

NAVAL POSTGRADUATE SCHOOL Monterey, California



MCTSSA Software Reliability Handbook

Volume IV

Schneidewind Software Reliability and Metrics Model Tool List

by

Norman F. Schneidewind

12 May 1997

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Prepared for: U.S. Marine Corps
Tactical Systems Support Activity
Camp Pendleton, CA 92244-5171

19970918 133

DTIC QUALITY INSPECTED 8

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This report was prepared for and funded by the U.S. Marine Corps, Systems Support Activity, Camp Pendleton, CA 92255-5171.

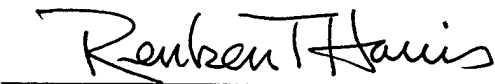
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REPORT DOCUMENTATION PAGE

Form Approved

OMB No 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE 12 May 1997	3. REPORT TYPE AND DATES COVERED Technical Report
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4. TITLE AND SUBTITLE MCTSSA Software Reliability Handbook Volume IV Schneidewind Software Reliability and Metrics Model Tool List	5. FUNDING RLACH
---	---------------------

6. AUTHOR(S) Dr. Norman F. Schneidewind
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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Department of Systems Management Naval Postgraduate School Monterey, CA 93943-5000
--

8. PERFORMING ORGANIZATION REPORT NUMBER NPS-SM-97-005

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) U.S. Marine Corps Tactical Systems Support Activity Box 555171 Building 31345 Camp Pendleton, CA 92255-5171
--

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
--

11. SUPPLEMENTARY NOTES The views expressed in this report are those of the authors and do not reflect the official policy or position of the Department of Defense or the United States Government.

12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution unlimited.
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12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words.) The purpose of this handbook is threefold. Specifically, it: o Serves as a reference guide for implementing standard software reliability practices at Marine Corps Tactical Systems Support Activity and aids in applying the software reliability model o Serves as a tool for managing the software reliability program o Serves as a training aid This handbook consists of four volumes. The content of each of the volumes is as follows: Volume I: Software Reliability Engineering Process and Modeling for a Single Function System Volume II: Data Collection Demonstration and Software Reliability Modeling for a Multi-Function Distributed System Volume III: Integration of Software Metrics with Quality and Reliability Volume IV: Schneidewind Software Reliability and Metrics Models Tool List

14. SUBJECT TERMS

15. NUMBER OF PAGES 23

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED

18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED
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19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED

20. LIMITATION OF ABSTRACT SAR

MCTSSA SOFTWARE RELIABILITY HANDBOOK

VOLUME IV

SCHNEIDEWIND SOFTWARE RELIABILITY AND METRICS MODELS TOOL LIST

12 May 1997

Revised: 15 July 1997

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INTRODUCTION

The following is a complete listing, as of this date, of *Schneidewind Software Reliability Model* equations and *Schneidewind Software Metrics Model* equations divided into tool implementation categories (i.e., *SMERFS*, *Statgraphics*, *Defect Control System Database*, and *Windows Calculator*). The purpose is to show which equations are implemented in which tool. The list is divided as follows:

o SOFTWARE RELIABILITY MODEL EQUATIONS

- NOTATION
- EQUATIONS IMPLEMENTED IN SMERFS
- EQUATIONS IMPLEMENTED IN STATGRAPHICS
 - * TABLE 1

o DISTRIBUTED SYSTEM MODEL EQUATIONS

- NOTATION
- EQUATIONS IMPLEMENTED USING DEFECT CONTROL SYSTEM DATABASE
- EQUATIONS IMPLEMENTED USING WINDOWS CALCULATOR
- EQUATIONS IMPLEMENTED IN STATGRAPHICS
 - * TABLE 2
- EQUATION IMPLEMENTED IN SMERFS

o METRICS MODELS EQUATIONS

- DISCRIMINATIVE POWER VALIDATION MODEL
 - * NOTATION
 - * EQUATIONS IMPLEMENTED IN STATGRAPHICS
 - * EQUATION IMPLEMENTED USING WINDOWS CALCULATOR
 - ** TABLE 3
- PREDICTABILITY VALIDATION MODEL
 - * NOTATION
 - * EQUATIONS IMPLEMENTED IN STATGRAPHICS
 - ** TABLE 4

The reason for TABLES 1...4 is that the syntax of the *STATGRAPHICS* equation editor does not correspond identically to that in the equation notation (e.g., no Greek symbols, subscripts, and superscripts available). Also the limited space available for a *STATGRAPHICS* equation definition does not always allow these definitions to be identical to the mathematical definitions. Thus in order to use the *STATGRAPHICS* package, it is necessary to see the equations as they are written, using its syntax. The tables define the syntax.

