

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.				
1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE 24 Dec 97		3. REPORT TYPE AND DATES COVERED
4. TITLE AND SUBTITLE ENDOGENOUS EFFORT, PROMOTION, RETIREMENT AND HETEROGENEOUS ABILITY IN A LABOR HIERARCHY			5. FUNDING NUMBERS	
6. AUTHOR(S) GERALD EDWIN SOHAN				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Texas A&M University			8. PERFORMING ORGANIZATION REPORT NUMBER 97-038D	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) THE DEPARTMENT OF THE AIR FORCE AFIT/CI, BLDG 125 2950 P STREET WPAFB OH 45433			10. SPONSORING/MONITORING AGENCY REPORT NUMBER	
11. SUPPLEMENTARY NOTES				
12a. DISTRIBUTION AVAILABILITY STATEMENT Unlimited Distribution In Accordance With AFI 35-205/AFIT Sup 1			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words)				
DTIC QUALITY INSPECTED 4				
14. SUBJECT TERMS			15. NUMBER OF PAGES 124	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT	

**ENDOGENOUS EFFORT, PROMOTION, RETIREMENT AND
HETEROGENEOUS ABILITY IN A LABOR HIERARCHY**

A Dissertation

by

GERALD EDWIN SOHAN

Major, United States Air Force

Texas A&M University
DOCTOR OF PHILOSOPHY

1997

Major Subject: Economics

124 pages

19980106 053

ABSTRACT

This work simultaneously attacks the questions of jointly endogenous effort, retirement, and promotions. Workers endowed with heterogeneous ability are employed within a firm's hierarchical promotion structure. Analyzing the situation step-by-step, worker retirement, consumption and effort decisions are examined first. Using savings, laborers store earnings for future use and eventually choose to retire, continuing to consume earnings from accumulated savings. In retirement, workers avoid disutility from working. Utility of laborers improves discretely upon retirement. Worker consumption tends to be constant over time. Effort during employment is efficient and independent of worker ability. Workers cannot be ordered by ability using observed consumption, effort or retirement. Next, the nature of hierarchical firms is examined. When manager decisions affect lower level worker productivity, firms desire higher ability workers as managers. A given worker prefers to work as a manager versus an underling. Firms must devise a promotion system to identify able employees for advancement since workers do not self-select. Promotion opportunities automatically create tournament situations. With promotion tournaments, effort is unevenly distributed across worker ability. Workers nearest the promotion cutoff work hardest. Without collusion, smaller promotion contest groups result in higher overall worker effort. Combining the two previous situations, worker effort, consumption and retirement in a promotion hierarchy is examined. Giving promoted workers residual claimancy on the firm can change the retirement decision, causing inefficiently early retirement. Retirement decisions of higher level workers create

externalities for lower level workers, thus affecting firm productivity. Firms have incentives to modify upper level workers behavior through varying retirement structure. Only with full knowledge of worker utility can firms completely correct for incentive distortions caused by hierarchies. Finally, the results of this analysis are applied to a military promotion system model. Care must be used in military force reductions to preclude adverse incentive affects. Some actions of the military during the recent force drawdown appear very effective while others appear ill-advised.

REFERENCES

- Asch, Beth J. and John T. Warner.** *A Theory of Military Compensation and Personnel Policy*, Santa Monica, CA: RAND, MR-439-OSD 1994.
- Becker, Gary S.** *Human Capital: A Theoretical and Empirical Analysis, With Special Reference to Education*. 3d ed. Chicago: University of Chicago Press, 1992.
- Ben-Porath, Yoram.** "The Production of Human Capital and the Life Cycle of Earnings," *Journal of Political Economy*, (August 1967), 75(2), pp. 352-65.
- Bull, Clive, Andrew Schotter, and Keith Weingelt.** "Tournaments and Piece Rates: An Experimental Study," *Journal of Political Economy*, (February 1987), 95(1), pp. 1-33.
- Burrelli, David F. and Elizabeth A. Dunstan.** "Military Retiree Health Care: Base Closures and Realignment," *Congressional Research Service*, adapted for "Base Closure and Retiree Health Care," *Air Force Magazine*, July 1995, pp. 74-79.
- Callander, Bruce D.** "A New Shot at the Promotion System," *Air Force Magazine*, July 1995, pp. 70-72.
- Campbell, J.B.** *The Emperor and the Roman Army: 31 BC - AD 235*. Oxford, UK: Oxford University Press, 1984.
- Demski, Joel S., and Gerald A. Feltham.** "Economic Incentives in Budgetary Control Systems," *Accounting Review*, (April 1978), 53(2), pp. 336-359.
- Dye, Ronald A.** "The Trouble with Tournaments," *Economic Inquiry*, (January 1984), 22(1), pp. 147-149.
- Ehrenberg, Ronald G., and Michael Bognanno.** "Do Tournaments Have Incentive Effects?" *Journal of Political Economy*, (December 1990), 98(6), pp. 1307-1324.
- Ellwood, David T.** "Pensions and the Labor Market: A Starting Point (The Mouse Can Roar)" with comment by Zvi Bodie in David A. Wise. ed., *Pensions, Labor, and Individual Choice*. Chicago: University of Chicago Press, 1985 pp. 19-54.
- Frederickson, James R.** "Relative Performance Information: The Effects of Common Uncertainty and Contract Type on Agent Effort," *Accounting Review*, (October 1992), 67(4), pp. 647-669.

Fullerton, Richard L. and R. Preston McAfee. "Auctioning Entry Into Tournaments," *Journal of Political Economy*, forthcoming (1997).

Gotz, Glenn and John J. McCall. *A Dynamic Retention Model for Air Force Officers: Theory and Estimates*, Santa Monica, CA: RAND, R-3028-AF, 1984.

Green, Jerry R., and Nancy L. Stokey. "A Comparison of Tournaments and Contracts," *Journal of Political Economy*, (June 1983), 91(3), pp. 349-364.

Holmstrom, Bengt. "Moral Hazard in Teams," *Bell Journal of Economics*, (Autumn 1982), 13(3), pp. 324-340.

Jovanovic, Boyan. "Job Matching and the Theory of Labor Turnover," *Journal of Political Economy*, (October 1979), 87(5), pp. 972-990.

_____. "Matching, Turnover, and Unemployment," *Journal of Political Economy*, (February 1984), 92(1), pp. 108-122.

Klein, Benjamin, Robert G. Crawford, and Armen A. Alchian. "Vertical Integration, Appropriable Rents, and the Competitive Contracting Process," *Journal of Law and Economics*, (October 1978), 21(3), pp. 297-326.

Lazear, Edward P. "Why is There Mandatory Retirement?" *Journal of Political Economy*, (December 1979), 87(6), pp. 1261-1284.

_____. "Pensions as Severance Pay," with comment by David A. Wise in Zvi Bodie and John S. Shoven, eds., *Financial Aspects of the United States Pension System*. Chicago: University of Chicago Press, 1984, pp. 57-89.

_____. "Incentive Effects of Pensions," with comment by Roger H. Gordon in David A. Wise, ed., *Pensions, Labor, and Individual Choice*. Chicago: University of Chicago Press, 1985, pp. 253-282.

_____. "Social Security and Pensions," *Research in Labor Economics*, (1986), 7, pp. 1-30.

Lazear, Edward P., and Sherwin Rosen. "Rank-Order Tournaments as Optimum Labor Contracts," *Journal of Political Economy*, (October 1981), 89(5), pp. 841-864.

McAfee, R. Preston, and John McMillan. "Auctions and Bidding," *Journal of Economic Literature*, (June 1987), 25(3), pp. 699-737.

