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6. AUTHOR(S) D.M. Pai				
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13. ABSTRACT (Maximum 200 words)				
<p>This report describes a wave-equation based inversion method in layered medium with potential applications to ocean acoustical tomography. This method caters specifically to the strong depth-dependence of the problem by building inversion on top of a depth-dependent background, as opposed to on a homogeneous background in traditional diffraction tomography methods. The method uses eigenmodes (defined with respect to the medium depth dependence) for data decomposition to mode-mode scattering amplitudes which are then used for medium extraction. This approach reduces the original 2-D medium inversion to a set of independent 1-D inversions.</p>				
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**Full-Wave Inversion for
Ocean Acoustical Tomography**

Final Technical Report

to the

Office of Naval Research

by

D. M. Pai

Department of Electrical and Computer Engineering

University of Houston

Houston, TX 77204-4793

Tel: (713) 743-4437 Fax: (713) 743-4444

E-mail: dpai@uh.edu

Background

The goal of this project is to develop a wave-equation based inversion method for extracting the ocean medium from the sound transmission data collected over a tomography acquisition system consisting of two vertical arrays, one source array and one receiver array, suspended in ocean at a fixed distance apart. Ocean acoustical tomography has been an ongoing interest to the underwater acoustics community. Part of the interests stem from the need of knowing the "ocean lens" for doing phase coherent processing on navy-interest acoustic signals.

Wave-equation based inversion methods, as opposed to travel-time based inversion methods, are of interests since diffraction effects are inevitably important for achieving

resolution imaging at a scale comparable to wavelength. However, traditional diffraction-type inversion methods (known as diffraction tomography) specialize mainly in imaging small scatterers buried in homogeneous backgrounds. For an ocean acoustics environment characterized by strong depth-dependence, such methods based on homogeneous background may not be most appropriate

Method of Attack

Our method of attack, therefore, caters specifically to the strong depth-dependence of the problem. It does so by building inversion on top of a depth-dependent background, as opposed to on a homogeneous background in traditional diffraction tomography methods. The medium in between the two arrays is assumed to be having a dominant depth-dependence and a variation (which has both lateral and depth dependence) on top of that background. Our task is to image, from the array transmission data, the depth-dependent background itself as well as the deviation on top of the background.

The method relies on the idea that, in a purely depth-dependent medium, the wave propagation problem is separable in terms of eigenstates (or modes) defined with respect to the medium depth dependence. The modes serve two purposes: they separate the wave equation and they form a set of orthogonal basis functions which we can use for expanding data collected over vertical arrays. In terms of the vertical mode framework, then, the deviation on top of the background causes mode-mode scatterings. Therefore, in inversion, we can decompose data into mode-mode scattering amplitudes, and then use the mode-mode scattering amplitudes to extract the deviation.

This method of medium extraction simplifies the inversion problem. As illustrated in Figure 1, since each mode travels sideways with a distinctive horizontal wave number, a

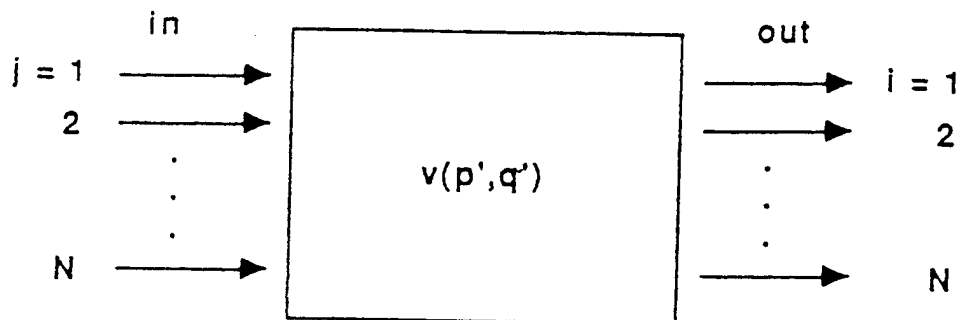


Figure 1 The medium $v(p', q')$ (where p' and q' are the horizontal and vertical wavenumbers, respectively) is probed with N in-modes, each of which is scattered into N out-modes. The scattering amplitude of one mode into another mode samples $v(p', q')$ at a particular p' (the difference between the p of the two modes) and a collection of q' . Inversion is done by grouping data of the same p' and solving for $v(p', q')$, at that p' , as a function of q' - namely, a 1-D inversion with respect to q' .

