDF			(Top Weibull++ Selection)						(Weibull++ Exponential)				
25 Data Points			High Level Fitting Parameters						Low Level Fitting Parameters				
Set1	Set2	Set3		Rep1	Rep2	Rep3			Rep1	Rep2	Rep3		
10.9449	3.7484	87.0939	Shape	0.7077	0.7423	0.705		Lambda	0.0005	0.0005	0.0004		
11.1556	45.4783	90.6331	Scale	1630.637	1816.511	2203.728		mean	2000	2000	2500		
11.6602	76.0758	124.7874	Location	0	0	62.81		Location	0	0	0		
49.9084	175.6294	149.4903											
153.5405	188.8683	162.0355											
176.5265	272.3074	196.7246											
468.0817	378.9127	411.433											
576.0046	526.0171	476.6189											
642.2621	853.6597	740.2461											
648.6122	861.7123	901.4276											
692.8479	941.2544	1142.9903											
788.1248	976.4346	1361.5264											
797.9105	1006.7838	1742.5392											
1019.872	1052.5932	1782.2856											
1162.9292	1494.4841	2169.7709											
1711.7281	1495.1746	2414.2543											
2346.9917	1590.8216	2528.401											
2932.8064	1779.3303	2761.1596											
3121.6927	2316.7388	3271.3115											
3265.6842	2880.537	4597.1368											
3767.835	3517.6496	5929.2883											
5175.5476	6082.2382	6308.3627											
5842.0775	6734.5327	8273.6667											
6673.5799	9184.2438	10776.82											
7470.7063	9901.7573	11152.869											

Repair												True Lognormal Mean: 1000	
Repair PDF			(Top Weibull++ Selection)						(Empirical)			True Lognormal St Dev: 100	
5 Data Points			High Level Fitting Parameters						Low Level Fitting Parameters			True Lognormal Variance: 10000	
Set1	Set2	Set3		Rep1	Rep2	Rep3			Rep1	Rep2	Rep3		
921.2770	820.7662	1047.2591	N Mean										
1026.9795	1007.9188	1086.8297	N S.D.										
1030.6159	1161.9647	1105.9727	LogN Mean	1	1	1						Mean for Normal variates: 6.90278	
1081.1291	1213.9549	1108.9158	LogN S.D.	0	0	0						Var for Normal variates: 0.00995	
1179.9064	1230.9343	1149.3595										St Dev for Normal Variates: 0.099751	
			Weibull Shape	3.5735	9.636	1099.667	Normal						
			Scale	299.5936	1150.825	33.1844	SD						
			Location	778.58	0								
Repair PDF			(Top Weibull++ Selection)						(Empirical)				
25 Data Points			High Level Fitting Parameters						Low Level Fitting Parameters				
Set1	Set2	Set3		Rep1	Rep2	Rep3			Rep1	Rep2	Rep3		
863.4720	816.3019	836.3563	N Mean		6.9115								
870.4745	836.0325	857.4328	N S.D.		0.111								
878.7886	888.8314	870.3992	LogN Mean	1	1009.954	1							
881.4143	889.2766	881.9993	LogN S.D.	0	112.4511	0							
883.4105	898.2122	921.5095											
891.1241	915.6568	923.8170	Weibull Shape	2.2105		1.8793							
892.3451	927.0283	924.6352	Scale	184.6464		227.2435							
930.7104	941.5621	930.0624	Location	813.92		807.18							
935.6024	956.2913	937.2837											
947.4867	967.8106	945.3822											
951.459	978.4647	950.9466											
966.2918	1017.5979	968.5811											
986.5928	1024.2288	978.9309											
989.2622	1029.7179	1014.9392											
994.1616	1032.0143	1020.9281											
1008.1781	1033.658	1031.0253											
1008.6267	1037.4829	1051.3581											
1011.5922	1051.5717	1085.0167											
1030.4598	1057.6908	1093.2055											
1037.8319	1086.4589	1107.9219											
1056.746	1088.9196	1114.7159											
1058.7383	1094.1714	1146.3594											
1068.7242	1123.3769	1160.0313											
1137.1276	1189.747	1177.6552											
1142.8991	1368.8295	1284.396											

Component 13:

IFR Failure												
Failure PDF												
5 Data Points												
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3		
1534.7392	896.028	2376.5041	Shape	1.0452	0.9373	1.4478	Lambda	0.0002	0.0002	0.0004	Weibull	1.3
2953.8106	1543.0735	3122.547	Scale	4776.894	3617.809	2920.016	mean	5000	5000	2500	Scale	4200
4334.0274	3307.1905	3895.0499	Location	1089.76	669.88	1841.89	Location	0	0	2058.92	Location	0
5885.3096	5861.0583	5460.114										
14192.729	10349.6589	7561.16	Exp. Lambda		Normal							
			mean	#DIV/0!	s.d.							
			Location									
Failure PDF												
25 Data Points												
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3		
303.173	364.9067	52.2914	Shape	1.099	1.5637	1.3457	Lambda	0.0003	0.0003	0.0003		
426.5639	471.2389	517.6315	Scale	3738.021	4208.958	3298.907	mean	3333.333	3333.333	3333.333		
716.8078	534.2293	536.4013	Location	185.79	0	0	Location	203.05	364.9067	52.29		
876.5694	692.8011	676.0851										
972.2855	835.1185	743.9198										
1083.6165	1157.1796	1353.7882										
1800.6851	1813.9937	1552.2205										
1888.3481	2007.6607	1642.183										
1970.5479	2590.7362	1664.7736										
2143.4951	3074.3267	2000.7745										
2294.3538	3756.3237	2496.9661										
2343.1229	3858.5608	2515.8117										
2403.8472	4232.3753	2581.3519										
2907.9941	4321.4384	2815.3824										
2996.2714	4630.4668	2816.2662										
3180.0696	4673.2074	3561.1123										
4185.0583	5054.6401	3664.1376										
4862.0909	5222.681	3734.7926										
5179.9238	5749.319	3843.1136										
5292.771	5893.3512	3992.5466										
6423.234	6030.9471	5862.6757										
8207.1477	6260.7526	5985.4605										
9439.1723	6386.9334	6126.255										
9843.7147	7641.1074	7591.0921										
13060.971	8022.6307	7761.8746										

DFR Failure												
Failure PDF												
5 Data Points												
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3		
821.429	1923.5228	505.3313	Shape	0.7792	0.4406	0.933	Lambda	0.0003	0.0002	0.0004	Weibull	0.86
1765.5065	2224.9034	1174.8287	Scale	2059.37	1153.917	2315.178	mean	3333.333	5000	2500	Scale	3591
1978.1663	2262.4041	1524.5589	Location	753.76	1918.814	333.62	Location	0	0	0	Location	0
2189.6663	2516.0291	3041.0487										
8948.9675	16093.1049	7387.8961										
Failure PDF												
25 Data Points												
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3		
36.3236	21.5841	49.1341	Shape	0.7608	0.855	0.9524	Lambda	0.0002	0.0002	0.0004		
188.6881	102.3341	118.9211	Scale	4714.48	5917.635	3668.353	mean	5000	5000	2500		
199.9364	498.039	274.2967	Location	11.09	0	0	Location	0	0	0		
222.5115	614.9749	393.2561										
266.6395	831.7648	485.8735										
567.3778	1468.5343	521.5053										
944.2548	1831.2292	545.8457										
1038.0092	2044.4742	566.8894										
1948.6666	2640.3637	1212.735										
2286.6888	2906.0534	1218.5584										
2759.1723	3043.1824	1545.1322										
2961.0126	3600.0084	1764.2993										
3450.8804	4234.745	1880.0662										
3551.7663	4412.2526	2024.5855										
3829.7049	5383.6358	2314.5517										
4366.7506	5846.0498	2387.5884										
5261.639	5928.3691	3156.058										
6222.2804	6579.8596	3413.1687										
6379.6793	8293.0474	3588.0676										
6982.4762	8534.6481	4226.6113										
8316.9783	10004.7272	4605.8261										
13430.928	11397.2067	5564.6401										
19695.835	19237.692	6450.3334										
20668.013	19319.2679	8800.8698										
22262.107	31005.7548	11070.3856										

Component 17:

[illegible][illegible]

