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13. ABSTRACT *(Maximum 200 words)*

The study of nonlinear vibrations, stability, and dynamics of structures underlies all applications in modern dynamical systems. Indeed, improving our understanding of the responses of nonlinear systems would contribute to the advancement of control systems, machine dynamics, acoustics, and noise control. It would also contribute to improving our national productivity and competitiveness by opening the design space.

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FINAL REPORT
SIXTH CONFERENCE ON NONLINEAR VIBRATIONS, STABILITY, AND DYNAMICS OF STRUCTURES

DAAH004-96-1-0021

June 9-13, 1996

Ali H. Nayfeh and Dean T. Mook
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March 11, 1997

19970514 176

Objectives

The Sixth conference was held on June 9-13, 1996 and had the following basic objectives:

1. Development of a better understanding of nonlinear dynamical phenomena,
2. Assessment of the state of the art of experimental, computational, and analytical techniques for nonlinear dynamics, including presentations from the leading international researchers,
3. Exchange of ideas among the leading national researchers,
4. Suggestions for directions of future research, and
5. Bringing graduate students in contact with the experts in nonlinear dynamics, giving some of them an opportunity to present their research, and helping them to know the state of the art.

Scope

The Sixth conference focused on the following topics:

1. Multibody dynamics,
2. Vehicle dynamics,
3. Rotorcraft dynamics,

4. Modal interactions
5. Nonlinear modes and localization,
6. Parametrically excited vibrations: single- and multi-frequency excitations of single- and multi-degree-of-freedom systems,
7. Analytical methods,
8. Computational techniques: efficient algorithms, use of symbolic manipulators, integration of symbolic manipulation and numerical methods, and use of parallel processors,
9. Experimental methods: benchmark experiments, measurements in hostile environments, and instrumentation techniques,
10. Structural control,
11. Identification of nonlinear systems,
12. Dynamics of composite structures,
13. Dynamics of adaptive structures, and
14. Fluid/structure interactions.

Report

The program consisted of invited and contributed papers. The leading researchers were identified and invited to give presentations. Potential contributors were asked to submit a 1000-word abstract by November 1, 1995. They were notified of their acceptance by January 10, 1996. Full-length papers were due on April 1, 1996. Ninety-seven papers were presented in either an oral or poster form. Two-page extended abstracts of the invited and the contributed presentations were published in a Proceedings that was available at the time of the conference. Please find attached ten copies of the Proceedings, including the Program.

The conference was well-received and attended by many prominent researchers. It provided a forum for many fruitful exchanges of ideas and for an assessment of the state of the art. The participants agreed that it was also very successful in: a) contributing to the development of a better understanding of nonlinear phenomena, b) providing an assessment of the state of the art of experimental, computational, and analytical techniques for nonlinear dynamics, and c) fostering an exchange of ideas among leading researchers and a discussion of priorities for future research.

One hundred seventy participants came from many countries, including Canada, Germany, Italy, Great Britain, Japan, Greece, Bulgaria, Jordan, Taiwan, Peoples Republic of China, Hong Kong, and Brazil. The papers were uniformly good; many were submitted for publication in the journal **Nonlinear Dynamics** and the **Journal of Vibration and Control.** Many papers were reviewed by participants who were in the audience during the presentation. This review process contributed to the lively discussions following many of the presentations and considerably reduced the time needed to evaluate submitted papers.

SIXTH CONFERENCE ON NONLINEAR VIBRATIONS, STABILITY, AND DYNAMICS OF STRUCTURES

June 9-13, 1996

Program

Partially Sponsored by the
United States Army Research Office
Virginia Polytechnic Institute and State University
Wolfram Research, Inc., Champaign, IL

In honor of Professor Milton Van Dyke
for his outstanding contributions
to the science of nonlinear phenomena

Chairmen:
Ali H. Nayfeh and Dean T. Mook
Department of Engineering Science and Mechanics
Virginia Polytechnic Institute and State University
Blacksburg, Virginia 24061-0219

Professor Milton Van Dyke

Sunday, June 9
1320-1330 **Opening Remarks**

Session 1. Analysis I

Chairmen: Y. Ishida, Nagoya University, Nagoya, JAPAN and L. N. Virgin, Duke University, Durham, NC

1330-1510

An Analysis of Roll/Sway/Heave Dynamics in Relation to Vessel Capsizing in Regular Beam Seas
S-L. Chen, S. W. Shaw, Michigan State University, East Lansing, MI, and A. W. Troesch, The University of Michigan, Ann Arbor, MI

Hydrodynamic and Geometric Stiffening Effects on the Out-of-Plane Waves of Submerged Cables
M. Behbahani-Nejad and N. C. Perkins, The University of Michigan, Ann Arbor, MI

Nonlinear Behavior of a Typical Airfoil Section with Control Surface Freeplay
M. D. Conner, D. M. Tang, E. H. Dowell, and L. N. Virgin, Duke University, Durham, NC

Analytical Methods for Rigid Body Dynamics with Friction and Unilateral Constraints
A. Sinopoli, Istituto Universitario di Architettura, Venezia, ITALY

Stability Boundaries for the Swinging Heart
J. A. Morillo and P. V. Bayly, Washington University, St. Louis, MO

1510-1530 **Break**

Session 2. Modal Interactions I

Chairmen: A. C. Soudack, The University of British Columbia, British Columbia, CANADA and H. W. Haslach, Jr., University of Maryland Baltimore County, Baltimore, MD

1530-1710

In-Plane and Out-of-Plane Oscillations of a Non Resonant Suspended Cable with a Longitudinal Control
M. Pasca, V. Gattulli, and F. Vestroni, Universitá di Roma "La Sapienza", Roma, ITALY

Nonplanar Oscillation of Wire Electrode and its Relaxation in System of Wire and Plate Electrodes
M. Yoshizawa, Y. Itoh, T. Tsuzuki, and S. Nakazato, Keio University, Yokohama, JAPAN

Slow Waves in a One-Dimensional Array of Soft-Stiff Pair of Oscillators
I. T. Georgiou, Naval Research Laboratory, Washington, DC and A. F. Vakakis, University of Illinois at Urbana-Champaign, Urbana, IL

Nonplanar Vibrations of a Forced, Buckled Beam
K. Yagasaki, Gifu University, Gifu, JAPAN

Internal Resonance of the Jeffcott Rotor (Critical Speed of Twice the Major Critical Speed)
Y. Ishida and T. Inoue, Nagoya University, Nagoya, JAPAN

1900-2100 **Reception**

Monday, June 10

Session 3. Control

Chairmen: A. Kurdila, Army Research Office, Research Triangle Park, NC and A. F. Vakakis, University of Illinois at Urbana-Champaign, Urbana, IL

0830-1010

Inertia Parameter Identification of a Rigid Body Using a Measurement Robot
H. Hahn, M. Niebergall, and F. Hecker, University of Kassel, Kassel, GERMANY

Dynamic Properties of Pseudoelastic Shape Memory Alloys
D. Z. Li and Z. C. Feng, Massachusetts Institute of Technology, Cambridge, MA

Design and Control of Nonlinear Structures
X. Song, M. J. Schulz, and P. F. Pai, North Carolina A&T State University, Greensboro, NC

Stabilization of a Nonlinear String by the Boundary Control
S. M. Shahruz, Berkeley Engineering Research Institute, Berkeley, CA

Energy-Based Control of Flexible Structures using a Digital Algorithm
H. L. Young and M. F. Golnaraghi, University of Waterloo, Waterloo, Ontario, CANADA

1010-1030 **Break**

Session 4. Bifurcations

Chairmen: A. K. Bajaj, Purdue University, West Lafayette, IN and S. W. Shaw, Michigan State University, East Lansing, MI

1030-1210

Chaotic Dynamics in the Cooperrider Truck
C. N. Jensen and H. True, The Technical University of Denmark, Vedbaek, DENMARK

Experimental Study of the Indeterminate Bifurcation Using a Gravity-Loaded 'Roller Coaster'
M. D. Todd and L. N. Virgin, Duke University, Durham, NC

Bifurcations and Chaos in a Forced Vibratory System with an Asymmetric Piecewise Linear-Restoring Force
M. Kuroda, M. Nakai, T. Hikawa, and Y. Matsuki, Ministry of International Trade and Industry, Ibaraki, JAPAN

Bifurcation Control of Parametrically Excited Duffing System by a Combined Linear-Plus-Nonlinear Feedback Control
H. Yabuno, University of Tsukuba, Tsukuba-City, JAPAN

Computation of a Hopf Bifurcation by Using an Asymptotic-Numerical Algorithm
M. E. H. Bensaadi, A. Tri, Université de Metz, Metz, FRANCE, B. Cochelin, Université de Marseille II, Marseille, FRANCE, and M. Potier-Ferry, Université de Metz, Metz, FRANCE

1210-1330 **Lunch**

Session 5. Poster Session

Chairmen: H. G. Davies, University of New Brunswick, Fredericton, New Brunswick, CANADA G. T. Flowers, Auburn University, Auburn, AL

1330-1510

Papers listed at end.

1510-1530 **Break**

Session 6. Computational Methods I

Chairmen: B. Sanders, Air Force Office of Scientific Research, Bolling Air Force Base, DC and D. E. Gilsinn, National Institute of Standards and Technology, Gaithersburg, MD

1530-1710

Symbolic Computation of Stability and Bifurcation Surfaces for Nonlinear Systems with Strong Parametric Excitation
S. C. Sinha and E. A. Butcher, Auburn University, Auburn, AL

Real Time Animation of Ocean Waves
B. T. Nohara, Mitsubishi Heavy Industries, Ltd., Nagoya, JAPAN

Real-Time Dimension Estimation Applied to the GearBox Dynamics
D. C. Lin and P. Fromme, University of Waterloo, Waterloo, Ontario, CANADA

On the Development of a New Finite Element Strategy to Study Nonlinear Dynamical Systems with Time Varying Coefficients
S. A. Q. Siddiqui, M. F. Golnaraghi, and G. R. Heppler, University of Waterloo, Waterloo, Ontario, CANADA

Forced Response Analysis of a Dry-Friction Damped Oscillator Using an Efficient Hybrid Frequency-Time Method
J. Guillen and C. Pierre, The University of Michigan, Ann Arbor, MI

Session 7. Multibody Dynamics I

Chairman and Chairwoman: F. Vestroni, Universitá di Roma "La Sapienza", Roma, ITALY and S. W. Smith, University of Kentucky, Lexington, KY

0830-1010

Dynamics of a Flexible Slider-Crank Mechanisms Driven by a Non-Ideal Source of Energy
J. Wauer and P. Bührle, Universität Karlsruhe, Karlsruhe, GERMANY

Dynamic Parameter Estimation of Mechanisms
S. S. Shome and D. G. Beale, Auburn University, Auburn, AL

Dynamics of CVT Gears
F. Pfeiffer, Technische Universität München, München, GERMANY

Comparison of Various Techniques for Modeling Flexible Beams in Multibody Dynamics
P. Fisette, J. C. Samin, and R. E. Valembois, University of Louvain, Louvain-la-Neuve, BELGIUM

Modelling of Spatial Mechanical Systems using Graph Theory and Branch Coordinates
J. McPhee, University of Waterloo, Waterloo, Ontario, CANADA

1010-1030 **Break**

Session 8. Friction and Dampers

Chairmen: K. Yasuda, Nagoya University, Nagoya, JAPAN and M. F. Golnaraghi, University of Waterloo, Waterloo, Ontario, CANADA

1030-1210

Friction in Nonlinear Dynamics: An Historical Review
B. Feeny, Michigan State University, East Lansing, MI and A. Guran, University of Southern California, Los Angeles, CA

Experimental and Numerical Study of Low Frequency Stick-Slip Motion
V. G. Oancea and T. A. Laursen, Duke University, Durham, NC

Adaptive Backlash Compensation
G. Tao, University of Virginia, Charlottesville, VA

Nutation Damping of Flow Induced Oscillations: Analysis and Experiments
V. J. Modi and M. L. Seto, University of British Columbia, Vancouver, British Columbia, CANADA

Nonlinear Vibration of a Rotating System with an Electromagnetic Damper and a Cubic Restoring Force
Y. Kligerman, O. Gottlieb, and M. Darlow, Technion-Israel Institute of Technology, Haifa, ISRAEL

1210-1330 **Lunch**

Session 9. Nonstationary and Random Vibrations

Chairmen: S. Hanagud, Georgia Institute of Technology, Atlanta, GA and M. Nakai, Kyoto University, Kyoto, JAPAN

1330-1510

Railway Wheel Squeal (Squeal of Disk Subjected to Random Excitation)
M. Nakai, Kyoto University, Kyoto, JAPAN and S. Akiyama, Kawasaki Heavy Industry Co., Ltd., Kobeshi, JAPAN

Nonlinear Random Response of Ocean Structures Using Second-Order Stochastic Averaging
M. Hijawi, R. A. Ibrahim, and N. Moshchuk, Wayne State University, Detroit, MI

Nonstationary Period Doubling Bifurcations
H. G. Davies and K. Rangavajhula, University of New Brunswick, Fredericton, New Brunswick, CANADA

Dynamic Bifurcations in Systems with 1:2 Internal Resonance under Nonstationary External Excitation
B. Banerjee, A. K. Bajaj, and P. Davies, Purdue University, West Lafayette, IN

On Non-Ideal Passage Through Resonance: Modeling and General Characteristics
J. M. Balthazar, State University of São Paulo at Rio Claro, Rio Claro, BRAZIL, H. I. Weber, State University of Campinas, Campina, BRAZIL, and D. T. Mook, Virginia Polytechnic Institiute and State University, Blacksburg, VA

1510-1530 **Break**

Session 10. Analysis II

Chairman and Chairwoman: M. R. M. Crespo da Silva, Rensselaer Polytechnic Institute, Troy, NY and A. Sinopoli, Istituto Universitario di Architettura, Venezia, ITALY

1530-1710

Dynamics of Delaminated Beams
H. Luo and S. Hanagud, Georgia Institute of Technology, Atlanta, GA

Chaotic Vibrations of a Cylindrical Shell-Panel with an In-Plane Elastic-Support at Boundary
T. Yamaguchi, Subaru Research Center, Co., Ltd., Gunma, JAPAN and K. Nagai, Gunma University, JAPAN

Asymptotic Research of Nonlinear Wave Processes in Saturated Porous Media
I. Edelman, Russian Academy of Sciences, Moscow, RUSSIA

Resonance Capture in a Three-Degree-of-Freedom System
D. D. Quinn, The University of Akron, Akron, OH

Aeroelastic Response of a Two-Dimensional Airfoil-Aileron Combination with Freeplay Nonlinearities
H. Alighanbari and S. J. Price, McGill University, Montreal, CANADA

1900 **Banquet**

Wednesday, June 12

Session 11. Modal Interactions II

Chairmen: D. Marghitu, Auburn University, Auburn, AL and M. Yoshizawa, Keio University, Yokohama, JAPAN

0830-1010

Evolution of Domains of Attraction of a Forced Beam with Two Mode Interaction
W. K. Lee, Yeungnam University, Gyongsan, KOREA and C. H. Kim, Korea Heavy Industry Co., Changwon, KOREA

Nonlinear Dynamics of Cyclically Coupled Duffing Oscillators
C. N. Folley, A. K. Bajaj, Purdue University, West Lafayette, IN, and O. D. I. Nwokah, Southern Methodist University, Dallas, TX

Nonlinear Modal Interaction and Chaos in an Experimental Cable/Mass Suspension
G. Rega, Universitá di Roma "La Sapienza", Roma, ITALY, F. Benedettini, Universitá dell'Aquila, L'Aquila, ITALY, and R. Alaggio, Universitá di Roma "La Sapienza", Roma, ITALY

Multi-Modal Nonlinear Dynamics in Machine Tool Cutting Processes
D. E. Gilsinn and M. Davies, National Institute of Standards and Technology, Gaithersburg, MD

On the Dynamics of Multiple Centrifugal Pendulum Vibration Absorbers
C-P. Chao, S. W. Shaw, and C-T. Lee, Michigan State University, East Lansing, MI

1010-1030 **Break**

Session 12. Computational Methods II

Chairmen: T. D. Burton, Texas Tech University, Lubbock, TX and V. J. Modi, University of British Columbia, Vancouver, British Columbia, CANADA

1030-1210

Nonlinear Dynamics of Shells Under Finite Deformations. A Finite Element of Approach, Integration Schemes, and Applications to the Chaotic Motion
C. Sansour and J. Sansour, Technische Hochschule Darmstadt, Darmstadt, GERMANY

A Time Integration Algorithm for Flexible Mechanism Dynamics: The DAE α-Method
L. Petzold, S. Raha, and J. Yen, University of Minnesota, Minneapolis, MN

Computer Implementation of Nonlinear Dynamical Problems with Maple
N. E. Sanchez, The University of Texas, San Antonio, TX

Karhunen-Loéve Decomposition in Dynamical Modeling
S. R. Sipcic, A. Benguedouar, and A. Pecore, Wolfram Research, Inc., Champaign, IL

Using the Karhunen-Loéve Decomposition to Examine Chaotic Snap-Through Oscillations of a Buckled Plate
K. D. Murphy, University of Nebraska, Lincoln, NE

1210-1330 **Lunch**

Session 13. Analysis III

Chairman and Chairwoman: H. True, The Technical University of Denmark, Vedbaek, DENMARK and P. Davies, Purdue University, West Lafayette, IN

1330-1510

Numerical Solutions of Second-Order Implicit Nonlinear Ordinary Differential Equations
C. Semler, W. C. Gentleman, and M. P. Paidoussis, McGill University, Montreal, CANADA

Model Reduction in Nonlinear Structural Dynamics
T. D. Burton, Texas Tech University, Lubbock, TX

Experimental Nonlinear Localization in a Vibro-Impact System
E. Emaci, T. A. Nayfeh, and A. F. Vakakis, University of Illinois at Urbana-Champaign, Urbana, IL

Free Vibration Analysis of Rotating Nonlinearly Elastic Structures with Symmetry: An Efficient Group-Equivariance Approach
T. M. Whalen, University of Illinois at Urbana-Champaign, Urbana, IL and T. J. Healey, Cornell University, Ithaca, NY

Dynamic Response to Singularities in the Potential for Mechanical Systems
H. W. Haslach, Jr., University of Maryland Baltimore County, Baltimore, MD

1510-1530 **Break**

Session 14. Rotors

Chairmen: N. C. Perkins, The University of Michigan, Ann Arbor, MI and S. R. Sipcic, Wolfram Research, Inc., Champaign, IL

1530-1710

On the Modeling and Response Analysis of Helicopter Rotor Blades
M. R. M. Crespo da Silva, Rensselaer Polytechnic Institute, Troy, NY

Stability and Control of a Parametrically Excited Flexible Rotating Beam
D. Boghiu, D. Marghitu, and S. C. Sinha, Auburn University, Auburn, AL

An Experimental Identification Technique for a Nonlinear Rotating Shaft System
K. Yasuda, K. Kamiya, and H. Yamauchi, Nagoya University, Nagoya, JAPAN

Use of a Flexible Internal Support to Suppress Vibrations of a Rotating Shaft Passing Through a Critical Speed
S. Suherman and R. H. Plaut, Virginia Polytechnic Institute and State University, Blacksburg, VA

A Study of Housing Effects on the Rotordynamics of a Shaft Supported by a Clearance Bearing: Analysis and Experiment
J. L. Lawen, Jr., General Electric Corporate Research and Development Center, Schenectady, New York and G. T. Flowers, Auburn University, Auburn, AL

Thursday, June 13

Session 15. Multibody Dynamics II

Chairmen: J. Wauer, Universität Karlsruhe, Karlsruhe, GERMANY and C. Pierre, The University of Michigan, Ann Arbor, MI

0830-1010

Impacts with Friction and Normal and Tangential Reversibility in Multibody Systems
M. Beitelschmidt and F. Pfeiffer, Technische Universität München, München, GERMANY

Impact Analysis with Friction in Open-Loop Multibody Systems
T. Shakil, H. M. Lankarani, Wichita State University, Wichita, KS, and M. S. Pereira, Technical University of Lisbon, Lisboa PORTUGAL

Impact Dynamics in Milling of Thin-Walled Structures
M. A. Davies, National Institute for Standards and Technology, Gaithersburg, MD and B. Balachandran, University of Maryland, College Park, MD

Multicriteria Optimization of a Machine Aggregate with Single State Spur Gears
B. Cheshankov, B. Belnikolovsky, and I. Jordanov, Sofia, BULGARIA

Utilizing Parallel Computing in the Combined Multibody, Control and Structural Dynamic Simulation Code FEDEM
O. I. Sivertsen, Norwegian University of Science and Technology, Trondheim, NORWAY and A. Marthinsen, SINTEF Industrial Mathematics, Trondheim, NORWAY

1010-1030 **Break**

Session 16. Analysis IV

Chairmen: G. Rega, Universita Degli Studi Di Roma "La Sapienza ", Roma, ITALY and D. G. Beale, Auburn University, Auburn, AL

1030-1210

Higher-Order Periodic Solutions and a Comprehensive Stability Analysis using N-Mode Harmonic Balance of an Unsymmetric Oscillator
P. Donescu and L. N. Virgin, Duke University, Durham, NC

Continuous Gyroscopic System Stability Near Critical Speeds
R. G. Parker, Ohio State University, Columbus, OH

Application of Phase Space Reconstruction to Fault Diagnosis
J. Xu and Y. Gong, Xi'an Jiaotong University, Xi'an, CHINA

Dynamic Stability of an Elastic Cylindrical Shell Being in External Contact with an Elastic Isotropic Medium with Respect to Nonstationary Subjections
Y. A. Rossikhin and M. V. Shitikova, Voronezh State Academy of Construction and Architecture, Voronezh, RUSSIA

A Route to Chaos in a Nonlinear Time-Delay System
J. Pratt and A. H. Nayfeh, Virginia Polytechnic Institute and State University, Blacksburg, VA

Session 17. Tutorial Sessions on the Use of Mathematica

Engineering Applications of Mathematica
C. Cetinkaya, Wolfram Research, Inc., Champaign, IL

Perturbation Methods
C. Chin, Virginia Polytechnic Institute and State University, Blacksburg, VA

Poster Session

Nonlinear Vibrations of Thin Elastic Plates by an Asymptotic-Numerical Method
L. Azrar and M. Potier-Ferry, Université de Metz, Metz, FRANCE

The Effects of Friction on the Nonlinear Behavior of an Impact Oscillator
C. J. Begley and L. N. Virgin, Duke University, Durham, NC

Nonlinear Control of Crane Operations at Sea
C. Chin and A. H. Nayfeh, Virginia Polytechnic Institute and State University, Blacksburg, VA

A Decrement Method for the Simultaneous Estimation of Coulomb and Viscous Friction
B. F. Feeny and J. W. Liang, Michigan State University, East Lansing, MI

An Experimental and Theoretical Investigation into the Infuence of Hysteretic Damping on the Dynamic Behavior of a Three-Beam Structure
L. Fiedler and A. H. Nayfeh, Virginia Polytechnic Institute and State University, Blacksburg, VA

Efficient Theoretical Modelling and Computer Simulation of a Planar Servo-Pneumatic Test Facility
F. Hecker, D. Fürst, and H. Hahn, University of Kassel, Kassel, GERMANY

Using a Multi-Axis Test Facility for the Identification of the Inertia Parameters of a Rigid Body
F. Hecker and H. Hahn, University of Kassel, Kassel, GERMANY

A Finite-Element-Based Methodology for Reducing Rattle in Structural Components
S-R. Hsieh, Ford Motor Company, Dearborn, MI, S. Shaw, Michigan State University, East Lansing, MI, V. Borowski, and J. Her, Ford Motor Company, Dearborn, MI

Experimental Control of Flexible Structures using Nonlinear Modal Coupling: Forced and Free Vibrations
A. Khajepour, M. F. Golnaraghi, and K. A. Morris, University of Waterloo, Waterloo, Ontario, CANADA

Experimental Survey of Nonlinear Response Behaviors Exhibited by a Model Solar Array
G. I. Knowles and S. W. Smith, University of Kentucky, Lexington, KY

Evidence for Chaos in the Ventilation Patterns of Resting Human Subjects
R. L. Leask, M. F. Golnaraghi, and R. L. Hughson, University of Waterloo, Waterloo, Ontario, CANADA

Nonlinear Dynamic Stability of Normal and Arthritic Greyhounds
D. B. Marghitu and P. Nalluri, Auburn University, Auburn, AL

On Discretization of Distributed-Parameter Systems with Quadratic and Cubic Nonlinearities
A. H. Nayfeh and W. Lacarbonara, Virginia Polytechnic Institute and State University, Blacksburg, VA

Design of an Optimal Vibration Isolation System Using Nonlinear Localization
T. A. Nayfeh and A. F. Vakakis, University of Illinois at Urbana-Champaign, Urbana, IL

Asymptotic Analysis of Dynamic Tension in Submerged Cables
B. L. Newberry and N. C. Perkins, University of Michigan, Ann Arbor, MI

A Control Method Based on the Saturation Phenomenon
S. Oueini and A. H. Nayfeh, Virginia Polytechnic Institute and State University, Blacksburg, VA

Nonlinear Vibrations of Chains
P. F. Pfeiffer, P. Fritz, and J. Srnik, Technische Universität München, München, GERMANY

Nonlinear Dynamics and Stability of a Machine Tool Traveling Joint
I. Ravve, O. Gottlieb, and Y. Yarnitzky, Israel Institute of Technology, Haifa, ISRAEL

An Energy-Momentum Method for the Nonlinear Dynamics of Rods of the Cosserat Type with Applications to Chaotic Motion
C. Sansour, J. Sansour, and P. Wriggers, Technische Hochschule Darmstadt, Darmstadt, GERMANY

Monitoring the Behavior of an Experimental Impacting System
K. N. Slade and L. N. Virgin, Duke University, Durham, NC

Sunday, June 9

1320-1330 Opening Remarks

1330-1510

Session 1. Analysis I

An Analysis of Roll/Sway/Heave Dynamics in Relation to Vessel Capsizing in Regular Beam Seas

Shyh-Leh Chen†, Steven W. Shaw†, and Armin W. Troesch‡
†Department of Mechanical Engineering
Michigan State University, East Lansing, MI 48824
‡Department of Naval Architecture and Marine Engineering
The University of Michigan, Ann Arbor, MI 48109

This study is concerned with the nonlinear behavior, especially the capsizing, of small fishing vessels. The 1-DOF roll results of [2] and [3] using dynamical systems theory are extended to a multi-DOF model. The model considered here is a 3-DOF beam sea model, one that considers roll, sway and heave motions occurring in a (vertical) plane. The focus is on the coupling effects of heave and sway to roll motions and their effects on capsize.

We start by deriving a ship model that takes into account regular wave excitation, hydrostatic and hydrodynamic forces, and wind forces. The procedure in the work of Thompson *et al.* [4] is generally followed, but we put it on a solid footing and include the effects of hydrodynamic coupling. Then, a singular perturbation formulation is sought through a nondimensionalization and rescaling process. It is shown that the roll/sway motions are slow compared to heave and hence their dynamics lie on a slow invariant manifold. Also, this slow manifold exists globally (up to the angles of vanishing stability) and is locally attractive. The slow dynamics turns out to be in the form of a slowly varying oscillator.

Next, using a fast manifold approach [1], we incorporate the two dimensional phase space transport theory with a Melnikov analysis for slowly varying oscillators to propose capsizing criteria for both biased and unbiased ships. The criteria are then compared to those obtained by 1-DOF roll models to examine the coupling effects. It is found that the coupling from heave has little effect on the propensity of a vessel to capsize, but the sway coupling tends to increase possibility of capsize. Therefore, for a particular ship under a given sea state, the 3-DOF model will propose a higher capsizing probability than does the 1-DOF model. The physical implication for this is that there are situations, predicted to be safe by the criterion using 1-DOF models, that are actually vulnerable to capsize. The results will be illustrated by numerical simulations.

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Hydrodynamic and Geometric Stiffening Effects on the Out-of-plane Waves of Submerged Cables

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ABSTRACT

This investigation addresses the need to understand the dynamics of submerged cables employed in ocean engineering applications including: towing and mooring systems, oceanographic and ranging systems, and tether and umbilical systems. In all of these applications, the submerged cable is excited by loads that originate from the surrounding fluid environment (e.g., currents and waves) and from the motion of attached bodies (e.g., motions of instruments, buoys, vessels). The ensuing cable motions may significantly influence the performance of the cable and the mission that it is designed to support. Thus, accurate predictions of dynamic response are often desired to design or to evaluate cable/system performance.

Predictions of dynamic cable response may, at times, follow from low-order cable models assuming the participation of a few selected cable modes. While computationally simple, this modal approach becomes less feasible for very long cable suspensions or for higher frequency/shorter wavelength dynamics due to high modal density. In such instances, an attractive alternative is to describe the cable response in terms of propagating waves, in lieu of standing waves (cable modes). To this end, this study contributes a fundamental understanding of nonlinear wave propagation along a fluid-loaded cable, specifically for the case of out-of-plane structural waves.

This study focuses on the relative importance of two sources of nonlinearities affecting submerged cable response. The first of these is the added fluid damping offered by the surrounding medium while the second is the geometric stiffening offered by the cable through finite extensions of its centerline. The contribution of each nonlinear effect, taken separately and in tandem, is evaluated herein through the study of structural waves that form in the (out-of-plane) direction normal to the cable equilibrium

plane. While these waves may, in practice, also induce in-plane cable response, the present study focuses exclusively on the simpler out-of-plane response. Taken with the recently developed *linear* theory governing the in-plane structural waves [1, 2], the present investigation on nonlinear cable waves provides a further step towards a complete nonlinear, three-dimensional theory.

The nonlinearities due to hydrodynamic drag and geometric stiffening render the cable/fluid model intractable by exact analytical methods. Numerical solutions are pursued herein using a range of finite difference algorithms. These algorithms are brought to bear on two nonlinear cable/fluid models including: 1) a non-linear submerged cable model in which hydrodynamic drag is the sole nonlinear mechanism (referred to herein as the *nonlinear drag model*); and 2) a nonlinear submerged cable model in which hydrodynamic drag and geometric stiffening are both active nonlinear mechanisms (the *nonlinear elastic-drag model*). Numerical solutions for propagating cable waves are developed for the case of a long suspension subjected to a concentrated harmonic excitation source. Conclusions are subsequently drawn regarding the spatial decay of the resulting out-of-plane waves and the dynamic cable tension induced by these waves. The effect of these two nonlinear mechanisms is further explored through the analysis of two additional, linear models: 3) a simple linear taut string model without drag (the *simple model*); and 4) a linear taut string model with linear drag (the *linear drag model*). The simple model and the linear drag model are classical and are evaluated using closed-form techniques. In particular, the linear drag model is evaluated by first linearizing the hydrodynamic drag force model over one excitation period and then, via a Green's function formulation, evaluating forced wave propagation solutions. Results of both linear and nonlinear cable/fluid models are critically compared.

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Nonlinear Behavior of a Typical Airfoil Section with Control Surface Freeplay

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Abstract

A standard state-space modeling technique has been combined with Henon's method for integrating systems of equations to a given surface of section to yield a robust numerical model for the three degree of freedom aeroelastic typical section with control surface freeplay. The model takes advantage of the piecewise linearity present in the system to enhance the computational efficiency and minimize the number of states necessary to accurately predict the behavior of the system. The system response is determined by time marching of the governing equations using a standard Runge-Kutta algorithm. The numerical model has been validated by an experimental model which closely approximates the three degree of freedom typical section in two-dimensional, incompressible flow. Consideration has also been given to modeling realistically the structural damping present in the experimental system. The development of the state-space model will offer advantages in future research which will investigate the effects of freeplay on the control of flutter in the typical section. Comparisons between theory and experiment will be presented.

Introduction

The general form of the equations of motion for the three degree-of-freedom aeroelastic typical section, shown below, in two-dimensional, incompressible flow was derived by Theodorsen [1]. Edwards et al. [2] proposed a state-space model for this linear system incorporating a two state approximation to Theodorsen aerodynamics to determine the unsteady aerodynamic loads. However, similar state-space models for piecewise linear systems, such as those with freeplay in the structural stiffness of one or more degrees of freedom, have typically not been exploited. Such systems are usually modeled in one of two ways: (1) using a describing function or harmonic balance approach to linearize the restoring force [3, 4, 5, 6] or (2) using a much more complicated model requiring the solution of finite-difference or vortex lattice equations in a time marching solution [4, 5, 7]. A typical curve relating the structural restoring moment in the control surface as a function of the control surface displacement is shown below. Within each of the three regions, the system is linear.

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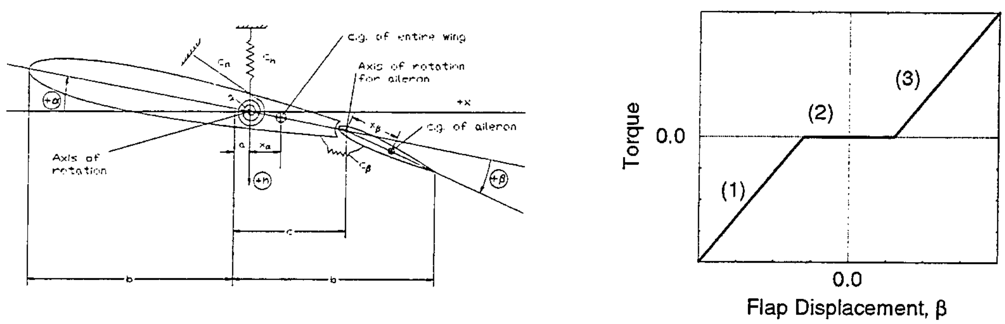
Recently, Lin and Cheng [3] have used a combination of linear state-space models to represent the distinct subdomains of a nonlinear aeroelastic system with freeplay. Their "time domain method" incorporates Padé approximants for the determination of the unsteady loading on the airfoil. A key issue in using several distinct piecewise linear models to simulate the freeplay system in the time domain is locating the exact point at which the system moves from one linear region into the next. Failure to capture this point accurately can result in a "round-off" error that may grow as the numerical integration proceeds and lead to numerical instability [3]. To avoid this type of instability, Lin and Cheng employ a numerical algorithm with a self-adjusting time step to ensure that the system does not skip a subdomain as a point of discontinuity is approached. Once it is determined that the system has moved into another subdomain, the method of bisection is used to accurately determine the switching point. An alternative means of accurately and efficiently determining the switching point in the time domain is presented in this paper.



Schematic of the aeroelastic typical section with control surface and structural restoring moment as a function of position for a control surface with freeplay.

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ANALYTICAL METHODS FOR RIGID BODY DYNAMICS WITH FRICTION AND UNILATERAL CONSTRAINTS

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ABSTRACT

In recent years, the dynamics of systems made of rigid elements has received an increasing amount of attention in the scientific literature. The applicative purposes of these studies are connected to various problems, though in most cases the main question is the dynamic behaviour of a system, which can be modeled as a rigid body in the presence of a boundary.

Consider the particular case of a rigid block free-standing with friction on a moving rigid ground. In order to obtain information about the features of the motion and its stability, numerous analytical investigations have been carried out only under restrictive assumptions [1]; i. e., by assuming a friction coefficient large enough to prevent slidings and by modelling the unilateral constraint as a bilateral one. But, if the constraints are unilateral, the connection between instantaneous forces, position, velocity and acceleration is determinant for the identification of the dynamic solution.

On the other hand, a consolidated procedure [2] to tackle the general problem of the dynamics with friction and unilateral constraints consists in formulating it as a linear complementarity problem according to the statements of the normal and tangential contact laws. In this case, numerical methods are generally used to follow the dynamic evolution, which can also exhibit phases of discontinuous motion and is characterized by the transitions from one mechanism to another, among the ones consistent with the constraint.

The aim of this paper is to investigate analytically the features of the motion of the block by means of a variational formulation and a geometric method recently proposed by the author[3]. The method allows the solution of the dynamic problem to be instantaneous determined in the configurations space without any direct evaluation of the contact forces; thus, different situations, depending on the instantaneous values of relative positions, velocities and active forces, can be identified.

The efficiency of the proposed formulation will be demonstrated by discussing and solving classical and well known questions.

Furthermore, applications of the method to problems concerning the protection and maintenance of the historical patrimony will be also analysed. Particular attention will be paid to the identification of the starting mechanisms and their maintenance during the dynamic evolution for the forced dinamics of systems which can be modelled as rigid bodies in the presence of Coulomb dry friction and unilateral constraints.

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Stability boundaries for the swinging heart

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Abstract The human heart may exhibit "swinging" motion within the chest if the fluid-filled space surrounding the heart (the pericardium) enlarges in response to injury or infection. In this condition, known as *pericardial tamponade*, period-2 swinging can lead to an alternating electrocardiogram and pulse. Subharmonic motion was clearly observed via echocardiography more than twenty years ago by Gabor and co-workers [1] but it remains incompletely understood.

A mechanism for the heart's motion during pericardial effusion was recently proposed by Rigney and Goldberger [2]. They modeled the heart as a pendulum excited by the pulsatile motion of blood entering and leaving the system. The resulting equation of motion is:

$$\ddot{\theta} + a_1 \sin\theta + a_2 \sin(\theta + \Phi) + a_3 \sin(\theta - \phi_l) + a_4 \sin(\theta + \Phi - \phi_l) + \dot{\theta}/\tau = 0, \tag{1}$$

where θ is the angular displacement of the heart from vertical. The parameters Φ, ϕ_l, and ϕ_r are geometrical variables; a_1 and a_2 contain the effects of gravity (including buoyancy); and a_3 and a_4 represent torques per unit mass due to fluid forces on the great vessels of the heart. Damping is represented by the time constant τ. The torque on the heart depends on whether the heart is contracting (in *systole*) or relaxing (in *diastole*), hence the parameters a_3 and a_4 are time-varying. Rigney and Goldberger assumed that parameters change instantly and discontinuously at a critical phase ψ in the cardiac cycle: $a_n = a_{ns}$, $0 < \psi < 0.6\pi$; $a_n = a_{nd}$, $0.6\pi < \psi < 2\pi$, $n = 3, 4$. They used numerical simulation to show that both period-1 and period-2 swinging motion can result at realistic parameter values.

In the present study, analytical methods are used to explain and extend the results of Rigney and Goldberger. The discontinuously varying parameters a_3 and a_4 were replaced by their zeroth and first Fourier components. A linearized equation for small oscillations about a nominal periodic solution was obtained in the form:

$$\gamma'' + M\gamma' + [\delta + \epsilon(e^{2j\tau} + e^{-2j\tau})]\gamma = 0. \tag{2}$$

This is the classical damped Mathieu equation, the stability of which is summarized by a Strutt diagram. This diagram consists of stability boundaries in the $\delta - \epsilon$ plane for given values of M. Using the Hill determinant method, period-2 motions observed by prior investigators are shown to occur within instability tongues in a Strutt diagram. In Figure 1 the stability boundaries for the heart model are shown for two heart rates used in simulations by Rigney and Goldberger [2]. Stability boundaries were then computed as a function of dimensional heart rate ω and damping $1/\tau$ (Figure 2). The existence of subharmonic periodic orbits in the instability regions was confirmed by numerical simulation and harmonic balance.

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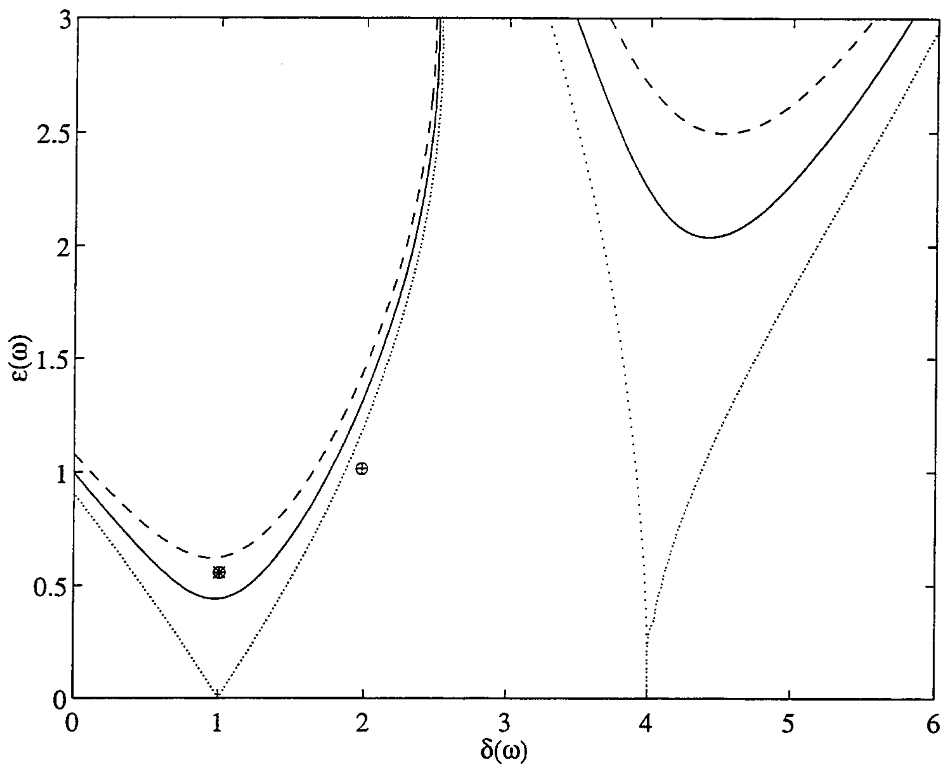

Figure 1: Stability boundaries for period-1 motion of the heart. The dashed curve is for a fixed heart rate of 75 beats per minute (bpm); the solid curve corresponds to a rate of 105 beats per minute; the dotted curve represents undamped motion. The circled +-marker represents stable period-1 motion at 75 bpm; the circled asterisk (*) corresponds to period-2 motion at 105 bpm.

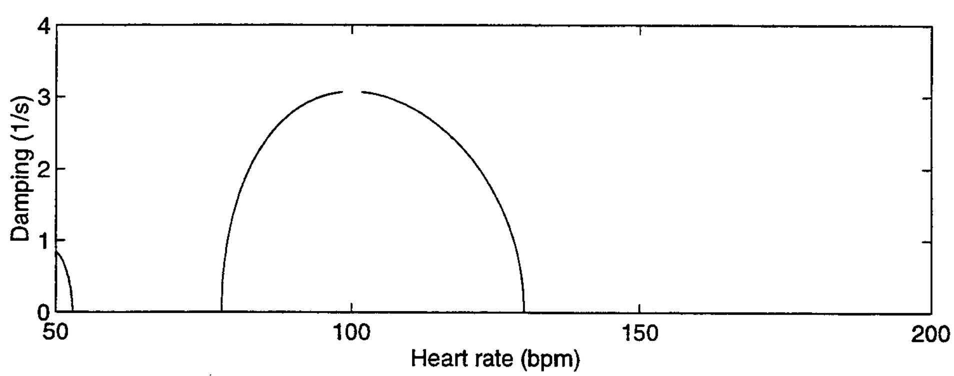

Figure 2: Stability boundaries as a function of heart rate ω and system damping $(1/\tau)$.

Sunday, June 9

1530-1710

Session 2. Modal Interactions I

In-plane and out-of-plane oscillations of a non resonant suspended cable with a longitudinal control

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Abstract

Strings and cables are very simple structural elements but intensively studied for their importance and the wide variety of nonlinear dynamical phenomena. They are mainly due to the presence of quadratic nonlinearities beyond cubic ones [1, 2]. A delicate point concerns an inherent light structural damping of these systems which makes transverse vibrations to be easily excited by wind or boundary motion. A certain number of papers have been devoted to analyze the use of passive dampers, which could be scarcely effective when applied to very long systems. An alternative attractive approach is represented by the active control realized through a longitudinal force or a support motion [3-5].

This paper deals with the use of a longitudinal control in order to reduce transverse vibrations of a suspended cable. As it is well known the oscillations of a suspended cable can be described by two ordinary integro-differential equations in the two transversal displacement components, with quadratic and cubic nonlinearities. The control is realized with an imposed support motion which is assumed as a function of observed response quantities: different situations are discussed concerning this dependence.

As already evidenced by other authors [3] and well known in nonlinear dynamics, the optimal energy dissipation can be reached when the support motion acts in condition of parametric resonance. This circumstance, although obtained within an harmonic phenomenon, suggests the kind of dependence of the control force on the measured quantities. Important aspect are the variables chosen in the feedback, the kind of dependence (polynomial, absolute value) and the order of the control law. A control law that is a linear function of observed quantities produces linear and quadratic terms in the in-plane equation, whereas only quadratic terms in the out-of-plane equation. A nonlinear control law produces only nonlinear terms in both equations. The linear control is very effective; however, together with the dissipative linear terms, nonlinear nondissipative terms arise which produce some undesirable effects. Thus, the order and the kind of the control law have been selected in such a way to guarantee dissipation of the prevailing components. It is not always easy to define a region of parameters where the control action produces damping on both in-plane

and out-of-plane motions.

The terms introduced by the control do not alter the already rich variety of nonlinearities which are present in the motion of uncontrolled cables. Accordingly, the same internal resonance conditions are found. While in most of the papers already published on cable nonlinear dynamics the internal resonance phenomena have been investigated, the present study examines the nonlinear coupling phenomena of cables not in conditions of internal resonance. In such conditions, the study is mainly devoted to the phenomenon of parametric excitation of in-plane motion versus out-of-plane oscillations, which has not received sufficient attention in the past.

Restricting the attention to a polynomial control law, amplitude and phase equations are obtained by means of multiple time scales method developed up to the fourth order which is suitable to describe modulations on the time scales associated with quadratic and cubic terms; fixed points and nonstationary motions are considered. The analysis is mainly performed by numerical integration of either the equations of motion or the amplitude and phase equations. In the absence of control the stability of the in-plane oscillations is investigated; a region of unstable motion is found where the one mode solution bifurcates in a steady-state two modes oscillation with the in-plane component oscillating with a frequency twice the out-of-plane one. The behavior is similar to the case of internal resonance conditions: different is the diagram of the in-plane amplitude versus the in-plane force intensity which does not exhibit the classical phenomena of saturation.

The effects of longitudinal control on reducing the in-plane and three dimensional oscillations of cables are investigated. The obtained results are in general quite satisfactory: however, a delicate calibration of the gain coefficients which affect the contribution of in-plane and out-of-plane components must be done for obtaining good results in widely different conditions.

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Nonplanar Oscillation of Wire Electrode and its Relaxation in System of Wire and Plate Electrodes

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The purpose of this paper is to clarify and suppress the induced lateral oscillation of a wire electrode in a system of wire and plate electrodes for the case of an electrostatic field with corona discharge. The relaxation of such an oscillation is very important, in order to use the electrostatic adhesion technique in a precipitator, a duplicator and the manufacturing process of polyester film. However there are still many unclarified problems regarding the growth and the characteristics of the above oscillation. Furthermore the wire oscillation was suppressed just only on the basis of the practical experiences.

First, experiments have been conducted to observe the induced lateral oscillation of the wire electrode when dc high electric voltage ϕ_0 is applied between a wire and a plate electrodes as shown in Fig.1, where mass m is fixed in this case.