mode-mode scattering amplitude samples a particular horizontal Fourier component (given by the difference of the horizontal wave numbers of the two modes) of the deviation. The key, then, is that, by collecting mode-mode scattering amplitudes which sample the same horizontal Fourier component, we can invert for the vertical Fourier components of the deviation for that particular horizontal Fourier component alone. Effectively, the original 2-D medium inversion is reduced to a set of independent 1-D inversions (one 1-D inversion for a particular horizontal Fourier component). This is a remarkable simplification.

The significance of the method is illustrated in Figure 2. Inversion in general involves a mapping from a data space D to a model space M . The two spaces, in general, are large and poorly connected (i.e. the matrix linking M to D is large and ill-conditioned and inversion

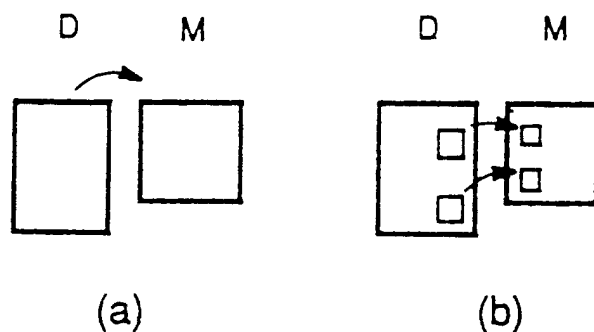


Figure 2 Data space D and model space M . (a) Full-space to full-space connection. (b) Sub-space to sub-space connection.

of that matrix is difficult and unstable). However, in the present method, both spaces are partitioned into sub-regions, each region characterized by a particular horizontal Fourier component of the medium, and only sub-regions of the same horizontal Fourier component are connected. The mapping from a sub-region in D to the corresponding sub-region in M is only a 1-D inverse problem.

The method was originally introduced by the PI [1] and was later developed by an Exxon scientist [2].

Results

The problem consists of two major steps. The first step is to extract, from data, a most appropriate depth-dependent background. The second step is to extract, as described above, from data and the recovered depth-dependent background, mode-mode scattering

amplitudes and then the variation on top of the background.

Let the source array and the receiver array each be consisting of N elements. The data, for each frequency component, can then be expressed as an $N \times N$ matrix, which we shall call the data matrix. Specifically, we shall call it the depth-domain data matrix, with the first column corresponding to the received data at the N receivers from the first source, etc.

Recover the depth-dependent background

To facilitate discussion, first we consider how to recover the depth dependence in a purely depth-dependent medium. In such a medium, a vertical mode (defined by the medium depth dependence) generated by the source array would arrive at the receiver array as the same mode. That is, the data matrix, which is non-diagonal in the depth-domain, would be purely diagonal in the vertical mode domain. Thus, by a numerical diagonalization of the $N \times N$ data matrix from the original depth-domain, one can recover from the diagonalizing matrix N vertical eigenmodes and from the resulting diagonal values the corresponding horizontal wavenumbers of the N modes. Each one of the eigenmode, together with its horizontal wavenumber, can then be used to generate the vertical velocity profile from the modal equation defining the eigenmodes.

Actually, the problem is a bit more complicated since, due to the finite array length and finite transducer spacings, the recovered eigenmodes from the diagonalization of the data matrix are non-exact. Further, at where eigenmodes vanish (nodal points), the modal equation defining the eigenmodes carries no information about the velocity. The rescue comes from data redundancy. An $N \times N$ matrix can generate N modes. With N_f frequency components, therefore, the data redundancy in determining the vertical dependence is $N \times N_f$ fold. The nodal points are no longer a problem since different modes vanish at different points. By demanding that the modal equation, in which the vertical dependence

is the unknown, are satisfied by all the non-exact modes as much as possible - basically a least square problem - one can solve for an optimum vertical velocity profile.