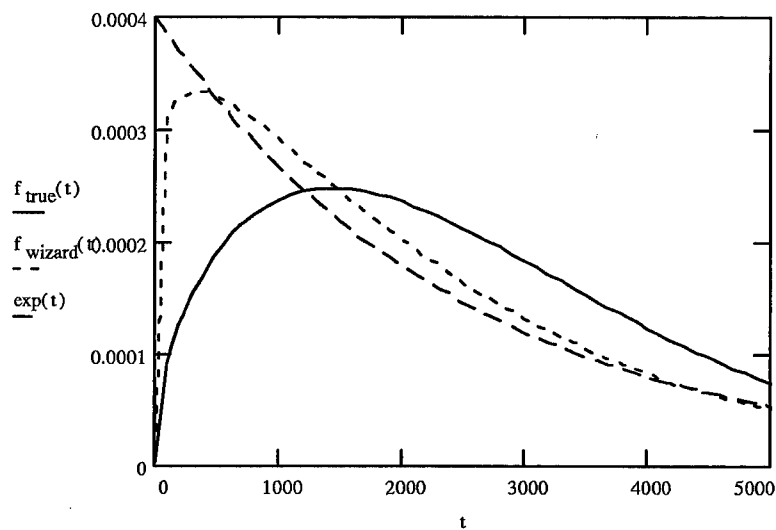
DFR Failure										TRUE DFR PARAM			
Failure PDF				(Top Weibull++ Selection)				(Weibull++ Exponential)				Weibull	
5 Data Points				High Level Fitting Parameters				Low Level Fitting Parameters				Shape	0.48
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3		Scale	829
19.2842	24.0392	0.1402	Shape	0.3007	0.4577	0.3947		0.0003	0.0028	0.0011		Location	0
45.759	32.7484	81.4712	Scale	676.804	160.2191	310.1298		mean	3333.333	357.1429	909.0909		
71.7595	148.4099	103.7989	Location	19.14	23.48	0		Location	0	0	0		
3121.848	219.2888	295.1515											
14259.96	1387.394	3958.93	p. Lambda										
			mean		#DIV/0!								
			Location										
Failure PDF				(Top Weibull++ Selection)				(Weibull++ Exponential)					
25 Data Points				High Level Fitting Parameters				Low Level Fitting Parameters					
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3			
0.0133	0.0458	9.4646	Shape	0.4801	0.4267	0.5169		0.0006	0.0003	0.0004			
8.8798	1.5861	23.4149	Scale	753.4989	1087.99	1253.119		mean	1666.667	3333.333	2500		
19.6567	11.611	24.6482	Location	0	0	8.45		Location	0	0	0		
20.7896	37.0668	33.4034											
45.0159	44.7426	75.794											
88.2708	64.8171	75.9579											
93.5473	79.0982	101.4868											
107.3531	166.1867	324.5026											
125.3961	211.427	343.8184											
162.2768	251.8883	360.2157											
187.0662	309.0835	372.9556											
317.6282	314.9474	387.9473											
404.9284	337.0527	483.8195											
420.0821	345.1927	523.1712											
420.7332	659.6077	541.1124											
551.4718	783.5122	734.027											
564.3061	832.9628	863.8202											
663.5022	1546.006	1164.115											
822.9871	1895.595	2691.308											
1084.615	2237.328	3127.158											
2198.133	3253.82	4729.701											
2527.125	3877.281	4765.47											
4800.574	6135.726	11599.86											
10587.4	9381.876	12590.5											
17009.16	59438.62	13556.88											

Repair										True Lognormal Mean: 280			
Repair PDF				(Top Weibull++ Selection)				(Empirical)				True Lognormal St Dev: 50	
5 Data Points				High Level Fitting Parameters				Low Level Fitting Parameters				True Lognormal Variance: 2500	
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3			
160.9515	263.4569	205.7862	N Mean										
207.5786	268.0693	261.3554	N S.D.										
249.9895	268.5295	293.0248	logN Mean	1	1	1						Mean for Normal variates: 5.619095	
259.1388	306.5290	324.6998	LogN S.D.	0	0	0						Var for Normal variates: 0.03139	
288.9591	354.9991	334.4042										St Dev for Normal Variates: 0.177172	
			Weibull Shape	6.497	0.6542	283.8541	Normal						
			Scale	251.3703	22.0338	46.7088	SD						
			Location	0	262.9								
Repair PDF				(Top Weibull++ Selection)				(Empirical)					
25 Data Points				High Level Fitting Parameters				Low Level Fitting Parameters					
Set1	Set2	Set3		Rep1	Rep2	Rep3		Rep1	Rep2	Rep3			
226.1870	192.9462	162.4683	N Mean			5.5349							
226.2906	200.9697	192.8644	N S.D.			0.1931							
233.5470	205.7703	198.9839	logN Mean	1	1	258.1508							
234.2416	219.1725	204.5344	LogN S.D.	0	0	50.31723							
237.7709	234.0049	204.7760											
238.4575	240.3753	218.1189	bul Shape	1.916	3.1754								
242.8845	244.4475	227.3584	Scale	74.3501	174.918								
250.1624	245.8746	232.3286	Location	210.45	132.27								
260.8614	257.2065	233.5123											
267.3973	279.2046	236.7337											
271.7493	284.9472	237.1974											
271.9375	293.7138	238.1687											
272.2074	298.8779	258.0996											
276.2545	301.0381	260.832											
280.349	307.0936	261.3496											
281.0325	307.0983	269.5956											
289.3342	321.275	274.1441											
294.0659	324.7649	298.4224											
295.8926	325.7814	300.6731											
300.4485	328.0474	306.8594											
301.0505	329.8898	309.0275											
326.5971	334.6501	313.1513											
327.3225	350.851	317.2415											
329.0394	377.8968	321.3988											
371.0237	408.8771	374.819											

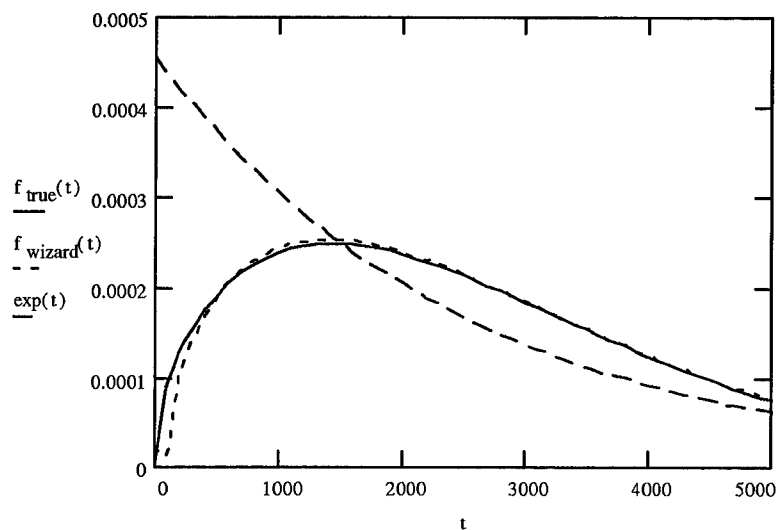
Appendix E: Data Fitting Graphs

Examples of True versus Weibull++ Wizard and Exponential Fitted Distributions for Component 1 (Final Experiment):

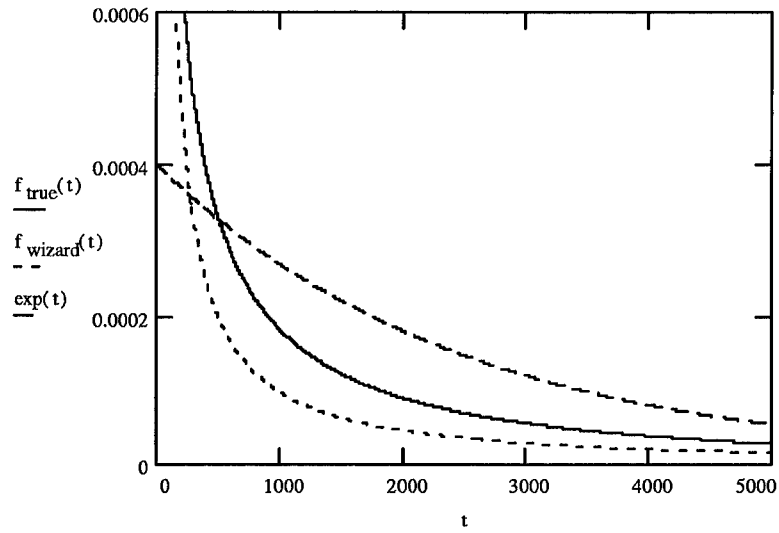
IFR Failure PDF (Weibull)
5 data points
Replication 3



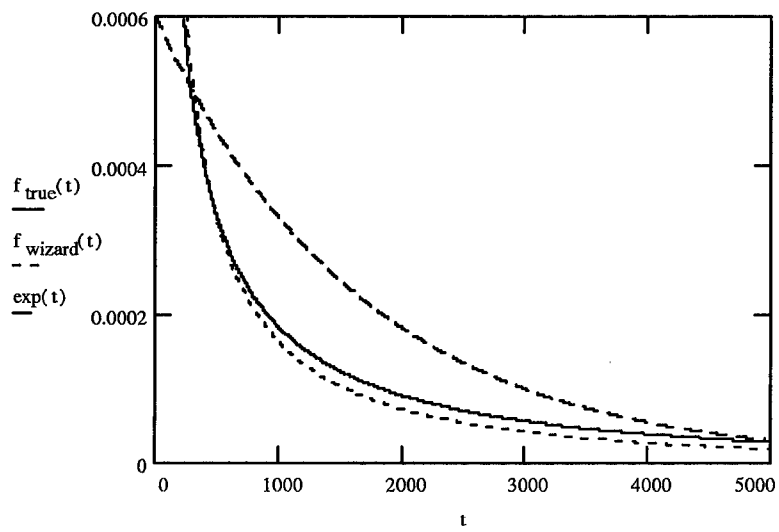
IFR Failure PDF (Weibull)
25 data points
Replication 3