- McLaughlin, Kenneth J.** "Aspects of Tournament Models: A Survey," *Research in Labor Economics*, (1988), 9, pp. 225-255.
- Mortensen, Dale T.** "Specific Capital and Labor Turnover," *Bell Journal of Economics*, Summer 1978, 9(3), pp. 572-586.
- Murphy, Kevin M., and Finis Welch.** "Empirical Age-Earnings Profiles," *Journal of Labor Economics*, (April 1992), 8(2), pp. 202-229.
- Nalebuff, Barry J., and Joseph E. Stiglitz.** "Prizes and Incentives: Towards a General Theory of Compensation and Competition," *Bell Journal of Economics*, Spring 1983, 14(1), 21-43.
- Nalebuff, Barry J., and Richard J. Zeckhauser.** "Pensions and the Retirement Decision," in David A. Wise, ed., *Pensions, Labor, and Individual Choice*. Chicago: University of Chicago Press, 1985, pp. 283-316.
- O'Flaherty, Brendan, and Aloysius Siow.** "Up-or-Out Rules in the Market for Lawyers," *Journal of Labor Economics*, October 1995, 13(4), pp. 709-735.
- O'Keefe, Mary, W. Kip Viscusi, and Richard J. Zeckhauser.** "Economic Contests: Comparative Reward Schemes," *Journal of Labor Economics*, April 1984, 2(1), pp. 27-56.
- Posner, Richard A.** *Aging and Old Age*. Chicago: University of Chicago Press, 1995.
- Pratt, John W.** "Risk Aversion in the Small and in the Large, reprinted from *Econometrica* January-April 1964, 32(1-2) in Peter Diamond and Michael Rothschild, eds., *Uncertainty in Economics: Readings and Exercises*. San Diego: Academic Press, Inc., 1989, pp. 61-75.
- Rosen, Sherwin.** "Authority, Control, and the Distribution of Earnings," *Bell Journal of Economics*, Autumn 1982, 13(3), pp. 311-323.
- _____. "Prizes and Incentives in Elimination Tournaments," *American Economic Review*, September 1986, 76(4), pp. 701-715.
- _____. "The Military as an Internal Labor Market: Some Allocation, Productivity, and Incentive Problems," *Social Science Quarterly*, June 1992, 73(2), pp. 227-237.

Stern, Steven. "Promotion and Optimal Retirement," *Journal of Labor Economics*, October 1987, 5(4, part 2), pp. S107-S123.

_____. "Ability, Promotion, and Optimal Retirement," *Journal of Labor Economics*, January 1994, 12(1), pp. 119-137.

United States Code Annotated. *Title 10, Armed Forces, 1-3000*. St. Paul, MN: West Publishing Co., 1983, with supplementary pamphlet from 1997.

United States General Accounting Office. "Military Personnel: High Aggregate Personnel Levels Maintained Throughout Drawdown," Report to the Chairman, Subcommittee on National Security, House of Representatives, GAO/NSIAD-95-97, June 1995.

_____. "Force Structure: Potential Exists to Further Reduce Active Air Force Personnel," GAO/NSIAD-97-78, March 1997.

Williamson, Oliver E. "Credible Commitments: Using Hostages to Support Exchange," *American Economic Review*, September 1983, 73(4), pp. 519-540.

**ENDOGENOUS EFFORT, PROMOTION, RETIREMENT AND
HETEROGENEOUS ABILITY IN A LABOR HIERARCHY**

A Dissertation

by

GERALD EDWIN SOHAN

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

December 1997

Major Subject: Economics

**ENDOGENOUS EFFORT, PROMOTION, RETIREMENT AND
HETEROGENEOUS ABILITY IN A LABOR HIERARCHY**

A Dissertation

by

GERALD EDWIN SOHAN

Submitted to Texas A&M University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Approved as to style and content by:

Finis Welch
(Chair of Committee)

Thomas E. Wehrly
(Member)

Donald R. Deere
(Member)

Dennis W. Jansen
(Head of Department)

Curtis R. Taylor
(Member)

December 1997

Major Subject: Economics

ABSTRACT

Endogenous Effort, Promotion, Retirement and Heterogeneous Ability in a Labor

Hierarchy. (December 1997)

Gerald E. Sohan, B.S., United States Air Force Academy;

M.S., Texas A&M University

Chair of Advisory Committee: Dr. Finis Welch

This work simultaneously attacks the questions of jointly endogenous effort, retirement, and promotions. Workers endowed with heterogeneous ability are employed within a firm's hierarchical promotion structure. Analyzing the situation step-by-step, worker retirement, consumption and effort decisions are examined first. Using savings, laborers store earnings for future use and eventually choose to retire, continuing to consume earnings from accumulated savings. In retirement, workers avoid disutility from working. Utility of laborers improves discretely upon retirement. Worker consumption tends to be constant over time. Effort during employment is efficient and independent of worker ability. Workers cannot be ordered by ability using observed consumption, effort or retirement. Next, the nature of hierarchical firms is examined. When manager decisions affect lower level worker productivity, firms desire higher ability workers as managers. A given worker prefers to work as a manager versus an underling. Firms must devise a promotion system to identify able employees for advancement since workers do not self-select. Promotion opportunities automatically create tournament situations. With promotion tournaments, effort is unevenly distributed across worker ability. Workers

nearest the promotion cutoff work hardest. Without collusion, smaller promotion contest groups result in higher overall worker effort. Combining the two previous situations, worker effort, consumption and retirement in a promotion hierarchy is examined. Giving promoted workers residual claimancy on the firm can change the retirement decision, causing inefficiently early retirement. Retirement decisions of higher level workers create externalities for lower level workers, thus affecting firm productivity. Firms have incentives to modify upper level workers behavior through varying retirement structure. Only with full knowledge of worker utility can firms completely correct for incentive distortions caused by hierarchies. Finally, the results of this analysis are applied to a military promotion system model. Care must be used in military force reductions to preclude adverse incentive affects. Some actions of the military during the recent force drawdown appear very effective while others appear ill-advised.

DEDICATION

To Donna and Jamie.

ACKNOWLEDGMENTS

Thanks go to my committee for all their help. Thanks to my fellow graduate students at Texas A&M for their empathy and help. Thanks, also, to the people of the Department of Economics and Geography at the United States Air Force Academy for allowing me this opportunity and their continued confidence in me. In particular, thanks to Colonel Raymond E. "Chip" Franck, Jr., Lieutenant Colonel Bruce Linster and Majors John Stallings and Rich Fullerton. They jointly served as both role models and guides.

Specials thanks go to my daughter Jamie for her patience while Dad was otherwise occupied. Her maturity never ceases to amaze me.

Most of all, thanks go to my wife, Donna Elizabeth Sohan. Thank you for supporting me, listening to me and for your unwavering faith in me. I love you.

TABLE OF CONTENTS

	Page
ABSTRACT.....	iii
DEDICATION.....	v
ACKNOWLEDGMENTS.....	vi
TABLE OF CONTENTS.....	vii
CHAPTER	
I INTRODUCTION.....	1
Retirement and Effort.....	1
Previous Work in This Area.....	2
Why and When to Retire.....	2
Promotions as Tournaments.....	4
Promotion and Retirement.....	5
A Generalized Look at Retirement.....	7
Heterogeneous Ability, Endogenous Effort and Retirement.....	7
Promotion and Endogenous Effort.....	9
Endogenous Retirement and Effort in a Hierarchical Setting.....	10
Effort and Promotion in an Internal Labor Market.....	11
II RETIREMENT AND HETEROGENEOUS ABILITY.....	12
Introduction.....	12
A Model of Temporary Discontent.....	14
Implications of the Model.....	29
Conclusion.....	33
III HETEROGENEOUS ABILITY IN HIERARCHICAL LABOR PROMOTIONS.....	36
Introduction.....	36
A Brief Look at Firm Structure.....	39
A Model of Hierarchical Promotions.....	50
Conclusion.....	56

CHAPTER	Page
IV EFFORT, PROMOTION AND RETIREMENT.....	59
Introduction.....	59
A Model of Promotion, Retirement and Effort.....	61
Conclusion.....	71
V PROMOTION AND RETIREMENT IN AN INTERNAL LABOR MARKET: INCENTIVES FOR AIR FORCE OFFICERS.....	73
Introduction.....	73
Air Force Officer Promotion and Retirement System.....	75
Promotion.....	76
Retirement.....	78
Recent Developments.....	79
A Simple Model of the Military Promotion System.....	81
Applications to Military Force Reductions.....	96
Conclusion.....	101
VI CONCLUSION.....	103
Summary.....	103
Retirement and Heterogeneous Ability.....	103
Heterogeneous Ability in Hierarchical Labor Promotions.....	105
Effort Promotion and Retirement.....	108
A Model of the Military Internal Labor Market.....	109
REFERENCES.....	112
APPENDIX 1.....	116
APPENDIX 2.....	119
VITA.....	124

CHAPTER I

INTRODUCTION

RETIREMENT AND EFFORT

Worker choice of retirement and effort is an enigma. Why would productive workers remove themselves completely from the labor market when they are still capable of positive marginal product and, thus, of increasing their level of consumption? What role do firms play in this? What role does worker effort play? Do workers only retire when they are not self-employed? Or, does retirement occur for all workers? How do firms structure retirement? Does this differ from the manner in which laborers would act if left to their own devices? Is retirement efficient? What purpose is served by mandatory retirement? How do promotions and the structure of the firm affect the individual retirement and effort decision? Since the vast majority of U.S. residents are covered by some sort of retirement plan, these questions are of great importance in today's society.

