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# On Assessing the Sufficiency of a Fiber Optic Distribution System for Communication Requirements

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# ON ASSESSING THE SUFFICIENCY OF A FIBER OPTIC DISTRIBUTION SYSTEM FOR COMMUNICATION REQUIREMENTS

## 1 Introduction

This report is motivated by the systems engineering task to ensure in the planning stage the sufficiency of the physical cabling plant in a building for the communications required in performing the mission. Fulfilling this task calls for several subtasks; i) determine what the facilities need to communicate with each other on a routine basis, ii) determine the required data rates, iii) determine the structure of the cabling plant and the capacity of physical communication media, etc. Then comes the process of assigning the physical resources in the cabling plant to the requirements in the most efficient way. If the most efficient assignment does not meet the communication requirement, the cabling plant cannot satisfy the communication requirement, and the plan must be revised. This report provides procedures to assign optical fibers to device-to-device connections in the most efficient way.

In establishing facilities (data processing center, network control center, etc.) with high-rate, heterogeneous, intra-building data communication requirements, the task of laying out optical fiber cabling plants (fiber optic cables, patch panels, fiber optic cabinets, fiber optic terminal housing, etc.) and the task of designing high-level network topology (e.g., electronic switches, computers, and their logical connection) are functionally decomposed. The practice of installing dedicated fiber cables for each new network is not adequate. Typically, the optical fiber cabling plants are often laid out during the building construction with the anticipation of future use of fibers. The cable plant is provided by the construction contractor. The communication network designers then make plans to connect geographically separated communication devices (end-user computers, switches, etc.) with one another by the fibers in the cabling plant to form the network topology (FDDI ring, mesh connection of ATM switches, point-to-point connections of Fiber Channel, etc.) The cabling plant specification includes the location of fiber distribution points such as Main Distribution Frames (e.g., Communication Rooms illustrated in Figure 1), Intermediate Distribution Frame (e.g., Corner Closets illustrated in Figure 2), distribution equipment such as LGX<sup>1</sup> and LIU<sup>2</sup>, patch panels, fiber optic enclosure, cabinets, pop-up boxes etc.) The specification also includes the number of fibers running from one fiber distribution point to another, etc. The communication network topology plan includes the location of hardware devices, which devices are to be directly connected together through the fiber, etc. To make an end-to-end connection between a pair of devices located in different locations, the network planner selects a fiber distribution point in the cabling plant for each

<sup>1</sup> AT&T's 324-pair central equipment room patch unit

<sup>2</sup> AT&T's six-pair remote patch unit; Light Interface Unit

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end device. Then, there must be a fiber that runs between the device and the distribution point. If the same distribution point is chosen for both devices, two fibers meet at the distribution point. To complete the device-to-device connection, two fibers can be connected at the distribution point through various connection methods (e.g., through ST or SC type connectors, through fiber splicing, etc.) If distinct distribution points are chosen for the two devices, more fibers need to be selected to complete the connection. If there is a cable running between these two fiber distribution points, the network planner can select a fiber in that cable and connect the ends of that fiber at those two distribution points to the two fibers attached to the end devices. Or, the physical fiber connection can be made with more distribution points in between. In short, the network planner must select the path (sequence of distribution points) between two end devices. This report addresses the problem of efficiently assigning the optical fibers in the cable plant to all pairs of devices that need direct fiber connection. Loosely stated, this report addresses the following questions:

- Given the structure of the physical cabling plant, the location of communication devices, and the list of devices that need to be connected with each other by fibers, how can we efficiently assign fibers in the cabling plant to such connections?
- How can we determine if the physical cabling plant is sufficient to make all required connections?

This report formulates these questions with a conceptual model of the cabling plant and the communication requirement, and provides algorithms that answer these questions.

This report is motivated by analyzing the data processing facility, whose building and cabling structures are illustrated in Figures 1 and 2. These figures exhibit the “hierarchical star” structure of the physical plant. The center of the hierarchical star is in Building A in Figure 1. The cables run from Building A to other buildings. The cable that runs from Building A to Building B terminates at the Communication Room in Building B, where fiber optic distribution products such as AT&T’s LGXs are present. From there, cables with a less number of fibers run to smaller fiber optic distribution products such as AT&T’s LIUs that are spread across each floor of the building. Devices in Building B are attached to the physical plant through the LIUs. The cabling running from Building A to Building C are again terminated in the Communication Room of Building C. From there separate cables run to corner closets of different floors. In each floor, cables run from corner closets to pop-up boxes spread across the floor. This report will address the problem within the scope of the “hierarchical star” structure of the physical plant. The hierarchical star structure is supported by TIA/EIA-568A Standard. [1] This report will focus on the case that the communication devices can be attached to the physical plant only through the distribution point with the lowest hierarchy (LIU or pop-up box), as will be explained in the next sections. A motivation for such a constraint is to make the maintenance of the physical plant simple and economical.

