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13. ABSTRACT *(Maximum 200 words)*

There has been a great deal of work on statistical pattern recognition, non-parametric estimation, and formal models of machine learning. Recent and classical work in these areas have provided fundamental results on the amount of data needed for classification, estimation, and prediction in a variety of non-parametric settings. The applicability of these paradigms is often limited by the assumptions on the data gathering mechanisms and the performance criteria.

Our work has had two primary goals. The first is to investigate extensions and new models which give results useful in broader applications. The second goal is to apply these learning results to other areas such as signal/image processing, geometric reconstruction, and system identification. We have studied a variety of problems and have been able to relax assumptions required on the observed data as well as on the success criteria while still obtaining positive results. Our results have provided new insights into classical work and have also suggested a number of directions for further work.

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Final Report for ARO Grant DAAL03-92-G-0320

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Summary of Completed Project

There has been a great deal of work on statistical pattern recognition, non-parametric estimation, and formal models of machine learning. Recent and classical work in these areas have provided fundamental results on the amount of data needed for classification, estimation, and prediction in a variety of non-parametric settings. The applicability of these paradigms is often limited by the assumptions on the data gathering mechanisms and the performance criteria.

Our work has had two primary goals. The first is to investigate extensions and new models which give results useful in broader applications. The second goal is to apply these learning results to other areas such as signal/image processing, geometric reconstruction, and system identification. We have studied a variety of problems and have been able to relax assumptions required on the observed data as well as on the success criteria while still obtaining positive results. Our results have provided new insights into classical work and have also suggested a number of directions for further work.

1 Brief Description of Results

Below we briefly describe some specific areas of research that have been pursued and the results obtained during the course of the project (namely 8/1/92 to 7/31/95). In the next section, a list publications resulting from this award is provided.

1.1 Class Identification and Discernibility

We have been investigating a general problem for class identification that consists of trying to identify to which of two classes A_0 or A_1 an unknown object or distribution belongs given a information presented sequentially. We have considered a variety of success criteria and have characterized when class identification can be performed. We have also considered applications to discrete geometry, system identification, and language identification. A paper on these results appeared in *Annals of Statistics* [6] and another paper appeared in *IEEE Transaction on Information Theory* [10]. We also investigated a problem which is a hybrid of standard PAC learning and the class identification problem. These results take steps towards establishing connections between the two areas. This work was presented at the 6th Annual Workshop on Computational Learning Theory [35]. We subsequently generalized these results as well as extended and provided a new proof technique for results of others. This work was presented at the 1995 Int. Symp. on Information Theory.

1.2 Learning Under a Family of Probability Measures

We have been investigating PAC learning with respect to a class of distributions. Near the start of this project, we resolved a conjecture regarding this problem that had been open for about five years. A paper on this work appeared in *IEEE Transactions on Information Theory* in May '94 [11]. During this period we also obtained some results which begin to characterize learnability under general classes of probability measures. This work was presented at the 1994 International Symposium on Information Theory [24] and also appeared in *IEEE Transactions on Information Theory* [7].

1.3 Regression Estimation for Arbitrary Processes

Our work in this area began by exploring on-line learning of functions in which predictions are made sequentially as data arrives and a cumulative success criterion is used. Results from this work were presented at the 1993 American Control Conference [33] and at the 6th Annual Workshop on Computational Learning Theory [34]. We then extended the analysis to a more general statistical framework with the aim of obtaining rates of convergence for nearest-neighbor estimation under arbitrary sampling. Some of these results were presented at the 1994 International Symposium on Information Theory [23] and the 1994 Information Theory and Statistics Workshop [27]. A paper on this work appeared in *IEEE Transactions on Information Theory* [5]. We then further extended the work to deal with a broad class of non-parametric estimators. We introduced a new notion of Cesaro consistency for which the time-average of the risk must tend to zero, and have shown that Cesaro consistency can be achieved under mild conditions for arbitrary processes. These results were presented at the 1995 International Symposium on Information Theory [20]. More recently, we used similar techniques for universal prediction of the output of nonlinear fading memory systems. These results were presented at the 1995 Conference on Decision and Control [15] and more refined results have also been submitted for publication in *IEEE Transactions on Automatic Control* [1].