In this dissertation, I try to shed some light on these issues. Much work has been done in this area, but there are quite a few areas which have been left unexplored or underexplored in the literature. In particular, the combination of factors mentioned in the title are candidates for further examination. No source that I could uncover has simultaneously attacked the questions of endogenous effort, retirement, and promotion choice. Nor has this been further combined with heterogeneous ability.

There is a reason for this omission. The combination of factors is extraordinarily complicated and it was my goal to simplify the analysis as much as possible while retaining

the gist of the questions I wished to address. In order to accomplish this, I first had to examine what has gone before.

PREVIOUS WORK IN THIS AREA

Why and When to Retire

The bulk of the literature views pensions as an alternate source of wages in contracts between firms and laborers. The intertemporal nature of pensions provides an alternative means with which to control worker behavior. While formalized private pensions have only existed since the 19th century,¹ public sector pensions and, in particular, military pensions have been around much longer. For instance, in ancient Rome, the older surviving soldiers were given defined benefit pensions upon reaching a certain age and rank.²

The firm or other entity which sponsors the pension may only do so up to the point where the workers are at least as well off as they would be without the pension. Nalebuff and Zeckhauser (1985) describe seven reasons why individuals would retire: decreasing productivity, increasing disutility of effort, outside employment, pension and other incentives, health information, labor indivisibility, and declining marginal utility of consumption. Each of these have been examined in economic literature.

It has been observed that measured earnings tend to increase over a lifetime up to a point and subsequently decrease.³ This pattern of earnings has driven the literature,

¹ See Nalebuff and Zeckhauser (1985).

² See Campbell (1984).

³ See, for example, Murphy and Welch (1990).

since any reasonable model must be able to explain this phenomenon.⁴ The standard reasoning is that retirement is due to changes in productivity, effort, or utility. These explanations are all essentially the same. Workers no longer value the results of employment as highly in later years and therefore choose to work more during their earlier years when the relative return is highest. Lazear (1979) added another possible dimension by assuming worker characteristics are constant, but wages are increasing. Thus, worker marginal productivity per dollar decreases over time, necessitating mandatory retirement or recontracting. This is one of many ways in which the structure of pensions can affect retirement behavior.

In a way, examining the structure of pensions to determine retirement behavior is putting the cart before the horse. Since a competitive firm facing a competitive labor market cannot introduce a retirement system which makes workers worse off than without it, the incentive to retire must be a causal factor in the structure of the pension system. However, once the incentive to retire has been demonstrated, it is important to examine ways in which firms may take advantage of the situation in their maximization of profits. For instance, Becker (1992) posits that specific human capital creates an incentive for workers or firms or both to cheat. Klein, Crawford, and Alchian (1978) call this cheating the appropriation of quasi-rents. Implicit in their contract enforcement mechanism is the idea of finite length employment contracts. Hence, contract theory assumes the existence of labor retirement in the area of contract enforcement. This is present throughout the literature.⁵ It does give a plausible explanation for the use of pensions by firms as a

⁴See Ben-Porath (1967) for an excellent discussion of the life cycle of earnings.

⁵See, for example, Williamson (1983). He uses "hostages" to ensure contract compliance. However,

contract enforcing device.

Many other factors affect pensions. Some of these include mandatory retirement,⁶ severance pay,⁷ government programs and social security⁸, and the firm's promotion system. To the extent that retirement is influenced by outside employment, most search literature also applies to retirement, since quitting a firm and retiring from a firm are no different from the firm's perspective.⁹

Promotions as Tournaments

Most firms have a hierarchical structure with some kind of promotion system. How the promotion system affects worker effort greatly impacts the efficiency of the firm. Lazear and Rosen (1981) examine the case of tournaments where workers' effort adds to their ability endowment. They showed that tournaments can produce the efficient level of effort. This helped spawn a flurry of literature. Nalebuff and Stiglitz (1983) examine the case of output being the product of effort and ability (multiplicative effort). Holmstrom (1982) shows that the addition of a tournament structure when ability is already known adds no new information to a system. Thus, a tournament only has value if it is used to extract information. This would seem to be a reasonable use for tournaments in a setting of heterogeneous workers.

implicit in this idea is the return of the hostage at the end of the contract. Pensions can thus be used as a manner for taking wages hostage until fulfillment of the labor contract, thus allowing firms to secure their investment in specific capital.

⁶Lazear (1979) examines mandatory retirement.

⁷See Lazear (1984).

⁸See Lazear (1986).

⁹See, for example, Mortensen (1978).

For the most part, heterogeneity has been dealt with by using handicapping, or leveling the playing field.¹⁰ Additionally, Rosen (1986) shows that, as long as the higher ability individual has a greater value of continuing, tournaments will select the best players. However, unless the prize structure is weakly increasing in the tournament level (prizes increase more than linearly at the top), effort decreases at the last level of the tournament with linear effort. O'Keefe, Viscusi, and Zeckhauser (1984) show that, with risk neutral workers, effort is increasing in ability. Thus, the higher ability individuals will be chosen. Interestingly, Demski and Feltham (1978) show that risk aversion is a necessary condition for a contract which compares a worker's output to a fixed standard to be Pareto improving. The tournament literature is similar to auctions,¹¹ although in the case of tournaments, the losers all pay in the form of *ex ante* effort.

The results of tournament theory have been examined empirically¹² and experimentally¹³ and much of the theory has been vindicated. It is interesting to note, however, that the theory and empirics have all dealt with the effort of individuals prior to the awarding of the prize. If the firm uses the tournament to award a promotion, they are additionally interested in the amount of effort promotees expend once the promotion has taken place.

Promotion and Retirement

Promotions by ability are also observed in a non-tournament setting.

¹⁰See Green and Stokey (1983).

¹¹See McAfee and McMillan (1987).

¹²See Ehrenberg and Bognanno (1990).

¹³See Bull, Schotter, and Weigelt (1987) and Frederickson (1992).

Unfortunately, these require some further assumptions. For example, Jovanovic (1979 and 1984) assumes that workers can only influence the firm's productivity through quitting. Once they are working, their output is determined by their ability and a random term only. Thus, even though the firm incompletely measures output, the individual can do nothing to influence the measurement. Not surprisingly, he finds that turnover decreases with tenure.

In a keen bit of insight, Stern (1987) determines that, if position within a firm affects productivity, then workers retire too late if left to their own devices. In Stern (1994), this is expanded by allowing for heterogeneous ability. However, he assumes perfect information on the part of the firm in measuring worker ability. In addition, he does not take into account the incentive effects of promotions on effort.

In examining the incentive effects of pensions, Lazear (1985) uses additive effort to examine the incentive effects of pensions. He determines that distortions caused by the pension can always be undone by a judiciously chosen wage function under risk neutrality. Defined benefit plans, which depend only upon years of service and not salary, are efficient. Even though workers may not explicitly take into account that wages are reduced by the amount of the pension, they do it implicitly. However conventional plans, which depend on an average salary and tenure, do not result in efficiency. He also discusses vesting, which results in subsidization of the stayers by the leavers, causing a distortion. Unfortunately, although he examines effort, he does not include promotions.