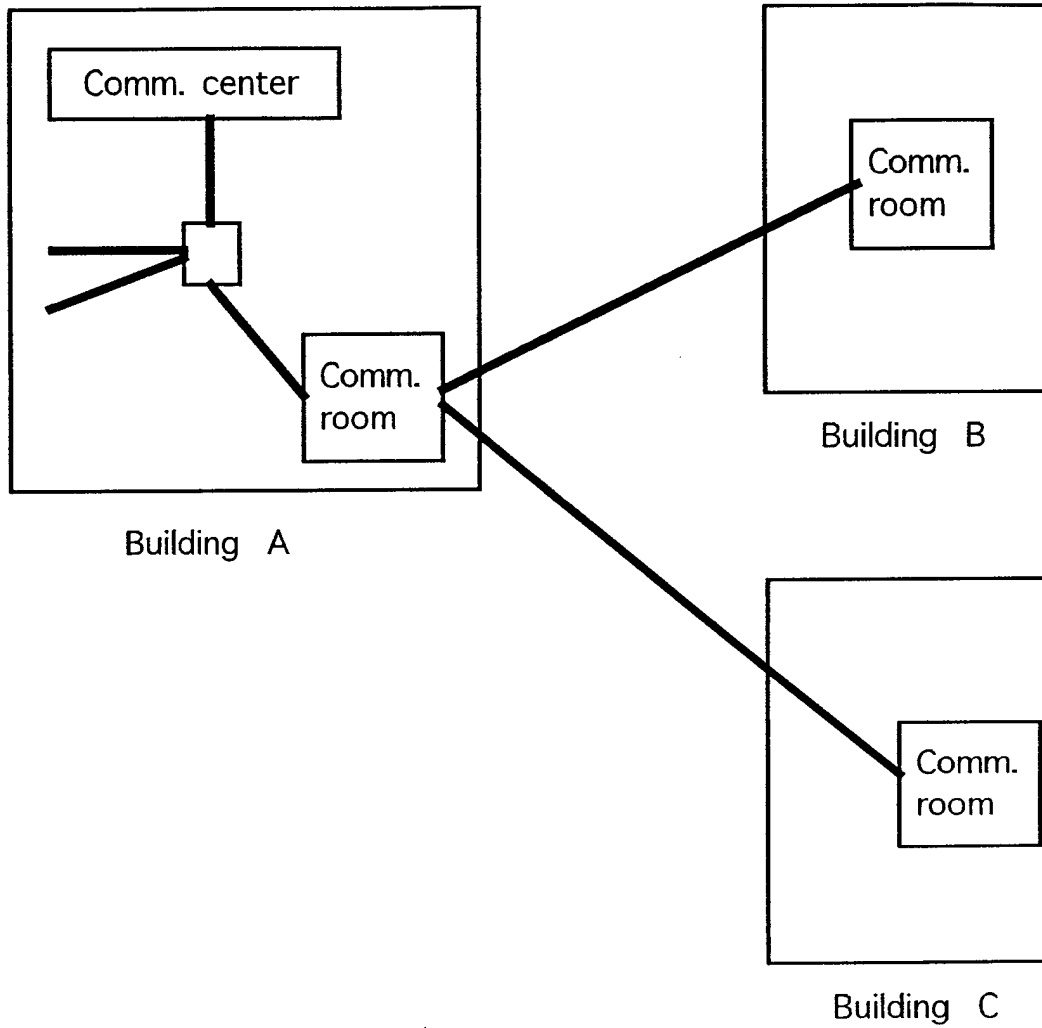


Figure 1: Overview of buildings and FODS cabling structure

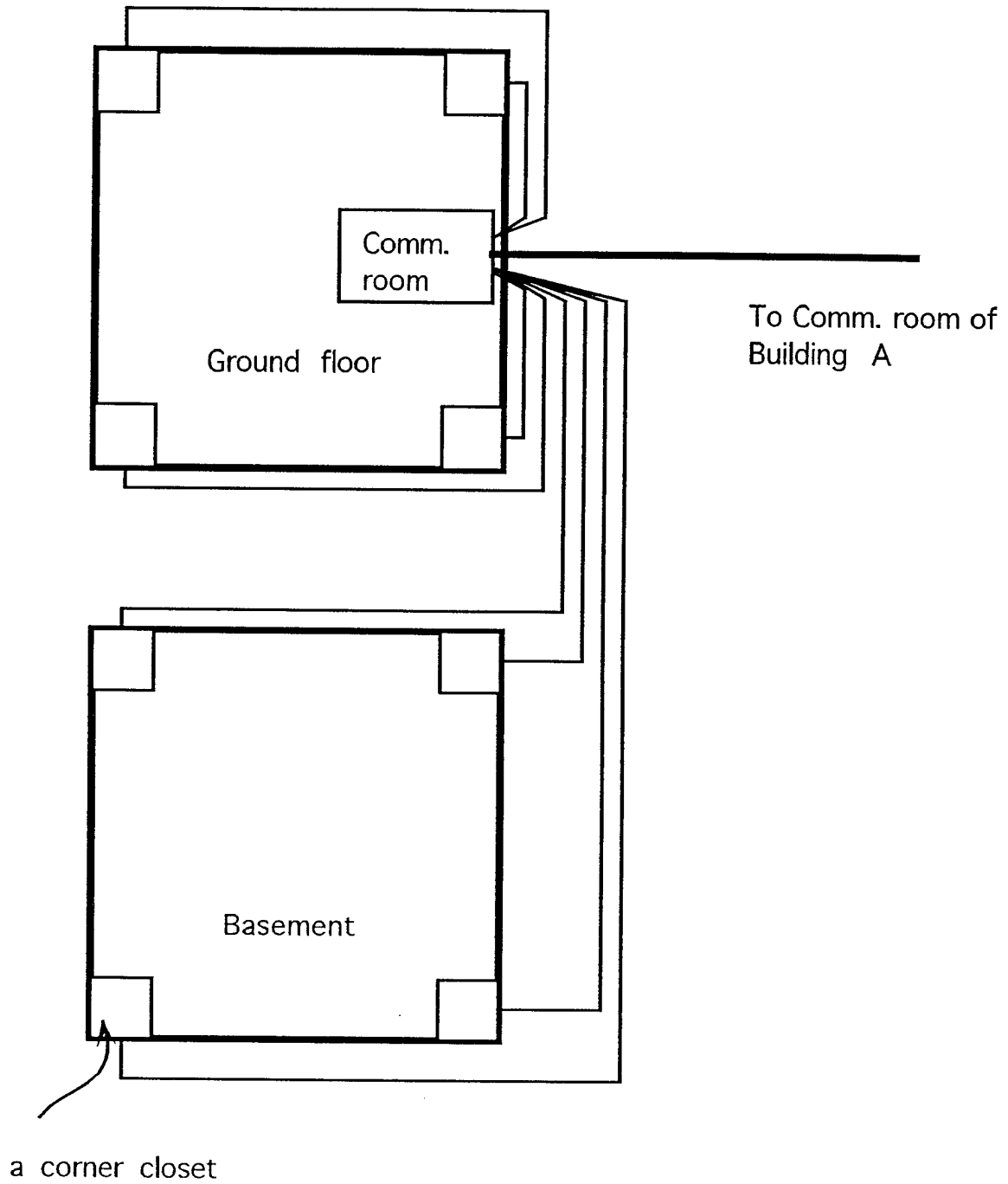


Figure 2: Physical cable plant of Building C

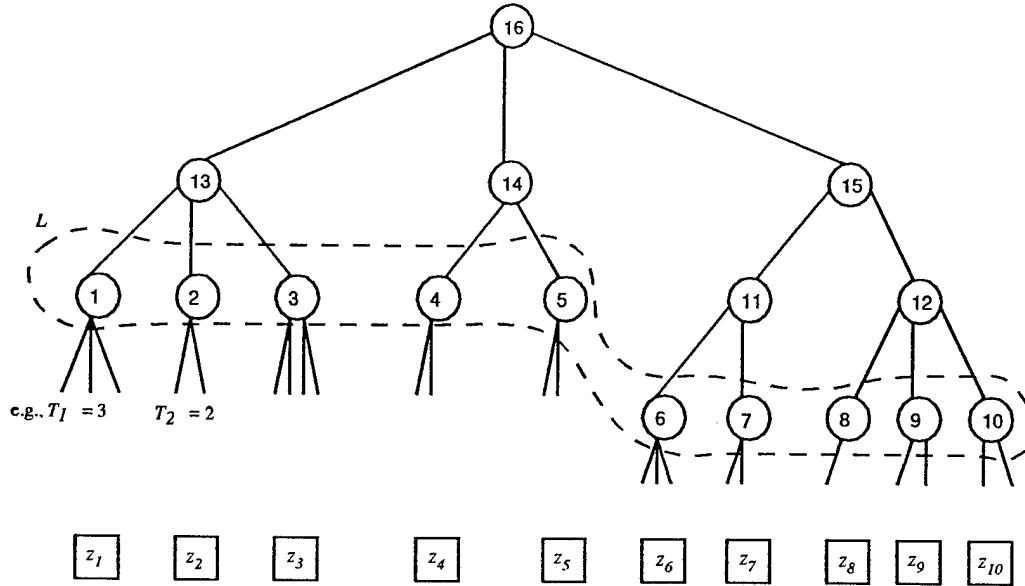
## 2 Formulation

### 2.1 Physical topology of the cabling plant

We represent the physical cabling plant or Fiber Optic Distribution System (FODS) by undirected graph  $G = (V, E)$ , wherein the nodes in  $V$  of the graph represent the fiber optic distribution points (e.g. corner closets of Building C illustrated in Figure 2, LGX, LIU, etc.), and the edges in  $E$  represent the fiber optic cables. The hierarchical star structure of the physical plant corresponds to the tree topology of graph  $G$ . We denote each node of the tree by a distinctive integer,  $i$ . We denote the set of leaves by  $L$ . Recalling that the tree represents the physical topology of the cabling plant, and that the leaves represent the extremity such as pop-up boxes where the communication (network) equipment accesses the cabling plant infrastructure, we associate with each leaf a geographical zone of the establishment. We associate leaf  $i$  with zone  $z_i$ . This represents the situation that each pop-up box or LIU is located at a certain geographic area, and it is primarily used to enable devices in that area to access the physical plant. Each leaf has fibers through which devices can be connected to the cabling plant. We refer to these fibers as “terminal fibers”. With each leaf  $i$  we associate  $T_i$ , the number of terminal fibers that run from leaf  $i$  to where devices are. With