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6. AUTHORS K.T. Arasu
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13. ABSTRACT (Maximum 200 words)
Time-discrete one and two-dimensional sequences and arrays with perfect autocorrelation functions are studied. Such sequences find applications in signal processing and as aperture functions for electromagnetic and acoustic imaging. Two dimensional perfect arrays are also applicable in 2-D synchronization and time-frequency coding. Mathematical tools from algebraic number theory, representation theory and group theory are employed to investigate the theory of their existence leading to new families of these arrays and some generalizations thereof.

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Progress report on

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High volume communication channels :

A mathematical investigation

for the period covering July, 1996 to August 1997

Principal Investigator: K.T.Arasu,
Department of Mathematics and Statistics,
Wright State University,
Dayton, OH 45435
Phone number: 937 775 3828
Fax Number: 937 775 2081
Email: karasu@desire.wright.edu

1. Objectives:

Time-discrete one and two-dimensional sequences and arrays with perfect autocorrelation functions are studied. Such sequences find applications in signal processing and as aperture functions for electromagnetic and acoustic imaging. Two dimensional perfect arrays are also applicable in 2-D synchronization and time-frequency coding. Mathematical tools from algebraic number theory, representation theory and group theory are employed to investigate the theory of their existence leading to new families of these arrays and some generalizations thereof.

2. Status of effort:

Periodic sequences and multidimensional arrays whose entries are either 0, 1 and -1 or the complex fourth roots of unity are studied. These sequences/arrays with low autocorrelation values are useful in reliable synchronization problems. Some new constructions of a class of such sequences have been obtained. A few structure theorems for these mathematical objects have also been proved, in a joint work with Ma and Voss. Another joint work with Seberry has resulted in classifying the existence status of periodic ternary sequences (i.e. with entries 0, 1 and -1) all of whose out-of-phase autocorrelations are zero. This work assumes the period of the sequences to be the number of points on a projective plane of certain finite order (an extremal case for this class of sequences). As a by-product of the methods, we were able to construct a new class of nested row-column designs based on a Galois Field, and these designs are useful in statistical design of experiments.

3. Accomplishments:

Periodic sequences with entries plus or minus one all but one of whose autocorrelation coefficients are zero were studied by Wolfmann. Using their equivalence to certain nice subsets in a cyclic group whose order equals the period of the corresponding sequence (these subsets are called divisible difference sets), we obtain sequences of period 8, 12 and 28. Further structure theorems using the factorization of ideals in a suitable algebraic number field are also proven. The theory developed in this paper [1] could be used for other types of sequences and arrays. This joint work with Ma and Voss has appeared in the Journal of Algebra.

In [2], (a joint work with Ma), we characterize those abelian

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groups which admit a McFarland difference set of order 81. These difference sets are variations of the well-known Hadamard difference sets which are equivalent to perfect binary arrays. Our methods do not make use of the widely made assumption known as "self-conjugacy". (This roughly means that the complex conjugation fixes the prime ideal factors of the principal ideal generated by the order of the corresponding difference set). We thus are able to fill two missing entries in Kopilovich's table. (Incidentally Kopilovich is a Ukrainian engineer who is writing up a monograph in this area that connects radar problems and "my" kind of mathematics. Our results will be cited in his book.) This paper [2] has been submitted to the journal Designs, Codes and Cryptography.

A square matrix W of order n with entries from $\{0, -1, +1\}$ satisfying $W W^t = k I_n$ is said to be a weighing matrix of order n with weight k . If the underlying matrix is also circulant, we therefore get a perfect ternary sequence of period n . Using very interesting facts about finite fields, we provide a construction of a family of circulant matrices of weight $33t$ and weight 25, for each positive integer t . Consequently we fill a missing entry in the CRC handbook (Section on weighing matrices by Craigen). While not a single matrix of order 33 and weight 25 was known before, we actually constructed a circulant one, to the surprise of experts in this field. Our new matrices also give a rise to a new class of orthogonal designs. This paper [3] has been accepted by the Journal of Combinatorial Designs.

In [4] Seberry and the P.I. investigate circulant weighing matrices of various weights whose order equal the number of points on a finite projective plane. This is an extremal case and the investigation is very theoretical. We have settled the existence question in orders upto 25. This paper has been submitted to the Australasian Journal of Combinatorics.

In [5], (joint work with Balasubramanian and Evans), we point out the connection between quadratic starters in a finite field and a class of nested row-column designs. As a consequence we provide an infinite class of such designs, generalizing a few sporadic examples obtained earlier by others. This paper has been accepted by the Journal of Combinatorics and Combinatorial Computing.

4. Publications:

1. On a class of almost perfect sequences, (with S.L.Ma and N.J.Voss), J. Algebra. 192, 641-650(1997).
2. Abelian difference sets without self-conjugacy, (with S.L.Ma), submitted to Designs, Codes and Cryptography.
3. New weighing matrices of weight 25, Journal of Combinatorial Designs, 5, 1-5(1997).
4. Circulant weighing matrices, (With Jennifer Seberry), submitted to Australasian J. Combinatorics.
5. A new family of nested row-column designs, (with Balasubramanian and A.B.Evans), to appear in Journal of Combinatorial Mathematics and Combinatorial Computing.

Personnel supported

Jeffrey Linthicum, Graduate research assistant

Conference Presentations

1. Invited lecture at the third International Conference of Combinatorics and Statistics at Portland, Maine, July 1997
2. Keynote address at the International conference on Combinatorics, May 1997, Hefei, China
3. Invited colloquium at the University of Hong Kong, May 1997
4. Invited colloquium at the Mathematical Institute, Madras, India, June 1997

Honors

I have been invited to write a chapter on Difference sets and its relation to Communication Engineering by Wiley Publishers. This 40 to 50 page article will be one chapter of an encyclopedia they are publishing on Computational Engineering.