This is the pattern in the literature. The separate topics have been covered extensively, but the combination of endogenous effort, retirement, promotion, and

heterogeneous ability has not been examined, to my knowledge. Yet, all these factors occur contemporaneously within firms and have been shown to interact when subsets are present simultaneously. This omission has not been ignored in the literature. Rosen (1992) mentions all these issues when describing the military, yet he develops no model¹⁴ in this paper. This then is an area in which an explanation is needed but has of yet not been forthcoming and at which I direct this dissertation.

A GENERALIZED LOOK AT RETIREMENT

Heterogeneous Ability, Endogenous Effort, and Retirement

Some people live to work. Others work to live. When considering the latter case, workers would obviously prefer to not work, yet they must work at some point in order to provide for their continued existence. Unless individuals are able to accumulate wealth or capital in some way, they must live hand-to-mouth, consuming as they produce. It is also the case that workers experience some disutility in working even prior to exerting productive effort. Merely showing up to work and having the opportunity to exert effort causes some fixed cost. A significant portion of time is spent allocated to an activity that the worker finds relatively distasteful, a rather Dickensian situation. With the presence of some sort of savings, laborers can store value for future use and, thus, may be able to cease working while continuing to consume. Workers may thus retire and discontinue paying the fixed cost of showing up to work. Thus the utility of the laborer changes discretely upon retirement.

¹⁴Rosen (1982) contains an interesting time allocation hierarchical model, but it, too, does not deal with retirement or endogenous effort.

In the second chapter of this dissertation, I show that, without firm-sponsored pensions, individuals would continue to operate in an individual-sponsored pension environment. In other words, they would save for retirement. Thus, the correct comparison is not between pension and no pension, but between firm-sponsored pensions and private savings. If firms provide pensions, they must make the individual no worse off than with private pensions. Otherwise, workers will not agree to work for a firm providing inferior pensions. The incentive to retire is the activity that the firm chooses to exploit by creating its own pension plan. The correct comparison is, therefore, not between non-retirement and firm-sponsored retirement, but between firm- and individual-sponsored retirement. In order for the worker to choose a firm's pension versus individual saving for retirement, the worker must be at least as well off in the firm's pension system as by individually saving for retirement. In this context, I then discuss some possible implications of firm-sponsored pensions, as well as the budgeting process for pensions which the firm may use.

When laborers find working distasteful or effort costly, they have an incentive to work early in life and save for retirement. What the worker loses in consumption by not working during the latter years of life is made up for by the fact that effort no longer needs to be expended in order to consume. When workers are self-employed or are exactly paid their marginal products and choose effort based upon their constant, exogenous, heterogeneous ability, consumption and time to retirement cannot be ordered by ability unless additional assumptions are made limiting the degree of risk aversion. Effort is independent of ability, since the level of effort chosen by the worker is driven by the

structure of the disutility. This model will use a dynamic setting with either infinitely lived individuals or, equivalently, individuals with a constant per-period probability of dying. In this model, ability is constant throughout worker life.

Promotion and Endogenous Effort

There must be someone in charge of a firm. A leader or manager provides direction. He is like the tip of a spear, giving it direction. The directions given by the upper level individuals in a firm increase the productivity of those in lower levels. Selection of a leader is thus extremely important. The better the leader, the more effective the firm at maximizing profits. Thus, the firm attempts to use the available information to choose the best individual for the highest position. However, when workers' actions affect the information available to the firm, promotion opportunities automatically create a tournament. Dye (1984) raises several objections to the use of tournaments as incentive devices. While he is essentially correct in pointing out the problems, my point is that tournaments cannot be avoided. If a promotion is beneficial to individuals they will modify their behavior accordingly, regardless of the effect on efficiency.

In the third chapter I model a firm with a labor force which is heterogeneous in ability and in which individuals determine their multiplicative effort endogenously based upon the wage structure and their privately known ability. The reason for using multiplicative effort is its intuitive nature. It seems reasonable to believe that output is zero when there is no effort. This does not occur with additive effort.

I then assess the effect of ability on effort and the ability of the firm to determine

the most able worker. The firm prefers to fill upper level positions with the most able workers. However, all individuals would prefer to be in the higher position rather than the lower. Also, the distribution of effort is unevenly distributed across worker ability. Those who are most likely to be near the promotion cutoff tend to work the hardest. In addition, I discuss the relative efficiency of the tournament structure, as unavoidable as it may be. It turns out that the smaller the group in which the competition is held, the higher the overall worker effort when workers are unable to directly observe each others' ability. This portion of the analysis is conducted in a two-period static setting.

Endogenous Retirement and Effort in a Hierarchical Setting

The time an individual chooses for retirement affects not only her own utility, but also that of the others in her organization, even if there is no correlation between workers' productivity. The choice of promotion rewards affects the retirement decisions of the promoted individuals. In order to induce a worker to fully internalize the total results of an upper level worker's decision, the worker must be made a residual claimant.

In the fourth chapter, I show that giving a promoted worker a residual claimancy in the firm can change the retirement decision and cause the worker to retire too early. By choosing to retire earlier or later, a worker in a higher position also changes the promotion probability of those in positions below. The higher level departure opens up a new slot for promotion. As the arrival rate of promotion opportunity varies, so does the incentive to exert effort. Thus, the upper level retirement decision creates an externality for the lower level workers which effects the productivity of the firm. The firm thus has an incentive to

modify the behavior of the upper level workers by varying the retirement structure. Only by knowing the upper level worker's degree of risk aversion, however, will the firm be able to completely correct for any incentive distortions.

Effort and Promotion in an Internal Labor Market

In the fifth chapter of the dissertation, I combine the results of the chapters II, III and IV and examine the effects of the hierarchical promotion system on the retirement and effort decisions of workers in an internal labor market. I create a situation which corresponds fairly closely to that of the military. I first describe the military compensation scheme in general and that of the Air Force in particular. Then I create a simplified model of the military system using the results of the previous chapters. I then use the situation of promotion of Air Force officers as an example. In particular, I examine implications of the recent military drawdown and their effects on the incentive structure in the military.

CHAPTER II

RETIREMENT AND HETEROGENEOUS ABILITY

INTRODUCTION

Pensions serve as an alternate source of wages in contracts between firms and laborers. The intertemporal nature of pensions provides an alternative means with which to control worker behavior. As mentioned in Chapter I, Nalebuff and Zeckhauser (1985) discuss seven possible reasons for retirement. Six of these reasons have something to do with the individual or the environment changing over time. The other reason is that labor is indivisible and work must be accomplished in discrete units. While all of these do provide explanations for retirement behavior, I propose to show that retirement would occur in the absence of any of these factors.

In particular, I will show that individuals with constant utility functions, constant productivity potential, infinite planning horizons with a constant knowledge about health, and constant outside opportunities may still save and retire. This occurs even with continuous labor divisibility and even without the presence of firm-sponsored pensions. Without firm-sponsored pensions, individuals would continue to operate in an individual-sponsored pension environment.

This incentive to retire is the activity that firms may choose to exploit by creating their own pension plans. However, retirement would still occur in the absence of firm-sponsored pensions. Hence, if one is examining the relative efficiency of firm-sponsored pensions, it should be with respect to individual retirement behavior as opposed to a no-retirement situation. In order for the worker to choose a firm's pension versus individual

saving for retirement, the worker must be at least as well off in the firm's pension system as by individually saving for retirement.

Therefore, it is important to understand the nature of retirement at the individual level in order to examine some of the options available to firms for controlling worker behavior. Inasmuch as individual retirement planning provides a boundary for firm-worker interaction, firm-sponsored pension contracts are limited in the extent to which they may manipulate worker behavior. The thrust of this chapter, therefore, will be to analyze worker retirement behavior and examine some of the implications it has on the firm.