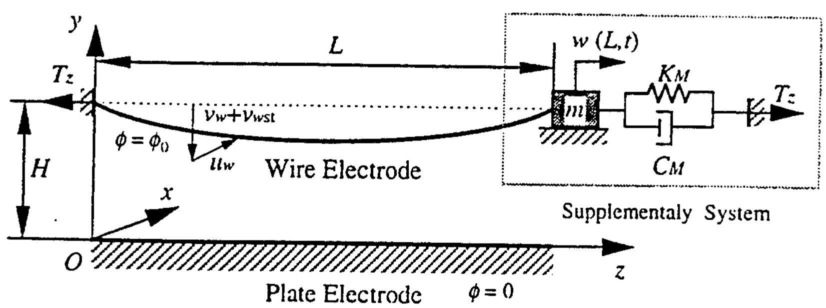

Fig.1 System of Wire and Plate Electrodes with Supplementary System

Experimental results showed that the nonplanar lateral oscillation of the wire electrode was excited with the fundamental natural frequency of the lateral oscillation, and the amplitude of the horizontal component u_w of the wire oscillation exceeds that of the vertical component v_w as shown in Fig.2. Moreover the fluid flow around the wire, of which the Reynolds number is about 20, depends on the Corona discharge from the moving wire electrode itself, and it curves perpendicularly along the plate electrode.

Second, the reason for the growth of the self-excited oscillation are physically discussed on the basis of the above experimental results and the theoretical results analyzed numerically the electrostatic fluid flow around the wire electrode. As a result, it was neither the

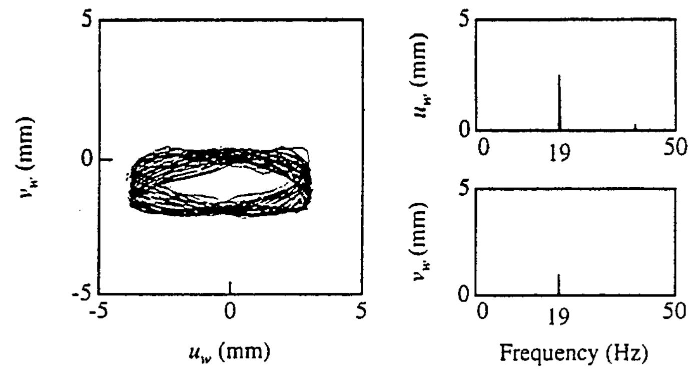

Fig.2 Motion of Wire Electrode in $x-y$ Plane

change of the electrostatic force due to periodic fluctuations on the applied voltage nor the change of the fluid force due to vortex shedding from the wake of the wire electrode. Furthermore, the basic equation governing the nonplanar lateral oscillation of the wire electrode was derived under the assumption that the horizontal component of the wire oscillation was excited by the galloping of the wire due to the unsteady drag force of the fluid.

Finally, it was proposed theoretically that the nonplanar lateral oscillation of the wire electrode would be decayed through the effect of damping force on the supplementary system of Fig.1. under the state of internal resonance between the wire oscillation and the oscillation of mass m. It is cleared from the dimensionless equations governing the nonlinear coupled oscillations between the wire and the supplementary systems :

$$\frac{\partial^2 u_w}{\partial t^2} - 2\hat{\mu}_h \frac{\partial u_w}{\partial t} - \left(c_z^2(t) + \frac{\gamma}{h} w(1,t) \right) \frac{\partial^2 u_w}{\partial z^2} = -\frac{4}{3}\hat{\alpha} \left(\frac{\partial u_w}{\partial t} \right)^3 \tag{1}$$

$$\frac{\partial^2 v_w}{\partial t^2} + 2\hat{\mu}_w \frac{\partial v_w}{\partial t} - \left(c_z^2(t) + \frac{\gamma}{h} w(1,t) \right) \frac{\partial^2 v_w}{\partial z^2} = 0 \tag{2}$$

$$\frac{d^2 w}{dt^2}(1,t) + 2\hat{\mu}_M \frac{dw}{dt}(1,t) + \omega_M^2 w(1,t) = -\frac{\gamma}{2Mh} \int_0^1 \left[\left(\frac{\partial u_w}{\partial z} \right)^2 + \left(\frac{\partial v_w}{\partial z} \right)^2 \right] dz \tag{3}$$

where $c_z^2(t) = 1 + (\gamma/2) \displaystyle\int_0^1 \{(\partial u_w/\partial z)^2 + (\partial u_w/\partial z)^2\} dz$. $u(z,t)$ and $v(z,t)$, which are the functions of the dimensionless coordinate z and time t, are the dimensionless horizontal and vertical displacement of the wire oscillation respectively, and $w(1,t)$ is the dimensionless displacement of mass m. M, h, $\hat{\alpha}$, γ, $\hat{\mu}_h$, $\hat{\mu}_w$ and ω_M, are the dimensionless parameters.

Furthermore, relaxation of the wire oscillation under the state of the internal resonance was confirmed by the experiment.

SLOW WAVES IN AN ONE-DIMENSIONAL ARRAY OF SOFT-STIFF PAIR OF OSCILLATORS

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Large-scale repetitive systems are used often in engineering applications. Truss-structures in aerospace applications, periodically stiffened plates and shells used as parts of airplane fuselages, or disk blade assemblies in turbine engines are examples. Linear periodic systems of infinite periodic extent possessing single of multiple coupling coordinates have been studied extensively in the literature [1]. Models of ordered or disordered linear periodic systems have been used to study localization of electron transport and spin diffusion in solid state physics, and mode localization in weakly coupled and weakly disordered structural assemblies with cyclic or noncyclic symmetry.

In a series of theoretical works, asymptotic and numerical techniques were used to study standing or traveling solitary waves, solitons and chaotic motions in free or forced nonlinear lattices [2-4]. In addition, asymptotic techniques for studying localized nonlinear normal modes in cyclic or noncyclic large-scale periodic systems with stiffness nonlinearities have been developed [5]. In [6] the method of multiple-scales was employed to study nonlinear traveling waves in a nonlinear periodic system. In [7] the nonlinear propagation and attenuation zones (PZs and AZs) of a nonlinear layered structure with weak coupling were examined. In this last work, a new asymptotic methodology based on the concept of nonlinear normal modes was presented, which enabled the analytic approximation of nonlinear standing waves inside AZs of the nonlinear layered system.

A plethora of periodic large scale structures in engineering applications and biology are composed of a *soft* substructure and a complementary *stiff* substructure. In this work we present a new approach for studying propagating and attenuating waves in large-scale *soft-stiff* nonlinear periodic systems, based on the concept of slow invariant manifold of dynamics. In particular, we show that when the nonlinear dynamics possess slow and fast scales, one can develop analytic approximations by restricting the motion to an invariant manifold where the slow dynamics are dominant; the fast dynamics provide small perturbations to the slow motions.

The slow invariant manifold approach was previously developed by Georgiou and co-workers [8,9,10,11] to investigate the dynamics of structural and mechanical systems composed of a *soft* (low frequencies) substructure and a *stiff* (high frequencies) substructure. According to this approach, the equations of motion of *soft-stiff* structural or mechanical dynamical system are formulated as a singular perturbation of the equations of motion of the *soft* substructure. The singular perturbation approach introduces naturally slow and fast invariant manifolds (normal modes) of motion. Whenever the coupling, measured by the singular perturbation parameter, is sufficiently small, the dominant dynamics of a *soft-stiff* dynamical system are carried by the slow manifold whose dimensions are identical to the dimensions of the phase space of the *soft* substructure.

We consider an one-dimensional linear spring-mass array coupled to an one-dimensional array of uncoupled pendula; and investigate its nonlinear dynamics in the limit of weak nonlinearities, i.e., when the (fast) nonlinear pendulum (*stiff* substructure) effects are small compared to the underlying (slow) linear dynamics of the linear spring-mass array (*soft* substructure). We approach the dynamics in the context of invariant manifolds of motion. In particular, we prove the existence of an invariant manifold containing the (predominantly) slow dynamics of the system, with the fast pendulum dynamics providing small perturbations to the motions on the invariant manifold. By restricting the motion on the slow invariant manifold and performing asymptotic analysis we prove that the nonlinear large-scale system possesses propagation and attenuation zones (PZs and AZs) in the frequency domain, similarly to the corresponding zones of the linearized system. Inside PZs nonlinear traveling wave solutions exist, whereas in AZs only attenuating waves are permissible.

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Nonplanar Vibrations of a Forced, Buckled Beam

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Holmes and Marsden [1] studied chaotic vibrations of a forced buckled beam such as shown in Figure 1. Their mathematical model for the beam was a partial differential equation for the transverse deflection $v(z,t)$ of the beam,

$$\ddot{u} + u'''' + \Gamma u'' - \kappa \left(\int_0^1 [u']^2 \mathrm{d}\zeta \right) = f_0 \cos \omega_0 t - \delta_0 \dot{u} \tag{1}$$

where dot and prime, respectively, denote differentiation with respect to t and z; δ_0, Γ and κ, respectively, represent the damping, axial compressive load and nonlinear membrane stiffness; f_0 and ω_0, respectively, represent the amplitude and frequency of the transverse sinusoidal acceleration of the base; and ϵ is a small parameter such that $0 < \epsilon \ll 1$. The chaotic dynamics they analyzed are essentially the same as those of the single mode approximation, i.e., the Duffing oscillator,

$$\ddot{x} + \delta \dot{x} - x + x^3 = \gamma \cos \omega t. \tag{2}$$

In this paper we consider a situation in which the buckled beam easily deforms in a non-planar manner. We assume that constants of stiffness in the u- and v-directions are of the same order, and only the first mode is excited by the forcing in each of the two directions. Applying the Galerkin procedure, we obtain a two-degree-of-freedom model for the forced beam

$$\ddot{x} + \delta \dot{x} - x + (x^2 + y^2)x = \gamma \cos \omega t, \quad \ddot{y} + \delta \dot{y} - \kappa y + (x^2 + y^2) y = 0. \tag{3}$$

We analytically and numerically study periodic and chaotic motions in (3) and discuss the forced vibrations of the buckled beam. A weak coupling version of (3)

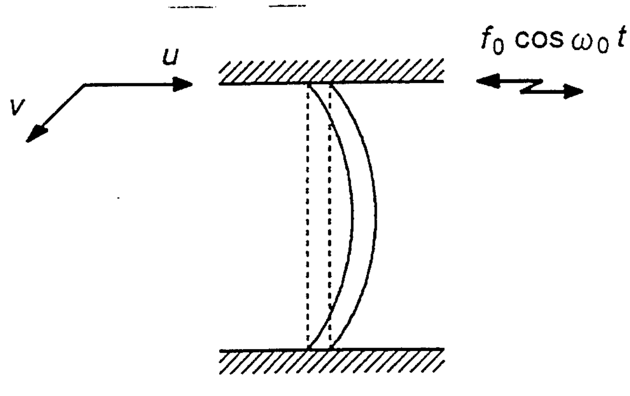

Fig. 1. A forced, buckled beam

was also analyzed in [2]. The strong coupling of the two oscillators complicates the problem.

We first use the averaging method and analyze periodic motions and their bifurcations. Second, we apply a version of Melnikov's method recently proposed by Yagasaki [3] and obtain a condition under which transverse homoclinic motions exist and consequently chaotic dynamics may occur. This version of Melnikov's method does apply to a wide class of perturbations of multi-degree-freedom Hamiltonian systems, and is an extension of Wiggins' version [4]. Numerical simulation results are also given and compared with the theoretical results. In particular, we will see that chaotic dynamics occurring essentially in two modes are much more complicated than in single mode.

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INTERNAL RESONANCE OF THE JEFFCOTT ROTOR
(Critical Speed of Twice the Major Critical Speed)

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1. Introduction

The Jeffcott rotor with a disk at the midspan is the most widely used model in the analysis of nonlinear resonances of rotors. As the gyroscopic moment does not act in the Jeffcott rotor, its forward natural frequency p_2 and backward natural frequency p_3 satisfy the condition of internal resonance, such as, $p_2 : p_3 = 1 : -1$ (Fig.1(a)). Therefore, when the Jeffcott rotor has nonlinear spring characteristics, some unique phenomena may appear due to the internal resonance. However, in almost all the previous studies on nonlinear resonances in the Jeffcott rotor, the authors paied no attentions to this internal resonance relation. In this paper, internal resonance phenomenon in the Jeffcott rotor is discussed at twice the major critical speed, where the forward and backward subharmonic resonances of order 1/2 and the combination resonance $[p_f - p_b]$ coicide.

2. Equations of Motion

In order to discuss the transition from internal resonance to ordinary nonlinear resonance, we use the following equations of motion for inclination oscillations :

$$\left.\begin{aligned} \ddot{\theta}_x + i_p\omega\dot{\theta}_y + c\dot{\theta}_x + \theta_x + N_{\theta x} = (1-i_p)\tau\omega^2\cos\omega t \\ \ddot{\theta}_y - i_p\omega\dot{\theta}_x + c\dot{\theta}_y + \theta_y + N_{\theta y} = (1-i_p)\tau\omega^2\sin\omega t \end{aligned}\right\} \tag{1}$$

where i_p is the ratio of the polar to diametral moments of inertia, τ is the dynamic unbalance, and ω is the rotational speed. Putting $i_p = 0$ gives the equations of motion of the Jeffcott rotor with nonlinear spring characteristics. The nonlinear terms $N_{\theta x}$ and $N_{\theta y}$ are given by

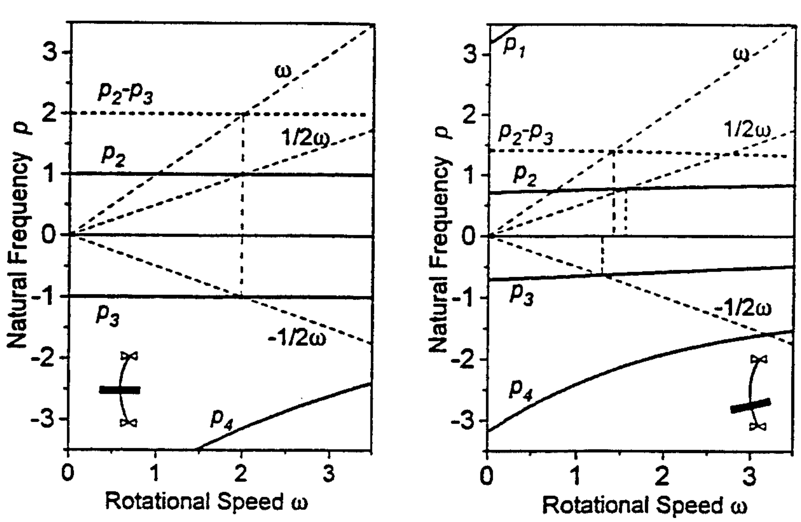
$$V = V_0 + V_N = \frac{1}{2}(\theta_x^2 + \theta_y^2) + V_N, \quad N_{\theta x} = \frac{\partial V_N}{\partial \theta_x}, \quad N_{\theta y} = \frac{\partial V_N}{\partial \theta_y} \tag{2}$$

After the transformation $\theta_x = \theta\cos\varphi, \theta_y = \theta\sin\varphi^{(1)}$, the potential energy V_N is expressed by

$$\begin{aligned} V_N = &(\varepsilon_c^{(1)}\cos\varphi + \varepsilon_s^{(1)}\sin\varphi + \varepsilon_c^{(3)}\cos 3\varphi + \varepsilon_s^{(3)}\sin 3\varphi)\theta^3 \\ &+(\beta^{(0)} + \beta_c^{(2)}\cos 2\varphi + \beta_s^{(2)}\sin 2\varphi + \beta_c^{(4)}\cos 4\varphi + \beta_s^{(4)}\sin 4\varphi)\theta^4 \end{aligned} \tag{3}$$

3. Subharmonic Resonance of Order 1/2 and Internal Resonance

When i_p is very small, the critical speeds of the subharmonic oscillation of order 1/2 of



(a) Disk at the mid span (b) Disk at an arbitrary position

Fig.1 4DOF Rotor Systems

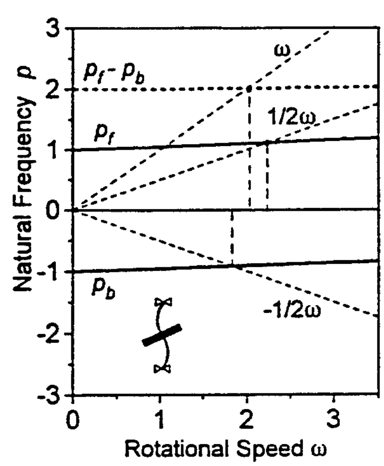


Fig.2 2DOF Theoretical model given by Eq.(1)

forward whirling mode and that of backward whirling mode, and the critical speed of the combination resonance almost coincide. In such a case, the oscillations with the frequencies $\omega_f = (1/2)\omega$ and $\omega_b = -(1/2)\omega$ occur simultaneously. The solution in the accuracy of $O(\varepsilon^0)$ is given by

$$\left.\begin{aligned} \theta_x &= R_f \cos(\omega_f t + \delta_f) + R_b \cos(\omega_b t + \delta_b) + P\cos(\omega t + \beta) + A_x \\ \theta_y &= R_f \sin(\omega_f t + \delta_f) + R_b \sin(\omega_b t + \delta_b) + P\sin(\omega t + \beta) + A_y \end{aligned}\right\} \quad (4)$$

Using harmonic balance method, we can obtain the expressions for the resonance curves. Although all the nonlinear components have influence to this resonance, we consider only the isotropic symmetrical nonlinear component $\beta^{(0)}$ and the unsymmetrical nonlinear components $\varepsilon_c^{(1)}, \varepsilon_s^{(1)}$ in the following. Resonance curves for R_f are shown in Fig.3. Figure 3(a) for $i_p = 0$ represents a result in the Jeffcott Rotor. The resonance curves have a unique shape due to internal resonance, but quasi-periodic motions do not appear. Figure 3(b) is the case with a very small i_p. The branch AB in Fig.3(a) is divided into two parts AI and JB, and quasi-periodic motions occur between Two branches.

Figure 3(c) shows the case where these critical speeds exist apart to some extent. As i_p increases from 0.01 to 0.1, the branch BCDJ move upword, AIE becomes small and then disappears, and FGH inclines to the higher speed side. For comparison, resonance curves obtained by the analysis considering no internal resonance are shown by chain lines.

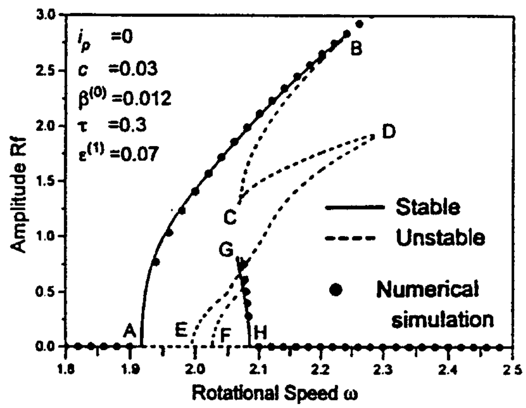


Fig.3(a) $i_p = 0$

4. Experimental Results

Figure 4 illustrates an experimental result obtained in the Jeffcott rotor shown in Fig.1(a).

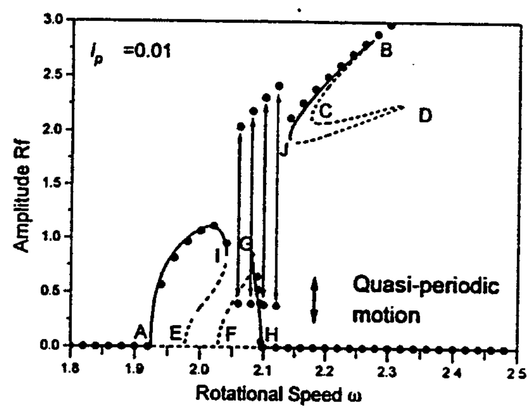


Fig.3(b) $i_p = 0.01$

5. Conclusions

The subharmonic resonance of order 1/2 of forward whirling mode, that of backward whirling mode, and the combination resonance $[p_f - p_b]$ coincide in the Jeffcott rotor with nonlinear spring characteristics. As the result of internal resonance, the shape of resonance curves change and the quasi-periodic oscillations appear.

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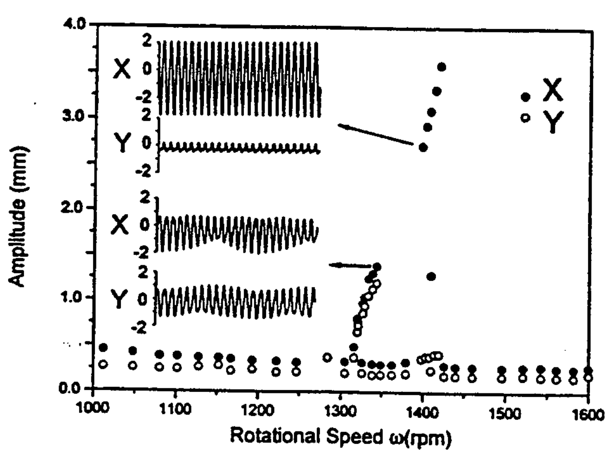

Fig.4 Experimental Result

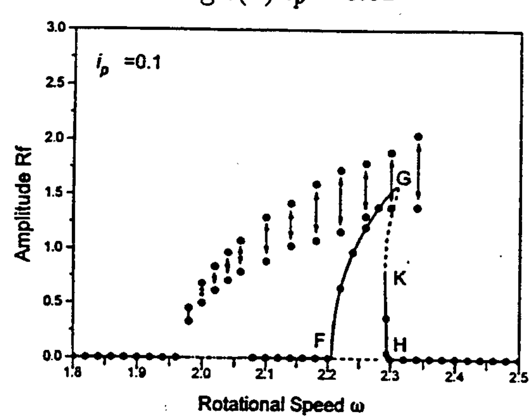

Fig.3(c) $i_p = 0.1$

Monday, June 10

0830-1010

Session 3. Control

INERTIA PARAMETER IDENTIFICATION OF A RIGID BODY USING A MEASUREMENT ROBOT

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Standard experiments for identifying inertia properties of a rigid body only provide a subset of the inertia parameters of the body [1], [2], [3], [4], [5], [6].

The objective of the work described in this paper is the simultaneous, automatic experimental identification of the ten inertia parameters of a rigid body.

This work may be roughly described in three steps:

1. The experimental identification of the ten inertia parameters of a rigid body will be performed automatically by a specially designed measurement robot. This test robot will be described in detail. It includes the following subsystems:
   - mechanical components of the robot inclusive the test body (rigid bodies),
   - driving systems,
   - sensing elements including
     - a force–moment sensor and
     - angle decoders.

   The test robot is used to perform the following tasks:
   - acceleration of the test body in space,
   - signal recording of the test body orientations (positions) and of the forces and moments acting on the test body, and
   - identification of the ten inertia parameters of the test body.
2. An extended nonlinear mathematical model of a rigid body under large spatial accelerated motion will be presented [7]. This model serves
   - as a model hypothesis in the identification process, and
   - as a basis for an invers computer simulation of the laboratory experiment, verifying this model hypothesis.
3. The results obtained
   - in laboratory experiments using the new test robot,
   - in accuracy tests of the laboratory experiments,
   - in computer simulations of the laboratory experiments, and
   - in the identification process

   will be discussed in several steps:

- In a first step measurement signals obtained in a laboratory experiment and in a corresponding invers computer simulation of spatial motions of the test body (with known inertia properties) will be discussed. This includes an accuracy test of the measurement systems.
- In a second step, the identification process and the identification results obtained in laboratory experiments and in invers computer simulations of the laboratory experiments will be discussed [8]. These results will be compared with theoretically calculated inertia parameters of a given test body with known inertia parameters.

The identified ten inertia parameters of the test body are in good agreement with its analytically calculated inertia parameter values. They prove, that the identification process based on an extended nonlinear mathematical model for spatial motions of a rigid test body and on the special construction of the test robot has been successful. There exist various industrial applications of this measurement robot.

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Dynamic Properties of Pseudoelastic Shape Memory Alloys

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Abstract

Shape memory alloys such as Nickel-Titanium and Copper-Zinc-Aluminum exhibit nonlinear mechanical properties. Specifically, in a temperature environment which is higher than the phase transformation temperature of the material, when an applied stress exceeds a certain threshold, stress induced phase transformation generates large strains in the material so that it appears to yield. On the unloading cycle, a reverse phase transformation takes place so that no permanent deformation is left. Moreover, the stress-strain relationship during this loading and unloading cycle shows hysteresis. Previous investigations show that the damping of shape memory alloys displays a peak and the elastic modulus demonstrates a trough in the vicinity of the phase transformation during the heating and cooling processes. Obviously, such mechanical properties associated with phase transformation are typical of material nonlinearity.

The lower elastic modulus and higher material damping of shape memory alloys are desirable characteristics of passive vibration control systems. However, since nonlinearity can cause complicated dynamics including deterministic chaos in a forced system, it is thus necessary to understand the dynamics of systems containing such nonlinear materials.

Experiments are conducted on pseudoelastic Nickel-Titanium (NiTi) shape memory alloy rods. NiTi rods are used as springs in our vibration transmissibility tests. To investigate how pseudoelasticity affects the transmissibility characteristics, a set of vibration transmission experiments under various base motion levels, which cause different strain levels in the NiTi rods, are conducted.

Our experimental results of tension-compression vibration show that the NiTi rods behave as spring with softening nonlinearity. Furthermore, the damping of the material is dependent on the base motion levels. In the transmissibility curves, the resonance peaks shift to lower frequencies at higher base motion amplitudes. Meanwhile, the height of the resonance peaks becomes lower for higher base motion amplitudes.

Since the base motion is also a source of parametric forcing of the bending modes, the amplitudes of tension and compression vibration are limited by the onset of parametric instability of the bending modes. To explore the dynamic properties of NiTi rods when large strains occur in the material, we conduct experiments on the bending vibrations of the rods. The results are similar to those of the tension-compression tests. However, for the bending mode, the material damping is even bigger due to the large strain amplitude that

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can be applied to the rod under bending. Since large strains lead to larger areas of the hysteresis loop on the stress-strain curve, higher damping occurs.

Our experiments also show that as a passive device, the NiTi shape memory alloys are not limited to low frequency applications. In fact, our experiments for the tension and compression vibration of the NiTi rods are conducted around resonance frequencies on the order of 1 kHz.

For qualitative understanding of the system dynamics, a one-dimensional constitutive model is used in a theoretical study of the nonlinear dynamics of a mechanical system containing pseudoelastic shape memory alloy rods. The theory predicts complicated dynamics including chaotic dynamics at small forcing amplitudes. The theory also predicts amplitude dependence of the resonance frequencies and the damping constants. The theoretical prediction is in qualitative agreement with the experimental results.

DESIGN AND CONTROL OF NONLINEAR STRUCTURES

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Abstract

A design technique to simultaneously optimize the structure and controller to suppress vibration on nonlinear structures is developed. Newmark-Beta explicit time integration is used to obtain a nearly exact solution to the simultaneous second-order nonlinear differential equations describing the closed loop system. The nonlinear structural model is assumed to be generated from a nonlinear finite element code. With this approach only the mass matrix that is often diagonal needs to be inverted in the integration solution, and the sparsity of the structural matrices is always preserved. The remaining computations in the integration only involve addition and multiplication of sparse matrices. Thus it is practical to carry out the solution for a large number of time steps. The optimization computations are streamlined by computing the gradients or sensitivities of the design variables in closed form, or semi-analytically depending upon the type of finite elements used in the model. Matrix inversion is needed to compute the gradients, but the gradient matrices are mostly sparse and easy to invert. For simple structural elements, the gradient is obtained exactly without any additional function evaluations or recursion that is normally required for nonlinear systems.

An iteration of the integration solution is done each timestep to obtain an equilibrium force balance that greatly improves the accuracy of the solution. The explicit Newmark-Beta integration method used is more accurate than Euler finite difference type methods, and it is simpler to derive analytic gradients and preserve the sparsity of the structural matrices than in the Runge-Kutta or implicit Newmark-Beta methods. Since the mass matrix is often constant, it only needs to be inverted once. In most nonlinear models the stiffness matrix is nonlinear and inversion is necessary to integrate the nonlinear equations. No inversion of the stiffness matrix is needed using the solution procedure presented here.

Structural Design and Controller Optimization

The structure and control optimization is performed by defining an objective function to try to drive the forced nonlinear system to its zero equilibrium solution using the minimum control force. The design variables in the optimization are the structural parameters and control gains and are bounded within desired ranges. No functional constraints are needed. The objective function minimizes the mean square velocities, displacements, and control forces. Constant gain output feedback control is used where the control gains are actually the coefficients of some polynomial function of the nonlinear displacements and velocities of the system. This paper investigates the use of linear and nonlinear control to suppress vibration for multi-degree-of-freedom systems. Collocated and noncollocated damping and position feedback are used.

Nonlinear Truss Model

A fully nonlinear dynamic truss model based on a geometrically-exact structural theory developed by Pai is used for application of the proposed method. The nonlinear differential equation of motion is

$$[\mathbf{M}]\{\ddot{\mathbf{X}}\}+[\mathbf{D}]\{\dot{\mathbf{X}}\}+[\mathbf{K}]\{\mathbf{U}\}=\mathbf{F}(t) \qquad (1)$$

where M is the mass matrix, D is the damping matrix, and [K]{U} is a nonlinear stiffness term. This dynamic equation is a fully nonlinear model for the truss. The analytic gradient is also derived for the truss model.

The computational nonlinear design technique is used to design an active controller and to simultaneously optimize the structural parameters to suppress the vibration of the truss. The optimization duration is 2.5 seconds. The step size for integration is 0.001 sec. for small loads, and 0.0001 sec. for large loads. External forces of $F(t) = 212\sin(\omega t)$, $21213\sin(\omega t)$, and $35355\sin(\omega t)$, where $\omega=2\pi*2$ are applied at the end of the truss. The actuator is installed near the truss support. Linear and nonlinear collocated and non-collocated controllers are designed and compared.

Summary of Results

It should be emphasized that the following results are problem-dependent, that is, other types of loading or different types of nonlinear structures may give somewhat different conclusions.

We find that after the optimization the cross-sectional area of the truss always reaches its upper bound. This is reasonable as the structural vibration will usually be suppressed with the increase of cross-section area. Overall, from the simulation results, it can be concluded that when the closed loop system operates near the linear range of displacement, the linear and nonlinear controllers are nearly the same. That is, the nonlinear terms in the nonlinear control law are very small and almost negligible. When the closed loop system operates in the range of very large displacements, the nonlinear controller has a small advantage over the linear controller. A universal control law can be designed by assuming the maximum load on the structure. We can say that low load is suitable for the design of linear structures. However, high load should be used for the design of control laws for nonlinear structures. This is because the nonlinear system may become unstable if the load is increased above the design point.

The fully nonlinear truss model reflects the physical characteristics of very flexible structures. In general, for a large scale structure, the design steps are:

(1) Build a nonlinear model for the structure.

(2) Consider a maximum load.

(3) Apply the proposed method to the design of a linear or nonlinear feedback control law.

(4) The form of the position feedback control law can be linear or nonlinear and collocated or non-collocated.

(5) The form of the velocity feedback control law (active damping) can be linear or nonlinear and collocated.

The proposed design technique is not at present applicable to discontinuous controllers. The gradient function is incorrect at the jump points and the optimization does not converge. Also, the non-collocated damping feedback law could not be used as it became unstable for this example.

Stabilization of a Nonlinear String by the Boundary Control

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Abstract

In this paper, we consider an elastic string of unit length and unit mass density represented by the following nonlinear partial differential equation:

$$y_{tt}(x,t) = \left[a + b \int_0^1 y_x^2(x,t)\,dx \right] y_{xx}(x,t)\,, \tag{1a}$$

for all $x \in (0,1)$ and $t \geq 0$. In (1a), $y(\cdot,\cdot) \in \mathbb{R}$ denotes the transversal displacement of the string, $y_{tt} := \partial^2 y/\partial t^2$, $y_x := \partial y/\partial x$, and $y_{xx} := \partial^2 y/\partial x^2$, and $a > 0$ and $b > 0$ are constant real numbers.

The boundary conditions of the string are

$$y(0,t) = 0\,, \qquad \left[a + b \int_0^1 y_x^2(x,t)\,dx \right] y_x(1,t) = u(t)\,, \tag{1b}$$

for all $t \geq 0$, where $u(\cdot)$ denotes the control input force applied transversally to the string at $x = 1$. The boundary conditions in (1b) imply that the string is fixed at $x = 0$ and is under the control input u at $x = 1$. We remark that the tension in the string represented by (1a) is not constant and is given by

$$T(t) = a + b \int_0^1 y_x^2(x,t)\,dx\,,$$

for all $t \geq 0$. Therefore, the boundary condition at $x = 1$ represents the balance of the transversal component of the tension and the control input u.

The initial displacement and velocity of the string are respectively

$$y(x,0) = f(x)\,, \qquad y_t(x,0) = g(x)\,, \tag{1c}$$

for all $x \in (0,1)$, where $y_t := \partial y/\partial t$. We assume that $f \in C^1[0,1]$, and that at least one of the functions f and g is not identically zero over $[0,1]$.

The nonlinear model of vibrating strings in (1a) was originally derived by Kirchhoff [1], and later rederived by Carrier [2], Oplinger [3], and Narasimha [4]. Researchers have studied the system (1a) from the physical and mathematical points of view; see [5] for an extensive list of references.

The control input u in (1b) is commonly known as the *boundary control*. In this paper, we study the stabilization of the string in (1a) by u. More precisely, we study a u that results in $y(x,t) \to 0$ as $t \to \infty$ for all $x \in [0, 1]$. As a stabilizing control input, we propose

$$u(t) = -k \; y_t(1, t) \,, \tag{2}$$

for all $t \geq 0$, where $k > 0$. With this choice of u, the boundary control is the negative feedback of the velocity of the string at $x = 1$, with the gain k. It is known that *linear* strings represented by (1), for which $b = 0$, can be stabilized by the control law in (2); see, e.g., [6-8]. Roughly speaking, the boundary control in (2) provides a dissipative effect in linear strings, because it is of the form of negative velocity feedback. This is in accordance with the well known fact that the negative velocity feedback increases damping in most finite dimensional inertial systems, such as, large flexible systems and robotic manipulators.

Our goal in this paper is to show that the boundary control u in (2) stabilizes the nonlinear string in (1), i.e., u results in $y(x,t) \to 0$ as $t \to \infty$ for all $x \in [0, 1]$.

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Energy-Based Control of Flexible Structures using a Digital Algorithm

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One useful area in which control techniques may be applied is in vibration suppression. Structural vibrations have long been known as a cause of material fatigue and thus a source of reduced machine lifetime. In addition, in some industrial applications vibration may reduce the accuracy with which parts may be fabricated. In this type of environment, it is common for a physical plant to experience both periodic and aperiodic forcing.

Energy based controllers have been successfully employed to reduce free and forced vibrations in a cantilever beam using piezo-ceramic actuators. Past applications of this control method have relied on analog circuitry to implement co-ordinate coupling and damp oscillations through a virtual controller. This control technique has been recently re-worked using a digital controller. The digital nature of the controller allows adaptations which can completely eliminatevibrations. Thus, the controller energy is successively reset to zero rather than differentially damped, as in the analog implementation.

Because the control algorithm is computer based, a variety of energy based controllers may be quickly developed. Linear strategies have focused on velocity dependent as well as energy dependent controllers, which have proven successful in free as well as forced vibrations. In addition, nonlinear control has been used to efficiently eliminate free vibration.

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Monday, June 10

1030-1210

Session 4. Bifurcations

Chaotic Dynamics in the Cooperrider Truck

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Abstract

The Cooperrider truck is a model of a truck for a railway passenger coach. The nonlinear dynamics of the truck running on a straight and horizontal track has been investigated by Cooperrider [1], Kaas-Petersen [2] and True [3]. True [3] is a survey of earlier work, where a detailed description of the model is presented.

The bifurcation diagram in [3] is incomplete. Chaos was found in two regions but the transitions to chaos were not investigated. We have therefore later continued our investigations of the dynamics of the truck concentrating on the transitions to chaos.

Two chaotic attractors were known above a speed V =203 m/s. They are reflections of each other in the track center line and we therefore only investigate one of them. The transition to chaos was investigated by Isaksen and True (article in preparation) and the results were presented at the IUTAM symposium in L'Aquila 1994.

Two other - symmetrically positioned - chaotic attractors exist around V =112 m/s in a speed range where the dynamics is complicated with many solutions - both stable and unstable. The transitions to those attractors is the objective of this presentation and again we deal with only one of them.

Kaas-Petersen [2] found that the bifurcation at point D in figure 1 at V = 112.59 m/s is a

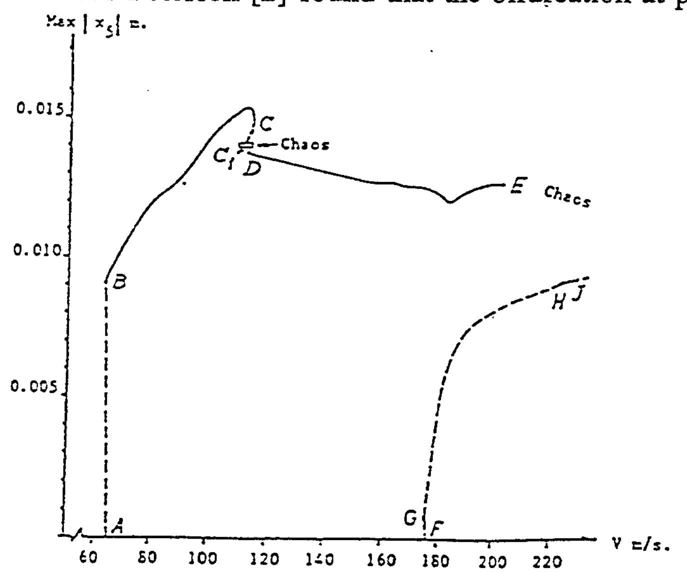

Fig.1 Bifurcation diagram for the truck from [3].

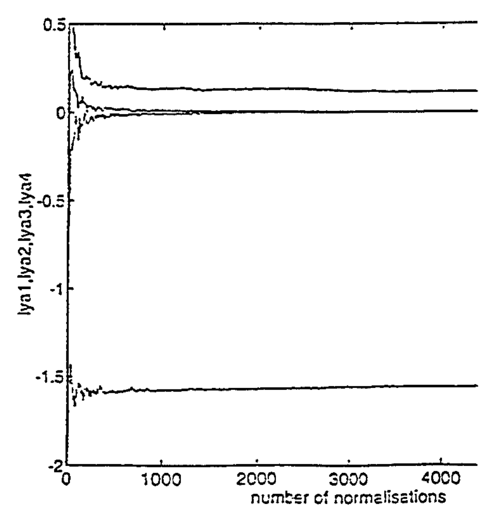

Fig. 2 Convergence of the four largest Lyapunov exponents at V =112 m/s.

bifurcation to a bi-periodic (possibly quasi-periodic) solution from an unstable asymmetric periodic solution. We have now found that the bi-periodic motion bifurcates supercritically, and it is unstable. It gains stability in a saddle-node bifurcation at V =113.19 m/s

and has then been followed in the parameter-phase space for decreasing speed values. The bi-periodic motion has a very dominating frequency around 4.5 Hz (the "hunting" frequency of the truck) and a modulation frequency around 0.2 Hz.

The bi-periodic motion undergoes a period doubling of the modulation frequency near 112.4 m/s, which is the start of a period doubling sequence that ends up in fully developed chaos just above V =112.27 m/s. The existence of the bi-periodic solutions is demonstrated by Poincare section plots of the solutions and a calculation of the four largest Lyapunov exponents for V =113.0 m/s. In the same way we verify the existence of the chaotic attractor, but in addition the Lyapunov dimension is estimated for a couple of values of the speed. The result of one of the calculations is shown on figure 2.

We followed the development of the chaotic attractor for decreasing values of the speed and calculated Poincare section plots and estimates of the four largest Lyapunov exponents for discrete values. We found two bi-periodic (possibly quasi-periodic) windows at V =112.24 m/s and 112.15 m/s. We also found that the erratic component of the motion becomes more pronounced for lower speeds although it remains small. As a result the Lyapunov dimension grows slightly from 3.03 to 3.09. We have further verified our claim that the attractor is asymmetric by calculating the quantity

$$\overline{x_5} = \lim \frac{1}{T} \int_{O}^{T} x_5(t)\, dt$$

and checking that the average does not converge to zero as T grows.

We have not been able to track the attractor below V =111.6 m/s. We have not found any bifurcation point that could explain the disappearance so we are forced to conjecture that the attractor disappears in a global bifurcation, where one of the invariant manifolds of the cyclic saddles in the neighbourhood plays a decisive role.

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Experimental Study of the Indeterminate Bifurcation Using a Gravity-loaded 'Roller Coaster'

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Many dynamic systems are subjected to wide ranges of operating conditions. Such occurrences often lead engineers to be interested in *global* dynamical properties of these systems, particularly from a predictability standpoint. In the case of nonlinear systems, one well-known property is the saddle-node bifurcation, associated with sudden jumps to and from resonance, bounding a region of hysteresis. Recently, attention has been given to the *indeterminate* bifurcation [1, 2, 3], where the appearance of fractal basin boundaries in association with this resonant jump results in indeterminacy regarding the attractor to which a given trajectory moves. Such behavior has direct consequence in eroding predictability, given the inevitable uncertainty in the various parameters and initial conditions of the system.

This study considers the indeterminate bifurcation associated with both the canonical escape system and the twin-well Duffing system (characterized by quadratic and cubic stiffness nonlinearities, respectively). Making use of the "ball-rolling-on-a-hill" concept, potential energy surfaces have been constructed upon which a "roller-coaster" cart is constrained to roll, thus mimicking a particle oscillating in a potential well. With the addition of computer-controlled excitation, these systems have been shown to provide a rich variety of nonlinear behavior, including jumps, flips, subharmonics, and chaos [4, 5, 6]. The experiment is controlled completely through the LabVIEW object-oriented, programmable interface. Experimental initial condition maps are generated by direct integration and stochastic interrogation [7].

A few preliminary numerical results are shown in Figure 1 below for the escape equation; the figures show the initial condition map and some corresponding representative time series at an intermediate level of forcing that gives rise to the indeterminate bifurcation. The unstable manifold of the resonant saddle (unstable solution in the hysteresis region) becomes heteroclinically tangled with the stable manifold of the hilltop saddle, and fractal escape boundaries are generated with an infinite number of fractal fingers accumulating. The fractal nature is represented by three trajectories which were started close to each other, yet eventually settled into three different outcomes. At lower forcing levels, the basin boundaries, while complicated, are not fractal, and at higher forcing levels, virtually all trajectories escape during the jump to resonance. Experimental results will be presented.

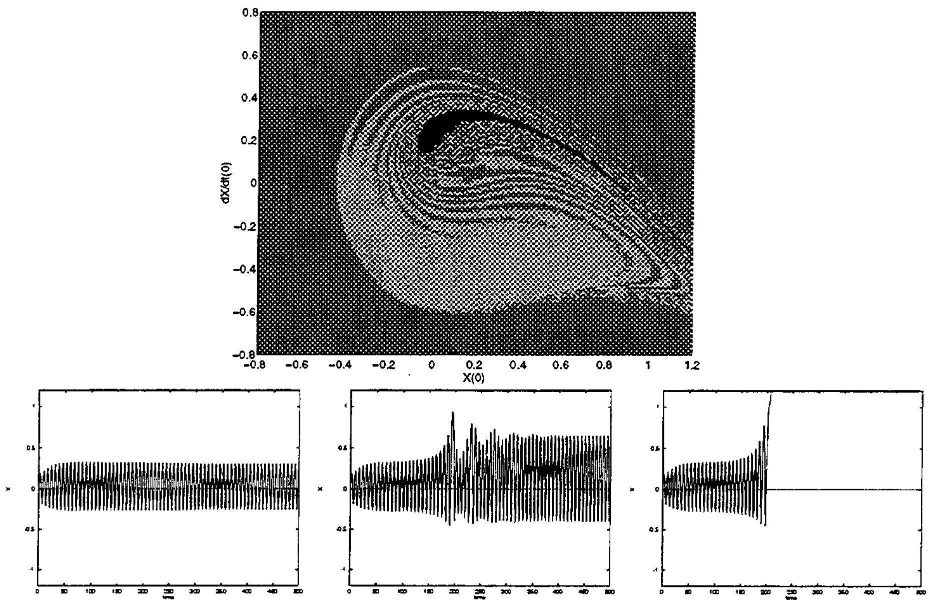

Figure 1: (Top) Initial condition map for the escape equation just before the jump to resonance; black indicates the non-resonant solution, light grey the resonant solution, and dark grey escape. (Bottom) Three slightly different initial conditions at the same forcing conditions, leading to the non-resonant solution, resonant solution, and escape, left to right.

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Bifurcations and Chaos in a Forced Vibratory System with an Asymmetric Piecewise Linear-Restoring Force

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There are a lot of piecewise linear equations of motion which describe physical phenomena such as the vibration caused by collision or occurring in real machinery containing gears. Since most of these equations are descriptive of chaotic vibration, it is necessary to investigate the generation mechanism of chaotic vibration thoroughly. This research is concerned with the gear-meshing vibration which is well known as a chaos-producing vibratory system as a result of collision. In this paper, the behavioral characteristics of bifurcation procedures occurring in a forced vibratory system with an asymmetric piecewise linear-restoring force were investigated, the system being derived from the circumferential-directional vibration of a gear containing a meshing error.

When an attractor transforms from being chaotic to being periodic, there can be two types of transformation. One is the "hysteresis" type in which the chaotic attractor transforms into the periodic solution situated far from the chaotic attractor in the phase plane; the other is that where the chaotic solution transforms into a periodic solution located within the chaotic attractor's boundaries before the transition.

In the case where an n-periodic solution changes into a chaotic solution via period-doubling, this is normally followed by a process in which the attractor's number of bands falls discretely and exponentially to 1. This sequence of band attractors has a close relation to the invariant curves which emanate from the inversely unstable fixed points of each period. In order to have an n-band attractor, the existence of a heteroclinic intersection between an out-set of the invariant curves from an inversely unstable fixed point of period-2n and an inset of the invariant curves from an inversely unstable fixed point of period-n is required. If an irrelevant directly unstable fixed point exists in the vicinity of any band attractor participating the above-mentioned process, and the attractor exhibits a heteroclinic intersection with an inset of the invariant curves starting from the directly unstable fixed point as the external angular frequency increases, an "interior catastrophe" makes the attractor expand, and it transforms suddenly into a 1-band attractor.

Around the first resonance, a series of the periodic solutions belonging to various super/subharmonic vibrations which appear irregularly as "windows" in the bifurcation diagram, comprise one "family", which can be categorized as being either type-I or type-II. Type-I is a family characterized by multifolding, namely, iterating fold-bifurcations (saddle-node bifurcations). The solution curve repeats the following pattern: directly unstable $\longrightarrow$ fold-bifurcation $\longrightarrow$ completely stable $\longrightarrow$ inversely unstable $\longrightarrow$ completely stable $\longrightarrow$ fold-bifurcation $\longrightarrow$ directly unstable. Type-II is a family which is fold-bifurcated only at both ends of its solution curve.

The intermittent bifurcations of the chaotic attractor can be called either "interior catastrophe"-like or "hysteresis"-like according to whether or not the bifurcated chaotic attractor remains within the same area in which the attractor was located in the phase plane before the bifurcation. Furthermore, it is shown that both these two kinds of bifurcation phenomena can occur in a forced vibratory system with a piecewise linear asymmetric restoring force.

When a 2^n-band attractor transforms into a 2^{n-1}-band attractor by intersecting an inset of the invariant curves from the inversely unstable fixed points of period-2^{n-1} without a fold-bifurcated directly unstable fixed point, and simultaneously an irrelevant directly unstable fixed point of a certain period is located close to an inset of the invariant curves from the inversely unstable fixed points of period-2^{n-1}, no stable 2^{n-1}-band attractor can be produced. The result of this case is that the 2^{n-1}-band attractor changes abruptly into a 1-band attractor after intersecting an inset of the invariant curves from the directly unstable fixed point.