SOFTWARE RELIABILITY MODEL EQUATIONS

NOTATION

α	failure rate at the beginning of interval s
β	negative of derivative of failure rate divided by failure rate (i.e., relative failure rate)
$F(i)$	predicted failure count in the range $[1, i]$; used in computing MSE_r
F_{ij}	observed failure count during interval j since interval i ; used in computing MSE_T
$F(t)$	predicted failure count in the range $[1, t]$
F_t	given number of failures to occur after interval t ; used in predicting $T_F(t)$
$F(t_1, t_2)$	predicted failure count in the range $[t_1, t_2]$
$F(\infty)$	predicted failure count in the range $[1, \infty]$; maximum failures over the life of the software
i	current interval
j	next interval $j > i$ where $F_{ij} > 0$
J	maximum $j \leq t$ where $F_{ij} > 0$
MSE_F	mean square error criterion for selecting s for failure count predictions
MSE_r	mean square error criterion for selecting s for remaining failure predictions
MSE_T	mean square error criterion for selecting s for time to next failure predictions
$p(t)$	fraction of remaining failures predicted at time t
$Q(t)$	operational quality predicted at time t ; the complement of $p(t)$; the degree to which software is free of remaining faults (failures)
r_c	critical value of remaining failures; used in computing RCM $r(t)$
$r(t)$	remaining failures predicted at time t
$r(t_t)$	remaining failures predicted at total test time t_t
$\Delta r(T_F, t)$	reduction in remaining failures that would be achieved if the software were executed for a time T_F , predicted at time t
RCM $r(t_t)$	risk criterion metric for remaining failures at total test time t_t
RCM $T_F(t_t)$	risk criterion metric for time to next failure at total test time t_t
s	starting interval for using observed failure data in parameter estimation
s^*	optimal starting interval for using observed failure data, as determined by MSE criterion
t	cumulative time in the range $[1, t]$; last interval of observed failure data; current interval
t_m	mission duration (end time-start time); used in computing RCM $T_F(t)$

t_t	total test time (observed or predicted)
$T_F(t)$	time to next failure(s) predicted at time t
$T_F(t_t)$	time to next failure predicted at total test time t_t
$T_F(\Delta r, t)$	time to next N failures that would be achieved if remaining failures were reduced by Δr , predicted at time t
T_{ij}	time since interval i to observe number of failures F_{ij} during interval j ; used in computing MSE_T
x_k	number of observed failures in interval k
X_i	observed failure count in the range $[1, i]$
X_{s-1}	observed failure count in the range $[1, s-1]$
$X_{s,i}$	observed failure count in the range $[i, s-1]$
$X_{s,i}$	observed failure count in the range $[s, i]$
$X_{s,t}$	observed failure count in the range $[s, t]$
X_{s,t_1}	observed failure count in the range $[s, t_1]$
X_t	observed failure count in the range $[1, t]$
X_{t_1}	observed failure count in the range $[1, t_1]$

EQUATIONS IMPLEMENTED IN SMERFS

Parameter Estimation

The log of the likelihood function is:

$$\log L = X_t [\log X_t - 1 - \log(1 - \exp(-\beta t))] + X_{s-1} [\log(1 - \exp(-\beta(s-1)))] \\ + X_{s,t} [\log(1 - \exp(-\beta))] - \beta \sum_{k=0}^{t-s} (s+k-1) x_{s,k}$$

This function is used to derive the equations for estimating α and β for each of the three methods. In the equations that follow, α and β are *estimates* of the population parameters.

Method 1

Use all of the failure counts from interval 1 through t ($s=1$). This method is used if it is assumed that all of the historical failure counts from 1 through t are representative of the future failure process. The following two equations are used to estimate β and α , respectively.

$$\frac{1}{\exp(\beta)-1} - \frac{t}{\exp(\beta t)-1} = \sum_{k=0}^{t-1} k \frac{X_{k+1}}{X_t}$$

$$\alpha = \frac{\beta X_t}{1 - \exp(-\beta t)}$$

Method 2

Use failure counts only in the intervals s through t ($1 \leq s \leq t$). This method is used if it is assumed that only the historical failure counts from s through t are representative of the future failure process. The following two equations are used to estimate β and α , respectively.

$$\frac{1}{\exp(\beta) - 1} \frac{t - s + 1}{\exp(\beta(t - s + 1)) - 1} = \sum_{k=0}^{t-s} k \frac{X_{s,k}}{X_{s,t}}$$

$$\alpha = \frac{\beta X_{s,t}}{1 - \exp(-\beta(t - s + 1))}$$

Method 2 is equivalent to Method 1 for $s=1$.

Method 3

Use the cumulative failure count in the interval 1 through $s-1$ and individual failure counts in the intervals s through t ($2 \leq s \leq t$). This method is used if it is assumed that the historical cumulative failure count from 1 through $s-1$ and the individual failure counts from s through t are representative of the future failure process. This method is intermediate to Method 1, which uses all the data, and Method 2, which discards "old" data. The following two equations are used to estimate β and α , respectively.

$$\frac{(s-1)X_{s-1}}{\exp(\beta(s-1)) - 1} + \frac{X_{s,t}}{\exp(\beta) - 1} - \frac{tX_t}{\exp(\beta t) - 1} = \sum_{k=0}^{t-s} (s+k-1)X_{s,k}$$

$$\alpha = \frac{\beta X_t}{1 - \exp(-\beta t)}$$

Method 3 is equivalent to Method 1 for $s=2$.