When laborers find working distasteful or effort costly, they have an incentive to work early in life and save for retirement. What the worker loses in consumption by not working during the latter years of life is made up for by the fact that effort no longer needs to be expended in order to consume. When workers are self-employed or are exactly paid their marginal products and choose effort based upon their constant, exogenous, heterogeneous ability, consumption is increasing in ability while effort is independent of ability. Time to retirement decreases with ability for workers with low relative risk aversion but increases for workers who are relatively risk averse. Firms are thus unable to sort workers by ability using time to retirement as a gauge.

Employment has certain binary characteristics. The nature of consumption changes discretely when an individual retires. While employed, a significant portion of a laborer's time is spent exerting effort in the pursuit of remuneration. This cost occurs prior to the exertion of any effort. It is the price paid for the opportunity to earn a wage, a fixed cost per period of remaining employed. It is similar to a worker renting the capital

used in production and becoming the residual claimant on the results of the use of the capital in production. If no effort occurs, nothing is produced, yet the worker is still responsible for paying the rental cost. It does not, however, have to be an explicit fee. There is a cost to the worker of being required just to show up, even though payment does not occur unless production occurs. If the fixed portion of disutility is allowed to be heterogeneously distributed throughout the labor force, effort is increasing in disutility and time to retirement is decreasing.

In the following section, I will develop a model which demonstrates all of these characteristics. Following that, I will examine some implications of this model. Then, I will conclude with a brief summary.

A MODEL OF TEMPORARY DISCONTENT

Consider the case of workers who are endowed with heterogeneous ability $a_i \in (0,1)$, where i is the index for individuals. Each individual has an infinite planning horizon with an internal discount rate of ρ . This return reflects the value of time and any perceived hazard of dying. ρ is assumed constant across individuals and over time. Each individual may also invest at a real rate of return r .

The analysis is conducted using continuous time. However, the conclusions are robust when evaluated using a discrete time model. At each point in time individuals have the choice of being employed or not. If they are employed, they simultaneously and endogenously choose the non-negative quantities or flows of effort, $z_i(t)$, and consumption, $c_i(t)$ for every time t . The price of output is normalized to one. The labor

market is assumed to offer no market power for the firm, so each worker is paid exactly her own marginal product flow. This assumption uncomplicates the analysis, since individuals now behave as though they are self-employed. One can think of the situation as a case in which workers rent capital from the firm at a fixed per period cost and become the residual claimants on any resulting production. Thus, individual efficiency corresponds to both production and consumption efficiency. Output is $Y_i(t)$, defined by the following equation.

$$(1) \quad Y_i(t) = a_i z_i(t).$$

The use of multiplicative effort in equation (1) seems intuitively reasonable, since zero output corresponds with the lack of effort. However, other studies¹⁵ have used additive effort. The additive aspect seems more akin to ability augmentation or investment in human capital than what is commonly meant by effort in the common vernacular. Other papers¹⁶ seem to support my view.

I assume that individuals have a utility function which is additively separable in consumption and effort. This ensures no income effect. The separated portions have the properties which ensure convex flow utility $U_i(t)$.

$$(2) \quad U_i(t) = u(c_i(t)) - d(z_i(t)).$$

¹⁵See Lazear and Rosen (1981), Lazear (1985), and Rosen (1986).

¹⁶See Nalebuff and Stiglitz (1983), O'Keefe, Viscusi, and Zeckhauser (1984), and McLaughlin (1988).

Both the utility portion of equation (2), $u(\cdot)$, and the disutility portion, $d(\cdot)$, have positive first derivatives, while $u(\cdot)$ has a negative and $d(\cdot)$ a positive second derivative.

I begin the analysis by assuming that the worker will choose to retire at some point, which turns out to be correct. Once the retirement decision has been made, the workers no longer exert effort. This simplifies the analysis. The workers simply choose $c_i(t)$ to be drawn from the accumulated savings, $K_i(t)$. The savings accumulates according to the state equation (3). The choice variable $z(t)$ is defined to be zero during retirement.

$$(3) \quad K'(t) = rK(t) + az(t) - c(t).$$

For a given ability endowment, a worker's choices are independent of other workers' actions. Therefore, I will be examining an individual worker. Since I will be examining an individual worker, the i subscripts will be dropped. The worker chooses $c(t)$ for each time period based upon the value function. R designates the worker's time from the beginning of employment until retirement and T designates the time at which the value function is evaluated.

$$(4) \quad V(T) = \max_{c,z,R} \left\{ \left(\int_T^R (u(c(t)) - d(z(t))) e^{-\rho t} dt \right) + \left(\int_R^\infty u(c(t)) e^{-\rho t} dt \right) \right\}.$$

The flow of consumption is determined using dynamic optimization. Since I am assuming the existence of a retirement date R , using backward induction I can evaluate the

value function at R prior to examining the more complicated left-hand integral in equation

(4). The optimization will be performed on the Hamiltonian function.

$$(5) \quad H = u(c(t))e^{-\rho t} + \lambda(t)K'(t).$$

$\lambda(t)$ is the intertemporal Lagrangean multiplier. At the optimum, at any point in time, $\lambda(t)$ can be thought of as the marginal value of an additional amount of savings.¹⁷

The optimization procedure returns the state equation (3) and the following two conditions.

$$(6a) \quad \text{multiplier equation:} \quad \lambda'(t) = -r\lambda(t).$$

$$(6b) \quad \text{optimization condition:} \quad u'(c(t))e^{-\rho t} - \lambda(t) = 0.$$

Equation (6a) says that the marginal value of savings declines over time. Equation (6b) equates the marginal utility of consumption to the marginal value of savings. The second order sufficiency condition for optimization is satisfied by the assumptions on the utility function. Combining equations (6a) & (6b) and solving, we come up with the intertemporal consumption relationship, equation (7).

¹⁷See Kamien and Schwartz (1981), part II, section 4.

$$(7) \quad \frac{u''(c)c'(t)}{u'(c)} = (\rho - r).$$

If $r \neq \rho$, then there is no steady state equilibrium in retirement. If $\rho < r$ then consumption increases throughout retirement since saving exceeds consumption. If the opposite is true, then consumption decreases asymptotically toward zero during retirement, since present consumption is more valued than future consumption. If the hazard rate of dying were increasing over a lifetime, which would result in ρ increasing, then we would see greater saving at earlier ages and greater consumption during retirement. This seems plausible. However, in equilibrium, these two parameters should, on average, be equal, in order that the capital markets will clear. I will assume that the two parameters are equal. In this case it is clear that consumption remains constant during retirement, i.e., $c'(t) = 0$. This is a situation with retirees living on a fixed income. Since, in this model, individuals have an infinite planning horizon, the stock of savings the individual holds must remain constant after retirement. Therefore, the former workers will only consume an amount equal to the interest earned at each point in time during retirement. If consumption were constant and greater than the earnings on savings, the stock of savings would diminish, and would eventually approach depletion. If consumption were less than earnings, the stock of savings would grow exponentially, and more could be consumed. With constant consumption, the per period retirement earnings are equal to the amount consumed. Let's call this amount the pension, $P = rK(R)$. This leads me to Proposition 1, the proof of which is in Appendix 1.

Proposition 1: If $\rho = r$ and equations (7) and (3) hold, then $rK(R) = c(t) = \text{constant}$ for all $t > R$.¹⁸

Since P also equals consumption in all retirement periods, the situation simplifies even further.

$$(8) \quad V(R) = \int_R^{\infty} u(P)e^{-\rho t} dt = u(P) \left(\frac{e^{-\rho R}}{\rho} \right).$$

Given the retirement date, R , in the right-hand side of equation (8) is a constant, determined by the savings accumulated prior to retirement. Now, when I consider the time prior to retirement, I can treat the entire retirement period, albeit infinitely long, as a constant determined entirely by actions during the productive employment periods.