Bifurcation Control of Parametrically Excited Duffing System by a Combined Linear-plus-nonlinear Feedback Control

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Abstract. Recently, significant attention has been focused on bifurcation control to transform the bifurcation behavior of the uncontrolled system into the desirable behavior (Nayfeh and Balachandran, 1995). Abed and Fu [Abed and Fu, 1986 and 1987] propose bifurcation control methods for Hopf bifurcation and for static bifurcation. Wang and Abed [Wang and Abed, 1992] clarify that a subcritical Hopf bifurcation of a power model (Dobson and Chiang, 1989) can be transformed into a supercritical Hopf bifurcation using a linear-plus-nonlinear control method. These bifurcation control methods are for autonomous systems, and the bifurcation control method for nonautonomous system has not been studied so far. In this article, we consider a parametrically excited Duffing system that is a nonautonomus system, and we propose a bifurcation control method for the system in the case of the

principal parametric resonance. It is recalled that the modulation equations qualitatively characterize the parametrically excited Duffing system. The bifurcation control method is analytically established by modifying the modulation equation of the uncontrolled parametrically excited Duffing system using a combined linear-plus-nonlinear feedback. The shift of the unstable region of the trivial steady state by the proposed bifurcation control stabilizes the trivial steady state in the nonstationary frequency response (the frequency and the amplitude of the parametric excitation are swept and constant, respectively) and in the quasistationary frequency responses (the frequency and the amplitude of the parametric excitation are swept much slowly and constant, respectively). Also the change of the nonlinear character of the bifurcation by the bifurcation control eliminates jump in the quasistationary force response (the frequency and the amplitude of the parametric excitation are constant and swept much slowly, respectively). Furthermore, a method to reduce the jump in the nonstationary fore response (the frequency and the amplitude of the parametric excitation are constant and swept, respectively) is discussed using numerical simulations.

COMPUTATION OF A HOPF BIFURCATION BY USING AN ASYMPTOTIC-NUMERICAL ALGORITHM

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An asymptotic-numerical method has been recently applied for computing nonlinear equilibrium paths for elastic structures. This method has been proposed by N. Damil and M. Potier-Ferry (1990) for computing perturbed bifurcation [1]. The non-linear branches are sought in the form of asymptotic expansions with respect to a control parameter. The interest is that a large number of terms of the series can be determined with cheap computations, and by using a classical finite element methods. The computed series are generally limited by finite radius of convergence. However, replacing the power series by rational fractions (Padé approximants), we are able to build up a continuation of the solution branch outside of the radius of convergence [2], [4]. The numerical efficiency of this method has been established by mean of numerical examples. However, the asymptotic-numerical method has also been efficiently used for the computation of the bifurcating branches, for the detection of stationary bifurcation points and for computing periodic solutions branches [3]. Here, our aim is to represent a new asymptotic-numerical method for the detection of Hopf bifurcation points that by on a linear or nonlinear solution branches. In this respect, we define a Hopf bifurcation indicator, which becomes zero only at bifurcation points. Our indicator noted by $\Delta\mu$ (a, ω) is a function which depends on two parameters ('a' control parameter and 'ω' pulsation of the oscillations). For characterising the Hopf bifurcation points, we introduce a harmonic perturbation force f in the nonlinear system and we define the following perturbed problem :

$$\mathbf{\Gamma(a,\omega)\ V(a,\omega)\ =\ \Delta\mu(a,\omega)\ f} \qquad \textbf{(1)}$$

$$\mathbf{\langle V(a,\omega) - V(0,0),\ V(0,0)\rangle\ = 0} \qquad \textbf{(2)}$$

where Γ (a,ω) is the tangent operator which depends analytically on 'a' and on 'ω' and V(a,ω) is the reponse to the perturbation. The equation (1) is obtained by linearising the nonlinear perturbed problem. The equation (2) is a supplementary condition on V in order to have a unique solution (V, $\Delta\mu$). Basing on some particular properties, the couple ($V(a,\omega)$, $\Delta\mu(a,\omega)$) is sought as follows :

$$\mathbf{V(a,\omega)} = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \mathbf{a}^{i} \ \omega^{j} \ \mathbf{V(i,j)} = \sum_{i=0}^{\infty} \sum_{j=0}^{i} \mathbf{a}^{i-j} \ \omega^{j} \ \mathbf{V(i-j,j)} \qquad (3)$$

$$\Delta\mu(\mathbf{a},\omega) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \mathbf{a}^{i} \ \omega^{j} \ \Delta\mu(\mathbf{i},\mathbf{j}) = \sum_{i=0}^{\infty} \sum_{j=0}^{i} \mathbf{a}^{i-j} \ \omega^{j} \ \Delta\mu(\mathbf{i-j},\mathbf{j}) \qquad (4)$$

By introducing (3) and (4) into (1) and (2) and grouping the terms according to powers of 'a' and 'ω', we get a sequence of linear problems for each couple (V(i, j), $\Delta\mu$(i, j)), that we have solved numerically by the finite element method.
Next, we have replaced the series (4) by rational fractions, whose orders have to be chosen carefully. The Hopf bifurcation points and the corresponding pulsations are determined by seeking the zeros of the indicator.
The efficiency of this asymptotic-numerical procedure has been tested on some ordinary differential equations with a few degrees of freedom [3]. For the large systems, we have adapted this technique to elastic structures subjected to non conservative loading. Because of this efficiency, we are applying this procedure to fluid-structures problems.

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Monday, June 10

1330-1510

Session 5. Poster Session

NONLINEAR VIBRATIONS OF THIN ELASTIC PLATES BY AN ASYMPTOTIC-NUMERICAL METHOD

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In this paper, we present an Asymptotic-Numerical Method for the determination of nonlinear dynamic response of thin elastic plates. A simple approach consists in assuming that the dependence of the solution in time is harmonic. By using a harmonic balance method, one can obtain a nonlinear boundary value problem in the space variables. This technique is largely used because it permits to transform the nonlinear dynamic problem into a nonlinear static one.

In general the computation methods used to solve this problem are the incremental-iterative methods. With a proper parametrisation of the branch, such algorithms are successful in determining a complete shape of solution that will be represented point by point. But this requires long computation times as compared to a linear problem and it is difficult to automatize. An alternative approach corresponds to analytical representation techniques such as the perturbation methods. In contrast to the incremental-iterative methods, perturbation methods have received much less attention from computational community. There are two main explanations for the poor success of such methods in computational context. The first one is the growing complexity of the right-hand sides of the linear problems obtained. The second is the thought that the analytical representation is valid only for very small values of the perturbation parameter.

An Asymptotic-Numerical Method based on perturbation techniques and finite element method has been developed for nonlinear problems. This method permits to remove some of the previous difficulties in the classical perturbation methods. It has been proposed by Damil and Potier-Ferry [1] and applied in Azrar et al [2] for computing the post-buckling behaviour of elastic plates and shells. Next, it has been extended with a continuation procedure by Cochelin et al [3]. These works have brought out the essential features that affect the practicability of the coupling of perturbation methods and finite element methods.

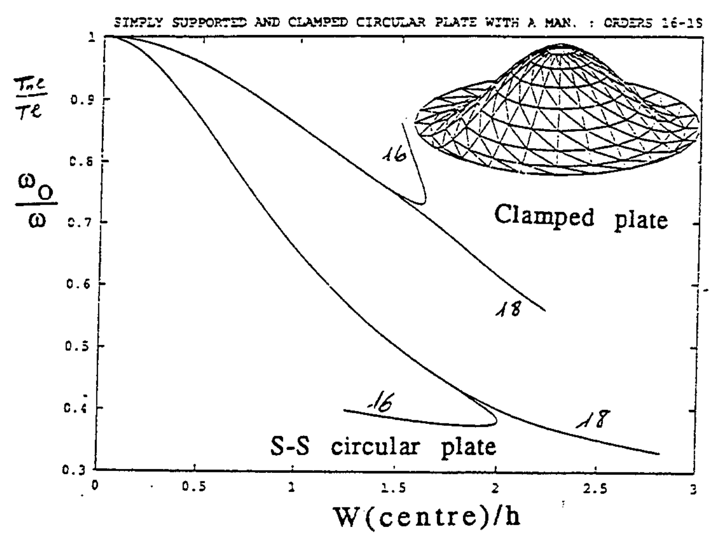
The differential equations of the motion governing the moderately large amplitude vibrations of thin elastic plates can be obtained from the Von Karman's deflection theory of plates. The displacement variational principle of these equations leads to a cubic nonlinearity. The mixed stress-displacement approach leads to an only quadratic nonlinearity. Then in view of applying the expansion procedure, the mixed approach is preferred here since it leads to a simple algebra. After the expansion process, we shall come back to a displacement formulation and use the very classical displacement finite element method. In view of using an operational notations the governing equation of free vibrations of thin elastic plates can be written as :

$$< L.U, \delta U > - (\omega^2 - \omega_o^2) < M.U, \delta U > + < Q(U,U), \delta U > = 0 \qquad (1)$$

where the linear operators L and M correspond to stiffness and the mass matrix respectively and Q is a quadratic one. $U = {}^t[u_1, u_2, w, N]$ is The mixed vector (displacement, stress), ω_o and ω are linear and nonlinear frequency. The unknowns are the mixed vector U and frequency ω. We assume that (U_o, ω_o) is a solution of equation (1) and that in the vicinity of this point the branch can be represented by power series with respect to a path parameter "a".

$$U = U_o + \sum_{p=1}^{+\infty} a^p\, U(p) \qquad \omega^2 - \omega_o^2 = \sum_{p=1}^{+\infty} a^p\, C(p) \qquad (2)$$

By introducing equation (2) into equation (1) and equating like powers of "a", we obtain a set of linear mixed problems at order p The principle of this numerical method is to compute successively a number of vectors U(p) and coefficients C(p) up to a given order n by solving linear problems. Since all these problems have the same stiffness matrix, only one matrix inversion is needed. The Asymptotic-Numerical solution (2) is truncated here at order 18. In the figure we present the ratio of nonlinear period to linear period versus the deflection at the centre of the plate for simply supported and clamped circular plate. It seen clearly that we obtain a large part of the backbone curve. We have studied free vibrations of different plates with various shapes and boundary conditions. The validity of the solution is limited by the radius of convergence which is indicated by the separation of the series for different orders. Different strategies could be used for increasing the zone of validity of the solution like Padé approximants and continuation technique. Last method is the best one [3]. It consists in applying the Asymptotic-Numerical Method iteratively in a step by step manner. Hence, all the nonlinear curve can be easily computed. Practically, the algorithm is very robust and completely automatic.



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The Effects of Friction on the Nonlinear Behavior of an Impact Oscillator

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Abstract

Experimental and analytical results are presented for a double-sided impact oscillator in the presence of dry friction. The simple experimental system, shown schematically in Figure 1, is intended to mimic contact behavior observed in several engineering applications, including mechanisms with clearances, gears with backlash, or valve/cam systems.

Analytical results and stability calculations are achieved via solution methods that take advantage of simplified piecewise linear impact and friction models. The normalized equation of motion may be written:

$$\ddot{\theta} + 2h\dot{\theta} + \theta + f\,\mathrm{sgn}(\dot{\theta}) + \begin{cases} 0 & |\theta| \le \sigma \\ \rho^2\theta - \sigma\rho^2\mathrm{sgn}(\theta) & |\theta| > \sigma \end{cases} = 2h\gamma\cos(\gamma\tau) + \sin(\gamma\tau)$$

This equation includes five parameters of interest: frequency ratio, γ, viscous damping, h, normalized friction force, f, impact locations, $\theta = \pm\sigma$, and the stiffness ratio, ρ.

The low frequency response tends to be dominated by friction effects, including stick-slip motion, while higher frequencies produce a variety of irregular behavior including chaos. Typical qualitative types of response, determined numerically and confirmed by experiment, include multiple impacts per response cycle, 'trapping' of the mass against an impact location for a finite duration, and mid-cycle sticking events.

Finally, the impact phenomenon gives rise to the potential for grazing bifurcations. The effects of friction on the evolution of these bifurcations is examined, as is the relationship between grazing boundaries in phase space and the eventual basins of attraction.

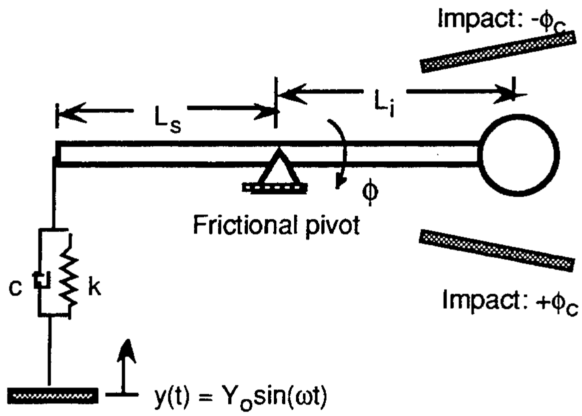

Figure 1. Schematic of experimental system.

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Nonlinear Control of Crane Operations at Sea

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Crane ships are used to transfer cargo from large, heavy ships to smaller, lighter landing craft utility ships (LCUs) at sea when a port is not available for the heavy ship. This transfer operation becomes dangerous when sea state 3 is reached. The sea state may not directly involve large motions of crane ships, but could indirectly cause large motions of the container being hoisted by creating a parametric instability of the load motion. This effect occurs due to the excitation of a parametric instability similar in form to that of the Mathieu instability. This phenomenon has been observed at full scale by crane operators and can arise in relatively mild sea states.

To illustrate how the parametric excitation and hence the instability arise, we consider a simple case in which the oscillations of the load are influenced by, but do not influence, the ship motion; that is, the coordinates x_a, y_a, and z_a of the boom tip are known functions of time t. We denote the coordinates of the load with mass m by x, y, and z. These depend on the response of the ship to the waves. The Lagrangian of the system is

$$L = \frac{1}{2}m\left[\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right] - mg(\ell_s - z + z_a) \tag{1}$$

where ℓ_s is the stretched length at equilibrium. When we place a controller at x_c, y_c, z_c on the hoisting rope assembly, the system is subjected to the constraint

$$(x - x_c)^2 + (y - y_c)^2 + (z - z_c)^2 = (\ell_s + r - \ell_1)^2 \tag{2}$$

where r is the coordinate of the load in the stretching direction of the rope and ℓ_1 is the distance between the controller and the boom tip.

To determine the equations of motion to third order in x and y, we let x_a, y_a, and z_a be $\leq O(x, y)$. The nondimensional forms of Euler Lagrange equations are

$$\begin{aligned}\ddot{x} + x = x_a + \ell_{1x} + (x - x_a)\Big[& r + \ddot{r} + \ddot{z}_a - \frac{1}{2}\ell_{1z} - C_R r - \frac{1}{2}r^2 + C_R r^2 \\ & -(r - \ell_{1z})(\frac{1}{2}r + \ddot{r} + \ddot{z}_a - \frac{1}{2}\ell_{1z}) - \frac{1}{4}\Gamma_1(x_c, y_c) - \Gamma_2(x_c, y_c) - \Gamma_3(x_c, y_c)\Big]\end{aligned} \tag{3}$$

$$\ddot{y} + y = y_a + \ell_{1y} + (y - y_a)\Big[r + \ddot{r} + \ddot{z}_a - \frac{1}{2}\ell_{1z} - C_R r - \frac{1}{2}r^2 + C_R r^2$$
$$-(r - \ell_{1z})(\frac{1}{2}r + \ddot{r} + \ddot{z}_a - \frac{1}{2}\ell_{1z}) - \frac{1}{4}\Gamma_1(x_c, y_c) - \Gamma_2(x_c, y_c) - \Gamma_3(x_c, y_c)\Big] \qquad (4)$$

where $x_c = x_a + \ell_{1x}$, $y_c = y_a + \ell_{1y}$, $z_c = z_a + \ell_{1z}$, and C_R is the nondimensional elastic coefficient of the hoisting rope assembly. Clearly, the linear natural frequencies of the load in the two orthogonal directions are the same, which produce a one-to-one autoparametric or internal resonance, which in turn lead to complex dynamics and an energy exchange between the two modes. Moreover, the vertical and lateral motions z_a, x_a, and y_a of the boom tip produce a parametric (multiplicative) excitation as well as an external (additive) excitation.

One of the cases we consider here is a principal parametric resonance such that

$$x_a = y_a = 0, \;\; z_a = \epsilon^2 w_a \cos \Omega t \qquad (5)$$

where $\Omega = 2 + \epsilon^2\sigma$. Using the method of multiple scales, we obtain the following equations describing the modulation of the complex amplitudes of the interacting modes:

$$i\dot{A}_1 = \epsilon^2(4\Lambda_1 A_1(A_1\bar{A}_1 + A_2\bar{A}_2) - 4\Lambda_2 A_2(A_1\bar{A}_2 - A_2\bar{A}_1) - f\bar{A}_1 e^{i\epsilon^2\sigma t}) \qquad (6)$$

$$i\dot{A}_2 = \epsilon^2(4\Lambda_1 A_2(A_1\bar{A}_1 + A_2\bar{A}_2) + 4\Lambda_2 A_1(A_1\bar{A}_2 - A_2\bar{A}_1) - f\bar{A}_2 e^{i\epsilon^2\sigma t}) \qquad (7)$$

where $\Lambda_i = \Lambda_i(C_R)$ and $f = f(C_R, z_a, z_c)$. We consider linear viscous damping and use the modern methods of nonlinear dynamics to analyze the equilibrium and dynamic solutions of the modulation equations (6) and (7) and their stability. The results show that the parametric excitation may produce an instability when the magnitude of the excitation exceeds a threshold. The threshold depends on the frequency content of the excitation. The next strong instability occurs when the frequency of the excitation is equal to or near the natural frequency of the load. The strongest instability occurs when the frequency of the excitation is equal or nearly equal to twice the natural frequency of the swinging load. The controller can be used to suppress this type of instability. Numerical results for both parametric as well as external excitations will be presented.

A DECREMENT METHOD FOR THE SIMULTANEOUS ESTIMATION OF COULOMB AND VISCOUS FRICTION

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In his *Theory of Sound*, Rayleigh (1877) noted that, for the free vibration of a linear damped oscillator, "the difference in the logarithms of successive extreme excursions is nearly constant, and is called the logarithmic decrement." In fact, the idea goes back to Hermann Helmhotz, who in 1863 applied the logarithmic decrement to determine frequency information in musical tones given a known damping coeffient (Helmholtz, 1954). This observation can be applied to estimate the damping factor from a free vibration of a single-degree-of-freedom linear oscillator.

Later, Lorenz (1924) had included the free-response solution of a mass and spring with Coulomb friction. He noted that the successive extreme excursions decrease at a constant rate. Thus, it is possible to extract the Coulomb friction from the constant decrement of a harmonic oscillator with constant frictional damping.

We present a method for extracting both Coulomb and viscous damping parameters by the examination of the free-vibration decrements of a system with both forms of dissipation. An algorithm uses the extreme excursions of the free-vibration solution to isolate and identify the viscous and Coulomb effects. In numerical tests, friction parameters are nearly exactly identified. The idea is also tested on an experimental mass-spring system. The method is applicable to linear single-degree-of-freedom systems when the damping is light enough that the free response undergoes a few oscillations.

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An Experimental and Theoretical Investigation into the Influence of Hysteretic Damping on the Dynamic Behavior of a Three-Beam Structure

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A theoretical and experimental investigation was conducted into the linear and nonlinear dynamic behavior of a frame structure, which is frequently used in engineering applications. The structure consisted of three continuous beams that were connected by two brass hinges.

A linear dynamic model of the structure was developed and linear vibration theory was applied. The hinges were assumed to rotate freely and were modeled by ideal pinned junctions. Experimental modal analysis indicated that the obtained natural frequencies were highly sensitive to very small changes in the oscillation amplitude. For small amplitudes, a relatively small increase in the oscillation amplitude led to a relatively large decrease in the natural frequencies. Thus, the amplitude range where linear responses can be expected was, in practice, very small. Instead, a strong nonlinear behavior was exibited by the experimentally obtained backbone curves for small vibrations. For the first and second modes, the backbone curves were convex, highly bent, and displayed almost zero slopes around the computed linear natural frequencies, indicating a strong softening-type behavior. Furthermore, the experimentally obtained frequency-response curves exhibited jump phenomena. Hence, linear vibration theory was insufficient to explain the dynamic behavior of the structure.

A nonlinear dynamic model that accounts for nonlinearities due to geometry, inertia, ma-

terial, damping, and nonfrictional coupling forces was developed, while the hinges were still assumed to rotate freely like ideal pinned junctions. We experimentally investigated whether the nonlinear dynamic characteristics of the structure were the results of modal interactions, such as internal or combination resonances. But neither one of these resonances could be activated. We concluded that the nonlinear coupling terms were very weak and hence not the source of the strong nonlinear behavior indicated by the experimentally obtained backbone curves.

Finally, we examined the dynamic characteristics of the hinges. We note that a preload was applied to the hinges when the structure was mounted. This preload led to geometric imperfections and a drastic increase in dry friction at the contact surfaces between the hinge elements. Thus, modeling the hinges as ideal pinned junctions was not justified and a detailed study of the nonideal dynamic characteristics of the hinges was undertaken. As a result of this study, we identified imperfect rotational slipping, preloaded backlash, and nonlinear flexibility of the connecting hinge elements as the three basic mechanisms leading to hysteretic damping in the hinges. We found that stiffness degradation hysteretic damping in the hinges is the best model that explains the observed nonlinear dynamic behavior. A multilinear stiffness degradation model was used to describe the overall hysteretic load-displacement relation. An approximate analytical approach was used to compute the steady-state response of the structure to a harmonic excitation. A good qualitative agreement between the computations and the experimental results was obtained.

We concluded that hysteretic damping greatly influenced the dynamics of the structure even for small vibrations. It could be used as a powerful damping mechanism to control and avoid dangerous nonlinear phenomena, such as internal and combination resonances, that were observed in a variety of structures.

EFFICIENT THEORETICAL MODELLING AND COMPUTER SIMULATION OF A PLANAR SERVO–PNEUMATIC TEST FACILITY

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Mathematical models and computer simulation models of technical processes (in our case planar multi–axis test facilities) strongly depend (a.o.)

- on the task to be performed,
- on the accuracy of the results to be achieved, and
- on the technology used to build these components.

In case of a planar multi–axis test facilities these models depend

- on the type and location of the sensing elements used in the control process, and
- on the type of the actuators used for driving the test facility.

In industrial practice, heavy bodies and large systems usually are tested by servo–hydraulic multi–axis test facilities. These test facilities are characterized by low masses of the actuators compared to the masses of the test table and payload. This implies that in the modelling of those test facilities the masses of the actuators can be neglected compared to the masses of the test table and of the specimen. In these cases **reduced models** of the test facilities have been successfully used. They include:

- one or two rigid bodies, and (in the planar case)
- three to four servo–hydraulic actuators, each coupled to the foundation and to the test table by joints.

Based on these reduced models various investigations and computer simulations have been done in the last years concerning high quality control concepts of those servo–hydraulic test facilities (a.o. [1], [2], [3]). The results obtained in those investigations depend on the assumption that the actuator masses can be neglected compared to the test table mass.

On the other hand there exist various applications, where these assumptions are not satisfied. For instance in experiments, where small structures and light test specimen have to be tested. In those situations electromagnetic, or more recently, servo–pneumatic test facilities are often used. Then the masses of both, the servo–pneumatic as well as the electromagnetic actuators can no longer be neglected compared to the masses of the test table and payload. This implies that the mechanical modelling and the control of those test facilities has to take into account the masses and the products and moments of inertia of the actuator components and their special coupling.

As a consequence **extended models** must be used. They include:

- seven up to ten rigid bodies, assotiated to
  - the test table and payload and to
  - the moving bodies of the actuators (housing, piston, etc.),
- coupled by various joints, and
- driven by dynamic models of the pneumatic or electromagnetic forces.

The paper presented includes the following topics:

- mathematical models of the test facility

- **reduced models**
  - models of the mechanical components,
  - models of the servo–pneumatic actuators and servo–valves, and
- **extended models**
  - models of the mechanical components,
  - models of the servo–pneumatic actuators and servo–valves, and

- computer simulation results obtained by a rigid body program, that demonstrate the necessity to use extended models in situations described above.

From the **point of view of mechanics** the modelling of those systems by rigid body software packages is standard.

From the **control theoretical point of view** the mathematical modelling of a multi–axis test facility in extended form described above leads to severe problems both, in the analytical derivation of nonlinear control algorithms as well as in the implementation of those algorithms in controller hardware. Servo–hydraulic multi–axis test facilities have been very efficiently and very accurately controlled by using exact linearization techniques ([2], [3], [4], [5]). These control algorithms are needed in symbolic form in order to later on adapt the controllers to practical situations by inserting relevant parameters of the test facility and of the test specimen into the control algorithms. These algorithms are already extrem lengthy and unwieldy in case of test facility models only including a single rigid body, i.e. in case of **reduced test facility models**.

Applying this control strategy to **extended test facility models** described in standard algebro–differential equations used in rigid body software packages would yield control algorithms of a complexity that can no longer be handled symbolically even by powerful modern computers.

To overcome these problems the extended model equations of the test facility have been reformulated in this paper. This has been done based on the following idea: Instead of modelling each of the rigid bodies of the actuators explicitly, the inertia forces and moments of these rigid bodies have been projected to the test table. This finaly will map the extended model equation into a form which is identical to the form of the reduced model equations, including more complex system matrices and vector functions. In the final paper these relations will be derived in detail. Based on this equations control algorithms have been developed that provide an accurate and robust control behavior of the test facility.

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USING A MULTI–AXIS TEST FACILITY FOR THE IDENTIFICATION OF THE INERTIA PARAMETERS OF A RIGID BODY

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The main **objectives** of the work described in this paper are:

- the experimental identification of the inertia parameters of a rigid body under planar motion and
- accuracy tests of the identification results, depending
  - on the type of test signals, i.e.
    - signal–form and
    - signal–frequency,
  - on the model hypothesis used in the identification process
    - extended models (non–linear/quasi–linear) and
    - reduced models (non–linear/linear), and
  - on the final application of the identified parameters
    - for a mechanical analysis and synthesis of the test body considered and
    - for fitting the parameters of sophisticated control algorithms [1] to the test facility.

The **methodology** used in the identification process will be described as follows:

- In **a first step**, the hardware of the test facility used for performing the identification experiments will be discussed. It includes the subsystems:
  - test table and payload (rigid bodies),
  - three servo–pneumatic actuators, each including
    - a cylinder and
    - a servo–valve,
  - sensing elements including
    - force transducers,
    - displacement sensors and
    - acceleration sensors and
  - controllers.
- In **a second step**, extended linear and nonlinear mathematical models as well as reduced linear and nonlinear mathematical models of the overall test facility including
  - the test table and payload,
  - the servo–pneumatic actuators,
  - the servo–valves and
  - the controller

  will be presented. They provide the model hypothesis used in the identification process, and they are the basis of the computer simulation of the test facility.

The **results** obtained in the laboratory experiments, in the computer simulations and in the identification process are discussed in several steps:

- In **a first step**, various laboratory experiments and computer simulations will be discussed. This includes a model validation by comparing the results of laboratory experiments with associated computer simulations.
- In **a second step**, the inertia parameters of the test table and/or of the payload have been identified [2], both, from laboratory experiments and from computer simulations. They will be compared with calculated inertia parameters of different test bodies. The identified inertia parameters are in good agreement with the calculated parameter values. The accuracy of these results depends
  - on the type of test signal selected and
  - on the model hypothesis used on the identification process.

  The best identification results have been obtained from the extended nonlinear model, using a test signal of a frequency of one hertz. Very poor results have been obtained from reduced linear models.
- In **a third step** various control results obtained by including identified inertia parameters of different quality into the control algorithms will be compared.

The investigations described in this paper have been obtained by a **planar test facility**. In a next step a **spatial multi–axis test facility** will be built,

- to identify the ten inertia parameters of a rigid body in space and
- to implement and to test sophisticated control algorithms [3] in a spatial laboratory test facility.

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A Finite-Element-Based Methodology for Reducing Rattle in Structural Components

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Abstract

Automobile noise has gained in importance recently due to rising consumer expectations. With the continual reduction of engine noise, powertrain noise, and tire noise, S&R (squeaks and rattles) have become a significant noise source in modern vehicles. Traditional efforts in improving S&R performance have focused on 'find and fix' techniques. There has been very little, if any, work done in the way of systematical design methodologies for these problems.

In this presentation, a recently developed finite-element-based technique for improving S&R performance is described. This technique is a combination of methods from commercial multibody dynamics solvers (such as DADS[1] and ADAMS[2]), finite element software (such as MSC/NASTRAN[3]), design sensitivity analysis, and random vibration theory. The ultimate goal of this technique is to minimize S&R as early as possible in the vehicle development process.

A single degree-of-freedom impact oscillator is used as a base model for the development of a quantitative measure of rattle frequency and intensity. An analytical expression, referred to as the 'rattle factor', of rattle frequency and intensity. An analytical expression, referred to as the 'rattle

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[1] DADS is a registered trademark of Computer Aided Design Software, Inc.
[2] ADAMS is a registered trademark of Mechanical Dynamics, Incorporated.
[3] MSC and MSC/ are registered trademarks and service makrds ot the MacNeal-Schwendler Corporation. NASTRAN is a registered trademark of the National Aeronautics and Space Adminstration.

factor' is derived by assuming that this impact oscillator is subjected to random excitation and that the rattle events are intermittent. This measure is then generalized to more complex situations and applied to study the rattle dynamics of the latch mechanism and corner rubber snubbers of a glove compartment. This study demonstrates how to incorporate the analytical expression together with a finite element solution package (MSC/NASTRAN) and a multibody dynamics simulation package (DADS) to provide a design sensitivity analysis for improvement of the rattle performance of the overall glove compartment. Results from analysis and simulation studies are compared, demonstrating the usefulness of the proposed method.

Experimental Control of Flexible Structures using Nonlinear Modal Coupling: Forced and Free Vibratios

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Abstract

There are several techniques available for vibration suppression of flexible structures. Application of these techniques however, may not cause the oscillatory systems to decay fast enough. What is the most important in vibration suppression for flexible structures is the rate at which the oscillations die out. In this paper we use modal coupling as a tool to control the free and forced vibrations induced in a flexible beam.

The experimental system comprises a piezo-actuated cantilever beam, controlled digitally using a personal computer. The controller is implemented on software using "C" code. The beam is modeled as a one-degree-of-freedom system and its natural frequency and damping ratio are obtained from the experimental data.

The experiment authenticates the theoretical results developed by the authors in [5], [6] and [7]. The results clearly demonstrate the advantage of the proposed controller over the conventional velocity feedback control. The proposed controller is able to produce a uni-directional input with the same performance as of a velocity feedback control with twice the peak to peak actuator effort. This is particularly useful for the cases where the actuators are uni-directional. Also, it seems beneficial from cost perspective.

EXPERIMENTAL SURVEY OF NONLINEAR RESPONSE BEHAVIORS EXHIBITED BY A MODEL SOLAR ARRAY

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With the advent of large, flexible substructures on–orbit, such as the radiators and solar arrays of the International Space Station (ISS) and the solar arrays of the Hubble Space Telescope (HST), the complexity of the dynamic response of these elements becomes a key issue in their performance and in their influence on the overall system performance. To understand potential issues, an experimental survey was conducted of a flexible solar array from the Dynamic Scale Model Technology (DSMT) program at NASA Langley Research Center (LaRC) [1]. The DSMT model was designed as a hybrid 1/10:1/5–scale (dimensional:dynamic) representative of the flexible solar array for the space station. In preliminary tests, this model demonstrated nonlinear response behaviors including superharmonic, subharmonic, and combination resonances, jumps, saturation, and chaos [2]. This presentation is a survey of modal changes due to variation of a lumped mass parameter and a selection of nonlinear response behaviors exhibited by this solar array phenomena model.

The modal survey was conducted with the DSMT solar array model mounted in a vertical cantilevered configuration. Responses were excited by impact with an instrumented hammer and were measured using accelerometers mounted in three orthogonal directions at twelve points. Testing was limited to the range 0–100 Hz which, due to the 1:5 dynamic ratio, corresponds to a range of 0–20 Hz for the full–scale solar array. Mode frequencies were measured for the bare model and for the model with 19 increments of mass, divided equally between four nodes located at the extreme tips of the crossbars. Results of the modal survey appear in Figure 1.

A nonlinear response survey was conducted with the DSMT model mounted vertically on a 1200 lbf exciter. Sine sweep and dwell tests were conducted to characterize nonlinear response behaviors. Again, measurements were taken using accelerometers. Harmonic resonances and related behaviors including jumps, saturation, and transition into chaos were observed.

Four examples of these nonlinear response behaviors were studied to determine the effect of added mass and changing modal frequencies. The first response, a superharmonic resonance of the third out–of–plane mode, was found not to be dependent on modal coupling. The second response, an interaction between the third and fourth out–of–plane bending modes occurs only when the these two modes exist near a 1:2 frequency ratio. The third response, a combination resonance involving the third and fifth in–plane modes and the fifth out–of–plane mode, also occurs only when the sum of the in–plane mode frequencies is near the out–of–plane mode frequency. The final nonlinear response behavior, a chaotic region, occurs only when a particular set of modes exist within a small region of frequencies. Plots of all four examples are presented in Figure 2.

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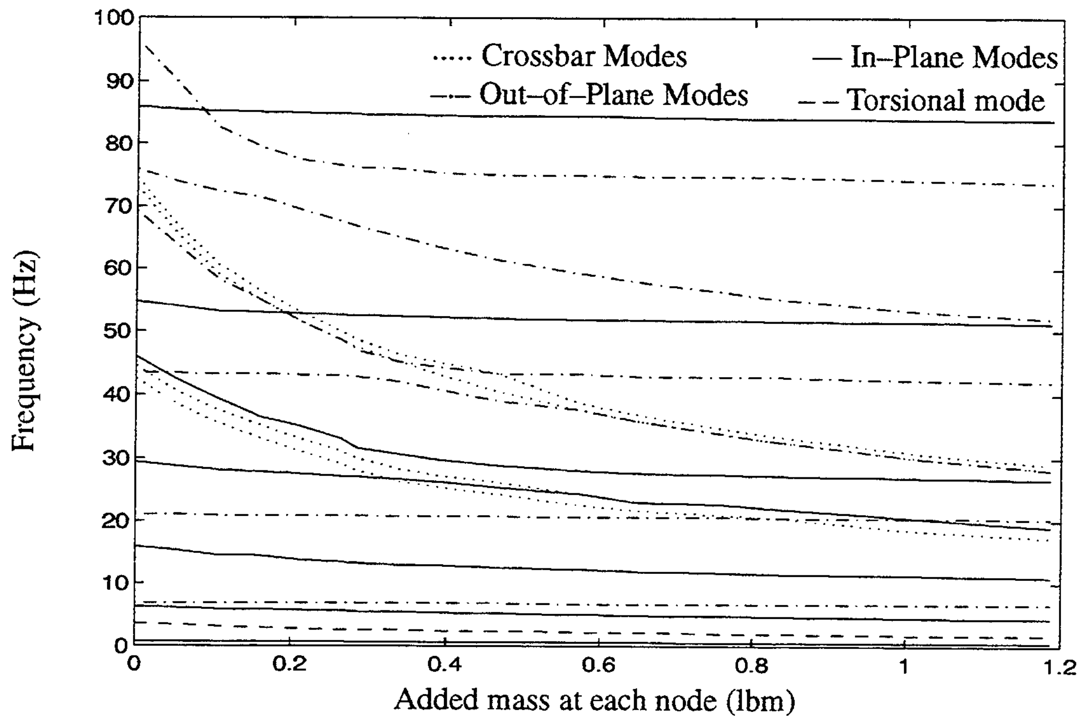

Figure 1 – Effects of added mass on solar array model natural frequencies

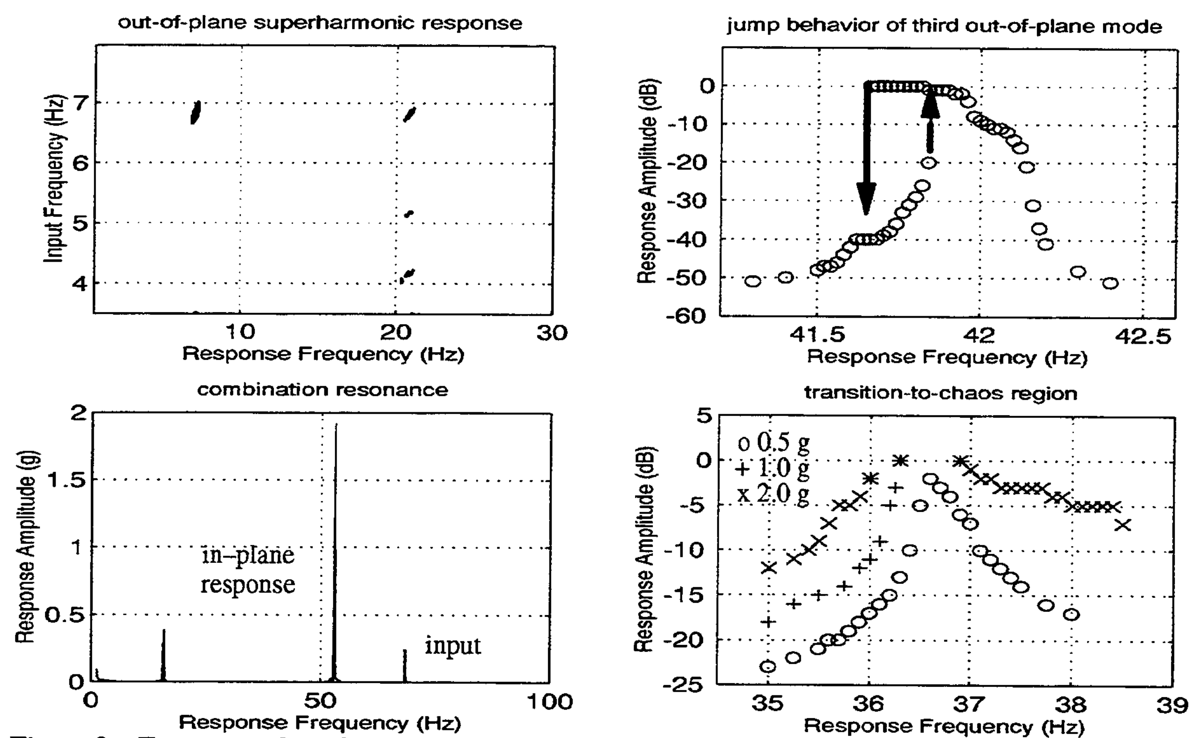

Figure 2 – Four examples of nonlinear response behaviors: a) Superharmonic resonance of third out–of–plane mode, b) jump behavior of the same mode, c) combination of third and fifth in–plane modes with an input at 68.6 Hz, and d) transition to chaos region for three input amplitudes.

Evidence for Chaos in the Ventilation Patterns of Resting Human Subjects

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Physiological systems generate highly complex and variable data sets. The idea that this data may represent the operation of a low dimensional dynamic system has far reaching implications for the interpretation of the data. Previous attempts to quantify breath-by-breath variations have ranged from calculation of the autocorrelation to more recent methods of spectral analysis. Recently, it has been hypothesised that resting breathing patterns may be described by a chaotic deterministic system. For a system to be chaotic, it must be governed by a nonlinear process and have sensitivity to initial conditions. The method of surrogate data has been applied to the ventilation data from resting humans to show evidence of a nonlinear process. Sensitivity to initial conditions is characteristic of at least one positive Lyapunov exponent. The results strongly suggest that resting breathing patterns are driven by a nonlinear "chaotic" system.

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Nonlinear Dynamic Stability of Normal and Arthritic Greyhounds

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The study of animal gait plays an important role in the development of techniques for both the diagnosis and treatment of its abnormalities. Commonly, experimentally acquired kinematic measures like joint rotations, cadence, stride length and joint velocity were used to characterize animal gait[1,2]. Later on, concepts like ground reaction forces, impulses, mean peak vertical forces, and ground force redistribution in limbs were used to evaluate gait in normal and pathological animals[3]. In recent years, researchers have incorporated the techniques of non-linear dynamics to develop qualitative and quantitative methods for human motion analysis. Classical methods like the phase plane portraits were used to compare the locomotion of neurologically impaired and normal individuals[4]. Relative phase angles, phase plane portraits[5], and reverse phase plane portraits[6] of joints were used to quantitatively describe multi-joint coordination during complex actions of humans. Phase plane portraits along with the first return maps were put into effective use to graphically highlight gait abnormalities in humans[7]. Further a scalar measure was proposed to compare the dynamic stability of normal and post-polio gait in humans[8].

In the present study, we analyze the gait stability of normal and arthritic dogs, based on an approach developed by Hurmuzlu[7,8]. The periodic motions that arise during quadrupedal locomotion formed the basis for this method. Specifically, we used phase plane portraits, first return maps and Floquet multipliers. Moreover, the proposed method did not depend on any assumptions regarding the inner structure of the system. The analysis was conducted by using displacement data of the animal's lower extremities obtained by means of a video based motion analysis system.

We made the assumption that the walking quadrupedal animal can be represented as a periodically forced non linear dynamical system, given by a set of n first order ordinary differential equations: $\dot{\mathbf{x}}(t) = \mathbf{F}(\mathbf{x}, t)$, where $\mathbf{x}$ is a vector in the phase space $\Re^n$. The vector field $\mathbf{F}(., t) = \mathbf{F}(., t+T)$ is periodic in time with the period T. A set of solutions representing the walking patterns of the animal would then be given by $\mathbf{x}(t) = \mathbf{x}(t+T)$. The animal gait can then be analyzed by studying the stability of the periodic solutions. For this we construct the phase plane portraits of the significant joints by plotting the joint velocities against the corresponding positions. This results in closed orbits on the phase plane and these orbits are stable if the neighbouring trajectories approach them asymptotically. To eliminate the additional factor of time and simplify the analysis we obtain discrete maps (Poincaré maps) by inserting fictitious planes transverse to the trajectories in phase space at specific instants of the gait cycle. Each cycle was defined as the motion between successive paw contact events of a particular limb. The local stability of the critical points of these discrete maps was analyzed using the Floquet theory. The state variables of the animal system were the relative joint rotations q_i and the joint angular velocities $\dot{q}_i$ (Fig. 1). The state vector $\mathbf{x}$ was defined as $\mathbf{x} = \{q_i, \dot{q}_i\}(i = 1, 2, 3)$. A discrete nonlinear map that represents the dynamics of the system was then given by $\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k)$ where, $\mathbf{x}_k$, and $\mathbf{x}_{k+1}$ represent the state vector sampled at the Poincaré section (paw strike PS) during the k-th and the $(k+1)$-th gait cycles and $\mathbf{f}$ is the discrete mapping function. Linearizing the map about the equilibrium points gives $\mathbf{x}_{k+1} = \mathbf{J}(\mathbf{x}_k)$ where $\mathbf{J}$ is called the Jacobian and is a constant coefficient matrix. The eigenvalues of this matrix are called the Floquet multipliers and for a stable system, the magnitudes of all these multipliers are less than unity. Also, for a stable system the largest of the Floquet multipliers dominates the system behaviour and can be used to as a stability index to compare different subjects.

The analysis was carried out in two distinct phases. The first phase involved the analysis of five healthy greyhounds. Kinematic data for each dog was averaged for 10 steady state gait cycles. For each joint, phase plane portraits and first return maps at the instant of paw strike were obtained. Floquet multipliers for each dog were computed from joint data extracted at paw strike. The magnitudes of the largest multipliers for all five dogs were averaged to obtain a representation of the normal group of dogs. For the second phase of the study, three dogs were selected from the above five and induced with acute and transient synovitis. Kinematic data acquired from these three dogs were then used to plot phase plane portraits and first return maps. Comparison of phase plane portraits and first return maps of the average normal and arthritic dogs graphically illustrated the differences in joint rotations and velocities (Fig. 2,3). The multipliers of the two arthritic dogs were larger than the average normal(Fig. 4). This indicated that their gait was less stable to the internal and external perturbations as compared to the gait of the normal dogs.

This study presents a superior objective measure to assist in clinical diagnosis and therapeutic decision making by addressing the mechanism of locomotion as a whole, including both continuous and discontinuous (foot impact and switching) phases of gait. The approach can be used in any clinical gait application that deals with a subject population that can generate a repetitive gait pattern.

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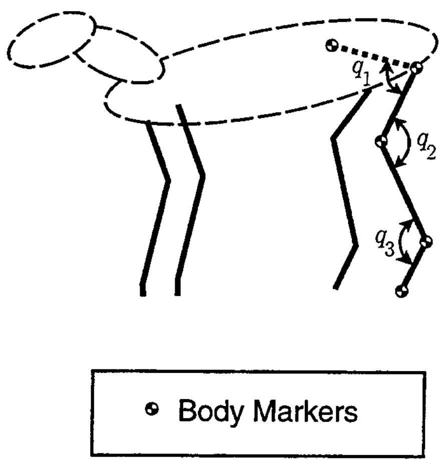

Figure 1 Quadrupedal model
q_1 Coxofemoral joint angle
q_2 Femorotibial joint angle
q_3 Tarsal joint angle

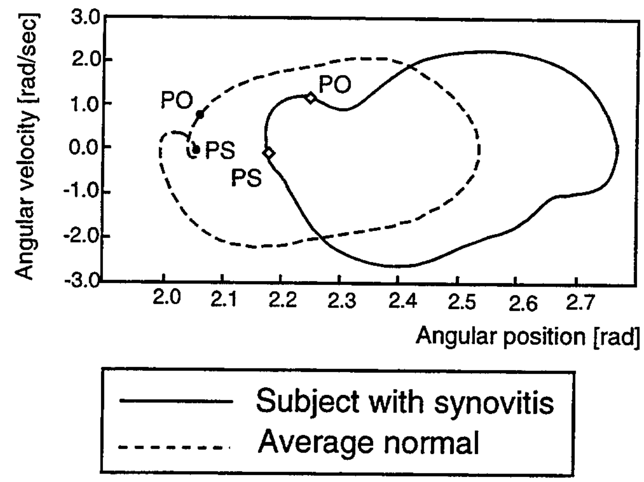

Figure 2. Phase plane portrait of the coxofemoral joint of a subject with synovitis along with that of the average normal . PS and PO represent events of paw strike and paw off respectively

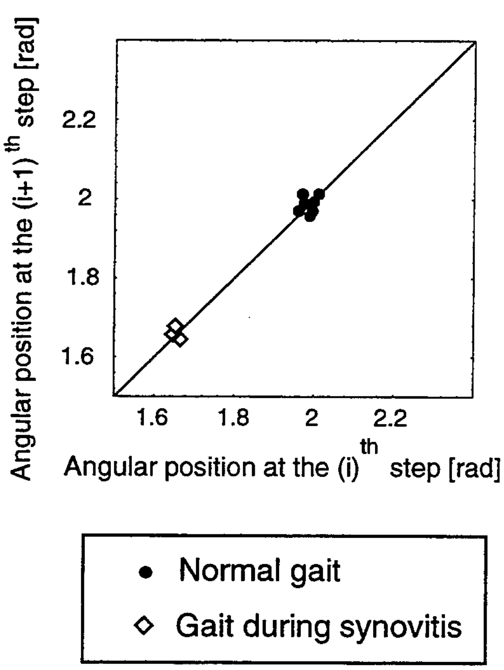

Figure 3. First return map of the coxofemoral joint

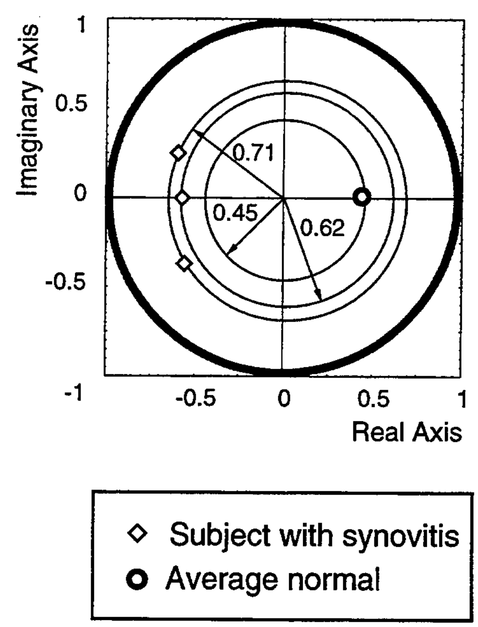

Figure 4 Largest multipliers for two arthritic dogs and that of the average normal.