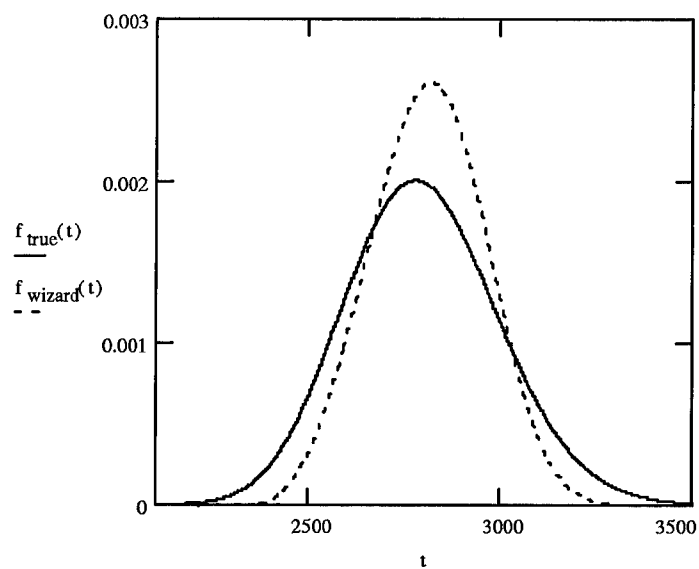
DFR Failure PDF (Weibull)
5 data points
Replication 3



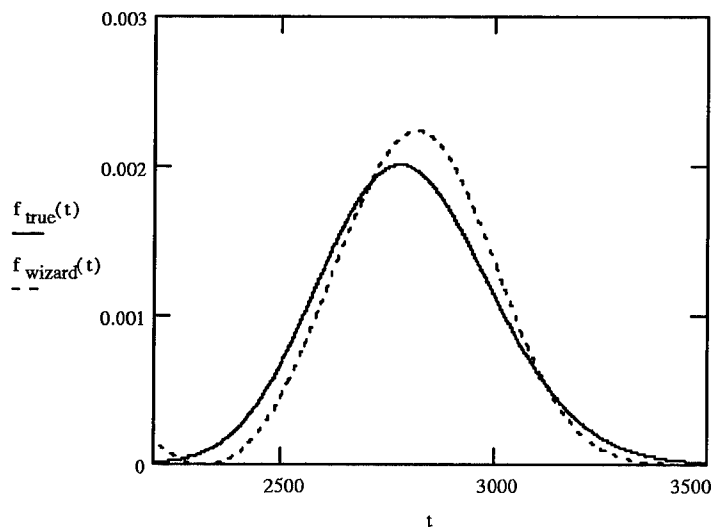
DFR Failure PDF (Weibull)
25 data points
Replication 3



Repair PDF (Lognormal)
5 data points
Replication 3



Repair PDF (Lognormal)
25 data points
Replication 3



Appendix F:

Birnbaum Structural Component Importance Measure Results for Final Experiment

Small / Series-Parallel Structure:

Component	Birnbaum Structural Importance Measure	Top 20%
1	.1875	
2	.1875	
3	.5625	√
4	.1875	
5	.1875	

Small / Complex Structure:

Component	Birnbaum Structural Importance Measure	Top 20%
1*	.410156	√
2	.410156	
3	.246094	
4	.410156	
5	.410156	

* Smallest MTTF/MRT ratio

Large / Series-Parallel Structure:

Component	Birnbaum Structural Importance Measure	Top 20%
1	.08832	
2	.08832	
3	.08832	
4	.206079	√
5	.206079	√
6	.08832	
7	.08832	
8	.08832	
9	.041216	
10	.041216	
11	.041216	
12	.041216	
13	.206079	√
14	.08832	
15	.08832	
16	.08832	
17	.206079	√
18	.08832	
19	.08832	
20	.08832	

Large / Complex Structure:

Component	Birnbaum Structural Importance Measure	Top 20%
1	.090469	√
2	.038773	
3	.064621	
4	.090469	√
5	.038773	
6	.042004	
7	.084007	√
8	.084007	√
9	.042004	
10	.015274	
11	.015274	
12	.045822	
13	.07637	
14	.015274	
15	.015274	
16	.07627	
17	.045822	
18	.024002	
19	.024002	
20	.024002	

Appendix G: Multivariate Analysis of RAPTOR Output

I. ANALYSIS TECHNIQUES

Overview

A main objective of this study was to provide insight for the reliability community in assessing differences in various systems of components through multivariate analysis of simulation output. Several multivariate techniques were applied, including discriminant analysis (DA), neural networks, logistic regression, principal component analysis (PCA), and factor analysis (FA).

Discriminant Analysis (DA)

A primary analysis objective was to discriminate between large versus small, complex versus series-parallel, and increasing failure rate (IFR) versus decreasing failure rate (DFR) component structures. Discriminant analysis was the key method to achieve this objective. Due to the relatively small size of the data set, the discriminant function was formed from the entire data set. Therefore, true validation cannot occur until the discriminant function is tested against future observations. As will be discussed later, the formatting of the data was a major difficulty in conducting discriminant analysis. Because of this, and as a learning exercise, DA was attempted on different forms of the data set, namely standardized data and transformed data (using a Box-Cox transformation). Furthermore, since the variance-covariance matrices were only statistically equal for the

IFR versus DFR case, discriminant functions were calculated using the within-class covariance matrices in addition to using the pooled matrices (for the large versus small and complex versus series-parallel cases). This was done mostly as a learning exercise to see what would happen and if any differences would occur in the discriminant results. In general, as detailed in the results section of this paper, significant success was achieved in discriminating between classes in all 3 cases.

Neural Networks

Since a quadratic discriminant function was the most effective for the complex versus series-parallel case, a neural network was also employed to assess its ability to discriminate between complex and series-parallel component structures. The neural net was trained on standardized data using back-propagation and sigmoidal processing with one hidden layer containing 20 nodes. A 'full' neural net was run using all the variables as well as a 'reduced' net containing only the 3 most salient variables. Good discriminant success was achieved (consistent with the DA results) for the training and validation sets for both the full and reduced models.

Logistic Regression

As an additional exercise, logistic regression was also tried in an attempt to discriminate between complex and series-parallel component structures. The models included a full model logistic regression of raw, standardized, and transformed data, without success. The software used in the logistic regression analysis (SAS and

JMP) could only produce a viable regression model on a reduced set of variables (the 3 most salient variables identified in the neural net analysis were used). Logistic regression proved to be the least powerful method for discriminating between complex and series-parallel component structures.

Principal Component Analysis (PCA)

Another analysis objective was to see if the majority of output variance could be adequately explained in smaller dimensions. To achieve this objective, principal component analysis (PCA) based on the correlation matrix was conducted. Although the loading structure was not completely clear-cut, by using Kaiser's criterion a reduction in the dimensionality of the data set to 3 components was achieved which explained a majority (82%) of the output data variance. Some success in discriminating between large versus small and IFR versus DFR structures using component score rankings was also achieved.

Factor Analysis (FA)

Our final analysis objective was to identify possible common underlying factors with common variance. Using factor analysis with varimax data rotation, 3 underlying factors were identified. The rotation produced much more clearly defined factor loadings. As with PCA, some success was achieved in discriminating between large versus small and IFR versus DFR structures using factor score rankings.

II. DATABASE

General Description

Multivariate analysis was conducted on output data produced by system component reliability models developed and run on the Rapid Availability Prototyping for Testing Operational Readiness (RAPTOR) software. RAPTOR, created by HQ AFOTEC/SAL, creates reliability, maintainability, availability (RM&A) and sparing models for various systems undergoing operational test and evaluation (OT&E).

Specific Output Measures

The specific output measures analyzed are defined below:

Availability: The ratio of the time the system is up (operational) versus total simulation time.

Mean Time Between Downing Events (MTBDE): The average time between events which bring the entire system down.

Mean Down Time (MDT): The average amount of time the entire system is down.

Mean Time Between Maintenance (MTBM): The average amount of time between any maintenance actions performed on any components of the system.

Mean Repair Time (MRT): The average amount of time it takes to repair any component in the system.

Analysis on the *standard deviations* of all of the above simulation output measures was also conducted.

Thirty-eight different system models with various characteristics were created and run on RAPTOR for a duration of 50,000 simulation time units per run. The three characteristics which define each system of components are structure type, failure probability density function (pdf) type, and system size. The breakdown for each category is as follows:

- Structure Type: Complex (non series-parallel) or Series-Parallel network
- Overall Component Failure pdf Type: Increasing Failure Rate (IFR) or Decreasing Failure Rate (DFR)
- Size: Large (20 components) or Small (5 components)

Two examples of structure types used in the study are shown in Figures F-1 and F-2.