Another complication is that the medium is not purely depth-dependent. As such, the data matrix is only nearly diagonal in the eigenmode domain. Thus, a numerical diagonalization of the data matrix from the depth domain yield non-exact eigenmodes. Again, data redundancy is the solution to this problem of non-exact eigenmodes.

The first example is illustrated in Figure 3. Two arrays, 90m in length, each consisting of 49 transducers with a spacing of 1.875m between each two transducers, are spaced 33.75m apart in a three layer medium (with velocities 1800m/sec, 2100m/sec and 2400m/sec). The figure shows simulated data using the finite-difference time-marching method (with absorbing boundary conditions), along with the time variation and the frequency spectrum of the sources. Figure 4 shows, as an illustration, for the frequency component 404.36Hz (corresponding to a wavelength about 5.2m for a velocity of 2100m/sec), the first three modes and the recovered vertical velocity profiles using each mode *individually*. Results show strong eigenmode oscillations in the lower velocity region and, correspondingly, as expected, poor results in the recovered velocity profile since, as explained, the nodal points (where eigenmodes vanish) do not carry information about the velocity. Notice that higher modes produce worse results because of the higher number of nodal points. These results illustrate the importance of using data redundancy. Figure 5 shows the obtained velocity profile using 15 modes (the first three modes at five frequencies ranging from 316.46Hz to 386.78Hz at 17.58Hz interval) *individually* and *collectively*. The obtained velocity profile using 15 modes collectively are all within 20m/sec for the three layers.

The second example, as illustrated in Figure 6, shows the study carried out for the same three layer medium, but now with a square-shaped scatterer embedded in the second layer. The reproduced velocity profile obtained using the same set of data (namely, the

first three modes from five frequency components as given above) agree with the lateral average of the actual velocity profile very well. This example illustrate that this algorithm is capable of producing perceptible difference in the background due to weak variations.

Recover the variation on top of the background

As a first example, we try to image the square-shaped scatterer shown in Figure 6 using the recovered background shown there. However, at this time, this work is still in progress.

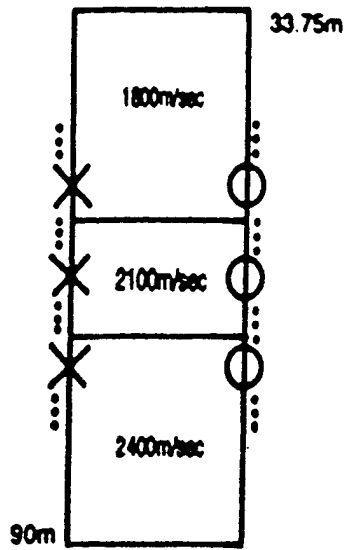
Conclusions

A method for diffraction tomography in layered medium has been proposed. The first part of the algorithm, namely recovering the layered background, has been illustrated. The second part, namely recovering the variation on top of the background, is still in progress.

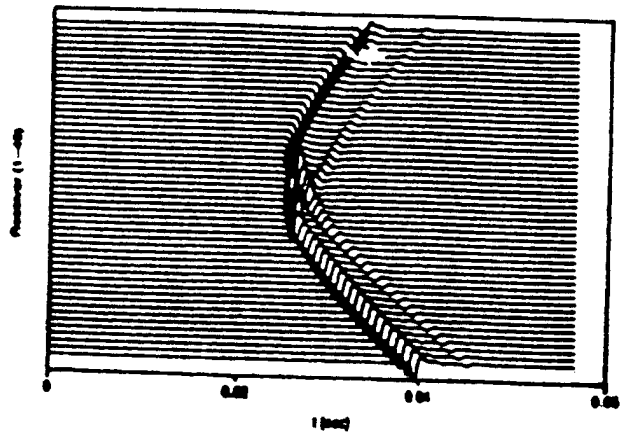
At this initial stage, in order to understand the method, we have studied using simple but unrealistic examples. After this initial stage, we hope to be able to extend the study to more realistic ocean acoustic situations.