On Discretization of Distributed-Parameter Systems with Quadratic and Cubic Nonlinearities

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Approximate methods for the study of vibrations of a hinged-hinged Euler-Bernoulli beam resting on a nonlinear foundation are discussed. The cases of primary resonance ($\Omega \approx \omega_n$) and subharmonic resonance of order one-half ($\Omega \approx 2\omega_n$), where ω_n is the natural frequency of the nth mode of the beam, are investigated. Approximate solutions based on discretization via the Galerkin method are contrasted with direct application of the method of multiple scales to the governing partial-differential equation and boundary conditions.

In nondimensional form, the transverse vibrations of the system are governed by

$$\frac{\partial^2 w}{\partial t^2} + \frac{\partial^4 w}{\partial x^4} + \alpha_2 w^2 + \alpha_3 w^3 + 2\mu \frac{\partial w}{\partial t} = F(x)\cos(\Omega t) \tag{1}$$

$$w(x,t) = 0 \quad \text{and} \quad \frac{\partial^2 w(x,t)}{\partial x^2} = 0 \quad \text{at} \quad x = 0 \quad \text{and} \quad 1 \tag{2}$$

where the α_i and μ are positive constants. For a single-mode Galerkin discretization, we let

$$w(x) \approx \Phi_n(x) q_n(t) \tag{3}$$

where $\Phi_n(x)$ is the nth linear mode and $q_n(t)$ is the associated generalized coordinate and obtain the equation governing the dynamics of the nth coordinate as

$$\ddot{q}_n + \omega_n^2 q_n + \delta_n q_n^2 + \frac{3}{2}\alpha_3 q_n^3 + 2\mu \dot{q}_n = f_n \cos(\Omega t) \tag{4}$$

We note that as a result of the discretization procedure, $\delta_n = 0$ for even modes.

Next, we use the method of multiple scales to determine approximate solutions of Equation (4) in the cases of primary resonance and subharmonic resonance of order one-half. Then, we compare these solutions with those obtained by attacking directly the partial-differential equation and boundary conditions given by Equations (1) and (2).

It is shown that the discretized equation fails to predict the correct dynamics of the original partial-differential system for two reasons: a) the dynamics are projected onto one eigenmode, and hence, no information on the spatial distribution generated by the quadratic nonlinearity is reflected by the discretization approach; b) the quadratic component of the nonlinearity in the original system is orthogonal to even linear normal modes.

In the case of primary resonance of even modes, the results totally disagree, since the discretization approach predicts symmetric and hardening-type responses regardless of the presence of the asymmetric nonlinearity. For odd modes, the results obtained with both approaches agree for the first mode, whereas the discretization results tend to deviate from the direct results for the higher modes.

In the case of subharmonic resonance of order one-half, it is shown that the discretization approach fails to predict the activation of subharmonic responses in even modes. On the other hand, for odd modes, the frequency-response curves obtained with both approaches are in agreement for the first mode, whereas they differ qualitatively as well as quantitatively for higher modes. We note that the frequency-response curves obtained with discretization are significantly narrower compared with those obtained with the direct treatment. The discretization approach underestimates the stability ranges for subharmonic responses.

Because, in general, nonlinear terms orthogonal to the linear mode shapes are likely to arise in most common nonlinear distributed-parameter systems, it can be concluded that discretization procedures, such as the Galerkin procedure, will generally yield inaccurate results.

Design of an Optimal Vibration Isolation System Using Nonlinear Localization

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There are many engineering structures and applications which contain vibrating machinery. In a large subset of such structures, the vibrations induced by such machinery may be detrimental to the function of the structure as a whole. For example, in a satellite the vibrations induced by momentum flywheels may adversely effect the ability of precision pointing devices to stay on target. It is desirable to minimize the transfer of such vibrations to the structure.

Traditional techniques for preventing the transfer of vibrational energy from components to the structure have included the use of vibration isolation padding, damping tape, and in many instances the use of active controllers. We propose a design technique where a passive vibration isolation system is created not as an external addition to the component, rather, it becomes an integral part of the design of the machine itself, and in the attachment of the machine to the structure.

In this study we investigate a three mass model (Figure 1), where mass m_1 represents the vibrating machine, mass m_2 the machine casing, and mass m_3 represents an intermediate grounding mass. The f(y) and g(y) represent general nonlinear stiffnesses, and the x_i represent the motion of each mass. In order to minimize the energy transfer to the structure, the motion of mass m_3 must be small.

We use the method of harmonic balance to derive the equations which govern the nonlinear normal modes of the system. The solutions to these equations are optimized using the method of constrained variations in order to insure that the motion of mass m_3 is minimal over a given frequency range. The coefficients of the damping, the nonlinearities, and linear stiffnesses are treated as design variables. These design variables are then determined by the optimization scheme. The result is an optimal nonlinear normal mode where the transfer of energy, even at resonance, is minimal.

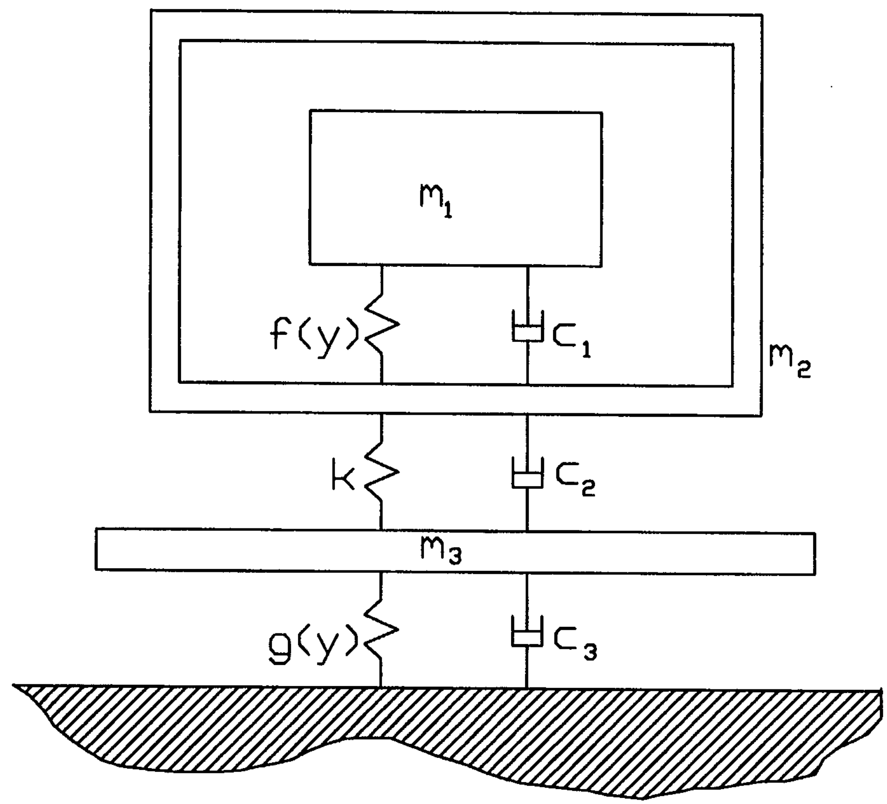

Figure 1: Schematic Drawing of the Model Under Consideration

Asymptotic Analysis of Dynamic Tension in Submerged Cables

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Abstract

Cables are used in a variety of ocean engineering applications and are often selected for their inherent flexibility. As compliant elements, cables possess vastly different stiffness properties in the tangential and lateral directions. For submerged cables, this stiffness anisotropy is augmented by drag anisotropy; heavy lateral drag verses light tangential drag. As a result of these effects, submerged cables behave very differently than cables in air, especially for moderate to high frequency excitation (Papazoglou *et al.*, 1990). Papazoglou *et al.* investigated the effects of direct tangential excitation on a submerged cable and found that for higher excitation frequencies the dynamic tension significantly increased. The mechanism which generated the increased tension was described only as an arresting of lateral motion by the fluid drag which, in turn, lead to cable response dominated by "elastic" stiffness (i.e. stretching). The excitation, however, need not be exclusively applied in the tangential direction to realize amplified dynamic tension. Recent numerical studies by the authors demonstrate an alternative tension resonance mechanism. This mechanism results from the internally resonant interaction of two in-plane cable modes, one of which induces elastic (tangential) cable response (Newberry and Perkins, 1996). This mechanism was identified therein through systematic inspection of the nonlinear fluid/cable model. The same mechanism is evaluated herein using asymptotic methods.

In this study, a minimum-order discretized model is presented which captures this resonant tensioning mechanism. As modeling elastic response is crucial, the usual assumption of quasi-static cable stretching must be relaxed. Comparison of dynamic tension from the minimum-order two degree-of-freedom model and from direct numerical simulation of the governing nonlinear partial differential equations show excellent qualitative and quantitative agreement. The minimum-order model likewise provides increased insight into the drag affecting the tangential response through coupling to the normal response; a key element in producing resonant tensioning.

Asymptotic analysis of the minimum-order model is also presented. This analysis provides understanding of the important subtleties in the perturbation procedure that must be correctly implemented if the resulting solution is to capture the physics of a tensioning resonance. Issues of particular importance include 1) the use of an alternative drag model to facilitate the asymptotic analysis while still accurately modeling the drag force, 2) the appropriate scaling of the drag terms to provide reasonable quantitative agreement between the asymptotic results and numerical simulations, and 3) the appropriate scaling of the stiffness terms to capture the essential coupling effects with minimum analytical effort. Comparisons are made between the numerical and asymptotic solutions.

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A Control Method Based on the Saturation Phenomenon

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We present a theoretical and experimental application of an active control strategy based on the saturation phenomenon to regulate the oscillations of flexible structures that are subjected to multifrequency excitations. The technique is based on introducing a series of second-order electronic circuits and coupling them to the structure's resonant modes through a quadratic feedback control law. When the structure is forced at its resonances and the circuits' natural frequencies are set to one-half the natural frequencies of the excited modes, the nonlinear coupling creates a unidirectional energy-transfer mechanism from the excited modes to the circuits.

We consider the problem of controlling the vibrations of a flexible cantilever beam that is attached to a shaker and actuated by piezoceramic patches attached near its root. The equation of motion of the beam can be written as

$$\rho A \frac{\partial^2 w}{\partial t^2} + C \frac{\partial w}{\partial t} + EI \frac{\partial^4 w}{\partial x^4} = -\rho A \frac{d^2 w_0}{dt^2} + \frac{\partial^2 M}{\partial x^2},$$

where $w(x,t)$ is the beam deflection, $w_0(t)$ is the shaker base motion, A is the crossectional area of the beam, ρ is the mass density, EI is the flexural rigidity, and $C(x)$ is a damping coefficient. The moment M generated by the piezoceramic patches is defined by

$$M(x,t) = \hat{M} V_a(t) \left[H(x - x_1) - H(x - x_2) \right],$$

where $\hat{M}$ is a constant, $V_a(t)$ is the control voltage, and $H(x)$ is the Heaviside step function.

In this analysis, we consider the case of primary resonances and define the shaker acceleration $\ddot{w}_0$ by

$$\ddot{w}_0 = \sum_{j=1}^{N} F_j \cos(\Omega_j t),$$

where the F_j and Ω_j are the amplitudes and frequencies of excitation, respectively, and N is the number of the resonant modes. We assume, without loss of generality, that the first N modes of the beam are excited. Therefore, we let $\Omega_j \approx \omega_j$, where ω_m represents the beam's m^{th} natural frequency. We introduce a series of controllers taking the form

$$\ddot{u}_j + 2\,\zeta_j\,\dot{u}_j + \lambda_j^2\,u_j = \alpha_j\,u_j\,e, \quad j = 1,\ldots,N$$

and a control signal

$$V_a(t) = \sum_{j=1}^{N} \gamma_j\,u_j^2,$$

where the controllers' natural frequencies λ_j are chosen such that $2\lambda_j \approx \omega_j$, ζ_j are the damping coefficients, and α_j and γ_j are constants. The feedback signal $e(x,t)$ is generated by a strain gage attached at the surface of the structure. Assuming simple beam theory, the strain measured by the gage is approximated by

$$e(x,t) = -\frac{1}{2}\frac{\partial^2 w}{\partial x^2}\,t_b,$$

where t_b is the thickness of the beam.

We present theoretical and experimental results of the application of the control strategy. First, the equations of motion are analyzed through perturbation techniques. Second, the controllers are built is electronic circuitry, and the strategy is tested by regulating the response of a cantilever beam that is subjected to single and two simultaneous resonant excitations.

Nonlinear Vibrations of Chains

Friedrich Pfeiffer, Peter Fritz, Jürgen Srnik, München

Typically chains consist of chain elements which might be connected by ideal joints, joints with backlash or by joints like journal bearing. Furthermore and considering Continuous Variable Transmission Systems (CVT) chains may be connected by rocker pins or more complicated pin systems. Chains come into contact with guides and sprocket wheels, or they have contact with controllable sheaves. In the case of vehicles they have sprocket and ground contact. In all cases contacts start by nullifying relative distances between guides, sprockets, sheaves and some chain element and contacts end by nullifying some force condition like normal forces or static friction forces. In between contacts possess sliding friction or stick-slip transitions, there might be local separation and closure again. Altogether all types of contact mechanical phenomena may occur between chains and their controlling elements.

In modeling chains and chain elements we first must choose convenient coordinate frames. We start with an inertial coordinate base chosen in such a way that the geometry to the element becomes as simple as possible, for example located in the centers of sprockets or of sheaves. We furtheron fix at each element (chain element, sheave, sprocket, guides) moving body-fixed coordinate system in a convenient point for instance in the mass-center. Sometimes it might be convenient to have a second body-related frame which is inertially fixed with one axis and rotates with a nominal speed, for example for shafts and wheels. Anyway, the system of coordinate frames must allow a complete and unambiguous geometrical description of all system elements.

Secondly, we must define a nominal motion, which is for chain dynamics nontrivial. We start with a prescribed (time-dependent or not) motion of the driving component, for instance one sprocket, one pulley. But then we have to decide how a nominal motion of the chain system might look like. One could go back to the string-like model as developed earlier which is a reasonable basis. A more advanced nominal motion can be evaluated by taking into account the discrete chain structure and thus the polygonial effects but by not considering the detailed friction and impact phenomena. By experience of the last years this approach seems to give good results and, moreover, a sound basis for more refined investigations.

Typical links and joints for chains are very similar in different applications like roller chains in cars, rocker pin chains in CVT-gears or even chains for tracked vehicles. The chain elements consist of links which are connected to the neighbouring link by a pin, a bushing, a rocker pin or similar. In all cases these connections possess backlash leading to relative motion within this connection. In roller chains or in CVT-chains the chain pin are lubricated by joil. They behave then like a journal bearing and can be modeled as such. In many cases it might be even sufficient to assume ideal joints being conveniently described by kinematical constraint equations.

CVT-chains are controlled by movable and non-movable sheaves, roller chains by sprockets and guides. The contact processes in the first case include impacts at the sheave entry and stick-slip phenomena during sheave contact. At the pulley exit we might have an impulsive effect, when the chain rocker pin leave the pulley. In the second case of roller chains we do not observe stick-slip processes but more sliding friction effects. Instead we have more impulsive processes on the guides and when entering or leaving the sprockets. Separation effects with subsequent contacts might happen along the guides. All effects are covered by the theory as presented in the paper.

In summarizing the contribution we consider nonlinear vibrations of chains by applying multibody theory in connection with contact mechanical properties. The problem of multiple and non-decoupled contacts being typical for chains affords a special treatment of multibody systems including aspects of linear and nonlinear complementarity theories. The availability of such tools is an indispensable prerequisite for modeling complex systems like chains with their numerous impulsive and friction-induced contact processes. The related theory is presented and applied to two cases, the case of rocker pin chains in CVT-gears and the case of roller chains in car applications. Results and comparisons with measurements confirm the way of modeling such systems. Further research has to be invested in numerical and computer time problems.

Nonlinear Dynamics and Stability of a Machine Tool Traveling Joint

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An increase in the manufacturing of machines and improvement of their quality present extremely high demands to the precision and productivity of machine tools. This, in turn, requires reduction of static and dynamic deformations in the traveling joints of the frame units. Static and dynamic behavior of a modern machine tool depends to a large extent on the design of guides, whose contribution to the total compliance of the machine may be crucial.

In this work, free and forced vibration of a lathe support in the horizontal plane will be considered. The external loads are static and dynamic components of the cutting force. The dynamic cutting force is caused by a small eccentricity of the workpiece geometric axis relative to the axis of its rotation.

The non-contact elastic deformations of sliding and stationary elements (support and bed) are assumed negligibly small with respect to the contact deformations in their joint. Thus, the contacting elements are considered rigid bodies with a thin elastic contact layer between their surfaces.

In recent years there has been progressive replacement of metal-metal guides by plastic-metal ones, especially in heavy machines. The reasons are more convenient technology of the guide surface manufacturing, better stick-slip conditions and lower friction coefficients. The static dependence "pressure - contact deformation" for both these types of material is essentially non-linear and can not be linearized for the small displacements.

Since the contacting bodies are absolutely rigid, the location and the orientation of the traveling unit in the horizontal plane may be described by three coordinates. However, for a symmetric location of the feed drive lead screw between the front and back guides, the equations of motion are reduced to two coupled damped highly nonlinear ordinary differential equations describing the lateral translatory and angular displacements of the support. The longitudinal translatory motion, which is decoupled from the other equations, does not influence compliance of the joint in the direction normal to the machining surface and only affects the wearing rate of the contacting materials.

The equilibrium equations of the dynamic system are highly nonlinear, but it can be shown that for any physical set of parameters, there is a unique fixed point. This fixed point can change its stability via two Hopf bifurcations defining an unstable domain in parameter space that is governed by the cutting tool coordinates. An approximate analytical stability criteria will be shown to control the unforced system response.

Self excited vibration evolves around an unstable fixed point. One or three limit cycles may coexist in the statically unstable domain. The subdomain of multiple limit cycles consists of two stable solutions separated by an unstable one. Furthermore, two limit cycles (stable and unstable) may coexist with the stable fixed point, beyond the statically unstable domain.

Due to the highly nonlinear nature of the system, a harmonic balance approach is used to estimate the frequencies and the amplitudes of the various limit cycles. Stability of approximate analytical solutions is obtained by Floquet analysis. The stable limit cycles and their frequencies are verified by numerical simulation and its Fourier transform.

Perturbation of the dynamic system with a weak periodic excitation results with further bifurcations including quasiperiodic and chaotic solutions. The various aperiodic solutions are obtained in different regions of parameter space where the excitation frequency and amplitude are defined by the lathe spindle running speed and the workpiece eccentricity respectively.

The relative motion between the cutting tool and the workpiece governs the precision of machining and surface roughness. The influence of self excited vibrations on wearing and fatigue strength of the system has an unfavorable effect on tool life. This analysis will enable the machine tool designer to develop a controlling set of parameters that will ensure prevention of dangerous large amplitude self excited and aperiodic forced vibrations.

An Energy-Momentum Method for the Nonlinear Dynamics of Rods of the Cosserat Type with Applications to Chaotic Motion

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Extended Abstract

Shell and rod theories which are usually used to describe thin bodies can be divided in general into two classes: 1) Theories derived from three-dimensional considerations using specific assumptions on the displacement field. 2) Theories derived by means of the so-called direct approach where the continuum under consideration is taken to be a one- or two-dimensional Cosserat continuum.

The paper is concerned with a dynamical formulation of rods when the corresponding theory is derived by means of the direct approach. In this case, a rotation tensor enters explicitly the formulation. Since Rotation tensors define a Lie group (the group SO(3)), the configuration space, that is the space of all admissible displacements and rotations, constitute a nonlinear manifold with the group operation being a multiplicative one. Explicitly, let $\mathbf{u}$ defines the displacement field and $\mathbf{R}$ defines the rotation tensor field. The configuration space

$$\mathcal{C} := (\mathbf{u}, \mathbf{R}), \mathbf{u} \in \mathcal{R}^3, \mathbf{R} \in SO(3) \tag{1}$$

is given as the set of all admissible pairs consisting of displacements and rotations. For two elements of the configuration space say $(\mathbf{u}, \mathbf{R})$ and $(\mathbf{w}, \mathbf{Q})$ the group operation is defined as $(\mathbf{u} + \mathbf{w}, \mathbf{QR})$. Now, within a time integration method one generates with the help of a given transversal velocity $\mathbf{v}$ and a given angular velocity $\mathbf{\Omega}$, the latter is an antisymmetric tensor, finite quantities which are superimposed on given displacements and rotations. For such an updating procedures, the following appears as a ntural formula:

$$\mathbf{u} = \mathbf{u_0} + \Delta T \mathbf{v}, \quad \mathbf{R} = \mathbf{R_0} \exp(\Delta T \mathbf{\Omega}) \tag{2}$$

when $\mathbf{u_0}, \mathbf{R_0}$ are given displacements and rotations and ΔT is the time step. The formula stands in accord with the group operation of the configuration space where the displacements are updated additively but the rotations multiplicatively. Moreover, the exponential map is a natural operation within Lie groups.

Although the exponential map seems to be a natural operation to be used in time integration schemes for the above models, the construction of energy-momentum methods as suggested by Simo & Tarnow [1] violates the use of the above map. Accordingly for the dynamics of rods, the mentioned authors used the Cayley transform for the updating procedures which is tailored to fulfill certain energy conserving propoerties.

In this paper we discuss the nonlinear dynamics of spatial rods and give an alternative energy-momentum method which is independent of the configuration space and, accordingly, applies when the exponential map is chosen to achieve the updating procedures. The rod model is derived by considering a one-dimensional Cosserat continuum. The model takes first order shear into account and is geometrically exact in the sense that arbitrary large deformations are admissible within the assumed class of constitutive behaviour. The latter is taken to be a linear one.

A hybrid finite element formulation is developed which proves efficient and free of so called locking phenomena. Different examples of rod dynamics at finite deformations including chaotic motion are presented.

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Remark. The second author will present the paper at the conference.

Monitoring the Behavior of an Experimental Impacting System

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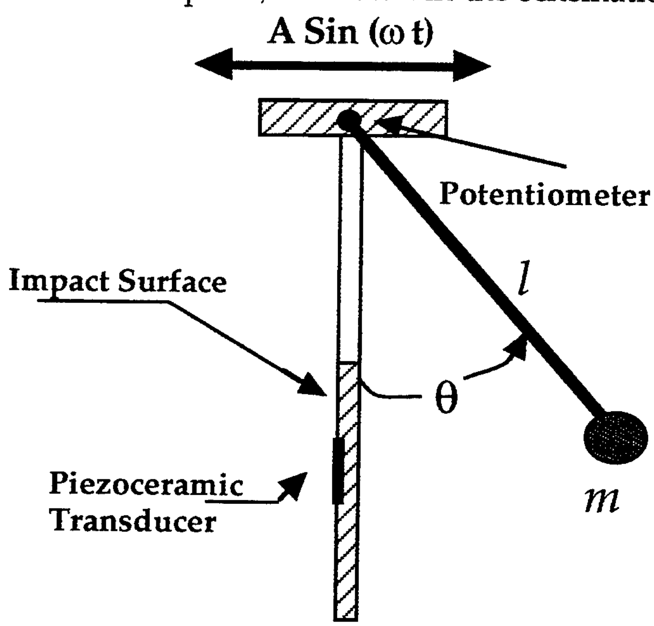
Since monitoring all the state variables in dynamic experiments may be difficult or even impossible, and since it is desirable to reduce the coupling between the system under study and the measuring device to as low a level as possible, it is useful to assess novel data acquisition techniques. Furthermore, it is well established that topological information can be obtained from delay coordinate embedding, and thus, not all of the state variables, or even a continuous set of a single state variable, need to be measured.[3] In the case of impacting systems, the impacts can be viewed as discrete events which can then be used to reconstruct more general features of the behavior. The success of such reconstruction techniques will be assessed in this project.

In previous studies of the dynamic behavior of an impacting pendulum, the experimental data was obtained through use of a rotational potentiometer placed at the pivot, as shown in the schematic below.[4]



This project examines the type of information which can obtained

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[2] Associate Professor and Warren Faculty Scholar, School of Engineering and Center for Nonlinear and Complex Systems

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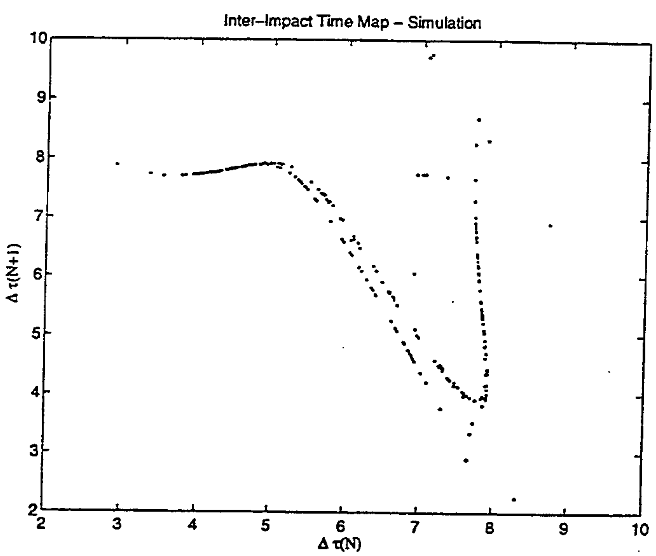
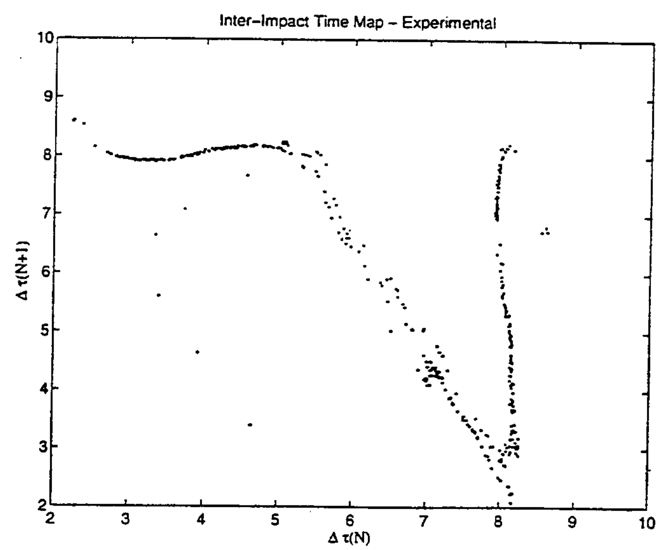
from the impacting pendulum system if only the impact times are recorded. This set of discrete events can then be embedded to produce a one-dimensional map. The ability of this map of inter-impact intervals to characterize the behavior of the system is examined, as well as the sensitivity of the measurement techniques to noise and repeated use.

Experimental data measuring the time of impact was gathered using a piezoceramic transducer placed at the impact point and by using a microphone to digitally record the sound of the impact. These alternative experimental techniques are compared to both results obtained by the standard potentiometer method and results obtained by numerical simulation. The behavior of the system is studied under both periodic and chaotic regimes. The following figures depict some typical initial results, comparing the map obtained by the piezoceramic transducer to numerical simulation.





These curves indicate the presence of chaotic behavior in the system, which in this case is being forced by sinusoidal base displacement at a frequency of 1.15 Hz. There is good qualitative agreement between the experiment and the simulation, as can be seen above. Current work in progress includes further mathematical analysis of the data in order to examine more fully the structure and symmetry of the maps produced under various forcing conditions, as well as understanding any inherent limitations on using only a subset of measurements to gain information.

Monday, June 10

1530-1710

Session 6. Computational Methods I

SYMBOLIC COMPUTATION OF STABILITY AND BIFURCATION SURFACES FOR NONLINEAR SYSTEMS WITH STRONG PARAMETRIC EXCITATION

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Abstract

An analysis technique is suggested for general time-periodic nonlinear dynamical systems of the form

$$\dot{x}(t) = f(x(t),t,s) = f(x(t),t+T,s) \qquad (1)$$

where $t \in \mathbb{R}^+$ denotes time, $\mathbf{x} \in \mathbb{R}^N$ is the state vector, $\mathbf{s} \in \mathbb{R}^L$ is a parameter vector, and $\mathbf{f}: \mathbb{R}^+ \times \mathbb{R}^N \times \mathbb{R}^L \rightarrow \mathbb{R}^N$ is analytic in the components of **x** and of **s**. Equation (1) may be expanded around a periodic solution to yield a quasilinear time-periodic system of the form

$$\dot{x}(t) = A(t,s)x(t) + f_2(x,t,s) + f_3(x,t,s) + \ldots + f_k(x,t,s) + O(|x|^{k+1},t) \qquad (2)$$

where A(t,s) is $n \times n$ and is periodic with principal period T, *i.e.* $A(t+T,s)=A(t,s)$, and where $f_k(\,)$ contain homogeneous T-periodic monomials in x of order k. For weakly excited systems, it has been possible to apply classical averaging (Sanders and Verhulst, 1985) and perturbation (Nayfeh and Mook, 1979) methods to equation (2) and obtain meaningful equivalent time-invariant systems from which one may obtain the bifurcation surfaces in the parameter space. In the event when the parametric excitation is strong, however, it has been shown that such techniques can not yield meaningful results, and other methods must be employed (Pandiyan, *et.al.*, 1994). In the method of analysis known as the *point mappings technique* (Bernussou, 1977; Flashner and Hsu, 1983), the original non-autonomous system (equation (1)) is replaced by a set of autonomous parameter-dependent *difference equations* which one may linearize and use to study the stability and bifurcation of the original system. However, the computational difficulties in the application of this technique are considerable. A practical alternative is suggested here whereby the Floquet Transition Matrix (FTM) of the linear part of equation (2) is obtained in terms of the parameter vector s. Since the FTM is just the system matrix for the Poincaré map, the same techniques of bifurcation and stability analysis that are used in the point mapping technique (including obtaining closed form expressions for the bifurcation surfaces and stability boundaries) may also be applied here, while the computational difficulties involved in obtaining the difference equations are completely avoided.

The stability and bifurcation of equation (2) may be studied first by considering the linear part of the equation, whose fundamental matrix solution $\Phi(t,s)$ satisfies $\Phi(0,s)=\mathbf{I}$. If the system is hyperbolic (the center manifold $W^c = \varnothing$), then the stability of equation (2) depends on the

eigenvalues (characteristic multipliers) of $\Phi(T,\mathbf{s})$, called the Floquet Transition Matrix (FTM). It is shown here that the FTM may be symbolically computed as a function of its parameters by first expanding the normalized periodic system matrix $\bar{A}(\tau,s)=\bar{A}(\tau+1,s)$ in shifted Chebyshev polynomials of the first kind, valid in the interval [0,1], and then computing successive approximations to the fundamental matrix to be evaluated at $\tau=1$. By employing the *integration* and *product operational matrices* for the shifted Chebyshev polynomials (Sinha and Wu, 1991) and truncating the expression after a finite number of terms, an approximate solution to any desired degree of accuracy is obtained. The Chebyshev expansion of the periodic matrix simply provides for the efficient symbolic computation of the fundamental matrix. Because the initial expansion, however, is in a power series (which is a generalization of the power series definition of the exponential of a matrix), the solution vector is not expanded in a Chebyshev series with unknown coefficients as was done in earlier studies (Joseph, *et.al.*, 1993; Sinha, *et.al.*, 1993).

The requirements of stability dictate that the eigenvalues (multipliers) of $\Phi(T,\mathbf{s})$ lie inside the unit circle in the complex plane. Since finding the roots of an equation with parameter-dependent coefficients is generally impossible except for the simplest cases, however, it is desired instead to employ a Routh-Hurwitz-type scheme whereby the stability conditions in terms of parameters are computed without actually finding the roots. Since the Routh-Hurwitz method is formulated to find stability conditions for autonomous differential equations, however, a complex linear fractional transformation (Jury, 1974) is employed which maps the unit circle of the complex plane to the left half plane so that the Routh-Hurwitz technique may be applied. The resulting inequalities thus guarantee the stability of equation (1) in terms of the parameter vector s. The bifurcation surfaces in parameter space may be similarly obtained for *tangent, period-doubling,* or *secondary Hopf* bifurcations as shown by Flashner and Hsu (1983).

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Real Time Animation of Ocean Waves

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ABSTRACT

This paper describes the real time animation technique of ocean waves, so-called short crested waves, which are characterized by directional spectra.

In recent years, the necessity of the studies on extreme environmental loads acting on ships as well as offshore structures has been pointed out. Because the demand of structural safety of floating structures in severe storm conditions has been increasing. So, the computer simulation of ocean waves is important.

In the Airy wave equations the water surface elevation of a fundamental wave is formed as a sine(or cosine) function. Therefore short crested waves of the ocean can be considered as a linear superposition of a large number of sine(or cosine) waves all traveling independent of one another in different directions with different frequencies. The spectral analysis based on linear theory describes how energy of each wave is distributed in direction and frequency.

Let (x, y) and t be the horizontal plane of ocean and time, then the water surface elevation of the ocean $\eta(x, y, t)$ is generally written by

$$\eta(x,y,t)=\sum_{n=1}^{N}\sum_{m=1}^{M}a_{nm}\cos(K_n x\cos\theta_m+K_n y\sin\theta_m+\sigma_n t+\varepsilon_{nm}),$$

where the subscription n and m indicate the values of n-th frequency and m-th direction, respectively, and N, M, a_{nm}, K_n, θ_m, σ_n, ε_{nm} denote the number of component waves in frequency, the number of component waves in direction, the amplitude of nm-th component wave, the wave number of n-th frequency wave, the m-th wave propagation angle, the angular frequency of n-th frequency wave, and the random phase lag defined from 0 to 2π, respectively.

So, the time consuming process is required to simulate real ocean waves even by a quite large computer. A well informed paper states that an EWS takes 72 hours for a 20-second animation every other 0.1 second.

The hardware configuration of the developed system, which is so-called a parallel computing system, consists of a PC and a wave calculation unit which is connected with a PC by a network line. The DOS based PC executes the pre-calculation of the parameters and drawing the wave animation. The wave calculation unit executes the real time calculation which is precisely described in this paper.

To achieve real time animation of ocean waves, the software takes two stages: the pre-calculation of the parameters of the wave and the real time calculation of the wave animation. The former stage is executed in the PC before real time calculation and drawing the wave animation. The latter stage is executed on the DSPs of a wave calculation unit.

The fast calculation algorithm for ocean waves and the hardware with a PC and some DSP(digital Signal Processor)s, both of which have been developed by the author, have made it possible to display the real time animation of many types of real ocean waves.

Real-Time Dimension Estimation Applied to the GearBox Dynamics

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ABSTRACT

Based on the Grassberger-Procaccia algorithm, we implement a real-time measurement for the dimension-like quantity from time-series data. In contrast to some previous attempts, our approach can be used to study high-dimensional cases and the dimension estimate is made continuously as the experiment runs. We like to propose such real-time measurement because, if the complicated signal is of mainly deterministic nature, this type of measurement can be used as a parameter for the dynamics diagnosis. To demonstrate this point, we apply our measurement procedure to the complicated gearbox dynamics. In the experiment, gears of different crack lengths are used in a series of tests conducted at different speeds. The variation of these system parameters is effectively picked up from the dimension measurement. Details of the experimental results are given and a video tape recording the on-line dimension estimation is shown.

[1] Department of Mechanical Engineering

On the Development of a New Finite Element Strategy to Study Nonlinear Dynamical Systems with Time Varying Coefficients

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University of Waterloo, Canada

A new nonlinear finite element formulation has been developed to solve nonlinear time varying coupled partial differential equations. As an example the dynamical interaction between a flexible cantilever beam and a mass constrained to move on the beam is investigated. The motion of the mass and the beam are coupled through nonlinearities arising due to Centripetal, Coriolis and other acceleration terms. This results in two coupled nonlinear partial differential equations of motion where the coupling terms have to be evaluated at the position of the mass. Using finite element discretization for the spatial coordinate the equations of motion are reduced to a system of n stiff ordinary nonlinear differential equations, where the coefficient matrices depend on the position of the mass and hence are time varying. These equations are solved numerically using a stiff solver.

Results show linear behaviour for small motions when the mass is small, and very complex dynamics for large oscillations and for larger moving mass. The presentation focusses on various behaviours of the system and the finite element formulation.

*Graduate Student

†Associate Professor Department of Mechanical Engineering

‡Associate Professor Department of Systems Design Engineering

Forced response analysis of a dry-friction damped oscillator using an efficient hybrid frequency-time method.

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Abstract

In this paper, the steady-state response to periodic excitation of a single-degree of freedom oscillator with an attached flexible dry friction damper is studied. One distinguishing feature of the work is that no further approximation of Coulomb's law is made in the friction model. Namely, the relative slip velocity at the frictional interface is taken to vanish when the damper is sticking. This is unlike many of the published works in the literature, where a piecewise linear relation between the friction force and the slip velocity is assumed. The solution procedure used in the study is a hybrid frequency-time domain algorithm, which is largely based upon the incremental multi-harmonic balance method. The method is significantly improved by the use of several efficient numerical techniques, such as Fast Fourier transforms and Toeplitz Jacobian matrices. In addition, new features are developed to achieve the fast generation of frequency responses, namely, adaptive step size and line search algorithms for automatic sweeps in amplitude and frequency.

The resulting solution procedure is highly performant and gives reliable and accurate results for a wide range of system parameters (friction damper stiffness, force amplitude, viscous damping ratio, etc.). It allows for the fast and automatic generation of steady-state frequency responses. Results obtained reveal interesting features of the nonlinear response. Of particular interest is the presence of two major resonant peaks, one for low levels of forcing (mostly sticking motion) and one for high levels of forcing (mostly slipping motion). Also noteworthy is the existence of sub-resonances where the multi-harmonic behavior of the system is very significant. Finally, it is interesting to see how, under certain conditions, the dry friction can actually lead, over a specific frequency range, to a saturation effect for the response amplitude.

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Tuesday, June 11

0830-1010

Session 7. Multibody Dynamics I

Dynamics of a Flexible Slider–Crank Mechanism Driven by a Non–Ideal Source of Energy

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The dynamics of rotors driven by non–ideal power supplies is well–understood. For linkage mechanisms in which combined rotational–translational motions appear, only for rigid multibody systems some experiences exist [1,2].

In the present contribution, an in–line planar slider–crank mechanism with an viscoelastic connecting rod (shear deformation and rotatory inertia neglected) is studied. The crank is assumed to be perfectly rigid, gravity influences are neglected and no external forces are applied to the piston (for which there is no clearance and offset but a velocity–dependent friction). After the formulation of the governing equations of motion based on finite deformations and a single mode truncation, the dynamic response of the mechanical system under the influence of an electric DC motor is examined in all details.

First, the steady–state dynamics is analyzed. In contrast to a simply supported rotor system, for the slider–crank mechanism no constant crank angular speed solution exist; fluctuations can not be avoided. In practical applications, the magnitude of this stationary speed variations is prescribed and the parameters of the driving engine have to be chosen such that the tolerated deviations are not exceeded. Due to the nonlinearities of the system (both inertial and geometric stiffening) interesting additional phenomena appear which are also addressed.

Then, the transient startup or rundown of such machines are dealt with by using a digital simulation of the complete dynamic model equations. Due to the flexibility, the system is able to vibrate so that the dynamics passing through resonances is an interesting feature where the primary and the principal parametric resonance are of special interest.

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Dynamic Parameter Estimation of Mechanisms

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1 Introduction

When building a model of a mechanical system, the engineer must input appropriate values of the dynamic parameters. Hence the dilemma : how to find the appropriate values of dynamic parameters, for a known or assumed form of the model.

Previous work on inertial parameter estimation has been done in the field of robotics mainly for model based control strategies. An, et. al. [1] developed a technique which involved joint torque sensing at all the joints and used ridge regression to solve the problem of ill conditioning. Gautier[2] used Singular Value Decomposition (SVD) and QR decomposition to numerically find the set of minimum parameters. Sheu and Walker [4] used the SVD to reduce the actual parameter set to the minimum parameter set. Explicit symbolic techniques to form a set of minimum linear combinations of the system parameters have also been developed. None of these techniques are applicable to completely general mechanisms.

The present work approaches the subject from the point of view of mechanical systems simulation engineering. Completely general equations have been developed, applicable to any multi-body system, using the library of standard constraints developed by Haug [3]. The equations have been derived such that, in the final form, they are suitable for convenient translation into computer code.

2 Equations of Motion

Consider a rigid body located in space by the vector $\mathbf{r}$ and a set of generalized coordinates that defines the orientation of the $x' - y' - z'$ body-fixed frame, relative to an inertial x-y-z reference frame. Let the orientation of the body be defined by the vector of the Euler parameter vectors $\mathbf{e} = [e_0, e_1, e_2, e_3]$. A vector $\sigma = [\xi\ ,\ \eta\ ,\ \rho]^T$ is defined which gives the center of mass of the body in the body fixed x'-y'-z' reference frame. Let $\mathbf{Q} = [\mathbf{F}\ ,\ n']^T$ be the vector of generalized non-gravitational applied forced acting on the body. This vector consists of 6 components, 3 forces parallel to the global coordinate axes and 3 torques about the local coordinate axes. The equations of motion may be written in a form which is linear in the inertial parameters :

$$\mathbf{C} \begin{bmatrix} m \\ m\sigma \\ \mathbf{p}_{J'} \end{bmatrix} = \begin{bmatrix} \mathbf{F} \\ \mathbf{n}' \end{bmatrix} \qquad (1)$$

where

$$\mathbf{C} = \begin{bmatrix} (\ddot{\mathbf{r}} - \mathbf{g}) & (\mathbf{A}\tilde{\omega}'^2 + \mathbf{A}\tilde{\dot{\omega}}') & \mathbf{0} \\ \mathbf{0} & (\mathbf{A}^T\tilde{\mathbf{g}}\mathbf{A} - \mathbf{A}^T\tilde{\ddot{\mathbf{r}}}\mathbf{A}) & (\overline{\overline{\dot{\omega}'}} + \tilde{\omega}'\,\overline{\overline{\omega'}}) \end{bmatrix}$$

Here $\mathbf{p}_{J'}$ is a vector consisting of the 6 constituents of the inertia matrix, $\mathbf{A}$ is the transformation matrix, $\mathbf{g}$ is the vector of acceleration due to gravity, and ω' is the angular velocity vector. The following operators have been used

$$\overline{\overline{\mathbf{a}}} = \begin{bmatrix} a_x & 0 & 0 & a_y & a_z & 0 \\ 0 & a_y & 0 & a_x & 0 & a_z \\ 0 & 0 & a_z & 0 & a_x & a_y \end{bmatrix}$$

$$\text{and } \tilde{\mathbf{a}} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

Equation (1) is true for any rigid body. Similar equations can, therefore, be written for each rigid body in the system and these can be summed up over n bodies.

The forces caused by non-inertial dynamic parameters, such as spring stiffnesses, damping coefficients and friction can also be written in a manner that is linear in the non-inertial dynamic parameters.

Hence, using the above and making use of constraint equations to represent the joints, the equations of motion for a multi-body system can be written in the form

$$\mathbf{A}\mathbf{x} = \mathbf{b} \qquad (2)$$

Here $\mathbf{A}$, parameter coefficient matrix, is expressed in terms of the independent displacements, velocities and accelerations. Vector $\mathbf{x}$ is made up of inertial and non-inertial dynamic parameters and $\mathbf{b}$ consists of the parameter independent, externally applied forces.

Though these equations are applicable to any mechanism, sepatate equations, which are easier to use, have also been developed for the special case of planar motion.

3 Linear Algebraic Analysis

Since equation (2) is linear in the parameters, well known linear algebraic techniques may be used investigate the influence of the parameter set on the equations of motion.

Suppose $\mathbf{x}$ is an $h \times 1$ vector, then measurements of displacements, velocities and accelerations can be taken at different instants of time so as to give f equations in h unknowns where $f \geq h$.

The matrix $\mathbf{A}$ can be decomposed into the following form using singular value decomposition.

$$\mathbf{A} = \mathbf{U}\Sigma\mathbf{V}^T$$
$$\text{where } \Sigma = \begin{bmatrix} \mathbf{S} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$$
$$\mathbf{S} = diag(\sigma_1, \sigma_2, ..., \sigma_k)$$

In this analysis, two subspaces of the matrix A in equation (2) are of importance :
(i) The nullsapce of A, $\mathcal{N}(\mathbf{A})$, which is made up of the set of solutions to the equations $\mathbf{Ax} = 0$.
(ii) The *essential parameter space* (EPS) of A, which is the smallest subspace which contains the solution to the equation $\mathbf{Ax} = \mathbf{b}$. This space will not contain any component of the nullspace and therefore will be perpendicular to the nullspace.

The component of the *actual parameter vector* (APV) which lies in the EPS, is the *essential parameter vector* (EPV) and is the only component of the APV to play a role in the equations of motion. The EPV may be found by using the concept of the pseudoinverse :

$$\mathbf{x}_E = \mathbf{V}_1 \mathbf{S}^+ \mathbf{U}_1^T \mathbf{b} \tag{3}$$

It can be seen that the number of nonzero elements in the EPV could be greater than k, the dimension of the EPS. It is possible, however, to obtain a *minimum parameter vector* (MPV) which has, at the most, k nonzero independent components.

A *minimum parameter vector* (MPV), $\mathbf{x}_M$, may be obtained by choosing an appropriate vector which lies in the nullspace and which, when added to the EPV, gives a vector which has at most k nonzero values as shown below.

$$\mathbf{x}_M = (\mathbf{I}_E - \mathbf{V}_2(\mathbf{F}_2^T\mathbf{V}_2)^{-1}\mathbf{F}_2^T)\mathbf{x}_E \tag{4}$$

where $\mathbf{I}_E$ is an identity matrix whose dimension is equal to the number of non-zero values of $\mathbf{x}_E$.

Using equations (3) and (4), the linear relationship between the MPV and the APV may also be obtained.

$$\mathbf{x}_M = \mathbf{H}\mathbf{x} \tag{5}$$

where $\quad \mathbf{H} = (\mathbf{I}_E - \mathbf{V}_2(\mathbf{F}_2^T\mathbf{V}_2)^{-1}\mathbf{F}_2^T)\mathbf{V}_1\mathbf{V}_1^T$

4 Results

For a spatial four bar mechanism having 32 actual dynamic parameters, 7 minimum parameters could be found. These estimated parameters gave good simulation results (which compared well with an ADAMS model) and were also close to the values of the actual physical parameters in linear combination as shown in Table (1)

Table 1 Estimated Minimum Parameters

no.	Actual	Estimated	Percentage Error
1	14.9416	15.0193	0.5202
2	15.1419	14.6794	3.0545
3	8.0271	7.8387	2.3468
4	4.3932	4.5489	3.5445
5	0.949	0.9428	0.644
6	-8.7163	-8.5645	1.7413
7	0.4972	0.5043	1.417

5 Conclusions

A simple and general technique using SVD has been developed to estimate the minimum parameters, and give the linear combinations of the actual parameters which make up the minimum parameters.