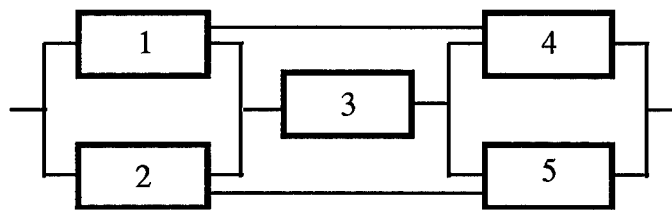


Figure F-1. Small / Complex Structure Type

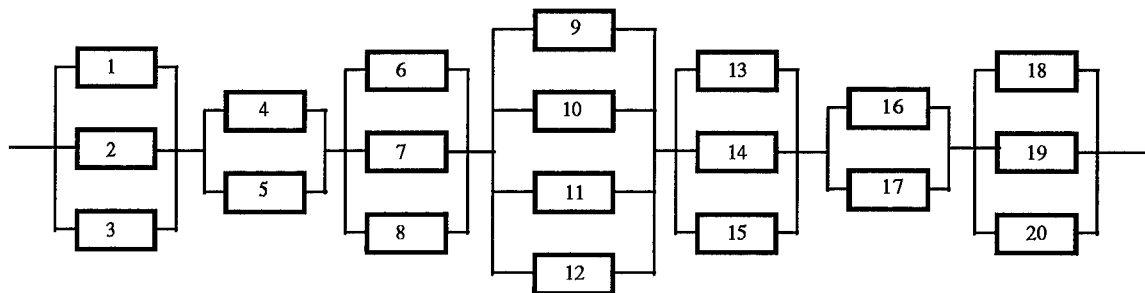


Figure F-2. Large / Series-Parallel Structure Type

Twelve basic structures/systems were developed: 3 large complex systems, 3 large series-parallel systems, 3 small complex systems, and 3 small series-parallel systems. The

parameters of the Weibull distributed failure rates for specific components in each system were varied, and 10 runs for each configuration measuring the outputs described above (averaged over the 10 simulation runs) were conducted. When re-configuring a component failure rate from IFR to DFR, the same *average* failure rate was maintained by adjusting the Weibull scale parameter. Therefore, when a component is altered from IFR to DFR (or vice versa), the only thing that changes is the fact that it's failure distribution is changed from Weibull IFR to Weibull DFR. Some runs were conducted with all component failure pdf's either exclusively IFR or DFR, and some were run where 20% of the component failure distributions were altered to the opposite type. To simplify the analysis, any system which had a predominant (80% or more) component failure distributions of IFR or DFR, was labeled as IFR or DFR, respectively. The final result was 38 total configurations. An entire overview of the structure types and simulation outputs is provided in Table F-1.

Structure	Failure					Simulation Output Parameters						
	PDF	Size	Ao	Ao S.D.	MTBDE	MTBDE S.D.	MDT	MDT S.D.	MTBM	MTBM S.D.	MRT	MRT S.D.
Complex	IFR	Large	0.62195	0.0317	831.36136	75.576532	506.489	69.955175	153.7701	6.125785	972.288	36.924329
Complex	DFR	Large	0.58173	0.0483	699.71808	96.007519	507.801	103.79795	139.7672	9.816596	990.091	76.259607
Complex	IFR	Large	0.63395	0.0373	880.95237	158.622887	505.67	87.795161	154.7226	9.789738	965.661	45.291379
Complex	DFR	Large	0.59402	0.0503	728.2015	125.372449	490.995	51.204153	139.7718	10.318022	990.298	90.250027
S-P	IFR	Large	0.84397	0.0239	1859.4221	185.712002	343.404	58.046129	160.8931	5.027334	934.313	19.928222
S-P	DFR	Large	0.82293	0.0515	1900.4104	682.441413	373.829	85.693138	151.0831	12.875395	924.663	71.292258
S-P	IFR	Large	0.8545	0.0273	2012.4874	488.187184	331.872	48.445802	162.9239	9.168588	927.149	42.11069
S-P	DFR	Large	0.83093	0.0649	1813.4586	468.103509	340.372	73.421084	152.8934	12.891715	935.142	51.081099
Complex	IFR	Small	0.79723	0.0411	2517.4118	600.101875	613.235	62.90322	607.9589	52.450363	962.774	40.528591
Complex	DFR	Small	0.77137	0.0709	2330.3009	1113.381932	603.902	153.02025	622.7378	176.694094	1050.93	96.956241
Complex	IFR	Small	0.79269	0.0727	2265.9171	623.983857	557.461	176.63768	591.8588	56.288879	972.672	102.08752
Complex	DFR	Small	0.78156	0.0535	2124.4053	546.198341	583.181	166.19736	567.2333	46.76866	964.838	111.4952
S-P	IFR	Small	0.64951	0.0356	1608.0276	186.849893	863.769	97.571768	594.1459	47.864929	1016.68	43.676779
S-P	DFR	Small	0.6214	0.0592	1336.2383	391.090585	788.749	128.67521	530.1731	83.196167	1097.2	140.45746
S-P	IFR	Small	0.64273	0.0302	1535.4473	265.502618	844.903	99.479778	580.5185	33.05382	1018.08	64.554289
S-P	DFR	Small	0.66742	0.0562	1658.0624	336.932573	812.166	150.83298	580.4298	117.845115	994.381	128.14823
Complex	IFR	Large	0.65005	0.0436	978.83652	182.752826	521.406	84.959954	152.2207	7.777956	987.486	33.613689
Complex	DFR	Large	0.65614	0.0592	993.56181	194.133449	511.751	85.010133	150.2238	16.701989	984.548	58.906297
S-P	IFR	Large	0.8629	0.0308	2562.1308	460.556013	396.451	66.725463	165.5002	6.53677	940.918	26.54521
S-P	DFR	Large	0.85516	0.0434	2318.5127	727.267637	377.627	130.5672	156.2256	12.517785	940.307	48.952462
S-P	IFR	Large	0.87645	0.0416	2751.387	1254.914262	341.681	124.08618	160.7838	12.007663	935.579	46.547722
S-P	DFR	Large	0.86522	0.0399	2793.5271	855.937428	417.518	121.19979	160.3137	10.684756	954.882	43.011271
Complex	IFR	Small	0.73506	0.0199	1771.9746	242.296206	636.269	81.977295	599.5027	26.616841	1001.37	62.470026
Complex	DFR	Small	0.74472	0.0884	1971.814	1053.069465	591.845	145.91455	568.734	87.423621	972.397	149.6474
Complex	IFR	Small	0.71677	0.051	1645.915	359.040565	633.405	89.892948	576.4315	38.333944	1039.75	99.237831
Complex	DFR	Small	0.74244	0.0324	1730.6547	489.090361	581.963	90.847932	558.1052	95.639121	973.869	84.669393
S-P	IFR	Small	0.78755	0.0218	3428.6584	502.256878	912.667	69.040485	601.2296	29.823233	931.045	42.499865
S-P	DFR	Small	0.77451	0.0418	2949.9511	514.004513	838.49	111.39209	578.9948	77.061728	993.46	116.64769
S-P	IFR	Small	0.76007	0.0584	3148.3652	987.897623	922.938	43.752593	607.5508	33.812394	968.5	60.030239
S-P	DFR	Small	0.78267	0.0314	3549.7397	741.920113	958.127	63.235759	571.3842	77.493459	978.117	113.25613
S-P	IFR	Small	0.65559	0.0172	1573.4532	157.864481	822.169	31.769495	599.1858	27.791716	980.591	27.843022
S-P	DFR	Small	0.58784	0.0772	1225.7376	388.082703	819.015	78.426313	539.2185	89.841237	944.66	111.6787
Complex	IFR	Small	0.65806	0.0219	1345.2839	117.197318	697.664	57.717031	572.4634	21.554528	980.978	60.770088
Complex	DFR	Small	0.67109	0.0622	1435.7829	365.305281	678.578	110.01039	589.7982	75.434413	1011.1	96.192313
S-P	IFR	Large	0.94244	0.0175	8397.0178	3438.66978	470.046	132.21821	163.135	5.218078	922.436	25.571554
S-P	DFR	Large	0.95354	0.0211	10122.331	6056.891589	389.935	111.03885	156.0759	11.852342	911.461	40.316235
Complex	IFR	Large	0.93384	0.0122	4155.5733	1040.334833	285.465	43.771342	162.8321	4.4553	912.337	27.591806
Complex	DFR	Large	0.9245	0.0354	3976.5947	3138.595902	225.842	64.706545	156.56	17.740922	933.373	39.069533

Table F-1. RAPTOR Output Database

III. ANALYSIS OBJECTIVES

Purpose of Investigation

The purpose of the investigation was to:

- 1) Ascertain whether one can distinguish between the complex versus series-parallel structures, IFR versus DFR configurations, and large versus small system sizes based on the simulation outputs;
- 2) Identify which output measures provide the most discriminant power;
- 3) See if one can adequately explain a majority of the output variance in smaller dimensions; and
- 4) Identify possible common underlying factors with common variance.

Variables Used

All 10 RAPTOR output variables were used in the analysis. In some cases, nearly equivalent results could be obtained by only using the most salient variables (this will be discussed in more detail in the results section of this report). Since there is a large disparity in magnitudes of the output variables, the variance-covariance matrix was sparse (contained many zeros). To alleviate computational problems resulting from this, standardized data was used for most analyses. The standardized data set is depicted in Table F-2.

When checking for multivariate normality for discriminant analysis, several of the variables did not pass the Shapiro-Wilk test for normality (at a 10% level of significance). In an attempt to achieve multivariate normality, a Box-Cox transformation was conducted

on all variables. The affects of the Box-Cox transformation on the passage of the Shapiro-Wilk test for each variable are shown in Table F-3.