1.4 Stochastic Approximation

We have analyzed the classical Robbins-Monro and Kiefer-Wolfowitz stochastic approximation algorithms under arbitrary disturbances. Our main result characterizes convergence in terms of a condition on the noise sequence which we call persistently disturbing, that turns out to be equivalent to the classical Kushner-Clark condition. Specifically, we show that the algorithms converge under each fixed noise sequence iff the noise is not persistently disturbing. Our result is stronger than previous results and in fact we can obtain a number of previous results as simple corollaries. Furthermore, our proof techniques are elementary. Portions of this work were presented at the 32nd IEEE Conference on Decision and Control [29], the 1994 American Control Conference [26], and the 1994 SIAM Annual meeting [25]. A paper on this work appeared in *IEEE Transactions on Automatic Control* [8]. In further work, we extended and showed the equivalence of various conditions for the convergence of the Robbins-Monro algorithm. This work was presented at the 32nd Allerton Conference [22] and appeared in *Advances in Applied Probability* [4].

1.5 Bandit Problems

The two-armed bandit is perhaps the simplest problem which exhibits the well-known exploration vs. exploitation (or identification vs. control) tradeoff. During this period we formulated a generalization of the classical two-armed bandit problem which allows the selection of which to play to depend on a side observation. Bounds on the achievable regret for this problem were obtained using results from PAC learning. Hence, this work also relates recent work in non-parametric estimation to classical bandit problems. This work was presented at the 31st Allerton Conference [30].

1.6 Geometric reconstruction

In previous work, we considered a problem in geometric reconstruction (recovering a curve from line crossing data) and used results from learning to obtain bounds on the data required for reconstruction to a prescribed accuracy. During this project, we applied similar techniques to the problem of convex set reconstruction from various types of data. This work was presented at the 1993 Conference on Information Sciences and Systems [32]. We also did some related work on the consistency of support plane data in higher dimensions. This appeared in *Journal of Mathematical Imaging and Vision* [9].

1.7 Hybrid Systems

There has been a great deal of recent interest in hybrid systems. One problem We studied is the reduction of timed automata. Previous results had shown that such systems could be reduced to a single finite state automaton for certain simple enabling regions. We have extended these results for quite general enabling regions. This work was presented at the 33rd Conference on Decision and Control [28] and a manuscript has been submitted for publication in *Discrete Event dynamic Systems* [2]. Another problem we have considered is the design of controller switching policies based on output prediction errors. We characterized memory requirements for convergent decision rules and showed their use in adaptive control based on switching between a set of pre-selected controllers. This result appeared in *IEEE Transactions on Automatic Control* [3].

2 Publications Resulting From This Award

Journals

[1] S.R. Kulkarni and S.E. Posner, "Nonparametric Output Prediction for Nonlinear Fading Memory Systems," submitted to *IEEE Transactions on Automatic Control.*

[2] S. Di Gennaro, C. Horn, S.R. Kulkarni, P.J. Ramadge, "Reduction of Timed Hybrid Systems," submitted to *Journal of Discrete Event Dynamic Systems.*

[3] S.R. Kulkarni and P.J. Ramadge, "Model and Controller Switching Policies Based on Output Prediction Errors," *IEEE Transactions on Automatic Control*, Vol. 41, No. 11, pp. 1594-1604, November, 1996.

[4] I.-J. Wang, E.K.P. Chong, and S.R. Kulkarni, "Equivalent Necessary and Sufficient Conditions on Noise Sequences for Stochastic Approximation Algorithms," *Advances in Applied Probability*, 1996.

[5] S.R. Kulkarni and S.E. Posner, "Rates of Convergence of Nearest Neighbor Estimation under Arbitrary Sampling," *IEEE Transactions on Information Theory*, Vol. 41, No. 4, pp. 1028-1039, July, 1995.

