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TACTS TRACKING ACCURACY ANALYSIS

WEAPON SIMULATION GUIDELINES

NOT FOUND ON CD

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SECTION 1

INTRODUCTION

This study establishes guidelines for determining how accurately aircraft position and velocity should be measured on a Tactical Aircrew Combat Training System (TACTS) range. The method of analysis is based on the sensitivities of Measures of Effectiveness (MOE) computed from TACTS weapon simulations to input aircraft position and velocity at the time of weapon launch. On the basis of desired MOE accuracies, maximum allowable errors (standard deviations) are determined for launch aircraft position and velocity estimates for a variety of weapon/target scenarios.

SECTION 2

APPROACH

TACTS ranges support the training of Navy pilots. One range function is to track the status of each aircraft, including position, speed, and attitude. The accuracy in estimated values for these parameters depends upon many factors, including geography, hardware, telemetry, and software for data processing.

What accuracies are required? In part, the answer lies in subsequent uses of parameter estimates. One such use is an input to weapon simulation programs which simulate weapon performance and calculate appropriate MOEs.

Aircraft position is typically specified by values for three coordinates, X, Y, Z, and speed is specified by three velocity components, V_x, V_y, V_z. The following analysis focuses on the required accuracy of these position and velocity component values.

Suppose there were no errors in the determination of position and velocity. Then, assuming that the weapon simulation programs are completely accurate, a correct MOE value would be computed in every instance. In actuality, there are errors in position and velocity estimates, and consequently weapon simulations do not always compute an accurate MOE value. In general, the larger the errors in input position and velocity values, the larger will be the error in computed MOE. The actual error sizes are not known in advance; if so, they could be eliminated in the processing of data. At best, error sizes can be characterized by using the language of probability (i.e., small errors imply highly accurate MOEs).

To formulate the analytical method for the determination of TACTS range accuracy requirements the following guidelines were established:

- The method should be general enough to apply to various aircraft/weapon/target combinations.
- The method should require only readily available input.
- The method should produce quantitative output which would support trade-off analysis and provide flexibility to the decision-making process.

The goal of this method was to estimate parametrically the amounts by which aircraft position and velocity component values, as determined by TACTS range telemetry and processing, could be in error while requiring that TACTS weapon simulations compute a reasonably accurate MOE value reasonably often.

As a first step in developing the analytical method, a Performance Goal was defined as follows:

Performance Goal - The accuracy with which the MOE is to be determined.

Maximum allowable uncertainties in position and velocity component values consistent with achievement of this Performance Goal were determined. For example, for target=point target and weapon=bomb, probability of kill (P_k) is an MOE. A Performance Goal might be a relative error of no more than ten percent (i.e., the difference between computed and correct values for P_k should be no larger than ten percent of the correct value for P_k). Achievement of this goal places restrictions on the magnitudes of errors in the values of position and velocity components.

The errors in position and velocity components are treated as unbiased, independent, normal random variables. Qualitatively this means that errors are just as likely to be positive as negative with the average error being zero. Further, the error in determining one position or velocity component value is not related to the error in determining a different component.

For errors which are normal, the likelihood of errors is determined by the distribution parameter "standard deviation" or sigma (σ). The larger the value of σ, the more likely will be the occurrence of large errors. The magnitudes of position and velocity component errors for achievement of a Performance Goal can be stated in terms of error distribution standard deviations.

A requirement that the Performance Goal must be achieved almost always will require that TACTS range position and velocity estimates be very accurate almost always. On the other hand, if achieving the Performance Goal less frequently is acceptable, then TACTS range estimates must be accurate less frequently (and consequently accuracy requirements would be relaxed). Thus, we define a Performance Level as follows:

Performance Level - The frequency (fraction of launches) that the Performance Goal is achieved.

Performance Levels are varied from .75 to .95 in this analysis. Both GDOP ≤ 5 and $5 <$ GDOP ≤ 12 are considered (GDOP stands for Geometric Dilution of Precision, a function of the TACTS range physical layout of position-locating equipment, reference a). Under favorable conditions (GDOP ≤ 5), vertical position (Z) and velocity (V_z) components are assumed to be determined with the same accuracy as horizontal (X/V_x and Y/V_y) values. Under less favorable conditions ($5 <$ GDOP ≤ 12), standard deviations for errors in vertical component determinations are assumed to be four times as large as the standard deviations in horizontal component determinations.

For each combination of target, weapon, and MOE, a parametric set of TACTS range accuracy requirements is generated by the following general procedure*:

1. Specify the Performance Goal: the accuracy with which the MOE is to be determined (e.g., computed P_k must be within ten percent of actual P_k).

2. Identify values for position and velocity components recommended by tactical considerations (these are "intended" values at weapon release).

* Documented more fully in Appendix A.

3. Using the weapon simulation:

   a. Simulate weapon launch with "intended" values and move target to weapon impact position.

   b. For each of several variations** (representing "pilot error") in position and velocity component values about their "intended" values:

      (1) Simulate weapon launch, determining computed MOE (the "correct" value).

      (2) In each of the six components (three position and three velocity), one at a time, determine the extent to which the value can be changed while still producing a computed MOE value which is within the accuracy of the "correct" value, specified in the Performance Goal (Step 1). This determination is done by running the weapon simulation with altered input values.

   c. At this stage, several component value intervals have been determined which represent allowable variability under the accuracy requirement specified in Step 1.

   d. Repeat from Step 2 if there are additional tactical cases of importance.

4. Specify the Performance Level (i.e., fraction of time that the computed MOE value is to be within the accuracy specified for the Performance Goal in Step 1).

5. Using the BASIC program DISCRETE_FIT***, compute combinations of standard deviations (σ's) for position and velocity component values for which errors within the bounds identified in Step 3.c occur with the frequency (fraction of time) specified in Step 4. The resulting σ's constitute TACTS range parametric accuracy requirements produced by this methodology for the selected combination of target, weapon, and MOE at the specified Performance Level.

6. Repeat from Step 4 for additional Performance Levels.

To summarize, the TACTS weapon simulations were used to determine in absolute terms how much variability can be tolerated in position and velocity component determination while still maintaining acceptable MOE accuracy. Treating these absolute quantities as random variables, the statistical approach was to compute corresponding standard deviations which assure that tolerable errors in position and velocity component determination are achieved with specified frequency.

Presented in the following sections is an extensive collection of computed results pertaining to the following weapon/target combinations:

** See Appendix B.
*** See Appendix C.

1. Bombing a point target (three cases)
2. Bombing a runway (one case)
3. Bombing a point target - retarded bomb (one case)
4. Firing a rocket at a point target (three cases)
5. Firing a gun in air-to-air combat (seven cases)

The different cases considered in each scenario represent different intended weapon launch conditions reflecting tactical options. Cases are defined, case data is reported, scenario MOEs and Performance Goals are given, and computed results are presented graphically, parametrized by Performance Level. Illustrations of the interpretation and use of these parametrically presented results are provided.

SECTION 3

TACTS RANGE ACCURACY GUIDELINES
Bombing a Point Target Scenario

The following conditions are assumed for the point target bombing scenario:

Target - Small fighter aircraft on the ground

Weapon - MK-82 Mod 1 (Std) electric fuse bomb

Aircraft - F/A-18

MOE - P_k

Performance Goal - No more than ten percent relative error (alternately, no more than five percent relative error).

Performance Levels - .75, .80, .85, .90, .95

Presented in Table 1 are input values for position, speed, pitch, heading, and the velocity components computed from the latter dynamic parameters for the three cases investigated in this scenario. The bomb drop parameters were chosen from a Tactical Manual (reference b).

In order to simulate pilot error in the attainment of "intended" launch parameter values, the method documented in Appendix B was employed which yielded the values presented in Table 1 for each case investigated.

Figures 1 and 2 present in graphical and parametric form scenario-specific guidance for assessing current TACTS range accuracies or establishing future accuracy requirements. Figures 1 and 2 pertain to GDOP <5 and to 5< GDOP ≤12, respectively. Figures 3 and 4 present guidance derived under the more stringent Performance Goal that there be no more than five percent relative error in the determination of P_k.

To use the figures, first select a value for V, the standard deviation for velocity component error, on the horizontal axis, and a value for L, the standard deviation for position component error, on the vertical axis. The corresponding point (V,L) defines the Performance Level; for instance, if (V,L) falls within (below) the curve labelled .85, then for TACTS range accuracies L and V, the Performance Level will be at least 85 percent. If (V,L) falls outside (above) the curve labelled .85, then for TACTS range accuracies L and V, the Performance Level will be less than 85 percent. Further, all (V,L) pairs within the .85 contour will yield Performance Levels of at least 85 percent.

Figures 1 through 4 provide a ready vehicle for establishing required TACTS range accuracies for a single target type under GDOP <5 or 5< GDOP <12 conditions. For instance, consider the following <u>typical conclusions</u> obtainable from Figure 1:

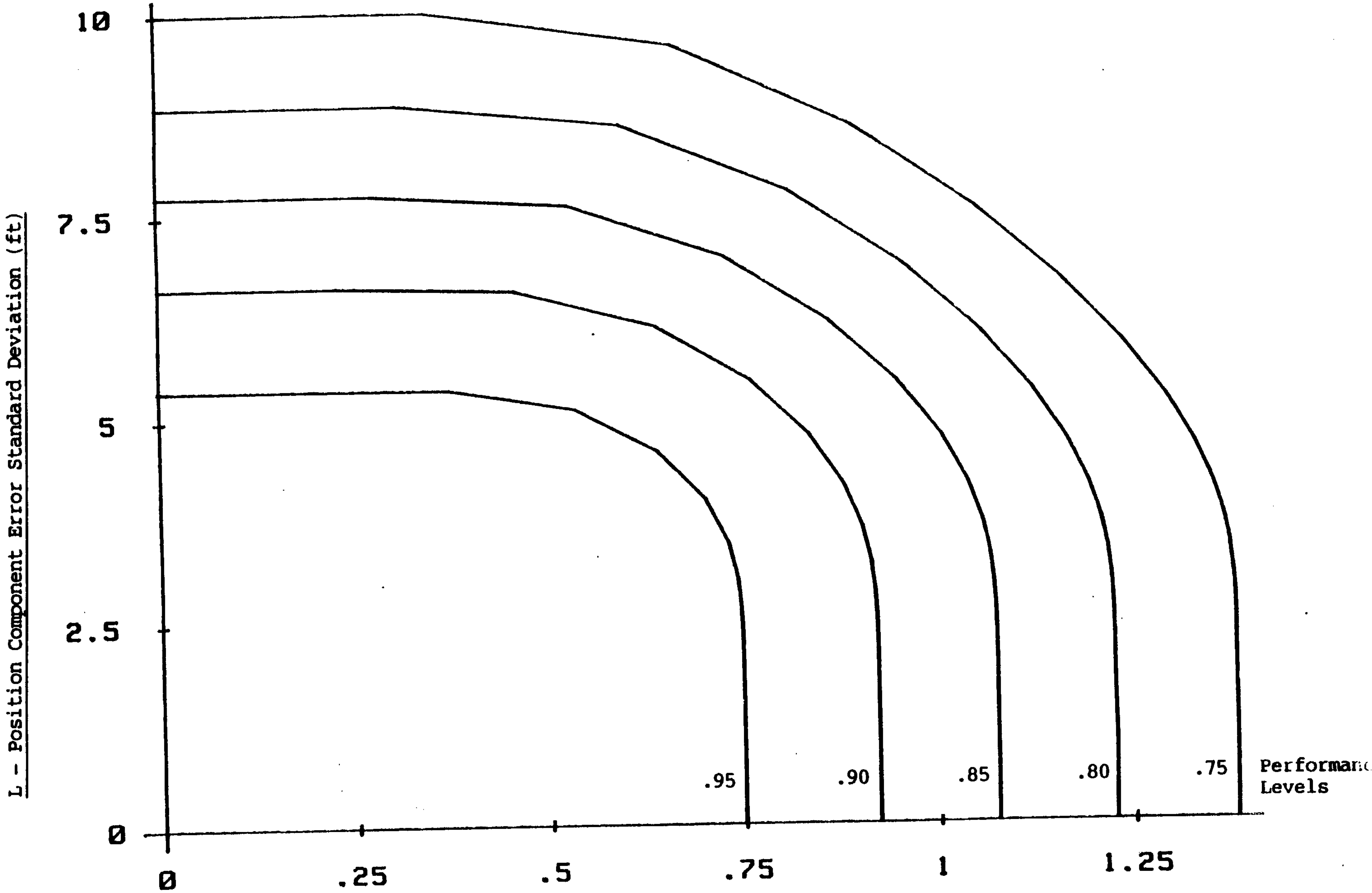
1. If telemetry leads to a standard deviation in aircraft velocity components of V = 1 ft/sec and a standard deviation in aircraft position components of L = 5 ft then the bomb simulation would produce a value for P_k whose relative error is no more than 10 percent between 85 percent and 90 percent of the time.

2. If velocity component error standard deviations are V $\geq$ 1 ft/sec, then the Performance Level can never be as large as 87.5 percent.

3. If position component error standard deviations are L = 1 ft, then velocity component error standard deviations (V) must not exceed approximately 6 ft/sec if a Performance Level of 85 percent is required.

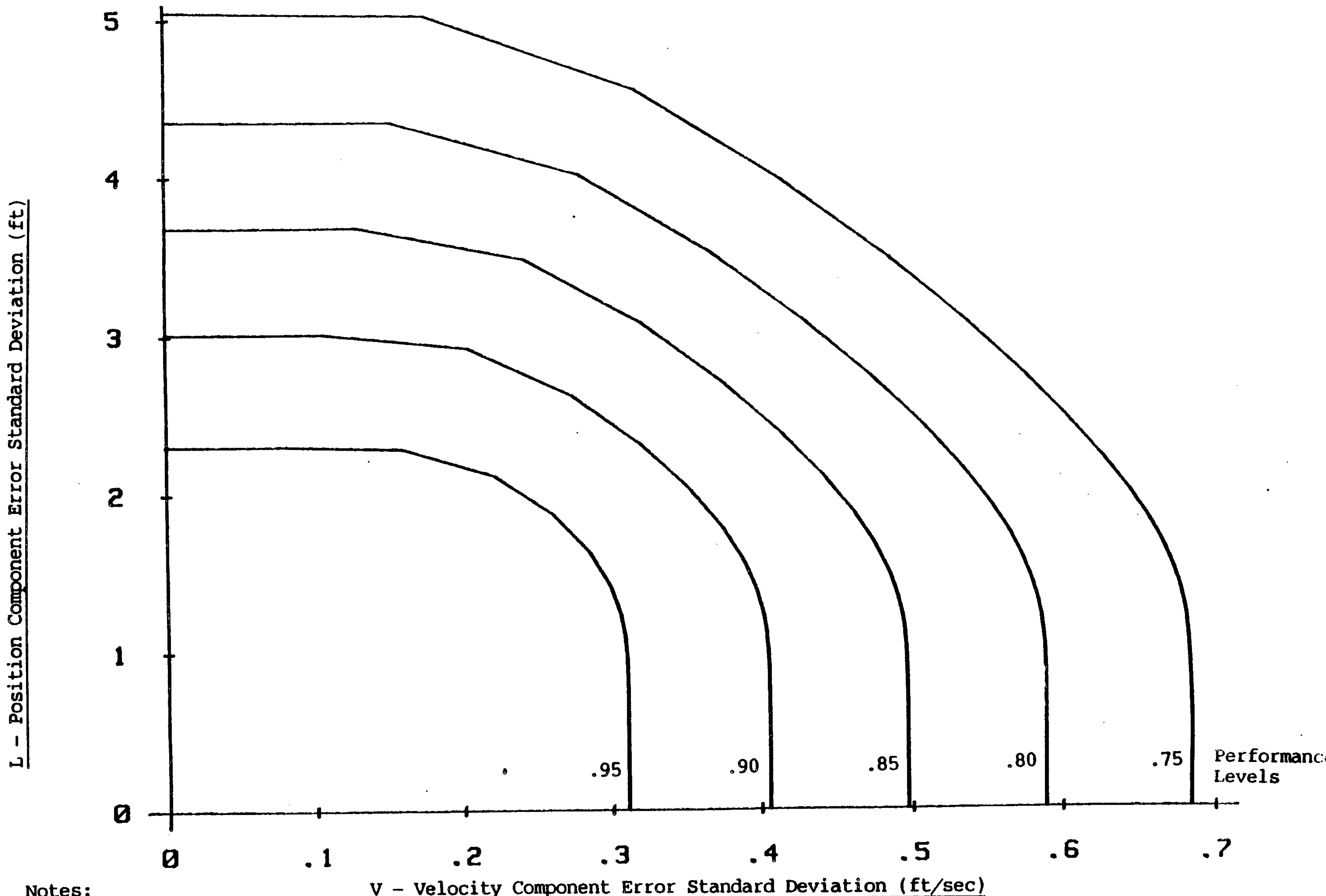
TABLE 1. CASE DATA - SIMULATED BOMB DROPS ON A POINT TARGET

Case	1	2	3
(1) Intended aircraft position relative to aim point on target at time of bomb drop			
Xpos	4129 ft	8346 ft	4790 ft
Ypos	0 ft	0 ft	0 ft
Zpos	5000 ft	2000 ft	2000 ft
(2) Intended dynamic parameter values at time of bomb drop			
Speed	802 ft/sec	760 ft/sec	760 ft/sec
Pitch	-45°	0°	-15°
Heading	0°	0°	0°
(3) Velocity components computed from dynamic data			
V_x	567.1 ft/sec	760 ft/sec	724.44 ft/sec
V_y	0 ft/sec	0 ft/sec	0 ft/sec
V_z	-567.1 ft/sec	0 ft/sec	-194.12 ft/sec
(4) Sigmas computed for pilot error			
σx	17.8 ft	41.85 ft	22.75 ft
σy	19.8 ft	25.9 ft	15.81 ft
σz	24.3 ft	20.3 ft	12.83 ft
σV_x	2.35 ft/sec	3.87 ft/sec	3.48 ft/sec
σV_y	2.7 ft/sec	2.35 ft/sec	2.4 ft/sec
σV_z	3.3 ft/sec	1.85 ft/sec	1.95 ft/sec



