Final Project Report

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Principal Investigator: Stephan Olariu

PI Institution: Old Dominion University

PI Phone Number: (757) 683-4417

PI email address: olariu@cs.odu.edu

1 Introduction

The PI was awarded the United States Office of Naval Research grant N00014-95-1-0779 titled "Algorithmic Issues on Locally Dense Networks" scheduled to extend from May 1, 1995 to August 31, 1996. The total amount of the award was $43,582.

Although the research supported by this grant encompasses a wide variety of issues and approaches, the unifying theme of our effort was the design of efficient algorithms for locally dense graphs – a class that extends and generalizes a number of classical classes of graphs with many practical applications.

The grant was specifically designed to investigate various classes of networks that feature locally dense behaviour. Examples include LANs as well as some classes of Wide Area Networks. Our goal was to provide a unifying look at these types of networks. Results were sought both at the structural and algorithmic level. At the structural level we were interested in identifying the "agent" responsible for the linear structure apparent in these networks. It is this very linear structure that is exploited, in one way or another, by all known algorithm on the underlying graph classes. Our goal was to identify a common algorithmic base that will result in simpler and more efficient algorithms for a large spectrum of computational problems.

In support of this goal we have established three major foci of effort for this project:

1. Our primary focus has been to further investigate the notion of domination in a network. For asteroidal-free networks the existence of dominating pairs of stations in the network was proved by the PI; our principle goal in this direction was to design an efficient algorithm returning a dominating pair in a connected asteroidal triple-free network.

2. Our second focus was to investigate the concept of p-connectedness in networks. This concept is more general than the usual connectivity, yet seems to capture well the local density property featured by the network classes mentioned above.

3. Our third focus was to obtain a characterizations of p-conected networks that could lead to a decomposition theorem and is likely to reveal the deep unifying features of the classes of interest.

The PI has obtained structural properties of asteriodal-free networks (ATN, for short) that were instrumental in a number of linear time algorithms for computational problems on this class of networks. These algorithms dramatically improve on the state of the art, showing that our approach is fruitful and worth pursuing.

In the next sections we discuss primary results in each of these areas of investigation.

2 Detailed Summary of Work

The PI has shown recently that every connected ATN contains a *dominating pair* – a pair of vertices such that *every* path connecting them is a dominating set. This novel and surprising property has a dramatic impact on *network communications.* Consider again a network whose underlying graph is asteroidal triple-free. The dominating pair property guarantees the existence of a pair of stations x, y in the network with the property that no matter what communication path is established between x and y, all the remaining stations are guaranteed to be adjacent to this "main" path, with the corresponding broadcasting delay minimized. In addition, the dominating pair property has a lot of implications from the fault-tolerance point of view, for if one of the paths cannot be used, in general other paths can be found preserving the communication properties of the network.

Thus, the task of determining a dominating pair in an ATN becomes of utmost importance. Quite recently the PI has devised an algorithm to solve this problem in time $O(|V| + |E|)$, where $G = (V, E)$ is the underlying graph. Clearly, this algorithm is optimal. We note here that the best previously-known algorithm for this task was running in $O(|V|^3)$ time. To summarize, the PI has obtained the following important results concerning dominating pairs in ATNs.

- a linear time algorithm for computing a dominating pair in a connected ATN (this result also provides a much simpler algorithmic proof of the existence of a dominating pair);
- a linear time algorithm for computing all dominating pairs in a connected ATN of diameter larger than three;
- a linear time algorithm for finding a minimum cardinality connected dominating set in a connected ATN;
- a linear time algorithm to triangulate an ATN

The following papers on the topic of ATNs acknowkeldge the present grant.

1. Asteroidal Triple-free Graphs, (joint work with D.G. Corneil and L. Stewart), *SIAM Journal of Discrete Mathematics*, to appear 1997.
2. A Linear Time Algorithm to Compute a Dominating Path in an AT-free Graph, (joint work with D. G. Corneil and L. Stewart), *Information Processing Letters*, 54, (1995), 253–257.
3. A Linear-Time Algorithm to Recognize P4-reducible Graphs, (joint paper with B. Jamison), *Theoretical Computer Science*, 145, (1995), 329–344.

4. Linear Time Algorithms for Dominating Pairs in Asteroidal Triple-free Graphs, (joint work with D.G. Corneil and L. Stewart), *SIAM Journal on Computing*, accepted for publication.