each edge  $(k, l)$ , we associate capacity  $C_{kl}$ , which represents the number of fibers running between nodes  $k$  and  $l$  of the tree-structured graph. Figure 3 illustrates these notations. We assume that devices can only contact terminal fibers, which stem from leaves. Regarding the number of terminal fibers stemming from leaf  $i$  and the number of fibers running between leaf  $i$  and its parent  $p$ , we first note the following simple fact:

*If  $C_{ip} > T_i$ , at least  $C_{ip} - T_i$  fibers of edge  $(i, p)$  are unused in efficient assignment of fibers to connections.*

The following is the argument. Each fiber in edge  $(i, p)$  used for a device-to-device connection can be connected at leaf  $i$  to either a terminal fiber or to another fiber in edge  $(i, p)$ . If the fiber in  $(i, p)$  is connected to another fiber in  $(i, p)$  at leaf  $i$ , both of these two fibers make paths from node  $p$  eventually to two devices, as illustrated in Figure 4. Then, we can make this device-to-device connection by connecting the two paths at node  $p$  and save two fibers in  $(i, p)$ . Therefore, for efficiency, one end of each fiber in edge  $(i, p)$  used for a device-to-device connection must be connected to a terminal fiber at leaf  $i$ . Therefore, any number of fibers in link  $(i, p)$  in excess of  $T_i$  are unused. This advises the designer of the cabling plant to make the number of fibers between a parent and a leaf no more than the number of terminal fibers from the leaf. For this reason we assume in this report  $C_{ip} \leq T_i$  for each leaf  $i$  and its parent  $p$ .