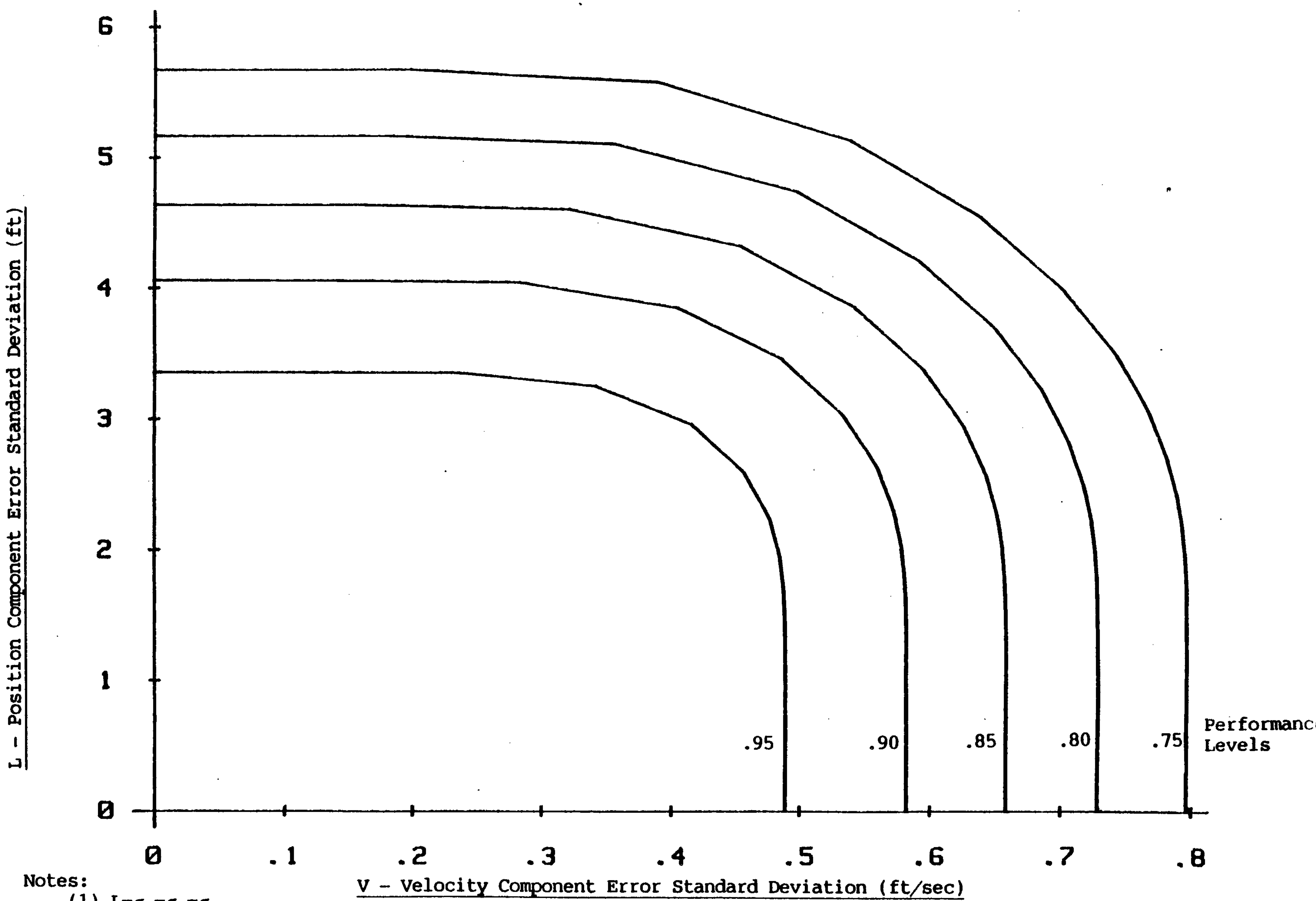
Notes:
(1) $L=\sigma_x=\sigma_y=\sigma_z$
(2) $V=\sigma V_x=\sigma V_y=\sigma V_z$

Figure 1. TACTS Range Accuracy - Bomb vs. Point Target
(GDOP $\leq$ 5, 10% Relative Error in P_k)



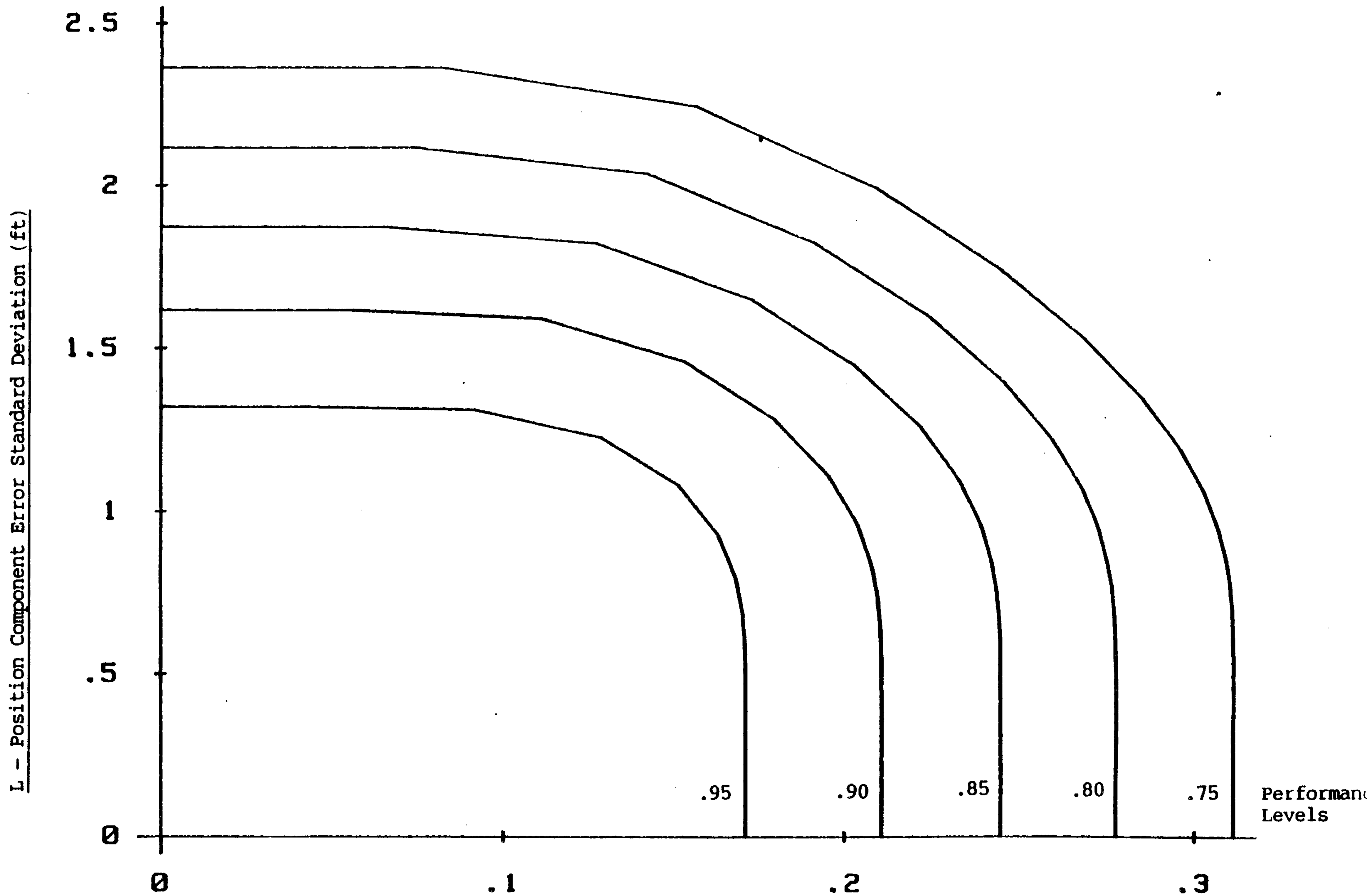
Notes:
(1) $L=\sigma_x=\sigma_y=\sigma_z/4$
(2) $V=\sigma V_x=\sigma V_y=\sigma V_z/4$

Figure 2. TACTS Range Accuracy - Bomb vs. Point Target
($5 < GDOP \leq 12$, 10% Relative Error in P_k)



Notes:
(1) $L=\sigma_x=\sigma_y=\sigma_z$
(2) $V=\sigma V_x=\sigma V_y=\sigma V_z$

Figure 3. TACTS Range Accuracy - Bomb vs. Point Target ($GDOP \leq 5$, 5% Relative Error in P_k)



Notes:
(1) $L=\sigma_x=\sigma_y=\sigma_z/4$
(2) $V=\sigma V_x=\sigma V_y=\sigma V_z/4$

Figure 4. TACTS Range Accuracy - Bomb vs. Point Target
(5 < GDOP $\leq$ 12, 5% Relative Error in P_k)

SECTION 4

TACTS RANGE ACCURACY GUIDELINES
Bombing a Runway Scenario

The following conditions are assumed for the runway bombing scenario:

Target - Long runway

Weapon - MK-82 Mod 1 (Std) electric fuse bomb

Aircraft - F/A-18

MOE - Crater Diameter

Performance Goal - Compute correct crater diameter exactly

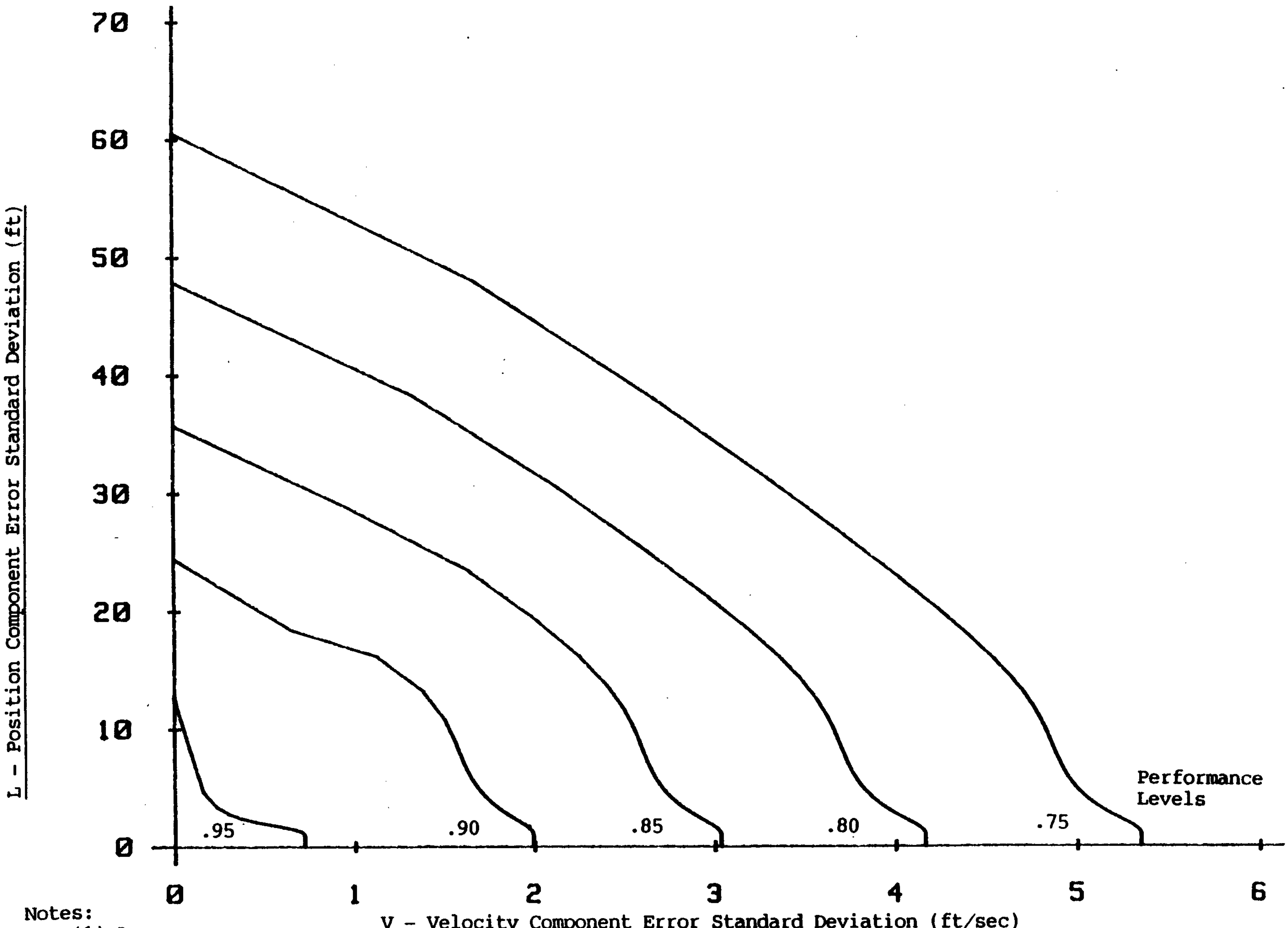
Performance Levels - .75, .80, .85, .90, .95

Presented in Table 2 are input values for position, speed, pitch, heading, and the velocity components computed from the latter dynamic parameters for the case investigated in this scenario. The parameters were chosen from a Tactical Manual (reference b). In addition, sigmas computed for pilot error are listed for the position and velocity components.

Figures 5 and 6 present in graphical and parametric form scenario-specific guidance regarding TACTS range accuracy. Figures 5 and 6 pertain to GDOP ≤ 5 and to $5 <$ GDOP ≤ 12, respectively.