Failures in an Interval Range

Predicted *failure count in the range* $[t_1, t_2]$:

$$F(t_1, t_2) = (\alpha/\beta)[1 - \exp(-\beta((t_2 - s + 1)))] - X_{s,t_1}$$

Maximum Failures

Predicted *failure count in the range* $[1, \infty]$ (i.e., *maximum failures* over the life of the software):

$$F(\infty) = \alpha/\beta + X_{s-1}$$

(Note: Implemented in SMERFS but the user must make the manual correction of adding X_{s-1} to the quantity α/β that SMERFS computes).

Remaining Failures

Predicted *remaining failures* $r(t)$ at time t :

$$r(t) = (\alpha/\beta) - X_{s,t} = F(\infty) - X_t$$

(Note: Implemented in SMERFS but the user must make the manual correction of adding X_{s-1} to the quantity $\alpha/\beta - X_t$ that SMERFS computes).

Time to Next Failure

Predicted *time for the next F_t failures* to occur, when the current time is t :

$$T_F(t) = [(\log[\alpha/(\alpha - \beta(X_{s,t} + F_t))])/\beta] - (t - s + 1)$$

$$\text{for } (\alpha/\beta) > (X_{s,t} + F_t)$$

Mean Square Error Criterion for Remaining Failures, Maximum Failures, and Total Test Time (For Method 2 and Method 1 (s=1))

Mean Square Error (MSE_r) criterion for number of *remaining failures*, etc.:

$$MSE_r = \frac{\sum_{i=s}^t [F(i) - X_i]^2}{t - s + 1}$$

where

$$F(i) = (\alpha/\beta)[1 - \exp(-\beta((i-s+1)))] + X_{s-1}$$

Mean Square Error Criterion for Time to Next Failure(s) (For Method 2 and Method 1 (s=1))

Mean Square Error criterion for *time to next failure(s)*:

$$MSE_T = \frac{\sum_{i=s}^{J-1} [([\log[\alpha/(\alpha - \beta(X_{s,i} + F_{ij}))])/\beta - (i - s + 1)] - T_{ij}]^2}{(J - s)}$$

$$\text{for } (\alpha/\beta) > (X_{s,i} + F_{ij})$$

EQUATIONS IMPLEMENTED IN STATGRAPHICS

Cumulative Failures

Predicted *failure count in the range [1, t]*:

$$F(t) = (\alpha/\beta)[1 - \exp(-\beta((t-s+1)))] + X_{s-1}$$

Remaining Failures

Predicted *remaining failures as a function of total test time t_t* :

$$r(t_t) = (\alpha/\beta)(\exp(-\beta[t_t - (s-1)]))$$

Fraction of Remaining Failures:

Fraction of remaining failures predicted at time t:

$$p(t) = r(t)/F(\infty)$$

Operational Quality

Operational quality predicted at time t:

$$Q(t) = 1 - p(t)$$

Total Test Time to Achieve Specified Remaining Failures

Predicted *total test time* required to achieve a specified *number of remaining failures* at t_t , $r(t_t)$:

$$t_t = [\log[\alpha/(\beta[r(t_t)])]]/\beta + (s-1)$$

Time to Next N Failures and Remaining Failures Tradeoffs

Time to next N failures that would be achieved if *remaining failures* were reduced by Δr , predicted at time

$$T_F(\Delta r, t) = (-1/\beta)[\log[1 - ((\beta\Delta r/\alpha)(\exp(\beta(t-s+1))))]]$$

for $((\beta\Delta r/\alpha)(\exp(\beta(t-s+1)))) < 1$.

Reduction in remaining failures that would be achieved if the software were executed for a time T_F predicted at time t :

$$\Delta r(T_F, t) = (\alpha/\beta)[\exp(-\beta(t-s+1))][1 - \exp(-\beta(T_F))]$$

Mean Square Error Criterion for Failure Counts (For Method 2 and Method 1 (s=1))

Mean Square Error criterion for *failure counts*:

$$MSE_F = \frac{\sum_{i=s}^t [\alpha/\beta(1-\exp(-\beta(i-s+1))) - X_{s,i}]^2}{t-s+1}$$

Criteria for Safety

1) predicted *remaining failures* $r(t) < r_c$,
where r_c is a specified critical value, and

2) predicted *time to next failure* $T_F(t) > t_m$,
where t_m is mission duration.