Structure	Failure				Simulation Output Parameters							
	PDF	Size	Ao	Ao S.D.	MTBDE	MTBDE S.D.	MDT	MDT S.D.	MTBM	MTBM S.D.	MRT	MRT S.D.
Complex	IFR	Large	-1.23508	-0.60383	-0.84725	-0.63232609	-0.377552	-0.65269	-1.05905	-0.85128284	0.020338	-0.88381
Complex	DFR	Large	-1.61179	0.30777	-0.91868	-0.6139671	-0.370949	0.286585	-1.12462	-0.75528568	0.469788	0.242433
Complex	IFR	Large	-1.12264	-0.29764	-0.82035	-0.55770183	-0.381668	-0.15756	-1.05459	-0.75598425	-0.14697	-0.64425
Complex	DFR	Large	-1.49669	0.418498	-0.90323	-0.5875802	-0.455488	-1.1731	-1.1246	-0.74224371	0.47501	0.643005
S-P	IFR	Large	0.844632	-1.03029	-0.28944	-0.53335994	-1.197926	-0.98321	-1.0257	-0.8798533	-0.93838	-1.37044
S-P	DFR	Large	0.647505	0.483103	-0.2672	-0.08700604	-1.044875	-0.2159	-1.07163	-0.67572702	-1.18199	0.100208
S-P	IFR	Large	0.943237	-0.84711	-0.20639	-0.2615601	-1.255933	-1.24966	-1.01619	-0.77214022	-1.11923	-0.73532
S-P	DFR	Large	0.722469	1.214216	-0.31438	-0.279607	-1.213178	-0.55649	-1.06316	-0.67530254	-0.91744	-0.47848
Complex	IFR	Large	-0.97185	0.049844	-0.76724	-0.53601902	-0.302513	-0.23624	-1.06631	-0.80831024	0.404019	-0.9786
Complex	DFR	Large	-0.91481	0.904462	-0.75925	-0.52579255	-0.351082	-0.23485	-1.07566	-0.57619818	0.329852	-0.25443
S-P	IFR	Large	1.021919	-0.65423	0.091846	-0.28638907	-0.931079	-0.74232	-1.00412	-0.84059321	-0.77163	-1.18099
S-P	DFR	Large	0.949401	0.039314	-0.04034	-0.04672584	-1.025772	1.029537	-1.04755	-0.68502837	-0.78705	-0.53942
S-P	IFR	Large	1.148794	-0.06083	0.194534	0.42740982	-1.206594	0.849663	-1.02621	-0.69829653	-0.90642	-0.60828
S-P	DFR	Large	1.043613	-0.15406	0.217399	0.06889498	-0.825104	0.769555	-1.02841	-0.73270504	-0.41908	-0.70953
S-P	IFR	Large	1.766935	-1.38288	3.257803	2.38970111	-0.560873	1.075359	-1.0152	-0.87489209	-1.23822	-1.20886
S-P	DFR	Large	1.870899	-1.185	4.193942	4.74239756	-0.96386	0.487548	-1.04825	-0.70233639	-1.51529	-0.78669
Complex	IFR	Large	1.686361	-1.67371	0.956433	0.23459184	-1.48938	-1.37939	-1.01662	-0.89473177	-1.49317	-1.15102
Complex	DFR	Large	1.598865	-0.40272	0.858321	2.12005904	-1.789305	-0.79836	-1.04599	-0.54917577	-0.96212	-0.82239
Complex	IFR	Small	0.406776	-0.08683	0.067581	-0.16099517	0.1594215	-0.84841	1.067802	0.35360887	-0.21986	-0.78061
Complex	DFR	Small	0.164557	1.545412	-0.03394	0.30023091	0.11247	1.652698	1.137008	3.58515997	2.005838	0.835017
Complex	IFR	Small	0.364287	1.646762	-0.06888	-0.13953516	-0.121144	2.308175	0.992409	0.4534478	0.030039	0.981935
Complex	DFR	Small	0.260034	0.593831	-0.14566	-0.20943211	0.0082353	2.018415	0.877094	0.20582907	-0.16774	1.251295
S-P	IFR	Small	-0.97687	-0.39093	-0.42584	-0.53233745	1.4196953	0.113784	1.003119	0.23434278	1.141103	-0.69048
S-P	DFR	Small	-1.24016	0.904078	-0.57331	-0.3488097	1.0423186	0.977027	0.70355	1.15330022	3.173793	2.08054
S-P	IFR	Small	-1.04043	-0.68653	-0.46522	-0.46166124	1.3247934	0.166739	0.939305	-0.15089079	1.176339	-0.09271
S-P	DFR	Small	-0.80916	0.739385	-0.39869	-0.39747531	1.1601118	1.591992	0.93889	2.05451145	0.578095	1.728104
Complex	IFR	Small	-0.17557	-1.24933	-0.33689	-0.48251419	0.2752868	-0.31902	1.028203	-0.31831514	0.754506	-0.15239
Complex	DFR	Small	0.08506	2.50533	-0.22846	0.24603499	0.0518184	1.455487	0.884121	1.26325533	0.023086	2.343666
Complex	IFR	Small	-0.34686	0.45623	-0.40528	-0.37760939	0.260883	-0.09933	0.920167	-0.01355597	1.723533	0.900343
Complex	DFR	Small	-0.10642	-0.56577	-0.35931	-0.26074851	0.0021111	-0.07283	0.834349	1.47693861	0.060263	0.483221
S-P	IFR	Small	0.316085	-1.14524	0.562016	-0.24891727	1.6656677	-0.67807	1.03629	-0.23491762	-1.02088	-0.72417
S-P	DFR	Small	0.194006	-0.04854	0.302273	-0.23836102	1.2925298	0.497352	0.93217	0.99374486	0.554858	1.398821
S-P	IFR	Small	0.058729	0.861685	0.409931	0.18747251	1.717337	-1.37991	1.065891	-0.13116045	-0.07528	-0.22224
S-P	DFR	Small	0.270356	-0.62149	0.627713	-0.03355934	1.8943498	-0.83918	0.896531	1.00497408	0.167492	1.301714
S-P	IFR	Small	-0.91996	-1.39872	-0.4446	-0.55838332	1.2104309	-1.71249	1.026719	-0.28775691	0.229945	-1.14383
S-P	DFR	Small	-1.55452	1.891527	-0.63327	-0.35151254	1.1945657	-0.41758	0.745908	1.32613697	-0.67716	1.256549
Complex	IFR	Small	-0.89678	-1.14206	-0.5684	-0.59492625	0.5841287	-0.99234	0.901585	-0.44998475	0.239736	-0.20106
Complex	DFR	Small	-0.77479	1.066304	-0.5193	-0.37198001	0.4881198	0.459005	0.98276	0.95141877	1.000255	0.813144

Table F-2. Standardized Data Set

Variable	Large		Small		Complex		S-P		IFR		DFR	
	Before	After	Before	After	Before	After	Before	After	Before	After	Before	After
Ao	Fail	Fail	Fail	Fail	Pass	Pass	Pass	Pass	Fail	Fail	Pass	Pass
Ao S.D.	Pass	Pass	Pass	Pass	Pass	Pass	Pass	Pass	Pass	Pass	Pass	Pass
MTBDE	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass
MTBDE S.D.	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass
MDT	Pass	Pass	Fail	Fail	Fail	Pass	Fail	Fail	Pass	Pass	Pass	Pass
MDT S.D.	Pass	Pass	Pass	Pass	Fail	Pass	Pass	Pass	Pass	Pass	Pass	Pass
MTBM	Fail	Pass	Pass	Pass	Fail	Fail	Fail	Fail	Fail	Fail	Fail	Fail
MTBM S.D.	Pass	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Fail	Fail	Fail
MRT	Fail	Fail	Pass	Pass	Pass	Pass	Fail	Fail	Pass	Pass	Pass	Pass
MRT S.D.	Pass	Pass	Pass	Pass	Pass	Pass	Fail	Pass	Fail	Pass	Pass	Pass

* Boldface cells note where improvement was achieved

Table F-3. Effects of Box-Cox Transformation on Shapiro-Wilk Normality Test for Each Variable

From Table F-3, it is apparent that an improvement in the overall normality of the data was achieved. Although not all variables passed the Shapiro-Wilk test after the transformation, the majority of the variables did pass. Therefore, the assumption of multivariate normality was reasonably justified for use in discriminant analysis.

IV. ANALYSIS RESULTS

Special Problems Encountered

The most difficult problem encountered was the formatting of the data. As discussed previously, the large scale differences in the data caused numerical problems, but this was overcome via standardization. Another problem was the lack of multivariate normality, which was addressed by the use of Box-Cox transformations. In the end, several different data formats were tried (raw, standardized, and transformed) in the discriminant analysis to see what type of results would be achieved with each format.

When conducting logistic regression, SAS and JMP could not produce a viable regression model using all variables. This was true using the raw simulation output data, standardized data, as well as transformed data. However, a viable model was produced when the set of variables was reduced (down to 3) to those that were identified as most salient in the neural network analysis.