The main advantage of this technique over existing techniques is that it is completely general and can be applicable to any multi-body system. What had hitherto been used in limited manner for robotics applications only, can now be extended to encompass mechanisms with all types of joints.

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Dynamics of CVT Gears

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Chains play an important role in drive train systems of cars and for continuous variable transmission systems (CVT-gears). In such CVT-systems a double-conical driving wheel powers the chain which transmits its energy to a double-conical driven wheel. By changing hydraulically the cone distances of the wheels a continuous variable transmission can be achieved.

Power is transmitted by friction. A chain element entering the wheel cone generates impacts with friction and in the following transits to stiction. But usually stiction cannot be maintained. Due to the impacts of entering chain elements and due to the fact that all contacts are coupled through the bolt-connected elements other chain elements in the wheel may start to slip again. Therefore, each chain element has to be modeled with sufficient degrees of freedom (1 to 3) and each contact has to be treated in all details. This leads to a large multibody system with an extremely high number of contacts and thus to a necessary application of the complementarity theory. The results for a real CVT-gear-system confirm the ideas of the story.

COMPARISON OF VARIOUS TECHNIQUES FOR MODELLING FLEXIBLE BEAMS IN MULTIBODY DYNAMICS

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The modelling of flexible elements in mechanical systems has been widely investigated through several methods issuing from both the area of structural mechanics and the field of multibody dynamics. As regards the latter discipline, beside the problem of the generation of the multibody equations of motion, the choice of a spatial discretization method for modelling flexible elements has always been considered as a critical phase of the modelling. Although this subject is abundantly tackled in the open-literature, the latter probably lacks for an objective comparison between the most commonly used approaches.
The goal of the present contribution is to highlight the relative advantages and drawbacks of a few methods to discretize flexible beams, in a pure multibody context.

For that purpose, it was first necessary to generalize our multibody formalism, initially developed for rigid bodies, for dealing with flexible elements. In this context, the capabilities of our multibody program ROBOTRAN [1] have thus been enlarged to compute in a symbolic way the equations of motion of multibody systems containing flexible beams. In particular, one notices that the symbolic approach is really suitable to ensure a high versatility for introducing the desired deformation (or "shape") functions. Without entering into details of the formalism, let us note that a recursive formulation in relative coordinates has been selected for setting up the multibody equations. As regards the beams, their flexibility is modelled by a 3 D approach including shear deformation and cross-section rotary inertia, and taking care of the so-called geometric stiffening phenomenon : indeed, being part of a multibody system, a beam is subjected to arbitrary motions whose dynamic effects must be taken into account to achieve a reliable modelling.
From a single symbolic data file, ROBOTRAN generates the equations of motion (joint and deformation equations) of the envisaged multibody system in a fully symbolic form[1], to be directly introduced as a "black-box" into a numerical simulation program.
As regards the choice of the shape functions, the following alternative is proposed :
1. the classical approach in which the shape functions and the corresponding integrals must be precomputed – using a modal analysis for instance – and stored by the user into unequivocal symbolic variables, the latter appearing in the equations of motion generated by ROBOTRAN,
2. the "Power-series" approach, where series of monomial functions are selected to discretize the deformations: this family of shape functions[2] allows ROBOTRAN to compute symbolically the various required integrals so that *a fully symbolic generation* is obtained in one execution without resort to a preliminary modal analysis.

[1] in FORTRAN or MATLAB syntax
[2] which was shown to be problem-independent (see [2])

With the help of ROBOTRAN, several experiments and numerical simulations were performed on various multibody systems with flexible beams. Both tree-like and closed-loop systems have been envisaged. For each of them, the spatial discretization of the beam's deformations was systematically achieved by using the following techniques :

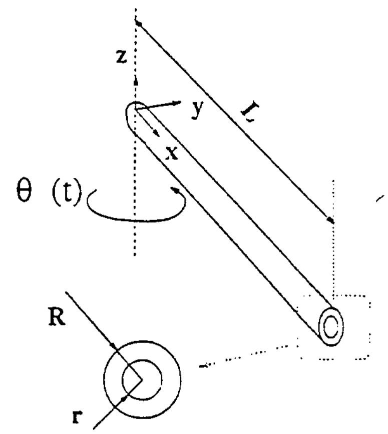
. An assumed modes method using the "power series" technique mentioned above,

. An assumed modes method using shape functions issuing from a preliminary modal analysis,

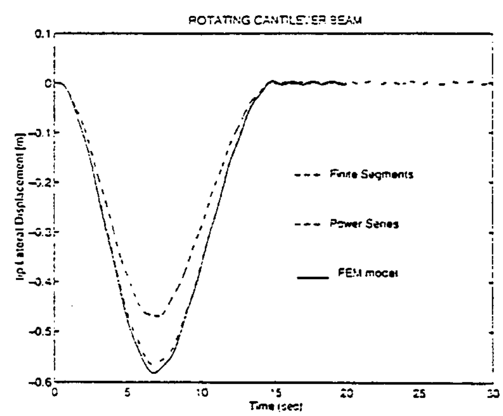
. A finite segment approach (see [3] for instance) in which the beam is replaced by a sequence of rigid segments interconnected by equivalent springs (ROBOTRAN is then used as a rigid multibody code),

. A fully finite element model, by using an appropriate FEM code.

To briefly illustrate the proposed comparison, let us give a typical result consisting of the lateral tip deflection of the rotating cantilever beam proposed in [4], using three of the envisaged methods.



A rotating cantilever beam



Tip lateral deflection

The full contribution will first review the main parts of the multibody formalism underlying this work and then compare the envisaged discretization techniques mentioned above in terms of accuracy, versatility, CPU time performances and user-friendliness.

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Modelling of Spatial Mechanical Systems using Graph Theory and Branch Coordinates

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ABSTRACT

A main goal of multibody dynamics research is to develop formulations that automatically derive the equations of motion for complex mechanical systems, given only a description of the system as input. By combining the computer implementation of such a formulation with numerical methods that generate approximate solutions to the governing differential-algebraic equations (DAEs) of motion, a powerful design tool is obtained. Essentially, these multibody programs relieve an engineer of the error-prone and tedious task of deriving these equations by hand. Furthermore, by automating the analysis procedure, more attention can be focussed on the *design* of the mechanical system. The widespread interest in multibody dynamics is evidenced by the existence of a large number of commercial software packages [1] and the recent appearance of several monographs [2]–[7] devoted to multibody dynamics formulations.

To be applicable to a wide range of mechanical systems containing both open and closed kinematic chains, a multibody formulation must incorporate general mathematical methods for representing the system topology (which is not known *a priori*) as well as the time-varying configuration. The representation of topology is efficiently and naturally handled using elements of linear graph theory [8], the branch of mathematics devoted to the study of topology. However, selecting a set of coordinates to represent the changing system configuration is a problem that is not so easily resolved. The two most popular sets in current use are absolute (Cartesian) coordinates [4] and joint (natural, relative) coordinates [6]; neither set of coordinates are independent for a multibody system with closed kinematic chains.

An alternative set of coordinates have recently been introduced [9], and called "branch coordinates" because they directly correspond to the branches making up the spanning trees of linear graphs representing the system topology. These branch coordinates were used to automatically generate the equations of motion for two-dimensional mechanical systems, using graph-theoretic methods and a symbolic computer implementation [10].

In this current paper, it is shown how this previous work can be extended to the kinematic and dynamic analysis of spatial multibody systems. To automatically generate the equations of motion in terms of branch coordinates, formal graph-theoretic methods are combined with orthogonal projection techniques [11] to obtain a dynamic formulation that is both unifying and simplifying.

It is unifying in the sense that it represents a generalization of conventional formulations, since both sets of absolute and joint coordinates are simply special cases of branch coordinates. It also leads to a smaller number of equations than would generally be obtained using either absolute or joint coordinates. Thus, the number of computations required to effect a numerical solution of the DAEs is reduced.

In the final paper, a brief review of graph-theoretic methods for spatial mechanical systems will be presented, followed by criteria for selecting trees that lead to the greatest reduction in the number of motion equations. This will be followed by a description of the graph-theoretic procedure for automatically generating the kinematic or dynamic equations for spatial multibody systems, including a discussion of the parameters used to represent three-dimensional rotations. Finally, two examples will be presented to illustrate the application of this new formulation procedure and its ability to produce directly, without additional user input or effort, a smaller set of equations than that obtained by traditional multibody formulations.

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Tuesday, June 11

1030-1210

Session 8. Friction and Dampers

FRICTION IN NONLINEAR DYNAMICS: AN HISTORICAL REVIEW

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Friction is abundant in machines, transportation systems, and other processes. It had been a topic of technological attention long before the dawn of science, and remains a hot topic in scientific and engineering research today. As a nonlinearity, it poses challenges to the dynamical systems researcher. Depending on the application, friction may be very difficult to model, since the underlying mechanism is not entirely understood. Additionally, dry-friction models are usually discontinuous, meaning that much of the dynamical-systems theory for smooth systems is not applicable to a frictional system. Thus, friction as a nonlinearity is the current focus of a great number of research activities throughout the world. In this presentation, we look at the history of developments associated with the nonlinearity called friction.

We start in prehistoric times. The manifestations of friction took the attention of the ancients as they developed critical technologies. Although the earliest exploitations of the awareness of friction were not accompanied by scientific explanation, it took incredible ingeniuty and creativity for the first people of a community to recognize properties of friction, and then impliment them in the design of a new machine. The resulting technological developments had a great impact on society. We touch upon some of the significant ancient developments, such as those related to firemaking, bowed musical instruments, sledges and skis, the wheel, early lubrication and bearings, the development of roads, the belt drive, the spouting fish basin, crayons and pencils, and frictional charging.

As time progressed, and people yearned to understand machines and nature, awareness of friction eventually made its way into scientific thought. Friction laws were formulated, and the ideas of dissipation and motion were conceived. Equivalents of Newton's laws of motion were stated by Aristotle and the *Mo Ching* with references to the effects of dissipation. Leonardo da Vinci formulated friction laws which are applied to this day. The development of analytical mechanics with contacts is still underway.

With friction laws included in system equations of motion, friction becomes an issue in nonlinear dynamics. We trace events including Galileo's comments on the fingered goblet, Chladni's figures, Helmholtz' description of stick-slip, the existence and uniqueness studies of Painlevé, the documentation of musical sands, and earthquake modeling. Friction now plays a significant role in modern dynamical systems, as can be seen in today's explosion of activity on friction-induced vibration, friction damping, and chaos.

Experimental and Numerical Study of Low Frequency Stick-Slip Motion

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Abstract

Although almost always undesirable, friction induced vibrations are very often encountered in practice, with stick-slip motion in particular being the subject of numerous studies [1, 2]. Countless physical observations indicate that many physical systems require more sophisticated and physically reasonable models after the onset of frictional sliding, particularly when one is interested in predicting potential slip instabilities. Often studied are effects related to frictional traction dependence on sliding velocity, changes in true area of contact and contact pressure, and their effect on the overall dynamics of the system in which the frictional interfaces are embedded.

In this work, a combined experimental and numerical study is performed to study the stick-slip motion in a simple spring-mass frictionally damped harmonically forced oscillator in which the only important nonlinearity in the system is due to friction. The slider (steel ball bearing) is cantilevered using an elastic beam with adjustable stiffness while the sliding surface (spring steel) is harmonically forced using an electromagnetic shaker, the friction force inducing the motion of the slider. The normal force, the amplitude and frequency of the forcing can be adjusted as well augmenting the number of control parameters to four.

Three types of tests were conducted. First, as a function of the total slip distance, several sliding regimes could be identified upon the frequency content of the induced vibrations in the elastic beam. It was found that after mild and severe wear occurs both lower (3-20 Hz) and higher frequencies (1-4 KHz) get excited in the vicinity of the first and second natural modes of the components of the system while coupling between tangential and normal directions occurs only at high frequencies. At early sliding stages smooth sliding occurs, the motion induced in the normal direction being negligible and the frequencies excited below 20 Hz. Metalographic photos of the worn surface consistently suggested that high frequencies are excited when, due to the wear process, the average distance between

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† Person to be contacted

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asperities is decreasing below a certain threshold.

Second, the effect of the four control parameters on the nature of stick transients was studied in the smooth sliding regime. It was found that for the same normal force, entering and exiting a stick phase does not occur at the same value of the friction force and such transients are a function of sliding history. Both clockwise and counterclockwise loops described by the apparent coefficient of friction were observed in the same test, which to our knowledge is an unprecedented observation.

Finally, all four control parameters were systematically varied to study the evolution of periodicity in the system. The appearance of a typical phase portrait in relative velocity - slider displacement coordinates includes a (stick) portion and a number of loops at higher relative velocities. The number of harmonics in the system strongly depends on all control parameters and is increasing for larger stiffnesses and forcing amplitudes and for smaller forcing frequencies and normal pressures. The forcing amplitude was very slowly and continuously varied to obtain bifurcation diagrams. Chaotic motion was seen to occur at the transition between permanent stick motion at small forcing amplitudes and multiperiod response and was always confined to a very narrow window. The motion consists of irregularly alternating stick and slip periods, the transient from one phase to the other occuring in an unpredictable manner. Qualitatively, the appearance of the attractor is very similar for many choices of the control parameters.

In the last decades, efforts in synthesizing viable frictional models have been numerous. Some authors lay more emphasis on surface energy as the main contributor to friction, and propose models based on the concept of renewal of population of contacting asperities over a critical sliding distance. Several experimental studies point out that the friction force is very sensitive to the normal separation, and provide a theoretical explanation based on upward impulses on asperities. Ruina [3] developed a state variable description friction model which describes the condition of the interface via an evolution law while Oden & Martins [4] propose a model in which high frequency normal vibrations induce low frequency oscillations in the tangential plane.

In this work, a pointwise phenomenological approach is adopted using a frictional model tangent to the interface proposed by the same authors [5] in a previous work. The friction law is described at a point while the interface is considered to be only mechanically coupled to its surroundings through slip displacement, normal and tangential stresses. The model features a state variable approach for description of long term effects and a viscoplastic regularization for instantaneous effects. General experimental observations about stick slip motion are verified: decrease of stick-slip amplitude with increase of driving velocity, increase of the spring stiffness and decrease of mass. Single degree of freedom simulations were run using both a simple Coulomb friction model and the state variable model described above. Tests included both modeling of the full dynamics of the system as well as simpler tests in which the measured relative velocity was used as input data to drive a one dimensional mass. Comparisons with experimental data are given for both the friction force evolution and for the bifurcation diagrams.

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Adaptive Backlash Compensation *

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Backlash is common in control systems and often severely limits system performance. Backlash characteristic is usually unknown and may vary with time. A desirable control scheme should be able to adaptively cancel the effects of backlash. This talk presents an adaptive inverse approach for achieving adaptive backlash compensation.

Backlash. Backlash has haunted the constructors of control systems for more than 50 years: from the servomechanisms in the 1940's to the modern high precision robotic manipulators. Typical backlash examples are gear trains and similar mechanical couplings.

A parametrized backlash modeled will be presented.

Backlash Effects. A further insight into the nature of backlash will be gained from the waveform of the backlash output when its input is a sawtooth signal. Two fundamental features of backlash will be shown: first, it introduces a phase delay, and second, it causes a loss of information by "chopping" the peaks of the input.

Influence of backlash on control systems. An example of systems with backlash will be shown in which the backlash is in the valve control mechanism and $G(s) = \frac{k}{s}$ is the transfer function relating the liquid level h with the difference between the controlled inflow u and the uncontrolled outflow d.

The presence of backlash adversely affects the static accuracy of feedback control systems. With a linear controller the static accuracy is limited by the width of the backlash. Attempts to improve it by increasing the gain of the feedback loop lead to sustained oscillations (limit cycles) which may cause rapid wear of gear-trains, valves and other components. Similar difficulty is encountered with integral action (PI control). This will be shown by simulation results.

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Backlash inverse. To cancel the effects of backlash, we use an adaptive backlash inverse whose characteristic will be described in detail. This inverse, when implemented with true parameters, can cancel the backlash effects, when implemented with parameter estimates, results in a control error expressible in terms of the parameter error and a bounded "disturbance", which is suitable for developing an adaptive law for updating the parameter estimates.

Adaptive backlash inverse control. The adaptive backlash inverse will be combined with a linear controller for the first-order example discussed above. An adaptive scheme will be developed for updating the backlash inverse parameter. Simulation results will be presented to show that an adaptive backlash inverse is able to adaptively cancel the effects of the unknown backlash so that the system tracking performance is significantly improved: the tracking error between the plant output and the desired output reduces to very small values, and the limit cycle, which would appear without an adaptive backlash inverse and with a linear controller alone, is eliminated.

The adaptive inverse approach. For the general case when an unknown backlash $B(\cdot)$ is at the input of an nth-order linear plant $G(s)$, we have developed an adaptive inverse approach which employs an adaptive backlash inverse to cancel the backlash and a fixed or adaptive linear controller structure for a known or unknown linear part. Main issues in developing this approach are: models of backlash and its inverse, control error equation, linear controller structures, new parametrizations, error equations, adaptive laws, performance evaluation. We will give an overview of this approach and introduce a new area of adaptive control: adaptive control of systems with unknown nonsmooth nonlinearities of which backlash is a representative.

Conclusions. The developed adaptive inverse approach is promising in handling unknown nonsmooth nonlinearities such as backlash in control systems. In this research direction, there are still many important open problems of urgent relevance to theory as well as applications.

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NUTATION DAMPING OF FLOW INDUCED OSCILLATIONS: ANALYSIS AND EXPERIMENTS

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Wind effects on tall buildings, bridges, smokestacks, transmission line conductors and similar aerodynamically bluff bodies have been of interest to scientists and engineers for a long time. The flow induced vibrations of concern are the vortex resonance and galloping type of instabilities. Of particular interest is the effectiveness of energy dissipation to damp the motion. Several methods have been developed over years and used in practice, however, the nutation damper with energy dissipation through sloshing liquid is particularly attractive due to its efficiency, simplicity in design, and ease of construction as well as maintenance (Refs. 1-4). Unfortunately, numerical approach to the nonlinear fluid dynamics of a rectangular nutation damper and fluid-structure interaction dynamics in presence of such energy dissipation mechanism have received little attention (Refs. 5-10). With this as background the paper investigates three distinct aspects associated with a comprehensive study aimed at damping of wind induced instabilities as indicated below. It involves original contributions in every phase.

(i) Numerical Simulation of Sloshing Liquid with Dissipation

To begin with, a nonlinear model of the sloshing liquid in a rectangular container, accounting for nonlinear kinematic and dynamical conditions so as to be applicable to large scale motions at resonance, is considered. It accounts for wave dispersion as well as energy dissipation through boundary-layers at the walls, floating particle interactions at the free surface, and wave breaking. The sloshing equations were solved numerically using a finite difference scheme. Effects of system parameters on the free surface modes were assessed and the wave breaking conditions established to achieve peak energy dissipation (Figure 1). The animation program was also written and a video was taken which will accompany the presentation.

(ii) Structural Response in the Presence of Nutation Damper

The next logical step was to assess effectiveness of the nutation damper in suppressing wind induced instabilities. To that end a model of the system comprised of a flexible vertical Euler-Bernoulli cantilever beam, representing a tall building, with a nutation damper at its free end was considered. The conjugate character of the model accounts for the effect of structural dynamics on sloshing as well as the influence of sloshing on the structural response. A finite difference parametric study showed the nutation damper to be quite successful in arresting both the vortex resonance (Figure 2) and self-excited form of the galloping instabilty.

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(iii) Validation of the Numerical Studies

Three different procedures, involving extensive experiments in conjunction with specially constructed facilities, were used to complement the numerical studies. The governing parameters were varied systematically resulting in a vast body of results which substantiated, rather dramatically, the numerical results:

(a) Sloshing Modes and Dissipation

A Scotch-Yoke mechanism driving a horizontal platform, supporting a nutation damper on a flexible instrumented beam, was constructed to measure reduced damping and added mass parameters. The experimentally obtained results are also presented in Figure 1 to establish accuracy of the numerical algorithm.

(b) Fluid – Structure Interaction Dynamics

Extensive tests were carried out in a closed circuit wind tunnel to assess effectiveness of the nutation damper in suppressing wind induced instabilities. Typical results are presented in Figure 2 which clearly substantiates the numerical model in such a challenging situation.

(c) Visualization of Sloshing Modes and Wave Breaking

The shaking table in (a) was also used to visualize the free surface waves as affected by the system parameters (frequency and amplitude of excitation, liquid height, damper aspect ratio, floating particle characteristics, etc.). Typical flow visualization results are compared with the numerical animated data in Figure 3. Indeed the correlation is exceptionally good. The presentation will also include a flow visualization and the fluid-structure interaction dynamics video.

Such a comprehensive study, representing innovation at every stage, has never been reported before, and represents an important step forward in understanding, at the fundamental level, this class of fluid-structure interaction problems.

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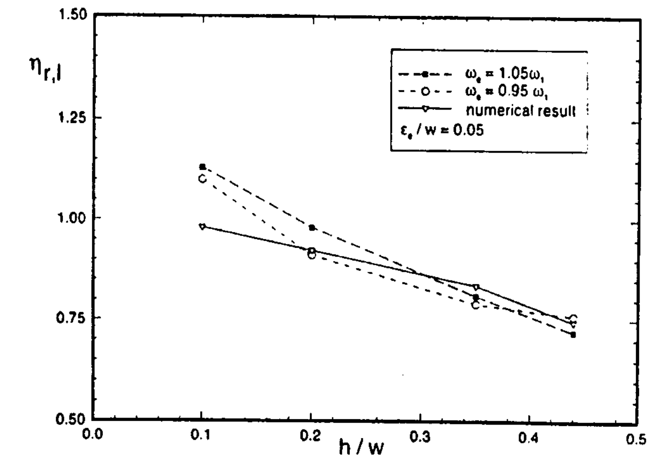

Fig. 1 Variation of the reduced damping parameter $\eta_{r,l}$ with the liquid height ratio h/w as predicted by the nonlinear, free surface sloshing wave model. Experimental data substantiate the results quite well: ω_e, excitation frequency; ω_l, liquid resonance frequency; ϵ_e, excitation amplitude; w, width of the damper.

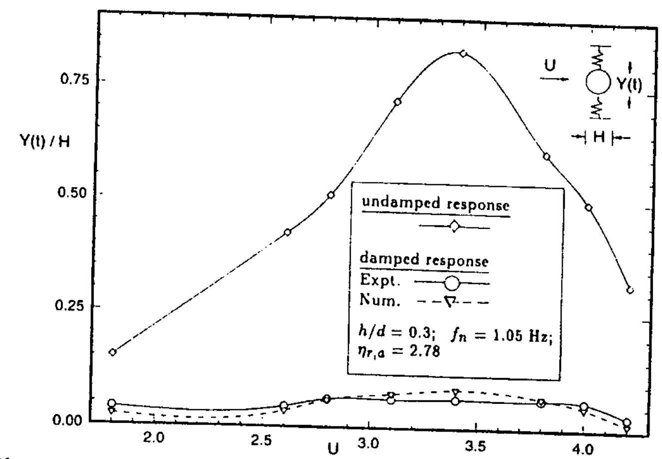

Fig. 2 A comparative study of numerical and experimental results showing effectiveness of a nutation damper in arresting the vortex induced instability: U, reduced wind speed; Y, displacement; H, diameter; h/d, liquid height parameter; f_n, structural natural frequency; $\eta_{r,a}$, aerodynamic damping parameter.

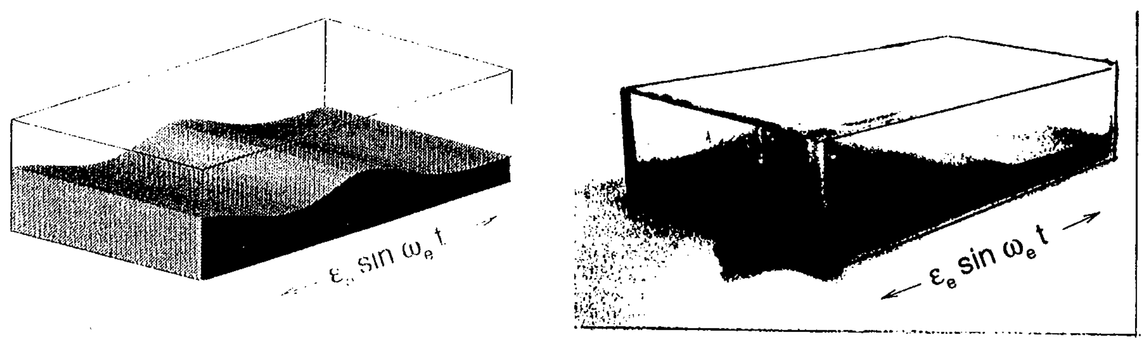

Fig. 3 A typical surface wave obtained numerically during sloshing condition of a rectangular damper. The numerical results were animated. The corresponding flow visualization picture tends to substantiate the numerical algorithm.

Nonlinear Vibration of a Rotating System with an Electromagnetic Damper and a Cubic Restoring Force.

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We investigate the nonlinear dynamics and stability of a rotating system with an electromagnetic non-contact damper. The damper dissipates energy through induced eddy currents generated in a small disk mounted to and rotating with the shaft.

There have been a number of papers published recently on the design and operation of electromagnetic eddy-current dampers for controlling lateral vibration of rotating machinery. These papers have primarily considered low-speed operations but there have been a few reports concerning high-speed (supercritical) operations. Some of the latter have indicated to instability problems but none have provided a valid analysis to account for the instability.

The results of the analytical development of the forces acting on the rotor in the electromagnetic eddy-current damper are summarized in this work. The eddy-current damper is represented by a thin nonmagnetic disk translating and rotating in an air gap of a direct current excited cylindrical electromagnet. The following assumptions govern the selected damper model: i) the air gap is narrow with respect to the internal radius of the electromagnet; ii) the electromagnetic field is characterized by a low value of the magnetic Reynolds number. The stability analysis of the rotating system with an eddy-current damper is carried out for a simple model consisting of a symmetric shaft and a lumped central mass. External nonmagnetic linear damping is provided at the rotor mass and the system stiffness is complemented by a cubic restoring force representing nonlinear behavior of shaft and boundary conditions.

Analysis of the rotating system equilibrium state reveals that the system becomes unstable via a Hopf bifurcation when reaching a specific supercritical angular velocity. The cause of instability is the rotation of the diamagnetic disk in the magnetic field while pure translation of the disk provides only positive damping enhancing system stability. The threshold of instability depends on the parameters of the rotating system and the eddy-current damper. Above the threshold of instability a self excited limit cycle representing nonsynchronous whirl of the rotordynamic system is generated.

A closed form solution for the radius of the limit cycle and the frequency of the self excited oscillation are obtained by the method of harmonic balance and are verified by numerical simulations. Stability of the self excited limit cycle is demonstrated by use of Floquet theory.

Forced vibration induced by the rotating system unbalance is investigated. System response includes periodic and quasiperiodic solutions. For low speed operation only a periodic solution with the system rotational frequency corresponding to synchronous whirl is obtained. For high speed operation quasiperiodic solutions govern system response. Further bifurcation of both periodic and quasiperiodic solutions will be discussed.

This analysis enables an explanation of the nonlinear dynamics and stability phenomena documented for rotating systems controlled by electromagnetic eddy-current dampers.

Tuesday, June 11

1330-1510

Session 9. Nonstationary and Random Vibrations

Railway Wheel Squeal (Squeal of Disk Subjected to Random Excitation)

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Abstract

In our previous paper, frictional experiments using a thin stationary steel disk and a rod and analysis were performed as a fundamental study of the squeal of a railway wheel running on a corrugated rail. A disk was subjected to periodic excitation in its axial direction. The results led to the conclusion that a squeal with more than three modes does not occur.

However, when a railway wheel and a rail are worn out, their surface roughnesses form mostly random waves. In this paper, experiments and numerical simulations are thus conducted on squeals produced when white noise excitation is applied to the stationary disk under the assumption that external forces due to surface roughnesses of the rail are random.

Moreover, whether squeals having multiple modes occur under white noise excitation is investigated by applying the method of stochastic averaging which has proved to be a very useful tool for deriving approximate solution to problems of band-broad random vibration.

An analysis was performed as follows: A rod is pressed at the point of the circumference of a disk subjected to white noise excitation and is rubbed against the disk in its axial direction under a contact load. In our analyses, the vibration of the rod, rotatory inertia and shear deformation of the disk are neglected. The equation of motion for displacement of the disk from the equilibrium can be obtained.

Applying Galerkin's method and the averaging method to the equation yields the equations for the amplitude envelope process and the phase process. From the Fokker-Planck equation, the response moment equations of any order can be obtained.

The steady state solutions of response-moment equation for different spectral amplitudes of the white noise were calculated in two and three-degree of freedom. These results suggest that a squeal with multiple modes occurs in the multi-degree of freedom system.

The Experiments were carried out: One side of a squared rod was rubbed against the circumference of a stationary disk which was clamped at the center with a free periphery, and squeal noises were generated when the rod was moved in the axial direction of the disk under a contact load. White noise excitation was applied at the distance of 2 cm from the contact point of the disk and the rod by an electrodynamic minishaker. How frequency components of these squeals change depending on the excitation level was investigated. A squeal with double modes occurred more frequently as the white noise excitation level increased. This result is the same as the numerical simulation and the practical railway wheel squeal.

Conclusion

The variation of the frictional force due to the surface roughnesses between the railway wheel and the rail is assumed to be a white noise excitation, and when it is applied to the disk rubbed by the rod, the response of squeal of the disk is determined. These results reveal that a squeal with only a single mode occurs at the small excitation level and squeals having multi-modes occur more frequently as the excitation level increases. These numerical simulations were verified by the experiments. Furthermore, the numerical results exhibit a good agreement with practical railway wheel squeals which include frequency components of multiple modes.

Nonlinear Random Response of Ocean Structures Using Second-Order Stochastic Averaging

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Abstract. First-order stochastic averaging has proven very useful in predicting the response statistics and stability of nonlinear ocean structures subjected to nonlinear hydrodynamic forces and parametric excitation. However, the influence of system stiffness or inertia nonlinearities is lost during the averaging process. These nonlinearities can be recaptured only if one extends the stochastic averaging to second-order analysis. In this paper, a second-order stochastic averaging is used to capture the influence of stiffness and inertia nonlinearities that were lost in the first-order averaging process. The results are compared with those predicted by Gaussian and non-Gaussian closures and by Monte Carlo simulation. In the absence of parametric excitation, the non-Gaussian closure solutions are in good agreement with Monte Carlo simulation. On the other hand, in the absence of hydrodynamic forces, second-order averaging gives more reliable results in the neighborhood of stochastic bifurcation. However, under pure parametric random excitation, the second-order stochastic averaging and Monte Carlo simulation predict the on-off intermittency phenomenon near bifurcation point, in addition to stochastic bifurcation in probability.

Nonstationary period doubling bifurcations

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We consider the nonstationary response of two types of nonlinear systems as a bifurcation parameter is varied about a period doubling bifurcation point. The systems are (i) a continuous dynamic system - Rössler's folded band attractor, and (ii) the iterated logistic map. A very similar map may be derived from the Rössler attractor by a suitable choice of Poincaré section, so the two systems considered are indeed somewhat related.

We revisit the Rössler attractor in the form

$$\dot{x} = -y - z$$
$$\dot{y} = x + \epsilon y$$
$$\dot{z} = \epsilon + z(x - \mu)$$

μ is a control parameter, and $0 < \epsilon < 1$ is a small fixed parameter. Trajectories in (x,y,z) phase space show typical period doubling and chaotic response as the control parameter μ is increased. Typical trajectories are shown, for example, in the text by Thompson and Stewart [1]. We consider both linear and sinusoidal variation of μ. The rate of variation is kept small relative to the fundamental period of a typical orbit. We are interested primarily in stability of the first period-1 and period-2 orbits.

Approximate representations of the period-1 orbits may be obtained by averaging, or by a multiple time scale expansion. The small parameter ϵ is used as the perturbation series parameter. For small ϵ, x(t) and y(t) are approximately sinusoidal, but the essential non-sinusoidal nature of z(t), typically $z(t) \sim \exp(\sin t)$, must be maintained for accuracy. z(t) acts as a "switch" to fold (x,y) trajectories back towards the origin. This folding in phase space is what leads to period doubling and chaos. The averaged solution yields results that agree well with numerical simulation. In addition, Floquet theory using the averaged solution as input gives a good indication of the stability. An alternative approach, using averaging to find the onset of a period-doubled solution gives less accurate results at present.

Analysis of an associated iterative map appears to be more tractable. A Poincaré section of the Rössler attractor taken at $y=0$, $x<0$ can be treated as a Lorenz map $x_n \rightarrow x_{n+1}$ that characterises the oscillator (the variation of $z(t)$ here is small). The resulting one dimensional map is smooth and unimodal, and with suitable scaling is remarkably well approximated by the logistic map

$$x_{n+1} = \mu_n \, x_x \, (1-x_n) \quad .$$

In the stationary case, $\mu_n = \hat{\mu} = 3$ corresponds to a period doubling bifurcation. Here again we treat the control parameter as varying, for example linearly in the form $\mu_{n+1} = \mu_n + v$ with $|v| << 1$. We take advantage of the work of Baesens [2] who defined adiabatic invariant manifolds for slow linear variations of μ_n. It is easy to see that for arbitrary initial conditions, trajectories converge towards the appropriate period -1 or period-2 manifolds for μ less than or greater than $\hat{\mu}$ respectively. For increasing $\mu > \hat{\mu}$ the trajectory can stay very close to the now unstable period-1 manifold before "jumping" to period-2. This apparent shift in the bifurcation value can be predicted. For decreasing μ from a period-2 to a period-1 region, the bridging manifold is obtained by matching asymptotic solutions. In the period-2 region, trajectories are also described well by a renormalised period-1 solution, so the results can be extended to the trajectories of period -4, -8, and so on. The results for linear variation of μ are extended to the periodic case, and a complete description of the various types of trajectories obtained is possible. These latter trajectories appear to have a remarkable global stability, even though in some regions they are locally unstable. Again, excellent agreement is obtained between calculated and simulated results.

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Dynamic Bifurcations in Systems with 1:2 Internal Resonance under Nonstationary External Excitation

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Considerable effort has been made to understand the dynamics of undamped single degree-of-freedom systems, as some parameter evolves through a resonance region (e.g. see Kevorkian [1982]), and the effects of slow evolution of parameters, through regions where bifurcations arise in the stationary case (Haberman [1979]). Neal and Nayfeh [1990] and Raman et al. [1996] have studied the dynamics of single degree-of-freedom damped oscillators, with nonstationary excitation. Balachandran and Nayfeh [1992] studied modal interactions in a two degree-of-freedom L-shaped structure subjected to nonstationary excitation, experimentally. Shyu et al. [1993] studied the passage through subharmonic resonance in a slender, cantilevered beam, and considered the effects of passage through resonance on multi-mode interactions.

As far as the authors know, theirs is the first attempt to study multi degree-of-freedom, damped, nonlinear, nonstationary systems, analytically. The objective of this work is to apply ideas from "Dynamic Bifurcation Theory" to these multi degree-of-freedom systems with slowly varying parameters that also exhibit internal resonances.

Many mechanical and structural systems can be modelled as two degree-of-freedom systems with quadratic nonlinearities. After appropriate scaling, the equations of motion for such systems can be averaged over the fast time scale for the condition of 1:2 subharmonic internal resonance between the two modes ($\frac{\omega_1}{\omega_2} = \frac{1}{2}$). It is also assumed that the excitation frequency in being linearly varied through resonance with the second mode. The first-order autonomous equations so obtained can be written in polar coordinates as

$$\begin{aligned} a_1' &= -a_1\xi_1 - \tfrac{a_1 a_2}{2}\sin(\beta_2 - 2\beta_1), \\ a_1\beta_1' &= a_1\tfrac{\sigma_2-\mu}{2} + \tfrac{a_1 a_2}{2}\cos(\beta_2 - 2\beta_1), \\ a_2' &= -a_2\xi_2 + a_1^2\sin(\beta_2 - 2\beta_1) - E_4\sin\beta_2, \\ a_2\beta_2' &= a_2\sigma_2 + a_1^2\cos(\beta_2 - 2\beta_1) - E_4\cos\beta_2, \\ \sigma_2' &= \sigma_{2r}. \end{aligned} \qquad (1)$$

where "$'$" denotes derivative with respect to the slow time "ϵt"; a_i, β_i (i=1,2) are the amplitudes and phases of the two oscillators; μ is the internal mistuning between the two modes; ξ_i (i=1,2) are the damping in the two modes; σ_2 is the mistuning from the linear natural frequency of the second mode; σ_{2r} is the rate of sweep of the mistuning σ_2.

A typical set of results for equations (1) for the stationary case ($\sigma_{2r} = 0$), is that the the single-mode solution ($a_1 = 0, a_2 \neq 0$) undergoes a pitchfork bifurcation to a coupled-mode solution ($a_1 \neq 0, a_2 \neq 0$), and the coupled-mode branch exhibits multiple solutions through a turning point. For nonstationary excitation ($\sigma_{2r} \neq 0$) numerical simulations of the averaged equations (1) show that closer the initial conditions are to the zero single mode equilibrium solution, the longer the response of the system stays near this zero equilibrium after the bifurcation. The faster the sweep rate, the greater the penetration into the region of instability and longer the time taken for the response to approach the slowly varying coupled-mode equilibrium solution.

Equations (1) are then analytically studied for their response for slow sweep rates. Center manifold reductions and asymptotic techniques of the inner and outer expansions (along the lines of Haberman [1979] and Raman et al. [1996]) are used to predict the afore described observations from the numerical simulations.

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On Non-Ideal Passage Through Resonance: Modeling and General Characteristics

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Nonstationary vibrations in rotordynamics have been the topic of much research. Here we are concerned with the nonstationary oscillations of such a system that develop as it passes through a resonance. In the traditional approach, the excitation is idealized to be independent of the response of the system; however, in some applications the power supply to the motor, or the size of the motor, may be so small that this ideal model is inadequate. Indeed, in such a real system, the power may be so limited that the system will not pass through resonance, but become stuck in it. Here, we consider a nonideal model in which an excitation of limited power interacts with, and depends on, the response it produces.

Kononenko (1969) and Nayfeh and Mook (1979) used asymptotic methods to analyze the steady-state responses of nonideal systems. They took the limited power of the excitation into account by adding an equation that relates the RPM of the motor to the load on the shaft and the input power. Several experiments were mentioned in both books. In some earlier studies of nonstationary responses of nonideal systems, Blekhman (1953) considered self-synchronization, Dimentberg and Frolov (1967) considered the Sommerfeld effect in a system with randomly varying natural frequencies, Rand et al. (1992) considered the passage through resonance, Krasnopolskaya et al. (1993) studied chaos in nonideal systems, Dimentberg et al. (1994) proposed a method for getting a nonideal system through resonance by switching stiffnesses, Nobrega and Mazzili (1994) studied autosynchronization in which the rotor speeds and phase differences of nonideal motors supported by elastic structures became quasi-stationary, and Brasil and Mook (1994) used numerical simulation to model the passage through resonance of a nonideal motor supported on a portal frame.

In the present study we consider the passage through resonance of system composed of a nonideal motor supported on the end of a cantilever beam. The beam equations account for the nonlinear effects of inertia and curvature (Crespo da Silva and Glynn, 1978). Nonstationary responses are studied by numerical integration of i) the modulation equations obtained by the method of multiple scales and ii) the original equations obtained by Galerkin's method. The analytical/numerical results are compared with the observations made in a companion experiment.

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Tuesday, June 11

1530-1710

Session 10. Analysis II

DYNAMICS OF DELAMINATED BEAMS

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ABSTRACT

Composite materials are used in structural design because of the promising characteristics, like high strength and stiffness, light weight, fatigue resistance, and damage tolerance. Despite these attractive properties, composites are very sensitive to the anomalies induced during their fabrication and service life. Delamination has been one of the major failure modes in these structures. In this research, we present a mechanics model to study the dynamics of composite beams with delaminations. The model includes the effects of shear, rotary inertia, the coupling of longitudinal and transverse deformations at the delamination boundaries. Nonlinear interaction represented by a piecewise linear spring model between the delaminated sublaminates is included. Based on the proposed model, delamination frequencies and delamination opening modes are predicted. Nonlinear dynamic response of the delaminated beam is calculated using nonlinear modal analysis. To verify analytical model, experiments on delaminated beams, with properly attached sensor and actuator, are designed, constructed, and carried out.

Chaotic Vibrations of a Cylindrical Shell-Panel with an In-plane Elastic-Support at Boundary

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ABSTRACT

Shells are used commonly as structural element in various vehicles, airplanes and space structures. To increase a structural rigidity of the shell within a limited amount of volume, curvatures and boundary conditions are a significant factor of improvement. Because, the shell is usually fixed on other elastic frames. When the shell is subjected to pressure, buckling phenomena occurs under ultimate level of pressure. As the shell is exposed in a periodic pressure, large amplitude vibration is generated by resonance. In a typical combination of exciting frequency and exciting amplitude, the resonance vibration will be taken place to chaotic vibration. Since the chaotic vibration generates a violent vibration, the shell will be damaged by cyclic fatigue.

Our main interests exist to clarify predominant factors for emergence of the chaos and the contribution of freedom of vibration on the chaos of the shell.

This paper presents analytical results on the chaotic vibrations of the shallow cylindrical shell-panel under harmonic lateral excitation. The shell with a rectangular boundary is simply supported for deflection and the shell is constrained elastically in an in-plane direction.

Using the Donnell equation modified with an inertia force, the basic equation is reduced to the nonlinear differential equation of a multiple-degree-of-freedom

system by the Galerkin procedure. The normal coordinates of the nonlinear equation are selected as natural modes of linear vibration. Further, a linear modal damping is involved into the equations.

To get fundamental characteristics of the shell, nonlinear restoring forces and natural frequencies of linear vibration are computed under three cases of the inplane elastic constraint.

Chaotic vibrations of the shell are expected to be generated in specific combinations of exciting frequency, exciting amplitude and the resonance response of the shell.

To estimate regions of the chaos, nonlinear response curves of steady state vibration are calculated by the harmonic balance method.

The time progresses of the chaotic response are obtained by the numerical integration technique of the Runge-Kutta-Gill method. The chaos of the shell is identified both by the Lyapunov exponent and the Poincare projection onto the phase space. The Lyapunov dimension is carefully examined by increasing the assumed modes of vibration. Detailed discussions are presented for the effects of the in-plane elastic constraint on the chaos of the shell.

It is found that the followings: when the shell is constrained with loosely in the in-plane direction that is in-plane stresses are free. Modal interactions in the chaos exceed more than six fundamental modes of vibration for the shell. When the boundary is fixed comparatively rigid in the in-plane direction, the coupled modes of vibration decrease less than four modes. On the condition the shell has a too rigid in-plane constraint, a characteristic of restoring force changes to a type of hard spring, therefore, it was hard to generate the chaotic response.

In addition, it is found that the shell with the looser in-plane constraint has many positive components in the Lyapunov spectrum. Furthermore the response of the chaos gives more complex projections in their Poincare maps.

Title: ASYMPTOTIC RESEARCH OF NONLINEAR WAVE PROCESSES IN SATURATED POROUS MEDIA

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Abstract. Numerous geophysical, geological and technological applications call for a theoretical description of wave processes in saturated porous media. It was formulated by continuous mechanics methods and researched 3-D mathematical model of nonlinear wave dynamics of thermo-visco-elastically deformed porous media[1]. The model is based on a classical Frenkel-Biot-Nikolaevskij theory about elastic waves propagation and is formulated as mass, momentum and energy differential conservation laws and closing rheological and thermodynamical relations. In suggested model porous medium consists of elastic skeleton, viscous liquid film which is bound by solid phase and fluid phase (weakly compressed liquid or perfect gas). Also it was researched wave dynamics of two-phase saturated (by liquid and gas) porous medium with consideration of capillary pressure. It is considered nonlinear, dispersive and dissipative factors (for a example, in contrast to a classical model it was taken into account viscous stresses inside a fluid phase). Carried out dimensions analysis allowed to obtain small dimensionless parameter which defines the scale of dispersion properties appearance and allows to formulate hyperbolic in principal part closed equations system w.r.t. unknown scalar, vector and tensor functions. The presence of small parameter in suggested equations system allowed to apply multiscaled expansions method and to distinguish two different scales motions: slow nonperiodic background motion and quick oscillating motion. It was obtained dispersive equation for a definition of the waves frequencies. In the case of one-phase porous medium saturation dispersive equation roots

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correspond to the frequencies of longitudinal waves of first and second kinds and transverse waves of different polarization but for the two-phase porous medium saturation additionally to the frequencies of interphase tension wave (longitudinal) which appearance is stipulated by the consideration of capillary pressure. An application of asymptotic method transforms at the second approach Cauchy problem for initial equations system to Cauchy problem for nonlinear evolutionary Korteweg-de Vries - Burgers equation for quick waves amplitudes and for nonhomogeneous hyperbolic set of equations for slow background motion resulting from nonlinear quick waves propagation. The coefficients of obtained generalized Korteveg-de Vries - Burgers equation depend on medium parameters, frequency and wave vector of propagating wave and parameters of equilibrium background medium state. This evolutionary equation describes the waves modulations and allows to obtain space-time distributions of the amplitudes of desired functions. Some kinds of stationary solutions of the Korteweg-de Vries - Burgers equation are studied numerically. Under real medium physical parameters it was shown an existence of a few types of solutions which realization mainly depends on a relation between the coefficients at second and third derivatives in the equation. In the case of one-phase porous medium saturation obtained evolutionary equation has two types of particular solutions: diffusional solutions with monotone decreasing amplitude and oscillating (with different frequencies) solutions. For two-phase porous medium saturation it was shown that effects of interaction and redistribution of fluid phases influence significantly on waves propagation dynamics and result in modification of longitudinal waves propagation regimes. In particular, aside from diffusional and oscillating regimes it proved to be possible soliton-like (as kink) propagation regimes of longitudinal waves of first and second kinds and interphase tension waves. Obtained results allowed to explain qualitatively experimental facts of waves frequencies transformation and appearance of envelopes oscillating waves (for example, a generation of ultrasonic waves by seismic waves).

Also it was constructed numerical stationary solutions of nonhomogeneous set of equations for slow background motion. Obtained results demonstrate nonlinear character of the accumulation of background functions amplitudes under the influence of elastic waves.

The construction of asymptotic solution was carried out with an application of a symbolic calculations software.

Resonance Capture in a Three-Degree-of-Freedom System

D. Dane Quinn *

1 Introduction

Resonance capture describes the attraction of a system and/or its mathematical model into a state of resonance. We study this phenomena in systems possessing components that slowly change in time. Because of the slowly varying nature of these systems, trajectories that are initially far from resonance can, under suitable initial conditions and parameter values, become attracted to a state of resonance for long times. This phenomena is referred to as "capture into resonance." In contrast, trajectories that are initially in a state of resonance, but at later times leave this state, are said to "escape".

We study a three-degree-of-freedom mechanical system consisting of an unbalanced rotor, attached to a frame, supported by two orthogonal, linearly elastic supports as shown in Figure 1. The rotor is subjected to a small applied torque. In addition, we assume that the linearly elastic supports are almost identical, *i.e.* $k_1 = k_2 + e$, with $e \ll 1$. If the system is started from rest, the angular velocity of the rotor increases due to the applied torque. However, as $\dot{\theta}$ approaches the natural frequency of the spring-mass system, there exists the possibility of capture into resonance. If this occurs, the angular velocity of the rotor becomes locked in the neighborhood of $\sqrt{k_1/(m_1+m_2)}$, while the amplitude of vibration of the frame increases.

