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Investigation of Potential Mapping Products Based on Acoustic Imagery from AN/SQS-53B ASW Sonar

CHARLES L. WALKER
MARIA T. KALCIC

Mapping, Charting, and Geodesy Branch
Marine Geosciences Division

FREDERICK A. BOWLES

Seafloor Sciences Branch
Marine Geosciences Division

DAVID HANDSCHUMACHER

University of New Orleans
New Orleans, LA

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Charles L. Walker, Maria T. Kalcic, Frederick A. Bowles, and David Handschumacher†

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†University of New Orleans, New Orleans, LA

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13. ABSTRACT *(Maximum 200 words)*

Naval Research Laboratory (NRL) personnel collected acoustic backscatter data using the AN/SQS-53B ASW sonar on board the *USS Monterey* (CG-61) during transits of the Red Sea in November and December 1995. This report presents preliminary results on the potential of this type of data as a source of acoustic imagery for seafloor mapping. The AN/SQS-53B sonar operates at a frequency of 3.5 kHz giving it the ability to operate at long ranges. Its 12 geo-stabilized beams facilitate collects of multiple aspect angle coverages of the same area of the seafloor that the ship transits. The NRL data acquisition system works in conjunction with the sonar operation without interference. A preliminary comparison of the AN/SQS-53B sonar data with multibeam bathymetry, taken simultaneously by the Naval Oceanographic Office, shows that acoustic backscatter generally correlates with seafloor relief in both deep and shallow water. This observation leads to the main conclusion that an acoustic backscatter mapping product based on data collected during routine operations could enhance the utilization of the sonar in its primary mission of ASW, i.e., determination of optimum look-direction for ASW sonars and detection targets in high bottom-reverberation areas. A further implication of these results is that U.S. combatants operating in unfamiliar or poorly charted areas will have an on board capability to detect shoals or other navigational hazards well in front of the ship's movement.

14. SUBJECT TERMS

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Background:

Over the past few years the Naval Research Laboratory has investigated the use of the AN/SQS-53B ASW sensor as a research tool in seafloor characterization and mapping. This program has resulted in the development of a data acquisition system which can be attached to the sonar receiver to obtain digital recordings of the pulse returns. The latest version of the system was installed on the USS Monterey (CG-61) prior to its planned transit of the Red Sea in November 1995.

The AN/SQS-53B has a number of operating modes designed to optimize its primary mission of detecting submarines in the deep ocean. The modes of most interest in acoustic seafloor mapping are the Convergence Zone (CZ), Bottom Bounce (BB), and Bottom Bounce / Tracking (BB/TK) modes. In the first two, the received reflected acoustic pulse is formed into twelve nominally conical beams each of which is approximately 10° wide in the horizontal plane. The receiver beams can be steered so that the entire 120° coverage is centered on any azimuth. The beams are stabilized so that once the sector center of the 120° coverage is set, each beam remains pointed in a constant direction, relative to true north, regardless of the ship's course. The beams can also be steered vertically down to 45° depression angle. The BB/TK mode is similar, with the exception that the projector beam is constrained to a 30° horizontal coverage directed along the sector center. This characteristic provides increased power in the direction of a previously located target for tracking.

The horizontal stabilization of the sonar beams (built into the on board electronic beamforming and the ship's inertial navigation) provides a capability for acoustic image mapping not found on conventional side-scan sonars or imagery derived from multibeam systems. In conventional side-scan systems the beam look-direction is always perpendicular to the tow ship track. The availability of multiple beams, each stabilized in a different geographic look-direction, allow the construction of an acoustic backscatter image for each look direction. In many areas of the world, backscatter from the seafloor is highly dependent on the azimuthal aspect angle of the incident acoustic signal. This fact has historically hindered the full exploitation of conventional side-scan sonar systems in bottom mapping. An example of the aspect dependence of the backscatter is in areas where the seafloor is highly delineated (such as in the Red Sea or the Mid-Atlantic Ridge). When the linear seafloor features are illuminated broadside, the acoustic return is much higher than when the illumination is along the linear direction. The development of an acoustic imagery mapping product requires a means of accounting for these large aspect angle dependencies. One solution, suggested here, is that separate maps be prepared for each available look direction. Such a data set would then prove useful in developing methods of combining look directions to categorize the backscatter.

The Naval Oceanographic Office conducted a multibeam bathymetry survey in the Red Sea during the same time the Monterey made its transit. The combination of this data with the AN/SQS-53 imagery data allows increased exploitation of both data sets. This fusion is possible because of the precise location of the AN/SQS-53 based on GPS.

Figures 1 and 2 show the location of an area of the Red Sea (Area 3 on the map) chosen for a preliminary analysis. The background bathymetry is at a 2-arc minute resolution (DBDBV) and the Naval Oceanographic Office multibeam data was process to a 100 meter resolution as was the sonar imagery.

Approach:

The NRL data acquisition system digitized data from each of the twelve receiver beams formed in the three modes. For each beam, data was acquired from two separate points in the receiver. These points were essentially before and after the Automatic Gain Control (AGC) amplifier but after the Time Varied Gain (TVG) amplifier in the receiver circuit. These points are designated Pre-AGC and Post-AGC channels. Thus, 24 channels of data were recorded. Previous NRL experiments indicated that Pre-AGC data was preferable to the Post-AGC data for bottom mapping.

Two sonar acoustic data sets, Pre- and Post-AGC, were digitized at a rate of 250 Hz per channel giving a range sampling distance of 3 meters. The data sets were recorded on optical disks in a "band-interleaved-by-pixel" format. Each ping or scan was preceded by a header containing ping number and time. Other fields in the ping header were used for additional information including navigation and sonar operating modes. During this deployment, however, the latter data was not readily available for digital recording. As a result, navigation was recorded separately (by hand, every 5 minutes) using a GPS receiver. Time differences between the GPS and the computer controlling the data acquisition were noted. In addition, sonar modes and sector center locations were recorded in the log book.

In order to produce an acoustic image map from the acquired data, it is necessary to locate the position of the ship at the time the projector transmit ping is made. This position, combined with the known direction of the beam and the water depth, allows the received ping to be geo-rectified on a map datum. Subsequent pings can thus be combined into a geo-rectified image. Due to the ping length and beamwidth combinations, the "raw" imagery has pixels with large differences in the extent of transmit direction and cross beam direction. A separate image can be formed from each beam. As noted, the image is analogous to a side-scan sonar image except that the look directions are constant (relative to true north) and the beams are not all directed at 90 degrees to the ship's course.

The initial step in the data analysis consists of de-multiplexing the time sequence for each beam and merging with the navigation and log data . This was accomplished based on the time of the ping and the time of the GPS fix. The latter was accurate to 1 microsecond.

Twelve separate "channels," or data files, were constructed for each ping. Each file contained a proto-image that was swept out as the ship moved along track. The approach to geo-rectification of each file is based on the geometry of a geo-stabilized and a relatively constant ship course. This approach has proved unsatisfactory for the beams pointed in the direction of the ship's heading but is well suited to the other beams. The details of the geo-rectification and the "slant range" correction using a flat bottom assumption are included in the Appendices. The procedure consists of placing each ping (considered as a line) in its proper geographic position based on the interpolated position of the start of the ping and the number of samples to the end giving the range to the end of the ping (assuming a constant sound velocity). The pings are thus aligned in a rectangular box by padding out the beginnings and ends with zeros. The location of the corners of the boxes are known. The aligned ping data is then resampled in a direction perpendicular to the ping direction, resulting in a file with equally spaced data in both directions. The "gridded" data is imported into an image processing computer program (ERDAS Imagine ver 8.1) and the geo-rectification completed.

Once the data is geo-rectified and put in image format, it can be compared to bathymetry and other data; a number of image enhancements can be performed to aid in data interpretation.

Results:

The processing steps described above are shown in Figures 3 through 5c. Figure 3 shows the raw data for a single beam. This particular data set contains pings #505 through #600 of file 27 (a Post-AGC set). Figure 4 shows the "gridded" data for all twelve beams. Figures 5a and 5b (different scales) show the geo-rectified Post-AGC imagery and Pre-AGC imagery, respectively, for a look direction of 52°. In Figures 5a-5c, the ship's track is from lower right to upper left. The ping origins are along the left (lower) edge of the data. The bright bands on the right (upper) edge of the image are artifacts that have not yet been corrected. Comparison of Figures 5a and 5b reveal differences in texture, i.e., some bottom features show up better in one or the other of these images. For example, the highlight returns in the lower right of Figure 5b originate from a seamount; however, in Figure 5a only the shadow of the seamount is evident (the highlights are obscured by the overall bottom reverberation). Figure 5c shows that the best aspects of both the Post-AGC (red) and Pre-AGC (green) images are enhanced when the images are merged.

It is difficult to display all twelve geo-rectified beams because they overlap each other. Figure 6a shows only ten look directions superimposed, but illustrates the difficulty of "seeing" multiple aspect-angles. For comparison, Figure 6b shows the details of one look direction at 52°. However, on the computer it is possible to flicker between two or more beams by varying the transparency. One method of simultaneously viewing up to three beams is to use color coding. In Figure 7a, three beams with look directions of 2° (upper), 52° (center) and 92° (lower) are superimposed. Inside the boxed area (Figure 7a), the 3 beams have been converted to color (Figure 7b) by assigning a different color gun to each beam: red for 92°, green for 52°, blue for 2°. For features common to all three beams the resultant color should be white (if the magnitudes are equal). If a feature occurs in only one beam then it will appear in the color assigned to that beam. As can be seen in Figure 7b, many of the ridge features are *not* common in all the beams. This is attributed to the different aspect angles. The use of color in this type of analysis requires further investigation.

Comparing the separate beams with bathymetry can be difficult. One method is to drape the imagery over the bathymetry and display the result in perspective shaded relief. A segment of the bottom bathymetry in the vicinity of image m27/700/52 (see Figure 1 for location) is shown in Figure 8 in shaded relief. Figure 9 shows the result of draping the imagery (m27/700/52) on the bathymetry. (In Figures 8 and 9 we are looking north). This overlay demonstrates that while there is a strong correlation of the backscatter with many bottom features, this is not always the case.

The sonar imagery in this area has also been compared to previously collected GLORIA data. GLORIA is a 3.5 kHz side-scan sonar that has been used world wide in acoustic bottom imaging. Figure 10 is a portion of the GLORIA data (a photo copy from a journal article) that has been geo-rectified after scanning. Preliminary analysis indicates that although the GLORIA along-track resolution is much better than that of the AN/SQS-53B, the AN/SQS-53B detected all the linear features that are evident in the GLORIA data.

The performance of the AN/SQS-53B in shallow water (less than 100 meter) was also investigated using the available data set. This data is shown in Figures 11 and 12a and 12b for the Strait of Bab al Mandab that connects the Red Sea and the Gulf of Aden. The sonar signals in Figures 11 and 12 (a and b) use slightly different beam combinations. Figures 11 shows beams at angles of 270° and 10°, and Figures 12a and 12b shows beams at angle of 10°. Figure 12a shows the acoustic returns from the shoaling areas around Perim Island. A bathymetric chart of this area was digitized and the acoustic imagery of Figure 12a superimposed on the chart (Fig. 12b). In Figure 12b strong returns are shown in black, which is the reverse of Figure 12a (and all the other figures). This is done to allow low returns to be "transparent." This overlay is much easier to interpret on the computer display since the transparency of the imagery can be readily controlled. A detailed comparison of the chart and the image shows a close correlation with all the charted features. However, some returns on the sonar imagery are not charted. This demonstrates the value for detecting possible hazards to navigation and for delineating bottom structure at high spatial resolution (compared to the chart) in shallow water. The data was obtained as the ship transited the Strait of Bab al Mandab at a speed of 25 knots. The capability demonstrated here suggests the use of this sonar and geo-referencing for aiding the planning and execution of detailed and accurate hydrographic surveying.