D. References

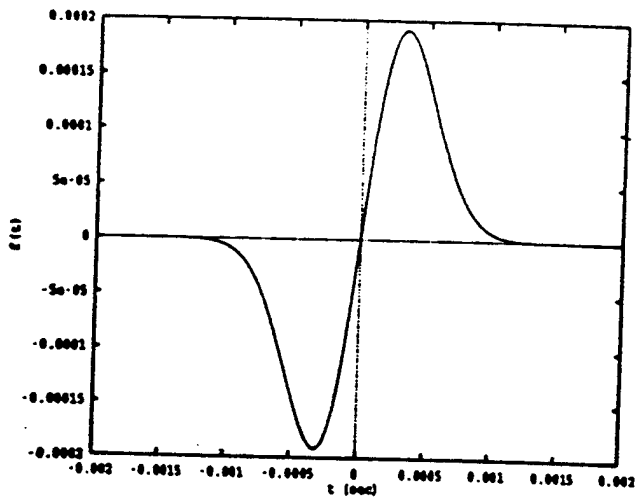
- [1] Pai, D. M., 1990, Crosshole seismics using vertical eigenstates, *Geophysics* 55, 815-820.
- [2] Dickens, T. A., 1994, Diffraction tomography for crosswell imaging of nearly layered media, *Geophysics* 59, 694-706.



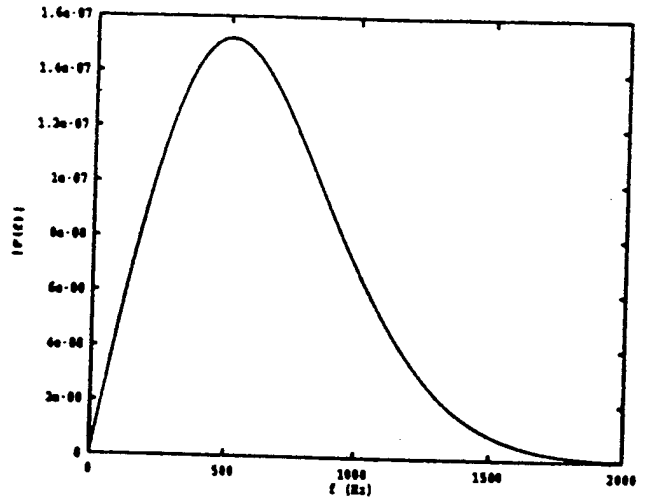
(a)



(b)

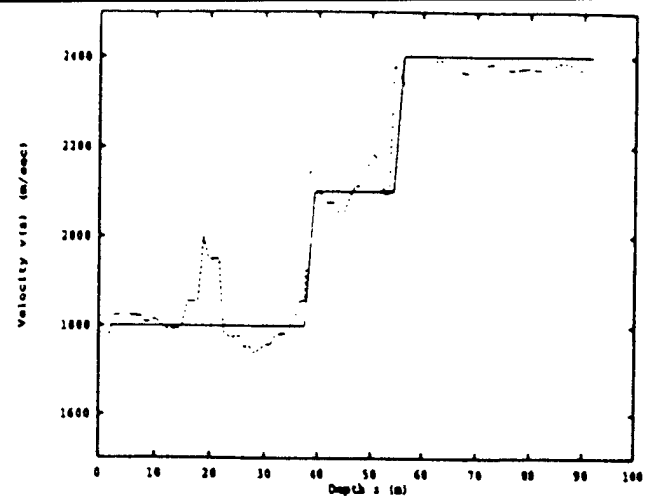
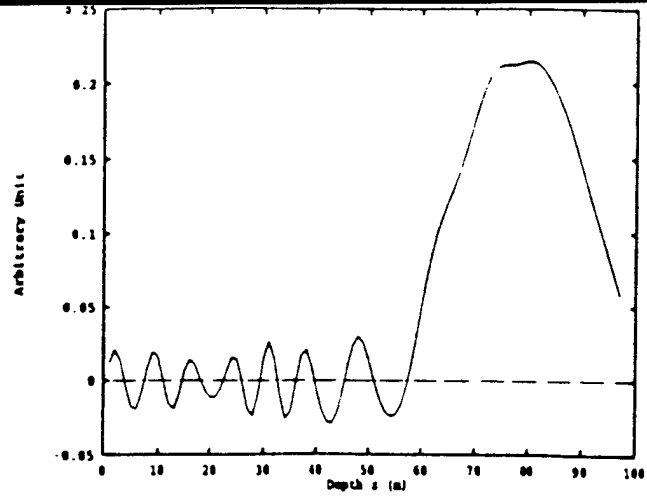