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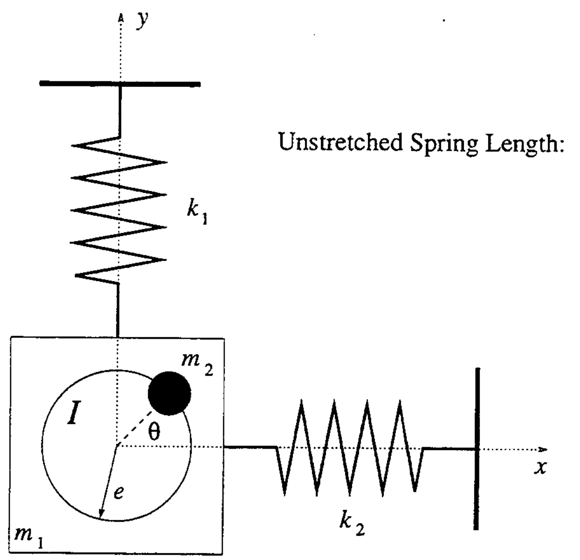

Figure 1: Mechanical system. The unbalanced rotor has a moment of inertia of I. The total mass of the system is $m_1 + m_2$, with m_2 the mass of the imbalance.

Past studies by Quinn and Rand [4, 5] have focused on this phenomena in a two-degree-of-freedom system, essentially obtained by restricting motion of the frame to the x direction only. However, the present work is complicated by the $1:1$ resonance between the motion of the frame in the x and y directions. There exists a four dimensional resonance region to which solutions can be attracted.

The angular velocity of the rotor is assumed to be near the natural frequency of the frame and method of averaging [2, 6] is used simplify the equations of motion for this system. This procedure leads to the following simplified equations:

$$\begin{aligned}\dot{q}_1 &= p_1, & (1.1)\\ \dot{p}_1 &= 1 - r_1\cos q_1 - r_2\cos q_2, & (1.2)\\ \dot{q}_2 &= p_2, & (1.3)\\ \dot{p}_2 &= 1 - r_1\cos q_1 - r_2\cos q_2, & (1.4)\\ \dot{r}_1 &= \varepsilon\cos q_1, & (1.5)\\ \dot{r}_2 &= \varepsilon\cos q_2. & (1.6)\end{aligned}$$

In this system, (q_1, p_1) and (q_2, p_2) represent the phase difference between the rotor and the two modes of vibration, while r_1 and r_2 characterize the amplitudes of oscillation in the x and y directions. The parameter ε is related to the imbalance of the rotor and the detuning from resonance appears in the restriction:

$$\begin{aligned}p_1(t) - p_2(t) &= p_1(0) - p_2(0),\\ &= 2\eta.\end{aligned}$$

where η is related to this detuning. This relationship is consistent with $\dot{p}_1 = \dot{p}_2$ in Eqs. (1).

We are interested in the effects of not only the detuning from the exact resonance (so that $\eta \neq 0$) [1, 3], but the effects of the slowly varying variables r_1 and r_2. For $\varepsilon \neq 0$, there exists the possibility that solutions are captured into resonance. This phenomena is studied numerically and via global perturbation methods.

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AEROELASTIC RESPONSE OF A TWO-DIMENSIONAL AIRFOIL-AILERON COMBINATION WITH FREEPLAY NONLINEARITIES

by

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Aeroelastic behaviour of a typical section of a wing and aileron subjected to incompressible flow has been studied. The typical section is a rigid three-DOF airfoil-aileron mounted via torsional and translational springs. Freeplay structural nonlinearities are considered in the aileron hinge and pitch directions. The aerodynamic response of the three-DOF airfoil performing an arbitrary unsteady motion is calculated from that of an oscillating airfoil by means of a Fourier analysis, and the aerodynamic forces are evaluated using Wagner's function. The resulting equations of motion are either integrated numerically using a finite difference method to give time histories of the airfoil motion, or solved in a semi-analytical manner using a describing function technique.

The linear flutter speed of the system is calculated using the finite difference method as well as the traditional P-k method. Both methods are in excellent agreement for the prediction of the linear flutter speed. Variation of the flutter speed via the aileron hinge stiffness shows three distinct regions of instability corresponding to bending-aileron, torsion-aileron and bending-torsion binary flutter.

Flutter analysis of the airfoil-aileron combination with a freeplay nonlinearity in the aileron hinge moment indicates limit-cycle solutions for velocities well below the linear flutter boundary. The existence of these solutions is explained by analysing the aforementioned regions of linear instability.

For some airspeeds there is more than one stable periodic solution, with possibly irrational frequency ratio, which may lead to quasi-periodic oscillations. The effect of different parameters has been investigated and, for some particular set of parameters, quasi-periodic and chaotic oscillations are obtained. It is also shown that the existence of stable equilibrium, periodic or aperiodic solutions depends on the initial conditions of the airfoil.

Since the limit-cycle oscillations are mostly of period-one, there is excellent agreement between the finite difference and describing function results in terms of both the existence and amplitude of the limit-cycle oscillations.

The flutter response of the system, containing a freeplay in the pitch direction, shows a considerable region of chaotic oscillations for the aileron centre of mass aft of the aileron hinge, $x_\beta > 0$. However, moving the centre of mass forward, $x_\beta < 0$, introduces more order into the system and eliminates chaos. The response of the airfoil-aileron combination with freeplay in both the pitch and aileron restoring moments has also been investigated and stronger chaotic motion over a large range of airspeeds is obtained. In all cases, a small amount of preload in the freeplay nonlinearity can substantially improve the stability boundary and eliminate chaos.

Wednesday, June 12

0830-1010

Session 11. Modal Interactions II

Evolution of Domains of Attraction of a Forced Beam with Two-Mode Interaction

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Abstract

When a dissipative dynamical system has multiple attractors, it is a task to determine and recognize the global domain of attraction of each attractor. In this paper we study the global behavior of a forced hinged-clamped beam with two-mode interaction. The equation of motion of the beam is reduced to four first-order autonomous ordinary differential equations. The system has an internal resonance condition of $\omega_2 \approx 3\omega_1$, where ω_1 and ω_2 denote natural frequencies of the first and second modes, respectively. When the excitation frequency is near ω_1, the system can have three equilibrium solutions, among which two are asymptotically stable and one is unstable. We examine how the domains of attraction of two stable equilibrium solutions evolve as the forcing frequency is varied across jump points. By using a special plane which contains all equilibrium points, called a principal plane, the global domains of attraction can be discussed more effectively. Results show that knowledge of this evolution helps us better understand the jump phenomenon.

NONLINEAR DYNAMICS OF CYCLICALLY COUPLED DUFFING OSCILLATORS

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Weakly coupled structural systems have received much attention in the engineering literature, particularly from the turbomachinery and aerospace engineering sectors. For the aerospace engineer, the vibratory characteristics of space antennas, and other flexible space structures, have been studied, particularly when shape control is of the utmost importance. For turbomachinery engineers, determining the cause of "rogue blade failure", where a small number of contiguous compressor blades fail while the remaining blades are well within fatigue and wear limits, is of interest. The literature for these investigations has predominately been concerned with linear oscillators where, given the parameters defining the components and the external excitation, a unique solution may exist for each sub-structure. When numerical studies are performed to determine the system response, the number of sub-structures must be specified. Our interest is in the dynamics of cyclically coupled nonlinear sub-structures for an arbitrary number (n) of such oscillators.

As a preliminary investigation, we consider each oscillator to be a damped, forced Duffing oscillator, and discuss small amplitude vibratory motions. These Duffing oscillators are coupled by weak linear springs, and identical external forcings, except possibly differing in phase values, are applied to each oscillator. It has been shown in previous studies that, for strong coupling of such Duffing oscillators, there are two system modes of vibration that are in 1:1 internal resonance, while for weak coupling, each oscillator is in resonance with the remaining (n-1) oscillators. Therefore, the dynamics in the weakly coupled case is quite rich. Since we are interested only in oscillators that are coupled in a cyclic manner such that they form an n-gon in a plane, we restrict the external forcing to the plane containing the oscillators. The tangential component of the external force produces a direct forcing of the system while the radial component produces a parametric forcing. We consider both these possibilities of excitation in the equations of motion.

This study starts with the development of the system model by applying Newton's Second Law for a system of cyclically coupled Duffing oscillators, coupled by weak linear springs, with the assumption that the damping, cubic order stiffness, and weak external harmonic forcing are of the same order as the linear coupling stiffness. The linear stiffness element is considered as the only strong parameter. Periodic solutions are studied by transforming the system to van der Pol coordinates, and averaging over the external forcing frequency, giving rise to a vector field describing the weak amplitude and phase dynamics. This 2n-dimensional vector field possesses a large amount of symmetry, and therefore we explore the fixed-point solutions by borrowing some results from Equivariant Bifurcation Theory.

By exploiting the symmetry of the system, we classify the fixed-point solutions of the averaged equations. These solutions constitute blocks of oscillators with definite phase characteristics relative to neighboring blocks such that the interior structure of each block is determined by the symmetry of each class. There are three categories of symmetric solutions that determine solution characteristics between blocks: In-Phase oscillations where each block of sub-structures oscillates in-phase with the surrounding blocks; Standing Waves where the phase difference between nearest-neighbor blocks is π; and Traveling Waves where the phase of each block is increased from its nearest-neighbor block by the amount $2\pi/n$. The traveling wave motions are produced by engine order excitation of the system where the forcing satisfies the same phase requirement as that of the traveling wave solution between blocks. It is shown that the smallest number of possible fixed-point solutions occurs generically for a prime number of oscillators while the largest number of such solutions occurs generically when the number of oscillators is a multiple of four. A linear stability analysis for the model is performed for two examples showing the connections of the various fixed-point solutions as a function of the system and excitation parameters.

Nonlinear modal interaction and chaos in an experimental cable/mass suspension

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The present work tackles an experimental model of elastic cable/mass suspension made by a nylon wire carrying eight equally spaced concentrated masses and hanging at supports which are given either in-phase or out-of-phase vertical harmonic motions. The system mechanical and geometrical parameters are adjusted to realize 1:1 internal resonance involving the first planar and nonplanar antisymmetric modes, and 2:1 internal resonance involving the former and the first nonplanar symmetric mode. Different ranges of excitation frequency are considered, which include meaningful external resonance conditions such as the one-half subharmonic, the primary, and the superharmonic resonances of the antisymmetric modes, and corresponding one-fourth subharmonic, one-half subharmonic, and primary resonances of the out-of-plane symmetric mode. Wide ranges of variations of the excitation amplitudes are considered, too.

Based on systematic measurements of the system response, overall behaviour charts in the excitation control parameter space are built around each resonance condition. A quite rich picture of regular response regions exhibiting contributions from different planar and nonplanar, symmetric and antisymmetric, cable modes is observed, as well as the occurrence of several regions of quasiperiodic and chaotic responses. The main and more robust classes of regular and nonregular motion of the system are identified.

Attention is focused on two main aspects. First, with reference to classes of regular motion, the extent and the features of the nonlinear interaction among different contributing modes, the competition between coexisting classes, and the relevant local bifurcations in various control parameter ranges, are analyzed in detail. Quite complex path pictures are highlighted by means of several amplitude-frequency and amplitude-forcing plots.

Second, interest is devoted to the analysis of bifurcations from simply-periodic to complex-QP and -chaotic motions, and to qualitative and, mostly, quantitative characterization of the latter. This is made by means of a delay-embedding procedure permitting the reconstruction, under certain assumptions, of the global properties of the attractor from a scalar experimental time series. In the reconstruction, particular attention is given to the improuvement of accuracy obtainable by using the rotation of the representation basis for pseudovectors, according to the singular value decomposition of the sample covariance matrix. The correlation dimension of the attractors in pseudo phase-spaces and the maximum Lyapunov exponent are computed. Calculation of the full spectrum of exponents is also planned.

Some experimental results are observed against the background of the nonlinear dynamic phenomena exhibited by a theoretical model of continuous cable with four degrees-of-freedom accounting for the first planar and nonplanar symmetric and antisymmetric modes. The relevant amplitude and phase modulation equations obtained with a multiple scales technique are used to obtain steady and nonstationary response. Qualitative comparison is performed between the classes of regular steady motions obtained with the experimental and theoretical models, as well as between the periodically and chaotically amplitude modulated motions of the four d.o.f. model arising after Hopf bifurcations in the steady amplitude system and the quasiperiodic and chaotic responses of the experimental model. Analysis of periodic-to-complex transitions is also conducted directly on the four-d.o.f. model by means of a path-following procedure, and by analyzing the stability of the obtained paths.

Multi-Modal Nonlinear Dynamics in Machine Tool Cutting Processes

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High precision machining of complex geometries, such as boring of long holes and milling of deep channels, has called for the use of long tools. These tools tend to be flexible and lead to dynamic instabilities. Current models for turning and milling, e.g. Hanna and Tobias [1], need to to be modified in order to account for the possibility of multi-modal vibrations. In this paper we examine a nonlinear cutting model with multiple degrees of freedom with forcing delay, called the regenerative effect. That is, the delay terms are due to the interaction between the tool and the previously cut surface. The model assumes a uniform tool that is decomposed into two equal masses with nonlinear structural coupling, forced by a delayed force due to chip width or depth of cut. The specific model is formulated as

$$\begin{aligned} m\ddot{x}_1 &= f_1(x_1,x_2) - c\dot{x}_1 - c(\dot{x}_1 - \dot{x}_2) \\ m\ddot{x}_2 &= f_2(x_1,x_2) - c(\dot{x}_2 - \dot{x}_1) + f_3(x_2,x_2(t-\tau)) \end{aligned} \qquad \textbf{(1)}$$

where $f_1(x_1,x_2)$ and $f_2(x_1,x_2)$ are nonlinear structural stiffness terms and $f_3(x_2,x_2(t-\tau))$ is the cutting term. These can be expanded in Taylor series to give a two-degree-of freedom nonlinear model. To analyze this system we follow the methods of Nayfeh, Chin and Pratt [2] by first developing the characteristic equation for (1). Since the resulting characteristic equation is similar to that developed by Hsu [3] in his study of a dynamic system subject to a retarded follower force, we use the tau-decomposition technique of Lee and Hsu [4] to compute the regions of asymptotic stability. Hopf bifurcations to limit cycles (chatter) can occur at the boundaries of stability. Normal forms are used to examine these limit cycles. The harmonic content of the limit cycles can be developed to a high order by using Galerkin's method. The analytic results are validated by numerical integration, using a Runga-Kutta scheme modified to interpolate for the time delay terms. A high order Hermite interpolation algorithm is used for this interpolation. This technique was suggested by the work of Oberle and Pesch [5].

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On the Dynamics of Multiple Centrifugal Pendulum Vibration Absorbers*

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In the dynamics of rotating and reciprocating machinery, forces are often generated which cause undesirable oscillatory torques at frequencies that are multiples of the nominal rotation rate, resulting in torsional oscillations which introduce roughness and fatigue difficulties. A centrifugal pendulum vibration absorber (CPVA) is a device used for reducing these torsional oscillations. This CPVA is a mass restricted to move along a prescribed path relative to the base rotating system. The absorber is driven by the centrifugal field generated by rotation, and its motion provides a restoring torque which, when the path is properly designed, reduces the level of torsional oscillations. Previous work has concentrated on analyzing the dynamics of these absorbers which use the easily-manufactured circular path (Newland, 1964 and Sharif-Bakhitar & Shaw,1992). More recent work considers the search for optimal paths that minimize the level of torsional oscillations. This can be done via intentional mistuning (DenHartog,1938) or by path generation through a nonlinear dynamic analysis (Denman, 1985 and Lee *et al*, 1994).

Due to spatial and balancing considerations, the implementation of CPVA's invariably requires that the total absorber inertia be divided into several absorber masses that are stationed about the center of rotation. In order to achieve the designed-for performance, such a system of CPVA's is expected to move in an exact unison response. The present work deals with the dynamic stability and bifurcation of this response.

In Part I, we determine the conditions under which the unison motion of a system of several identical CPVA's is dynamically stable. This is done for the special case of tautochronic absorbers applied to a system that is subjected to a purely harmonic torque. The stability criterion is obtained by the method of averaging. The small parameter employed is the ratio of the total moment of inertia of all absorbers to that of the base rotating system — a number that is nearly always satisfied in practice. This technique allows for results to be obtained for large amplitude motions. The procedure exploits certain symmetries in the equations of motion and the unison response and results in a stability criterion expressed in terms of a critical torque level $\hat{\Gamma}^*$. For small levels of absorber

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damping (a conditions required for satisfactory performance), i.e., $\hat{\mu}_a << 1$, it is found that $\hat{\Gamma}^*$ is given by,

$$\hat{\Gamma}^* = \sqrt{2b_0 \, n \, \hat{\mu}_a}. \qquad (1)$$

where b_0 is the ratio of the total moment of inertia of all absorbers to that of the base rotating system is and n is the order of the applied torque. The result is verified by numerical simulations of the system near the critical parameter conditions.

In Part II, we investigate the post-critical response. This is important in that the instability will affect the performance of the absorber system and will reduce the effect torque range of operation. The analysis is carried out by a special modal coordinate transformation, the method of averaging (in this case based on amplitude expansions), and symmetric bifurcation theory. It is determined that, while there are many possible solutions in the post-critical stage, in this range the amplitude of the oscillatory angular acceleration of the system is independent of the type of solution encountered. Furthermore, the rather remarkable result is obtained that this torsional oscillation level is reduced in the post-critical range as compared with the unison response, and this is also independent of the particular solution encountered. However, the post-critical response has at least one absorber whose amplitude is larger than that given by the unison response. This results in a reduction in the effective torque range of operation, an estimate of which can be obtained from the analysis. These analytic results are verified by numerical simulation of the full system over a range of operating conditions.

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Wednesday, June 12

1030-1210

Session 12. Computational Methods II

Nonlinear Dynamics of Shells Under Finite Deformations. A Finite Element Approach, Integration Schemes, and Applications to the Chaotic Motion.

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Recently, in [1], a shell theory with seven degrees of freedom has been proposed which enjoys the following features:

1. It is geometrically exact in the sense that the formulation is capable to catch arbitrary large deformations within the class of the assumed material behaviour.
2. It allows for the application of three-dimensional constitutive laws. That is, within the range of very thin shells there is no need to simplified constitutive assumptions such as that of plane stress.
3. It is free of a rotation tensor, hence it preserves the vector-space nature of the configuration space.
4. Numerically, it is free of ill-conditioning in the very thin regime.

In this paper we present 1) a dynamical formulation of the above theory, 2) corresponding finite elements, 3) efficient numerical integration schemes, and 4) applications to long term computations such as the detection of chaotic motion.

The shell theory is based on the basic kinematical assumption of a quadratically varying displacement field over the shell thickness. First order shear deformation as well as thickness change are taken into account which allows for the consideration of a three-dimensional constitutive law. Although the latter law is taken to be a linear one, it should be mentioned that the theory is applicable to large strain formulations as well as to elasto-viscoplastic formulations where the dissipation is of a pure physical nature.

Let $\mathbf{E}$ define the Green strain tensor of the shell space. In the light of the basic geometric assumption, $\mathbf{E}$ can be written down in a serie with respect to the co-ordinate z perpendicular to the shell midsurface: $\mathbf{E} = \mathbf{E^0} + z\mathbf{K} + z^2\mathbf{L} + \cdots\cdots$. For the dynamical formulation only the first two strain tensors $\mathbf{E^0}, \mathbf{K}$ are considered essential and are contained in the formulation. Let the vectors $\mathbf{u}, \mathbf{w}$ as well as the scalar λ be the external kinematical fields pertinent to the shell model under consideration and let T be the kinetic energy. We assume the existence of a free energy function $\psi_{int}(\mathbf{E^0}, \mathbf{K})$ depending on the first two strain tensors, as well as the existence of an external potential

$\psi_{ext}(\mathbf{u}, \mathbf{w}, \lambda)$ related to the non-dissipative forces. The dynamics is then given by means of the following action

$$\delta \int_{t_0}^{t_1} \left[\int_{\mathcal{B}} \left(T(\dot{\mathbf{u}}, \dot{\mathbf{w}}, \dot{\lambda}) - \psi_{int}(\mathbf{E^0}, \mathbf{K}) - \psi_{ext}(\mathbf{u}, \mathbf{w}, \lambda) \right) dV \right] dt + \int_{t_0}^{t_1} \left[\int_{\mathcal{B}} (\mathbf{P} \cdot \delta \mathbf{u} + \mathbf{M} \cdot \delta \mathbf{w}) dV \right] dt = 0 \qquad (1)$$

Here $\mathbf{P}, \mathbf{M}$ are dissipative forces and moments with work conjugates $\delta\mathbf{u}, \delta\mathbf{w}$.

For the finite element formulation it proves more appropriate to modify the internal potential to arrive at a mixed form of the Hellinger-Reissner type. The resultant stress tensor related to the shell midsurface is taken as an independent variable for which interpolation functions are to be considered. On the basis of the above modified functional a four-node element is developed.

A fundamental issue of the paper is the integration of the resulting ordinary differential equations. Two possible integration schemes are presented and compared which fall within the class of energy-momentum conserving algorithms. The first is due to Simo & Tarnow which is valid for configuration spaces with vector-like structure, the second is new and is valid for configuration spaces with linear and nonlinear structure as well. Nonlinear configuration spaces appear when the shell theory incorporates explicitly a rotation tensor which is the case when the shell model is that of the Cosserat surface.

Various examples of shell dynamics including cases of large overall motion, large deformation, and chaotic motion are presented. Due to the finite element formulation, it is possible to consider a large number of degrees of freedom allowing for a more realistic modelling of continuous systems.

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Remark: The first author will present the paper at the conference.

A Time Integration Algorithm for Flexible Mechanism Dynamics: The DAE α-method *

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Abstract

This paper introduces a new family of second-order methods for solving the index-2 DAE equations of motion for flexible mechanism dynamics. These methods, which extend the α-methods for ODEs of structural dynamics to DAEs, possess numerical dissipation that can be controlled by the user. Convergence and stability analysis is given and verifies that the DAE α-methods introduce no additional oscillations and preserve the stability of the underlying ODE system. Convergence of the Newton iteration, which can be a source of difficulty in solving nonlinear oscillatory systems with large stepsizes, is achieved via a coordinate-split modification to the Newton iteration. Numerical results illustrate the effectiveness of the new methods for simulation of flexible mechanisms.

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Computer Implementation of Nonlinear Dynamical Problems with Maple

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Abstract

The successful application of computer algebra to compute limit cycles and in general invariant tori of solutions makes this tool basic for the analysis of nonlinear dynamical problems. In the past, I used computer algebra in a large number of applications including the nonlinear dynamics and control of a four-wheel steering vehicle and a number of other cases. Other authors have also used it in a number of situations, in Maple, and other languages such as Macsyma. The power of personal computers and the possibility of doing extended computations in a very short time opens up the opportunity of having the computer run algebraic and numerical computations. The presentation of a code in Maple to perform algebraic and numerical analysis of nonlinear dynamical problems will be the objective of this paper. The paper will present the analyses done in order to find limit cycles and then a number of examples will be presented involving algebra and numerical evaluation of the solution. The code will be available via Internet for any interested in the topic. The future saves for us the simple use of complicated software developed with the goal of understanding and quantifying nonlinear systems.

KARHUNEN-LOÉVE DECOMPOSITION IN DYNAMICAL MODELING

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1 Introduction

Characteristics of the Karhunen-Loéve (KL) decomposition technique are investigated in the context of fluid structure interaction problems. First, the dynamics of an elastic panel subjected to an axial load and a fluid flow along its surface is studied using a highly accurate spectral solution procedure based on the Galerkin projection operator. The KL technique is then applied to the data generated from the spectral analysis, resulting in a set of KL eigenfunctions and eigenvalue spectra. Next, the dynamics of the panel is studied using the KL eigenfunctions as a base in the Galerkin method. Solutions based on the KL decomposition are compared with spectral solutions. The extent to which the KL eigenfunctions have spanned the space of admissible solutions for a range of parameters is studied. For a given set of parameters the convergence of the KL solution procedure is investigated. The numerical results obtained demonstrate the ability of the KL procedure to provide an efficient base for extracting low-dimensional dynamical models for infinite-dimensional systems. Furthermore, in the full length paper [1], it is shown that one severely truncated KL base can capture the qualitative behavior of systems with widely varying parameters.

2 The Karhunen-Loéve Procedure

In the following, brief outline of the discrete Karhunen-Loéve procedure, assume existence of an ensemble of M snapshots of a random variable, i.e., $u^{(m)} = u(x, t_m)$ where $m = 1, \ldots, M$, that has been extracted from some process. As is customary, we assume that the mean has been separated from the data so that $\overline{u} = < u^{(m)}(x) > = 1/M \sum_m u(x, t_m) = 0$. Given the random variable u, we ask which element φ is most similar to the members of u on average. Mathematically speaking, we seek a function φ which is most nearly parallel to the field u in function space. This is a classical problem in the calculus of variations whose solution is a discrete set of the eigenfunctions $\varphi(x)$ of the covariance $C(x, x') = \; < u(x, t_m)\, u(x', t_m) > \; = \frac{1}{M} \sum_m u(x, t_m)\, u(x', t_m)$, i.e.,

$$\int C(x, x')\, \varphi(x')\, dx' = \lambda\, \varphi(x) \tag{1}$$

where the integration is over the domain of interest. Equation (1) generates a complete set of eigenfunctions $\{\varphi_i\}_{i=1}^{\infty}$. The field at any instant can be expanded in terms of these eigenfunctions as

$$u(x, t) = \sum_i a_i(t)\, \varphi_i(x) \tag{2}$$

where $a_i(t) = \int u(x, t)\, \varphi_i(x)\, dx$.

Suppose that $u(x, t)$ is a velocity field. Then it can be shown, that KL decomposition is optimal in the sense that among all linear decompositions with a given number of modes, the KL will capture the most possible kinetic energy.

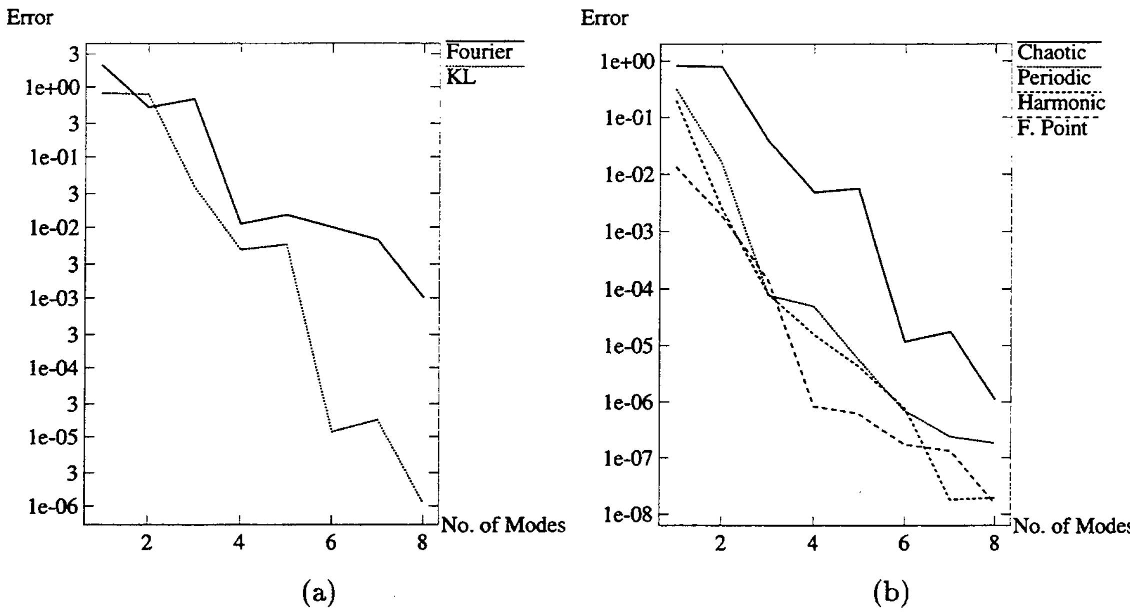

Figure 1: Comparison of Convergence Rate for: (a) the KL vs. the Fourier Chaotic Responses and (b) the Different Types of KL responses.

3 The Dynamic Model & Convergence Analysis

As an example, we will examine the motion of a simply supported elastic panel subjected to an axial load R_x and a supersonic stream of fluid along its surface. The fluid velocity is characterized in terms of the dynamic pressure λ. The dynamics of the panel is governed by the equation

$$w_{tt} + w_{xxxx} + \lambda w_x + \alpha w_t - \gamma w_{xx} \int_0^1 (w_x)^2 dx - R_x w_{xx} = 0 \qquad (3)$$

where $\alpha > 0$ represents fluid damping and $\gamma > 0$ is a measure of the nonlinear axial (membrane) restoring forces generated in the plane due to transverse displacements. The response of this system is primarily dependent on parameters R_x and λ.

The convergence rates of the KL base and the Fourier base are compared for different responses. Four response types are investigated: fixed point, simple harmonic, periodic but not simple harmonic, and chaotic. At a particular set of parameters, R_x and λ, a spectral solution, $w(x,t)$, is built from a 20-mode Fourier base. One thousand snapshots of that solution served as the ensemble from which the KL base is derived. The convergence of the truncated KL and Fourier solutions, $w_N(x,t)$ (N is number of modes), is quantified by the norm of the difference between the spectral solution and the approximate solutions for the same parameters and initial conditions, $Error = \int_0^1 [w(x,t) - w_N(x,t)]^2 \, dx$. The convergence analysis is performed at $t = 4$. The result for a chaotic response is given in Fig.1(a). One can see that the solutions obtained using the KL base converge faster than the solution obtained from the Fourier base for the same number of modes. The same trend is observed for all other types of responses. The influence of the qualitative type of a response on the convergence of the KL solutions is given in Fig.1(b). It can be seen that more complicated responses require more modes for the same accuracy.

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Using the Karhunen-Loève Decomposition to Examine Chaotic Snap-Through Oscillations of a Buckled Plate

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Recent studies on the snap-through oscillations of buckled plates have focused on behavior in the time and frequency domain as well as in the parameter space [1], [2]. However, another equally important issue relates to the spatial behavior of the plate as it undergoes snap-through. In the aforementioned studies, the partial differential equation of motion is discretized using the Galerkin method. The out-of-plane motion $w(x, y, t)$ is expanded as

$$w(x,y,t) = \sum_{i=1}^{n} a_i(t)\Phi_i(x,y) \tag{1}$$

where the $a_i(t)$ are time dependent modal coefficients and $\Phi_i(x, y)$ are a set of basis functions. Such an expansion raises the question: "is there a smaller set of basis functions which could be used to describe the spatial behavior of the motion?" If such a set exists, the order of the system and, hence, the number of equations could be reduced. This, in turn, would significantly reduce computational time as well as provide insight to the spatial dimension of the response. To address this question, the Karhunen-Loève (KL) decomposition is used.

The KL decomposition attempts to determine the directions within the n-dimensional displacement space which make the largest contributions to the "energy" of the response [3]. These directions constitute the KL modes for the motion. The original motion, obtained using Equation (1), may then be projected onto the KL modes to demonstrate the extent to which the KL subspace captures the relevant dynamics. Figure 1 shows a) the original motion using $n = 9$ and b) the projection of the motion onto two KL modes.

Next, the 2 dominant KL modes are used as basis functions for an expansion similar to Equation (1). The Galerkin method is then applied using this new expansion and a smaller set of ODE's result. Comparisons are then made between the dynamic response of the original set of 9 ODE's and the KL set of 2 ODE's. The results show qualitative similarities and a sizable computational savings with the KL approach.

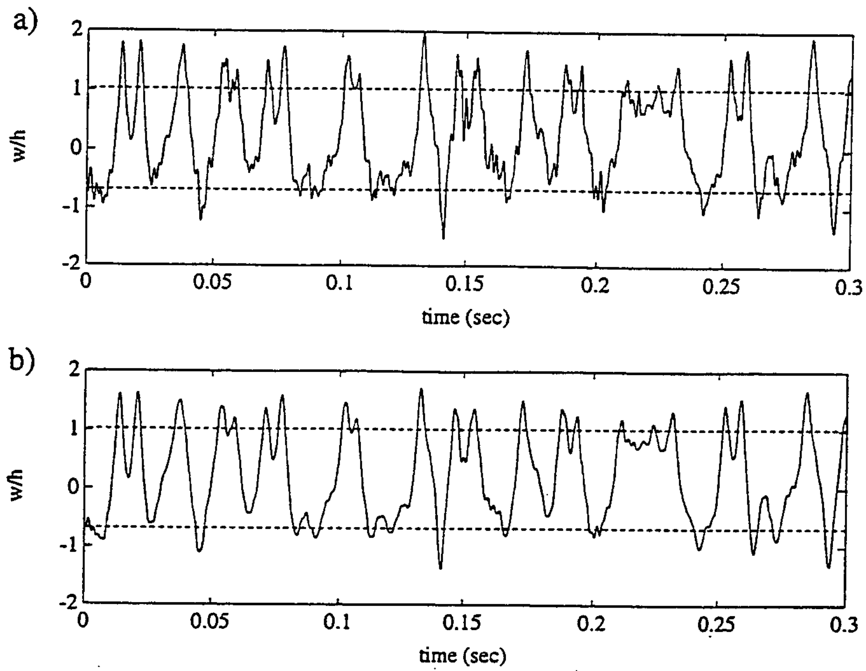

Figure 1: A chaotic snap-through vibration of an imperfect plate obtained by a) using Equation (1) with $n = 9$ b) projecting the original motion onto the two dominant KL modes. The dashed lines indicate the stable static equilibria.

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Wednesday, June 12

1330-1510

Session 13. Analysis III

NUMERICAL SOLUTIONS OF SECOND-ORDER IMPLICIT NONLINEAR ORDINARY DIFFERENTIAL EQUATIONS§

by

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The existing literature usually assumes that second-order ordinary differential equations can be put in first-order form, and this assumption is the starting point of most treatments of ordinary differential equations. This study examines numerical schemes for solving second-order *implicit* nonlinear differential equations, of the type

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F}(\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}, t),$$

where $\mathbf{F}$ is a nonlinear function and $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the $N \times N$ mass, damping and stiffness matrices associated with the linear part of the system, N being the number of degrees of freedom. It is assumed that the *nonlinear* inertial terms $\ddot{\mathbf{x}}$ in $\mathbf{F}$ cannot be removed or transformed.

Based on a literature review, three specific methods have been selected here, as follows. (a) Picard iteration using Chebyshev series as first described by Clenshaw and Norton (1963) and as recently used by Sinha *et al.* (1993) to predict periodic and chaotic responses of single and multi-degree of freedom nonlinear systems. (b) The Incremental Harmonic Balance (IHB) method, based on the multi-harmonic balancing procedure (Urabe 1965), as developed by Lau *et al.* (1981); it can be referred to as a combined Incremental Ritz-Galerkin Harmonic Balance method or as a Harmonic Balance Newton-Raphson method; in both Picard and IHB methods, the nonlinear differential equation is transformed into a set of algebraic ones that are solved iteratively. (c) A 4th-order (Houbolt 1950) and an 8th-order backward finite difference method (FDM). Each method is presented and applied to specific examples.

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It is shown that (i) the Picard method is not valid for solving implicit equations containing large nonlinear *inertial* terms, (ii) the IHB method yields accurate periodic solutions, together with the frequency of oscillation, and the dynamic stability of the system may be assessed very easily, and (iii) both Houbolt's and the 8th-order scheme can be used to compute time histories of initial value problems, if the time step is properly chosen. Finally, it is also illustrated how the combination of IHB and FDM can be a powerful tool for the analysis of nonlinear vibration problems defined by implicit differential equations (including also explicit ones), since bifurcation diagrams of stable and unstable periodic solutions can be computed easily with IHB, while periodic and non-periodic stable oscillations may be obtained with FDM.

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MODEL REDUCTION IN NONLINEAR STRUCTURAL DYNAMICS

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Introduction: The formulation of modern finite element models in linear structural dynamics nowadays may require tens of thousands of degrees of freedom in order to capture necessary system detail. The response of such structures would typically be analyzed using some type of *reduced* dynamic model, generally one of two types: 1) a *modal*-based model, in which a number of the lower mode shapes are used to obtain a reduced model in terms of a set of modal coordiantes, or 2) "*direct*" reduction, such as the Guyan reduction(1965), in which an approximate reduced model is obtained in terms of a subset (so called master degrees of freedom) of the original coordinates. Burton and Young (1994) have presented an exact (eigenstructure preserving) version of the Guyan reduction.

Model reduction for *nonlinear* vibratory systems may be approached in several ways, including the following: 1) use of the modal matrix of the undamped *linear* version of the system model, resulting in an *approximate* reduction in terms of a set of modal coordinates, 2) a direct model reduction of the linear version of the system model, followed by simply adding in the terms representing the nonlinearities, and 3) calculation of the exact nonlinear modal manifolds, associated with the so called nonlinear normal modes (NNM's), followed by projection onto the desired "nonlinear modal subspace." The latter process is preferable because it is exact: the effect of the nonlinearities on the individual and combined nonlinear modes is rigorously taken into account. Prescriptions for calculation of the nonlinear normal modes in discrete and continuous systems have been presented by Shaw and Pierre(1993, 1994), Burton and Young(1994), King and Vakakis(1993), and Nayfeh(1995). In these references the emphasis is on the calculation of individual nonlinear normal modes.

Objective: The objective of this work is to explore efficient, reasonably accurate ways to obtain reduced models of discrete *nonlinear* systems having the complexity and number of degrees of freedom that are typically encountered nowadays in *linear* analysis. The system model considered as a starting point for this work is taken in the form

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} + \{n(x,\dot{x})\} = \{F(t)\} \qquad (1)$$

in which $\{x\}$ is the n-vector of coordinates, $\{n(x,\dot{x})\}$ is an n-vector of nonlinear functions of generally position and velocity, and the other terms have their usual meaning. It is assumed that n is large.

In the context of the stated nonlinear model reduction problem, the following comments are relevant: 1) the exact NNM-based calculation, although desirable because it is an "exact" reduction, may be computationally prohibitive for large systems, even if symbolic manipulators are utilized; 2) the linear-modal-based type of reduction is desirable from a computational standpoint (essentially the same amount of work as doing a linear modal analysis; the problem is that one does not know how much information has been lost because of the approximations inherent in the method.

Result: We have obtained the following useful result for an undamped system with static nonlinearities: the linear-modal-based reduction yields an approximate reduced nonlinear model which, while differing from the exact, NNM-based reduced model, nevertheless yields the *same* leading order nonlinear frequency-amplitude dependence as the exact model. This result provides some justification for using the linear-modal-based reduction to do model reduction in large nonlinear vibratory systems.

Illustration: The details of the analysis will be presented in the full paper. Here we illustrate the result by example. We consider the two degree of freedom system analyzed by Shaw and Pierre (1993) and by Burton and Young (1994). The system consists of two unit masses, restrained by three unit linear springs, grounded on each end, with a cubic nonlinear spring connecting the left hand boundary to mass 1. The equations of motion are

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \begin{Bmatrix} \varepsilon x_1^3 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{2}$$

Exact result: The calculation of the first nonlinear modal manifold leads to the result (see Burton and Young(1994)), exact through cubic terms,

$$x_2 = x_1 + \frac{1}{3}\varepsilon x_1^3 + \frac{1}{2}\varepsilon x_1 \dot{x}_1^2$$

which in turn leads to the following reduced model in terms of the coordinate x_1, exact through cubic terms:

$$\ddot{x}_1 + x_1 + \varepsilon[\frac{2}{3}x_1^3 - \frac{1}{2}x_1\dot{x}_1^2] = 0 \tag{3}$$

Linear-modal reduction: The first linear mode shape is $[1 \quad 1]^T$, i.e., the masses move in phase with equal amplitudes in the first linear mode. Thus, define a modal coordinate y_1 by

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} y_1$$

substitute into (2), then premultiply by the row vector $[1 \quad 1]$ to obtain the linear-modal-based reduced model as

$$\ddot{y}_1 + y_1 + \frac{1}{2}\varepsilon y_1^3 = 0 \tag{4}$$

Note that the two reduced models (3) and (4) differ. If, however, one uses the method of harmonic balance with a one term approximation ($x_1 = a\cos(\omega t)$, $y_1 = a\cos(\omega t)$), then one obtains the following results for the undamped free vibration frequency-amplitude dependence:

NNM based (from (3)):

$$\omega^2 = \frac{1 + \frac{1}{2}\varepsilon a^2}{1 + \frac{1}{8}\varepsilon a^2} + O(\varepsilon^2 a^4) \tag{5}$$

Linear-modal-based (from (4)):

$$\omega^2 = 1 + \frac{3}{8}\varepsilon a^2 + O(\varepsilon^2 a^4) \tag{6}$$

Through terms of $O(\varepsilon a^2)$, these two relations are identical. In the full paper simulations and other examples involving more general versions of the nonlinear system will be presented.

Conclusion: The linear-modal-based reduction of an undamped nonlinear vibratory system with static nonlinearities yields a reduced model which, to leading order, has the same frequency-amplitude dependence as the exact, NNM-based reduced model. In view of the vast difference in computation required to do the two kinds of reduction, the result may be of practical value in justifying the simple linear-modal-based reduction as a practical way to do model reduction in nonlinear systems.

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Experimental Nonlinear Localization in a Vibro-Impact System

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Abstract

Experimental and numerical analysis is performed on a nonlinear flexible assembly with vibro-impacts. Nonlinear motion confinement phenomenon is observed. The assembly consists of two coupled cantilever beams whose motion is constrained at the ends by rigid barriers. In the theoretical model, the impact nonlinearities are simulated by clearance nonlinearities with steep stiffness characteristics. In construction of the model consideration is also given to the inelastic impacts which dissipate energy in the system. The theoretical results corroborate the experimental observations that the vibro-impact system possesses motion confinement properties. Transient and steady state motions of the system are investigated and it is shown that under certain conditions the vibrational energy of the system is passively confined to only one of the two beams. These essentially nonlinear motions exist even though there exists direct coupling between the two beams; this has no counterpart in linear theory. Experimental results confirm the theoretical predictions for transient and steady state responses.

Free Vibration Analysis of Rotating Nonlinearly Elastic Structures with Symmetry: an Efficient Group-Equivariance Approach

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In this work we consider the use of group-invariance techniques in nonlinear structural dynamics. The use of such ideas leads to powerful and efficient solution methods for both static and dynamic bifurcation problems. More specifically, we consider an important class of problems involving rotating structures which possess significant gyroscopic effects. We treat lumped-mass space trusses possessing n-fold rotational and reflectional symmetry (*i.e.*, D_n symmetry).

Our primary interest is in finding periodic modal solutions relative to a steadily rotating, symmetric equilibrium, given the (constant) global angular momentum of the structure. The steady rotational motion introduces gyroscopic terms into the equations of motion which break the reflection symmetries. Accordingly, the relevant symmetry group for such problems is C_n, the n-fold group of rotations. We treat both the linearized problem, for which the computational rewards of a group-theoretic approach (*viz.*, block diagonalization) are often tremendous, and the problem of small amplitude solutions of the nonlinear problem. For the latter, we shall show that it is the combined space-time symmetry group which plays an important role in the analysis.

Our first example is concerned with finding the vibrational modes of a rotating three mass truss element having an equilateral triangle as its undeformed shape. This example is provided primarily for illustrative purposes, since the problem is of low enough dimension to be handled without recourse to sophisticated procedures. After deriving the fully general governing equations for the truss element, we introduce the main concept of *group equivariance* (along with the relevant group-theoretic concepts) and demonstrate that the static force operator of the truss is equivariant under the group D_3. We then specialize the equations of motion to the case of motion relative to a rotating equilibrium and show via calculations that the gyroscopic terms present in the new EOM prevent equivariance under reflections (*i.e.*, the equivariance group is now C_3). With this knowledge and further results from group representation theory, we derive a transformation of the original coordinates of the problem into "symmetry modes", with the consequence that the 9×9 matrix equation of motion is *block-diagonalized* into two smaller problems of dimensions six and three. In the appendix, we give a proof that this block-diagonalization can be performed using the structural symmetry group D_3 instead of C_3. (The proof is in fact more general, showing that D_n symmetry information may always be used to help analyze a C_n equivariant problem.) The two block problems are then easily solved for the linear vibration modes and natural frequencies.

Having illustrated how group theoretic considerations can be used to help draw conclusions about the dynamics of a low degree of freedom problem, we next examine the application of these ideas to higher degree of freedom problems. Since these problems are seldom amenable to direct

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analysis, the emphasis is placed upon finding efficient algorithms for solving the equations of motion numerically. Block diagonalization via a symmetry mode transformation allows for a significant amount of decoupling in the linearized dynamics problem, leading to a substantial drop in computation time. Taking as an example the problem of finding the vibrational modes for a rotating two layer hexagonal array (*i.e.*, the space antenna), we follow the same formulation and analysis procedure as was used for the truss element. We consider the case of slow rotation and derive an appropriate set of linearized governing equations for this situation. The resulting 57×57 matrix problem is solved numerically by computing the required matrices, block-diagonalizing them according to the symmetry information (the relevant group is D_6), setting up each of the four individual block problems (having sizes 9×9, 10×10, 18×18, and 20×20), and solving them for the natural frequencies and normal modes. A comparison of this technique with traditional computational methods shows that use of symmetry methods reduces the computation time by 52 percent.

The solutions to the linearized equations discussed previously are often sufficient for engineering applications. Nevertheless, it is natural to wonder what relationship these solutions have with the nonlinear problem. There has been much interest in this question, going back at least to Liapunov, who answered it for non-resonant systems. The presence of symmetry in a problem, however, causes the non-resonance condition to be violated, since symmetry forces multiplicity of frequencies. However, if the degeneracy is due solely to symmetry, then techniques of symmetry-breaking bifurcation theory can be employed advantageously in such problems. The key idea here is to exploit an expanded spatio-temporal symmetry group of the differential equations. We employ this approach in this work and demonstrate its use in finding solutions for the nonlinear vibration problems of the truss element and the space antenna. By rescaling time t according to $t = \tau/\nu$, the differential equations for these two problems are recast as "bifurcation" equations with bifurcation parameter ν. Taking the steadily rotating state as the trivial solution, we show that the relative motion equations are equivariant under the group $C_n \times \mathcal{O}(2)$, which denotes a combined C_n spatial symmetry group ($n = 3$ or 6 depending on the problem) and an $\mathcal{O}(2)$ temporal symmetry group. Moreover, the linearized bifurcation problem has nontrivial solutions whenever ν takes on the value of a natural frequency. Although a multiplicity of solutions exist as each "bifurcation point", we demonstrate that the class of solutions possessing a combined spatio-temporal reflection symmetry can be associated with one-dimensional bifurcation problems. We analyze this case and determine that all solutions obeying a simple transversality condition correspond to a pitchfork bifurcation of nontrivial solution branches having the normal modes as their first order approximations. (For the space antenna, the transversality condition is computed numerically.)

In summary, we show in this work that use of group-theoretic ideas can greatly simplify the procedure for finding both linear vibration modes and frequencies and locating solution branches for nonlinear vibration problems. These methods can be employed in both analytic and numerical settings, and they lead to dramatic increases in solving efficiency. For the class of problems studied here (in which gyroscopic effects are present), we demonstrate that the original structural symmetry group can be employed to analyze the dynamics of the system despite the loss of reflection symmetry. This allows for a simpler and more intuitive implementation of the group representation techniques. In addition, we explain how ideas from static bifurcation theory can be encorporated with symmetry analysis to prove the existence of solution branches for nonlinear vibration problems. These examples provide great insight to the many advantages of using group-theoretic methods in structural vibrations problems.