The particular geo-rectification technique used up to this point does not show returns from directly in front of the ship well. This is illustrated in Figure 13 which shows the beam with a look direction of 320°. In the lower segment a fairly good image results from the ship track being about 340°. However, when the ship turns left to a course of about 315° the resultant image is poor. The fix for this situation is a "full-up" geo-rectification for each pixel on the bottom where the cumulative effects of all pings incident from a particular aspect angle are accounted for and the beam pattern of the sonar is included in the calculation. This is a more computer intensive method than the one used, but can be expected to give superior results. Alternate means of handling multi-aspect acoustic imagery are being examined, including 3-color composites and controlled transparency. Further efforts in this area are needed to fully exploit the information in the imagery.

Summary:

Preparation of geo-rectified AN/SQS-53B acoustic imagery, using data collected during November and December 1995 transits of the Red Sea, has been completed and superimposed with Naval Oceanographic Office multibeam bathymetry collect during the same period. Preliminary comparison of AN/SQS 53B sonar data with the multibeam bathymetry shows positive correlations between backscatter and seafloor relief (i.e., slope aspect) in both deep and shallow water. High reverberation levels are associated with deep-water linear ridges when insonified normal to their strike. In contrast, low levels result when the look direction parallels the lineations. In addition, features associated with island shoals were evident in reverberation data collected at a distance of 4 miles from the island. It is demonstrated that the AN/SQS-53B acoustic backscatter data can be collected, geo-rectified and archived into a potentially valuable product.

Conclusions:

The AN/SQS-53B is the primary active sonar submarine detection equipment in the Navy for surface combatants and is installed in numerous ships including cruisers, destroyers and frigates. The normal deployment of these ships could allow the collection of a huge amount of acoustic seafloor backscatter data over time. A digital acoustic imagery product, populated by operational AN/SQS-53B sonars, can be developed to fully exploit the capabilities of this sonar. In addition, on board processing can be developed to allow the direct generation of geo-rectified imagery for comparison to archived NIMA data. The advent of accurate GPS positioning for the ship combined with the computing power required for image processing has made the concepts outlined here feasible.

Acknowledgements

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List of Figures

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Appendices

1. Conversion from range to horizontal distance under a flat bottom assumption.
2. Specialized AN/SQS-53B Geo-rectification.

AN/SQS-53B / Red Sea

Bathymetry and Sonar coverage

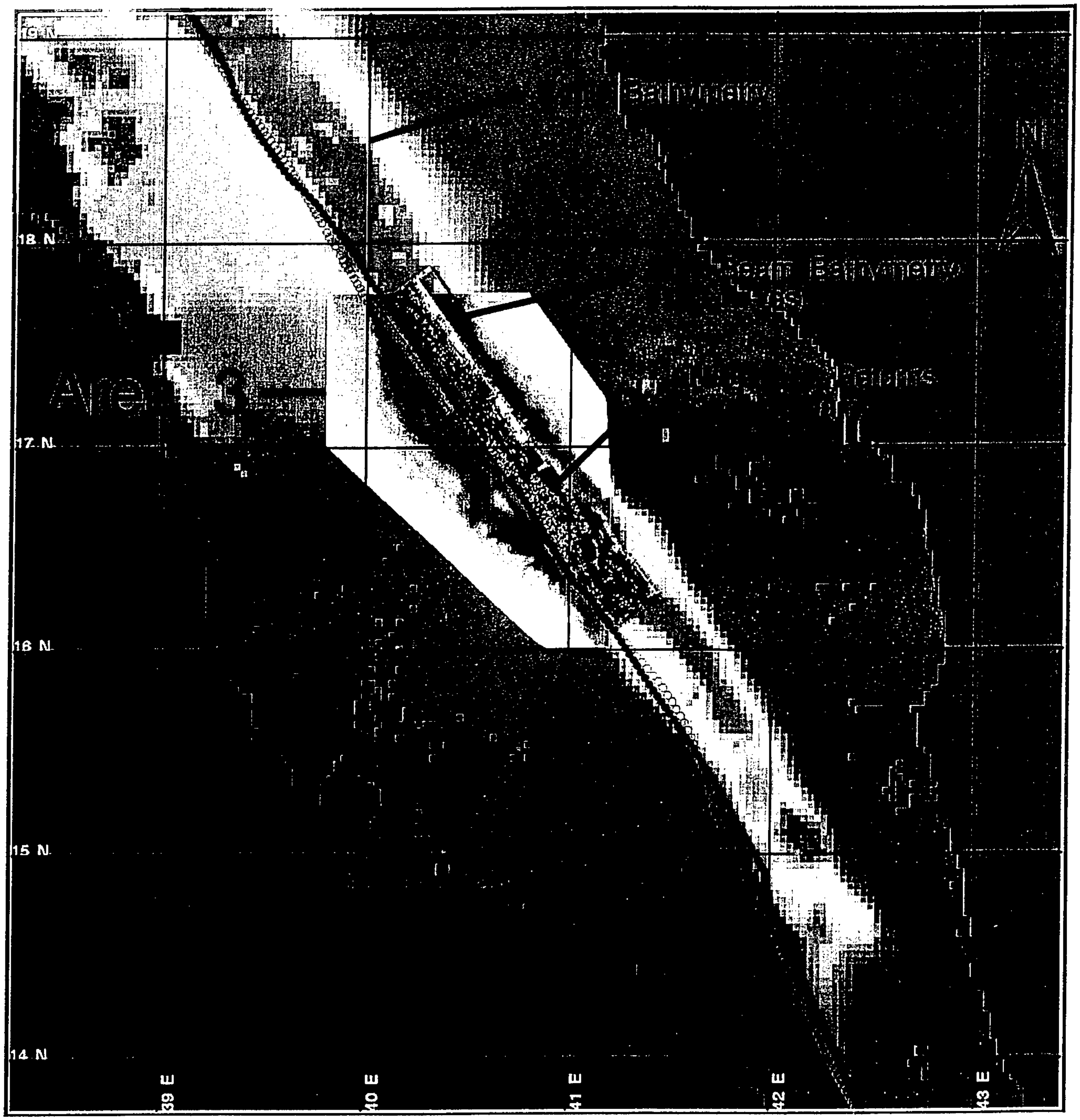
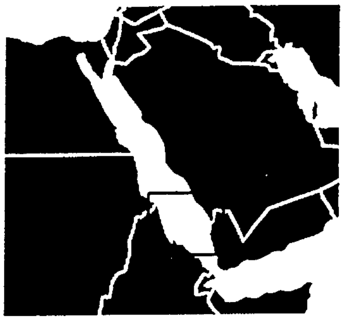


Figure 1. AN/SQS-53B sonar imagery (black and white) superimposed onto multibeam bathymetry (Color). Bottom depths range from shallow water (red) to deep water (dark blue). Solid circles mark the northward course of the ship and open circles mark the southward course.

AN/SQS-53B Sonar / Red Sea Bathymetry



USS MONTEREY
NOV-DEC 1995

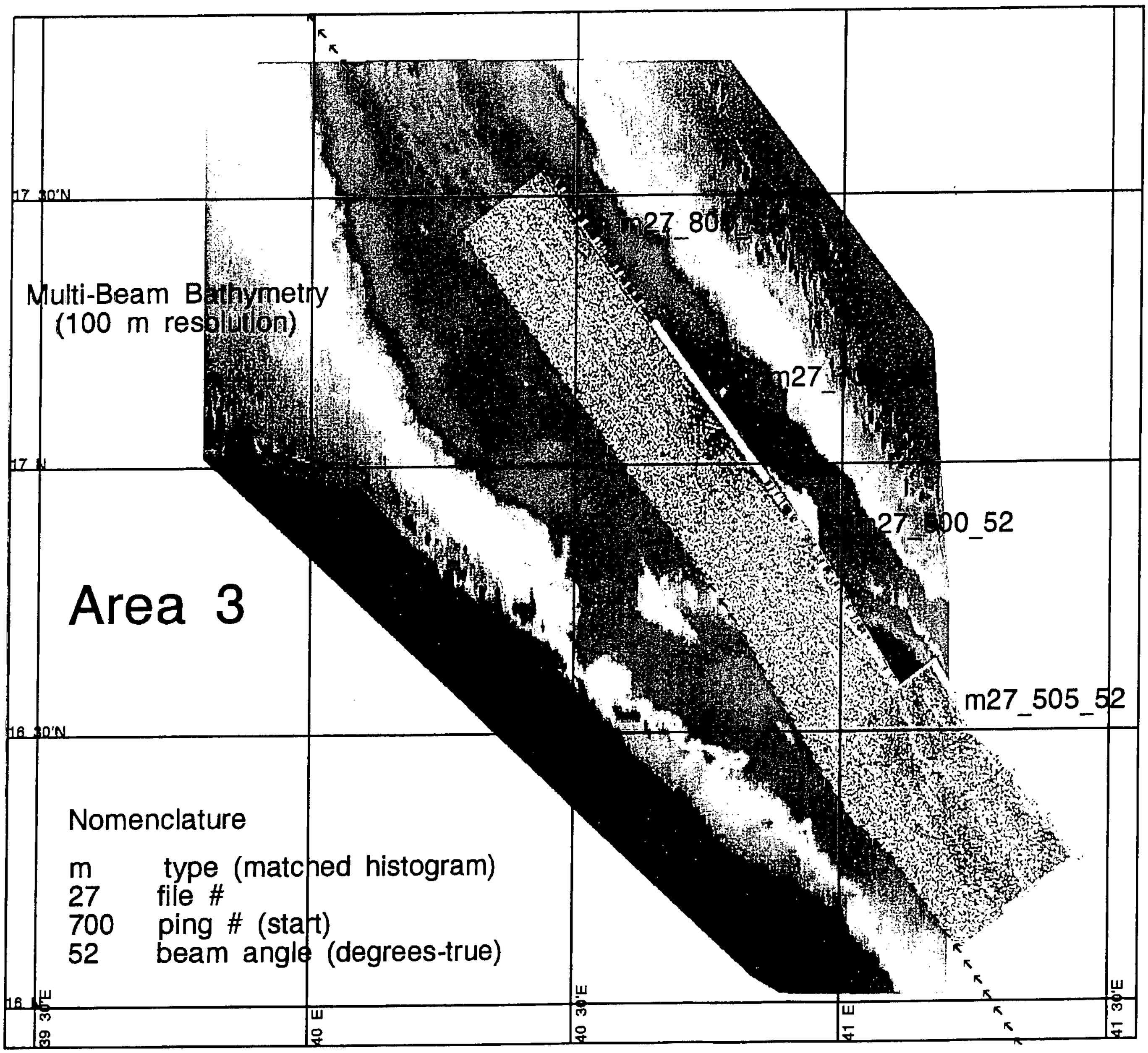


Figure 2. Enlargement of Area 3 shown in Figure 1. Bottom depths range from shallow water (red) to deep water (dark blue). Arrows indicate the ship's course and heading, and origin of the outgoing pings.