Risk Assessment

Risk criterion metric for *remaining failures* at total test time t_t :

$$RCM\ r(t) = (r(t) - r_c) / r_c = (r(t) / r_c) - 1$$

Risk criterion metric for *time to next failure* at total test time t_t :

$$RCM\ T_F(t) = (t_m - T_F(t)) / t_m = 1 - (T_F(t)) / t_m$$

Note: Although *Criteria for Safety* and *Risk Assessment* equations are not covered in the other volumes of the handbook, they are listed here because they are part of the *Schneidewind Software Reliability Model*. These items are covered in: Norman F. Schneidewind, "Reliability Modeling for Safety Critical Software", IEEE Transactions on Reliability, Vol. 46, No.1, March 1997, pp.88-98.

**TABLE 1: STATGRAPHICS (SGPLUS) EQUATION IMPLEMENTATIONS
SOFTWARE RELIABILITY MODEL EQUATIONS**

Math Notation	Sgplus Notation	Statgraphics Definition	Sgplus Function
α	alpha	Beginning failure rate	From SMERFS
β	beta	Relative failure rate	From SMERFS
Δr	deltaR	Delta Remaining Failures	Given value
$f(t)$	d	Predicted Failure Rate	$(\alpha) * (\text{EXP}(-(\beta * (i - (s - 1))))))$
$F(i)$	f	Predicted Cumulative Failures	$((\alpha / \beta) * (1 - \text{EXP}(-(\beta * ((i - s) + 1)))) + Xs$
F_{ij}	Fij	Number of failures at j since i in MSETf	From failure data
F_t	Fij	Number of failures to occur after interval t in tf	Given value
$F(t)$	Ft	Predicted Maximum Failures	$(\alpha / \beta) + (Xs)$
i	i	Execution time index	From failure data
J	J	Maximum $j \leq t$ where $F_{ij} > 0$	From failure data
$m(i)$	mi	Predicted failures in intervals	$(\alpha / \beta) * (\text{EXP}(-(\beta * (i - s)))) * (1 - \text{EXP}(-(\beta)))$
MSE_F	MSE	MSE: Cumulative Failures	$(\text{SUM}(((\text{EVAL } f) - X_{si})^2)) / ((t - s) + 1)$
MSE_r	MSEr	MSE: Remaining Failures	$\text{SUM}(((\text{EVAL } f) - X_t)^2) / ((t - s) + 1)$
MSE_T	MSETf	MSE: Time to Failure	$(\text{SUM}(((\text{EVAL } tf) - T_{ij})^2)) / ((J - s))$
$p(t)$	p	Fraction Remaining Failures	$(Rt) / (\text{EVAL } Ft)$
$Q(t)$	Q	Predicted Program Quality	$(1 - (\text{EVAL } p))$
r_c	Rc	Remaining Failures Criterion	Given value

$r(t)$	r	Predicted remaining failures using X_t	$(\alpha/\beta)-(X_{st})$
$r(t_t)$	rt	Predicted remaining failures, given tt	$(\alpha/\beta)*(EXP(-\beta*(tt-(s-1))))$
None	R	Predicted remaining failures using p	$p*(EVAL Ft)$
None	R_{tt}	Number of remaining failures in computing p and tt	Given value
$\Delta r(T_F, t)$	dR	Predicted delta Remaining Failures	$(\alpha/\beta)*(EXP(-(\beta*(i-(s-1)))))* (1-(EXP(-(\beta*TR))))$
RCM $r(t)$	$riskR$	Risk of Remaining Failure	$((EVAL rt)-R_c)/R_c$
RCM $T_F(t)$	$riskT$	Risk of Time to Failure	$(tm-(EVAL tf))/tm$
s	s	First failure interval	From SMERFS
t	t	Execution time	From failure data
t_t	tt	Predicted Total Test Time, given R_{tt}	$((LOG(\alpha/(\beta*R_{tt}))/\beta)+(s-1))$
$T_F(t)$	tf	Predicted Time to Failure	$((1/\beta)*(LOG(\alpha/(\alpha-(\beta*(X_{si}+F_{ij})))))) - (i-(s-1))$
$T_F(\Delta r, t)$	Tf	Time to Failure for delta Remaining Failures	$(-1/\beta)*(LOG(1-((\beta/\alpha)*(\Delta R)*(EXP(\beta*(i-(s-1)))))))$
T_{ij}	T_{ij}	Time since i to fail at j	From failure data
t_m	tm	Time to Failure Criterion	Given value
T_F	TR	Given Tf for Predicted delta Remaining Failures	Given value
X_{s-1}	X_s	Observed failure count in the range $[1, s-1]$	From failure data
$X_{s,i}$	X_{si}	Observed failure count in the range $[s, i]$	From failure data
$X_{s,t}$	X_{st}	Observed failure count in the range $[s, t]$	From failure data
X_t	X_t	Observed failure count in the range $[1, t]$	From failure data