DYNAMIC RESPONSE TO SINGULARITIES IN THE POTENTIAL FOR MECHANICAL SYSTEMS

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Singularities in the potential function for a conservative system affect its dynamic response. The extent to which the potential controls the dynamic response is investigated for certain families of one degree of freedom simple mechanical systems. For the vectorfields considered, the Jacobian at the singularity is nilpotent and so has codimension two. However, this does not distinguish different types of singularities. A normal form is obtained in terms of the universal unfolding of the potential function. A more difficult question is the extent to which the potential function organizes the response of such a mechanical system with damping. In this larger family, lines of Hopf bifurcation points with respect to damping are generated if the potential has a singularity.

The one degree of freedom undamped mechanical system considered has Hamiltonian which splits into a quadratic function of $\dot{x}$ and the static potential, $V(x;P)$, which depends on the control parameter, P,

$$H(x,\dot{x};P) = \alpha\dot{x}^2 + V(x;P). \qquad (1)$$

This family might be called a Hamiltonian potential system. It is known that, in these systems, the potential determines the fixed points and their stability. The fixed points are in a one-to-one correspondence with the static equilibria. The statically stable equilibria correspond to centers, and the statically unstable equilibria correspond to saddle points.

The normal form of the associated vectorfield in a neighborhood of a singular fixed point is obtained from the universal unfolding of the potential, $V(x;P)$. Denote x by x_1 and $\dot{x}$ by x_2. Also put $V_1 = \partial V/\partial x$ and $V_{11} = \partial^2 V/\partial x^2$.

Theorem 0.1 *A family of Hamiltonian vectorfields*

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= -\frac{V_1(x_1;P)}{M}, \end{aligned} \qquad (2)$$

where M is a constant, with a singular fixed point at $x=0$ and some value of P, has normal form in a neighborhood of the singularity

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= -\frac{1}{M}\frac{dV^*}{dx_1}, \end{aligned} \qquad (3)$$

where $V^*(x_1) = a_1x_1 + \ldots + a_{n-2}x_1^{n-2} + a_nx_1^n$ is a universal unfolding for the potential $V(x_1)$.

The behavior of this family of dynamic systems and its perturbations near its degenerate singularity is completely determined by the universal unfolding of the potential function.

The codimension is defined to be that of the universal unfolding, which equals the codimension of the family of vectorfields in the space of all vectorfields. This contrasts with the codimension of the single nilpotent vectorfield at the singularity defined as the codimension of its Jacobian in the space of 2×2 matrices.

Damping is introduced into the family by the linear damping coefficient, c.

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= -\frac{V_1}{M} - \frac{c}{M}x_2. \end{aligned} \qquad (4)$$

Let $\bar{x}$ be a minimum for $V(x)$. The eigenvalues at the associated fixed point in the dynamical system are purely imaginary if $c=0$. The derivative of the real part of the eigenvalue, $\lambda = \left(-c \pm \sqrt{c^2 - 4V_{11}M}\right)/2M$, at $c=0$ is non-zero. The fixed point is therefore a Hopf bifurcation with respect to the damping coefficient. The Bendixson criterion in the plane implies that there can be no closed orbits unless $c=0$.

Theorem 0.2 *Let $\bar{x}$ be a minimum for $V(x)$ for some choice of system parameters. The Hopf bifurcation at the fixed point, $x=\bar{x}, \dot{x}=0$, is degenerate.*

To illustrate this behavior, a vertical rigid rod of mass, m, and of length, L, is loaded at the top end by a vertical dead weight, P, and is supported at the bottom end by a frictionless pin. The additional support provided by various spring configurations determines the singularity order of the potential. The system is called perfect if the rod is initially vertical as the load P is increased from zero. When the load is larger than some load, P_c, the rod will deflect to one side, the analog to column buckling.

One configuration is a torsional spring at the pin end. The angle, θ, between the vertical and the rod is the only degree of freedom. The torsional spring responds as $k_1\theta + k_3\theta^3$ where k_1 and k_3 are constants, and so the potential energy for the perfect conservative system is

$$V(\theta) = \frac{1}{2}k_1\theta^2 + \frac{1}{4}k_3\theta^4 - \bar{P}L[1 - \cos\theta], \qquad (5)$$

where $\bar{P} = P + mg/2$, the control parameter. The Taylor series expansion of $V(\theta)$ to sixth order is

$$V(\theta) = \frac{1}{2}(k_1 - L\bar{P})\theta^2 + \frac{1}{24}(6k_3 + L\bar{P})\theta^4 - \frac{1}{720}L\bar{P}\theta^6. \quad (6)$$

The Hamiltonian is of the form of Eqn. (1). If $k_3 = 0$, so that the spring is linear, the potential is a cusp and so has codimension two. The pitchfork in $\theta - \bar{P}$ space has stable outer prongs.

Allowing the torsional spring to be non-linear increases the order of the singularity. If the spring softens and if $k_3 = -k_1/6$, then, at the critical load, the coefficients of θ^2 and θ^4 simultaneously vanish. The first non-zero term in the Taylor series expansion about the degenerate critical point for the potential is of sixth order and has a positive coefficient. The universal unfolding, the butterfly catastrophe, requires θ and θ^3 terms in addition to the existing θ^2 and θ^4 terms; the codimension of the function θ^6 is four. The nonlinearity in the spring inverts the pitchfork so that the non-vertical static equilibria positions are unstable and the corresponding fixed points are saddles.

The rod is, alternatively, supported at its top on either side by two horizontal springs so that the rod is in the vertical position when the springs are unstretched. The spring constants are each k. The potential energy is

$$V(\theta) = 2\left(\frac{1}{2}kL^2 \sin\theta\right) - PL(1 - \cos\theta), \qquad (7)$$

where θ is the angle between the rod and the vertical. This is a dual cusp; the equilibria form an inverted pitchfork with bifurcation at $P_c = 2kL$. Those equilibria defined by $P = 2kL\cos\theta$ for $P < 2kL$ and $\theta \neq 0$ are unstable. The fixed points for the dual cusp and the above subfamily of the butterfly are similar. But the butterfly accepts additional imperfection parameters.

An asymmetric example arises when the rod is supported by a single spring attached to its top. The other end of the spring is attached to the ground with which it makes a forty-five degree angle when unstretched. The potential is

$$V(x) = kL^2\left[(1+x)^{1/2} - 1\right]^2 - \bar{P}L[1 - (1-x^2)^{1/2}], \quad (8)$$

where xL is the horizontal distance through which the rod tip moves. The expansion of $V(x)$ to the third order is

$$V(x) = \left(\frac{1}{4}kL^2 - \frac{1}{2}\bar{P}L\right)x^2 - \frac{1}{8}kL^2x^3. \qquad (9)$$

This is a fold catastrophe with singularity at $P_c = kL/2$. Again, the Hamiltonian is of the form of Eqn. (1). Nonlinear damping, $(c + x)\dot{x}$, produces a Bogdanov-Takens bifurcation [1].

The example of a ball moving in a rotating hoop, given by Marsden [2, p. 192ff], is topologically equivalent to the linear torsional spring supported rod, but its behavior is not completely determined by the potential. Let ω be the constant angular velocity of the hoop, R be the hoop radius, m be the mass of the ball, ν be the friction, and the position of the ball be given by θ. The potential function, $V(\theta) = -mgR\cos\theta$, has no singularities. The Hamiltonian is

$$H = \frac{m}{2}\left[R^2\dot{\theta}^2 + (\omega R\sin\theta)^2\right] - mgR\cos\theta. \qquad (10)$$

Taking the Taylor series of the Hamiltonian up to order four and including the damping, ν/m, produces a system equivalent to that of the rod, in which ω^2 corresponds to $\bar{P}$.

$$\ddot{\theta} = \left(\frac{g}{6R} - \frac{2}{3}\omega^2\right)\theta^3 + \left(\omega^2 - \frac{g}{R}\right)\theta - \frac{\nu}{m}\dot{\theta}.$$

In this system, the kinetic energy term, $m(\omega R\sin\theta)^2/2$, induces the singularity.

At equilibrium, $R\omega^2\cos\theta\sin\theta = g\sin\theta$. There are always fixed points at $\theta = 0$ and $\theta = \pi$. Viewing ω as a parameter, there is a bifurcation if $\omega = \sqrt{g/R}$, where the coefficient of θ is zero, so that when $\omega > \sqrt{g/R}$, there are two additional fixed points, a Hamiltonian pitchfork bifurcation. The universal unfolding has unfolding parameters, ω^2 and an initial imperfection in the angle.

This method also applies to similar two degrees of freedom systems. The behavior of families of systems whose potential has an umbilic or double cusp singularity is characterized by the universal unfolding of the potential, and lines of degenerate Hopf bifurcations exist when linear damping is included.

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Wednesday, June 12

1530-1710

Session 14. Rotors

On the Modeling and Response Analysis of Helicopter Rotor Blades

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Because of the complexity of the governing differential equations of motion for helicopter rotor blades, it has been common practice in the rotorcraft dynamics literature to first expand the full nonlinear differential equations, to a pre-determined degree in a small parameter, about the undeformed state of the system. The resulting differential equations, which contain polynomial nonlinearities truncated to the desired order, are then used to analyze the aeroelastic response of the blade. Since the undeformed state is not an equilibrium solution to the full nonlinear differential equations of motion, one must be careful when using such expanded equations in order to obtain results that are both consistent with the approximating assumptions and are not physically incorrect. Due to a number of incorrect approximations and expansions, mathematically inconsistent equations and solutions were obtained in [1] for example. Such inconsistencies were identified in [2]. The work in [2] also showed that the inclusion of higher order terms in the expanded differential equations are needed in order to circumvent such inconsistencies.

An analysis of the response of a rotor blade in hover was presented in [3] by making use of Galerkin's method with a set of eigenfunctions for a non-rotating beam. In such methodology, the number of Galerkin coefficients depends on the number, N, of eigenfunctions used and on the order of truncation of the expanded differential equations of motion. As shown in [3], such number is greatly increased when one increases the truncation order of the expanded equations from quadratic to cubic. Also, as shown in [4] for a cantilever with a tip mass, even an analysis based on a higher approximation to such equations may yield, in some cases, very innacurate results.

In the present work, the problems involved in formulating the differential equations that govern the flap-lag-torsional-extensional aeroelastic response of a helicopter rotor blade, and in the analysis of the response for the case of hover, are presented and discussed. First, the *full non-linear* partial differential equations of motion for the blade are formulated based on the work presented in [5]. The equilibrium solution exhibited by the full nonlinear differential equations of motion are then determined by numerically solving a nonlinear two-point boundary value problem. The equilibrim solution is then perturbed and the stability of infinitesimally small motions about the equilibrium is analyzed in detail. In contrast with other work presented in the literature dealing with the dynamics of helicopter rotor blades, the analysis presented here does not rely on the use of ordering schemes and on an approximate equilibrium solution determined by expanding the full non-linear differential equations about the undeformed state of the blade (which is not an equilibrium solution to the system) and, then, expanding the expanded equations again about the approximate equilibrium to investigate stability.

With primes denote differentiation with respect to the spatial variable x (which is distance along the undeformed blade's reference axis, normalized by the blade's undeformed length R) and considering, for simplicity, a homogeneous blade of distributed mass m Kg/meter, the normalized differential equations of motion are

$$\begin{aligned} G_v' &\triangleq \left[\frac{A_{\theta_z}}{(1+e_0)\cos\theta_y\cos\theta_z} + (\tan\theta_z)\int_1^x f_u\,dx\right]' = \ddot{v} + 2\dot{u}\cos\beta - 2\dot{w}\sin\beta - v - Q_v \\ G_w' &\triangleq \left[G_v(\sin\theta_z)\tan\theta_y + \frac{A_{\theta_y}}{(1+e_0)\cos\theta_y} + (\tan\theta_y\cos\theta_z)\int_1^x f_u\,dx\right]' \\ &= \ddot{w} + 2\dot{v}\sin\beta - w\sin^2\beta + (x+u)(sin\beta)\cos\beta - Q_w \\ A_{\theta_x} &= Q_{\theta_x} \end{aligned} \tag{1}$$

where $f_u = \ddot{u} - 2\dot{v}\cos\beta + w(\sin 2\beta)/2 - (x+u)\cos^2\beta - Q_u$, while Q_u, Q_v, Q_w and Q_{θ_x} are the generalized aerodynamic forces whose virtual work is $Q_u\delta u + Q_v\delta v + Q_w\delta_w + Q_{\theta_x}\delta\theta_x$. The expressions for the generalized aerodynamc forces are given in [3]. The expressions for $A_{\theta_x}, A_{\theta_y}$ and A_{θ_z} are given in [5]. In Eqs. (1), dots denote partial differentiation with respect to time, normalized by the rotational angular speed Ω of the rotor shaft. The equilibrium solution and the stability of the motion governed by the above differential equations, which are of the form $\underline{y}' = \underline{f}\left(\underline{y}, \underline{\dot{y}}, \underline{\ddot{y}}, \underline{\dot{y}}', \underline{\ddot{y}}'\right)$, are determined in a rigorous manner. Conditions for nonlinear resonances, which are prone to occur for certain ranges of values of the system parameters, are also discussed.

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Stability and Control of a Parametrically Excited Flexible Rotating Beam

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The dynamic of flexible bodies attached to moving supports has been studied in the past [1,2] in connection with a number of diverse disciplines, such as machine design, robotics, aircraft dynamics and spacecraft dynamics. In particular, beams attached to moving bases have received attention in many technical papers. In this paper the stability and control problems of an elastic rotating beam attached to an oscillatory base, as shown in Fig.1, is considered. The base (of length L_b, moment of inertia J_b and mass m_b) has a known sinusoidal movement in the horizontal plane, of amplitude L_0 and angular velocity ω. The base can perform small rotation around its vertical axis of symmetry, and this angle is denoted by θ. An elastic beam (of length L, flexural rigidity EI and uniform mass distribution ρ) is cantilevered in the moving base and the whole system can rotate in the same horizontal plane (no gravity force) with a constant rotational frequency Ω. The flap motion of the elastic beam (the deflection y is perpendicular on the x_0Oy_0 plane) is analyzed by considering the first three modes of the beam.

Applying Lagrange method, the nonlinear equations of motion are obtained. If these equations are linearized around the equilibrium point then, one can obtain the following matrix differential equation:

$$\boldsymbol{M}\, d^2\boldsymbol{x}(t)/dt^2 + \boldsymbol{C}\, d\boldsymbol{x}(t)/dt + (\boldsymbol{K}_0(\gamma) + \boldsymbol{K}_1(\gamma, \omega, \varepsilon, t))\, \boldsymbol{x}(t) = \boldsymbol{B}\, \boldsymbol{u}(t) \qquad (1)$$

where ε is the normalized amplitude of the base excitation motion and γ is the normalized rotational frequency. In the Eq. (1) $\boldsymbol{M}$ is the mass matrix, C is the damping matrix, $\boldsymbol{K}_0(\gamma)$ is the time invariant term and $\boldsymbol{K}_1(\gamma, \omega, \varepsilon, t)$ is the periodic term of the stiffness matrix. $\boldsymbol{u}(t)$ is the control vector applied to the system.

As it can be seen, Eq. (1) contains periodic coefficients and its stability can be analyzed with Floquet theory. Thus, the system is stable if the eigenvalues of the fundamental matrix evaluated at the end of the principal period are located inside the unit circle. Stability charts were drawn in the (ω–ε) plane for several possible cases of damping-rotation combinations. Such a stability chart, for an undamped and nonrotating system is shown in Fig. 2. If the natural frequencies of such a system are denoted by λ_1, λ_2, λ_3 and λ_4, then various combination resonance were found, such as $2\lambda_1$, $2\lambda_2$, $2\lambda_3$, $\lambda_1+\lambda_2$, $\lambda_2+\lambda_3$, $\lambda_1+\lambda_3$, $\lambda_1+\lambda_4$, $(\lambda_1+\lambda_2)/2$, $(\lambda_2+\lambda_3)/2$, $(\lambda_1+\lambda_3)/2$, etc. Next, the Lyapunov-Floquet transformation (see [3,4]) is used to design a controller and an observer for the system. Simulations are performed for different values of the excitation frequency ω.

It is shown that the controllers and observers can be designed by time invariant methods. Because the control and observer gains can be computed off-line and then stored in the computer memory, this method is suitable for real time implementation.

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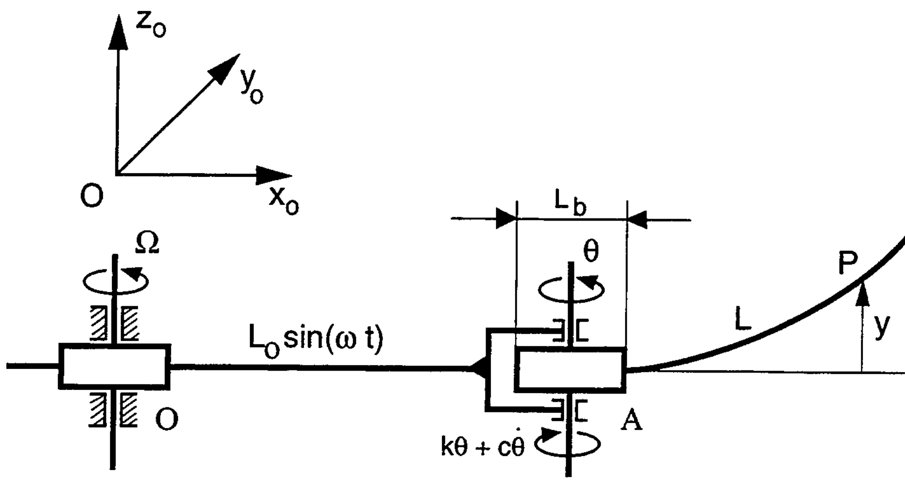

Fig. 1. The system model

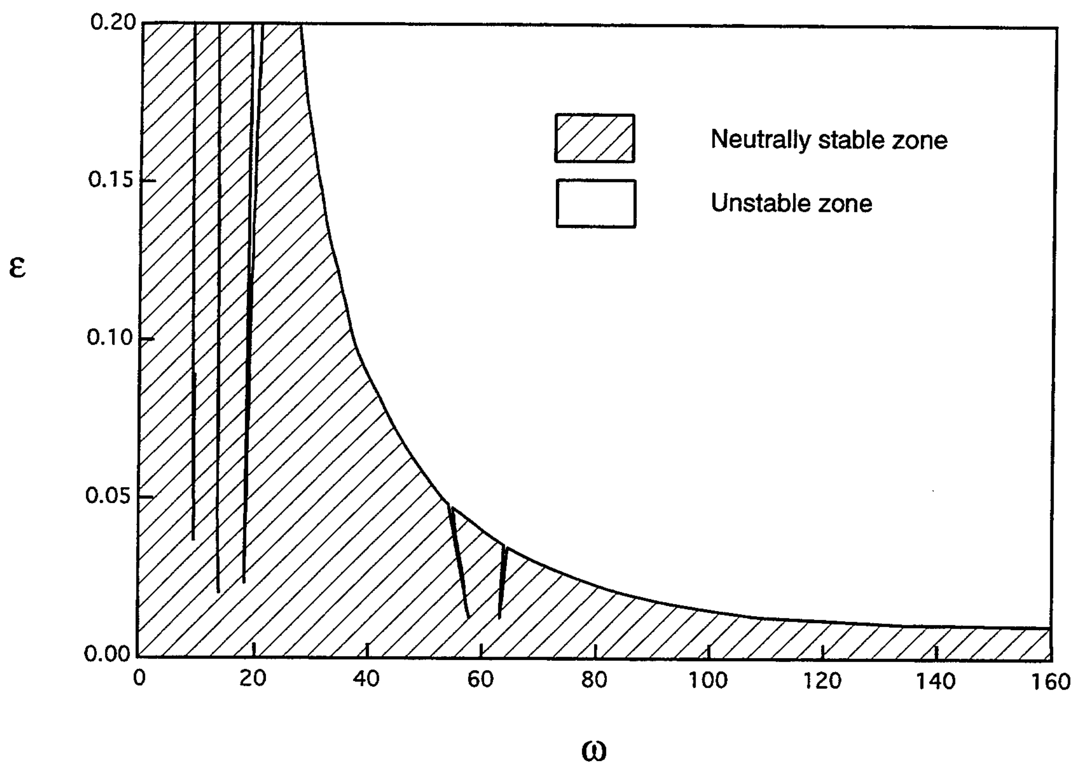

Fig. 2. Undamped and nonrotating system

An Experimental Identification Technique for a Nonlinear Rotating Shaft System

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Introduction

The identification techniques for a dynamic system through the use of experimental data of the response of the system are of interest to many investigators because of the increased importance in applied mechanics.

In this paper, as a basis for developing experimental identification techniques for rotating machines, we propose a technique for a simple rotating shaft system composed of an elastic shaft with a disk. Rotating machines often reveal nonlinearity, so we consider the case in which the rotating shaft system is nonlinear. In rotating machines, their unbalances produce a whirling motion. In proposing an identification technique, we attempt to use this motion.

Formulation of the problem

To propose a technique, we express the equations of motion of a system in complex form, which is convenient for expressing characteristics of the system. The result is

$$\begin{aligned} &m\ddot{z} + c_{zz}\dot{z} + c_{z\theta}\dot{\theta} - F_z\left(z,\bar{z},\theta,\bar{\theta}\right) = m\varepsilon_z\omega^2 e^{j\omega t} \\ &I\ddot{\theta} - j\omega I_p\dot{\theta} + c_{z\theta}\dot{z} + c_{\theta\theta}\dot{\theta} - F_\theta\left(z,\bar{z},\theta,\bar{\theta}\right) = \left(I - I_p\right)\tau_\theta\omega^2 e^{j\omega t} \end{aligned} \tag{1}$$

where z and θ are the complex displacement and inclination defined by

$$z = x + jy, \theta = \theta_x + j\theta_y \tag{2}$$

where F_z, F_θ are the complex restoring force and restoring moment defined by

$$F_z = F_x + jF_y, F_\theta = F_{\theta x} + jF_{\theta y} \tag{3}$$

and where ε_z and τ_θ are the complex static and dynamic unbalances defined by

$$\varepsilon_z = \varepsilon_x + j\varepsilon_y, \tau_\theta = \tau_x + j\tau_y \tag{4}$$

We assume that F_z, F_θ are nonlinear and are derivable from a potential of the form

$$V = \frac{1}{4}k_{zz}z^2 + \frac{1}{2}k_{z\bar{z}}z\bar{z} + \frac{1}{2}k_{z\theta}z\theta + \cdots + \frac{1}{6}\alpha_{zzz}z^3 + \frac{1}{2}\alpha_{zz\bar{z}}z^2\bar{z} + \frac{1}{2}\alpha_{zz\theta}z^2\theta + \cdots \tag{5}$$

by the formula

$$F_z = -2\frac{\partial V}{\partial \bar{z}}, F_\theta = -2\frac{\partial V}{\partial \bar{\theta}} \tag{6}$$

Then the problem of identification is reduced to determination of the unknown parameters $m, I, I_p, c_{zz}, c_{z\theta}, c_{\theta\theta}, \varepsilon_x, \varepsilon_y, \tau_x, \tau_y$ and $k_{zz}, k_{zzz}, \alpha_{zzz}, \alpha_{zzzz}, \alpha_{zz\theta} \cdots$.

Proposition of an identification technique

Now we propose a technique for determining the unknown parameters. In many cases, some of the parameters, for example the mass m and the moments of inertia I, I_p, can be obtained accurately and easily. First we propose a technique for the case in which some of the parameters are known. We measure for a certain rotating speed ω the whirling motion of the system for one period, and obtain data of time history of z and θ. Then we develop the obtained z and θ into complex Fourier series of the form

$$\begin{aligned} z &= \cdots + Z_{-2}e^{-2j\omega t} + Z_{-1}e^{-j\omega t} + Z_0 + Z_1 e^{j\omega t} + Z_2 e^{2j\omega t} + \cdots \\ \theta &= \cdots + \Theta_{-2}e^{-2j\omega t} + \Theta_{-1}e^{-j\omega t} + \Theta_0 + \Theta_1 e^{j\omega t} + \Theta_2 e^{2j\omega t} + \cdots \end{aligned} \tag{7}$$

Using the data of z and θ, we also develop into complex Fourier series all the terms $z^2, z\theta, \theta^2, \cdots$, that appear in E.(1) with F_z, F_θ given by Eq.(6). We substitute these expressions into Eq.(1). We apply to the resulting equations the principle of harmonic balance, i.e. we equate the coefficients of the whirling components of the same order in both sides of the equations. Then we have simultaneous equations with respect to the unknown parameters. We repeat the same procedures for various values of ω, and bring together all the equations to obtain the equations of the form

$$[A]\{S\} = \{Q\} \tag{8}$$

where $\{S\}$ is a vector, whose components are unknown parameters. Finally, we apply to the resulting equations the least square method to yield

$$\{S\} = \left([A]^T[A]\right)^{-1}[A]^T\{Q\} \tag{9}$$

This completes identification.

Next we consider the case in which no parameters are known beforehand. In this case, we measure the whirling motion, first without, and then with attaching known appropriate unbalance masses to the system. Combining these data, and following the same procedures as above, we can determine all the parameters of the system.

Numerical simulation and experiments

To check the applicability of the proposed technique, we conducted numerical simulation for a typical case using the data generated numerically from the equations of motion. We also applied the technique to identify an experimental setup. The results of the numerical simulation as well as the experiments confirm the applicability of the technique.

USE OF A FLEXIBLE INTERNAL SUPPORT TO SUPPRESS VIBRATIONS OF A ROTATING SHAFT PASSING THROUGH A CRITICAL SPEED

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As is widely known, large vibrations may occur when a rotating shaft passes through one of its critical speeds. In order to suppress these large motions, it may be possible to modify the system as it is accelerating or decelerating in such a way as to avoid this phenomenon. Here it is assumed that a flexible internal support can be activated or deactivated during run-up or coast-down of the shaft. Even though transient motions are caused by these actions, the resulting maximum displacement may be significantly less than that for the system passing through a critical speed.

This study is related to some earlier investigations. Nagaya et al. (1987) considered a flexible vertical shaft with an unbalanced disk at its end. The shaft rotated with a fixed angular velocity, and there were two internal supports whose stiffnesses could be switched from one value to another. A procedure for avoidance of critical speeds during run-up or run-down was discussed, and some experiments were described. Viderman and Porat (1987) investigated a similar problem, with a single spring or dashpot that could be activated for a while as the system began to increase its angular velocity, and then deactivated after the critical speed of the original system had been passed. Segalman et al. (1993) treated a one-degree-of-freedom system consisting of a rigid disk pinned to a rotating rigid shaft with a torsional spring whose stiffness could be changed. The shaft speed was increased linearly and the spring stiffness was changed from one value to a lower value to avoid passage of the critical speed.

Turkstra and Semercigil (1993a) considered a simply-supported beam with an attached unbalanced rigid rotor. The rotor was accelerated at a constant rate past the first two natural frequencies of the beam. A flexible support was active initially, then deactivated, and then activated again, to suppress the vibrations. Transient responses were determined numerically and experimentally. In Turkstra and Semercigil (1993b), a similar beam-rotor system was considered and the possibility of avoiding resonances by applying axial tension to the beam was discussed. Dimentberg et al. (1994) investigated the one-dimensional motion of a rigid base flexibly attached to the ground and excited by an unbalanced rigid rotor. The support stiffness was switched from a high value to a low value as the rotational speed of the rotor increased. The rotor speed was not specified; instead, the motor was assumed to be "nonideal", with a given motor-torque characterestic, and the rotor speed was governed by a differential equation.

A nonideal system also was investigated by Wauer and Suherman (1996). They considered a flexible, simply-supported, vertical shaft, with viscoelastic and external damping. It was assumed that the bending stiffness of the shaft could be switched from one value to another. As the rotational speed of the shaft increased, the bending stiffness was reduced at a certain time to avoid passage of a critical speed. The transient motion induced by this change of stiffness could be large, so the decrease in maximum response may not always be significant.

In the present study, a flexible, simply-supported, horizontal shaft with a central disk is examined. Eccentricity of the disk, gravitational forces, and internal and external damping are included. The equations of motion and boundary conditions are derived by Hamilton's Principle, and the Extended Galerkin Method is applied to eliminate the spatial dependence. Both run-up and coast-down past the lowest critical speed are treated, and results are obtained for both ideal and nonideal motors.

In order to suppress the vibrations, it is assumed that a flexible, axisymmetric, internal support can be activated at a certain position along the shaft to change the stiffness of the system. The nonstationary motions are computed by numerical integration of the equations of motion. Various properties of the system are investigated, such as the times of activation and deactivation of the support, the location of the support, the motor characteristic, and the damping magnitudes. It is shown that the maximum displacement of the shaft during run-up or coast-down past a critical speed can be reduced significantly with the use of the additional flexible support.

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A Study of Housing Effects on the Rotordynamics of a Shaft Supported by a Clearance Bearing: Analysis and Experiment

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One of the most innovative recent developments in the field of rotordynamics is the active magnetic bearing (AMB). It provides a noncontacting means of rotor support, eliminating bearing friction and offering the potential to actively suppress rotor vibrations. A particular area of concern with magnetic bearings is the auxiliary bearing which protects the soft iron core of the magnetic bearing and provides rotor support during a critical operating condition. Typically, the auxiliary bearings have small clearances so that the rotor does not contact the magnetic bearings if power loss or bearing failure occurs. Due to these small clearances, contact between the rotor system and the auxiliary bearings can occur during standard operation of the magnetic bearings. When this happens, load sharing between the magnetic bearings and the auxiliary bearing results, and the rotor system interacts with its auxiliary bearing. The dynamics associated with this load sharing must be understood in order to properly design the auxiliary bearing system for protection of the magnetic bearings.

There has been quite a bit of recent work that is specifically concerned with auxiliary bearings in magnetic bearing supported rotor systems. In one of the earliest studies specifically concerned with auxiliary bearings, Gelin et al. (1990) studied the transient dynamic behavior of rotors on auxiliary bearings during coast down. Ishii and Kirk (1991) and Kirk and Ishii (1993) investigated the transient responses of a flexible rotor during rotor drop after the magnetic bearings become inactive. Their work figures prominently in the literature on the subject. Schweitzer, et al. (1994) presents a good overall discussion and review of issues related to the touch-down dynamics of rotors on auxiliary bearings.

While these studies have greatly enhanced the understanding of the dynamics of rotors supported by auxiliary clearance bearings, most have been performed from the perspective that the rotor will be shut down if failure of one or more of the magnetic bearings occurs. As a result, work in this area has concentrated on transient dynamical behavior for a rotor with a decreasing spin speed. From a practical perspective, such an assumption is appropriate for noncritical applications, such as power generators and compressors. However for critical applications, such as jet engines, safety is a major concern and continued operation of the rotor (after AMB failure or overload) may be required. In light of this previous work, the present research consists of parallel experimental/simulation studies aimed at investigating the coupled dynamics of a magnetic bearing supported rotor system interacting with its auxiliary clearance bearing.

A schematic diagram of the experimental test rig is shown in Figure 1. It has two basic components: a flexible shaft and an auxiliary clearance bearing rig. The shaft is made of steel and is 0.374 inches in diameter and 18.0 inches in length. It is supported at 1.0 inch from the right end by ball bearings suspended in a frame by four springs and at 1.0 inch from the left end by a bushing with a tight clearance. These supports represent the magnetic bearings. The stiffness of the left support is 101.5 lb/in for both the horizontal and vertical directions. It is used to somewhat isolate the rotor from the effects of the flexible coupling which attaches the rotor to the motor and to enforce low amplitude vibration at this location to protect the motor. The stiffness of the right support, for both horizontal and vertical directions, is 14.5 lb/in. This lower stiffness allows for significant vibration of the rotor in the speed range of the motor (0-10,000 rpm). A rigid

[1] Work performed while at Auburn University

disk with holes for placing imbalance screws is placed at the midpoint of the bearing span. The imbalance screw mass is adjustable for studies of imbalance effects.

Figure 2 provides a schematic of the bearing rig and details of its components. It consists of a bushing suspended in a frame by four springs and is situated at the right end of the rotor. The auxiliary bearing housing is modeled using an aluminum disk. Housing stiffness is modeled using linear springs and is variable by interchanging these springs. Housing mass can be added by attaching extra mass to the aluminum disk with brass weights.

Using this experimental facility and simulation models, a study of the dynamical behavior of a flexible rotor supported by linear bearings (representing a set of magnetic bearings) and an auxiliary bearing with clearance has been performed. Studies have been conducted for a variety of parametric configurations. Specifically, the influence of housing stiffness and mass and disk imbalance were examined. The polar receptances and associated phases of the combined rotor-bearing-housing system were also used to predict possible interaction zones of coupled dynamics [based upon the work of Black (1968)]. The simulation model shows good agreement with experimental results and an interaction model is shown to be a good predictor of rotor/stator interaction as verified by plots of the bearing and housing responses. In general, the dynamic behavior of a flexible rotor interacting with an auxiliary bearing depends very strongly on the structural parameters of the auxiliary bearing and associated housing. Some guidelines were developed for the selection of bearing/housing parameters to achieve acceptable rotordynamical behavior.

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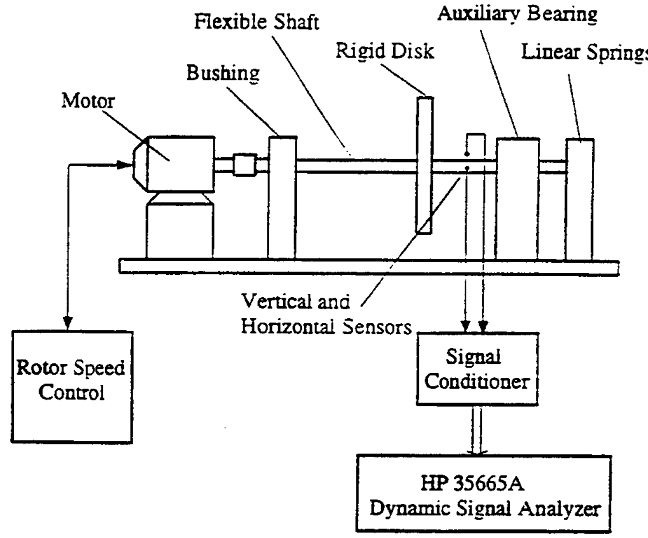

Figure 1: Schematic Diagram of Experimental Test Facility

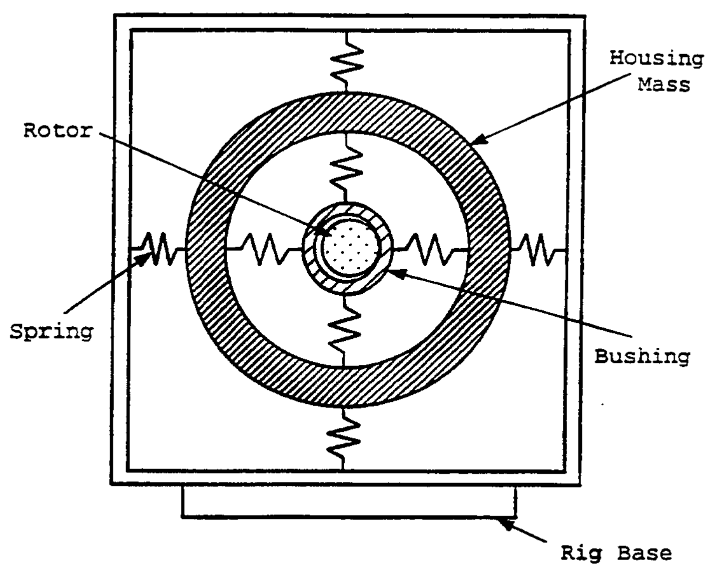

Figure 2: Auxiliary Bearing Test Facility

Thursday, June 13

0830-1010

Session 15. Multibody Dynamics II

Impacts with Friction and Normal and Tangential Reversibility in Multibody Systems

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1 Introduction

Impacts with friction can occur in all technical systems with any type of clearance or backlash. To enable an efficient numerical solution of the equations of motion of such systems, it is important to use an impact model that bases on rigid bodies and calculates the state of the whole system after the impact depending from the state before. If one of the bodies that performes an impact is of a rubber-like material, not only friction but also storing and restoring energy in tangential direction has to be taken into account.

In the following article there will be given the impact equations of a multibody system, including tangential effects. The impact classification known for single impacts will be generalized for multiple impacts. The problem of choosing coefficients of restitution for the tangential direction is shown and the solution for a simple example will be compared with experiments.

2 Formulation of the equations

An impact occurs if the relative distance between two bodies of the system vanishes and the normal relative velocity is less than zero. Because all other closed unilateral contacts of the system can be influenced by this event, the impact equations have to contain the whole system dynamics and all other closed contacts.

For this multiple impact we make the following assumptions:

- The duration of the impact is very short.
- The positions of the body remain constant during the impact.
- The compression phase is finished at all contact points at the same time.
- No wave effects are taken into account.

For all contacts participating at the impact we have to formulate the relative velocities $\dot{g}_{Ni}$ in normal and $\dot{g}_{Ti}$ in tangential direction depending from the generalized state $(\boldsymbol{q}, \dot{\boldsymbol{q}})$ of the system and the matrix $\boldsymbol{G}$ using the Jacobians $\boldsymbol{W}$ and the mass matrix $\boldsymbol{M}$ of the system:

$$\begin{aligned} \dot{\boldsymbol{g}}_N &= \boldsymbol{W}_N^T \dot{\boldsymbol{q}} + \tilde{\boldsymbol{w}}_N \\ \dot{\boldsymbol{g}}_T &= \boldsymbol{W}_T^T \dot{\boldsymbol{q}} + \tilde{\boldsymbol{w}}_T \end{aligned} \quad ; \qquad \boldsymbol{G} = \begin{pmatrix} \boldsymbol{W}_N^T \\ \boldsymbol{W}_T^T \end{pmatrix} \boldsymbol{M}^{-1} (\boldsymbol{W}_T \quad \boldsymbol{W}_T). \tag{1}$$

Using $\boldsymbol{G}$, the dynamic equation for the compression phase can be formulated (index A: start of impact, index C: end of impact):

$$\begin{pmatrix} \dot{\boldsymbol{g}}_{NC} \\ \dot{\boldsymbol{g}}_{TC} \end{pmatrix} = \boldsymbol{G} \begin{pmatrix} \boldsymbol{\Lambda}_{NC} \\ \boldsymbol{\Lambda}_{TC} \end{pmatrix} + \begin{pmatrix} \dot{\boldsymbol{g}}_{NA} \\ \dot{\boldsymbol{g}}_{TA} \end{pmatrix}. \tag{2}$$

To solve this equation the contact law consisiting of a unilateral normal part and coulombs law on impulse level must be taken into account. The dynamic equation for the expansion phase is identical to that for the compression phase, if you change the indices $A \to C$ and $C \to E$ (E: end of impact), respectively. Here the contact law is more complicated: In normal direction it is a complementary formulation of Poisson's law with the coefficient of restitution ϵ_{Ni} for each contact. In tangential direction there is used Coulomb's law expanded by a reversible tangential impulse Λ_{Si} which enables storing and restoring energy in tangential direction, too. To calculate Λ_{Si} two new tangential coefficients of restitution ϵ_{Ti} and ν_i are introduced:

$$2\Lambda_{Si} = \epsilon_{Ni}\epsilon_{Ti}\Lambda_{TCi} + \nu_i\mu_i \operatorname{sign}(\Lambda_{TCi})\Lambda_{NEi}. \tag{3}$$

The problem is to choose the tangential coefficients for any impact configuration and material of the colliding bodies.

3 Classification of Impacts

To classify the different types of multiple impacts, we look at the matrix $\boldsymbol{G}$ consisting of four submatrices $\boldsymbol{G}_{NN}$, $\boldsymbol{G}_{NT}$, $\boldsymbol{G}_{TN}$ and $\boldsymbol{G}_{TT}$ and analyze its structure. If the matrix has the dimension (2×2), we call it a single, otherwise a multiple impact. If $\boldsymbol{G}$ is a block diagonal matrix, it is a central impact. If the submatrices do not have off-diagonal elements, the contacts are decoupled and if $\boldsymbol{G}$ does not have full rank the contacts are called dependent.
Using these criterions, we can sort the impacts into eight classes. The choice of the parameters ϵ_{Ti} and ν_i is dependent from the class of the actual impact.

4 The Single Central Impact

For this type of impact all possible state transitions during the impact were examined and rules for the choice of the tangential impact parameters can be given. We proofed that ν must be equal to 1 what coincides with the result that a reversion of the direction of the tangential impulse does not occur here.
This impact can be described well using dimensionless velocities and impulses. We divide all velocities $\dot{g}_i; i \in \{NC,NE,TA,TC,TE\}$ by $-\dot{g}_{NA}$ and gain γ_i, especially $\gamma = \gamma_{TA}$. The impules are multiplied with $-G_{NN}/\dot{g}_{NA}$ and we obtain the dimensionless impulses $\Lambda_i^*; i \in \{NC,NE,TC,TE\}$. At last we introduce the mass-geometry-constant $\Gamma = G_{NN}/G_{TT}$ that is always > 0 because $\boldsymbol{G}$ is a positiv definite matrix. After the end of the compression phase we get $\Lambda_{NC}^* = 1$ and $\gamma_{NC} = 0$ always. For the tangential direction we have two possible cases:

$$\begin{array}{lll} \gamma_{TC} = 0, & \Lambda_{TC}^* = \frac{\gamma}{\Gamma} & \text{if} \quad |\gamma| < \mu\Gamma \\ \gamma_{TC} = \Gamma\mu - \gamma, & \Lambda_{TC}^* = \mu & \text{if} \quad |\gamma| \geq \mu\Gamma \end{array} \tag{4}$$

After the end of the impact phase we get $\Lambda_{NE}^* = \epsilon_N$ and $\gamma_{NE} = \epsilon_N$ always. For the tangential direction we have four possible cases:

$$\begin{array}{lll} \gamma_{TE} = \epsilon_T\epsilon_N\gamma, & \Lambda_{TE}^* = \epsilon_T\epsilon_N\Gamma\gamma, & \text{if} \quad |\gamma| < \mu\Gamma \\ \gamma_{TE} = (1+\epsilon_T\epsilon_N)\gamma\mu - \gamma, & \Lambda_{TE}^* = \epsilon_T\epsilon_N\mu & \text{if} \quad |\gamma| < \mu\Gamma(1+\epsilon_T\epsilon_N) \\ \gamma_{TE} = 0 & \Lambda_{TE}^* = -\mu + \frac{\gamma}{\Gamma} & \text{if} \quad |\gamma| < \mu\Gamma(1+\epsilon_N) \\ \gamma_{TE} = -\mu\Gamma(\epsilon_N+1) - \gamma, & \Lambda_{TE}^* = -\mu\epsilon_N & \text{all other cases} \end{array} \tag{5}$$

With the knowledge of the impulses transferred in the contact it is possible to calculate the state transition of the system from the moment immediately before the impact to the moment at the end of the impact. It is remarkable that the whole behaviour depends only on the relation of the relative velocities in the contact before the impact and some system and material specific constants.

5 Experiment

The most simple experiment to perform a single central impact with friction is to throw a superball to the ground and observe the impact there. The movement of the ball was photographed under stroboscopic exposure in exact time intervalls in a dark room. To enable also a measurement of the angular velocity of the ball a disk element cutted out from a ball with marks on one of its end faces was used for the experiments. From the two images immediately before and the two after the impact the velocities, the location and the time of the impact can be calculated.

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IMPACT ANALYSIS WITH FRICTION IN OPEN-LOOP MULTIBODY SYSTEMS

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Previous studies of the frictional impact analysis in jointed multibody mechanical systems have shown energy gains in the results partly due to the failure of recognition of the correct impact mode; i.e., sliding, sticking, and reverse sliding, and partly due to the inherent problem in the use of Newton's hypothesis for the definition of the coefficient of restitution. The objective of this paper is to develop a formulation for the analysis of impact problems with friction in jointed open-loop multibody systems using the Poisson's hypothesis and thus avoiding energy gains inherent with the use of Newton's hypothesis. The formulation shrinks the large system impact problem to a more manageable form that resembles a two-body frictional impact problem. The method also recognizes the correct mode of impact based on the initial slip velocity and the positions of the impacting bodies at the time of impact.

The formulation is developed using a canonical form of the equation of motion using joint coordinates and joint momenta, which results in a natural way of balancing the momenta. The use of joint coordinates eliminates the complications arisen from the kinematic constraint equations. The canonical-joint form of the equation of motion for an open-loop system is:

$$\mathbf{p} = \mathbf{M}\dot{\boldsymbol{\theta}} \qquad (1)$$

$$\dot{\mathbf{p}} = \mathbf{f} + \dot{\mathbf{M}}\dot{\boldsymbol{\theta}} \qquad (2)$$

where $\mathbf{M}$ is the coordinate dependent generalized mass matrix, $\mathbf{f}$ is the generalized force vector, $\dot{\boldsymbol{\theta}}$ is the vector of joint velocities, and $\mathbf{p}$ is the joint momenta. A normal tangential **n-t** coordinate system is established at the point of impact, perpendicular and parallel to the impacting surfaces. The system relative departing velocities of the contact points after impact, v_n^+ and v_t^+, are obtained in terms of the relative velocities before impact v_n^- and v_t^- as

$$v_n^+ = v_n^- + m_{nn}\pi_n + m_{nt}\pi_t \qquad (3)$$

$$v_t^+ = v_t^- + m_{nt}\pi_n + m_{tt}\pi_t \qquad (4)$$

where $m_{nn} = \mathbf{c}_n^T\mathbf{M}^{-1}\mathbf{c}_n$, $m_{nt} = \mathbf{c}_n^T\mathbf{M}^{-1}\mathbf{c}_t$, $m_{tt} = \mathbf{c}_t^T\mathbf{M}^{-1}\mathbf{c}_t$ are three generalized impact parameters, which are functions of the inertia properties and the configuration of the system at the time of impact; $\mathbf{c}_n$ and $\mathbf{c}_t$ are the composite normal and tangential vectors for all the coordinates; and $\pi_n=\pi_n(t_f)$ and $\pi_t=\pi_t(t_f)$ are the normal and tangential impulses at the end of the period of contact t_f. Variables m_{nn} and m_{tt} are always positive while m_{nt} could have either sign. In order to update the relative departing velocities v_n^+ and v_t^+, impact parameters m_{nn}, m_{nt}, m_{tt}, and also the composite normal-tangential vectors $\mathbf{c}_n$ and $\mathbf{c}_t$ are evaluated in terms of the system state at the time of impact. The impulses π_n and π_t are determined based on the Routh's graphical method, which is described next.

The Poisson's hypothesis is used for the definition of coefficient of restitution 'e' as the ratio of the impulse at the end of the restitution phase to the impulse at the end of the compression phase. The accumulated impulse during the contact period is plotted on the π_n-π_t plane for different modes of impact, as shown in Figure 1. In case of '*sliding*' impact, π_n is related to π_t from Coulomb's law as

$$\pi_t = i\mu_k\pi_n \qquad (5)$$

where $i= -v_t^- /|v_t^-|$ is used for enforcing opposite directions for π_t and slip velocity v_t^-, and μ is the kinetic coefficient of friction. A plot of Eq. (5) is represented by the line of limiting friction (line F) in Figure 1. In case of '*sticking*' impact; i.e., when $v_t^+=0$, Eq. (4) reduces to:

$$v_t^- + m_{nt}\pi_n(t_f) + m_{tt}\pi_t(t_f) = 0\ , \qquad (6)$$

which is represented by the line of sticking (line S) in Figure 1. Another mode of impact is the '*reverse sliding*' mode, when the slip velocity changes its direction during the impact. This could happen if after following line F, the impact reaches line S and the incremental impulse $d\pi_t$ on line S, is greater than $\mu_s d\pi_n$ (μ_s is the static coefficient of friction), as shown in Figure 1. The reversal of slip velocity also causes the reversal in the direction of $d\pi_t$. Thus in case of reverse sliding, Coulomb's law governs as

$$\pi_t - \pi_{ts} = - i\mu_k(\pi_n - \pi_{ns}) \qquad (7)$$

where π_{ns} and π_{ts} are the values of π_n and π_t at the intersection of lines F and S (for sticking).

