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Final Report: Unstructured h-p Finite Elements for Unsteady High Speed Flows

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There is currently a great interest in predicting the motion and corresponding loads of highly maneuverable aircraft capable of controlled flight at very high angles of attack. The design of air-to-air missiles could also benefit greatly from a better understanding of high angle of attack supersonic flight. Unsteady separation and formation of strongly asymmetric vortices from such bodies can introduce substantial lateral forces and yawing moments that must be counteracted or possibly used intelligently to aid in controlling the flight of the body. The majority of recent numerical studies has focused on steady and relatively low Mach number flows. Special algorithms have been developed for the design of high quality meshes to ensure convergence of the numerical solution in these complex configurations.

The objective of this project is to develop hybrid DNS-LES numerical capabilities to predict forces and details of flow structure for *unsteady viscous* flows around complex-geometry three-dimensional aerodynamic configurations at high speed. The fundamental idea is based on a new class of spectral methods for unstructured meshes we have developed for general computational mechanics problems. A particularly attractive property of this new method is that it achieves exponential convergence rate, which is essentially independent of the smoothness and the general quality of the unstructured mesh employed in the discretization.

We have worked **18 months** for this project. At this time we have accomplished the following:

- Formulation and implementation of a discontinuous Gelerkin algorithm for hyperbolic conservation laws in one- and two-dimensions. In particular, we have used our orthogonal spectral basis for triangles and developed interface flux conditions based on characteristic decomposition and appropriate Riemann solvers. A novel aspect of this algorithm is the use of multi-dimensional limiters developed by Prof. Shu who is a co-PI in the project; these limiters are particularly suited for p-type methods. A more recent development is the construction of variable order elements that allow from zero-order (i.e. constant) to high-order expansions. If constant state elements are used to discretized discontinuities the Gibbs phenomenon is avoided monotonicity is preserved. This in turn implies that there is no longer the need for limiters in our spectral/hp codes.

- Formulation and implementation of a mixed type Galerkin formulation for treating the viscous contributions in the Navier-Stokes equations. Here the velocity is discontinuous but is tested with continuous test functions while an auxiliary field (the flux) is continuous across element interfaces with discontinuous corresponding functions. This algorithm has been tested in a number of cases with functions of varying degree of continuity in order to evaluate its robustness. A new theoretical finding is the construction of appropriate polynomial spaces for the velocity and flux that guarantee stability of the approximations. Equal polynomial degree representation leads to unstable approximations.

- Implementation of a 2D Navier-Stokes solver for subsonic and supersonic flows. Here we used the developments in the previous two tasks and implemented a code which can treat both continuous as well as discontinuous (supersonic) velocity fields. The code has been using an explicit time integration for both convective and diffusive contributions. Several test cases were considered, and the exponential convergence rate has been verified in smooth regions. Particular emphasis was placed on unsteady solutions around thick airfoils at a large angle of attack.

- Dynamic simulations of 2D unsteady airfoil flows at a large angle of attack. Here we have obtained very high quality results using dynamic adaptive strategies by *steering* the resolution resources to regions in the domain with large gradients. Both supersonic and subsonic flows have been simulated in order to demonstrate the capability of the code. Unlike low-order methods used in adaptive refinement in the to demonstrate the capability of the code. Unlike low-order methods used in adaptive refinement where continuous modification of the mesh is required, in the method we have developed, a two-level refinement/unrefinement hierarchy exists that allows for selective refinement without the need for changing the mesh avoiding costly computational overhead. In addition, in the current method we have developed error estimators that guarantees accuracy of the solution with computation on a single mesh.

- A systematic validation of the spectral accuracy and third-order time accuracy of the code has been completed using analytical Navier-Stokes solutions and comparisons with a different spectral element/collocation code we have developed. In particular, unsteady flow past a cylinder was compared in detail, both the averaged fields as well as instantaneous fields and spectra.

- First three-dimensional simulation of flow past a thick airfoil at a large angle of attack. This was done with a parallel version of the spectral element-Fourier code developed under this project. These results were communicated to Dr. L. Sakell (scientific officer) in October 1996.

Personnel: Besides the PI, one graduate student, two postdocs, one visiting Professor, and Prof. Shu have been working on this research project.

Publications

1. Lomtev I., Quillen, C. and Karniadakis, G.E., "Spectral/hp algorithms for unstructured meshes for compressible viscous flows", J. Comp. Phys., submitted.

2. Lomtev I., and Karniadakis, G.E., "Simulations of viscous supersonic flows on unstructured hp meshes", AIAA, Reno, Jan. 1997.

3. Sherwin, S.J. and Karniadakis, G.E., "Triangular and Tetrahedral Spectral Elements", Houston Journal of Mathematics, p. 509, 1995.

4. Sherwin, S.J. and Karniadakis, G.E., "A New Triangular and Tetrahedral Basis for High-Order Finite Elemenet Methods", Int. J. Num. Meth. Eng., vol. 38, p. 3775, 1995.

5. Warburton, T.C, Sherwin, S.J. and Karniadakis, G.E., "Hierarchical Refinement Using Spectral/hp Triangles and Prisms", Proc. 6ISCFD, Lake Tahoe, Nevada, September 4-8, 1995.

6. Sherwin, S.J. and Karniadakis, G.E., "Adaptive hp finite elements on unstructured meshes", IX Int. Conf. on Finite Elements in Fluids: New Trends and Applications, October 15-21, 1995, Venice, Italy.

7. Sherwin, S.J. and Karniadakis, G.E., "Tetrahedral hp Finite Elements: Algorithms and Flow Simulations", J. Comp. Phys., vol. 124, p. 14, 1996.

8. Crawford, C.H, Evangelinos, C., Newman, D.J. and Karniadakis, G.E., "Parallel Computations of Turbulence in Complex Geometries", Computer & Fluids, to appear, 1996.

9. Evangelinos, C. and Karniadakis, G.E., "Parallel CFD Benchmarks on Cray Computers", Parallel Algorithms and Applications, vol. 9, pp. 273-298, 1996.

Interactions/Transitions

The PI was invited in the past year to present the AFOSR-sponsored research at: • MIT • Cornell Theory Center • Boston University • Virginia Polytechic Institute • Hong Kong University • International Conference on Parallel Algorithms and

Applications (Wuhan, China) • Institute for Scientific Computing, Chinese Academy of Sciences • Institute of Mechanics, Chinese Academy of Sciences • University of Tokyo • University of Notre Dame • University of California Santa Barbara • Wright Patterson Air Force Base • City College/Levich Institute • AFOSR Boiling AFB DC • Nuclear Regulatory Commission • National Institute of Standards and Technology • AIAA Conference New Orleans.

Discussions have started with commercial CFD software companies to transfer the new discretizations to industrial users. Similarly, discussions are ongoing with the academic and national labs community to replace the older generation of spectral methods currently in use with the new generation of dynamic spectral methods on unstructured grids.