DISTRIBUTED SYSTEM MODEL EQUATIONS

NOTATION

System Nodes

N_{cc} : Number of Critical Client nodes

$N_{nc}(t)$: Number of Non-Critical Client nodes

N_{cs} : Number of Critical Server nodes

$N_{ns}(t)$: Number of Non-Critical Server nodes

$N(t) = N_{cc} + N_{nc}(t) + N_{cs} + N_{ns}(t)$: Total number of nodes

Node Failure Probabilities

p_{cc} : probability of a software defect causing a critical client node to fail

p_{nc} : probability of a software defect causing a non-critical client node to fail

p_{cs} : probability of a software defect causing a critical server node to fail

p_{ns} : probability of a software defect causing a non-critical server node to fail

p_{sw} : probability of a node failure due to software

Node Failure Count

i : identification of an interval of operating time of the software

$f_{cc}(i)$: critical client node failure count in interval i

$f_{nc}(i)$: non-critical client node failure count in interval i

$f_{cs}(i)$: critical server node failure count in interval i

$f_{ns}(i)$: non-critical server node failure count in interval i

$d(i)$: total defect count in interval i

D : total defect count across all intervals

Types of Software Defects (Examples Only)

S: Software Defect

G: General Protection Fault

N: Network Related Defect

C: System Crash

System Failure Probability Components

t : cumulative time in the range $[1, t]$; last interval of observed failure data; current interval

P_{cc} : probability that **one or more** critical clients N_{cc} fail, given that the software fails

$P_{nc}(t)$: probability that **all** non-critical clients $N_{nc}(t)$ have failed by time t , given that the software fails

p_{cs} : probability that **one or more** critical servers N_{cs} fail, given that the software fails

$P_{ns}(t)$: probability that **all** non-critical servers $N_{ns}(t)$ have failed by time t , given that the software fails

System Failure Probability

$P_{sys}/node\ fails(t)$: probability of a system failure by time t , *given that a node fails*

EQUATIONS IMPLEMENTED USING DEFECT CONTROL SYSTEM DATABASE (Examples Only)

Node Failure Count

$f_{cc}(I)$ =COUNT as failures WHERE $(S \wedge G \wedge N \wedge not C)$ in interval I

$f_{nc}(I)$ =COUNT as failures WHERE $(S \wedge G \wedge not N \wedge not C)$ in interval I

$f_{cs}(I)$ =COUNT as failures WHERE $(S \wedge not G \wedge N \wedge C)$ in interval I

$f_{ns}(I)$ =COUNT as failures WHERE $(S \wedge not G \wedge not N \wedge C)$ in interval I

$d(I)$ =total defect count in interval I

$$D = \sum_i d(I)$$

EQUATIONS IMPLEMENTED USING WINDOWS CALCULATOR

Node Failure Probabilities

Probability of a software defect causing a critical client node to fail:

$$p_{cc} = \sum_i f_{cc}(I) / D$$

Probability of a software defect causing a non-critical client node to fail:

$$p_{nc} = \sum_i f_{nc}(I) / D$$

Probability of a software defect causing a critical server node to fail:

$$p_{cs} = \sum_i f_{cs}(I) / D$$

Probability of a software defect causing a non-critical server node to fail:

$$p_{ns} = \sum_i f_{ns}(I) / D$$

Probability of a node failure due to software:

$$P_{sw} = p_{cc} + p_{nc} + p_{cs} + p_{ns}$$

System Failure Probability Components

Probability that **one or more** critical clients N_{cc} fail, given that the software fails:

$$P_{cc} = 1 - (1 - p_{cc})^{N_{cc}}$$

Probability that **all** non-critical clients $N_{nc}(t)$ have failed by time t , given that the software fails:

$$P_{nc}(t) = (p_{nc})^{N_{nc}(t)}$$

Probability that **one or more** critical servers N_{cs} fail, given that the software fails:

$$P_{cs} = 1 - (1 - p_{cs})^{N_{cs}}$$

Probability that **all** non-critical servers $N_{ns}(t)$ have failed by time t , given that the software fails:

$$P_{ns}(t) = (p_{ns})^{N_{ns}(t)}$$

EQUATION IMPLEMENTED IN STATGRAPHICS

System Failure Probability

Probability of system failure, by time t , given a node failure:

$$P_{sys}/node\ fails(t) = [P_{cc}] [P_{nc}(t)] + [P_{cs}] [P_{ns}(t)] =$$

$$[1 - (1 - p_{cc})^{N_{cc}}] [(p_{nc})^{N_{nc}(t)}] + [1 - (1 - p_{cs})^{N_{cs}}] [(p_{ns})^{N_{ns}(t)}]$$