Thus the impact will follow one of the lines F, S, or RS depending on the mode of impact corresponding to sliding, sticking, or reverse sliding respectively. To determine the accumulated impulse, a line C, which represents the end of the compression phase in Figure 1, is used. Line C is represented by taking $v_n^+=0$ in Eq. (3), which gives:

$$v_n^- + m_{nn}\pi_n(t_c) + m_{nt}\pi_t(t_c) = 0\ . \qquad (8)$$

The intersection of either line F, S, or RS with line C, will give the normal impulse at the end of the maximum compression phase, $\pi_n(t_c)$. From the Poisson's definition of coefficient of restitution, the total accumulated normal impulse for the entire period of contact, $\pi_n(t_f)$, is evaluated as

$$\pi_n(t_f) = (1+e)\pi_n(t_c) \qquad (9)$$

The total accumulated tangential impulse $\pi_t(t_f)$ is then evaluated from Eqs. (5), (6), or (7) depending on the mode of impact. The relative departing velocities are then evaluated from Eqs. (3) and (4). The change in joint momenta $\Delta\mathbf{p}$ due to impact is obtained by integrating Eq. (2), whose final from will be,

$$\Delta\mathbf{p} = \mathbf{c}_n\pi_n + \mathbf{c}_t\pi_t\ . \qquad (10)$$

The momenta after impact are updated as

$$\mathbf{p}^+ = \mathbf{p}^- + \Delta\mathbf{p}\ . \qquad (11)$$

The joint velocities $\dot{\theta}$ are also updated from Eq. (1), and the analysis of the system state is resumed with the new momenta (or velocities).

Two classical impact problems are solved using the developed formulation. The problem of impact of a falling rod with the ground (a single object impact) is addressed, and the results obtained are verified with the results from other studies that have addressed single object frictional impact. A second problem studied is the impact of a double pendulum striking the ground, as shown in Figure 2, (a multibody system impact). Kane used this problem to point out the energy gain obtained by using the formulation based on the Newton's hypothesis. A sample of the results for the energy loss in terms of the coefficients of restitution and friction are shown in Figure 3. The results obtained from the present study proves that by considering the correct mode of impact and using the Poisson's instead of the Newton's hypothesis, energy gain can be avoided. It is also observed that only when the slip velocity does not change its direction during the impact, the approaches based on Newton's or Poisson's hypothesis give identical results.

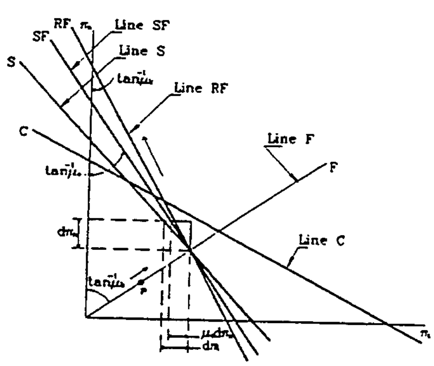

Fig.1 Impact process diagram.

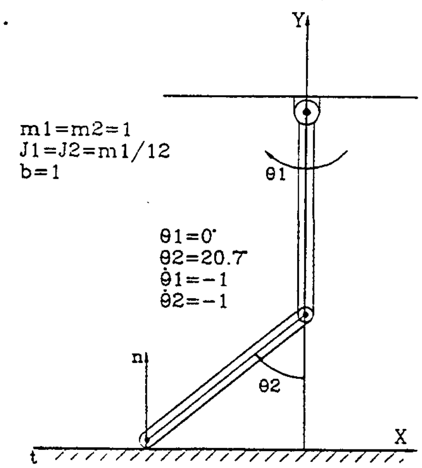

Fig. 2 Double pendulum impact.

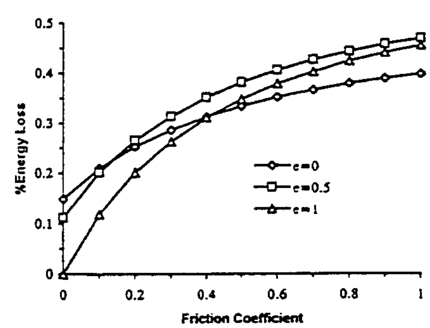

Fig.3 Resulted energy loss.

IMPACT DYNAMICS IN MILLING OF THIN-WALLED STRUCTURES

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With improvements in machine and spindle technology, high-speed milling is becoming an increasingly viable industrial process. In the aerospace industry, manufacturers have machined thin-walled components by using machines with spindle speeds in excess of 30,000 rpm. An important issue in high-speed milling is the *dynamic tool/workpiece interaction*. A thorough understanding of this challenging issue is yet to be attained. Due to the intermittent nature of the cutting process and other factors, the dynamics of milling is nonlinear and difficult to model. In most of the related research, the focus is on stability predictions by using retarded dynamical systems (e.g., [1-3]), which do not properly account for the intermittent nature of the cutting process. This aspect is taken into account in the current work, where the focus is on the nonlinear impact dynamics inherent in the milling of thin-walled workpieces.

End milling experiments were conducted with a thin-walled cantilevered workpiece on a conventional CNC milling machine by using a two-fluted cutter. The workpiece was instrumented with strain gages, to get a measure of bending and torsional vibrations. The strain-gage signals collected during upmilling and downmilling were analyzed by using the following tools: 1) Fourier spectra, 2) phase-plane plots, 3) Poincaré sections, 4) dimension calculations, and 5) wavelet transforms. A phenomenological model was developed to explain the nonlinear impact dynamics observed in the experiments. In Figure 1, an experimentally obtained phase portrait is presented along with the corresponding phase portrait obtained from the analytical model. The qualitative features of the experimental results are well captured by the developed model. In Figure 2, a Poincaré section of the workpiece motion predicted by the model is illustrated. This section has a discernible fractal character. These and other results indicate that the motions of thin-walled workpieces during high-speed milling can be quite complex. Other characteristics will be discussed in the full paper.

The experiments and analysis discussed in this work clearly point out the need for considering nonlinear impact dynamics during milling of thin-walled structures. It is expected that this consideration will contribute to gaining a better understanding of workpiece/tool interactions during high-speed milling.

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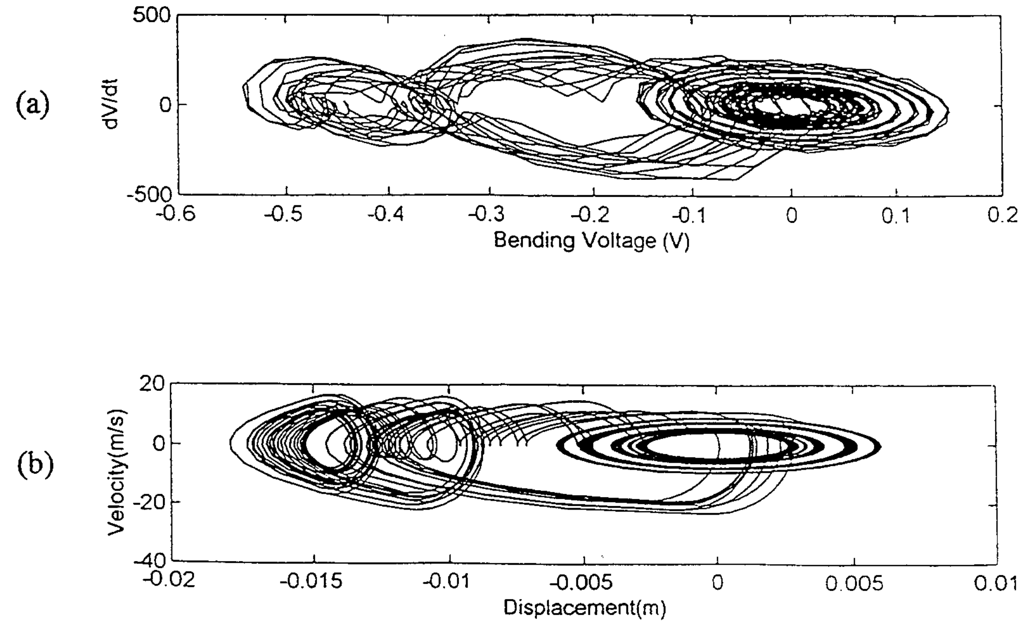

Figure 1: Phase Portraits: a) milling experiments and b) analytical model.

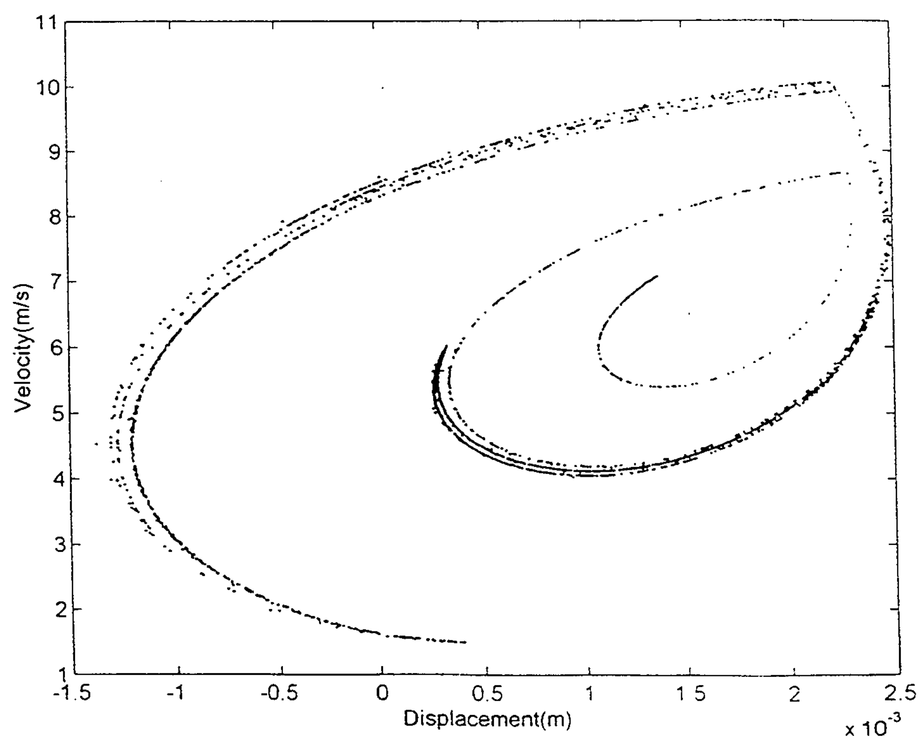

Figure 2: Poincaré section of a predicted workpiece motion.

MULTICRITERIA OPTIMIZATION OF A MACHINE AGGRAGATE WITH SINGLE STAGE SPUR GEARS

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Teeth meshing is the major factor by which mechanisms with gears differ from the other rotating mechanisms. It mainly controls the dynamic behavior of such mechanisms, since almost all sources of excitations proceed from teeth meshing. Among the large number of studies devoted to the dynamic investigation of the mechanisms of interest are those which deal with simulation of the dynamic process in teeth meshing. In a previous paper by the authors a new model for the simulation of the dynamic process in teeth meshing of spur gears was developed. This modeling was employed later to study the nonlinear vibrations of machine aggregate with single stage spur gears and to investigate the effect of a large number of external parameters, such as inertial, elastic, damping, etc. on the dynamic loads formation in different elements of the aggregate.

The aim of the presented paper is to determine optimal values of ranges of some of the parameters of the considered aggregate that will give minimum dynamic factors in all parts of the mechanism. The dynamic load factors are found analytically by modeling the mechanism as a six degree of freedom vibration system. The optimization problem is considered as a multucriterial. The optimization is done through an numerical technique using the method of sounding . In addition an approach for obtaining approximate Pareto- surfaces is proposed.

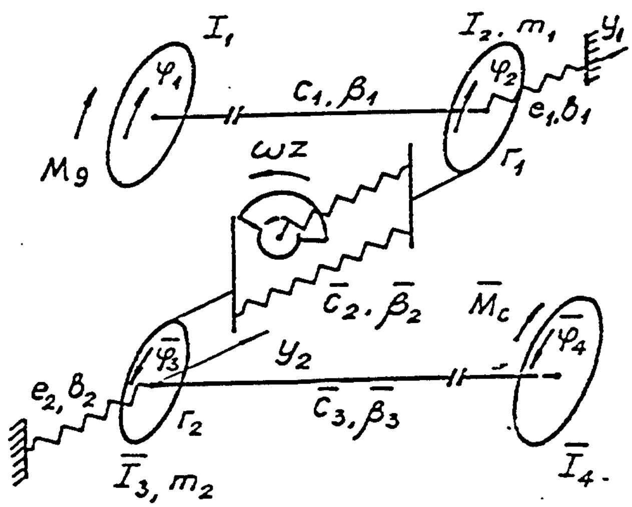

Fig. 1

A machine aggregate with single stage spur gear shown in Fig.1 is considered. It consists of driving motor, driving machine, two shafts and two gears. The flexibility of supporting bearings for each gear together with the flexural flexibility of the caring shaft is represented by an equivalent linear spring in the direction of the teeth meshing.. The gear meshing for contact ratio between 1 and 2 is modeled by the cam-follower mechanism as illustrated in Fig.1. The first linear spring in that mechanism represents the stiffness of the already existing pair of teeth in mesh. The second spring to which the cam is attached represents each new pair coming into engagement, while the cam simulates the interference between gear teeth caused by the effective tooth error. With the use of such mechanism most of the dynamic properties of meshing expressed by the variable stiffness of meshing, tooth error, tooth deflection and tooth modification may be represented. The cam rotates with a speed equal to the frequency of the excitation ωz. The magnitude of pitch error and tooth deflection and the shape of its rise and fall is determined by tooth modification.

Symbols appearing in the dynamic model denote as follows: b_1, b_2 - damping in support, $C_1, \overline{C}_3$ - stiffness of driving and driven shafts, e_1, e_2 - stiffness of supports, $I_1, I_2, \bar{I}_3, \bar{I}_4$ are respectively mass moments of inertia of motor, driving gear, driven gear and machine, M_g, M_c - turning torques of motor and machine , m_1, m_2 - masses of gears, r_1, r_2 - base circle radii of gears, $\beta_1, \overline{\beta}_2\ \overline{\beta}_3$ - damping in shafts and meshing.

The impact force between teeth at the beginning of each contact is also considered. The force is regarded as impulsive and is approximated to that occurring between two equivalent masses with initial impact velocity equal to the velocity due to the effective tooth error plus the velocity of vibration of the gear at the moment of impact.

The differential equation of motions with respect to coordinates considered about the equilibrium position of the system are given by

$$
\begin{aligned}
&I_1\ddot{\varphi}_1 + C_1(\varphi_1 - \varphi_2) = -\beta_1(\dot{\varphi}_1 - \dot{\varphi}_2)\\
&I_2\ddot{\varphi}_2 - C_1(\varphi_1 - \varphi_2) + C_2(t)\varphi = -F_d r_1 + \beta_1(\dot{\varphi}_1 - \dot{\varphi}_2) - \beta_2\dot{\varphi}\\
&I_3\ddot{\varphi}_3 + C_3(\varphi_3 - \varphi_4) - C_2(t)\varphi = F_d r_1 + \beta_3(\dot{\varphi}_3 - \dot{\varphi}_4) - \beta_2\dot{\varphi},\\
&I_4\ddot{\varphi}_4 - C_3(\varphi_3 - \varphi_4) = \beta_3(\dot{\varphi}_3 - \dot{\varphi}_4)\\
&I_5\ddot{\varphi}_5 + C_5\varphi_5 - C_2(t)\varphi = F_d r_1 - \beta_5\dot{\varphi}_5 + \beta_2\dot{\varphi},\\
&I_6\ddot{\varphi}_6 + C_5\varphi_6 + C_2(t)\varphi = -F_d r_1 - \beta_5\dot{\varphi}_5 - \beta_2\dot{\varphi}.
\end{aligned} \tag{1}
$$

The relative displacement φ and exciting force F_d in the teeth meshing are given by

$$
\begin{aligned}
&\varphi = (\varphi_2 - \varphi_3 - \varphi_5 + \varphi_6),\\
&F_d = P(t) + f_i(t) + C_2(t)\frac{\Delta_i(t)}{r_1},
\end{aligned} \tag{2}
$$

where $C_2(t), P(t) f_i(t)$ and $\Delta_i(t)$ are respectively the variable stiffness of the meshing, the impact force, the interference excitation between teeth (cam action) and the accumulative pitch error. The differential equations of motion (1) are solved for the steady state conditons using approximate analytical methods.

The main objective functions to be minimized are the dynamic load factors excited in the gear teeth, input and output shafts. The expressions of these functions, respectively are as follows:

$$
F_1 = 1 + \frac{C_2(t)\varphi + F_d r_1}{P_{ct} r_1}, \qquad F_2 = 1 + \frac{C_1(\varphi_1 - \varphi_2)}{P_{ct} r_1}, \qquad F_3 = 1 + \frac{C_3(\varphi_3 - \varphi_4)}{P_{ct} r_1}.
$$

It is possible to consider also another objective functions but the chosen three function describe the most important dynamic behavior of the mechanism.

The basic design parameters to be changed and optimized are the following:

C_0- stiffness of teeth pair in mesh,

C_1 , C_3 - torsional stiffness of shafts,

I_2 , I_3 - moments of inertia of the gear wheels.

These parameters actually could be changed in wide ranges, while the remaining parameters normally are fixed according to some specific reasons and could not be changed. So the moments of inertia of the motor and the driven machine are assumed to be given. Other system parameters, such as gear radii, number of teeth, modul, contact ratio etc. may also be assumed given unless are to be determined by some another algorithm. The angular speed and the transmitted load are to be taken into accordance to the required running conditions of the mechanism.

The optimization problem to be solved is to minimize the three objective functions (multicriteria optimization) for the chosen above five basic parameters.

Utilizing Parallel Computing in the Combined Multibody, Control and Structural Dynamic Simulation Code FEDEM[1]

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EXTENDED ABSTRACT

The Multidisciplinary simulation system FEDEM (Finite Element Dynamics of Elastic Mechanisms) is based on a nonlinear Finite Element (FE) formulation and analogue/digital control modelling for multibody simulation (MBS). The FEDEM system contains a comprehensive library of MBS elements as Links, Joints, Springs, Dampers, Forces, Drive elements, Gears, Coulomb friction and Control elements. However, the links in FEDEM are modelled as FE substructures and reduced to super elements by Component Mode Synthesis (CMS). The super nodes, called triads in FEDEM, are used for modelling of the mechanism elements referred to above. The formulation adopted in FEDEM makes modelling of flexible telescopic joints a standard option for the user. If no explicit FE models has been made for the links, super elements are generated automatically from beam elements. The user has, however, a comprehensive library of Beam, Shell and Solid Finite Elements available for modelling. With relatively stiff links and/or many super nodes on the link, the vibration modes calculated for the CMS transformation may have very high frequencies, and consequently need not be included in the super element for that link. The flexibility for a super element is, however, included in the simulation also if no vibration modes are included through the CMS transformation. Also, this approach to MBS makes stress retracking a natural part of the system. With stress retracking specified, animation of mechanism motion including stress contour plots on the links, is an option for the user. A comprehensive coupling is being made to I-DEAS Master Series, making the supplier of the FEDEM system a "Master Solution Vendor".

Computational times in FEDEM are very competitive to that of other MBS systems, however, with the FE technology as a modelling tool in FEDEM a mechanical/mechatronic product may be modelled in much more detail, for example

* through modelling of flexible telescopic joints,
* through modelling of parallel joints between links,
* through inclusion of distributed environmental forces on the links,

and the models may consequently be quite large depending on the detailing level. Because of this, parallel computing is an interesting option that has been tested.

The three modules *prepro, simule* and *stress* are the most computational intensive ones in FEDEM.

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prepro has a library of finite elements that are used for building substructures for the different links in a simulation model. For each substructure, with external nodes fixed, a number of eigenvalues and corresponding eigenvectors may be calculated for the internal degrees of freedom (DOF) followed by CMS transformation of super element mass and stiffness matrices. These are prepared for inclusion in the simulation model at system level. In *simule* the multidisciplinary simulation model is assembled from the super element matrices generated by the *prepro* module and from a number of system components, as mentioned above, entered from an entity oriented input format. Simulation is carried out by time integration in *simule* using the Newmark method. Vibration analysis at system level through eigenvalue analysis may be specified for certain positions during a simulation, and the resulting elastic vibration modes may be animated on the workstation. The *stress* module is optional for the user. As the module name indicates, this module retracks finite element stresses for the different substructures used in the simulation. Following a simulation in *simule*, the user specifies for which positions stress retracking is required. The evaluated stresses may be visualized on the links in a mechanism motion animation at system level or the stresses may be post-processed for one substructure at a time for instance in I-DEAS Master Series.

Parallel computing is utilized in all these three FEDEM modules. The analysis in *prepro* is carried out prior to the start of the *simule* module. Computations in *prepro* are in nature parallel over the different substructures in a simulation model and this parallelisation is quite easy to implement by distributing the substructures on the available computer nodes. This may be optimal if there are many almost equally sized substructures. However, with one or a few very large substructures compared to the rest of the substructures, this parallelisation strategy is poor and a method for completely distributed computation (parallel linear equation solution and CMS transformation) is adopted for the large substructures. Active column or skyline storage format is adopted in FEDEM and in the parallel implementation the columns in the system matrices are built distributedly on the computer nodes. The distributed system matrices are then used for distributed matrix operations performing linear equation solution and CMS transformation.

Time integration in *simule* is in nature a serial operation, but for very large models parallel equation solution will speed up the analysis significantly. The same is the case for eigenvalue analysis at system level. As in *prepro*, for large simulation models the system matrices are built distributedly and all the analysis is carried out distributedly.

The stress retracking in *stress* is, as for *prepro*, parallel in nature at substructure level and the substructures are distributed onto different computer nodes. As for *prepro*, when one or a few of the substructures are very large compared to the rest, another parallelisation strategy is sought. In this case, parallelisation of the large substructures is done at primary element level, i.e. each primary element is distributed onto different computer nodes.

Another parallelisation potential is in running the time integration (*simule*) in parallel with the stress retracking (*stress*) and simultaneously displaying animation of mechanism motion including stress contour plots on the mechanism links.

Finite element computations are usually well suited for parallelisation. Parallel linear algebra routines are in many cases very efficient, and construction of system matrices (stiffness and mass matrices, together with right hand side vectors) may be done distributedly. In addition, evaluation of displacements and retracking of stresses may be done in parallel for each element in the model.

When doing parallel computations, the data distribution strategy will often have a large impact on the performance of the parallel code. For an algorithm to perform efficiently, data must be available to each processor when needed, and the need for communication should be minimized. If parallelisation is considered when writing the program from scratch, the data structures may be designed to optimize efficiency on distributed memory platforms. If, on the other hand, data structures have been designed to suit the needs of a vector machine, much effort is needed to efficiently parallelise the code. In the industrial codes available today, the last case is dominating. FEDEM also conforms to this picture.

During the parallelisation work, the development of a library called *psky* has been initiated. The library contains routines for doing distributed linear algebra on matrices stored in a new distributed skyline storage format. The library is based on the SAM library (SAM is developed at SINTEF and the Norwegian University of Science and Technology). The part of *psky* used in FEDEM includes *pskysol* and matrix multiplication routines. *pskysol* solves a set of linear equations, (**Ax=b**), by use of the $\mathbf{LDL}^T$ factorization. Benchmarking of the *pskysol* routine will be presented.

The parallel implementation of FEDEM was done in an CEC ESPRIT project (EUROPORT/EP-FEDEM) that was completed by the end of 1995. In this project, three industrial applications of the parallel version of FEDEM were conducted: a large, a medium sized and a small simulation model was used from different industrial companies. The small simulation problem was a satellite deployment simulation from Dornier GmbH in Germany, the medium sized problem was an industrial robot from ABB Robotics Products AB in Sweden, while the large problem was a model of a personnel bridge between two offshore platforms in Norway (Statoil).

Parallel computations on different parallel computers and with different number of computing nodes were carried out for the three industrial applications. The results will be presented to show scalability of the parallel code (i.e. we show how the computation times are reduced as a function of number of computing nodes used in the calculations). Also lessons learned during the parallelisation project and the overall benefits from the parallelisation of the code will be discussed.

Thursday, June 13

1030-1210

Session 16. Analysis IV

Higher-order periodic solutions and a comprehensive stability analysis using n-mode harmonic balance of an unsymmetric oscillator

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This paper presents a systematic procedure to explicitly determine the algebraic equations arising from the method of harmonic balance with an arbitrary number of modes in the assumed solutions and a comprehensive stability analysis for an unsymmetric nonlinear oscillator.

The technique can be used for a wide variety of nonlinear oscillators (including systems of ordinary differential equations) and is illustrated in the case of second order differential equations with nonlinear restoring force.

Explicit formulation of the algebraic equations for the Fourier coefficients can be easily obtained for the case of the ordinary differential equation:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t), \quad \mathbf{x} \in \mathbf{D} \subset \Re^n, \;\; \mathbf{f} : \mathbf{D} \times \Re_+ \rightarrow \Re^n \tag{1}$$

under the assumption that each component of $\mathbf{f}$ is a polynomial in $\mathbf{x}$, i.e.

$$f_l(x,t) = a_l + \sum_{j_1=1}^{n} a_{l\,j_1} x_{j_1} + \sum_{j_1,j_2=1}^{n} a_{l\,j_1\,j_2} x_{j_1} x_{j_2} + \ldots + \sum_{j_1,\ldots,j_m=1}^{n} a_{l\,j_1\,j_2\ldots\,j_m} x_{j_1} x_{j_2} \ldots x_{j_m} \tag{2}$$

where the coefficients $a_{l\,j_1\,j_2\ldots}$ are periodic functions of time with period $T = \frac{2\pi}{\omega}$.

Assuming the subharmonic of period s approximated by a truncated Fourier series containing p modes:

$$x_l = \sum_{k=-p}^{p} \alpha_{kl} e^{ik\frac{\omega}{s}t} \tag{3}$$

a system of $(2p+1)n$ complex equations is obtained.

The above methodology can be successfully applied to second order ordinary differential equations like Duffing, Van der Pol, etc. and is exemplified here on the following nonlinear oscillator:

$$\ddot{x} + \beta\dot{x} + x - x^2 = F \sin \omega t \tag{4}$$

which has been extensively studied numerically by Thompson, 1989 and Foale and Thompson, 1991. It has been shown that this oscillator exhibits a well defined period-doubling cascade (Thompson, 1989).

The methodology allows the efficient determination of higher-order subharmonics and the Feigenbaum ratios $(6.665, 5.03667 \rightarrow \delta = 4.669201\ldots)$. Flip bifurcations with the frequency and force amplitude as parameters are computed and the results compare very favorably with published numerical simulations.

Bifurcation	**F numerical**	**F analytical**
period 1− > 2	0.1005	0.100490
period 2− > 4	0.1073	0.107355
period 4− > 8	0.10839*	0.108385
period 8− > 16	0.10859*	0.1085895

Table 1: Comparison of flip bifurcations obtained numerically and analytically for $\omega = 0.85$; * - values read from a diagram (Thompson, 1989).

Although numerical methods have been employed to solve the resulting systems of algebraic equations, the general approach is analytic. As such, this study confirms independently (i.e. nonsimulation) the period-doubling cascade of an escape equation including the bifurcation universal scaling laws.

Also bifurcations (flip, folds) have been studied using truncation of the infinite Hill determinant. It has been found by numerical experiments that by considering one more mode in the expansion of periodic part of the perturbation not only there is an improvement in the approximation of the bifurcation but also flip-folds bifurcations respect the correct sequence (Fig. 2) (they can occur only in pairs as at least one characteristic multiplier should remain inside the unit circle).

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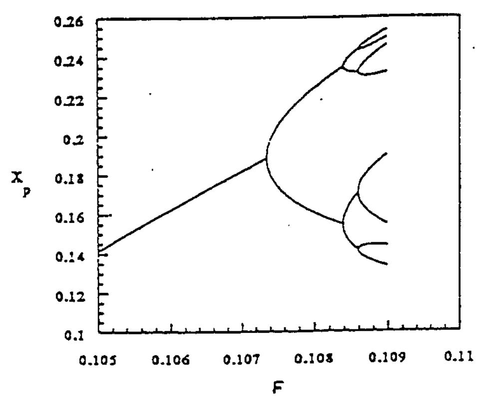

Figure 1: Bifurcation diagram based on a Poincaré section (zoom for periods 2, 4, 8, 16): $F - x_P$.

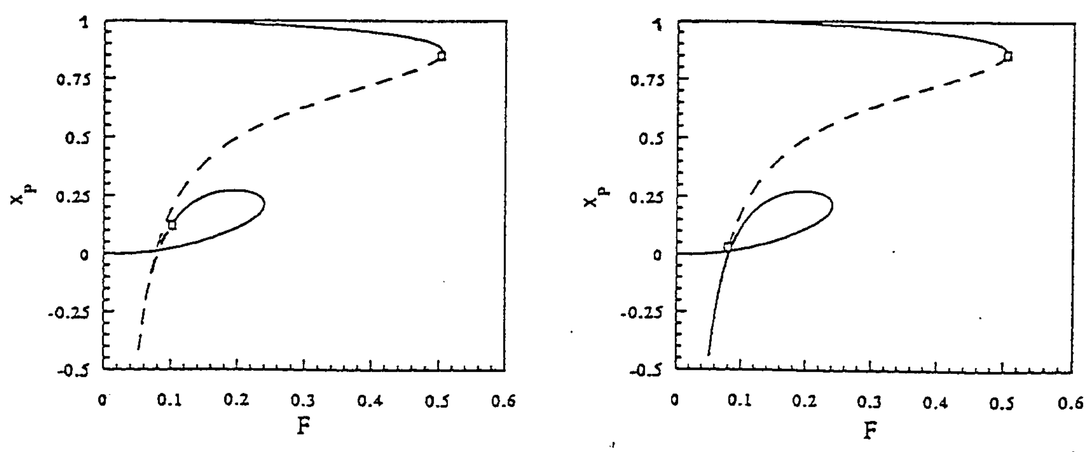

Figure 2: Stable and unstable solutions using Poicaré sections for $\omega = 0.6$: (a) two mode solution / two mode perturbation; (b) two mode solution / three mode perturbation;

Continuous Gyroscopic System Stability Near Critical Speeds

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Abstract

Continuous gyroscopic systems such as rotating disks, rotating beams, axially moving materials, and fluid-conveying pipes are subject to instability associated with vanishing eigenvalues at critical values of the rotation or translation speed (critical speeds). Identification of these critical speeds is essential for stability analysis, and they can frequently be calculated analytically. Their determination, however, provides an incomplete picture of system stability. Gyroscopic systems may experience divergence and flutter instabilities, and these can not usually be identified analytically for continuous systems. In this paper, the stability of continuous gyroscopic systems in the vicinity of critical speeds is examined. The two possible stability pictures near critical speeds are dramatically different. In one case, the system eigenvalues are imaginary in the speed region immediately above a critical speed and the system is stable. Axisymmetric, rotating plates and translating strings exhibit such behavior (Mote, 1970; Archibald and Emslie, 1958, Wickert and Mote, 1990). Alternatively, an eigenvalue of positive real part can occur in a neighborhood of the critical speed and divergence instability results. This is the case, for example, for translating beams and pipes conveying fluid (Wickert and Mote, 1990; Paidoussis and Issid, 1974). Knowledge of gyroscopic system stability at speeds other than critical speeds typically requires discretization of the continuous system to find the system eigenvalues. Such techniques are tedious, computationally intensive, and, if speed-dependent eigenfunctions are not used for the discretization, subject to poor convergence (Wickert and Mote, 1991).

This paper presents a perturbation analysis that predicts continuous gyroscopic system eigenvalue behavior in the vicinity of critical speeds without use of discretization. The results are in terms of the critical speed eigenfunctions and the continuous system differential operators. The eigenvalue behavior near the critical speeds is predicted by the sign of the first order eigenvalue perturbation. Additionally, an analogous perturbation of the stationary system eigenvalues allows construction of a piecewise linear approximation of the eigenvalue loci in the eigenvalue-speed plane at speeds away from the critical speeds. Based on an insightful examination of the gyroscopic system eigenvalue problem, Renshaw and Mote (1996) give a conjecture for predicting eigenvalue behavior in the vicinity of critical speeds. Mathematical justification and quantitative eigenvalue predictions are not presented. Although gyroscopic systems are subject to flutter instability at supercritical speeds, these are not addressed herein.

The approach presented depends on a formulation of the gyroscopic system eigenvalue problem in terms of λ^2, where λ is the eigenvalue. This idea was introduced by Huseyin and Plaut for discrete gyroscopic systems (Huseyin and Plaut, 1974; Huseyin, 1976; Plaut, 1976). The essential feature of this formulation in the current work is that λ^2 is smooth at a critical speed whereas λ is not. This is evident in Figure 1 for a rotating beam. This smoothness allows application of perturbation analysis to obtain an asymptotic expansion for λ^2 in the vicinity of a critical speed. λ does not vary smoothly in the neighborhood of a critical speed, and Renshaw and Mote (1996) note that formal perturbation of critical speed eigenvalues always predicts imaginary eigenvalues. Figure 1 also indicates the stability behaviors noted above. The axisymmetric rotating beam has stable, imaginary eigenvalues ($\lambda^2 < 0$) above the critical speeds. In contrast, asymmetric rotating beams have a region of divergence instability ($\lambda^2 > 0$) in the neighborhoods of the critical speeds. The presented asymptotic perturbation analysis predicts such stability behavior without solution of the gyroscopic system eigenvalue problem.

To demonstrate the method, the case of a traveling, tensioned beam is analyzed. The predicted stability behavior at the lowest two critical speeds matches results from numerical discretization. Eigenvalue perturbations are also determined for perturbation about the zero transport speed eigenvalues, and a piecewise linear approximation is constructed for the traveling beam eigenvalues.

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Application of Phase Space Reconstruction to Fault Diagnosis

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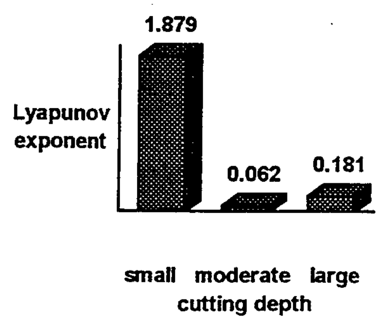
Now , more and more attentions have been absorbed to the applications of chaotic dynamics . In the study of surface topography , fractal-based techniques are applied to wear prediction [1]. Fractal is also applied to determine the character of fracture surfaces of metals [2]. Though only a start has been made in these fields up to now , approaches of dynamical system are more simple and clear compared with the traditional ones ,so they show great potentials .

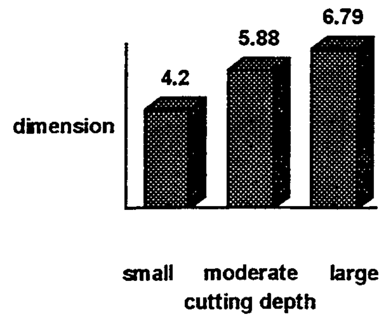
It is sure that chaos also exists widely in mechanical systems . I.Grabec first studied chaotic dynamics of cutting process [3]. He set up the model for the cutting system , based on the analyses of this model , conclusions were drawn that the displacement of the tool 、 acceleration 、 cutting force and energy dissipation due to the deformation of materials (acoustic emission) etc. may exhibit the properties of chaos . As he pointed out at the end of his paper that , although the model was very simple , it could be imagine in the real complex mechanical system , chaotic movements were universal .

In this paper , the theories of chaotic dynamics were used to the mechanical fault diagnosis . At first we will give a brief introduction to phase space reconstruction method of the chaotic time series analysis . Phase space reconstruction first proposed by N.H.Packard et al and proved mathematically by F.Takens individually is a powerful method especially in the identification and detection of the experimental data measured from an unknown complex system [4][5].

Then the feasibility of applying the characteristic exponents of dynamical system including Lyapunov exponent and correlation dimension to the system identification and diagnosis will be discussed in detail . Next , the experimental setup and procedures for the measurement of the acceleration signals of vibrations from the gearbox on a hobbing machine with fault will be described . Results of the exponents associated with three different kinds of cutting depth for the hob will be given to differentiate the characters of the gear defect . The changes of Lyapunov exponents and correlation dimensions with the increase of cutting depth for the hob are illustrated in Fig.1 (a) and (b) to reflect the alteration of the stability and the complexity of the mechanical system separately .



(a)



(b)

Fig.1. (a) The change of Lyapunov exponent with the cutting depth . (b) The change of correlation dimension with the cutting depth .

At the end of this paper , we will generalize the application of the quantities of dynamical system to the life estimation for the mechanical system , this idea was inspired by the wear prediction of material surface [1]. As we have known that the life of a mechanical system is roughly divided into three stages , including : run in 、 normal work and fault . In different stages , the properties of the system will be distinguishable , and the quantities describing the properties such as Lyapunov exponent and correlation dimension will also change with distinction . As a result , the possibility of predicting the life of the mechanical system is provided . Fig.2 shows how Lyapunov exponent of a mechanical system evolves in different stages , then the life of the system can be estimated from the time evolution curve of this exponent .

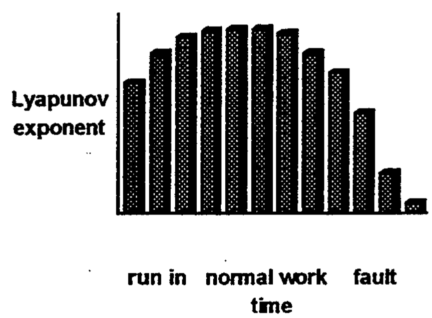


Fig.2. The time evolution of Lyapunov exponent at different life stages of a mechanical system .

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Dynamic Stability of an Elastic Cylindrical Shell Being in External Contact with an Elastic Isotropic Medium with Respect to Nonstationary Subjections

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The problem on the dynamic stability of an infinite plate of constant thickness resting on an elastic foundation with respect to nonstationary subjection was first considered by Rossikhin [1]. An anisotropic elastic half-space with an absolutely smooth boundary was used as the foundation, such that friction between the plate and the half-space boundary was absent. It was assumed also that tangential and normal forces acted in the plate mid-plane. The dynamic behavior of the plate was described by the classical Kirchhoff-Love equation. Nonstationary vibrations were excited by snap-action loads, resulting in three types of plane shock waves propagating in the elastic anisotropic half-space. The solution behind the wavefronts up to the contact boundary was constructed by using ray series [2]. Coefficients of the ray series, within an accuracy of arbitrary functions dependent on two coordinates on the plate mid-plane, were determined from recurrent differential equations of the ray method, which had been derived from the resolving system of equations for the anisotropic half-space using the theory of discontinuities [3]. Arbitrary functions, in their turn, were found from the condition of absence of tangential stresses and the condition of continuity for normal displacements and normal stresses on the contact boundary of the plate and the half-space, as well as from the initial conditions. The time-dependence and the mid-plane coordinates dependence of the plate deflection were obtained as a result of calculations. It was shown that with time the behavior of the plate deflection is dependent on the values of the compression force involved in the plate mid-plane. If these values exceeded the critical values (these critical values were less the critical values of the compression forces under which the plane form of equilibrium of the plate became unstable), then the normal displacements of some set of points possessed by the plate might take on rather big values in a small instant of the time.

In the identical definition and using the same method, Rossikhin and Shitikova [4] have solved the problem on dynamic stability of an elastic transversely isotropic plate being in welded or smooth contact with an elastic isotropic half-space. Stability analysis has been performed for two types of the plate: a relatively heavy and rigid plate and a relatively lightweight and complaint plate with respect to the half-space material.

In the present paper, the problem on nonstationary vibrations of an infinitely long cylindrical shell of constant thickness, whose internal side is suddenly subjected to tangential and normal displacement velocities but its external side is in welded or smooth contact with an elastic medium, is considered. Besides, the shell is kept under uniform axial pressure. Nonstationary subjection to the shell gives rise to two types of cylindrical shock waves in the contacting elastic isotropic medium. The solution behind the wavefront up to the contact boundary is constructed by using ray series. Coefficients of the ray series are found from recurrent differential equations of the ray method within the accuracy of arbitrary functions dependent on angular and axial coordinates. Arbitrary functions, in their turn, are determined from the condition of continuity for displacements and stresses on the contact boundary of the shell and the medium, as well as from the initial conditions. Calculations have been carried out with due account for three terms of the ray series for the functions to be found. The time dependence of the tangential and normal components of the displacement vector for the cylindrical shell have been obtained. It has been shown that with time the behavior of the shell's normal displacement is dependent on the magnitude of the axial pressure: under axial pressure in excess of the critical value the normal displacements of some set of points possessed by the shell may increase essentially in a small instant of the time.

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A Route to Chaos in a Nonlinear Time-Delay System

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The stability and complicated dynamics of a nonlinear single-degree-of-freedom system subject to time-delay feedback is explored via analog-computer simulation. The system modeled is taken from machine-tool dynamics but is analogous to many other practical engineering systems. The experimental results validate the analytical and computational methods developed by Nayfeh, Chin, and Pratt[2]. Good agreement between the theory and experiment is shown. A cyclic-fold bifurcation that leads to large-amplitude limit cycles is observed, as predicted. Also, evidence of torus breakdown route to chaos is presented. The dynamics of the system is analyzed using a variety of techniques, including phase portraits, time traces, Poincaré sections, power spectra, pseudo-state space construction, and pointwise dimension calculations. Digitization of the time-delay signal is shown to affect the experimental stability of solutions. Regions where digitization caused minimal error are identified.

Introduction

The stability of systems with time delay is of importance in a wide variety of practical engineering applications. Recent examples from the literature include machine-tool dynamics[1,2], liquid level process control[3], and vibration control of structural systems[4]. Stepan[5] cites human-machine interactions, such as crane operation, where human sensing and response can contribute up to 0.1 seconds of time delay to the control system.

It is well-known that delay, or transport lag, in a system can result in instability[6]. Merritt[7] exploited this concept to model the linear stability of the cutting process. His model is a single-degree-of-freedom linear oscillator (the machine-tool structure) with time-delay feedback proportional to the oscillator displacement (the cutting process). Using this approach, he determined the maximum gain (limit width of cut) for stable operation. This linear approach predicts unbounded oscillations once the limiting gain is exceeded. However, what is typically observed in systems is limit-cycle behavior. To capture this dynamics requires the inclusion of nonlinearities.

Analog-Computer Experiments

Experiments were carried out using an analog computer to model machine-tool vibrations using the following nonlinear single-degree-of-freedom system[1]:

$$\ddot{x} + 2\xi\dot{x} + p^2(x + \beta_2 x^2 + \beta_3 x^3) = -p^2 w[x - x_\tau + \alpha_2(x - x_\tau)^2 + \alpha_3(x - x_\tau)^3] \tag{1}$$

where

$$x_\tau = x(t - \tau) \tag{2}$$

Here, x is the displacement normal to a machined surface, damping is of the hysteretic type and ξ is inversely proportional to the chatter frequency ω, p is the natural frequency of the cutter, w is the width of cut, β_2 and β_3 are two constants describing the nonlinear stiffness of the machine-tool structure, α_2 and α_3 are nonlinear constant coefficients of the cutting force function, and the time delay τ is the period of one revolution. The analysis made use of the following parameter values: $p = 1088.56$ rad/sec, $\xi = 24792/\omega$ rad^2, $\beta_2 = 479.3$ 1/in, $\beta_3 = 264500$ 1/in^2, $\alpha_2 = 5.668$ 1/in, and $\alpha_3 = -3715.2$ 1/in^2. Output of the analog computer is fed back through a nonlinear control law using a data-acquisition board and a PC to impose the time delay.

Fig. 1 shows the experimentally determined bifurcation diagrams for two different time delays along with the corresponding results of a previous analysis done using the methods of multiple scales, harmonic balance, and digital computer simulation[2]. Experiment and theory match well for $\tau = 1/75$ but are seen to diverge for $\tau = 1/60$. This discrepancy is attributed to the digitization of the time-delay signal. Other experimental results will show the evolution of the dynamics from periodic to chaotic using time traces, phase portraits, Poincaré sections, power spectra, and dimension calculation. For example, Fig. 2 illustrates the transition from periodic to quasiperiodic motion as the bifurcation parameter w increased from 0.060 to 0.136.

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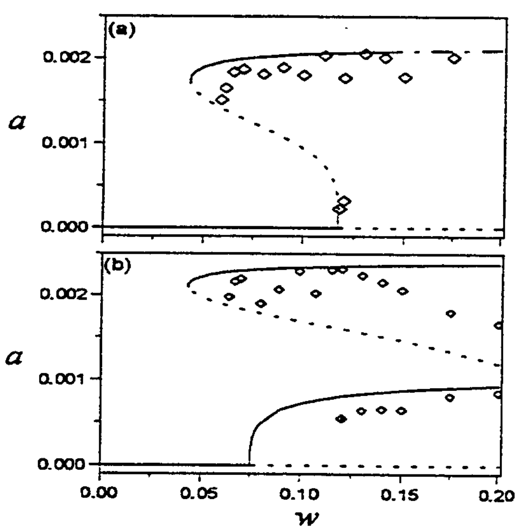

Fig. 1 Variation of amplitude a with width w for (a) $\tau = 1/75$ and (b) for $\tau = 1/60$. Solid lines denote stable periodic solutions and dotted lines denote unstable periodic solutions determined in the previous analysis. Diamonds denote points obtained via analog computer.

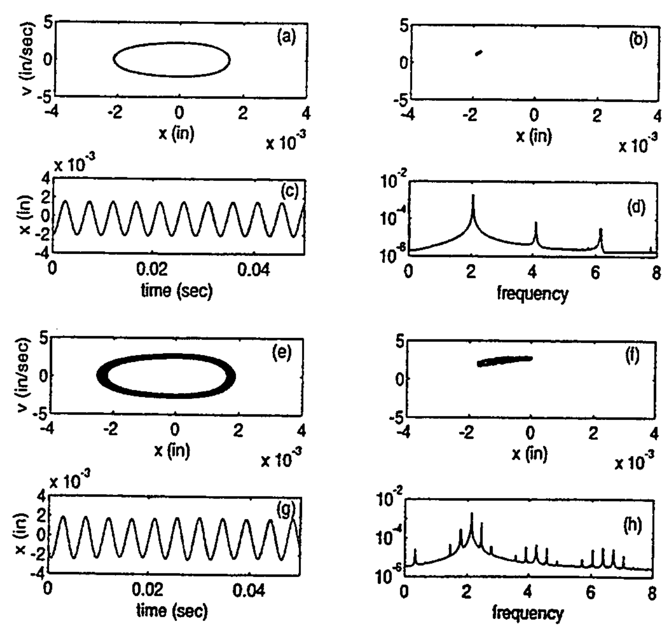

Fig. 2. The phase portraits (a) and (e), the Poincaré sections (b) and (f), the time histories (c) and (g), and the power spectra (d) and (h) of the attractors obtained when $\pi = 1.75$ and $w = 0.060$ and $w = 0.136$, respectively.

Thursday, June 13

1330-1510

Session 17. Tutorial Sessions on the Use of Mathematica

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