$$V = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\},$$

$$L = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

Figure 3: Tree representing the physical cable plant

## 2.2 Connection

Suppose that two devices, Device 1 and Device 2, need to be physically connected by fibers. If Device 1 is to make a connection through leaf  $i$  and Device 2 through leaf  $j \neq i$ , the fibers must run between leaf  $i$  and leaf  $j$  to complete the connection. Due to the tree structure of the graph, there is a unique path that does not contain a cycle between  $i$  and  $j$ . From each edge constituting the path, one fiber can be taken, and these fibers can be connected to complete the path. It is conceivable that the fibers running between two leaves result in a path that contains a cycle. In such a case fibers are used wastefully. Therefore, we assume that such a practice is not allowed.

## 2.3 Communication requirement

Each zone has communication devices located in it. For example, in a satellite operation data processing facility, several different functions (Mission Planning, Calibration, Data Processing, Performance Assessment) may be located in different zones. Each function owns computers, switches, and other communication devices. We can imagine that a device located in zone  $z_i$  and another

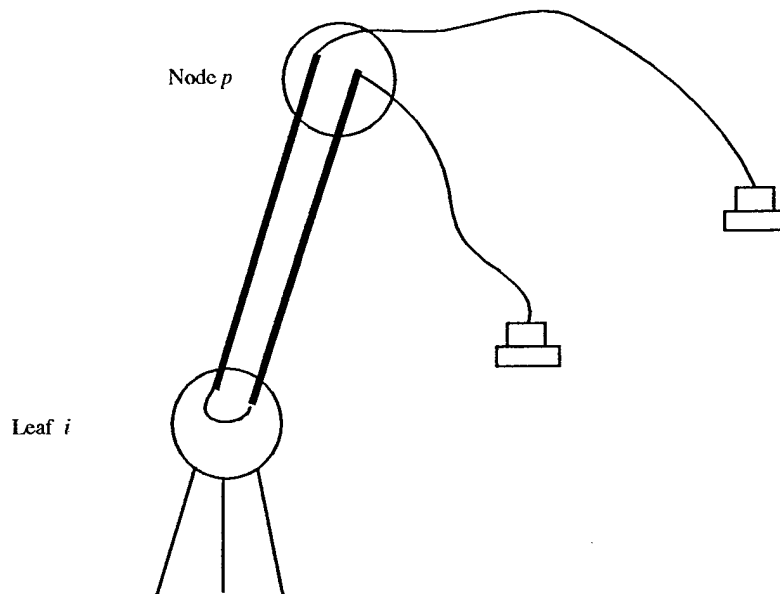


Figure 4: A case of inefficient use of fiber

device located in zone  $z_j$  need to be physically connected. We denote by  $r_{ij}$  the number of such pairs. Thus, communication requirement is specified by the set of numbers  $\{r_{ij} | i, j \in L\}$ . (We do not differentiate the direction of the connection, so  $r_{ij} = r_{ji}$ , and set  $\{r_{ij} | i, j \in L\}$  has redundant information.) For example, zone  $z_1$  holds Data Processing function, and zone  $z_2$  holds Calibration function. Then  $r_{11}$  pairs of devices (computers) within the Data Processing function are required to have physical, optical-fiber connection. Also, a certain device in data Processing Function needs to be directly connected with a device in Calibration. There are  $r_{12}$  such pairs of required connections. Note that for a given triplet of the graph  $G = (V, E)$ , the set of capacities  $\{C_{ij} | (i, j) \in E\}$ , and the numbers of terminal fibers  $\{T_i | i \in L\}$ , some requirements cannot be satisfied no matter how one assigns the fibers to the required connections. We say that the requirement is “infeasible” in that case. Note that in this report the communication requirement is solely represented by what device needs to be connected with what. This report does not address the issue of whether an optical fiber can support the required data rates of the point-to-point communication. This report assume that each fiber connection from device to device has enough capacity to satisfy the required rate of reliable data transfer.

### 3 Strict zone to leaf adherence

In this section we consider the fiber assignment problem under the constraint that devices in zone  $z_i$  can be attached to the physical cabling plant only at leaf  $i$ . In other words, devices located in

zone  $z_i$  can contact only the terminal fibers stemming from leaf  $i$ . Devices in zone  $z_i$  cannot use terminal fibers going to a leaf intended for (or associated with) a geographically neighboring zone, say  $z_j$ , even if the number of devices in zone  $z_i$  is more than the number of terminal fibers  $T_i$ . (Obviously, the requirement is infeasible in this case.) Under this constraint, we can find a simple algorithm that, if feasible, assigns physical fibers to the required connections and, if infeasible, decides infeasibility. The simplifying nature of this problem is that there is a unique path between any pair of leaves. Define a variable  $d_{kl}$  for each edge  $(k, l)$  of the tree and  $u_i$  for each leaf  $i$  of the tree.

#### Algorithm 1

- *Step 0: For each leaf  $i$  and each edge  $(k, l)$ , set  $u_i = 0$ ,  $d_{kl} = 0$*
- *Step 1: For each pair of zones  $(z_i, z_j)$ ,  $i \leq j$ , do*

$$u_i \leftarrow u_i + r_{ij}$$

$$u_j \leftarrow u_j + r_{ij}$$

$$d_{kl} \leftarrow d_{kl} + r_{ij} \text{ for each edge } (k, l) \text{ along the path from } i \text{ to } j$$

At the end of this procedure, if  $u_i > T_i$  for any leaf  $i$ , or if  $d_{kl} > C_{kl}$  for any edge  $(k, l)$ , then the communication requirement is infeasible. Otherwise, the fiber assignment is to assign one fiber along the path from  $i$  to  $j$  and two terminal fibers for each required end-to-end connection.