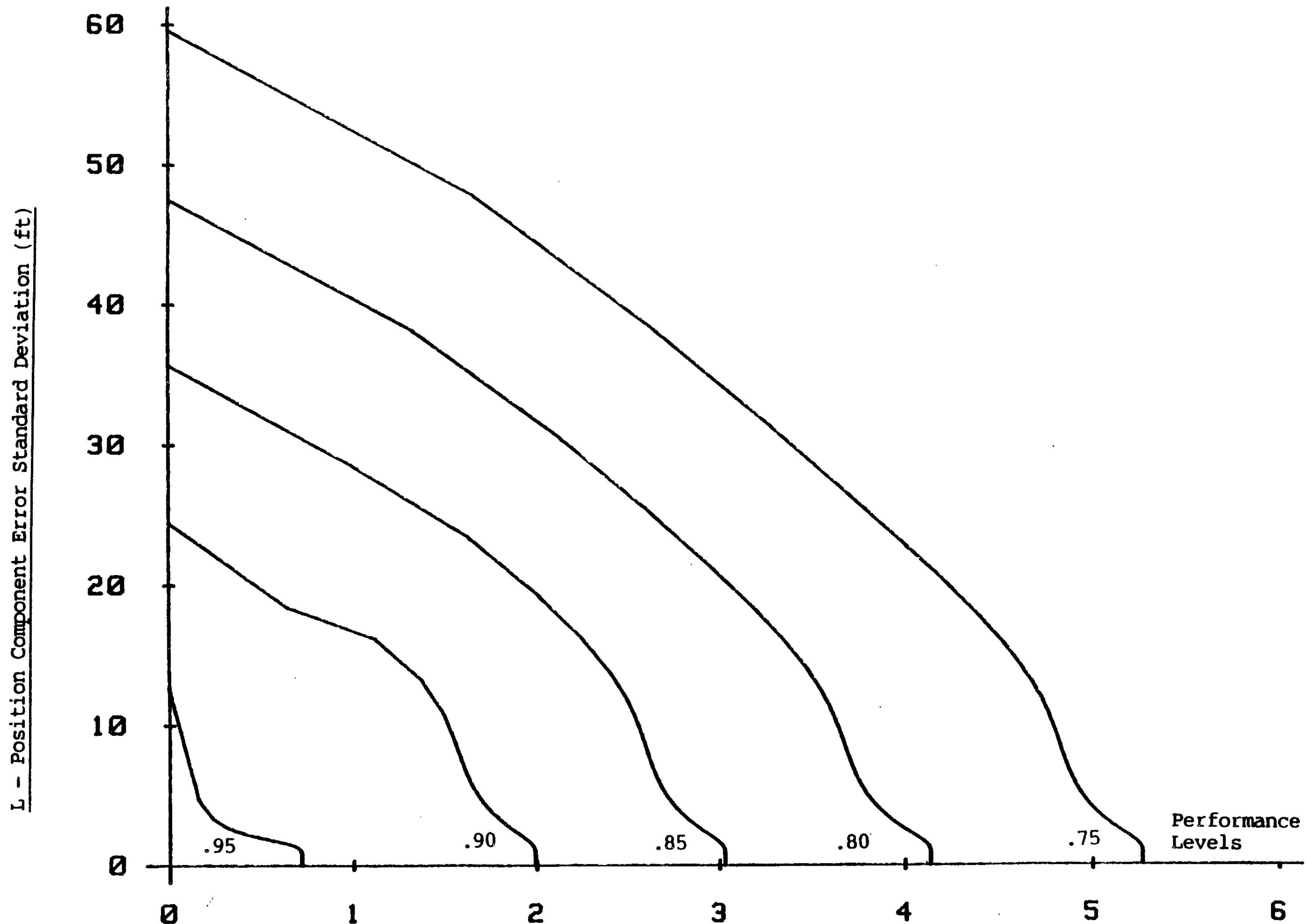
TABLE 2. CASE DATA - SIMULATED BOMB DROPS ON A RUNWAY

Case	1
(1) Intended aircraft position relative to aim point on target at time of bomb drop	
Xpos	6206 ft
Ypos	0 ft
Zpos	8000 ft
(2) Intended dynamic parameter values at time of bomb drop	
Speed	844 ft/sec
Pitch	-45°
Heading	0°
(3) Velocity components computed from dynamic data	
V_x	596.8 ft/sec
V_y	0 ft/sec
V_z	-596.8 ft/sec
(4) Sigmas computed for pilot error	
σx	25.8 ft
σy	30.9 ft
σz	41.1 ft
σV_x	2.4 ft/sec
σV_y	2.9 ft/sec
σV_z	3.9 ft/sec



Notes:
(1) $L=\sigma_x=\sigma_y=\sigma_z$
(2) $V=\sigma V_x=\sigma V_y=\sigma V_z$

Figure 5. TACTS Range Accuracy - Bomb vs. Runway Target (GDOP $\leq$ 5, Correct Crater Diameter)



Notes:
(1) $L=\sigma_x=\sigma_y=\sigma_z/4$
(2) $V=\sigma V_x=\sigma V_y=\sigma V_z/4$

Figure 6. TACTS Range Accuracy - Bomb vs. Runway Target
($5 < GDOP \leq 12$, Correct Crater Diameter)

SECTION 5

TACTS RANGE ACCURACY GUIDELINES
Bombing a Point Target With Retarded Bomb Scenario

The following conditions are assumed for the bombing of a point target with a retarded bomb scenario:

Target - Small fighter aircraft on the ground

Weapon - MK-82 Snakeye retarded bomb

Aircraft - F/A-18

MOE - P_k

Performance Goal - No more than ten percent relative error

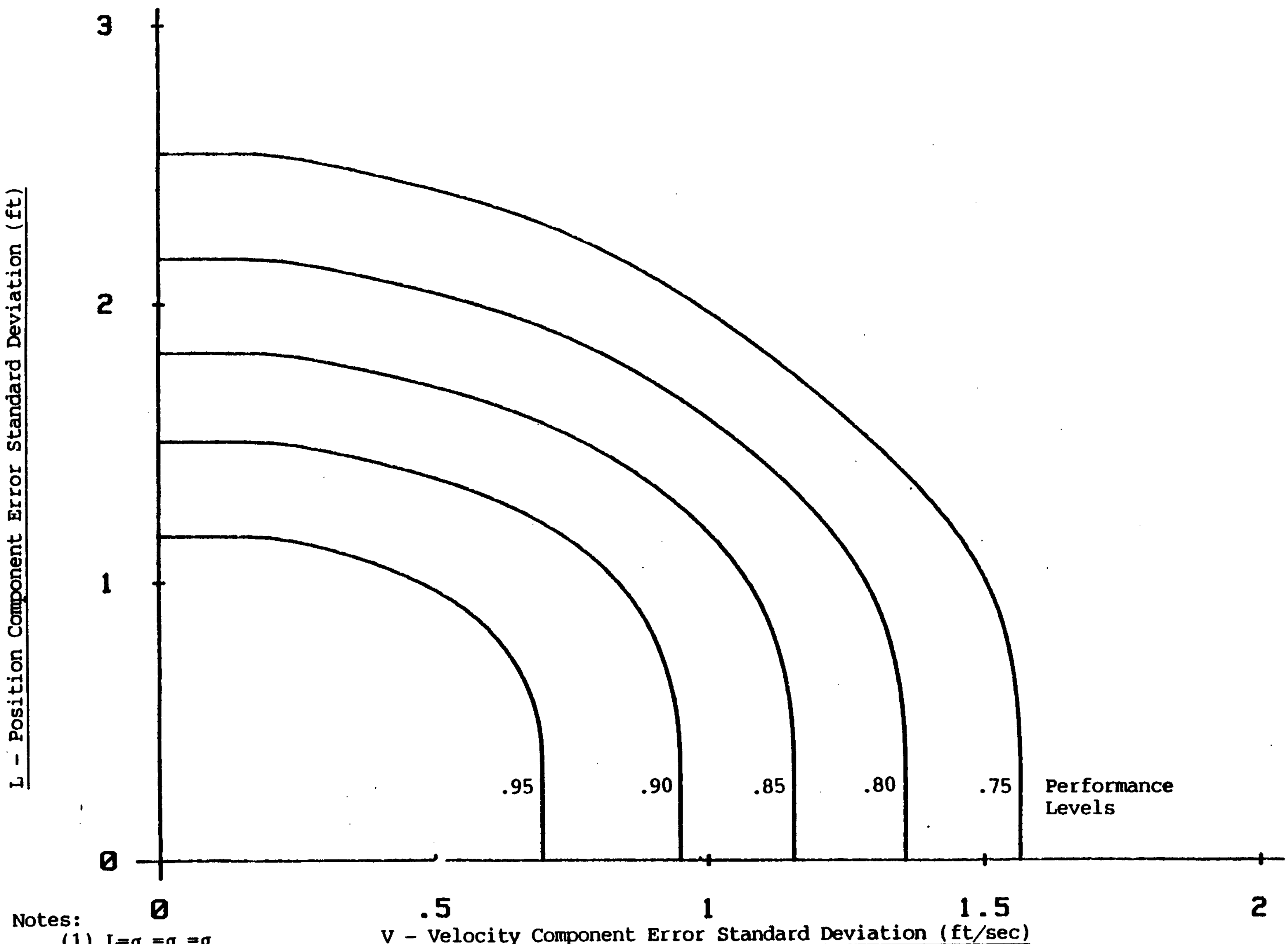
Performance Levels - .75, .80, .85, .90, .95

Presented in Table 3 are input values for position, speed, pitch, heading, and the velocity components computed from the latter dynamic parameters for the one case investigated in this scenario. Pilot error sigmas for position and velocity components are also shown. The initial input parameters were chosen from a Tactical Manual (reference b). A retarded bomb was used in this scenario for compatibility with the low-altitude, high-speed "pop-up" delivery technique.

Figures 7 and 8 present in graphical and parametric form scenario-specific guidance regarding TACTS range accuracy guidelines. Figures 7 and 8 pertain to GDOP ≤ 5 and to $5 <$ GDOP ≤ 12, respectively.

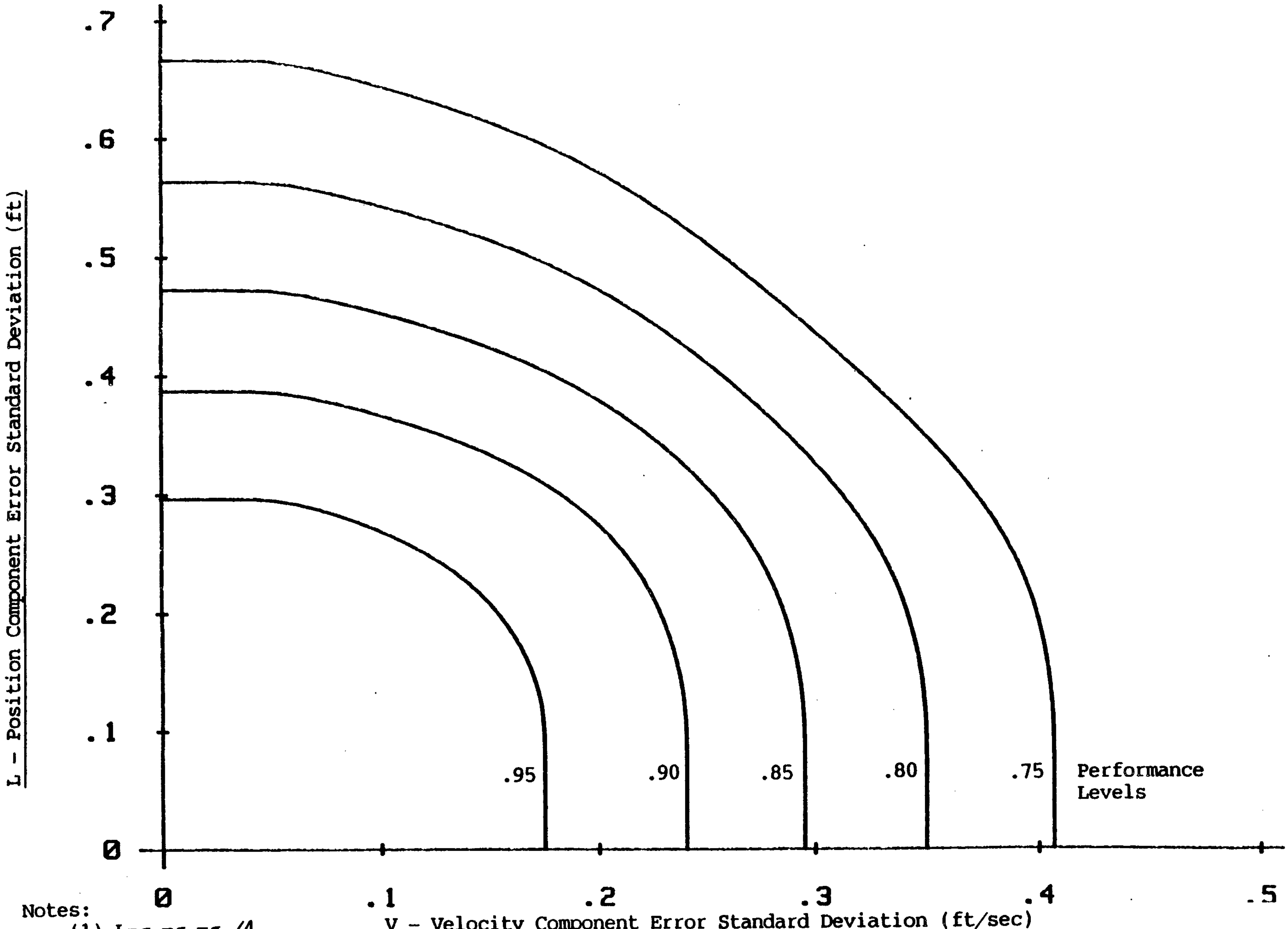
TABLE 3. CASE DATA - SIMULATED BOMB DROPS ON A POINT TARGET - RETARDED BOMB

Case	1
(1) Intended aircraft position relative to aim point on target at time of bomb drop	
Xpos	1468 ft
Ypos	0 ft
Zpos	500 ft
(2) Intended dynamic parameter values at time of bomb drop	
Speed	760 ft/sec
Pitch	-15°
Heading	0°
(3) Velocity components computed from dynamic data	
V_x	724.44 ft/sec
V_y	0 ft/sec
V_z	-194.2 ft/sec
(4) Sigmas computed for pilot error	
σx	12.9 ft
σy	5.14 ft
σz	3.78 ft
σV_x	5.16 ft/sec
σV_y	2.04 ft/sec
σV_z	1.5 ft/sec



Notes:
(1) $L=\sigma_x=\sigma_y=\sigma_z$
(2) $V=\sigma V_x=\sigma V_y=\sigma V_z$

Figure 7. TACTS Range Accuracy - Retarded Bomb vs. Point Target (GDOP $\leq$ 5, 10% Relative Error in P_k)



Notes:
(1) $L=\sigma_x=\sigma_y=\sigma_z/4$
(2) $V=\sigma V_x=\sigma V_y=\sigma V_z/4$

Figure 8. TACTS Range Accuracy - Retarded Bomb vs. Point Target ($5 < GDOP \leq 12$, 10% Relative Error in P_k)

SECTION 6

TACTS RANGE ACCURACY GUIDELINES
Rocket Firing at a Point Target Scenario

The following conditions are assumed for the rocket at a point target scenario:

Target - Small fighter aircraft on the ground

Weapon - MK-4/MK-67 WP/M 427 2.75" Folding Fin Aircraft Rocket (FFAR)

Aircraft - F/A-18

MOE - Miss Distance (In current rocket simulations, the post impact scoring routine is not invoked; therefore, P_k is not calculated.)

Performance Goal - No more than ten percent relative error

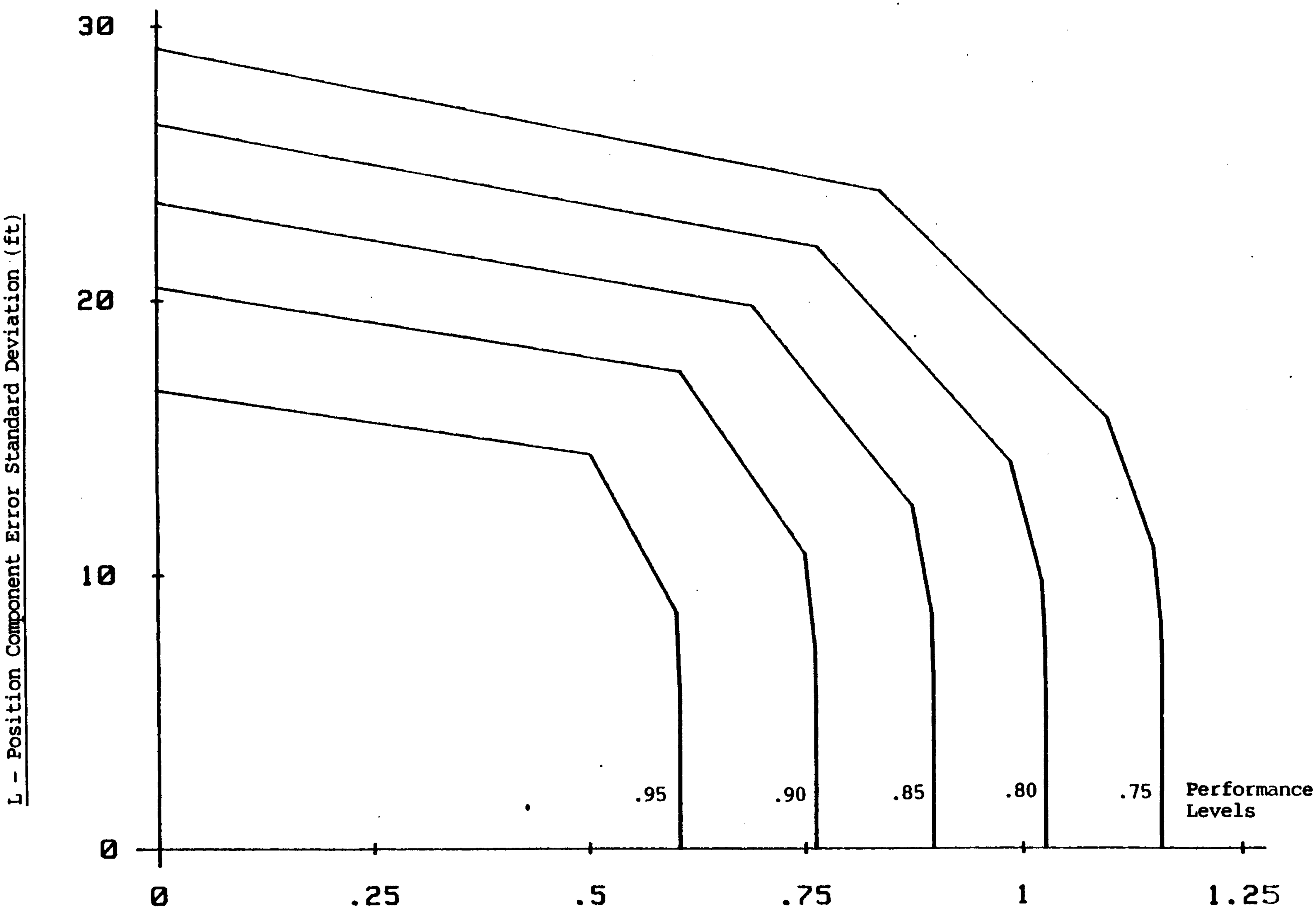
Performance Levels - .75, .80, .85, .90, .95

Presented in Table 4 are input values for position, speed, pitch, heading, and the velocity components computed from the latter dynamic parameters for the three cases investigated in this scenario. Pilot error sigmas for position and velocity components are also shown. The initial parameters were obtained from Tactical Manuals and represent a varied set of realistic conditions (reference b).