(c)

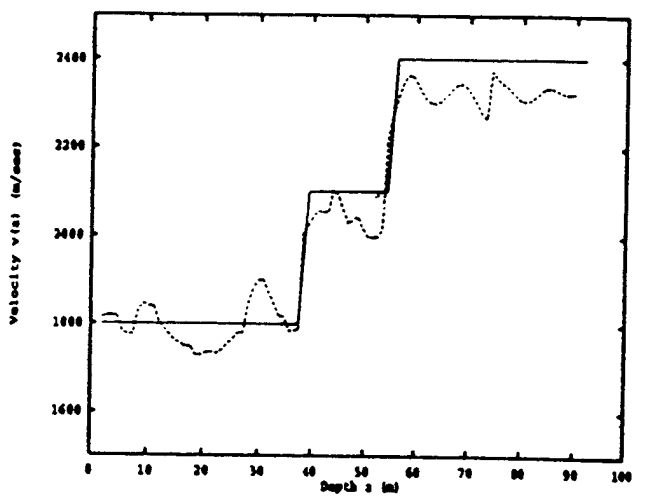
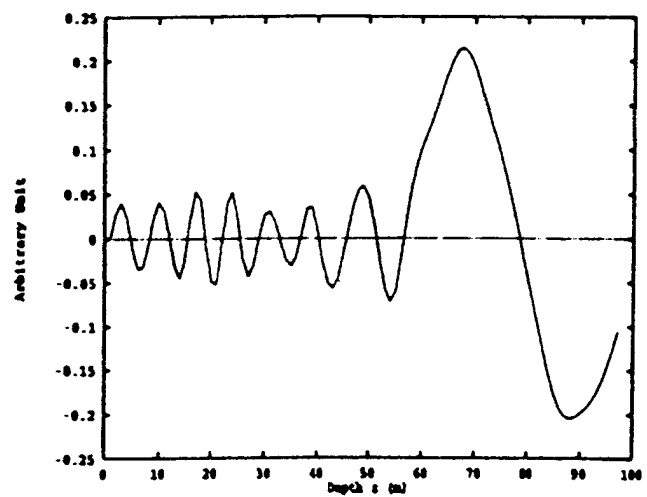