Another problem was the difficulty in interpreting the principal components. A 'clean' separation in the principal component loadings was not apparent, making the analysis challenging. Although principal components were defined from this analysis, the interpretation may be subject to debate due to the ambiguity in component loadings. However, after varimax rotation of the data, a much clearer loading structure was revealed in the subsequent factor analysis.

Discrimination Between Categories of Component Structures

Several multivariate techniques were used in an attempt to discriminate between large versus small, complex versus series-parallel, and increasing failure rate (IFR) versus

decreasing failure rate (DFR) component structures: DA, neural nets, logistic regression, as well as score rankings resulting from PCA and FA. The overall discriminant results for all methods are shown in Table F-4 for direct comparison.

Discriminant Results (percentages show classification accuracy)				
Data	Method	Large/Small	Complex/S-P	IFR/DFR
Standardized	SAS Pooled	100% / 100%	94% / 85%	95% / 95%
Transformed	SAS Pooled	100% / 100%	94% / 90%	89% / 95%
	SAS Unpooled	100% / 100%	100 % / 100%	----
	JMP	100% / 100%	94% / 90%	89% / 95%
Standardized	Full Neural Net: Training	----	93% / 100 %	----
	Full Neural Net: Validation	----	100% / 100%	----
	Reduced Neural Net: Training	----	98% / 100 %	----
	Reduced Neural Net: Validation	----	100% / 100%	----
Raw	Reduced Logistic Regression	----	67% / 85%	----
	Component Score Ranking	89% / 90%	----	84% / 74%
	Factor Score Ranking	100% / 100%	----	84% / 95%
	Best Discriminant Function	Linear	Quadratic	Linear
	Best Discriminant Variable(s)	MTBM	MTBDE Ao MRT MDT	MRT SD Ao SD MTBM SD MDT SD

Table F-4. Classification Accuracy for all Methods Used for Discrimination

For the most part, the results were consistent across methods with excellent discriminant success. There was strong agreement between methods on which variables served as the best discriminants (e.g. discriminant loadings, neural net salient variables, and components/factors which best discriminated for each category showed strong agreement). This general consistency across methods provided greater confidence in the overall analysis. The classification accuracy percentages for DA may be inflated because the entire data set was used. Logistic regression proved to be the weakest discriminant tool in the complex versus series-parallel case.

Neural Net Results

To help identify the variables which contributed most in discriminating between classes in the neural net, several graphical outputs produced by the Statistical Neural Network Analysis Package (SNNAP) software were reviewed. As an example, the following derivative graphs help show which variables had the greatest discriminant power. Looking at Figure F-3, the graphs with the more 'pointed' curves identify the more salient variables (A_0 , MDT, and MRT).

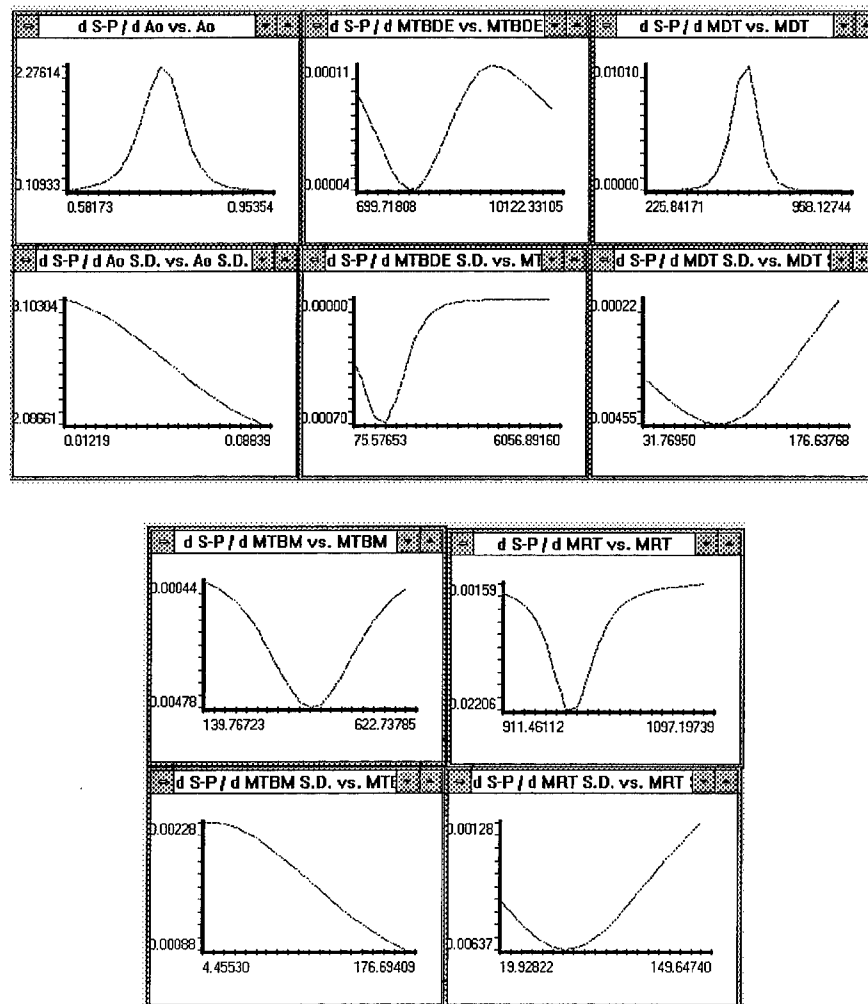


Figure F-3. Neural Net Derivative Saliency Graphs

Reduction in Dimensionality (PCA)

The objectives of performing a PCA on the database were to reduce the dimensionality of the data and to further attempt to discriminate between structure (by type, failure pdf, and size). Due to the difference in the units of the data, the PCA was performed using the data's correlation matrix (see Table F-5).

Variable	Ao	Ao S.D.	MTBDE	MTBDE S.D.	MDT	MDT S.D.	MTBM	MTBM S.D.	MRT	MRT S.D.
Ao	1	-0.318	0.7206	0.6327	-0.5495	0.0577	-0.332	-0.2459	-0.658	-0.3873
Ao S.D.	-0.3177	1	-0.371	-0.2081	0.1285	0.4976	0.2352	0.5285	0.344	0.692
MTBDE	0.7206	-0.371	1	0.9219	-0.1915	0.1148	-0.173	-0.1442	-0.472	-0.2649
MTBDE S.D.	0.6327	-0.208	0.9219	1	-0.3184	0.169	-0.255	-0.1122	-0.41	-0.1986
MDT	-0.5495	0.1285	-0.192	-0.3184	1	0.0143	0.8225	0.5198	0.544	0.4617
MDT S.D.	0.0577	0.4976	0.1148	0.169	0.0143	1	0.1952	0.455	0.268	0.5232
MTBM	-0.3315	0.2352	-0.173	-0.2554	0.8225	0.1952	1	0.7034	0.547	0.5575
MTBM S.D.	-0.2459	0.5285	-0.144	-0.1122	0.5198	0.455	0.7034	1	0.549	0.7272
MRT	-0.6576	0.3435	-0.472	-0.4099	0.5439	0.2676	0.5474	0.5487	1	0.5559
MRT S.D.	-0.3873	0.692	-0.265	-0.1986	0.4617	0.5232	0.5575	0.7272	0.556	1

Table F-5. Data Correlation Matrix

JMP software calculated the principle components. Three components were retained based on Kaiser's criterion. As Table F-6 indicates, these components accounted for 81.85% of the data set variation.

EigenValue:	4.6365	2.19	1.363	0.5431	0.4333	0.3292	0.2094	0.1776	0.101	0.0219
Percent:	46.3649	21.859	13.626	5.4311	4.3333	3.2919	2.0938	1.7758	1.006	0.2187
Cum Percent	46.3649	68.224	81.85	87.2808	91.614	94.906	96.9998	98.7756	99.78	100

Table F-6. Component Eigenvalues and Percentages

Using the eigenvalues and eigenvectors (eigenvector multiplied by the square root of the corresponding eigenvalue), JMP calculated the loadings matrix. As shown in Table F-7, only the first three loadings were analyzed.

	Component 1	Component 2	Component 3
Availability	-0.7264	0.49766	0.01967
Ao S.D.	0.615163	0.28342	-0.59
MTBDE	-0.607411	0.6781	0.35319
MTBDE S.D.	-0.572701	0.7072	0.16203
MDT	0.71309	-0.01032	0.6206
MDT S.D.	0.33791	0.6865	-0.3972
MTBM	0.74412	0.21079	0.52521
MTBM S.D.	0.75028	0.47291	0.0949
MRT	0.81152	-0.05729	0.067
MRT S.D.	0.79726	0.39407	-0.1759

Table F-7. PCA Loadings Matrix

After careful examination of the above loading matrix, in conjunction with knowledge of the database, each component was labeled based on the bold numbers in the respective column of the matrix.

- Component 1 → Maintenance Index
- Component 2 → Deviation Down Time Index
- Component 3 → Down Time Average Index

After successfully reducing the dimensionality of the database from ten to three, component scores were calculated to see if they were effective at discriminating a given structure into the following attributes:

- Type: Complex or Series-Parallel
- Failure pdf: Increase Failure Rate (IFR) or Decreasing Failure Rate (DFR)
- Size: Large or Small

Each vector of component scores was sorted in descending order to look for a pattern. The noticeable patterns appear in Table F-8.