Figures 9 and 10 present in graphical and parametric form scenario-specific guidance regarding TACTS range accuracy guidelines. Figures 9 and 10 pertain to GDOP ≤ 5 and to $5 <$ GDOP ≤ 12, respectively.

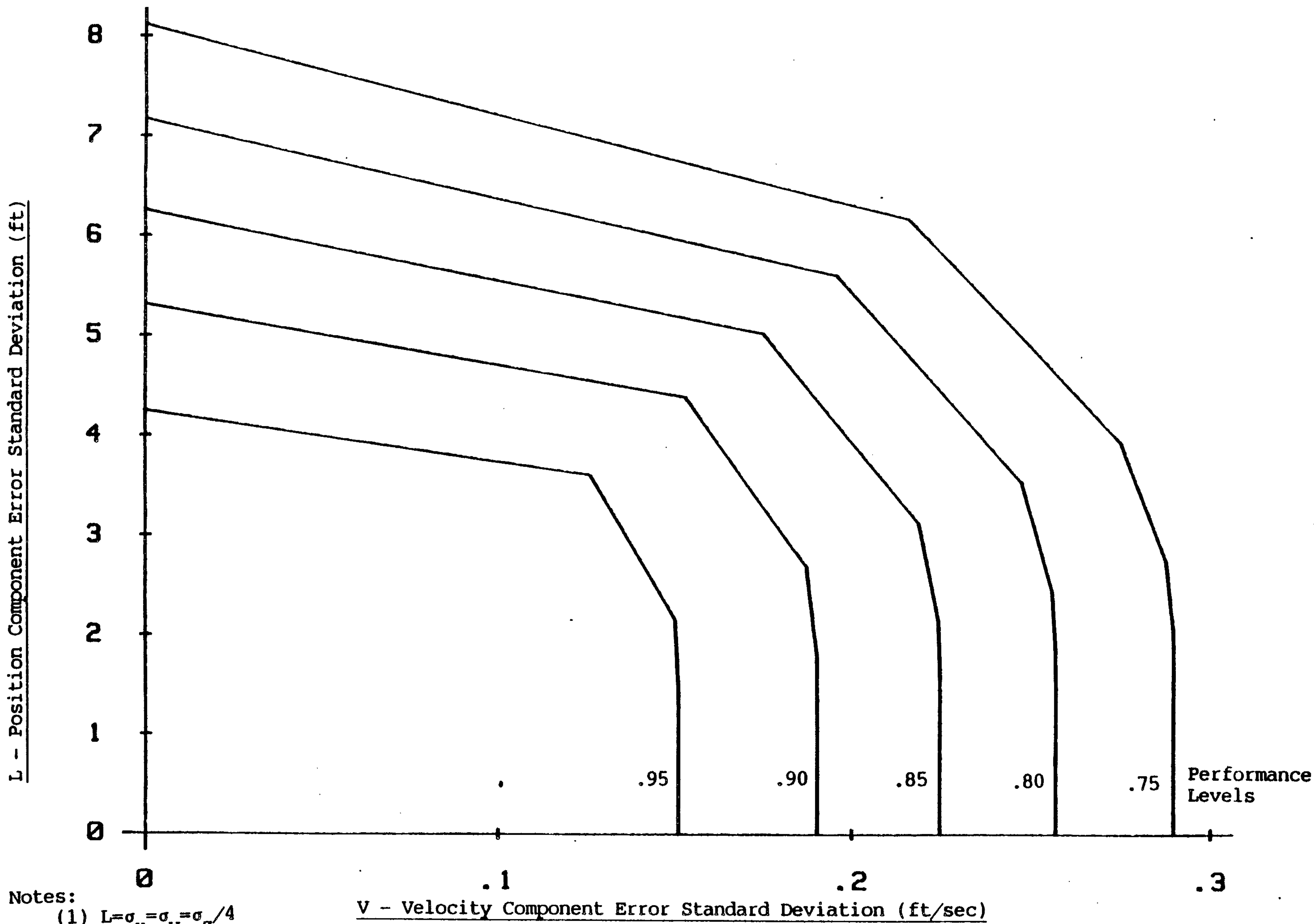
TABLE 4. CASE DATA - SIMULATED ROCKET FIRING AT A POINT TARGET

Case	1	2	3
(1) Intended aircraft position relative to aim point on target at time of rocket firing			
Xpos	0 ft	0 ft	0 ft
Ypos	0 ft	0 ft	0 ft
Zpos	5000 ft	3000 ft	1000 ft
(2) Intended dynamic parameter values at time of rocket firing			
Speed	759.9 ft/sec	675.5 ft/sec	665.2 ft/sec
Pitch	0°	0°	0°
Heading	-30°	-45°	-10°
(3) Velocity components computed from dynamic data			
V_x	658.1 ft/sec	477.65 ft/sec	665.2 ft/sec
V_y	-379.9 ft/sec	-477.65 ft/sec	-117.2 ft/sec
V_z	0 ft/sec	0 ft/sec	0 ft/sec
(4) Sigmas computed for pilot error			
σx	69.2 ft	76.1 ft	120.8 ft
σy	68.3 ft	56.2 ft	38.6 ft
σz	57.8 ft	54.1 ft	23.9 ft
σV_x	7.3 ft/sec	5.2 ft/sec	33.4 ft
σV_y	3.17 ft/sec	3.8 ft/sec	2.3 ft/sec
σV_z	2.2 ft/sec	2.17 ft/sec	1.4 ft/sec



Notes:
(1) $L = \sigma_x = \sigma_y = \sigma_z$
(2) $V = \sigma V_x = \sigma V_y = \sigma V_z$

Figure 9. TACTS Range Accuracy - Rocket vs. Point Target
(GDOP $\leq$ 5, 10% Relative Error in Miss Distance)



Notes:
(1) $L=\sigma_x=\sigma_y=\sigma_z/4$
(2) $V=\sigma V_x=\sigma V_y=\sigma V_z/4$

Figure 10. TACTS Range Accuracy - Rocket vs. Point Target
(5 < GDOP ≤ 12, 10% Relative Error in Miss Distance)

SECTION 7

TACTS RANGE ACCURACY GUIDELINES
Gunfire In Air-to-Air Combat Scenario

The following conditions are assumed for the gunfire in air-to-air combat scenario:

Target - Small fighter aircraft

Weapon - M61A1 20 mm gun

Aircraft - F-14

MOE - P_k

Performance Goal - No more than ten percent relative error

Performance Levels - .75, .80, .85, .90, .95

Presented in Table 5 are input values for position, speed, pitch, heading, and the velocity components computed from the latter dynamic parameters for the seven cases investigated in this scenario. Pilot error sigmas for position and velocity components are also shown. The parameters were chosen from a Tactical Manual (reference b). Sigmas for x position and x velocity parameters are not presented since the analysis indicated total MOE insensitivity to any change in x-direction components.

Figures 11 and 12 present in graphical and parametric form scenario-specific guidance regarding TACTS range accuracy guidelines. Figures 11 and 12 pertain to GDOP ≤ 5 and to $5 <$ GDOP ≤ 12, respectively. The standard deviations presented pertain to the differences between fighter and target position components, and between fighter and target velocity components.

For scenarios involving an airborne target, both launcher and target aircraft position and velocity are estimated. Under the assumption that estimation errors are independent and statistically the same for both aircraft, the error standard deviations for both the launcher and target are each $1/\sqrt{2}$ times the error standard deviation in the estimate of the difference of like parameter values. For instance, if X_L and X_T are the errors in estimates of launcher and target X-component values at time of weapon firing, then $\sigma_{x_L-x_T}^2 = \sigma_{x_L}^2 + \sigma_{x_T}^2$. If $\sigma_{x_L}^2 = \sigma_{x_T}^2 = \sigma_x^2$, then $\sigma_{x_L-x_T}^2 = 2\,\sigma_x^2$ and $\sigma_x = 1/\sqrt{2}\,\sigma_{x_L-x_T}$.

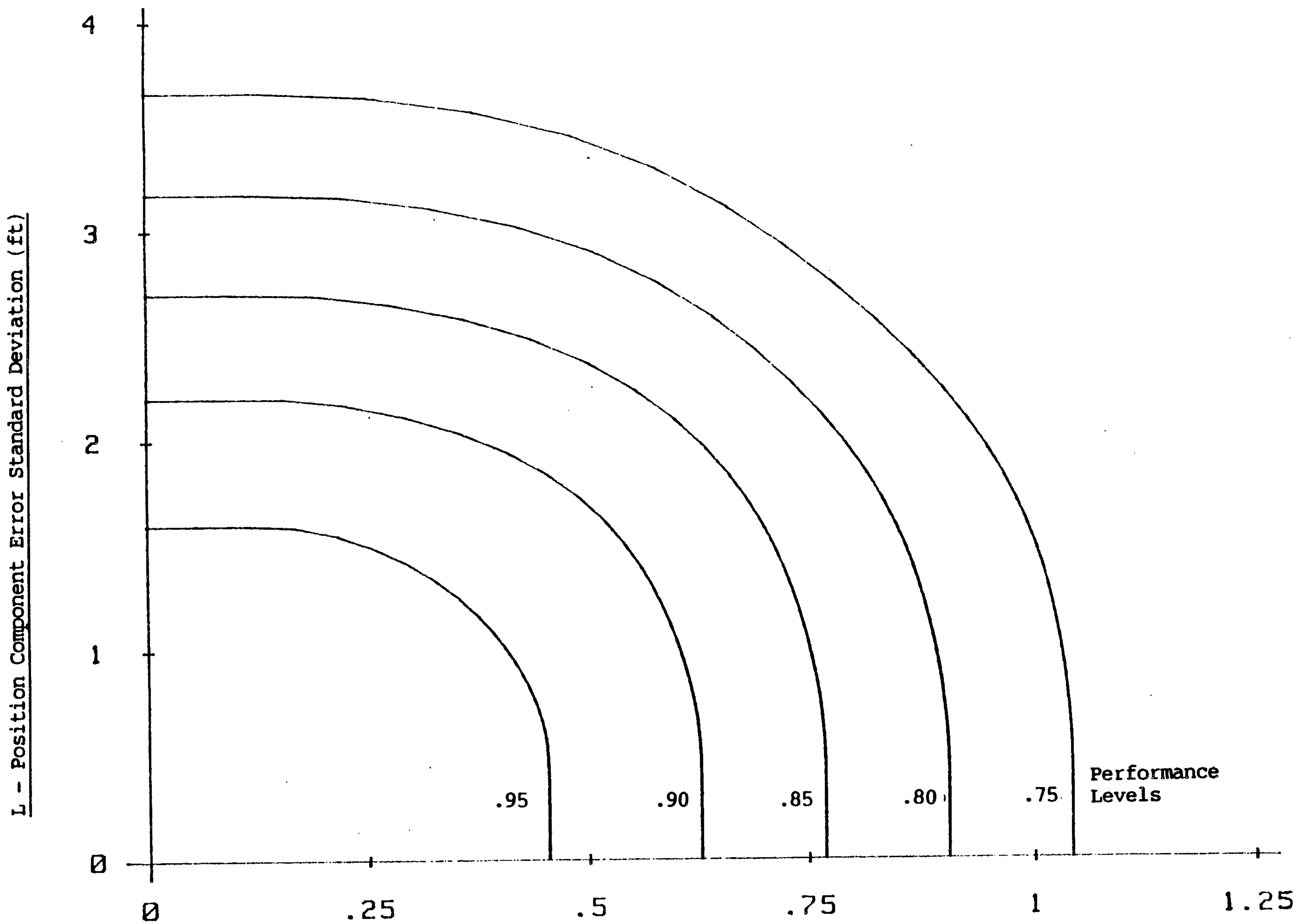
Guidance for accuracy requirements based upon airborne target scenarios is presented in Figures 11 and 12 in terms of paired values $(\sigma_{v_L-v_T}, \sigma_{x_L-x_T})$. If a particular numerical pair (V,L) assures a Performance Level of .90, then the required single aircraft error standard deviation pair is $(V/\sqrt{2}, L/\sqrt{2})$.

TABLE 5. CASE DATA - SIMULATED AIR-TO-AIR GUNFIRE

Case	1	2	3
(1) Intended aircraft position at time of firing			
Launcher:			
Xpos	0 ft	0 ft	0 ft
Ypos	0 ft	0 ft	0 ft
Zpos	15000 ft	15000 ft	15000 ft
Target:			
Xpos	1500 ft	500 ft	1500 ft
Ypos	0 ft	0 ft	0 ft
Zpos	15000 ft	15000 ft	15000 ft
(2) Intended dynamic parameter values at time of firing			
Launcher:			
Speed	500 ft/sec	500 ft/sec	500 ft/sec
Pitch	-2^{o}	-2^{o}	0^{o}
Heading	0^{o}	0^{o}	6^{o}
Target:			
Speed	500 ft/sec	500 ft/sec	500 ft/sec
Pitch	-2^{o}	-2^{o}	0^{o}
Heading	0^{o}	0^{o}	45^{o}
(3) Velocity components computed from dynamic data			
Launcher:			
V_x	499.69 ft/sec	499.69 ft/sec	497.26 ft/sec
V_y	0 ft/sec	0 ft/sec	52.27 ft/sec
V_z	-17.44 ft/sec	-17.44 ft/sec	0 ft/sec
Target:			
V_x	500 ft/sec	500 ft/sec	353.33 ft/sec
V_y	0 ft/sec	0 ft/sec	353.55 ft/sec
V_z	-17.44 ft/sec	-17.44 ft/sec	0 ft/sec
(4) Sigmas computed for pilot error			
σy	5.5 ft	1.84 ft	5.35 ft
σz	5.6 ft	1.84 ft	5.36 ft
σV_y	12.08 ft/sec	12.08 ft/sec	12.09 ft/sec
σV_z	12.07 ft/sec	12.07 ft/sec	12.07 ft/sec