Accordingly, the value function at any point prior to retirement becomes:

$$(9) \quad V(T) = \max_{c,z,R} \int_T^R (u(c) - d(z))e^{-\rho t} dt + u(P) \left(\frac{e^{-\rho R}}{\rho} \right).$$

The state equation (3) now has a non-zero value for $z(t)$ for $t < R$. This entails the Hamiltonian:

¹⁸For a formal proof, see Appendix 1.

$$(10) \quad H = (u(c) - d(z))e^{-\rho t} + \lambda(t)K'(t) + u(P)\left(\frac{e^{-\rho R}}{\rho}\right).$$

The resulting optimization returns equations (3), (6a), and (6b) in addition to one more optimization equation and two transversality equations. I do not explicitly optimize with respect to R . This is taken care of by the transversality conditions. Second order sufficient conditions are again guaranteed by the assumed utility and disutility structure.

$$(11a) \quad \text{optimization condition:} \quad -d'(z)e^{-\rho t} + a\lambda(t) = 0.$$

$$(11b) \quad \text{transversality condition (i):} \quad \lambda(R) = \left(\frac{r}{\rho}\right)u'(P)e^{-\rho R}.$$

$$(11c) \quad \text{transversality condition (ii):} \quad -d(z(R))e^{-\rho R} + \lambda(R)K'(R) = 0.$$

Equation (11a) equates the marginal disutility of effort per unit of ability to the marginal value of an unit of savings. Again, I assume the equality of r and ρ . Then equation (11b) equates the marginal value of an additional unit of savings during the retirement period to the marginal utility of increasing consumption during the retirement period. Equation (11c) sets the disutility of work at the time of retirement to be equal to the change in value an additional amount of time on the job would add. Thus, the optimal retirement date occurs when exerting effort for an additional unit of time is exactly compensated for by the value that the delay in retirement would add to consumption.

Since the level of effort is constant over time, the cost of an additional time period of effort remains constant while the marginal value of additional savings is decreasing. The existence of a retirement date is assured, which validates that previously stated assumption.

Combining equations (6a) and (11a) results in the disutility counterpart to equation (7) (which is still valid, also). An examination of equations (7) and (12) shows that, if $r \neq \rho$, effort and consumption move in opposite directions over time. In particular, if $\rho < r$ and ρ is increasing over time as previously discussed, then effort decreases over time at a decreasing rate and may eventually start to increase while consumption increases at a decreasing rate and may start to decrease if ρ becomes greater than r . This magnifies the savings in earlier periods, but does little else to aid in the analysis of effort and retirement as a function of ability and makes computation far more difficult. Therefore, I will assume equality between ρ and r henceforth. This implies, through equations (7) and (12), that effort and consumption are constant over time in the productive phase of the workers existence.

$$(12) \quad \frac{d''(z)z'(t)}{d'(z)} = (\rho - r).$$

To determine the relationship between consumption and effort, combine equations (6b) and (11a). We see that the marginal utility of consumption equates to the marginal disutility per unit of ability. Ability acts like a wage which the worker is paid for exerting effort. It acts like an inverse price for translating effort into consumption. Higher ability

workers not only can be producing more at any point in time, they may also be consuming more. Thus, higher ability workers have a higher value of time. The measure of ability serves as a link between the value of time for workers of different ability. At every instant in time the laborer maximizes the return on ability. This foreshadows the relationship, or lack thereof, between effort and ability.

$$(13) \quad u'(c) = \frac{d'(z)}{a}.$$

Combining equations (6b) and (11b) shows that Proposition 1 still holds at time R .¹⁹ Using this relationship between savings and accumulated consumption and inserting equations (3) and (11b) where appropriate into equation (11c) leads to the following characterization of the equilibrium level of effort.

$$(14) \quad \left. \frac{d'(z)z(t)}{d(z)} \right|_{t=R} = 1.$$

Since we are assuming equality between ρ and r , both z and c are constant. Thus, equation (14) also holds for all $t < R$. Hence, the optimal value of effort, z^* conditional upon $t < R$, is independent of ability. The multiplicative nature of ability assures this. Given identical disutility functions $d(\cdot)$, optimal effort is invariant across individuals. My assumption of non-zero disutility at zero effort ensures an interior solution. This

¹⁹ Actually, the transversality condition ensures that $K(t)$ is left-continuous at R . It turns out that K is both

discussion, then, serves as my heuristic proof of Proposition 2.

Proposition 2: Given the assumptions in Proposition 1, the optimal level of consumption c^* is constant over time and is determined by the equation $c^* = rK(R)$.

This completes the basic model of temporary disutility. However, my previous assumptions allow me to greatly simplify the comparative statics once I note one simple relationship. The savings stock at retirement is the accumulated total from a constant flow of savings. To wit, since optimal effort and consumption are constant over time, so is the quantity $(az - c)$ before retirement. Knowing that, we can write the equation for optimal savings at retirement as follows. Operation at the optimum level is assumed.

$$(15) \quad rK(R) = r \int_0^R (az - c)e^{rt} dt = (az - c)(e^{rR} - 1).$$

Time 0 is defined as the start of employment with $K(0) = 0$. Since proposition 2 also holds, we can replace $rK(R)$ with c in equation (15) and solve for c .

$$(16) \quad c = az(1 - e^{-rR}).$$

Thus, consumption is a constant proportion of individual output. Looking back at the original optimization problem, knowing the final answer, I reconsider the problem as one

left- and right-continuous.

of simply choosing two quantities optimally at the initial time of employment; effort z and time to retirement R . If $r \neq \rho$, this is similar to choosing a flow of effort and consumption with the relationship indicated by equation (13) holding at all times. When $r = \rho$, both consumption and effort are constant during the employment period and both sides of equation (13) remain constant. This simplifies the situation considerably. Since I know that z and R are both constants when optimally chosen, I reduce the optimization to a static problem subject to equation (16).

$$(17) \quad V(0) = \max_{z,R} \int_0^R (u(c) - d(z))e^{-\rho t} dt + u(P) \left(\frac{e^{-\rho R}}{\rho} \right) = \max_{z,R} \left(\frac{(u(c) - d(z)(1 - e^{-\rho R}))}{\rho} \right).$$

This approach is only valid when evaluated at time 0. If the individual is given a savings endowment or a lump sum payment, the time to retirement will change. For instance, if an individual is given a lump sum payment which brings the total accumulated savings to κ , it is as though the worker were transported forward to time T , defined implicitly in the following equation.

$$(18) \quad r\kappa = (az - c)(e^{rT} - 1).$$

Using the results of equation (17) we may solve for the modified time to retirement $R-T$ which may be negative if $\kappa > rK(R)$.

$$(19) \quad (R-T) = -\frac{\ln\left(\frac{rK}{az} + e^{-rR}\right)}{r}.$$

Thus, the time remaining until retirement depends upon the stock of accumulated savings, the ability endowment, and the equilibrium values for effort and total time to retirement determined by solving equation (17). If an individual is given a lump sum which brings the total accumulated savings above the optimal retirement savings, then the individual retires immediately.

The first order conditions for equation (17) are similar to the conditions for the dynamic optimization. I will denote the partial first and second derivatives of the value function by V subscripted with the applicable arguments. Whenever possible, I use the equality of r and ρ to simplify the notation.

$$(20) \quad V_z = \left(\frac{1}{\rho}\right) \left(u'(c) a (1 - e^{-rR}) - d'(z) (1 - e^{-\rho R}) \right) = 0.$$

$$(21) \quad V_R = u'(c) a z r e^{-rR} - d(z) \rho e^{-\rho R} = 0.$$

Equation (20) gives a relationship similar to that of equation (13), while equation (21) is akin to equation (11c). Combining the two returns equation (14). Thus, we learn nothing new from the static model so far. However, the advantage lies in the relative ease in computing the comparative statics.