Component 1		Component 2	
Size	Score	Failure pdf Type	Score
Small	3.9875798	DFR	4.2453607
Small	3.27563	IFR	2.7344832
Small	3.1916125	DFR	2.6642524
Small	2.5766016	DFR	2.2635533
Small	2.5624751	IFR	1.9337528
Small	2.3625727	DFR	1.4825866
Small	1.8048802	DFR	1.4374841
Small	1.7765076	DFR	1.0900088
Small	1.5808509	DFR	0.9908706
Small	1.5494862	DFR	0.6643567
Small	1.5341994	DFR	0.622185
Small	1.256253	IFR	0.5673482
Small	1.2471147	DFR	0.354911
Small	1.0250701	DFR	0.2859006
Small	0.6167541	DFR	0.2601498
Large	0.5833746	DFR	0.2469027
Small	0.4965454	DFR	0.0693093
Large	0.4641742	IFR	-0.134626
Small	0.4333046	IFR	-0.141776
Small	0.2313683	DFR	-0.239745
Large	0.2225142	IFR	-0.251629
Small	-0.096977	IFR	-0.376745
Large	-0.292476	DFR	-0.504691
Large	-0.394207	IFR	-0.505771
Small	-0.451488	IFR	-0.810156
Large	-0.550943	IFR	-0.821429
Large	-1.330643	IFR	-0.847647
Large	-1.357726	IFR	-0.899332
Large	-1.53975	IFR	-1.165003
Large	-1.637563	DFR	-1.242413
Large	-1.977937	DFR	-1.399575
Large	-2.244068	IFR	-1.485876
Large	-2.327493	IFR	-1.587855
Large	-2.39671	IFR	-1.683993
Large	-3.35344	IFR	-1.697677
Large	-3.691377	DFR	-1.879922
Large	-4.077877	IFR	-2.083892
Large	-5.058193	IFR	-2.153662

Table F-8. Component Scores

Even though the component scores do not discriminate completely, there appears to be some usefulness in these scores in determining the attributes of a given structure using the following formulas:

- If Component 1 Score $\geq 0 \rightarrow$ Classify the Structure as Small
- If Component 1 Score $< 0 \rightarrow$ Classify the Structure as Large
- If Component 2 Score $\geq 0 \rightarrow$ Classify the Structure as DFR
- If Component 2 Score $< 0 \rightarrow$ Classify the Structure as IFR

The component score 3 did not appear to have any discriminating power.

Identification of Underlying Factors (FA)

Factor analysis was performed on the database for two reasons: to identify any possible underlying factors and to use these factors to discriminate between the attributes of a given structure. Using SAS and varimax rotation, a rotated factor pattern was obtained. As can be seen in Table F-9, the underlying factors fell out very well.

	Factor 1	Factor 2	Factor 3
Avail	0.79513	-0.37048	-0.07841
Ao S.D.	-0.30561	0.02208	0.84431
MTBDE	0.9701	-0.01508	-0.11017
MTBDE S.D.	0.91457	-0.12722	0.04128
MDT	-0.21567	0.92026	-0.01965
MDT S.D.	0.21902	0.05537	0.832
MTBM	-0.09176	0.91097	0.18895
MTBM S.D.	-0.02499	0.64731	0.61313
MRT	-0.47654	0.57019	0.33782
MRT S.D.	-0.19501	0.46513	0.75333

Table F-9. Rotated Factor Pattern (from SAS with Varimax Rotation)

- Factor 1 → Functionality
- Factor 2 → Repair
- Factor 3 → Variance

The common variance contributions for each factor can be seen in Figure F-4.

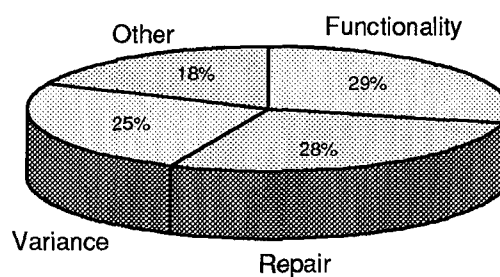


Figure F-4. Common Variance Contributions by Factor

Using standardized data, the factor scores were obtained for each of the three factors.

As with PCA, an attempt was made to discriminate a given structure by one of its three

attributes by sorting each factor score in descending order. As seen in Table F-10, factor scores 2 and 3 were very good at discriminating respectively between structure size and its failure rate pdf.

Size	Factor 2	Failure pdf Type	Factor 3
Small	1.754919849	DFR	2.375509734
Small	1.262692335	DFR	2.108713858
Small	1.257212778	IFR	1.879361215
Small	1.248601501	DFR	1.412724627
Small	1.198436102	DFR	1.338927349
Small	1.171639447	DFR	1.319662935
Small	1.160513013	DFR	0.809561313
Small	1.148780849	DFR	0.752946945
Small	1.070286433	DFR	0.476564177
Small	1.041964654	IFR	0.402846391
Small	0.803497969	DFR	0.39528509
Small	0.781802256	DFR	0.351588765
Small	0.708731145	DFR	0.312586852
Small	0.645365044	IFR	0.298188266
Small	0.636668185	DFR	0.287840003
Small	0.616611494	DFR	0.235440949
Small	0.581937118	DFR	0.217068964
Small	0.058035318	DFR	0.0410742
Small	0.045624986	DFR	-0.116572823
Small	-0.135169869	DFR	-0.154178944
Large	-0.157786373	DFR	-0.209412144
Large	-0.304504477	IFR	-0.331720076
Large	-0.712301993	IFR	-0.357517609
Large	-0.754459578	IFR	-0.382424315
Large	-0.796558728	DFR	-0.52311732
Large	-0.818000077	IFR	-0.624436134
Large	-0.874071803	IFR	-0.652270592
Large	-0.917883041	IFR	-0.658484054
Large	-0.925917777	IFR	-0.728537638
Large	-0.975227776	IFR	-0.751696689
Large	-1.066756545	IFR	-0.788207166
Large	-1.081593052	IFR	-0.84117866
Large	-1.104270837	IFR	-0.871777043
Large	-1.109476158	IFR	-1.021398497
Large	-1.301022087	IFR	-1.239891531
Large	-1.306582726	IFR	-1.303301235
Large	-1.327933971	IFR	-1.459488901
Large	-1.523803607	IFR	-2.00028026

Table F-10. Factor Scores

- If Factor Score 2 ≥ -0.15 → Classify the Structure as Small
- If Factor Score 2 < -0.15 → Classify the Structure as Large
- If Factor Score 3 ≥ -0.30 → Classify the Structure as DFR
- If Factor Score 3 < -0.30 → Classify the Structure as IFR

Factor score 1 did not appear to have any discriminating power.

Insights

Several useful conclusions can be drawn from this study. First, it was demonstrated (using a moderately small sample size) that successful discrimination can occur between large versus small, complex versus series-parallel, and IFR versus DFR component structures based on RAPTOR simulation output. All multivariate techniques demonstrated were moderately-to-highly successful in discriminating between the defined classes. Through the discrimination analysis, it was discovered that predominantly DFR structures display a relatively higher simulation output variability. Therefore, RAPTOR availability model output variability serves as a good discriminant for IFR versus DFR structures. Furthermore, Mean Time Between Maintenance (MTBM) is an excellent discriminant variable for the large versus small structure classification case. This conclusion makes intuitive sense, since one would expect a decrease in the average time between maintenance actions on components as the number of components in the structure increases. The analysis provides empirical support to this intuitive assessment. Additionally, it was discovered that neural nets can be used to effectively discriminate when the discriminant function may be of a higher order.

Additionally, the analysis revealed that the RAPTOR simulation output variance can be explained via 3 principal components: a maintenance index, a deviation down time index, and a down time average index. A majority of the output variance (82%) is explained by these three components. By using a rank order of the maintenance index

(component 1) scores and deviation down time index (component 2) scores, reasonable discrimination between large and small structures, and IFR and DFR structures respectively, can be achieved.

Finally, three underlying factors were identified by use of factor analysis. The first factor, functionality, relates to the structure's ability to get the job done in an efficient manner. The second factor, repair, reflects the maintenance and down time which is inherent in the structure. The third factor, variance, refers to the inherent variability of the output variables measured for each structure. Some success was also achieved in discrimination between large versus small structures and IFR versus DFR structures by using a rank order of the repair factor (factor 2) scores and variance factor (factor 3) scores respectively.

Throughout the discrimination analysis, consistency in the results was observed for each of the various methods used: similar classification accuracy and similar best discriminant variable selections. This consistency was further highlighted when component/factor score rankings were used as a discriminant. For example, based on the DA observations one would expect the factor which represents maintenance/repair (factor 2) to be the best in large versus small discrimination. This in fact was the case, with the factor 2 scores being the best large/small discriminant among all factor scores. The same proved true for factor 3 (variability) and IFR versus DFR discrimination. This consistency in results provided increased confidence in the conclusions.