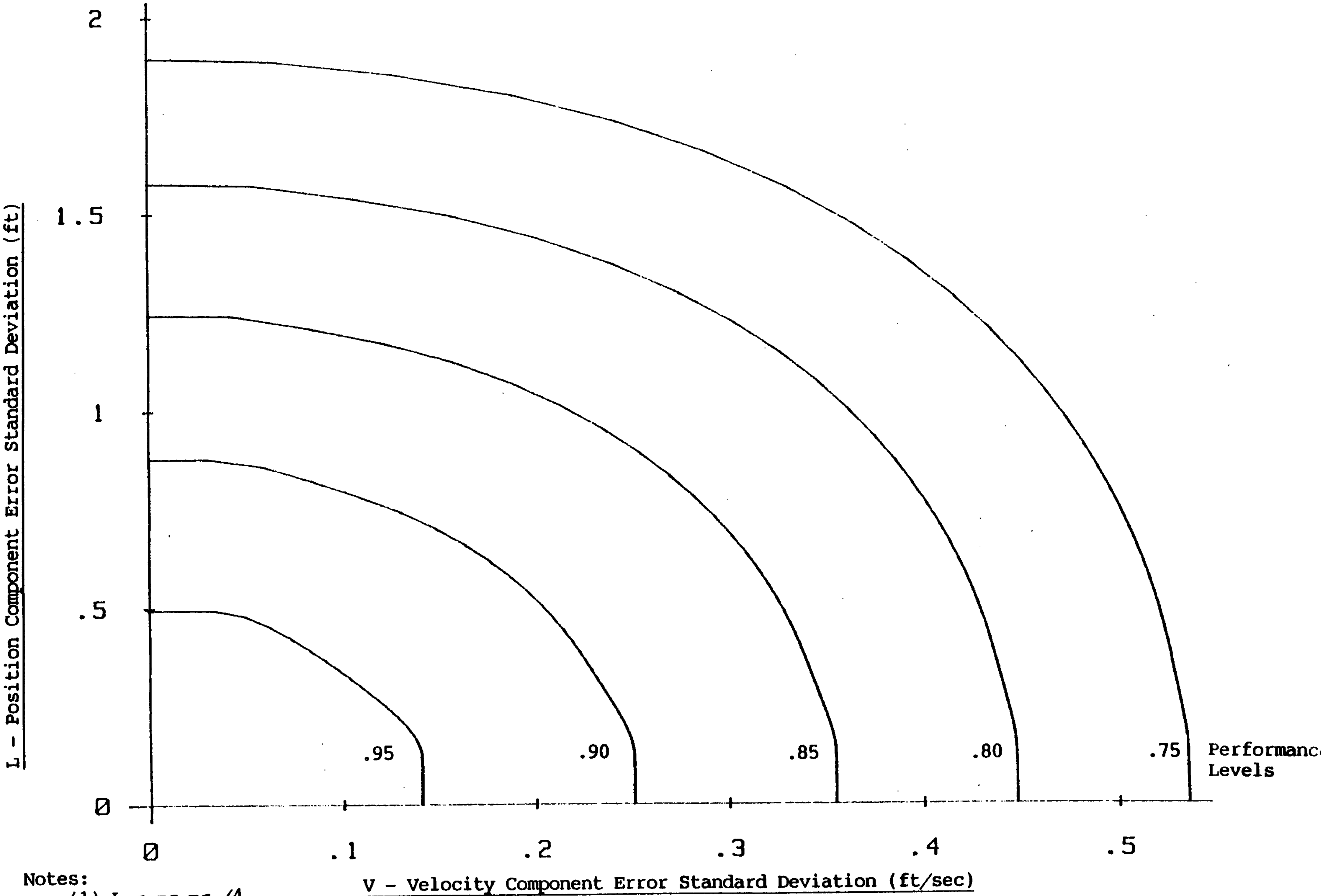
TABLE 5. CASE DATA - SIMULATED AIR-TO-AIR GUNFIRE (Cont.)

Case	4	5	6	7
(1) Intended aircraft position at time of firing				
Launcher:				
Xpos	0 ft	0 ft	0 ft	0 ft
Ypos	0 ft	0 ft	0 ft	0 ft
Zpos	15000 ft	15000 ft	15000 ft	15000 ft
Target:				
Xpos	500 ft	1500 ft	500 ft	1500 ft
Ypos	0 ft	0 ft	0 ft	0 ft
Zpos	15000 ft	15000 ft	15000 ft	15000 ft
(2) Intended dynamic parameter values at time of firing				
Launcher:				
Speed	500 ft/sec	500 ft/sec	500 ft/sec	500 ft/sec
Pitch	0°	0°	0°	0°
Heading	6°	9°	9°	0°
Target:				
Speed	500 ft/sec	500 ft/sec	500 ft/sec	500 ft/sec
Pitch	0°	0°	0°	0°
Heading	45°	90°	90°	180°
(3) Velocity components computed from dynamic data				
Launcher:				
V_x	497.26 ft/sec	493.84 ft/sec	493.84 ft/sec	500 ft/sec
V_y	52.26 ft/sec	78.21 ft/sec	78.21 ft/sec	0 ft/sec
V_z	0 ft/sec	0 ft/sec	0 ft/sec	0 ft/sec
Target:				
V_x	353.55 ft/sec	0 ft/sec	0 ft/sec	-500 ft/sec
V_y	353.55 ft/sec	500 ft/sec	500 ft/sec	0 ft/sec
V_z	0 ft/sec	0 ft/sec	0 ft/sec	0 ft/sec
(4) Sigmas computed for pilot error				
σy	1.76 ft	4.82 ft	1.6 ft	4.17 ft
σz	1.77 ft	4.83 ft	1.6 ft	4.17 ft
σV_y	12.08 ft/sec	12.09 ft/sec	12.08 ft/sec	12.08 ft/sec
σV_z	12.07 ft/sec	12.07 ft/sec	12.07 ft/sec	12.07 ft/sec



Notes:
(1) $L = \sigma_x = \sigma_y = \sigma_z$
(2) $V = \sigma V_x = \sigma V_y = \sigma V_z$

Figure 11. TACTS Range Accuracy - Air-to-Air Gunfire
($GDOP \leq 5$, 10% Relative Error in P_k)



Notes:
(1) $L=\sigma_x=\sigma_y=\sigma_z/4$
(2) $V=\sigma V_x=\sigma V_y=\sigma V_z/4$

Figure 12. TACTS Range Accuracy - Air-to-Air Gunfire
($5 < GDOP \leq 12$, 10% Relative Error in P_k)

SECTION 8

COMPARISONS

TACTS range accuracy guidelines for five different weapon/target scenarios were presented in the previous five sections. For comparison, the 90 percent Performance Level curves under GDOP ≤ 5 conditions for each of these scenarios are presented in Figure 13. Note that the V and L scales are NOT the same for all five curves.

The most demanding scenarios, in terms of TACTS range accuracy guidelines, are firing a gun in air-to-air combat and bombing a point target with a retarded bomb. The least demanding is bombing a runway. In the former scenarios, small errors in position and velocity component values cause large variations in P_k. In the latter scenario, the simulation is more tolerant of errors in position and velocity component values. In order of simulated weapon effectiveness sensitivity to TACTS range accuracy, the five scenarios are ranked from most demanding to least demanding as follows:

Firing a gun in air-to-air combat or
Bombing a point target with a retarded bomb
Bombing a point target
Firing a rocket at a point target
Bombing a runway

Generally, the more sensitive weapon effectiveness is in position and velocity component values at weapon launch, the more stringent are the TACTS range accuracy requirements. Thus, the results of this analysis do conform qualitatively with operational realities.

The TACTS range accuracy requirements should NOT be combined by simple averaging. Any requirement consolidation should include, at a minimum, a determination of the relative frequency with which each scenario is to be exercised. If the TACTS range were instrumented to conform with a "consolidated accuracy requirement," then high Performance Levels would be realized for less demanding scenarios and low Performance Levels for the more demanding scenarios. If different scenarios are to be exercised on the same portion of the TACTS range, then requirement consolidation is unavoidable.

The results of this study also can be used to assess how well a particular range instrumentation would perform. For instance, suppose that TACTS range instrumentation allows for determination of position components with error standard deviation of 1 ft and velocity components with error standard deviation of 2.5 ft/sec. Then computed values of P_k (with GDOP < 5) will have a relative error of no more than 10 percent for: (a) between 75 percent and 80 percent of all retarded bomb drops on point targets (Figure 7); (b) between 85 percent and 90 percent of all standard bomb drops on a point target (Figure 1); and (c) between 75 percent and 80 percent of all gun firings against an airborne target (Figure 11).

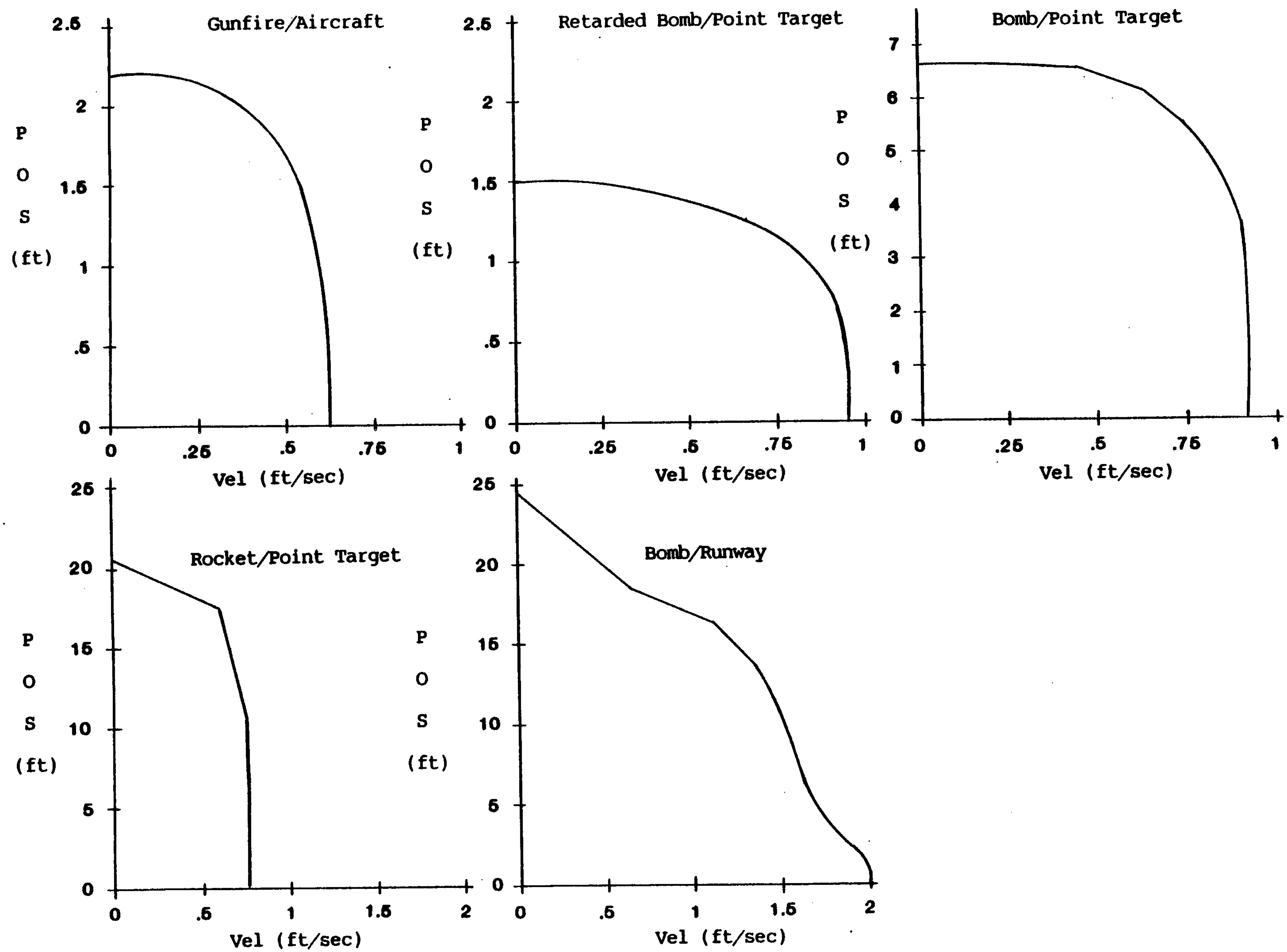


Figure 13. TACTS Range Accuracy Requirements Determined from Five Weapon/Target Combinations (90% Performance Level/5 $\leq$ GDOP)

Some options for determining specific TACTS range accuracy requirements are as follows:

a. Accommodate the most stringent accuracy requirements to satisfy a desired Performance Level for the most demanding scenario. Then desired Performance Levels for other scenarios will be exceeded.

b. Implement the best accuracy requirements consistent with technological and financial resources. Then there could be a disparity in Performance Levels for certain scenarios.

c. Implement different accuracy requirements for different multi- and single-purpose sites on the TACTS range. Then range-wide compliance with Performance Level requirements might be achievable within the constraints of technological and financial resources.

SECTION 9

CONCLUSIONS

Specification of TACTS range accuracy will depend in part upon available range instrumentation technology. The figures presented provide a vehicle for trading off position and velocity accuracy requirements for areas of the range used for different weapon/target pairs.

Large values for Performance Level (.90 to .95) lead to stringent TACTS range accuracy requirements. Smaller values for Performance Level (.75 to .90) accommodate the realistic possibility that under some circumstances aircraft position and velocity component values cannot be determined accurately.

Because it is not ideal to aggregate the accuracy requirements over all scenarios investigated, the individual scenario results should each be applied to the particular portions of the TACTS range on which the scenario weapon and target are to be exercised.

REFERENCES

Reference a - "System Specification for the Advanced Tactical Aircrew Combat Training System (TACTS)," SP514-1B

Reference b - "Tactical Manual Ballistic Tables," NAVAIR 01-1C-1T

Reference c - "ACEVAL - AIMVAL Test Plan" Volume X, Missile Simulation/Validation Report

APPENDIX A

GENERAL METHOD FOR DETERMINING
TACTS RANGE ACCURACY GUIDELINES

The general method for determining TACTS range accuracy guidelines is presented in this appendix. The method implements the following goal:

> Estimate the amounts by which aircraft position and velocity component values, as determined by TACTS range telemetry and processing, could be in error while requiring that TACTS weapon simulations compute a reasonably accurate value for Measure of Effectiveness (MOE), reasonably often.

The method is applicable to both discrete and continuous MOEs.

Figure A-1 depicts, mathematically, the relationship between launch parameter values and computed MOE (Score) as determined by a TACTS weapon simulation.

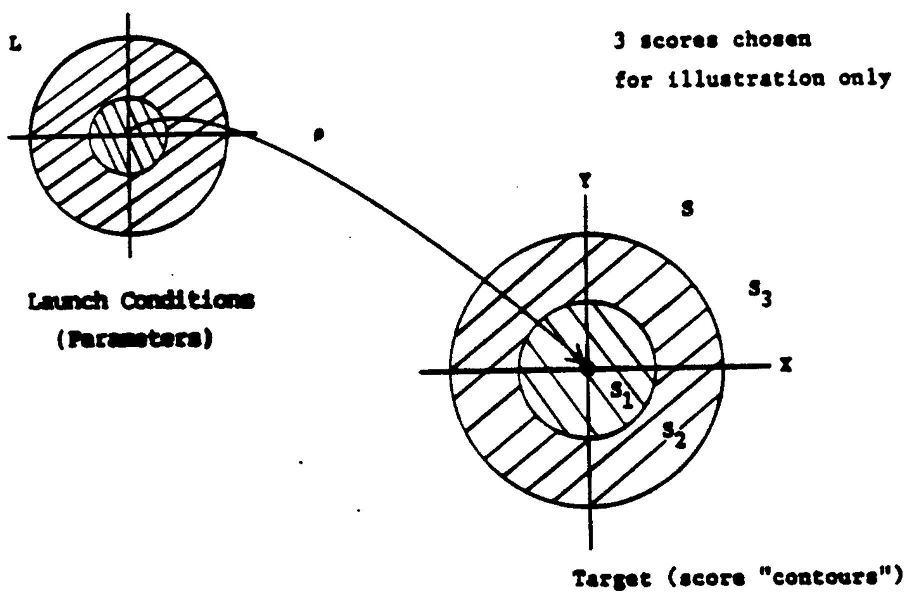


Figure A-1. Relationship Between Launch Condition and Score

$\rho : L \rightarrow S$ is a many-to-one mapping of launch conditions into scores. This function ρ is evaluated by the TACTS weapon simulations.

Denote $\ell \varepsilon L$ and $\delta \ell$ the error in ℓ at the time of weapon launch. The correct score for launch condition ℓ is given by $\rho(\ell)$. The computed score resulting from the TACTS range-measured data $\ell + \delta\ell$ is $\rho(\ell + \delta\ell)$. Conditions that these two scores are close may be written

$$|\rho(\ell+\delta\ell) - \rho(\ell)| \leqslant \varepsilon \qquad (A-1)$$

or

$$|\rho(\ell+\delta\ell) - \rho(\ell)| \leqslant \varepsilon \cdot \rho(\ell) \qquad (A-2)$$

in which $\varepsilon(\geqslant 0)$ is a prescribed tolerance. The first of the conditions places an absolute constraint on accuracy, independent of ℓ; the second specifies a relative accuracy (i.e., $\rho(\ell + \delta\ell)$ is to differ from $\rho(\ell)$ by no more than 100ε percent). Note that if the scores $\rho(\ell + \delta\ell)$ and $\rho(\ell)$ must match exactly, then choose $\varepsilon = 0$. Condition (A-1) is probably more appropriate for discrete MOEs (such as crater diameter, number of hits, etc.); the second condition (A-2) is more appropriate for continuous MOEs (such as P_k, miss distance, percent of target destroyed, etc.).