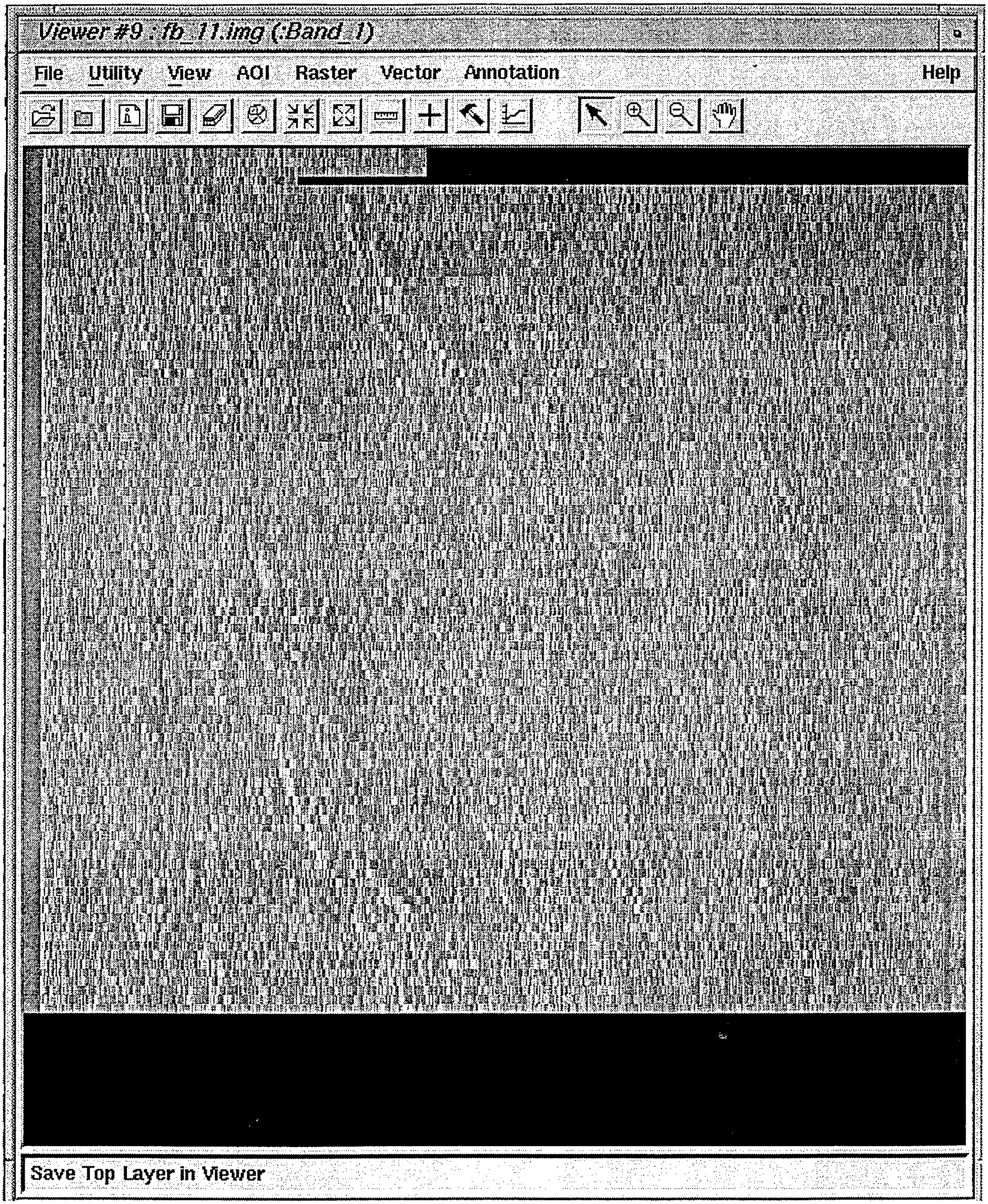


Figure 3. Single-ping, AN/SQS-53B raw sonar data for ping numbers 505-606.

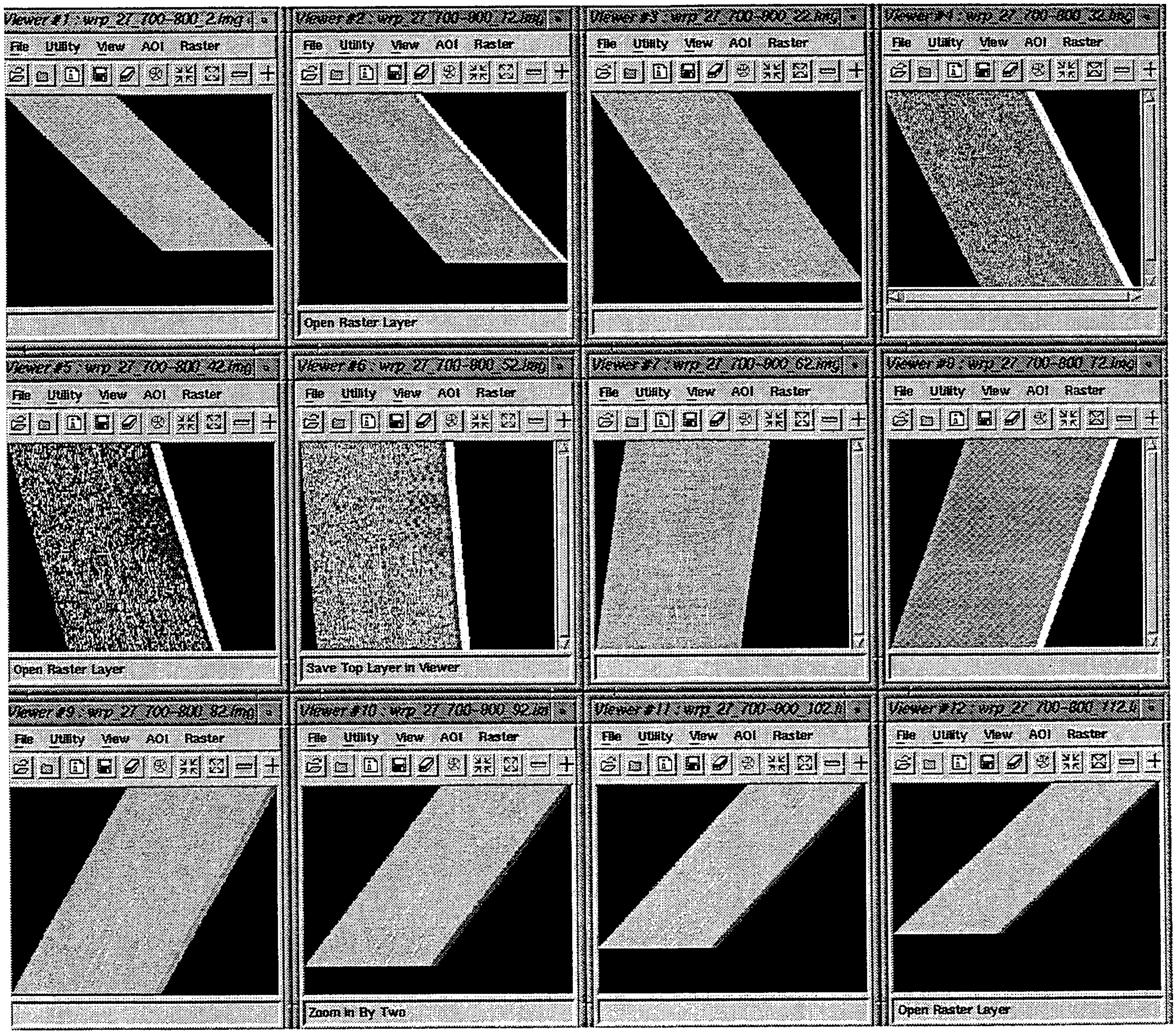


Figure 4. Gridded AN/SQS-53B sonar data for all twelve beams.

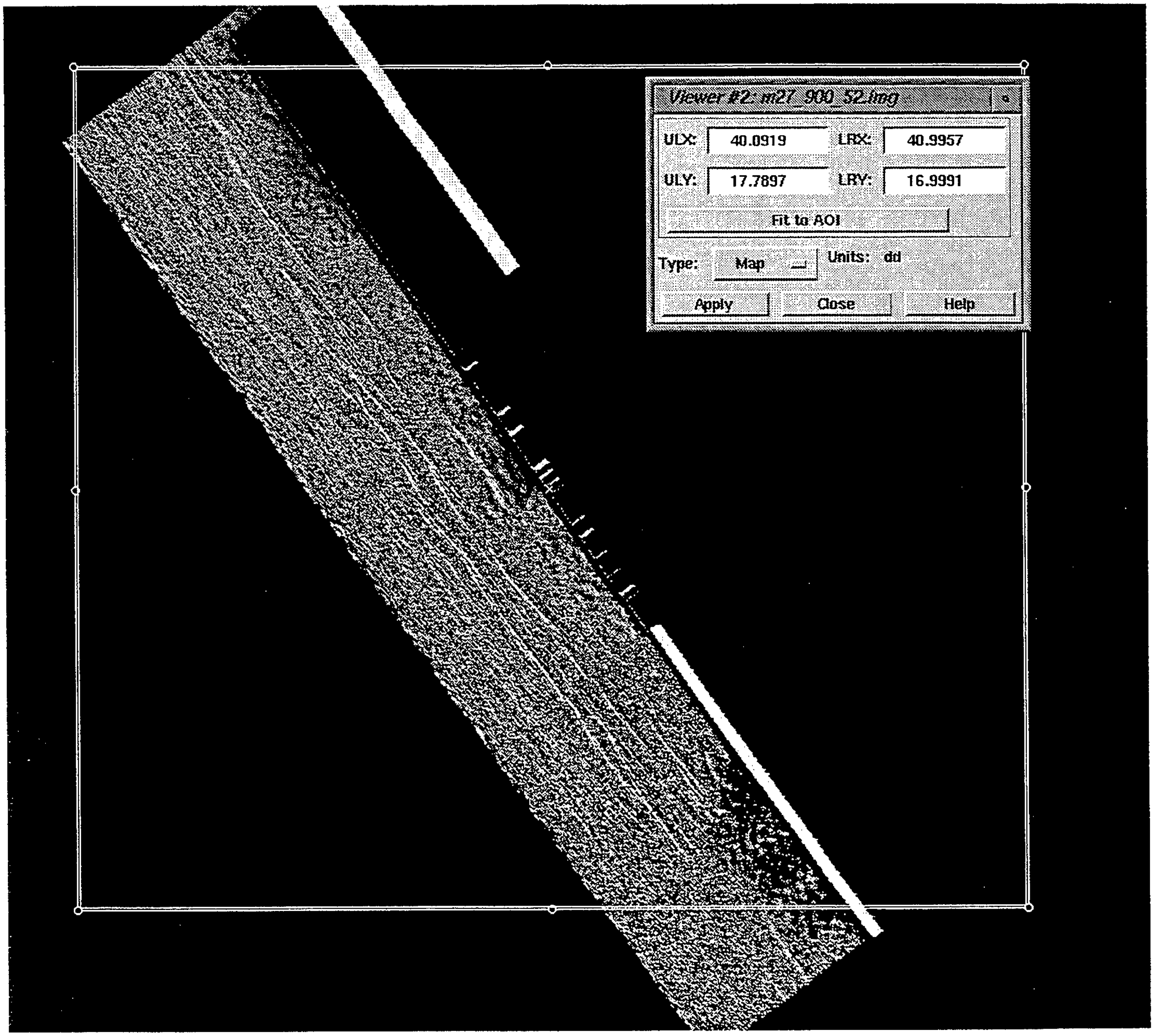


Figure 5a. Geo-rectified, Post-AGC, AN/SQS-53B imagery for 052° look direction. The ship's heading (relative to the box) is from lower right to upper left and the ship's track coincides with the left (lower) side of the imagery. For reference with Figure 2, the coordinate for the lower right corner of the box is 17°N/41°E.