[6] S.R. Kulkarni and O. Zeitouni, "A General Classification Rule for Probability Measures," *Annals of Statistics*, Vol. 23, No. 4, pp. 1393-1407, 1995.

[7] S.R. Kulkarni and M. Vidyasagar, "Learning Decision Rules for Pattern Classification Under a Family of Probability Measures," *IEEE Transactions on Information Theory*, Vol. 43, No. 1, pp. 154-166, January, 1997.

[8] S.R. Kulkarni and C. Horn, "An Alternative Proof for Convergence of Stochastic Approximation Algorithms," *IEEE Transactions on Automatic Control*, Vol. 41, pp. 419-424, Mar., 1996.

[9] W.C. Karl, S.R. Kulkarni, G.C. Verghese, and A.S. Willsky,"Local Tests for Consistency of Support Hyperplane Data," *Journal of Mathematical Imaging and Vision*, Vol. 6, pp. 249-267, June, 1996.

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Conferences

[12] J. Hocherman-Frommer, S.R. Kulkarni, P.J. Ramadge, "Time Varying Switched Control Based on Output Prediction Errors," *Proc. 34th Conference on Decision and Control*, pp. 2316-2317, Dec., 1995.

[13] S.R. Kulkarni and P.J. Ramadge, "Prediction Error Based Controller Selection Policies," *Proc. 34th Conference on Decision and Control*, pp. 3211-3216, Dec., 1995.

[14] S.R. Kulkarni and P.J. Ramadge, "On the Existence and Complexity of Convergent On-line Decision Rules," *Proc. 34th Conference on Decision and Control*, pp. 3022-3027, Dec., 1995.

[15] S.R. Kulkarni and S.E. Posner, "Universal Prediction of Nonlinear Systems," *Proc. 34th Conference on Decision and Control*, pp. 4024-4029, Dec., 1995.

[16] S.R. Kulkarni and C.S. Horn, "Necessary and Sufficient Conditions for Convergence of Stochastic Approximation Algorithms Under Arbitrary Disturbances," *Proc. 34th Conference on Decision and Control*, pp. 3843-3845, Dec., 1995.

[17] I.-J. Wang, E.K.P. Chong, S.R. Kulkarni, "On the Equivalence of Some Noise Conditions for Stochastic Approximation Algorithms," *Proc. 34th Conference on Decision and Control*, pp. 3849-3856, Dec., 1995.

[18] S.R. Kulkarni and O. Zeitouni, "On the Existence of Strongly Consistent Rules for Estimation and Classification," *Proc. Int'l Symposium on Information Theory*, p. 255, Sept., 1995.

[19] S.E. Posner and S.R. Kulkarni, "Some Results on Nonparametric Regression Estimation for Arbitrary Processes," *Proc. of Allerton Conference*, Sept., 1995.

[20] S.E. Posner and S.R. Kulkarni, "Nonparametric Regression Estimation for Arbitrary Random Processes," *Proc. Int'l Symposium on Information Theory*, p. 251, Sept., 1995.

[21] S.E. Posner and S.R. Kulkarni, "Consistent Regression Estimation Under Arbitrary Sampling," *Proc. Information Theory Workshop*, Rydzyna, Poland, June, 1995.

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[30] S.R. Kulkarni, "On Bandit Problems with Side Observations and Learnability," *Proc. 31st Allerton Conference*, Univ. of Illinois, pp. 83-92, 1993.

[31] S.R. Kulkarni, "A Review of Some Models for Machine Learning," *Proc. Silver Jubilee Workshop on Computing and Intelligent Systems*, pp. 122-137, I.I.Sc., Bangalore, India, 1993.

[32] S.R. Kulkarni, "Applications of PAC Learning to Problems in Geometric Reconstruction," *Proc. Twenty-seventh Annual Conf. on Information Sciences and Systems*, Johns Hopkins University, pp. 229-234, March, 1993.

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