Denote the absolute difference between $\rho(\ell + \delta\ell)$ and $\rho(\ell)$ by

$$D(\ell, \delta\ell) = |\rho(\ell + \delta\ell) - \rho(\ell)| \qquad (A-3)$$

Then the statistical requirement for TACTS range accuracy may be written

$$\Pr\ \{D(\ell, \delta\ell) \leqslant \varepsilon\} \geqslant .95 \qquad (A-4)$$

or

$$\Pr\ \{D(\ell, \delta\ell) \leqslant \varepsilon \cdot \rho(\ell)\} \geqslant .95 \qquad (A-5)$$

Note that the .95 is chosen arbitrarily for illustration.

Let $f(\ell,\delta\ell)$ be the joint probability density function for launch conditions ℓ and the TACTS range error in launch conditions $\delta\ell$. Requirements (A-4, -5) are equivalent to

$$\int_R f(\ell,\delta\ell)\,d\ell\,d(\delta\ell) \geq .95 \qquad \text{(A-6)}$$

in which region $R = \{(\ell,\delta\ell)\,|\,D(\ell,\delta\ell) \leq \varepsilon\}$

or alternately, $R = \{(\ell,\delta\ell)\,|\,D(\ell,\delta\ell) \leq \varepsilon\;\rho(\ell)\}$.

The value of the integral in (A-6) depends upon $f(\ell,\delta\ell)$; in particular, if $\sigma_{\delta\ell} = 0$ (and $\mu_{\delta\ell} = 0$ so that TACTS range errors are unbiased), then the value of the integral is 1; therefore, all scores are correctly determined by the weapon simulation because all input is correct.

As $\sigma_{\delta\ell}$ increases, the integral in (A-6) decreases in value. There is a maximum value of $\sigma_{\delta\ell}$ for which condition (A-6) holds. For the set of launch platform/pilot/target/environmental specifics represented in f(.,.), that maximum σ_{δ} is the TACTS range accuracy required to guarantee condition (A-6).

As a practical matter f(.,.) cannot be known, so a reasonable approximation must be employed for the determination of the integral in (A-6) as a function of $\sigma_{\delta\ell}$. Or more correctly, a reasonable approximation for the integral in (A-6) as a function of $\delta\ell$ must be used, and from that approximation, the value of $\sigma_{\delta\ell}$ determined.

One such method of approximation is as follows: let $\ell \varepsilon L$ be a typical set of actual launch conditions. Such actual launch conditions represent a realistic variation on intended launch conditions recommended by tactical manuals and other sources of tactical guidance. Evaluate $\rho(\ell)$. Then in one component of ℓ at a time find the largest and smallest variations $\delta\ell$ for which $D(\ell,\delta\ell) \leq \varepsilon$ [or $D(\ell,\delta\ell) \leq \varepsilon\;\rho(\ell)$].

For several values of ℓ representing realistic variations in launch conditions, determine lower and upper bounds on each of the position and velocity parameter values. The lower bound is the amount by which the parameter can be decreased without changing the value of ρ substantially - similarly for the upper bound.

For convenience denote the parameters by ℓ_i; $i = 1,\ldots 6$, with $\ell_1 = x$, $\ell_2 = y$, $\ell_3 = z$, $\ell_4 = v_x$, $\ell_5 = v_y$, $\ell_6 = v_z$. Let the lower and upper bounds be ℓ_i^- and ℓ_i^+. Let values for the r^{th} set of launch conditions be denoted $\ell_i^-(r)$, $\ell_i^+(r)$, $i = 1,\ldots,6$.

Suppose that the TACTS range standard deviations for the determination of these parameter values are σ_i, $i = 1,\ldots.6$.

If the errors are unbiased, normal, and independent, then

$$M(r) = \Pr\{D(\ell,\delta\ell) \leqslant \varepsilon\} \text{ (or } \Pr\{D(\ell,\delta\ell) \leqslant \rho(\ell)\}\text{)}$$

$$\approx \prod_{i=1}^{6} \left[\Phi\left(\frac{{}_{i}^{+}(r)}{\sigma_i}\right) - \Phi\left(\frac{{}_{i}^{-}(r)}{\sigma_i}\right) \right] \qquad \text{(A-7)}$$

in which $\underline{\overline{0}}$ is the cumulative normal distribution function.

The factor $\Phi\left(\frac{\ell_i^{+}(r)}{\sigma_i}\right) - \Phi\left(\frac{\ell_i^{-}(r)}{\sigma_i}\right)$ is the probability that the TACTS range determines a launch platform x-coordinate value between $\ell_1^-(r)$ and $\ell_1^+(r)$ and similarly for other position and velocity component values. The product determines the joint probability that all 6 values are determined with sufficient accuracy.

If the computed simulation score is to be accurate 95 percent of the time, then set

$$PL = \frac{1}{Nr} \sum_{r=1}^{Nr} M(r) = .95 \qquad (A-8)$$

in which Nr = number of simulation runs, PL = Performance Level (.95 is for illustration only), and M(r) is given in (A-7). The function PL decreases with increasing σ_i. Search (numerically) for combinations of σ_i's for which (A-8) holds. These combinations of values specify accuracy required on the TACTS range. Appendix C presents the listings of BASIC programs with the numerical procedures for computing desired combinations of σ_i's from weapon simulation data.

APPENDIX B
ESTIMATION OF VARIANCES IN LAUNCH PARAMETERS FROM MEASURES OF ACCURACY

In order to simulate actual launch parameter values, estimates for pilot ability to attain intended launch parameters must be available. This Appendix describes a method of estimation employed in the TACTS weapon simulations for purposes of this analysis. The method is described in the context of bombing a fixed target, and later applied in air-to-air gunnery.

Denote:

$$\underline{W} = (x, y, z, \theta, \psi, s)$$
$$\underline{X} = (x, y, z, V_x, V_y, V_z),$$

each a vector of launch parameters at the time of bomb release. Coordinate x is the horizontal distance between the aircraft and target; y is the aircraft's horizontal position perpendicular to the x-axis; and z is the vertical distance between the aircraft and the target. Dynamic parameters θ, ψ, and s are aircraft pitch, heading, and speed respectively, and velocity components V_x, V_y, and V_z are determined by values for θ, ψ, and s, and conversely. A pilot's ability to attain intended launch parameter values is represented by the variances in the differences between actual and intended values.

Let ΔX_h and ΔY_h be the differences between actual and intended *hit* position coordinates. If intended launch parameter values are attained, then ΔX_h and ΔY_h will each be zero. But in general,

$$\Delta X_h \approx \frac{\partial X_h}{\partial x}\Delta x + \frac{\partial X_h}{\partial y}\Delta y + \frac{\partial X_h}{\partial z}\Delta z + \frac{\partial X_h}{\partial V_x}\Delta V_x + \frac{\partial X_h}{\partial V_y}\Delta V_y + \frac{\partial X_h}{\partial V_z}\Delta V_z$$
$$\Delta Y_h \approx \frac{\partial Y_h}{\partial x}\Delta x + \frac{\partial Y_h}{\partial y}\Delta y + \frac{\partial Y_h}{\partial z}\Delta z + \frac{\partial Y_h}{\partial V_x}\Delta V_x + \frac{\partial Y_h}{\partial V_y}\Delta V_y + \frac{\partial Y_h}{\partial V_z}\Delta V_z,$$

in which $\Delta\underline{X} = (\Delta x, \Delta y, \Delta z, \Delta V_x, \Delta V_y, \Delta V_z)$ are the errors in $\underline{X}$ at launch.

The method which follows estimates variances for the errors in $\Delta\underline{X}$ subject to the "known" conditions on the hit position errors ΔX_h, ΔY_h. The "known" conditions are that bombing accuracy is "5 mils" and gunning accuracy is "14 mils".

As a first step the partial derivatives $\frac{\partial X_h}{\partial x}, \ldots, \frac{\partial Y_h}{\partial V_z}$ are estimated. Let $\underline{W}_I$ denote a set of *intended* launch parameter values as determined by tactical guidance. Run the bomb simulation to obtain bomb impact position - call it (X_h, Y_h) - to be thought of as the *intended hit* position.

Then for each component in $\underline{X}$ (one at a time), increase the parameter by a small amount and rerun the bomb simulation with the perturbed launch parameter value. (Perturbations in V_x, V_y, V_z must be converted to perturbations in θ, ψ, and s before running). Obtain the bomb impact positions under these six variations. (Variations in x and y need not be run because the bomb trajectory is simply translated so that miss distances on the ground are exactly the values in x and y variation.)

Denote the *changes* in impact positions (X_h, Y_h) resulting from parameter variations by

$$\Delta X_h(\Delta x)[=\Delta x], \Delta Y_h(\Delta x)[=0], \Delta X_h(\Delta y)[=0], \Delta Y_h(\Delta y)[=\Delta y],$$
$$\Delta X_h(\Delta z), \Delta Y_h(\Delta z)[=0], \Delta X_h(\Delta V_x), \Delta Y_h(\Delta V_x)[=0], \qquad \text{(B–1)}$$
$$\Delta X_h(\Delta V_y)[\approx 0], \Delta Y_h(\Delta V_y), \Delta X_h(\Delta V_z), \Delta Y_h(\Delta V_z)[=0].$$

Values known in advance are given in "[ ]".

Then as a first-order approximation,

$$\Delta X_h \approx \Delta x + \left\{\frac{\Delta X_h(\Delta z)}{\Delta z}\right\} \cdot \Delta z + \left\{\frac{\Delta X_h(\Delta V_x)}{\Delta V_x}\right\} \cdot \Delta V_x + \left\{\frac{\Delta X_h(\Delta V_z)}{\Delta V_z}\right\} \cdot \Delta V_z$$
$$\Delta Y_h \approx \Delta y + \left\{\frac{\Delta Y_h(\Delta V_y)}{\Delta V_y}\right\} \cdot \Delta V_y \tag{B-2}$$

in which the terms in "{ }" are the numerical estimates of partial derivatives. Alternative numerical estimation formulas for partial derivatives may be substituted for those used to obtain more accurate estimates.

To simplify notation, abbreviate

$$\left\{\frac{\Delta X_h(\Delta z)}{\Delta z}\right\} = Dz, \left\{\frac{\Delta X_h(\Delta V_x)}{\Delta V_x}\right\} = DV_x, \left\{\frac{\Delta X_h(\Delta V_z)}{\Delta V_z}\right\} = DV_z, \text{and } \left\{\frac{\Delta Y_h(\Delta V_y)}{\Delta V_y}\right\} = DV_y.$$

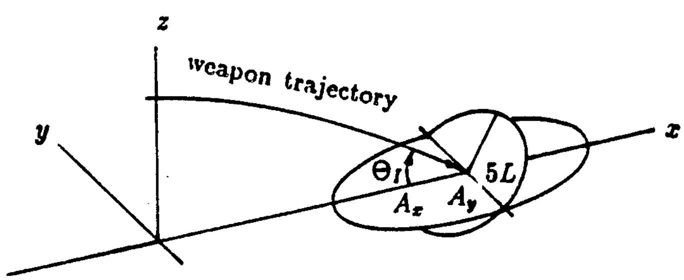
If it is assumed that the errors $\Delta \underline{X} = (\Delta x, \Delta y, \Delta z, \Delta V_x, \Delta V_y, \Delta V_z)$ in $\underline{X}$ at launch are independent normally distributed random variables with 0 means, then $\sigma^2_{\Delta x} = E[\Delta x^2]$, $\sigma^2_{\Delta y} = E[\Delta y^2]$, $\sigma_{\Delta x \Delta y} = E[\Delta x \Delta y](= 0)$, etc., in which $E[\cdot]$ is the "expectation" function. Consequently, ΔX_h, ΔY_h in equation (B–2) are normal and independent,

$$E[\Delta X_h] = E[\Delta Y_h] = 0,$$

and

$$\sigma^2_{\Delta X_h} \approx \sigma^2_{\Delta x} + Dz^2 \cdot \sigma^2_{\Delta z} + DV_x{}^2 \cdot \sigma^2_{\Delta V_x} + DV_z{}^2 \cdot \sigma^2_{\Delta V_z}$$
$$\sigma^2_{\Delta Y_h} \approx \sigma^2_{\Delta y} + DV_y{}^2 \cdot \sigma^2_{\Delta V_y}. \tag{B-3}$$

Suppose that bombing accuracy is 5 mils. Then the *median* miss distance in the plane perpendicular to the weapon trajectory at the point of impact is $5L$ feet as shown in the following Figure, where L = trajectory length in thousands of feet (e.g., $L = 2$ for a 2000 ft trajectory).



The elliptical area on the ground plane which contains 50 percent of the bomb hits has axes

$$A_y = 5L \quad \text{and} \quad A_x = \frac{5L}{\sin \Theta_I}$$

in which Θ_I = weapon impact angle. Thus

$$\frac{\sigma_{\Delta X_h}}{\sigma_{\Delta Y_h}} = \frac{A_x}{A_y} = \frac{1}{\sin \Theta_I}. \tag{B-4}$$

The probability within a circle of radius r centered at the mean for a circular normal random variable (U, V) with $\sigma_U = \sigma_V = \sigma$ is given by $1 - exp(-r^2/2\sigma^2)$. If this has value .5, then the corresponding $r = \sigma\sqrt{2 \cdot \ln 2}$ is the *median* miss distance, where $\sqrt{2 \cdot \ln 2} \approx 1.177$.

Thus for $CEP = 5L$, the corresponding standard deviation in the plane perpendicular to the trajectory at impact is $\sigma = 5L/1.177$. Thus

$$\sigma_{\Delta Y_h} = \frac{5L}{1.177} \quad \text{and} \quad \sigma_{\Delta X_h} = \frac{5L}{1.177} \cdot \frac{1}{\sin \Theta_I}.$$

Now abbreviate:

$$\begin{aligned} C_x &= \frac{5L}{1.177} \cdot \frac{1}{\sin \Theta_I} \\ C_y &= \frac{5L}{1.177}. \end{aligned} \tag{B-5}$$

Then equations (B-3) and (B-5) together imply the constraints

$$\begin{aligned} \sigma^2_{\Delta x} + Dz^2 \cdot \sigma^2_{\Delta z} + DV_x^2 \cdot \sigma^2_{\Delta V_x} + DV_z^2 \cdot \sigma^2_{\Delta V_z} &= C_x^2 \\ \sigma^2_{\Delta y} + DV_y^2 \cdot \sigma^2_{\Delta V_y} &= C_y^2 \end{aligned} \tag{B-6}$$

on the six error standard deviations.

Under the normality and independence assumptions on the components if $\Delta\underline{X}$, the joint probability density function for the six error components is proportional to Q:

$$Q = \exp\left[-\frac{1}{2}\left((\frac{\Delta x}{\sigma_{\Delta x}})^2 + (\frac{\Delta y}{\sigma_{\Delta y}})^2 + (\frac{\Delta z}{\sigma_{\Delta z}})^2 + (\frac{\Delta V_x}{\sigma_{\Delta V_x}})^2 + (\frac{\Delta V_y}{\sigma_{\Delta V_y}})^2 + (\frac{\Delta V_z}{\sigma_{\Delta V_z}})^2\right)\right] \tag{B-7}$$

Among all volumes which contain the six error values $(\Delta x, \Delta y, \Delta z, \Delta V_x, \Delta V_y, \Delta V_z)$ 50 percent of the time (so that the resulting bombing accuracy is 5 mils), the smallest volume is defined by $Q \geq K_1$ for some constant K_1 (whose actual value is irrelevant).

To confirm that $Q \geq K_1$ defines the smallest volume, note that $f(\underline{x})(\equiv f(x_1, \ldots, x_n))$ is a probability density function for random variable $\underline{X}$ if and only if $f(\underline{x}) \geq 0$ for all $\underline{x}$ and $\int_{E^n} f(\underline{x})dV = 1$. E^n is all of n-dimensional space $(= \{(x_1, \ldots, x_n) \,|\, -\infty < x_i < +\infty,\ i = 1, \ldots, n\})$ and dV is the volume differential $(= dx_1 dx_2 \cdots dx_n)$.