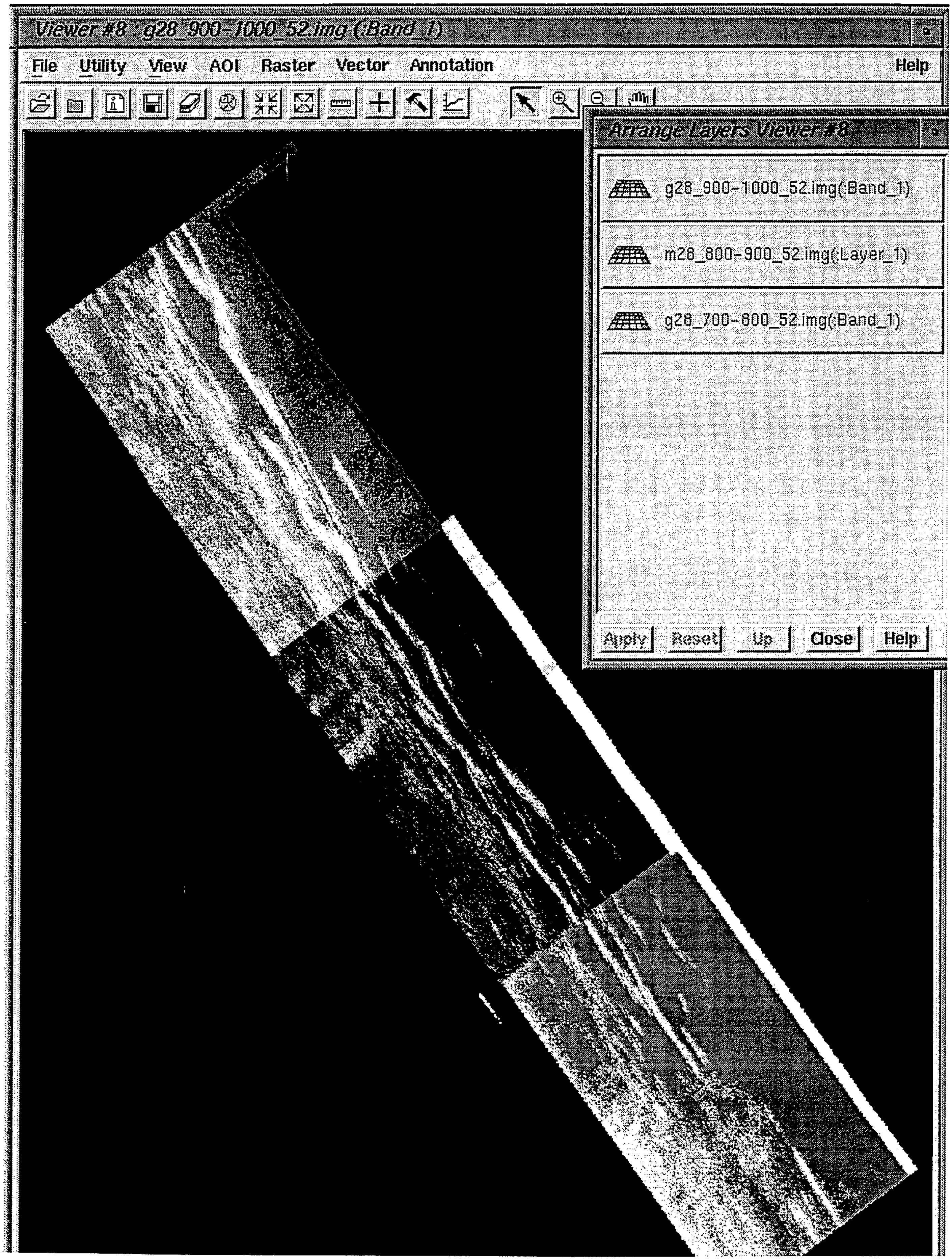


Figure 5b. Geo-rectified, Pre-AGC, AN/SQS-53B imagery for 052° look direction. The ship's heading is from lower right to upper left and the ship's track coincides with the left (lower) side of the imagery.

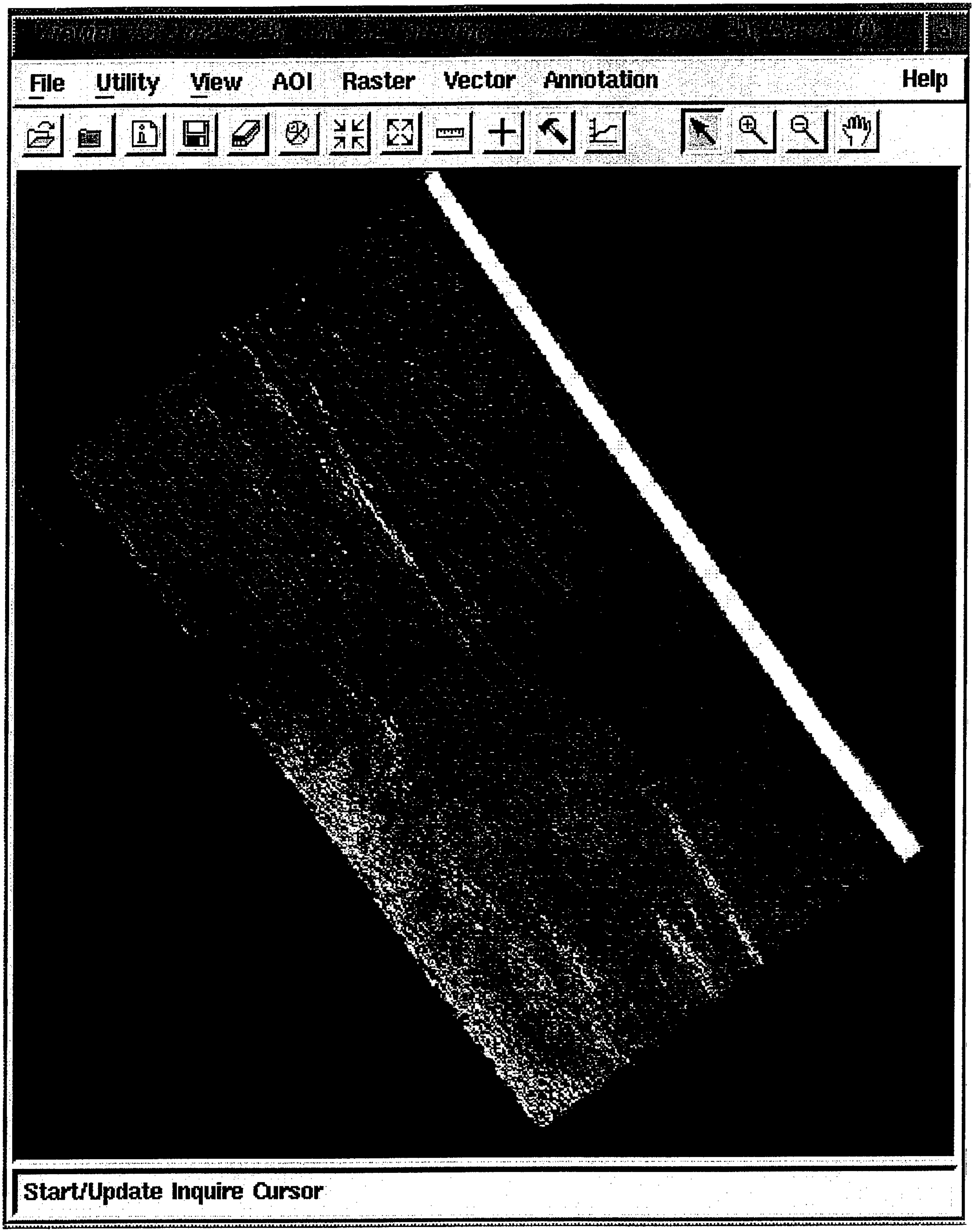


Figure 5c. Geo-rectified, merged Pre- and Post-AGC, AN/SQS-53B imagery for 052° look direction.

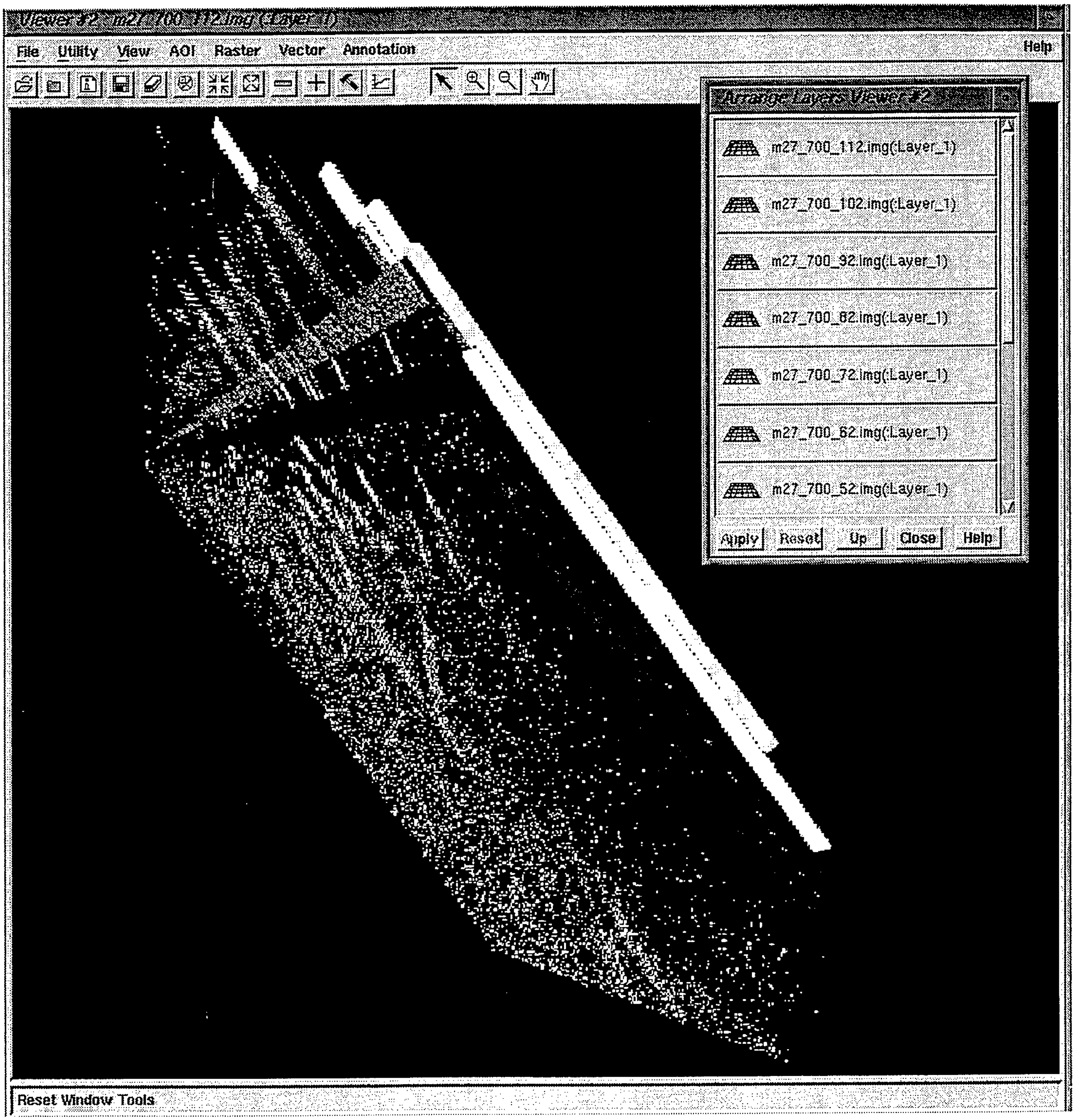


Figure 6a. Geo-rectified, AN/SQS-53B sonar imagery for ten look directions. The ship's heading is from lower right to upper left and the ship's track coincides with the left (lower) side of the imagery.