Probability of Client Failure Probability of Server Failure

**TABLE 2: STATGRAPHICS (SGPLUS) EQUATION IMPLEMENTATIONS
DISTRIBUTED SYSTEM MODEL EQUATIONS**

Math Notation	Sgplus Notation	Statgraphics Definition	Sgplus Function
N_{cc}	Ncc	Number of critical clients	From system configuration (constant)
$N_{nc}(t)$	Nnc	Number of non-critical clients	From system configuration (vector as a function of time)
N_{cs}	Ncs	Number of critical servers	From system configuration (constant)
$N_{ns}(t)$	Nns	Number of non-critical servers	From system configuration (vector as a function of time)
P_{cc}	pcc	Probability of critical client failure	From Windows Calculator
$p_{nc}(t)$	pnc	Probability of non-critical client failure	From Windows Calculator
P_{cs}	pcs	Probability of critical server failure	From Windows Calculator
$p_{ns}(t)$	pns	Probability of non-critical server failure	From Windows Calculator
$P_{sys}/node\ fails(t)$	Psys	Probability System Failure/Node Failure	$((1-(1-pcc)^{Ncc})*((pnc)^{Nnc}))+$ $((1-(1-pcs)^{Ncs})*((pns)^{Nns}))$

EQUATION IMPLEMENTED IN SMERFS

Time to Failure Prediction

Predicted time for the next F_i failures to occur, when the current time is t, for each of the four types of node failures::

$$T_F(t) = [(\log[\alpha / (\alpha - \beta(X_{s,t} + F_i)]) / \beta)] - (t - s + 1)$$

for $(\alpha / \beta) > (X_{s,t} + F_i)$

METRICS MODELS EQUATIONS

DISCRIMINATIVE POWER VALIDATION MODEL

NOTATION

Defined in Table 3.

EQUATIONS IMPLEMENTED IN STATGRAPHICS

Maximum vertical difference between the CDFs of two samples (e.g., the CDFs of M_{ij} for $drcount \leq F_c$ and $drcount > F_c$):

$$K-S(M_{c_j}) = \max \{ [CDF(M_{ij}/(F_i \leq F_c))] - [CDF(M_{ij}/(F_i > F_c))] \}$$

Module count, based on BDFs of F_i and M_{ij} , that are calculated over the n modules for m metrics:

$$C_{11} = \text{COUNT FOR } ((F_i \leq F_c) \wedge (M_{i1} \leq M_{c1}) \dots \wedge (M_{ij} \leq M_{cj}) \dots \wedge (M_{im} \leq M_{cm}))$$

$$C_{12} = \text{COUNT FOR } ((F_i \leq F_c) \wedge (M_{i1} > M_{c1}) \dots \vee (M_{ij} > M_{cj}) \dots \vee (M_{im} > M_{cm}))$$

$$C_{21} = \text{COUNT FOR } ((F_i > F_c) \wedge (M_{i1} \leq M_{c1}) \dots \wedge (M_{ij} \leq M_{cj}) \dots \wedge (M_{im} \leq M_{cm}))$$

$$C_{22} = \text{COUNT FOR } ((F_i > F_c) \wedge (M_{i1} > M_{c1}) \dots \vee (M_{ij} > M_{cj}) \dots \vee (M_{im} > M_{cm}))$$

Proportion of Type 1 Misclassifications:

$$P_1 = C_{21}/n$$

Proportion of Type 2 Misclassifications:

$$P_2 = C_{12}/n$$

Proportion of Type 1+Type 2 Misclassifications:

$$P_{12} = (C_{21} + C_{12})/n$$

Proportion of low quality (i.e., $drcount > 0$) software correctly classified:

$$LQC = C_{22}/n_2$$

Remaining Factor RF (e.g., remaining *dr count*). This is the sum of F_i not caught by inspection:

$$RF = \sum_{i=1}^n F_i \text{ FOR } (F_i > F_c) \wedge (M_{i1} \leq M_{c1}) \dots \wedge (M_{ij} \leq M_{cj}) \dots \wedge (M_{im} \leq M_{cm})$$

Proportion of RF, where TF is the total F_i prior to inspection:

$$RFP = RF/TF$$

$$TF = \sum_{i=1}^n F_i$$

Density of RF:

$$RFD = RF/n$$

Proportion of modules remaining that have $F_i > F_c$:

$$RMP = RFM/n,$$

where RFM is given by:

$$RFM = \text{COUNT}_{i=1}^n \text{ FOR } ((F_i > 0) \wedge (M_{i1} \leq M_{c1}) \dots \wedge (M_{ij} \leq M_{cj}) \dots \wedge (M_{im} \leq M_{cm}))$$