A subset $B \subset E^n$ is a confidence volume for $\underline{X}$ at confidence level C, $0 \leq C \leq 1$, if $\int_B f(\underline{x})dV = C$. Define

$$A(k) = \{\underline{x} \,|\, f(\underline{x}) \geq k\}.$$

Then $A(0) = E^n$ and $A(k + \epsilon) \subseteq A(k)$ for all $\epsilon \geq 0$. As k increases, $A(k)$ gets smaller;that is $\lim_{k\to\infty} A(k) = \emptyset$, the empty set: further,

$$\Pr(A(k)) = \int_{A(k)} f(\underline{x})dV \geq \int_{A(k)} k\, dV = k \int_{A(k)} dV = k \cdot Vol(A(k))$$

and because $\Pr(A(k)) \leq 1$, then $\lim_{k\to\infty} k \cdot Vol(A(k)) \leq 1$ and therefore $\lim_{k\to\infty} Vol(A(k)) = 0$. Thus $\lim_{k\to\infty} \Pr(A(k)) = 0$. Consequently, for fixed C, $0 < C < 1$, there is a constant $k = k(C)$ such that $\int_{A(k(C))} f(\underline{x})dV = C$, provided that f is continuous with non-zero partial derivatives.

The set $A(k(C))$ (abbreviated A) has smallest volume of all confidence volumes at level C, as shown by the following (B is any *other* confidence volume at level C):

$$\int_A f(\underline{x})dV = \int_{A\cap B} f(\underline{x})dV + \int_{A\cap \bar{B}} f(\underline{x})dV = C,$$

$$\int_B f(\underline{x})dV = \int_{A\cap B} f(\underline{x})dV + \int_{\bar{A}\cap B} f(\underline{x})dV = C,$$

so that by subtraction,

$$\int_{A\cap\bar{B}} f(\underline{x})dV = \int_{\bar{A}\cap B} f(\underline{x})dV.$$

But

$$\int_{A\cap\bar{B}} f(\underline{x})dV \geq \int_{A\cap\bar{B}} k\,dV = k\,Vol(A\cap\bar{B})$$

$$\int_{\bar{A}\cap B} f(\underline{x})dV \leq \int_{\bar{A}\cap B} k\,dV = k\,Vol(\bar{A}\cap B).$$

Thus

$$Vol(A\cap\bar{B}) \leq Vol(\bar{A}\cap B),$$

and

$$Vol(A\cap B) = Vol(A\cap B),$$

so adding,

$$Vol(A) \leq Vol(B).$$

The conclusion is that of all confidence volumes at confidence level C, the one of smallest volume is $A(k(C))$.

The shape of that smallest volume is an ellipsoid. Set $Q \geq K_1$ (Q is defined in equation (B–7)) and take the natural logarithm, obtaining the volume defined by

$$(\frac{\Delta x}{\sigma_{\Delta x}})^2 + (\frac{\Delta y}{\sigma_{\Delta y}})^2 + (\frac{\Delta z}{\sigma_{\Delta z}})^2 + (\frac{\Delta V_x}{\sigma_{\Delta V_x}})^2 + (\frac{\Delta V_y}{\sigma_{\Delta V_y}})^2 + (\frac{\Delta V_z}{\sigma_{\Delta V_z}})^2 \leq K_2^2$$

for some constant K_2. This volume is an ellipsoid in the 6-dimensional space of errors (whose points are labeled $(\Delta x, \Delta y, \Delta z, \Delta V_x, \Delta V_y, \Delta V_z)$), with semi-axes proportional to $\sigma_{\Delta x}$, $\sigma_{\Delta y}$, $\sigma_{\Delta z}$, $\sigma_{\Delta V_x}$, $\sigma_{\Delta V_y}$, and $\sigma_{\Delta V_z}$.

The volume of $A(k(C))$ is proportional to the product of the ellipsoid semi-axes, namely;

$$V = K_3 \cdot \sigma_{\Delta x}\sigma_{\Delta y}\sigma_{\Delta z}\sigma_{\Delta V_x}\sigma_{\Delta V_y}\sigma_{\Delta V_z} \tag{B–8}$$

for a constant K_3 (whose value is irrelevant).

The two constraints of equation (B–6) do not provide sufficient information for the determination of the six error standard deviations. Further, specific information regarding relative sizes of the six error standard deviations is not available. Consequently, a mathematical optimization is employed in which values for $(\sigma_{\Delta x}, \sigma_{\Delta y}, \sigma_{\Delta z}, \sigma_{\Delta V_x}, \sigma_{\Delta V_y}, \sigma_{\Delta V_z})$ are chosen to maximize the volume in equation (B–8) subject to the constraints of equation (B–6). The optimization is performed analytically using the method of Lagrange multipliers. Notice that the optimization

$$\max_{\{\sigma's\}} V = \max_{\{\sigma's\}} K_3 \cdot \sigma_{\Delta x}\sigma_{\Delta y}\sigma_{\Delta z}\sigma_{\Delta V_x}\sigma_{\Delta V_y}\sigma_{\Delta V_z}$$

is equivalent to the two optimizations

$$\max_{\{\sigma_{\Delta x},\sigma_{\Delta z},\sigma_{\Delta V_x},\sigma_{\Delta V_z}\}} \sigma_{\Delta x}\sigma_{\Delta z}\sigma_{\Delta V_x}\sigma_{\Delta V_z}$$

$$\text{S.T.}\quad \sigma_{\Delta x}^2 + Dz^2 \cdot \sigma_{\Delta z}^2 + DV_x^2 \cdot \sigma_{\Delta V_x}^2 + DV_z^2 \cdot \sigma_{\Delta V_z}^2 = C_x^2$$

and

$$\max_{\{\sigma_{\Delta y}, \sigma_{\Delta V_y}\}} \sigma_{\Delta y}\sigma_{\Delta V_y}$$

$$\text{S.T.} \quad \sigma^2_{\Delta y} + DV_y^2 \cdot \sigma^2_{\Delta V_y} = C_y^2$$

because the objective function V and the constraints separate into two independent sets.

Each of these optimizations has the form

$$\max_{\{X_i\}} \prod_{i=1}^{n} X_i$$

$$\text{S.T.} \quad \sum_{i=1}^{n} a_i^2 X_i^2 = b^2$$

which is a constrained optimization problem. This can be solved by maximizing the Lagrangian function

$$H(X_1, \ldots, X_n, \lambda) = \prod_{i=1}^{n} X_i - \lambda\left(\sum_{i=1}^{n} a_i^2 X_i^2 - b^2\right).$$

Differentiation yields

$$\frac{\partial H}{\partial X_i} = \prod_{j=1, j\neq i}^{n} X_j - 2\lambda a_i^2 X_i = 0, \quad i = 1, \ldots, n$$

$$\frac{\partial H}{\partial \lambda} = \sum_{i=1}^{n} a_i^2 X_i^2 - b^2 = 0.$$

The first yields (multiplying by X_i)

$$\prod_{j=1}^{n} X_j = 2\lambda a_i^2 X_i^2, \quad i = 1, \ldots, n$$

so that all terms $a_i^2 X_i^2$ are equal. Let their common value be u^2; then from the condition $\frac{\partial H}{\partial \lambda} = 0$ above,

$$\sum_{i=1}^{n} u^2 - b^2 = 0, \quad nu^2 = b^2, \quad u = \frac{|b|}{\sqrt{n}},$$

and consequently

$$\hat{X}_i = \frac{|b|}{|a_i|\sqrt{n}}, \quad i = 1, \ldots, n.$$

Apply this general result to the first optimization above by letting $X_1 = \sigma_{\Delta x}$, $X_2 = \sigma_{\Delta z}$, $X_3 = \sigma_{\Delta V_x}$, $X_4 = \sigma_{\Delta V_z}$, $a_1^2 = 1$, $a_2^2 = Dz^2$, $a_3^2 = DV_x{}^2$, $a_4^2 = DV_z{}^2$, $b^2 = C_x^2$, and $n = 4$. Similarly, apply this general result to the second optimization above by letting $X_1 = \sigma_{\Delta y}$, $X_2 = \sigma_{\Delta V_y}$, $a_1^2 = 1$, $a_2^2 = DV_y{}^2$, $b^2 = C_y^2$, and $n = 2$. As a result, the following estimates are obtained:

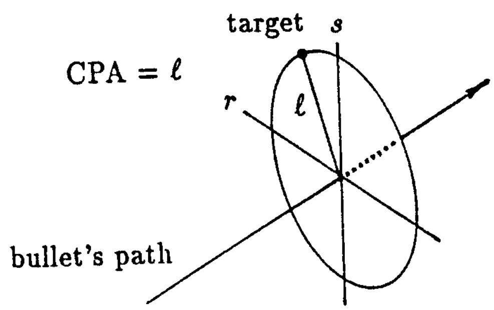
$$\hat{\sigma}_{\Delta x} = \frac{C_x}{2}; \quad \hat{\sigma}_{\Delta y} = \frac{C_y}{\sqrt{2}}; \quad \hat{\sigma}_{\Delta z} = \frac{C_x}{2|Dz|}; \tag{B-9a}$$

$$\hat{\sigma}_{\Delta V_x} = \frac{C_x}{2|DV_x|}; \quad \hat{\sigma}_{\Delta V_y} = \frac{C_y}{\sqrt{2}|DV_y|}; \quad \hat{\sigma}_{\Delta V_z} = \frac{C_x}{2|DV_z|} \tag{B-9b}$$

These error standard deviation estimates in equations (B-9) are used to simulate actual launch conditions with respect to stationary targets.

A similar approach is used to determine variances in launch parameter values for air-to-air gunnery.

Assume that the Closest Point of Approach (CPA) occurs in the r-s plane perpendicular to the bullet's path. At CPA the target is at position $(0, r, s)$ as shown in the following figure, where CPA = ℓ, and $\ell^2 = r^2 + s^2$.



Assume at CPA that r, s are values of independent, normal random variables, each with means $\mu = 0$ and equal variances σ^2. Then $(\ell/\sigma)^2$ is a random variable with Chi-square distribution with 2 degrees of freedom (abbreviated χ^2, 2*df*). The density function for this random variable is

$$f_{\chi^2}(x) = \frac{1}{2}\exp(-\frac{x}{2}), \quad x \geq 0.$$

If m is the *median* miss distance, then $\Pr(\ell \leq m) = .5$; that is, $\Pr(\ell^2 \leq m^2) = .5$ or $\Pr((\ell/\sigma)^2 \leq (m/\sigma)^2) = .5$. Thus $(m/\sigma)^2$ is the median M for the χ^2 random variable with 2*df*. The numerical value M is determined as follows; if

$$\int_0^M \frac{1}{2}\exp(-\frac{x}{2})dx = .5,$$

then

$$1 - \exp(-\frac{M}{2}) = .5 \quad \text{and} \quad M = \ln 4 \approx 1.386.$$

The *Air Combat/Missile Evaluation Study*, from 1976 (reference c) suggests that $m = 14L$ (a "14-mil error") so that

$$\left(\frac{14L}{\sigma}\right)^2 = \ln 4 \quad \text{and} \quad \sigma^2 = \frac{(14L)^2}{\ln 4}, \tag{B–10}$$

where σ^2 is the common variance for r and s.

Next, determine the variance for ℓ;

$$\sigma_\ell^2 = E[(\ell - \mu_\ell)^2] = E[\ell^2] - \mu_\ell^2. \tag{B–11}$$

From $E[(\frac{\ell}{\sigma})^2] = 2$ (the expected value of a χ^2 random variable = *df*) there follows

$$E[\ell^2] = 2\sigma^2 \quad \text{(using linearity of the expectation function).} \tag{B–12}$$

Further, $\mu_\ell = E[\ell]$. Let $w = (\ell/\sigma)^2$; then w is a χ^2 random variable with $2df$; $\ell = \sigma\sqrt{w}$; and $\mu_\ell = \int_0^\infty \sigma\sqrt{w} f_{\chi^2}(w)dw$ which can be simplified to yield

$$\mu_\ell = \frac{\sigma}{2}\sqrt{2\pi}. \tag{B-13}$$

Putting results in equations (B-10) through (B-13) together yields

$$\sigma_\ell^2 = 2\sigma^2 - \frac{2\pi\sigma^2}{4} = \frac{4-\pi}{2}\sigma^2,$$

and

$$\sigma_\ell^2 \approx C_\ell^2 = \frac{4-\pi}{2}\frac{(14L)^2}{\ln 4}, \tag{B-14}$$

in which C_ℓ^2 is an abbreviation for the constant given. Standard deviation σ_ℓ in miss distance ℓ is a function of deviations in parameters x, y, z, V_x, V_y, V_z at the time of gun firing; the actual miss distance $\Delta\ell$ is given by

$$\Delta\ell \approx \frac{\partial\ell}{\partial x}\Delta x + \frac{\partial\ell}{\partial y}\Delta y + \frac{\partial\ell}{\partial z}\Delta z + \frac{\partial\ell}{\partial V_x}\Delta V_x + \frac{\partial\ell}{\partial V_y}\Delta V_y + \frac{\partial\ell}{\partial V_z}\Delta V_z.$$

Under the assumption that the above parameter deviations $\Delta\underline{X}$ are unbiased and independent,

$$\sigma_\ell^2 \approx E[\Delta\ell^2],$$

or

$$(\frac{\partial\ell}{\partial x})^2\sigma_x^2 + (\frac{\partial\ell}{\partial y})^2\sigma_y^2 + (\frac{\partial\ell}{\partial z})^2\sigma_z^2 + (\frac{\partial\ell}{\partial V_x})^2\sigma_{V_x}^2 + (\frac{\partial\ell}{\partial V_y})^2\sigma_{V_y}^2 + (\frac{\partial\ell}{\partial V_z})^2\sigma_{V_z}^2 = C_\ell^2. \tag{B-15}$$

As in the treatment of stationary targets, abbreviate the estimate for $\frac{\partial\ell}{\partial x}$ as Dx, ..., and for $\frac{\partial\ell}{\partial V_z}$ as DV_z. Then perform the following optimization:

$$\max_{\{\sigma's\}} \sigma_{\Delta x}\sigma_{\Delta y}\sigma_{\Delta z}\sigma_{\Delta V_x}\sigma_{\Delta V_y}\sigma_{\Delta V_z}$$

$$\text{S.T.}\quad Dx^2\cdot\sigma_{\Delta x}^2 + Dy^2\cdot\sigma_{\Delta y}^2 + Dz^2\cdot\sigma_{\Delta z}^2 + DV_x^2\cdot\sigma_{\Delta V_x}^2 + DV_y^2\cdot\sigma_{\Delta V_y}^2 + DV_z^2\cdot\sigma_{\Delta V_z}^2 = C_\ell^2,$$

obtaining the estimates

$$\hat{\sigma}_{\Delta x} = \frac{|C_\ell|}{|Dx|\sqrt{6}};\quad \hat{\sigma}_{\Delta y} = \frac{|C_\ell|}{|Dy|\sqrt{6}};\quad \hat{\sigma}_{\Delta z} = \frac{|C_\ell|}{|Dz|\sqrt{6}}, \tag{B-16a}$$

$$\hat{\sigma}_{\Delta V_x} = \frac{|C_\ell|}{|DV_x|\sqrt{6}};\quad \hat{\sigma}_{\Delta V_y} = \frac{|C_\ell|}{|DV_y|\sqrt{6}};\quad \hat{\sigma}_{\Delta V_z} = \frac{|C_\ell|}{|DV_z|\sqrt{6}}. \tag{B-16b}$$

If an estimate $|Dp|$ for a partial derivative $|\frac{\partial\ell}{\partial p}|$ for a parameter p is nearly 0 (as is the case for parameters x and V_x), then ℓ is insensitive to p. In this instance, we may take $\hat{\sigma}_p = 0$, because the $\hat{\sigma}_p$'s are employed in simulating errors in p at the time of gun firing, and the error in p would be inconsequential insofar as simulating bullet CPA and measures of effectiveness depending upon CPA.

These error standard deviation estimates in equations (B-16) are used to simulate actual launch conditions with respect to airborne targets.

APPENDIX C

PROGRAM LISTINGS

Presented in this appendix are listings of two BASIC programs used in the determination of TACTS range accuracy requirements.

Program DISCRETE_FIT implements the methodology of Appendix A for the computations of position and velocity component standard deviations. Equations (A-7) and (A-8) are involved, together with data produced by TACTS weapon simulation runs.