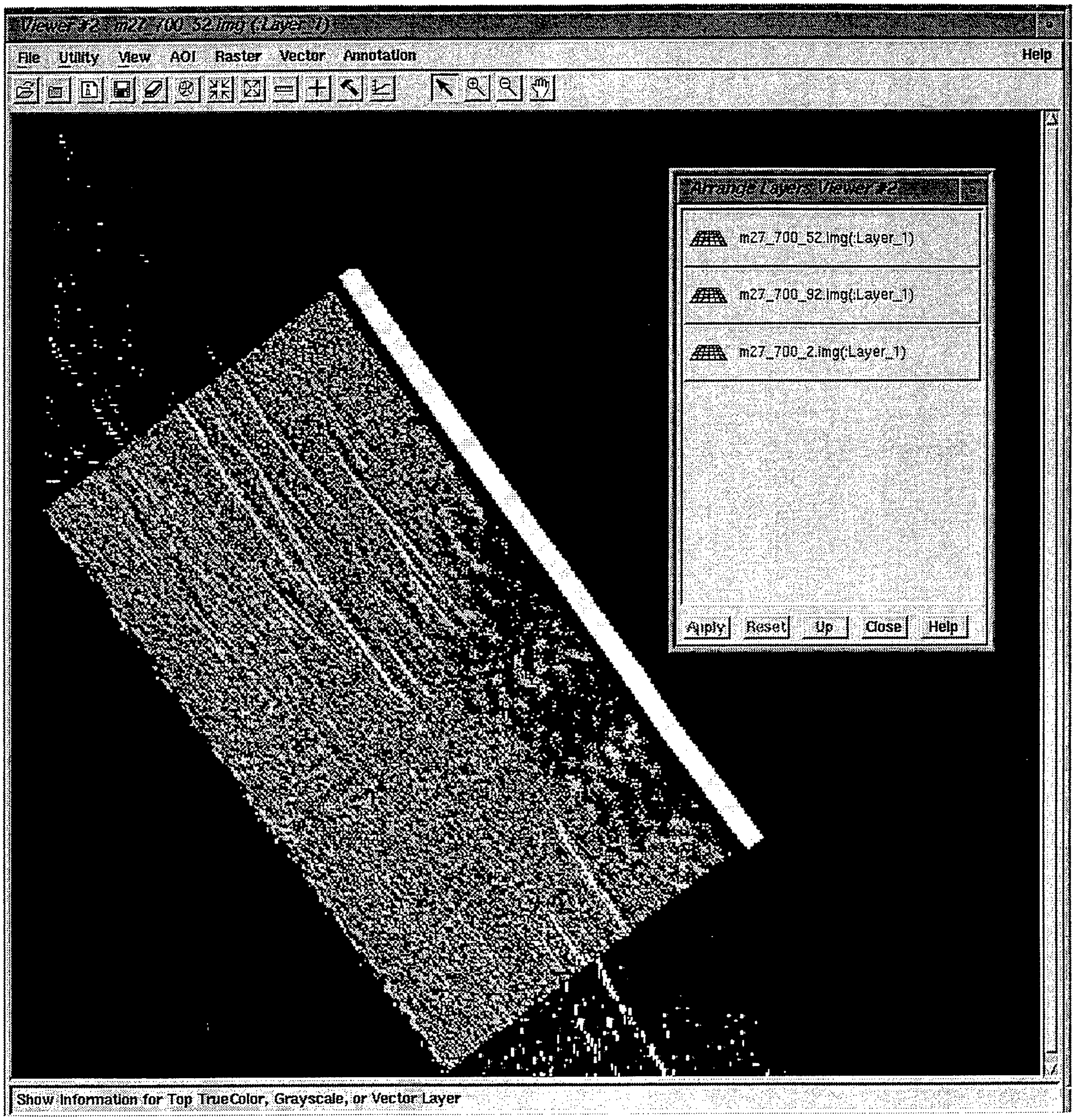


Figure 6b. Geo-rectified AN/SQS-53B sonar imagery for approximately the same field shown in Figure 6a, but for a look direction of primarily 052° (rectangular area). Portions of the 002° and 092° look directions show at the top and bottom edges of the rectangle, respectively. The ship's heading is from lower right to upper left and the ship's track coincides with the left (lower) side of the imagery.

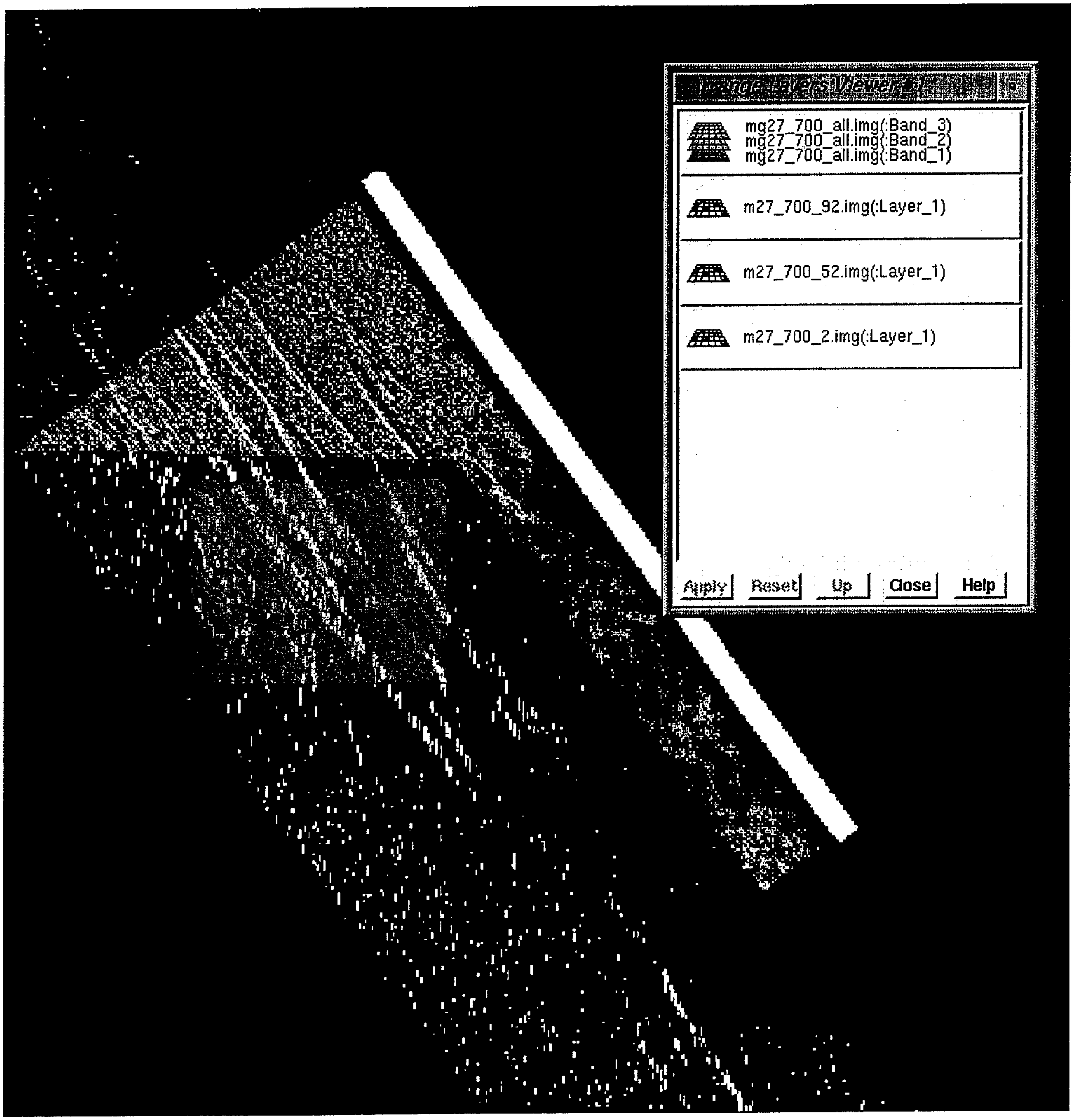


Figure 7a. Geo-rectified AN/SQS-53B sonar imagery for 002° (upper), 052° (center) and 092° (lower) look directions (field is essentially the same as in Figures 6a and 6b). Rectangular area shows a composite of the three beams.

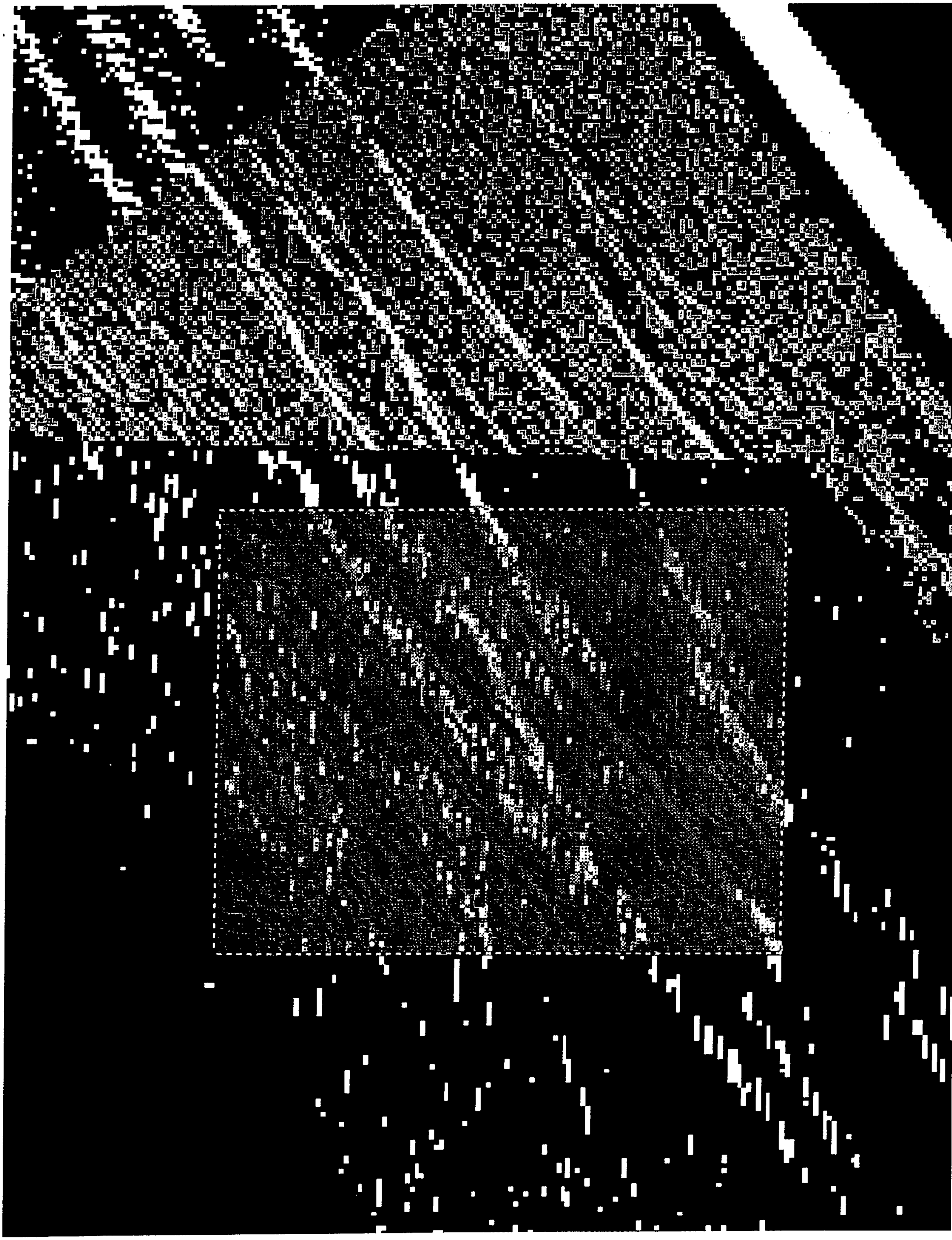


Figure 7b. Enlargement of rectangular area in Figure 7a. The colors represent the different geo-rectified beams or look directions: red for 092°, green for 052° and blue for 002°.