But there is more. The local density properties evident in the network classes mentioned above, and whose elucidation provided the motivation for the current research effort, suggested that the set of vertices belonging to dominating pairs should somehow be linked to the set of vertices achieving the diameter of the network. Unfortunately, this "intuition" in false in general. The problem of finding all the dominating pairs in an ATN seems to be hard. However, some progress has been made in the last month or so. Specifically, the PI has showed that in an ATN of diameter larger than three (which is the case of the vast majority of computer networks), the vertices that are in dominating pairs are restricted to two sets that can be easily described. In addition, for these networks, the set of all dominating pairs can be computed in $O(|V| + |E|)$ time, where $G = (V, E)$ is the underlying graph. This result has been reported in

5. Computing a Dominating Pair in Linear Time, (joint work with D. G. Corneil and L. Stewart), *Proc. of WADS'95*, Kingston, Ontario, August 1995. Also in S. G. Akl, F. Dehne, J.-R. Sack, and N. Santoro, Eds. *Algorithms and Data Structures*, Lecture Notes in Computer Science, 955, Springer-Verlag, Berlin, 1995,

6. On the Isomorphism of Graphs with Few P4s, (joint work with L. Babel), *Discrete Applied Mathematics*, accepted for publication 1997.

7. A Parallel Recognition Algorithm for P4-sparse Networks, (joint work with R. Lin), *Discrete Applied Mathematics*, accepted for publication 1997.

3 Future Directions

There is a whole lot more to do. Each of the foci of effort outlined in the Introduction needs more work. Although the goals of the first focus have been achieved, it is still important to look at other applications of our linear time algorithm to compute a dominating pair in a connected ATN. It is quite conceivable that many other applications could benefit from this algorithm by using it as a stepping stone.

Likewise, there is still a lot of work to do on the second major focus of this research effort. It is this PI's belief that the ATNs have very interesting properties related to secure communications and to fault tolerance. Yet another new development in this direction is the concept of *strong ATN network* put forth by the PI. In essence, this is strict subclass of ATNs defined in terms of the existence of a linear order on the vertex set. The PI has recently shown that the class of strong ATN is of import since it features interesting fault-resistant properties.

Relevant results acknowledging this grant can be found in

8. On the Homogeneous Decomposition of Graphs, (joint work with B. Jamison), *SIAM Journal of Discrete Mathematics*, 8, (1995), 448–463.

9. Simple Linear Time Recognition of Unit Interval Graphs, (joint work with D. G. Corneil, H. Kim, S. Natarajan, and A. Sprague), *Information Processing Letters*, 55, (1995), 99–104.

4 Other Results Acknowledging this Grant

In addition to work on structural and algorithmic issues dealing specifically with local density properties in networks the PI has devoted a substantial amount of work to developing sequential and parallel algorithms for a number problems that use the concepts discussed above. These results have been reported in the following papers and book chapters.

10. Parallel Graph Algorithms, in A. Zomaya, Ed., *Handbook of Parallel and Distributed Computing*, McGraw-Hill, New York, October 1996.

11. Time-Optimal Computations on Meshes with Multiple Broadcasting, in D. J. Evans, Ed., *Advances in Parallel Algorithms*, Wiley and Sons, New York, 1996.

12. Time Optimal Digital Geometry Algorithms on Enhanced Meshes, (joint work with V. Bokka, H. Gurla, I. Stojmenović, and J. L. Schwing), *Parallel Image Analysis: Theory and Applications*, L. S. Davis *et al.* Eds., Series in Machine Perception and Artificial Intelligence, vol. 19, World Scientific, London, 1996.

13. Average Waiting Time Profiles of the Uniform DQDB, (joint paper with K. Maly, N. Rao, S. Dharanikota, L. Zhang, and D. Game), *IEEE Transactions on Parallel and Distributed Systems*, 6, (1995), 1068–1084.

14. Square Meshes are not Optimal for Convex Hull Computation, (joint paper with Bhagavathi, Gurla, Schwing, and Zhang), *IEEE Transactions on Parallel and Distributed Systems*, 7, (1996), 545–554.

15. A New Deterministic Sampling Scheme with Applications to Broadcast Efficient Sorting on the Reconfigurable Mesh, (joint work with J. Schwing), *Journal of Parallel and Distributed Computing*, 32, (1996), 215–222.

16. Convexity Problems on Meshes with Multiple Broadcasting, (joint work with D. Bhagavathi, J. L. Schwing, W. Shen, L. Wilson, and J. Zhang), *Journal of Parallel and Distributed Computing*, 27, (1996), 142–156.

17. Time-Optimal Proximity Problems on Meshes with Multiple Broadcasting, (joint work with I. Stojmenović), *Journal of Parallel and Distributed Computing*, 36, (1996), 144–155.

18. Fairness in DQDB Revisited: a New Solution, (joint work with K. Maly, N. Rao, and L. Zhang), *Proc. 14-th Annual IEEE International Phoenix Conf. on Computers and Communications*, Phoenix, Arizona, March 1996, 198–204.