Proportion of modules that must be inspected:

$$I = (C_{12} + C_{22})/n$$

Wasted inspection:

$$RI = C_{22}/C_{12}$$

EQUATION IMPLEMENTED USING WINDOWS CALCULATOR

Quality Inspection Ratio:

$$QIR = (|\Delta RFP| / RFP_i) / (\Delta I / I_i)$$

TABLE 3: STATGRAPHICS (SGPLUS) AND WINDOWS CALCULATOR EQUATION IMPLEMENTATIONS SOFTWARE METRICS MODELS EQUATIONS

DISCRIMINATIVE POWER VALIDATION MODEL

Math Notation	Sgplus Notation	Statgraphics Definition	Sgplus Function
C_{11}	C11	Module count for C11	SUM ((drcount LE Dc) AND (M1 LE M1c) AND (M2 LE M2c) AND (M3 LE M3c) AND (M4 LE M4c) AND (sample EQ Ss))
C_{12}	C12	Module count for C12	SUM ((drcount LE Dc) AND ((M1 GT M1c) OR (M2 GT M2c) OR (M3 GT M3c) OR (M4 GT M4c)) AND (sample EQ Ss))
C_{21}	C21	Module count for C21	SUM ((drcount GT Dc) AND (M1 LE M1c) AND (M2 LE M2c) AND (M3 LE M3c) AND (M4 LE M4c) AND (sample EQ Ss))
C_{22}	C22	Module count for C22	SUM ((drcount GT Dc) AND ((M1 GT M1c) OR (M2 GT M2c) OR (M3 GT M3c) OR (M4 GT M4c)) AND (sample EQ Ss))
F_c	Dc	Quality factor critical value	Given value
F_i	drcount (example)	Vector of quality factor values	From quality factor data
I	I	Proportion of modules that must be inspected	$((\text{EVAL } C12) + (\text{EVAL } C22)) / n * 100 \%$
ΔI	None	Difference in two successive values of I	Windows Calculator computation
i	Module name	Module index	From metrics file
j	Metric name	Metric index	From metrics file
$K-S(M_{cj})$	maxcdfdiff	Maximum vertical difference between two CDFs	MAX (EVAL (cdfdiff)), where cdfdiff= (ABS(m1-m2))/100 & m1, m2=metric vectors
LQC	LQC	Proportion of low quality software correctly classified	$((\text{EVAL } C22) / (\text{EVAL } n2)) * 100 \%$
M_{cj}	M1c...M4c	Vector of j metric critical values	From metrics data and K-S test
M_{ij}	M1...M4	Matrix of modules and metrics	From metrics data and K-W test
N_1	N1	Count of accepted modules	SUM ((M1 LE M1c) AND (M2 LE M2c) AND (M3 LE M3c) AND (M4 LE M4c) AND (sample EQ Ss))

N_2	N2	Count of rejected modules	SUM (((M1 GT M1c) OR (M2 GT M2c) OR (M3 GT M3c) OR (M4 GT M4c)) AND (sample EQ Ss))
n	n	Number of modules in sample	Given value
n_1	n1	Count of high quality modules	(EVAL C11)+(EVAL C12)
n_2	n2	Count of low quality modules	(EVAL C21)+(EVAL C22)
P_1	P1	Proportion of Type 1 misclassifications	((EVAL C21)/n)*100 %
P_2	P2	Proportion of Type 2 misclassifications	((EVAL C12)/n)*100 %
P_{12}	P12	Proportion of Type 1+Type 2 misclassifications	((EVAL C12)+(EVAL C21))/n)*100 %
QIR	None	Quality Inspection Ratio	Windows Calculator computation
RF	RF	Remaining Quality Factor	SUM (drcount SELECT ((M1 LE M1c) AND (M2 LE M2c) AND (M3 LE M3c) AND (M4 LE M4c) AND (drcount GT Dc) AND (sample EQ Ss)))
RFD	RFD	Density of RF	(EVAL RF)/n
RFP	RFP	Proportion of RF	((EVAL RF)/(EVAL TF))*100 %
ΔRFP	None	Difference in two successive values of RFP	Windows Calculator computation
RFM	RFM	Count of modules with Remaining Quality Factor	SUM ((drcount GT Dc) AND (M1 LE M1c) AND (M2 LE M2c) AND (M3 LE M3c) AND (M4 LE M4c) AND (sample EQ Ss))
RMP	RMP	Proportion of RFM	((EVAL RFM)/n)*100 %
RI	RI	Wasted Inspection	(EVAL C22)/(EVAL C12)
None	Ss	Sample Identification	Given value
TF	TF	Total Quality Factor	SUM (drcount SELECT sample EQ Ss)
χ_c^2	χ_c^2	Critical value of Chi-Square	Function of C11, C12, C21, and C22