## 4 Sharing leaves within the branch

As an example, we imagine an overcrowded zone. Suppose that in zone  $z_i$ , more than  $T_i$  devices need to make a connection. With the constraint of attaching these devices only through leaf  $i$ , this communication requirement is infeasible. In this section, we relax this constraint. This section reflects the practical situation of allowing the device to connect through the fiber optic enclosure built for the neighboring geographic zone. In order to formulate the fiber assignment problem, we partition the set of leaves  $L$  into subsets; namely we group leaves with the same parent. We refer to the set of leaves with an identical parent as “branch”. A branch is denoted by  $B$ . When distinguishing different branches, we use notation  $B$  with a subscript. We denote by  $N_b$  the total number of branches in the tree  $G$ . Thus, we denote branches by  $B_1, B_2, \dots, B_{N_b}$ , and we then have  $L = \cup_{i=1}^{N_b} B_i$ . In this section we consider the fiber assignment problem with the following constraint:

*Devices in zone  $z_i$  can contact terminal fibers leading to any leaf  $k \in B_j$  as long as  $i \in B_j$ .*

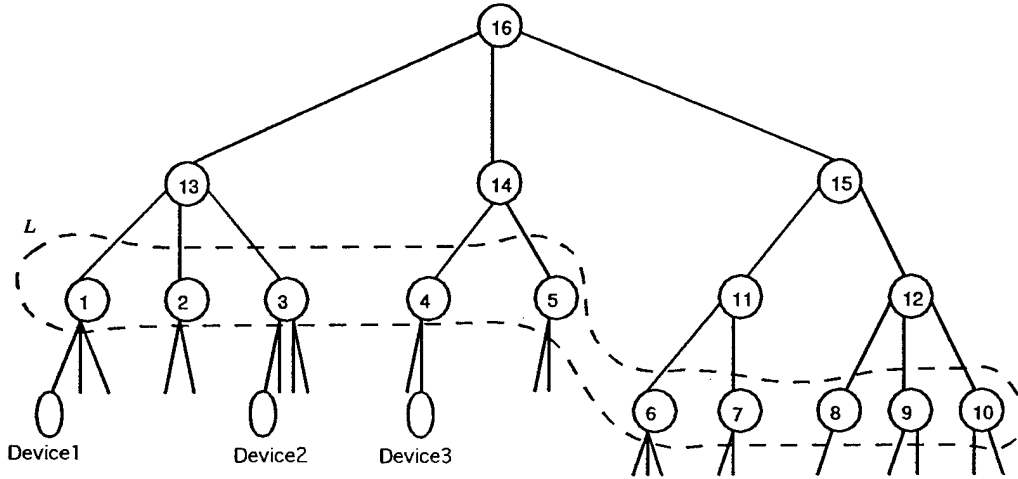


Figure 5: Inter-branch and intra-branch connections

#### 4.1 First phase: assignment of fibers running between branch parents

We can make a few important observations in designing an algorithm that assigns fibers to connections or decides infeasibility. We focus on the connections that originate from each individual branch  $B_i$ ; i.e., connections that have at least one device located in a zone associated with a leaf in branch  $B_i$ . Connections originating from a branch can be classified into two kinds: inter-branch and intra-branch connections. An inter-branch connection between branches  $B_i$  and  $B_j$  is an end-to-end connection whose one end is the device located in a zone associated with a leaf in branch  $B_i$  and whose other end is the device located in a zone associated with a leaf in another branch  $B_j$ . For example, in Figure 5 connection between Device 2 and Device 3 would be an inter-branch connection.  $R_{ij}$  denotes the number of inter-branch connections that are required to be made between  $B_i$  and  $B_j$ . An intra-branch connection within  $B_i$  is an end-to-end connection whose both ends are devices located in zones (or an identical zone) associated with leaves (or a leaf) in branch  $B_i$ . For example, in Figure 5 connection between Device 1 and Device 2 would be an intra-branch connection. We denote by  $\rho_i$  the number of required intra-branch connections within branch  $B_i$ . We denote the parent node of  $B_i$  by  $p_i$ . Then, an intra-branch connection within  $B_i$  can be made only with terminal fibers or with both terminal fibers and fibers connecting node  $p_i$  and its children in  $B_i$ . Such a connection does not need a fiber running from branch parent  $p_i$  to nodes not in  $B_i$ . An