The second order sufficiency condition for optimization requires a positive determinant of the matrix of second partial derivatives, which is guaranteed by the assumptions on utility.²⁰

$$(22) \quad \det \begin{vmatrix} V_{zz} & V_{zR} \\ V_{Rz} & V_{RR} \end{vmatrix} = - \left(\frac{d''(z)u''(c)a^2 z^2 r^2 (1 - e^{-\rho R})e^{-2rR}}{\rho^2} \right) \equiv \Delta > 0.$$

Using equation (22) and Cramer's rule, we may now examine the effect of ability on effort and time to retirement. The results are shown in equations (23) and (24).

$$(23) \quad \frac{\partial z}{\partial a} = 0.$$

$$(24) \quad \frac{\partial R}{\partial a} = \frac{u'(c) + u''(c)c}{u''(c)a^2 z r e^{-rR}}.$$

Taking the partial derivative of equation (16) with respect to a , using the results of equations (23) and (24) gives the reaction of consumption to a change in ability.

$$(25) \quad \frac{\partial C}{\partial a} = \frac{u'(c) + 2u''(c)c}{u''(c)a}.$$

²⁰See Appendix 1 for a listing of second derivatives.

The signs of both equations (24) and (25) depend upon the magnitude of the coefficient of relative risk aversion, A_R .²¹ Below a certain level ($A_R < 1$ in this setup) of relative risk aversion, time to retirement decreases with ability. Above the level, R increases with ability. Consumption operates in a similar manner. However, the cutoff level is higher. Hence, it is not generally possible to order individuals by ability using the retirement decision as a metric unless more is known or assumed about the underlying utility function.

$$(26) \quad A_R \equiv -\frac{u''(c)c}{u'(c)} > 1 \Leftrightarrow R_a > 0.$$

$$(27) \quad A_R > 2 \Leftrightarrow c_a > 0.$$

In this case, the risk is the possibility of living a long life after retirement without allowing for a high enough pension. The risk premium is paid through the increased disutility of effort during the working years.

The use of A_R in this case may seem a little out of place, since the model itself contains no risk. Risk aversion, in the sense I now mean it, merely indicates the degree of concavity of the utility function. For want of a better measure and since the measure of risk aversion is common and convenient I will continue to use this terminology throughout the analysis.

Now I will examine the disutility function more closely. Consider a situation in

²¹ See Pratt, 1964.

which we give a more explicit form for the disutility function $d(\cdot)$. In particular, let δ be a strictly positive parameter, which may be heterogeneously distributed amongst individuals as indicated in equation (26). $D(\cdot)$ is a constant function across individuals and has the same structure accorded to $d(z)$ with the exception that $D(0) = 0$.

$$(28) \quad d_i(z) \equiv \delta_i + D(z).$$

This does not alter any of the previous analysis, since the curvature of $d(\cdot)$ remains the same. However, now δ provides a specific measure of inherent disutility of work.

Comparative statics can now be accomplished on this new parameter.²²

$$(29) \quad \frac{\partial z}{\partial \delta} = \frac{1}{d''(z)z} > 0.$$

$$(30) \quad \frac{\partial R}{\partial \delta} = -\frac{u''(c)a^2(1-e^{-rR})-d''(z)}{d''(z)u''(c)a^2z^2re^{-rR}} < 0.$$

$$(31) \quad \frac{\partial c}{\partial \delta} = \frac{1}{u''(c)az} < 0.$$

Thus, those who dislike work more will work harder for a shorter period of time and will choose to consume at a lower level. Their saving rate will be higher, but individuals with

²²Again, see Appendix 1 for a list of second partial derivatives.

higher disutility of effort will choose to retire when their savings has accumulated to a lower level than workers who are relatively content.

IMPLICATIONS OF THE MODEL

While the firm is not explicitly modeled, its presence in the background supports the assumption of a fixed cost of effort associated with employment as the laborer rents capital from the firm at a fixed per period cost. The worker then optimizes within this framework. If employment also entails some firm-specific capital investment in the worker, a firm-sponsored pension may be used as an incentive to ensure that the workers do not depart the firm prior to the firm recouping its investment. In order for this to be viable, the firm must offer a pension which makes a worker no worse off than with an individual-sponsored pension or savings plan. Also, the firm must have some way to withhold the pension should the worker depart the firm prematurely. This is the reason for vesting. A non-vested pension serves as a hostage to ensure the worker's contractual compliance²³ and prevents the worker from appropriating the quasi-rents associated with the firm retrieving its investment in the worker.²⁴

If all workers were identical, they would all choose to consume and exert effort at the same levels and retire after the same amount of time has been spent in the labor force. In such a situation, the firm has an infinite choice set. In particular, it could set a wage structure which holds back a portion of compensation as a hostage eventually to be returned in the form of a pension which is contingent upon the worker fulfilling the

²³See Williamson (1983).

²⁴See Klein, Crawford, and Alchian (1978).

contract. The withheld wages could even be larger than the planned savings amount in the individual retirement system, since I have implicitly assumed that workers may borrow and save at the same rate. Workers then may borrow against future earnings in order to consume at a constant level. Once the hostage is large enough to ensure the workers' compliance, wages may be raised to marginal product. This would explain the increase in wages with tenure that is commonly observed. Partial vesting would also occur in some instances as the value of the accumulated retirement savings held by the firm increase beyond that required as a hostage. Mandatory retirement could also be used, since everyone would choose to retire at the same point anyway. This, however, is not the only possibility. Lazear (1978) has an alternative explanation which does not involve retirement, but uses wages which differ systematically from workers' marginal product.

Once we allow for heterogeneous workers, the firm may also use retirement to sort workers. If workers are relatively risk averse, in the sense of making equation (24) positive, then higher ability workers will have a longer time until retirement. Thus, higher ability will be associated with tenure. The converse is true if workers are relatively less risk averse. If some types of firms attract individuals who are relatively less risk averse, it may be necessary to institute a system of mandatory retirement in order to get rid of the dead wood, so to speak. The longer an individual remains with a firm, the lower her expected ability. Higher ability individuals would have a higher turnover. Retirement vesting would be used to counteract this phenomenon. A firm's retirement structure would be set up to take all of this into account. Any government intervention restricting mandatory retirement would cause firms in such situations to spend relatively more time

and energy directly evaluating individuals in order to terminate them for cause or, perhaps, to initiate a retirement incentive program which encourages earlier retirement. Thus, it should not be uncommon for persons who retire earlier to receive an actuarial advantage. Since I am restricting myself to the case of zero profit, this added retirement incentive must come from somewhere else in the compensation structure. It would be expected, in general, to decrease the overall pre-retirement wage structure. Thus, a restriction on mandatory retirement may provide a windfall for current potential retirees, but it also merely rearranges the wage/retirement structure in the long run.²⁵ I discuss this further when I examine the savings rate.

Looking at the retirement decision from a savings perspective gives a few more insights. Equation (16) gives consumption as a constant proportion of output. Another way to examine this is by considering the savings rate, s .

$$(32) \quad s = \frac{az - c}{az} = e^{-rR}.$$

As can be seen in equation (32), the savings rate is extraordinarily simple. Savings is a constant proportion of earned income. Assuming that firms attempt to mirror the actions of their employees with respect to setting up firm-sponsored pension plans, it should not be unusual to see firms allotting a constant proportion of workers' wages toward pensions. Even if the pension calculation is not directly tied to the individual workers' entire wage history, budgeting a constant proportion of all workers' salary

²⁵This idea is discussed in the final chapter of Posner (1995).

toward the firm's pension fund would be equivalent to saving a constant proportion of expected permanent income.

The savings rate is a function of the interest rate, which equals the required internal rate of return, and the time to retirement. Performing comparative statics, using equations (24) and (30), I come up with the following results.

$$(33) \quad \frac{\partial s}{\partial a} = -re^{-rR} R_a = -\frac{u'(c) + u''(c)c}{u''(c)a^2 z}.$$

$$(34) \quad \frac{\partial s}{\partial \delta} = -re^{-rR} R_\delta = \frac{u''(c)a^2(1 - e^{-rR}) - d''(z)}{u''(c)d''(z)a^2 z^2} > 0.$$

The savings rate acts in the opposite direction to that of the time to retirement. The longer the time to retirement, the lower the savings rate. More risk averse individuals have a savings rate which is decreasing in ability. The more risk averse individuals work longer and, thus, need to put away a lower percentage of income toward retirement. Likewise, the more an individual dislikes working, the more that individual will save, since this shortens the time to retirement.