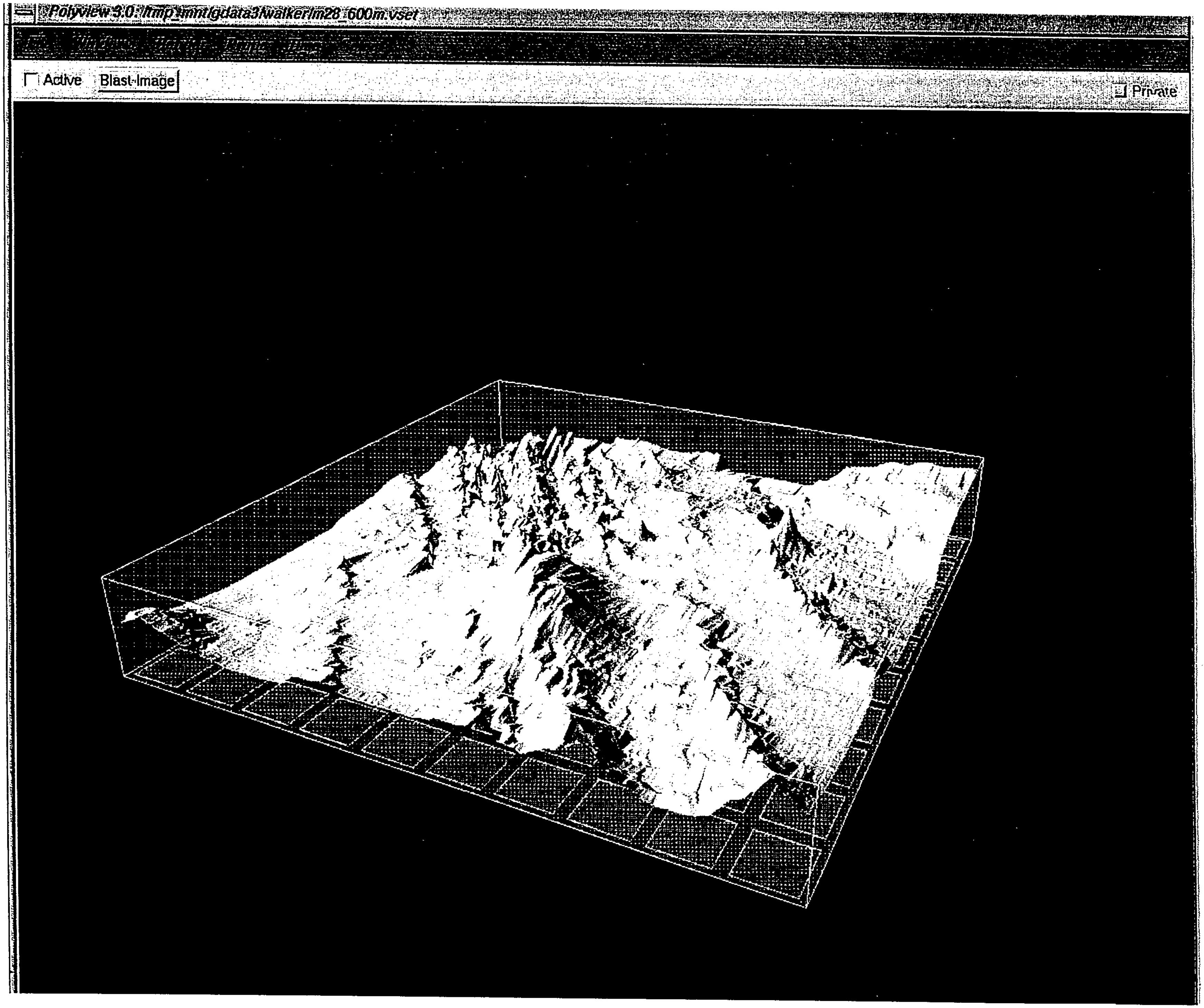


Figure 8. Three-dimensional, shaded bathymetry in the vicinity for image M27/700/52 (looking north).

Figure 9. AN/SQS-53B sonar image M27/700/52 draped over shaded bathymetry shown in Figure 8 (looking north).

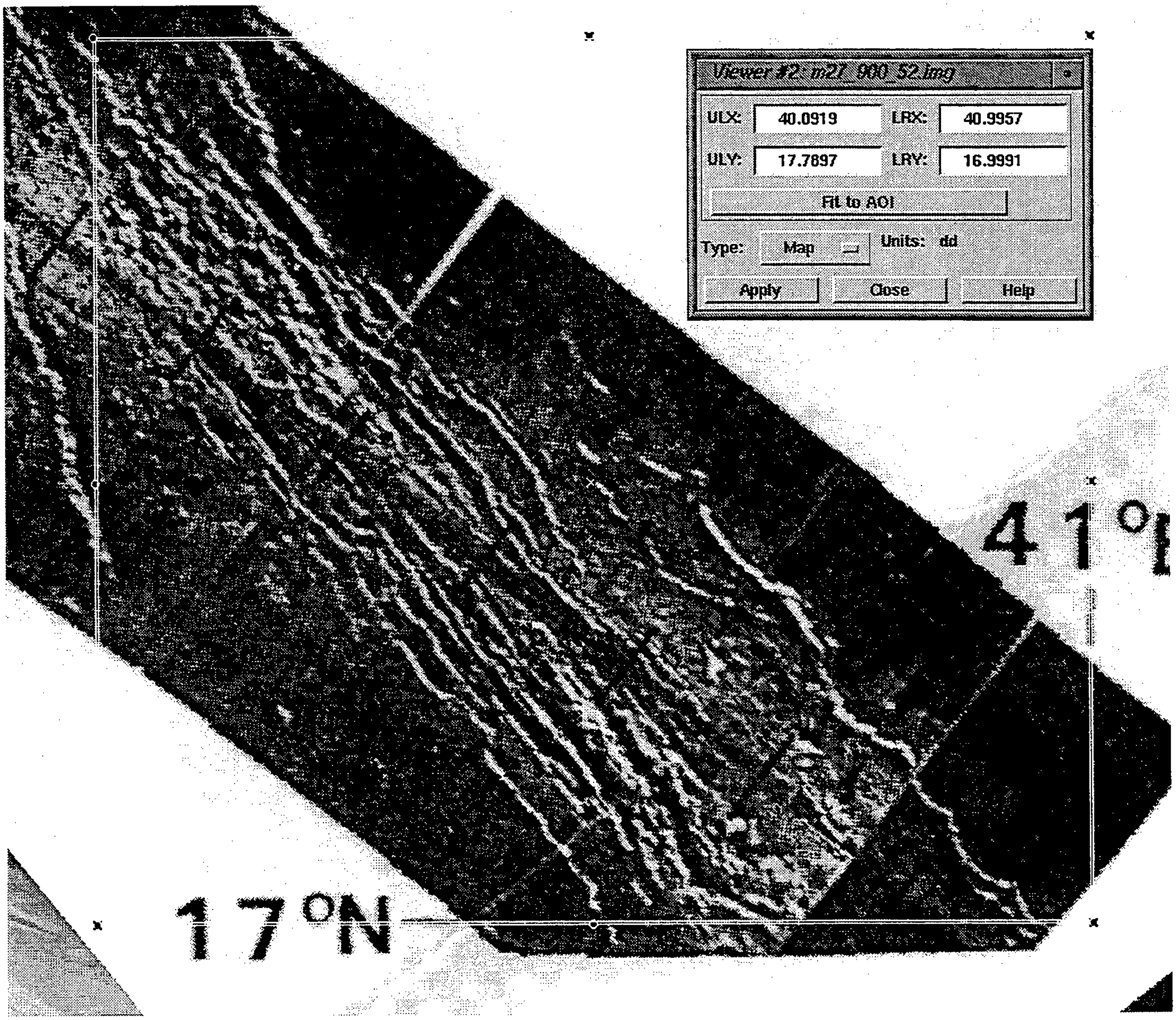
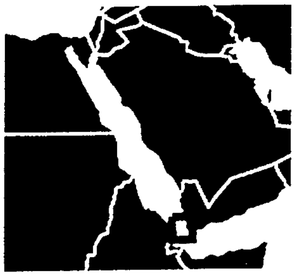


Figure 10. Geo-rectified GLORIA data (3.5 kHz) for the same area shown in Figure 5a.

AN/SQS-53B / USS Monterey transit of Bab al Mandab



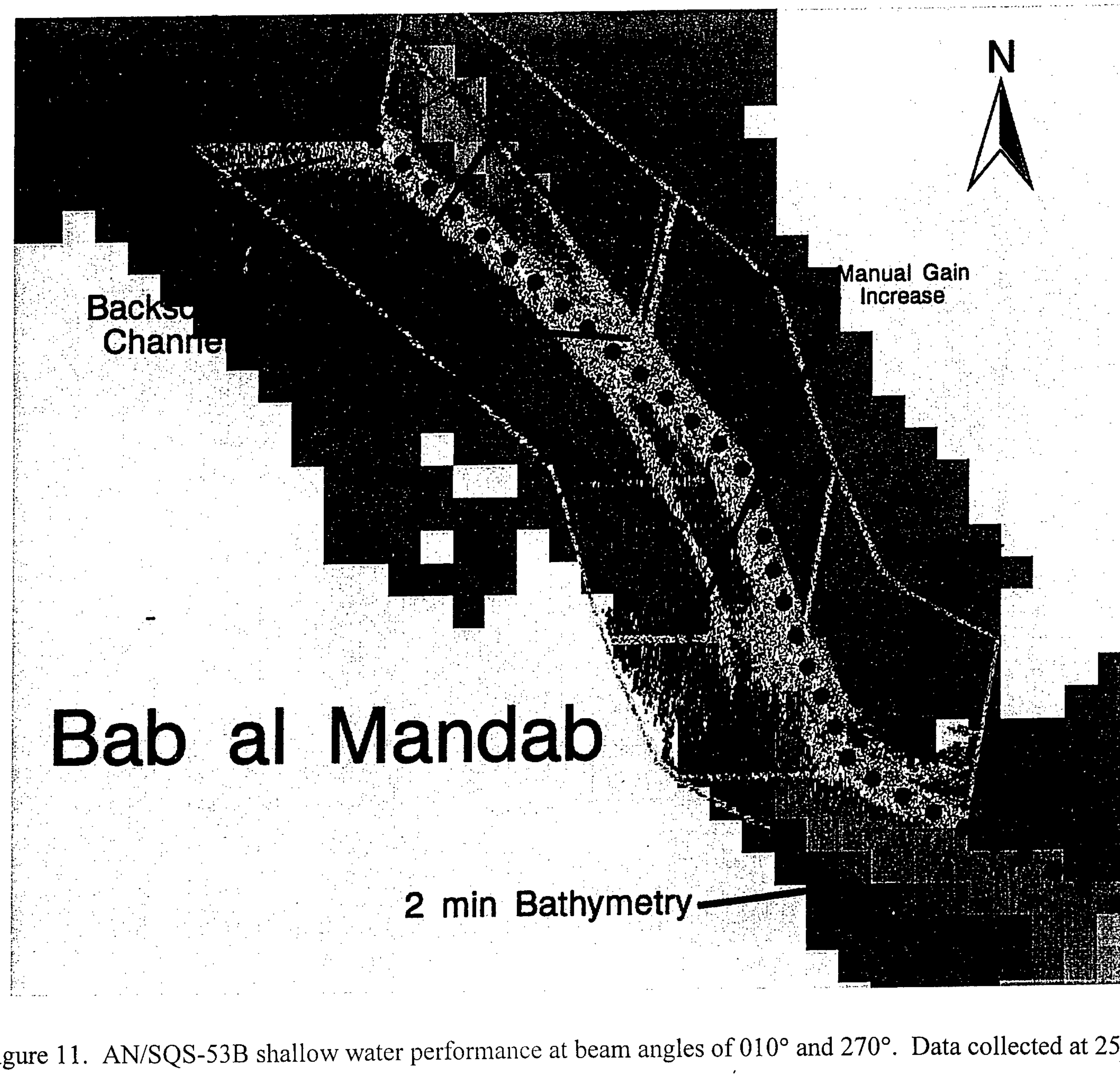


Figure 11. AN/SQS-53B shallow water performance at beam angles of 010° and 270°. Data collected at 25 kts.