PREDICTABILITY VALIDATION MODEL

NOTATION

Defined in Table 4

EQUATIONS IMPLEMENTED IN STATGRAPHICS

Proportion of modules with $F_i > 0$ in the Validation Sample prior to inspection and correction of defects:

$$p_n = (\text{COUNT FOR } F_i > 0) / n$$

Two-sided confidence limits of p_n , used as predicted limits of p_n' in the Application Sample:

$$CLp_n = p_n \pm Z_{\alpha/2} \sqrt{\frac{(p_n)(1-p_n)}{n}}$$

Proportion of modules **not flagged** for inspection (i.e., contained in N_1) with $F_i > 0$ in the Validation Sample:

$$pN_1 = \text{RFM} / N_1$$

One-sided upper confidence limit of pN_1 , used as predicted limit of pN_1' in the Application Sample:

$$ULpN_1 = pN_1 + Z_{\alpha} \sqrt{\frac{(pN_1)(1-pN_1)}{N_1}}$$

Proportion of modules **flagged** for inspection (i.e., contained in N_2) with $F_i > 0$ in the Validation Sample:

$$pN_2 = ((p_n)(n) - \text{RFM}) / N_2$$

One-sided lower confidence limit of pN_2 , used as predicted limit of pN_2' in the Application Sample:

$$LLpN_2 = pN_2 - Z_{\alpha} \sqrt{\frac{(pN_2)(1-pN_2)}{N_2}}$$

Proportion of quality factor that occurs on modules **not flagged** for inspection (i.e., contained in N_1) in the Validation Sample:

$$d_1 = \text{RF} / \text{TF} \text{ (same as RFP if RFP is expressed as a proportion)}$$

One-sided upper confidence limit of d_1 , used as predicted limit of d_1' in the Application Sample

$$ULd_1 = d_1 + Z_{\alpha} \sqrt{\frac{(d_1)(1-d_1)}{\text{TF}}}$$

Proportion of quality factor that occurs on modules **flagged** for inspection (i.e., contained in N_2) in the Validation Sample:

$$d_2 = 1 - d_1$$

One-sided lower confidence limit of d_2 , used as predicted limit of d_2' in the Application Sample:

$$LLd_2 = d_2 - Z_{\alpha} \sqrt{\frac{(d_2)(1-d_2)}{TF}}$$

Expected quality factor count (e.g., *drcount*) that occurs on modules **not flagged** for inspection (i.e., contained in N_1') in the Application Sample:

$$D_1 = (RF/N_1)(N_1')$$

Expected quality factor count (e.g., *drcount*) that occurs on modules **flagged** for inspection (i.e., contained in N_2') in the Application Sample):

$$D_2 = ((TF - RF)/N_2)(N_2')$$

**TABLE 4: STATGRAPHICS (SGPLUS) EQUATION IMPLEMENTATIONS
SOFTWARE METRICS MODELS EQUATIONS**

PREDICTABILITY VALIDATION MODEL

Math Notation	Sgplus Notation	Statgraphics Definition	Sgplus Function
p_n	pn	Proportion of modules with $F_i > 0$	(SUM((drcount GT 0) AND (sample EQ Ss)))/n
CLp_n	CLpn	Two-sided confidence limits of pn	((EVAL pn)+(Z*(SQRT (((EVAL pn)* (1-(EVAL pn)))/n))))
pN_1	pN1	Proportion of modules not flagged for inspection	(EVAL RFM)/(EVAL N1)
$ULpN_1$	ULpN1	Upper Confidence limit of pN1	((EVAL pN1)+(Z*(SQRT (((EVAL pN1)* (1-(EVAL pN1)))/(EVAL N1))))))
pN_2	pN2	Proportion of modules flagged for inspection	((n*(EVAL pn)-(EVAL RFM))/(EVAL N2)
$LLpN_2$	LLpN2	Lower confidence limit of pN1	((EVAL pN2)-(Z*(SQRT (((EVAL pN2)* (1-(EVAL pN2)))/(EVAL N2))))))
d_1	d1	Proportion of quality factor count that occurs on modules not flagged for inspection	(EVAL RF)/(EVAL TF)
ULd_1	ULd1	Upper confidence limit of d1	((EVAL d1)+(Z*(SQRT (((EVAL d1)* (EVAL d2))/(EVAL TF))))))
d_2	d2	Proportion of <i>drcount</i> that occurs on modules flagged for inspection	(1-EVAL (d1))
LLd_2	LLd2	Lower confidence limit of d2	((EVAL d2)-(Z*(SQRT (((EVAL d1)* (EVAL d2))/(EVAL TF))))))
D_1	D1	Expected quality factor count that occurs on modules not flagged for inspection	((EVAL RF)/(EVAL N1))*N1a
D_2	D2	Expected quality factor count that occurs on modules flagged for inspection	((EVAL TF)-(EVAL RF))/(EVAL N2))*N2a
N_1'	N1a	Count of accepted modules in <i>Application Sample</i>	SUM ((M1 LE M1c) AND (M2 LE M2c) AND (M3 LE M3c) AND (M4 LE M4c) AND (sample EQ Ss))
N_2'	N2a	Count of rejected modules in <i>Application Sample</i>	SUM (((M1 GT M1c) OR (M2 GT M2c) OR (M3 GT M3c) OR (M4 GT M4c)) AND (sample EQ Ss))
Z_α	Z	Standardized difference between variable and mean of normal distribution	Given value based on choice of α

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