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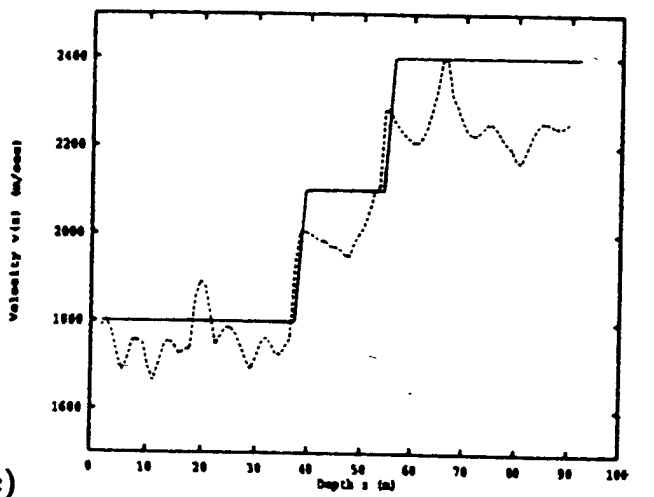
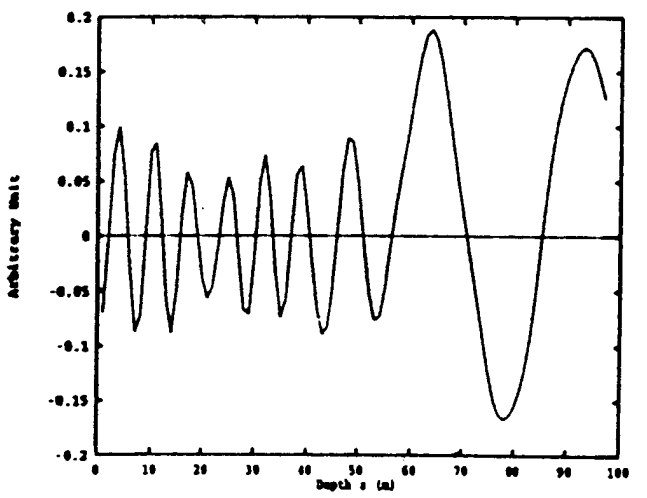
Figure 3 A three layer model and simulated data. (a) The model geometry. (b) The received signal at 49 receivers from the central source. (c) The time dependence of the source. (d) The frequency spectrum of the source.



(a)



(b)



(c)

Figure 4 Left panel shows the recovered eigenmode and the right panel shows the corresponding inverse velocity profile (dashed line), compared with the actual velocity profile (solid line). (a) The first mode. (b) The second mode. (c) The third mode.

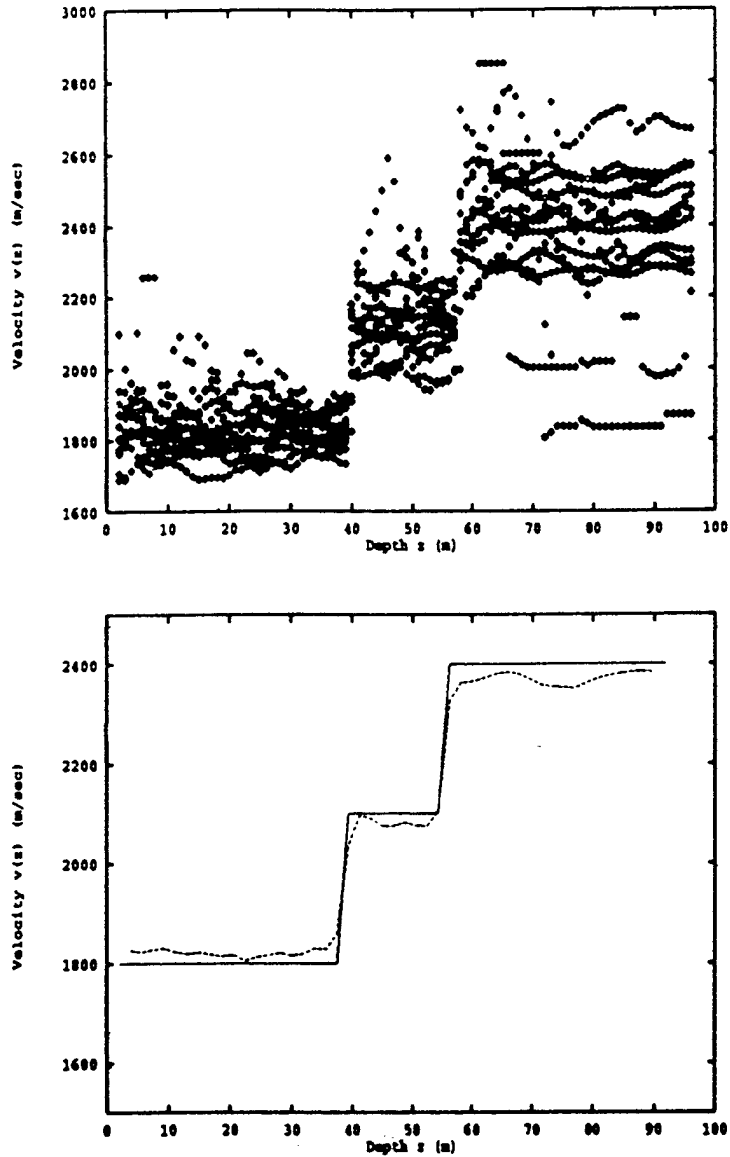
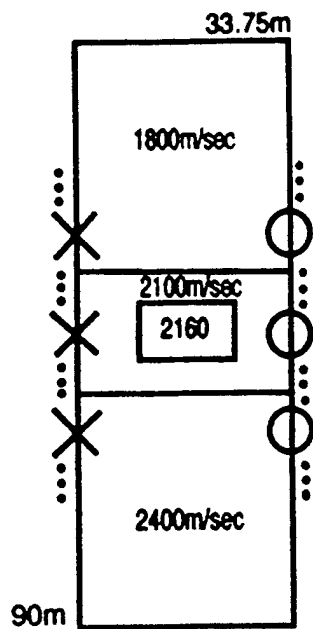
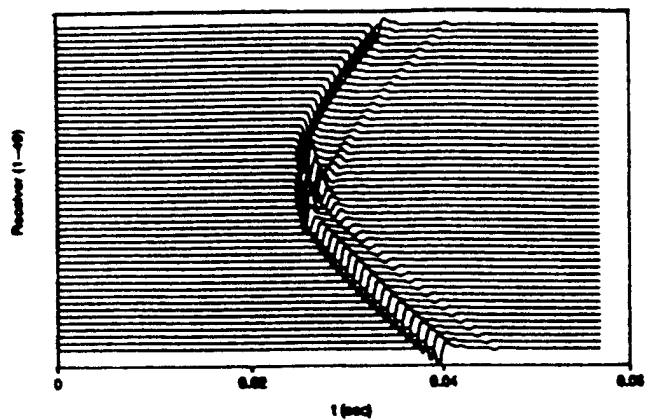