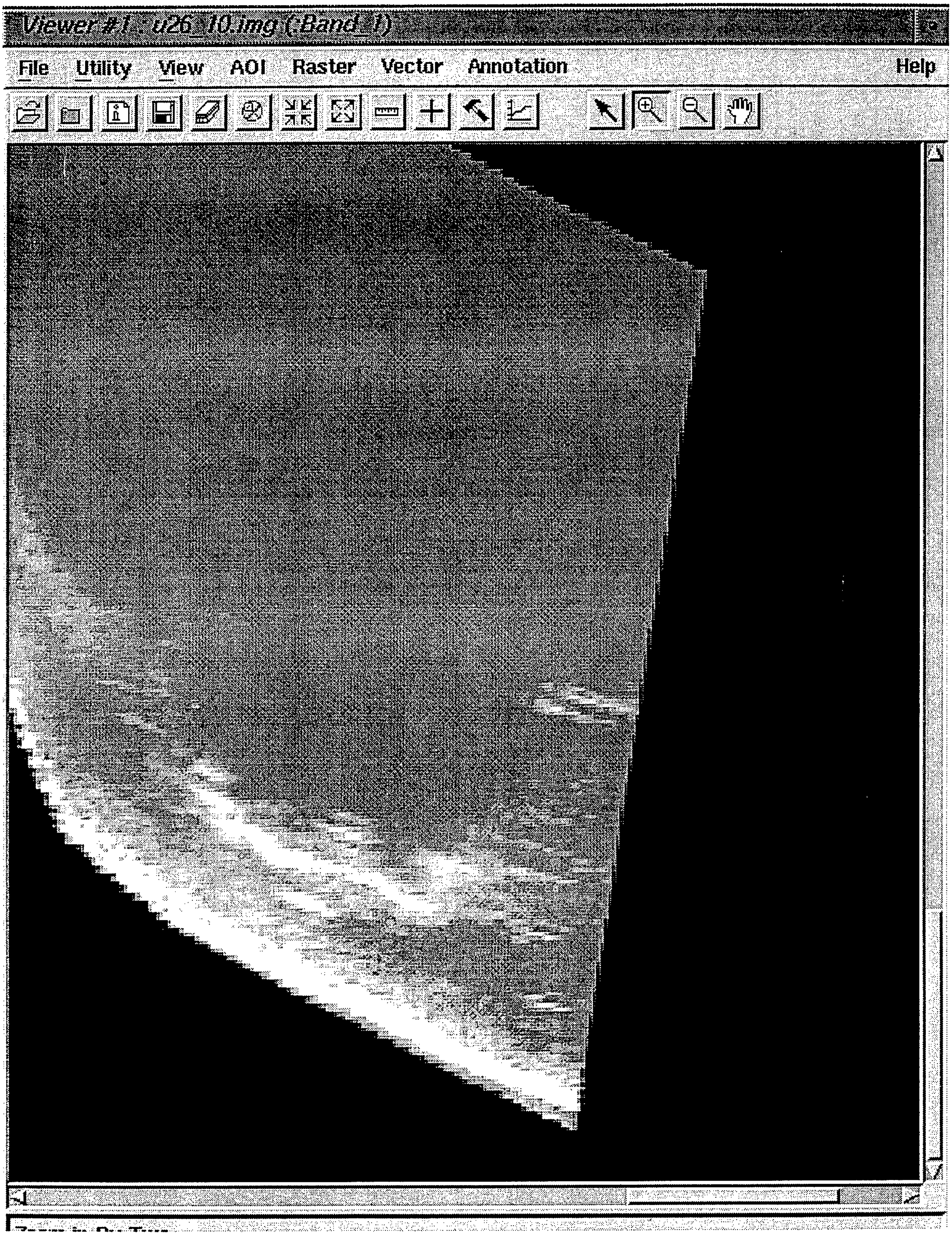


Figure 12a. AN/SQS-53B shallow water performance at beam angle of 10° showing acoustic returns (bright areas) from the shoal areas surrounding Perim Island.

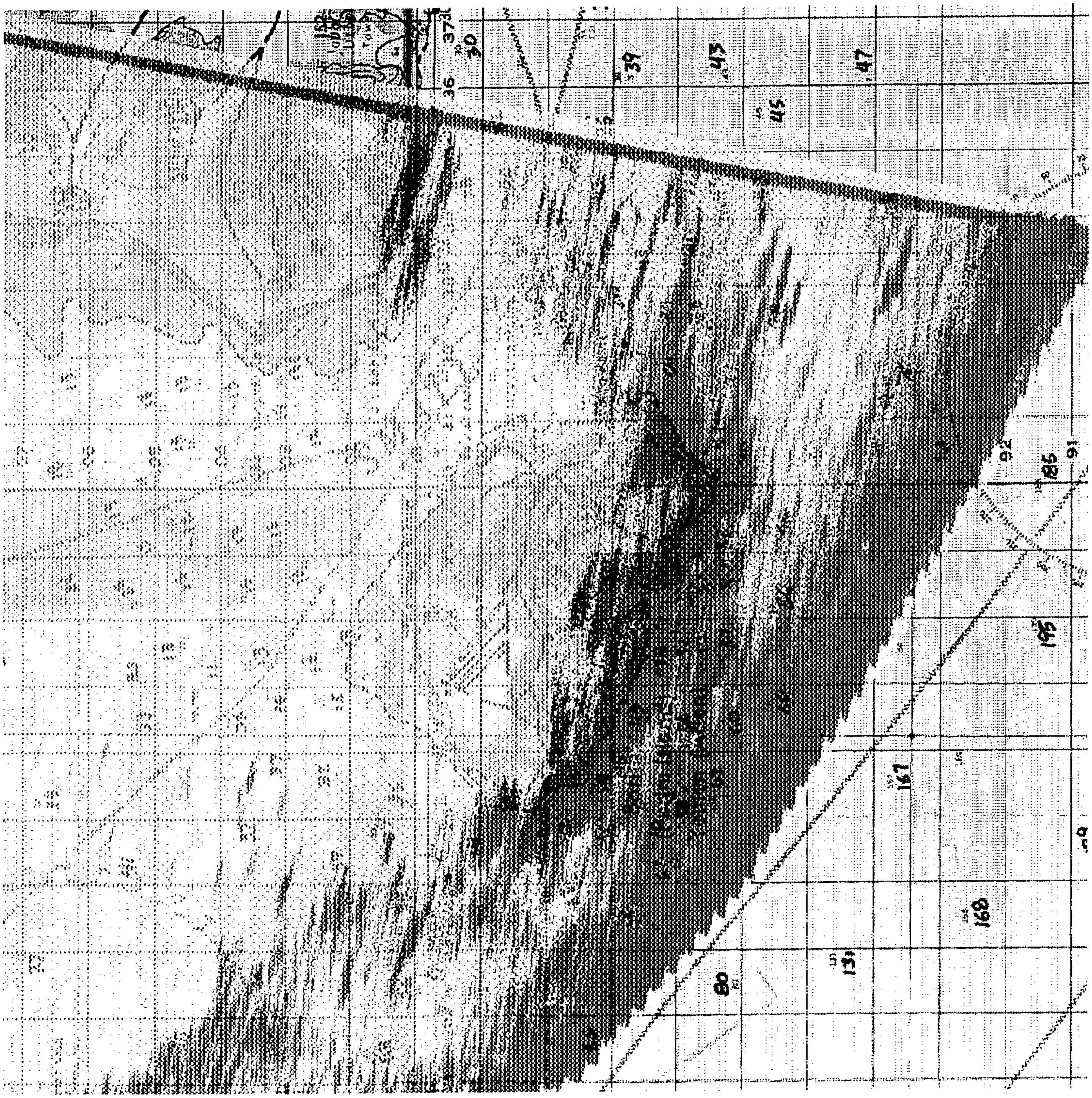
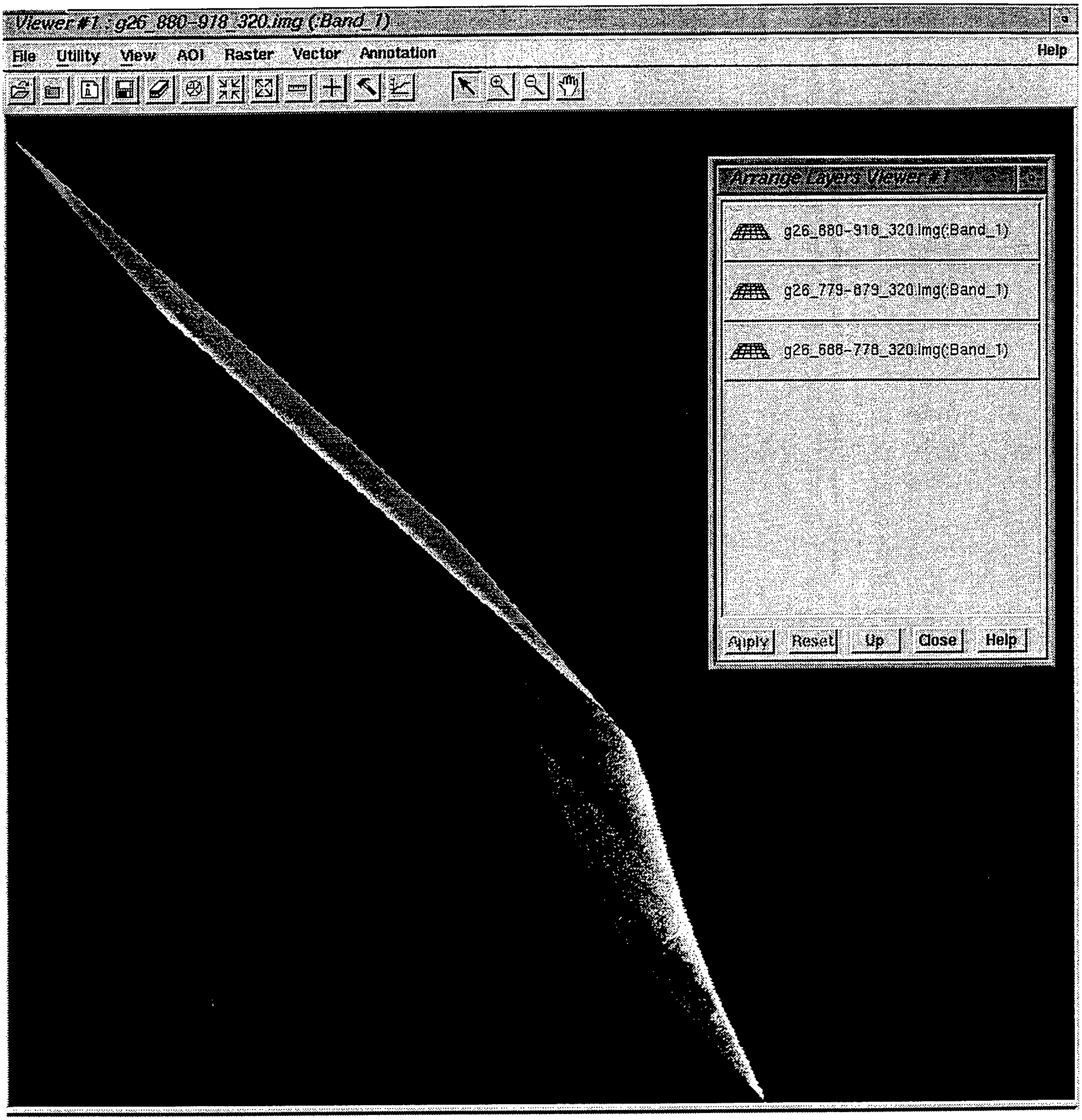


Figure 12b. Acoustic returns shown in Figure 12a (but shown here as dark areas) superimposed on bathymetric chart of the area surrounding Perim Island.