References

1. Kapur, K. C. and L. R. Lamberson. *Reliability in Engineering Design*. New York: John Wiley & Sons, Inc., 1977.
2. Boyd, Mark A. and Salvatore J. Bavuso. "Simulation Modeling for Long Duration Spacecraft Control Systems," *Proceedings of the Annual Reliability and Maintainability Symposium*. 106-112. IEEE, 1993.
3. Hoyland, Arnljot, and Marvin Rausand. *System Reliability Theory: Models and Statistical Methods*. New York: John Wiley & Sons, Inc., 1995.
4. Birnbaum, Z. W. "On the Importance of Different Components in a Multicomponent System," *Multivariate Analysis II* (P.R. Krishnaiah, Ed): 581-592. New York: Academic Press, 1969.
5. Boland, Phillip J. and Emad El-Newehi. "Measures of Component Importance in Reliability Theory," *Computers and Operations Research*, 22-4: 455-463 (1995).
6. Papastavridis, Stavros. "The Most Important Component in a Consecutive-k-out-of-n: F System," *IEEE Transactions on Reliability*, R-36: 266-268 (June 1987).
7. Xie, M. "On Ranking of System Components with respect to Different Improvement Actions," *Microelectronics and Reliability*, 29-2: 159-164 (1989).
8. Barlow, Richard E. and Frank Proschan. "Importance of System Components and Fault Tree Events," *Stochastic Processes and Their Applications*, 3: 153-173 (1975).
9. Aven, Terje. "On the Computation of Certain Measures of Importance of System Components," *Microelectronics and Reliability*, 26-2: 279-281 (1986).
10. Finkelstein, M. S. "Once More on Measures of Importance of System Components," *Microelectronics and Reliability*, 34-9: 1431-1439 (1994).
11. Hunter, J. S. and T. H. Naylor. "Experimental Designs for Computer Simulation Experiments," *Management Science*, 16-6: 422-433 (March 1970).
12. Law, Averill M., and W. David Kelton. *Simulation Modeling and Analysis*. New York: McGraw-Hill, Inc., 1991.
13. Smith, Dennis E. and Carl A. Mauro. "Factor Screening in Computer Simulation," *Simulation*: 49-54 (February 1982).

14. Box, G. E. P. and R. Daniel Meyer. "Finding the Active Factors in Fractionated Screening Experiments," *Journal of Quality Technology*, 25-2: 94-105 (April 1993).
15. Hamanda, Michael. "Using Statistically Designed Experiments to Improve Reliability and to Achieve Robust Reliability," *IEEE Transactions on Reliability*, 44-2: 206-215 (June 1995).
16. Montgomery, Douglas C. *Methods for Factor Screening in Computer Simulation Experiments*. Contract N00014-78-C-0312. Technical Report, Georgia Institute of Technology, March 1979.
17. Hamanda, Michael and C. F. J. Wu. "Analysis of Designed Experiments with Complex Aliasing," *Journal of Quality Technology*, 24-3: 130-137 (July 1992).
18. Plackett, R. L. and J. P. Burman. "The Design of Optimum Multifactorial Experiments," *Biometrika*, 33: 305-325 (1946).
19. Lin, Dennis K. J. and Norman R. Draper. "Projection Properties of Plackett and Burman Designs," *Technometrics*, 34: 423-428 (November 1992).
20. Myers, Raymond H. and Douglas C. Montgomery. *Response Surface Methodology: Process and Product Optimization Using Designed Experiments*. New York: John Wiley & Sons, Inc., 1995.
21. Box, George E. P., William G. Hunter, and J. Stuart Hunter. *Statistics for Experimenters*. New York: John Wiley & Sons, Inc., 1978.
22. John, Peter W. M. "Three-Quarter Replicates of 2^4 and 2^5 Designs," *Biometrics*, 17: 319-321 (June 1961).
23. Wolf, James R. *Sensitivity of Space System Availability Predictions to Underlying Component Reliability Estimates*. MS Thesis, AFIT/GSO/ENS/89D-17. School of Engineering, Air Force Institute of Technology (AU), Wright-Patterson AFB OH, December 1989 (AD-A215535).
24. Edgar, John F. and Tony Bendell. "The Robustness of Markov Reliability Models," *Quality and Reliability Engineering, International*, 2: 117-125 (1986).
25. Hwang, C. I., Frank A. Tillman, and M. H. Lee. "System-Reliability Evaluation Techniques for Complex-Large Systems-A Review," *IEEE Transactions on Reliability*, R-30, 5: 416-422 (December 1981).

26. Mortin, David E., Jane G. Krolewski, and Michael J. Cushing. "Consideration of Component Failure Mechanisms in the Reliability Assessment of Electronic Equipment - Addressing the Constant Failure Rate Assumption," *Proceedings of the Annual Reliability and Maintainability Symposium*. 54-57. IEEE, 1995.
27. Kline, M. B. "Suitability of the Lognormal Distribution for Corrective Maintenance Repair Times," *Reliability Engineering*, 9: 65-80 (1984).
28. Webb, Timothy S. and Kenneth W. Bauer, Jr. "Comparison of Analysis Strategies for Screening Designs in Large-Scale Computer Simulation Models," *Proceedings of the 1994 Winter Simulation Conference*. 305-311. IEEE, 1995.
29. Jacobson, David W. and Sant Ram Arora. "A Nonexponential Approach to Availability Modeling," *Proceedings of the Annual Reliability and Maintainability Symposium*. 253-260. IEEE, 1995.
30. Air Force Operational Test and Evaluation Center, *Rapid Availability Prototyping for Testing Operational Readiness (RAPTOR) Version 2 Software User's Manual*, 1995.
31. ReliaSoft, Inc., *Weibull++ Version 4 User's Manual*, 1995.
32. Box, George E. P., and Norman R. Draper. *Empirical Model-Building and Response Surfaces*. New York: John Wiley & Sons, Inc., 1987.
33. Banks, Jerry, John S. Carson II, and Barry L. Nelson. *Discrete-Event System Simulation*. Upper Saddle River: Prentice Hall, Inc., 1996.
34. SAS Institute, Inc., *JMP Statistics and Graphics Guide, Version 3.1*, 1995.

Vita

Major Darren P. Durkee [REDACTED] In June 1983, he received his Bachelor of Science Degree in Operations Research from the United States Air Force Academy. Upon graduation, he attended Undergraduate Navigation Training (UNT) and Electronic Warfare Training at Mather AFB, CA. He subsequently served in several flying assignments compiling over 4000 hours as a navigator and electronic warfare officer (EWO), including tours flying the RC-135 Rivet Joint at Offutt AFB, NE, the EC-130H Compass Call at Sembach AB, Germany, and the E-3 Airborne Warning and Control System (AWACS) at Tinker AFB, OK. During his flying tours, he served in several staff positions at the Wing and Air Division level in the areas of training, standardization and evaluation, requirements, and weapons and tactics. Major Durkee was selected to attend the AFIT Graduate Program in Operations Research in 1995. Upon graduation from AFIT in March 1997, he was assigned to the Air Staff as an operations analyst in the Air Force Studies and Analysis Agency, Pentagon, Washington, D.C.

[REDACTED]

[REDACTED]

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.				
1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE February 1997	3. REPORT TYPE AND DATES COVERED Master's Thesis		
4. TITLE AND SUBTITLE SENSITIVITY OF AVAILABILITY ESTIMATES TO INPUT DATA CHARACTERIZATION		5. FUNDING NUMBERS		
6. AUTHOR(S) Darren P. Durkee, Major, USAF				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Air Force Institute of Technology/ENS 2750 P Street Wright-Patterson AFB, Ohio 45433-7765		8. PERFORMING ORGANIZATION REPORT NUMBER AFIT/GOR/ENS/97-06		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) HQ AFOTEC/SAL 8500 Gibson Blvd. SE Kirtland AFB, NM 87117		10. SPONSORING/MONITORING AGENCY REPORT NUMBER		
11. SUPPLEMENTARY NOTES				
12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for Public Release; Distribution is Unlimited		12b. DISTRIBUTION CODE		
13. ABSTRACT (Maximum 200 words) Reliability analysts are often faced with the challenge of characterizing the behavior of system components based on limited data. Any insight into which model input data is most significant and how much data is necessary to achieve desired accuracy requirements will improve the efficiency and cost effectiveness of the data collection and data characterization processes. This thesis assesses potential significant factors in the probabilistic characterization of component failure and repair behavior with respect to the effect on system availability estimates. Potential factors were screened for significance utilizing fractional factorial and Plackett-Burman experimental designs for several system models developed using an AFOTEC simulation program entitled RAPTOR. Two input data characterization factors were found to have a significant affect on availability estimation accuracy: the size of the structure and the number of data points used for component failure and repair distributional fitting. Estimation error was minimized when the structures analyzed were small and many data points (in this case, 25) were used for the distributional fittings. Assuming constant component failure rates and using empirical repair distributions were found to be equally effective component characterization methods (pertaining to model availability estimation error) compared to using automated software fitting tools (or 'wizards'). The results of this study also indicate that there is no apparent benefit in concentrating on 'important' components for the highest fidelity distributional fittings.				
14. SUBJECT TERMS Availability Estimation, Fractional Factorial Experiment, Component Reliability, Distributional Assumptions		15. NUMBER OF PAGES 134		
		16. PRICE CODE		
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT UL	