The other program, DFIT_BAT_PLOT performs the computations of DISCRETE_FIT automatically for a variety of Performance Levels and data from one or several TACTS weapon simulation runs. Analyzed results are stored in data files for subsequent graphics presentations.

```
29 Jan 1987          10:31:09

100  REM DISCRETE_FIT
110  !
120  REM       WRITTEN: AUGUST 14, 1986
130  REM LAST MODIFIED: SEPTEMBER 4, 1986
140  !
150  COM /Consts/ P1,B(5)
160  COM /Ratios/ Ratio(1:6)
170  COM /Bounds/ Lower,Upper
180  COM /Output/ Sig(1:6)
190  COM /Intervals/ Nruns,X(1,6.100)
200  DIM Sig$(1:6),Nin$[80]
210  !
220  PRINT PAGE
230  GOSUB 350 ! INITIALIZATION
240  GOSUB 660 ! READ SIMULATION DATA
250  GOSUB 860 ! INPUT PRECISION
260  GOSUB 950  ! PRINT HEADER
270  WHILE FNPlevel(0,Sigv)>Despl
280   GOSUB 1140! BISECTION SEARCH
290   PRINT USING "6(2X,4D.2D,X)";Sig(*)
300   Sigv=Sigv+Delsigv
310  END WHILE
320  PRINTER IS CRT
330  END
340  !
350  REM INITIALIZATION
360  !
370  DATA .2316419
380  DATA 0,.31938153,-.356563782,1.781477937,-1.821255978,1.330274429
390  READ P1
400  FOR Param=0 TO 5
410   READ B(Param)
420  NEXT Param
430  !
440  Lower=0
450  Upper=1
460  !
470  Epsilon=.00001
480  Sigv=0
490  Delsigv=.01
500  !
510  Sig$(1)=" SIGMA-X  "
520  Sig$(2)=" SIGMA-Y  "
530  Sig$(3)=" SIGMA-Z  "
540  Sig$(4)=" SIGMA-Vx "
550  Sig$(5)=" SIGMA-Vy "
560  Sig$(6)=" SIGMA-Vz "
570  !
580  Ratio(1)=1
590  Ratio(2)=1
600  Ratio(3)=4
610  Ratio(4)=1
620  Ratio(5)=1
630  Ratio(6)=4
```

```
640  RETURN
650  !
660  REM READ SIMULATION DATA
670  !
680  REM  X(1,6,100) ! 1 = Lower,Upper
690                  ! 6 = X,Y,Z,Vx,Vy,Vz
700                  ! 100 = Runs
710  !
720  PRINT " INPUT NAME OF DATAFILE: ";
730  INPUT "",Nin$
740  PRINT Nin$
750  PRINT
760  ASSIGN #1 TO Nin$
770  READ #1;Nruns
780  FOR R=1 TO Nruns
790   FOR Param=1 TO 6
800    READ #1;X(0,Param,R),X(1,Param,R)
810   NEXT Param
820  NEXT R
830  ASSIGN #1 TO *
840  RETURN
850  !
860  REM INPUT PRECISION
870  !
880  PRINT
890  PRINT "INPUT DESIRED PERFORMANCE LEVEL (Between 0 & 1) = ";
900  INPUT "",Despl
910  PRINT Despl
920  PRINT
930  RETURN
940  !
950  REM PRINT HEADER
960  !
970  PRINT "HARDCOPY (Y or N): ";
980  INPUT "",Hc$
990  IF Hc$="Y" THEN PRINTER IS 402
1000 PRINT PAGE
1010 PRINT "NAME OF DATAFILE: ";Nin$
1020 PRINT
1030 PRINT "DESIRED PERFORMANCE LEVEL = ";Despl
1040 PRINT
1050 PRINT "      POSITION ERROR RATIO -     X:Y:Z = ";
1060 PRINT USING "D,A,D,A,D";Ratio(1);":";Ratio(2);":";Ratio(3)
1070 PRINT "      VELOCITY ERROR RATIO - Vx:Vy:Vz = ";
1080 PRINT USING "D,A,D,A,D";Ratio(4);":";Ratio(5);":";Ratio(6)
1090 PRINT
1100 PRINT USING "2X,6(10A)";Sig$(*)
1110 PRINT
1120 RETURN
1130 !
1140 REM BISECTION SEARCH
1150 REM  FNPLEVEL(.,.) IS A DECREASING FUNCTION IN BOTH ARGUMENTS
1160 !
1170 Sigllo=.01
1180 WHILE FNPlevel(Sigllo,Sigv)<Despl
1190  Sigllo=Sigllo/2
```

```
1200  END WHILE
1210  Siglhi=1
1220  WHILE FNPlevel(Siglhi,Sigv)>Despl
1230   Siglhi=2*Siglhi
1240  END WHILE
1250  Siglmid=.5*(Sigllo+Siglhi)
1260  Plmid=FNPlevel(Siglmid,Sigv)
1270  WHILE ABS(Plmid-Despl)>Epsilon
1280   IF Plmid>Despl THEN
1290    Sigllo=Siglmid
1300   ELSE
1310    Siglhi=Siglmid
1320   END IF
1330   Siglmid=.5*(Sigllo+Siglhi)
1340   Plmid=FNPlevel(Siglmid,Sigv)
1350  END WHILE
1360  RETURN
1370  !
1380  END
1390  !
1400  DEF FNPlevel(Sx,Sv)
1410   COM /Bounds/ Lower,Upper
1420   COM /Ratios/ Ratio(*)
1430   COM /Output/ Sig(*)
1440   COM /Intervals/ Nruns,X(*)
1450   DIM S(1:6)
1460   Prob_sum=0
1470   S(1)=Sx*Ratio(1)
1480   S(2)=Sx*Ratio(2)
1490   S(3)=Sx*Ratio(3)
1500   S(4)=Sv*Ratio(4)
1510   S(5)=Sv*Ratio(5)
1520   S(6)=Sv*Ratio(6)
1530   FOR Run=1 TO Nruns
1540    Prob=1
1550    FOR Param=1 TO 6
1560     IF S(Param)=0 THEN
1570      Factor=1
1580     ELSE
1590      Factor=FNP(X(Upper,Param,Run)/S(Param))
1600      Factor=Factor-FNP(X(Lower,Param,Run)/S(Param))
1610     END IF
1620     Prob=Prob*Factor
1630    NEXT Param
1640    Prob_sum=Prob_sum+Prob
1650   NEXT Run
1660   Plevel=Prob_sum/Nruns
1670   MAT Sig=S
1680   RETURN Plevel
1690  FNEND
1700  !
1710  DEF FNZ(X)
1720   IF ABS(X)>5 THEN
1730    Z=0
1740   ELSE
1750    Z=(1/SQR(2*PI))*EXP(-X*X/2)
```

```
1760   END IF
1770   RETURN Z
1780  FNEND
1790  !
1800  DEF FNP(Xin)
1810   COM /Consts/ P1,B(*)
1820   IF Xin>0 THEN
1830    X=Xin
1840   ELSE
1850    X=-Xin
1860   END IF
1870   T=1/(1+P1*X)
1880   Temp=0
1890   FOR I=5 TO 0 STEP -1
1900    Temp=T*Temp+B(I)
1910   NEXT I
1920   P=1-FNZ(X)*Temp
1930   IF Xin<0 THEN P=1-P
1940   RETURN P
1950  FNEND
```

29 Jan 1987 10:33:02

```
100  REM DFIT_BAT_PLOT
110  !
120  REM        WRITTEN: SEPTEMBER 11, 1986
130  REM LAST MODIFIED: OCTOBER 2, 1986
140  !
150  COM /Consts/ P1,B(5)
160  COM /Ratios/ Ratio(1:6)
170  COM /Bounds/ Lower,Upper
180  COM /Output/ Sig(1:6)
190  COM /Intervals/ Nruns,X(1,6,100)
200  DIM Sig$(1:6),Indexfile$[30],Nin$[80]
210  DIM Xout(0:50),Yout(0:50)
220  !
230  PRINT PAGE
240  GOSUB 710 ! INITIALIZATION
250  GOSUB 1010! DETERMINE CASES TO BE RUN
260  FOR Case=1 TO Ncases
270   GOSUB 1210! READ SIMULATION DATA
280   FOR Despl=.95 TO .7 STEP -.05
290    GOSUB 1870! BISECTION SEARCH FOR Maxsigv
300    Neval=25    ! NUMBER OF EVALUATIONS EQUALLY SPACED
310    Extra=4     ! EXTRA EVALUATIONS TO SMOOTH UP GRAPH
320    Delsigv=Maxsigv/Neval
330    GOSUB 1390! PRINT HEADER
340    FOR Eval=0 TO Neval+Extra
350     SELECT Eval
360     CASE =Neval+Extra
370      GOSUB 1790
380     CASE ELSE
390      IF Eval<Neval THEN
400       Sigv=Eval*Delsigv
410      ELSE
420       Sigv=(Neval-1)*Delsigv+(Eval+1-Neval)*Delsigv/(Extra+1)
430      END IF
440      GOSUB 1550! BISECTION SEARCH
450     END SELECT
460     PRINT USING "6(2X,4D.2D,X)";Sig(*)
470     Xout(Eval)=Sig(4)
480     Yout(Eval)=Sig(1)
490    NEXT Eval
500    GOSUB Putoutplotdata
510    IF NOT Hc THEN WAIT 1
520   NEXT Despl
530  NEXT Case
540  PRINT PAGE
550  PRINTER IS CRT
560  END
570  !
580 Putoutplotdata:  !ESTABLISH PLOTDATAFILES
590  !
600  Pname$=Casename$(Case)&"_"&VAL$(Ratio(3))&"_"&VAL$(100*Despl)
610  CREATE DATA Pname$,20
620  ASSIGN #1 TO Pname$
630  PRINT #1;Neval+Extra+1
```

```
640  FOR Eval=0 TO Neval+Extra
650   PRINT #1;Xout(Eval),Yout(Eval)
660  NEXT Eval
670  ASSIGN #1 TO *
680  !
690  RETURN
700  !
710  REM INITIALIZATION
720  !
730  DATA .2316419
740  DATA 0,.31938153,-.356563782,1.781477937,-1.821255978,1.330274429
750  READ P1
760  FOR Param=0 TO 5
770   READ B(Param)
780  NEXT Param
790  !
800  Lower=0
810  Upper=1
820  !
830  Epsilon=.00001
840  Delsigv=.05
850  !
860  Sig$(1)=" SIGMA-X  "
870  Sig$(2)=" SIGMA-Y  "
880  Sig$(3)=" SIGMA-Z  "
890  Sig$(4)=" SIGMA-Vx "
900  Sig$(5)=" SIGMA-Vy "
910  Sig$(6)=" SIGMA-Vz "
920  !
930  Ratio(1)=1
940  Ratio(2)=1
950  Ratio(3)=1
960  Ratio(4)=1
970  Ratio(5)=1
980  Ratio(6)=1
990  RETURN
1000 !
1010 REM DETERMINE CASES TO BE RUN
1020 !
1030 PRINT "INPUT INDEX FILE NAME OF CASES TO BE RUN: ";
1040 INPUT "",Indexfile$
1050 PRINT Indexfile$
1060 ASSIGN #1 TO Indexfile$
1070 READ #1;Ncases
1080 FOR Case=1 TO Ncases
1090  READ #1;Casename$(Case)
1100 NEXT Case
1110 ASSIGN #1 TO *
1120 !
1130 PRINT "HARDCOPY (Y or N): ";
1140 INPUT "",Hc$
1150 PRINT Hc$
1160 Hc=(Hc$="Y")
1170 IF Hc THEN PRINTER IS 401
1180 !
1190 RETURN
```

```
1200  !
1210  REM READ SIMULATION DATA
1220  !
1230  REM  X(1,6,100) ! 1 = Lower,Upper
1240                  ! 6 = X,Y,Z,Vx,Vy,Vz
1250                  ! 100 = Runs
1260  !
1270  Nin$=Casename$(Case)
1280  PRINT
1290  ASSIGN #1 TO Nin$
1300  READ #1;Nruns
1310  FOR R=1 TO Nruns
1320   FOR Param=1 TO 6
1330    READ #1;X(0,Param,R),X(1,Param,R)
1340   NEXT Param
1350  NEXT R
1360  ASSIGN #1 TO *
1370  RETURN
1380  !
1390  REM PRINT HEADER
1400  !
1410  PRINT PAGE
1420  PRINT "NAME OF DATAFILE: ";Nin$
1430  PRINT
1440  PRINT "DESIRED PERFORMANCE LEVEL = ";Despl
1450  PRINT
1460  PRINT "     POSITION ERROR RATIO -     X:Y:Z = ";
1470  PRINT USING "D,A,D,A,D";Ratio(1);":";Ratio(2);":";Ratio(3)
1480  PRINT "     VELOCITY ERROR RATIO - Vx:Vy:Vz = ";
1490  PRINT USING "D,A,D,A,D";Ratio(4);":";Ratio(5);":";Ratio(6)
1500  PRINT
1510  PRINT USING "2X,6(10A)";Sig$(*)
1520  PRINT
1530  RETURN
1540  !
1550  REM BISECTION SEARCH
1560  REM  FNPLEVEL(.,.) IS A DECREASING FUNCTION IN BOTH ARGUMENTS
1570  !
1580  Sigllo=.01
1590  WHILE FNPlevel(Sigllo,Sigv)<Despl
1600   Sigllo=Sigllo/2
1610  END WHILE
1620  Siglhi=1
1630  WHILE FNPlevel(Siglhi,Sigv)>Despl
1640   Siglhi=2*Siglhi
1650  END WHILE
1660  Siglmid=.5*(Sigllo+Siglhi)
1670  Plmid=FNPlevel(Siglmid,Sigv)
1680  WHILE ABS(Plmid-Despl)>Epsilon
1690   IF Plmid>Despl THEN
1700    Sigllo=Siglmid
1710   ELSE
1720    Siglhi=Siglmid
1730   END IF
1740   Siglmid=.5*(Sigllo+Siglhi)
1750   Plmid=FNPlevel(Siglmid,Sigv)
```

```
1760  END WHILE
1770  RETURN
1780  !
1790  REM SIG(NEVAL) VALUES
1800  !
1810  MAT Sig=(0)
1820  FOR I=4 TO 6
1830   Sig(I)=Maxsigv*Ratio(I)
1840  NEXT I
1850  RETURN
1860  !
1870  REM BISECTION SEA
1880  REM  FNPLEVEL(.,.
1890  !
1900  Sigllo=.01
1910  WHILE FNPlevel(0,
1920   Sigllo=Sigllo/2
1930  END WHILE
1940  Siglhi=1
1950  WHILE FNPlevel(0,S
1960   Siglhi=2*Siglhi
1970  END WHILE
1980  Siglmid=.5*(Sigllo
1990  Plmid=FNPlevel(0,S
2000  WHILE ABS(Plmid-De
2010   IF Plmid>Despl TH
2020    Sigllo=Siglmid
2030   ELSE
2040    Siglhi=Siglmid
2050   END IF
2060   Siglmid=.5*(Sigll
2070   Plmid=FNPlevel(0,
2080  END WHILE
2090  Maxsigv=Siglmid-Ep
2100  RETURN
2110  END
2120  !
2130  DEF FNPlevel(Sx,Sv
2140   COM /Bounds/ Lowe
2150   COM /Ratios/ Rati
2160   COM /Output/ Sig(
2170   COM /Intervals/ N
2180   DIM S(1:6)
2190   Prob_sum=0
2200   S(1)=Sx*Ratio(1)
2210   S(2)=Sx*Ratio(2)
2220   S(3)=Sx*Ratio(3)
2230   S(4)=Sv*Ratio(4)
2240   S(5)=Sv*Ratio(5)
2250   S(6)=Sv*Ratio(6)
2260   FOR Run=1 TO Nruns
2270    Prob=1
2280    FOR Param=1 TO 6
2290     IF S(Param)=0 THEN
2300      Factor=1
2310     ELSE
```

ARGUMENTS

```
2320      Factor=FNP(X(Upper,Param,Run)/S(Param))
2330      Factor=Factor-FNP(X(Lower,Param,Run)/S(Param))
2340     END IF
2350     Prob=Prob*Factor
2360    NEXT Param
2370    Prob_sum=Prob_sum+Prob
2380   NEXT Run
2390   Plevel=Prob_sum/Nruns
2400   MAT Sig=S
2410   RETURN Plevel
2420  FNEND
2430  !
2440  DEF FNZ(X)
2450   IF ABS(X)>5 THEN
2460    Z=0
2470   ELSE
2480    Z=(1/SQR(2*PI))*EXP(-X*X/2)
2490   END IF
2500   RETURN Z
2510  FNEND
2520  !
2530  DEF FNP(Xin)
2540   COM /Consts/ P1,B(*)
2550   IF Xin>0 THEN
2560    X=Xin
2570   ELSE
2580    X=-Xin
2590   END IF
2600   T=1/(1+P1*X)
2610   Temp=0
2620   FOR I=5 TO 0 STEP -1
2630    Temp=T*Temp+B(I)
2640   NEXT I
2650   P=1-FNZ(X)*Temp
2660   IF Xin<0 THEN P=1-P
2670   RETURN P
2680  FNEND
```

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