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Appendices

1. Conversion from range to horizontal distance under a flat bottom assumption.

2. Specialized AN/SQS-53B Geo-rectification.

1. Conversion from range to horizontal distance under a flat bottom assumption

Let r_i be the range for sample i given by $r_i = i\Delta r$ where Δr is the sample spacing given by

$$\Delta r = \left(\frac{c\Delta t}{2}\right)$$

where

$$\begin{aligned} c &= \text{speed of sound} \\ \Delta t &= \text{samping period} \end{aligned}$$

For a set of samples equally spaced in range, $r_{i,}$ and a water depth z, a set of corresponding horizontal distances, x_i, is given by

$$x_i = r_i\sqrt{1 - \left(\frac{z}{r_i}\right)^2}$$

However, these x_i are not evenly spaced as desired. In fact

$$\begin{aligned} \Delta x_i &= \Delta r\left(\frac{r_i}{x_i}\right) \\ &= \Delta r\frac{1}{\sqrt{1 - \left(\frac{z}{r_i}\right)^2}} \end{aligned}$$

In order to have a set of uniformly spaced horizontal values (spaced at Δr to agree with the long range case) we must interpolate between the x_i to get a set x_i' which are equally spaced. Let $\Delta x' = \Delta r = \left(\frac{c\Delta t}{2}\right)$. Choose a large $i = m$ such that

$$1 - \frac{\Delta x_m}{\Delta r} < \epsilon$$

for some arbitrarily small ϵ. Values of the acoustic signal at the new sample points are obtained through interpolation (including the special case of "nearest neighbor"). The following algorithm can be used:

$$x_m' = r_m$$

where m is determined as above.

$$\begin{aligned} \text{for } i &= m; x_i' \geq 0; i--\{ \\ x_i' &= x_{i+1}' - \Delta x' \\ r_i' &= \sqrt{(x_i')^2 + z^2} \end{aligned}$$

Find j such that

$$r_j \leq r_i' \leq r_{j+1}$$

Then

$$\begin{aligned} A\left(r_i'\left(x_i'\right)\right) &= A\left(r_j\right) + \left(r_i' - r_j\right) \frac{\left(A_{\bar{j}+1} - A_j\right)}{\left(r_{\bar{j}+1} - r_j\right)} \\ & \} \end{aligned}$$

where $A\left(r_j\right) = A_j$ is the signal amplitude at sample j at range r_j.

1. The geometry is shown in Figure

2. Specialized AN/SQS-53 Georectification

Since all the pings for a given beam are geostabilized and thus parallel (for fixed Sector Center direction), simplifications are possible in performing the georectification of the pings The basic idea is to construct an orthogonal coordinate system (x, y) such that the x-axis is parallel to the Nth beam pings. Assuming that the water depth correction (flat-bottom or exact) has been made, the indexing of each ping is modified so that for all samples (of pings $\cdots j, j+1, j+2, \cdots$)which have the same x-coordinate have the same index, i. Figure shows the concept. In this figure the (U, V) axes are geodetic, e.g. UTM while the (X, Y) axes are rotated such that X is in the direction of the Nth beam. Once the indices of each ping is adjusted, we construct a grid with a uniform spacing along the Y-axis by interpolation. The resulting set of pings will then be gridded or equally spaced in X and Y although the spacing in $X, \Delta x$, and in $Y, \Delta y$ will not necessarily be the same. The geographic coordinates of the four corners of this grid are known which allows for a trivial georectification using an internal module of most any image processing software package. The geometry is shown in Figure

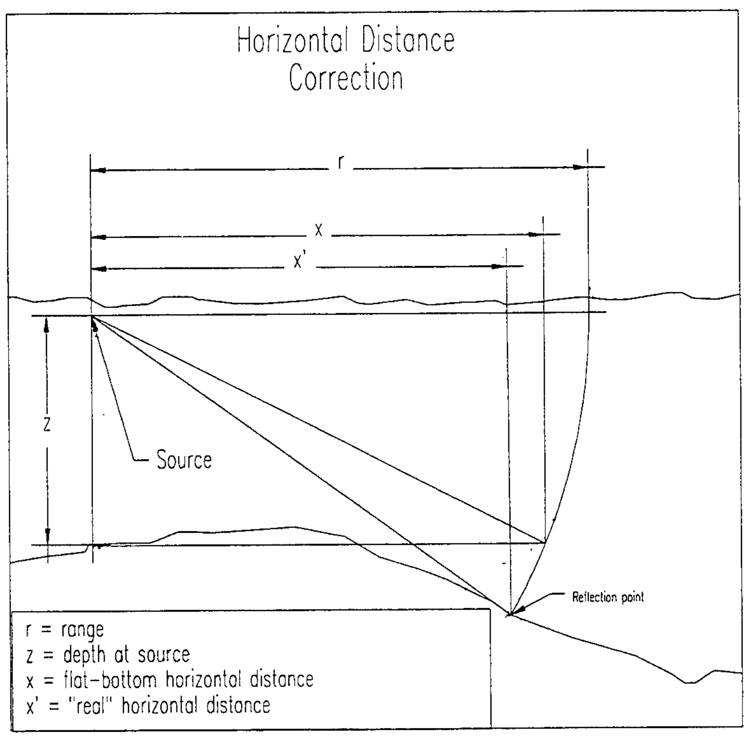
Horizontal Distance
Correction
r
x
x'
z
Source
Reflection point
r = range
z = depth at source
x = flat-bottom horizontal distance
x' = "real" horizontal distance

Figure 1.1:

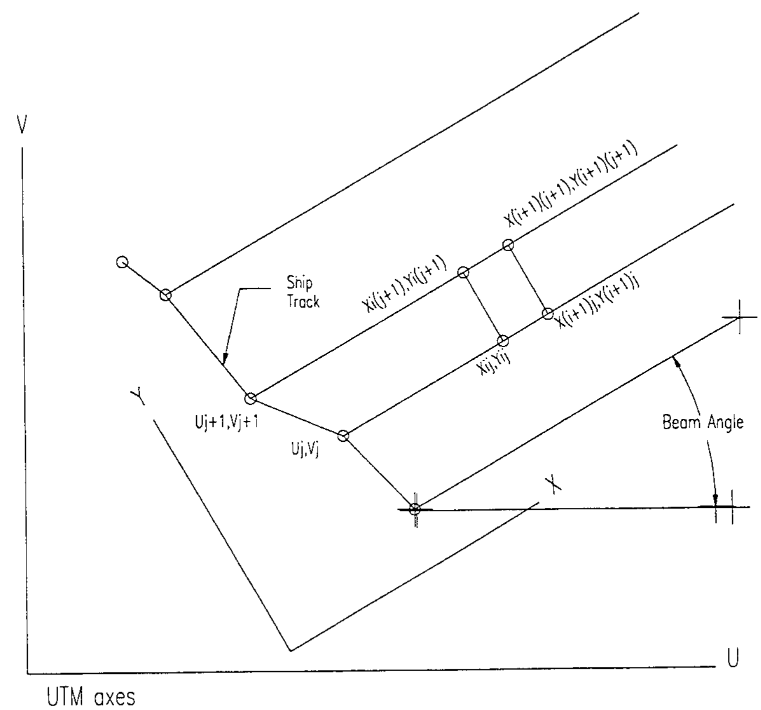
V
Ship
Track
Xi(j+1),Yi(j+1)
X(i+1)(j+1),Y(i+1)(j+1)
X(i+1)j,Y(i+1)j
Xij,Yij
Uj+1,Vj+1
Uj,Vj
Y
X
Beam Angle
U
UTM axes

Figure 2.1:

2.1. Along ping georectification

Assume the ship position at the start of ping j is given by (u_j, v_j) and that the angle of the Nth beam is $\theta = 90° - B_N$ as shown in the figure. Let the pings be indexed $\{0, 1, 2, \cdots, j, j+1, \cdots, J\}$. Begin the procedure by translating the (U, V) origin to (u_0, v_0). the beginning of the first ping.

$$\begin{aligned} u' &= u - u_0 \\ v' &= v - v_0 \end{aligned}$$

Then rotate the axes by θ to obtain

$$\begin{aligned} u' &= x\cos\theta - y\sin\theta \\ v' &= x\sin\theta + y\cos\theta \end{aligned}$$

or

$$\begin{bmatrix} u' \\ v' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

The new coordinate values are given by

or

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}^{-1} \begin{bmatrix} u' \\ v' \end{bmatrix}$$

or

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} u' \\ v' \end{bmatrix}$$

or

$$\begin{aligned} x &= u'\cos\theta + v'\sin\theta \\ y &= -u'\sin\theta + v'\cos\theta \end{aligned}$$

or

$$\begin{aligned} x &= (u - u_0)\cos\theta + (v - v_0)\sin\theta \\ y &= -(u - u_0)\sin\theta + (v - v_0)\cos\theta \end{aligned}$$

In the new (X, Y) system we find the ping-start with the minimum x-coordinate, $j = M$ and the maximum x-coordinate, $j = Q$ if $-90^{\circ} \leq \theta \leq +90^{\circ}$. (If the pings point in the negative x-direction pick the maximum and minimum, respectively).We then translate the x-axis to x_m:

$$\begin{aligned} x &= x' + x_M \\ y &= y' \end{aligned}$$

With these transforms the locations of every ping sample for the particular beam N should be positive. Let $a_{ij} = a(x_{ij}, y_{ij}))$ be the value of the ith sample of the jth ping located at the point (x_{ij}, y_{ij}). Let $a'_{nj} = a(x'_{ij}, y'_{ij})$ be the value of the sample located at the ith sample of the jth ping in the primed coordinates (x', y'). Recall that in the (x, y) coordinates the first sample occurs at the location (x_{0j}, y_{0j}) and that for all pings except for $j = M$, $x_{0j} > x_{0M}$. Let the sampling interval along the x-axis be Δx. If the jth ping begins at (x_{0j}, y_{0j}) then the coordinate of the ith sample is $(x_{0j} + i\Delta x, y_{0j})$. In the primed system let the values of the samples for $x'_{ij} < x_{0j} - x_M$ be zero. Let

$$k_j = \frac{x_{0j} - x_M}{\Delta x}$$

so that the index, k_j, corresponds to the first non-zero value of ping j.

Thus we can index the values a'_{ij} in the rotated and shifted system (x', y') as follows:

$$a'_{ij} = \begin{cases} 0 & 0 \leq i < k_j \\ a_{(i-k_j)j} & k_j \leq i \leq k_j + I_{\max} \end{cases}$$

where $I_{\max}$ is the number of samples in the original (depth-corrected) pings (all equal). If we require that each ping have the same number of samples then we must add enough zero values to each ping to give a total of

$$I'_{\max} = \frac{(x_Q - x_M)}{\Delta x} + I_{\max}$$

With this notation we have

$$a'_{ij} = \begin{cases} 0 & 0 \leq i < k_j \\ a_{(i-k_j)j} & k_j \leq i \leq k_j + I_{\max} \\ 0 & k_j + I_{\max} < i \leq I'_{\max} \end{cases}$$

2.2. .Ping-to-ping rectification

The gridding along the y-axis consists of choosing a Δy and calculating a new set of equally spaced pings at

$$y''_m = y_0 + m\Delta y$$

Since the x'_{ij} are already aligned it only remains to interpolate in the y-direction. The algorithm for linear interpolation follows:

$$\begin{aligned}
\text{for}(m &= 0; m < m_{\max}; m++\{ \\
\text{find } j \text{ such that } y_j &\leq y_0 + m\Delta y < y_{j+1} \\
\text{for } (i &= 0; i \leq i_{\max}; i++\{ \\
a''_{ij} &= a'_{ij} + (y_m - y_j)\frac{\left(a'_{ij} - a'_{i(j+1)}\right)}{(y_j - y_{j+1})} \\
&\} \\
&\}
\end{aligned}$$

where the ith subscript on y is omitted since it is redundant.