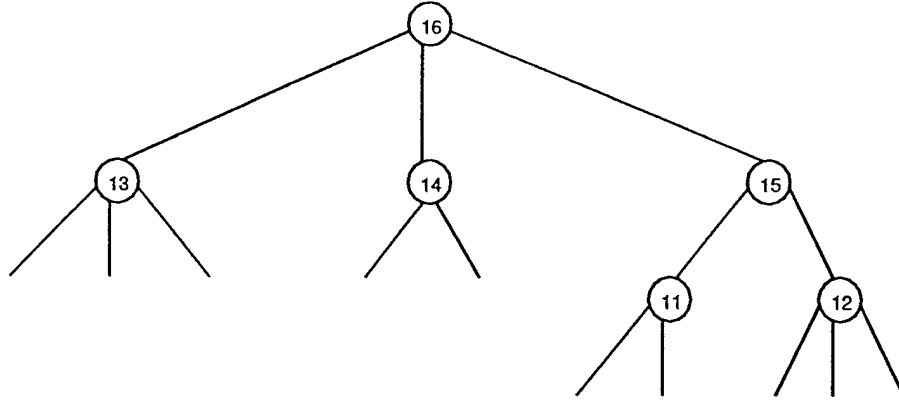


Figure 6: Truncation of the tree

inter-branch connection between a device associated with branch  $B_i$  and a device associated with branch  $B_j$  must include fibers running between branch parents  $p_i$  and  $p_j$ . Therefore,  $R_{ij}$  separate fiber connections must be made between node  $p_i$  and  $p_j$  for each pair of branch parents  $p_i, p_j, i \leq j$  in order to satisfy the inter-branch connection requirements. Due to the hierarchical tree structure, the path between  $p_i$  and  $p_j$  is unique. Now consider the truncated physical topology obtained by pruning all the leaves in  $L$ . For example, the truncated topology obtained from the tree in Figure 3 is illustrated in Figure 6. In the truncated physical topology, the fibers running between a branch parent and its leaves become terminal fibers. We can observe now that the problem of assigning fibers between branch parents to inter-branch connections has the structure identical to the case of “strict zone to leaf adherence” in the previous section.

To solve the fiber assignment problem, we can use a two-phase approach. The first phase assigns fibers running between parents to each inter-branch connection. We can use Algorithm 1, with graph  $G$  replaced by the truncated physical topology and with requirement  $\{r_{ij} | i \in L, j \in L\}$  replaced by the inter-branch connection requirement  $\{R_{ij} | 1 \leq i, j \leq N_b\}$ . If this algorithm decides “infeasibility”, then the communication requirement  $\{r_{ij} | i \leq j\}$  is infeasible, and the second phase is not necessary. If this algorithm successfully assigns fibers running between branch parents to the required inter-branch connections, the second phase must enter. In the second phase, the fiber connection running between parents must extend all the way to leaves and terminal fibers in order to complete each inter-branch connection. Also, intra-branch connections must be assigned with fibers within each branch. The following section describes the second phase.

## 4.2 Second phase: assignment within the branch

In this subsection, we discuss assigning to connections the terminal fibers and the fibers running between leaves of a branch and their parent. We denote by  $B_k$  an arbitrary branch;  $1 \leq k \leq N_b$ . We denote by  $\mathcal{R}_k$  the number of inter-branch connections whose one end is associated with branch  $B_k$ ; thus,

$$\mathcal{R}_k \equiv \sum_{l \neq k, 1 \leq l \leq N_b} R_{kl}$$

Recall that  $\rho_k$  denotes the number of intra-branch connections within branch  $B_k$ , both end of whom are associated with branch  $B_k$ . Thus, we have

$$\rho_k \equiv \sum_{i \leq j, i \in B_k, j \in B_k} r_{ij}$$

We now define some decision variables  $x_i, i \in L$  that are to be decided in the second phase;  $x_i$  is the number of inter-branch connections that uses terminal fibers from leaf  $i$  to contact their end devices. Note that we have assumed  $T_i \geq C_{ip}$  for each leaf  $i$  and its parent  $p$ .

### 4.2.1 Necessary condition for feasibility

In some cases the infeasibility of the requirement can be trivially decided. Obviously, there must be enough number of fibers running between leaves and their parent to take care of both inter-branch and intra-branch connections. Also, there must be enough number of terminal fibers.

#### Necessary condition

$$\begin{aligned} \sum_{i \in B_k} C_{ip} &\geq \mathcal{R}_k \\ \sum_{i \in B_k} T_i &\geq \mathcal{R}_k + 2\rho_k \end{aligned}$$

In this condition,  $p$  refers to the parent of leaves in  $B_k$ . For a requirement to be feasible, this condition must be met in each branch  $B_k$  of the tree  $G$ . Regarding the term  $2\rho_k$  in the second inequality, note that the two terminal fibers are needed in branch  $B_k$  in order to make one intra-branch connection.

### 4.2.2 Sufficient conditions for feasibility

In some cases determining the feasibility of the requirement and assigning fibers are rather simple. If the following condition is satisfied in each branch  $B_k$ , then the requirement is feasible.

**Sufficient condition 1**

$$\begin{aligned}\sum_{i \in B_k} C_{ip} &\geq \mathcal{R}_k \\ \sum_{i \in B_k} T_i &\geq \mathcal{R}_k + 2\rho_k + |B_k|\end{aligned}$$

When this sufficient condition is satisfied, we can assign fibers in the following way in each branch  $B_k$ .

- Step 1: For inter-branch connections, set  $x_i$  with integers for each  $i \in B_k$  such that  $x_i \leq C_{ip}$  and  $\sum_{i \in B_k} x_i = \mathcal{R}_k$ .
- Step 2: Do the following with each leaf's  $T_i - x_i$  terminal fibers left unassigned after Step 1. For each leaf  $i$  in  $B_k$ , repeat the procedure of pairing up two terminal fibers stemming from leaf  $i$ , connecting them at leaf  $i$ , and assigning that pair to an intra-branch connection; until there is only one or no terminal fiber from  $i$  left unassigned.