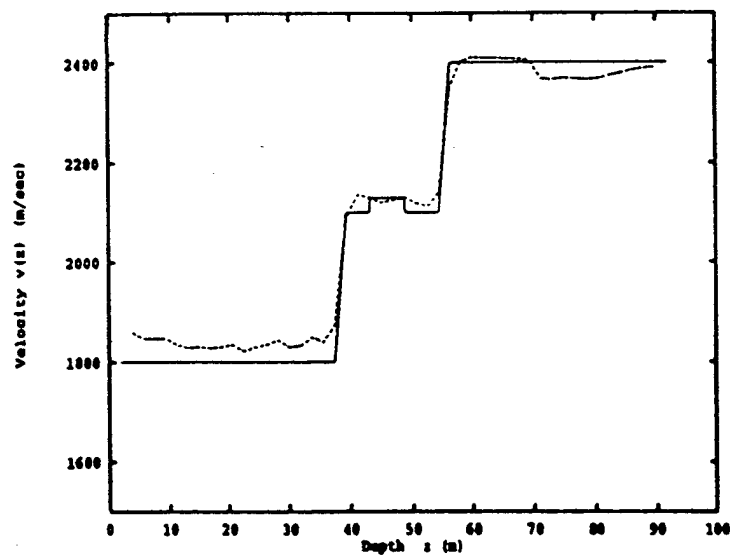
Figure 5 The top diagram shows the inverse velocity profile using 15 modes individually. The bottom diagram shows the inverse velocity profile using the same 15 modes collectively.



(a)



(b)



(c)

Figure 6 A three layer model embedded with a scatterer. (a) The model geometry. (b) The received signal at 49 receivers from the center source. (c) The inverse velocity profile (dashed line) compared with the actual velocity profile (solid line).