**Proof**

Step 1 takes care of all the inter-branch connections. For leaf  $i$  for which  $T_i - x_i$  is even, all  $T_i - x_i$  terminal fibers can be used to make  $(T_i - x_i)/2$  intra-branch connections by connecting two fibers at leaf  $i$ . For leaf  $i$  for which  $T_i - x_i$  is odd,  $T_i - x_i - 1$  terminal fibers can be used to make  $(T_i - x_i - 1)/2$  intra-branch connections by connecting two fibers at leaf  $i$ . Denote by  $o_k(x)$  the number of leaves in  $B_k$  for which  $T_i - x_i$  is odd. Then,  $\sum_{i \in B_k} (T_i - x_i)/2 - o_k(x)/2$  intra-branch connections within  $B_k$  can be made.

$$\begin{aligned}\sum_{i \in B_k} (T_i - x_i)/2 - o_k(x)/2 &= \sum_{i \in B_k} T_i/2 - \mathcal{R}_k/2 - o_k(x)/2 \\ &\geq \sum_{i \in B_k} T_i/2 - \mathcal{R}_k/2 - |B_k|/2 \\ &\geq \rho_k\end{aligned}\tag{1}$$

Inequality (1) is equivalent to the second inequality of Sufficient condition 1. Thus, the requirement can be satisfied. **Q.E.D.**

There is another sufficient condition for feasibility, under which the fiber assignment can be accomplished easily. If the following condition is satisfied in each branch  $B_k$ , then the requirement is feasible.

## Sufficient condition 2

$$\begin{aligned} \sum_{i \in B_k} (C_{ip} - 1) &\geq \mathcal{R}_k \\ \sum_{i \in B_k} T_i &\geq \mathcal{R}_k + 2\rho_k \end{aligned} \quad (2)$$

When this sufficient condition is satisfied, we can assign fibers in the following way in each branch  $B_k$ .

- Step 1: For inter-branch connections, arbitrarily pick integers  $x_i$  for each  $i \in B_k$  such that  $x_i \leq C_{ip} - 1$  and  $\sum_{i \in B_k} x_i = \mathcal{R}_k$ . (Then,  $\sum_{i \in B_k} T_i - \mathcal{R}_k$  terminal fibers are available for intra-branch connections. Also, at least one fiber in each edge  $(i, p)$  is available for the intra-branch connections.)
- Step 2: For each leaf  $i$  in  $B_k$ , repeat the procedure of pairing up two terminal fibers stemming from leaf  $i$ , connecting them at leaf  $i$ , and assigning that pair to an intra-branch connection; until there is only one or no terminal fiber from  $i$  left unassigned.  
If we do not make all the required intra-branch connections this way, go to Step 3. (In this case, each leaf  $i$  with odd  $T_i - x_i$  is left with one terminal fiber unassigned.)
- Step 3: Pair up leaves  $i, j$  with odd  $T_i - x_i$  and  $T_j - x_j$ . Serially connect one terminal fiber from leaf  $i$ , one fiber in edge  $(i, p)$ , one fiber in edge  $(j, p)$ , and one terminal fiber from leaf  $j$ . Assign these fibers to an intra-branch connection. Repeat this procedure with other leaves in  $B_k$

### Proof

Step 1 successfully assigns to all inter-branch connections the terminal fibers and fibers running between leaves and their parent. In Step 2 and Step 3, if  $(\sum_{i \in B_k} T_i) - \mathcal{R}_k$  is even,  $\{\sum_{i \in B_k} T_i - \mathcal{R}_k\}/2$  intra-branch connections can be made through branch  $k$  and possibly their parent  $p$ . If  $\sum_{i \in B_k} T_i - \mathcal{R}_k$  is odd,  $\{\sum_{i \in B_k} T_i - \mathcal{R}_k - 1\}/2$  intra-branch connections can be made. In either case, the number of intra-branch connections that can be made is at least  $\rho_k$  from Inequality (2). Therefore, the requirement is satisfied. **Q.E.D.**

### 4.2.3 Other conditions

Now, we need to discuss the case:

$$\begin{aligned} \mathcal{R}_k &= \sum_{i \in B_k} (C_{ip} - 1) + u, \quad 1 \leq u \leq |B_k| \\ \sum_{i \in B_k} T_i &= 2\rho_k + \mathcal{R}_k + v, \quad 0 \leq v \leq |B_k| - 1 \end{aligned} \quad (3)$$

In this case, we define  $\mathcal{E} \equiv \{i \in B_k \mid T_i - C_{ip} \text{ is even.}\}$ , and the following procedure determines the feasibility of the requirement. During the procedure, if all fibers in edge  $(i, p)$  are assigned, we will say that the edge is “saturated”. The procedure also provides the fiber assignment for feasible requirements.

**Algorithm 2**

- Step 1: For each leaf  $i \in B_k$ , do

$$x_i \leftarrow C_{ip} - 1$$

(In words, for each leaf  $i \in B_k$ , give  $C_{ip} - 1$  terminal fibers from leaf  $i$  and  $C_{ip} - 1$  fibers in edge  $(i, p)$  to  $C_{ip} - 1$  inter-branch connections. Each of these connections take one terminal fiber and one fiber in  $(i, p)$ . Note that there are still  $u$  more inter-branch connections to be assigned with such fibers after Step 1. Step 2 starts assigning the remaining  $u$  individual inter-branch connections.)

- Step 2: If  $u \leq |\mathcal{E}|$ , the requirement is feasible. Select any  $u$  leaves from  $\mathcal{E}$ . For each leaf  $i$  of these selected leaves, do

$$x_i \leftarrow x_i + 1$$

Go to Step 3 to do the assignment of intra-branch connections.

If  $u > |\mathcal{E}|$ , for each leaf  $i \in \mathcal{E}$  do

$$x_i \leftarrow x_i + 1$$

Go to Step 5.

- Step 3: For each leaf  $i$  in  $B_k$ , repeat the procedure of pairing up two terminal fibers stemming from leaf  $i$ , connecting them at leaf  $i$ , and assigning that pair to an intra-branch connection. Do this until there is only one or no terminal fiber from  $i$  left unassigned. If the intra-branch connections run out, terminate the fiber assignment in branch  $B_k$ .
- Step 4: Pair up leaves  $i, j$  with odd  $T_i - x_i$  and  $T_j - x_j$ . Serially connect one terminal fiber from leaf  $i$ , one fiber in edge  $(i, p)$ , one fiber in edge  $(j, p)$ , and one terminal fiber from leaf  $j$ . Assign these fibers to an intra-branch connection. Repeat this procedure with other leaves in  $B_k$ .  
Terminate the fiber assignment in branch  $B_k$ .
- Step 5: Assign the remaining  $u - |\mathcal{E}|$  individual inter-branch connections to fibers from distinctive leaves not in  $\mathcal{E}$ .
- Step 6: For each leaf  $i$  in  $B_k$ , repeat the procedure of pairing up two terminal fibers stemming from leaf  $i$ , connecting them at leaf  $i$ , and assigning that pair to an intra-branch connection. Do this until there is only one or no terminal fiber from  $i$  left unassigned.

If there is an intra-branch connection left unassigned with fibers, the requirement is infeasible.  
(This happens if  $v < u - |\mathcal{E}|$ .)

**Proof**

We first consider the case  $u \leq |\mathcal{E}|$ . After Step 2 each leaf  $i$  in  $\mathcal{E}$  such that  $x_i = C_{ip}$  has an even number of unassigned terminal fibers; namely,  $T_i - C_{ip}$ . Each leaf  $i$  not in  $\mathcal{E}$  also has an even number of unassigned fibers; namely,  $T_i - (C_{ip} - 1)$ . These fibers can be paired up within their associated individual leaves. Each leaf  $i$  in set  $\{i \in \mathcal{E} \mid x_i = C_{ip} - 1\}$  has an odd number of unassigned fibers, but it has one unassigned fiber in edge  $(i, p)$ . Step 4 utilizes terminal fibers stemming from these leaves, which are still unpaired after Step 3. If  $|\{i \in \mathcal{E} \mid x_i = C_{ip} - 1\}| = |\mathcal{E}| - u$  is even, the number of terminal fibers left unassigned after Step 2, which is  $\sum_{i \in B_k} T_i - \mathcal{R}_k$ , is even. These  $\sum_{i \in B_k} T_i - \mathcal{R}_k$  fibers can be used for intra-branch connections. Thus, in that case up to  $(1/2)(\sum_{i \in B_k} T_i - \mathcal{R}_k)$  intra-branch connections can be supported. From Eq. (3) we have

$$(1/2)(\sum_{i \in B_k} T_i - \mathcal{R}_k) \geq \rho_k,$$

so all the intra-branch connections are supported. If  $|\{i \in \mathcal{E} \mid x_i = C_{ip} - 1\}| = |\mathcal{E}| - u$  is odd,  $\sum_{i \in B_k} T_i - \mathcal{R}_k$  is odd, and up to  $(1/2)(\sum_{i \in B_k} T_i - \mathcal{R}_k - 1)$  intra-branch connections can be supported. Eq. (3) also implies

$$(1/2)(\sum_{i \in B_k} T_i - \mathcal{R}_k - 1) \geq \rho_k$$

(Note that in the case  $v = 0$ , Eq. (3) implies that  $\sum_{i \in B_k} T_i - \mathcal{R}_k$  is even.) Thus, all the intra-branch connections are supported.

Now we consider the case  $u > |\mathcal{E}|$ . In this case, after Step 5 each leaf  $i \in \mathcal{E}$  has an even number (namely,  $T_i - C_{ip}$ ) of terminal fibers still unassigned. Each leaf  $i$  in set  $\{i \notin \mathcal{E} \mid x_i = C_{ip} - 1\}$  has an even number (namely,  $T_i - C_{ip} + 1$ ) of terminal fibers still unassigned. All these terminal fibers can be used for intra-branch connections. Each leaf  $i$  in set  $\{i \notin \mathcal{E} \mid x_i = C_{ip}\}$  has an odd number (namely,  $T_i - C_{ip}$ ) of terminal fibers still unassigned after Step 5, and edge  $(i, p)$  is saturated for such leaf  $i$ . Therefore, one terminal fiber for each leaf  $i$  in  $\{i \notin \mathcal{E} \mid x_i = C_{ip}\}$  is unusable for assigning to an intra-branch connection. The total number of such unusable fibers is  $|\{i \notin \mathcal{E} \mid x_i = C_{ip}\}| = u - |\mathcal{E}|$ . Except for such unusable terminal fibers, all the terminal fibers still unassigned after Step 5 can be used for intra-branch connections. Therefore, if  $\sum_{i \in B_k} T_i - \mathcal{R}_k - (u - |\mathcal{E}|) \geq 2\rho_k$ , or equivalently  $v \geq u - |\mathcal{E}|$ , then the requirement is satisfied. Otherwise (if  $\sum_{i \in B_k} T_i - \mathcal{R}_k - (u - |\mathcal{E}|) < 2\rho_k$ , or equivalently  $v < u - |\mathcal{E}|$ ), the requirement cannot be satisfied. **Q.E.D.**

## 5 Discussion

For network planning, this report presents systematic procedures of determining whether the physical cabling plant is sufficient for the communication requirement of the facility. In mathematical formulation, we also gain simple insights that should guide the design of the physical plant; e.g. Section 2.1. Although the problem in this report was motivated by the use of fibers in network planning, the algorithms and principles presented in this report may be applied to the assignment of any communication media (e.g., channels of time division multiplexing or frequency division multiplexing in point-to-point links) constituting the hierarchical tree structure.

## References

- [1] Electronic Industries Association. Commercial building telecommunications wiring standard. Revision A, October 1995.