If some firms have relatively less risk averse employees, a pension plan which saves a large portion of income would attract higher quality individuals, since they would choose to save a higher proportion of their income. Lower quality individuals, however, save a small portion of income and would be put off by a pension plan which saves at a higher rate than they desire. Hence, we should see relatively risky occupations which have

fairly low wages but a relatively generous pension system. In particular, this method of sorting could be used in lieu of mandatory retirement if restrictions on it are imposed. Hence, a predicted consequence of the Age Discrimination Act of 1980 would be a stagnation of direct wages but an increase in retirement benefits and an increase in the number of individuals covered by firm-sponsored pension plans.

If it is possible to monitor effort, the fact that effort is constant across individuals would provide an easy way for firms to pay a salary which does not depend directly upon output. This is important, for instance, in cases where workers operate in teams and individual output is not directly measurable. There remains, however, the problem of differentiating workers by ability. For, although all workers exert the same effort, the marginal value products differ across individuals according to ability. Thus, greater ability individuals require higher salaries in order to remain with the firm.

CONCLUSION

Individuals will retire even without firm-sponsored pension plans. Thus, the institution of a firm-sponsored pension plan need not alter the activity of individuals, although it may help facilitate reaching an efficient outcome. Firms may hold pensions as hostages in order to enforce the implicit labor contract and provide incentives for individual laborers to remain with the firm, thus allowing the firm to recoup any firm-specific investment. In particular, vesting allows employers to withhold previously earned income until the firm's investment in the worker has panned out.

The particular assumption which ensures that individuals will retire is the existence

of a fixed per-period disutility of employment effort. In an efficient capital market individual consumption remains constant throughout the workers' lifetimes, both before and after retirement.

Even with heterogeneous ability, optimum effort is constant across individuals. This could be useful for situations in which individual contribution to productivity is difficult to measure directly. However, there still remains the problem of determining individual wages.

It is not possible, in general, to order individuals by ability using time to retirement, consumption, or the savings rate since none of these variables vary monotonically with ability. However, given a measure of individual relative risk aversion, a firm may order individuals by ability.

For example, if a firm tends to attract individuals who are less relatively risk averse, time to retirement decreases with ability and the savings rate increases with ability. Actually, this assumption seems fairly innocuous. Relatively risky occupations should tend to attract individuals who are less relatively risk averse, since they have a comparative advantage in these occupations and would demand a relatively lower wage. It is probably no coincidence that firm-sponsored pensions, as mentioned in Chapter I, first appeared in the military and in relatively risky occupations.

Mandatory retirement may be called for to help eliminate low quality hangers on. Any restrictions on mandatory retirement would cause a shift in firm retirement policy toward alternative ability sorting methods. For instance, firms would hold a larger portion of earnings in the form of savings toward retirement, since this is positively related to

ability. Apparent wages would decrease, but would be compensated by the increase in retirement benefits for higher quality individuals. Firms would, essentially, take over more of the responsibility for retirement saving from individuals.

In this discussion, I have steadfastly ignored the question of why a firm would care about individual ability per se, considering output is all that directly matters in this chapter. I will leave this discussion to Chapter III where I will also examine the competitive promotion process.

CHAPTER III

HETEROGENEOUS ABILITY IN HIERARCHICAL LABOR PROMOTIONS

INTRODUCTION

The tournament structure of labor promotions has been analyzed to a great extent, but there exists a problem in perspective in the analysis. While much of the literature has focused on the attempt to use tournaments as a method for ensuring an efficient level of effort, little attention has been devoted to the productivity of the winner after the results of the tournament has been determined and the promotion has occurred.

Most firms have a hierarchical structure with some kind of promotion system. How the promotion system effects worker effort greatly impacts the efficiency of the firm. Lazear and Rosen (1981) examine the case of tournaments where workers' effort adds to their ability endowment. They showed that tournaments can produce the efficient level of effort. This helped spawn a school of literature. Nalebuff and Stiglitz (1983) examine the case of output being the product of effort and ability (multiplicative effort). Holmstrom (1982) shows that the addition of a tournament structure when ability is already known adds no new information to a system. Thus, a tournament only has value if it is used to extract information. This would seem to be a reasonable use for tournaments in a setting of heterogeneous workers.

For the most part, heterogeneity has been dealt with by using handicapping, or leveling the playing field.²⁶ Additionally, Rosen (1986) shows that, as long as the higher ability individual has a greater value of continuing, tournament will select the best players.

However, unless the prize structure is weakly increasing in the tournament level (prizes increase more than linearly at the top), effort decreases at the last level of the tournament with linear effort. O'Keefe, Viscusi, and Zeckhauser (1984) show that, with risk neutral workers, effort is increasing in ability. Thus, the higher ability individuals will be chosen. Interestingly, Demski and Feltham (1978) show that risk aversion is a necessary condition for a contract which compares a workers output to a fixed standard to be Pareto improving. The tournament literature is similar to auctions,²⁷ although in the case of tournaments, the losers all pay in the form of *ex ante* effort.

The results of tournament theory have been examined empirically²⁸ and experimentally²⁹ and much of the theory has been vindicated. It is interesting to note, however, that the theory and empirics have all dealt with the effort of individuals prior to the awarding of the prize. If the firm uses the tournament to award a promotion, they are additionally interested in the amount of effort promotees expend once the promotion has taken place.

In many sports settings, the tournament determines the most able participant. Once the competition is over, however, nothing more is expected of the winner. In contrast, many other productive enterprises use tournaments both as an effort-inducing device and as a means for sorting participants by ability. In addition, a hierarchical structure may be the result of technological constraints. The hierarchy may be required

²⁶See Green and Stokey (1983).

²⁷See McAfee and McMillan (1987).

²⁸See Ehrenberg and Bognanno (1990).

²⁹See Bull, Schotter, and Weigelt (1987) and Frederickson (1992).

for organizational purposes.³⁰ The tournament structure itself may only be the result of the hierarchical structure of the firm. Once the hierarchy exists, given that higher levels in the hierarchy provide a higher return for individuals, a tournament is necessarily introduced, irrespective of whether or not the firm requires the incentive producing or sorting ability inherent in the tournament structure.

Any look at the problems with tournaments³¹ must, therefore, be tempered with the fact that the tournament itself may be unavoidable. An incentive may have been introduced for individuals to modify their behavior in an effort to achieve a higher employment status in the hierarchy. Hence, it is important to understand the nature of promotion tournaments in order to better influence and modify the structure of the firm to incorporate any such unavoidable tournaments.

That being said, the question remains as to wherefore the hierarchical structure exists. Why does there need to be only one or few individuals at the top? What is behind the pyramidal firm structure so commonly seen?³² It must be the case that the activity of workers effects the productivity of other workers. This externality can be used to help explain the hierarchical structure consisting of a decreasing number of individuals at each rung of the hierarchical ladder.

In this chapter, I will first examine some aspects of the firm which may explain the use of a hierarchical labor structure. This sets the stage for further examination of a promotion scheme with heterogeneous ability workers competing for promotion, providing a closer look at what factors may effect a firm's promotion structure.

³⁰ Rosen (1982) points this out in his examination of a hierarchical control model.

³¹ See, again, Dye (1984).

Examining what affects individual action in tournaments allows me then to attempt an explanation and prediction of what types of promotion structure may be empirically observed.

A BRIEF LOOK AT FIRM STRUCTURE

When a firm employs multiple individuals, there must exist some sort of organizational structure which defines the areas of individual responsibility. Although it is conceivable that all laborers within a firm act as equals in decision-making and in dividing areas of responsibility, this seems to defeat the purpose of having a firm. Why bring multiple individuals together into a firm when they already individually choose to align themselves into the various taskings without the help of a firm's structure? For, if individuals in a firm independently choose the optimal actions, the firm itself becomes redundant. Thus, the mere existence of firms implies that firms must provide some sort of structure to the labor input.

Given that the firm provides structure which would not exist independent of the firm, the individual laborers within a firm must not all be perfectly satisfied with their intra-firm position. However, the workers must not be too dissatisfied, otherwise they would depart their present firm for another in which they would be less dissatisfied. Therefore, a better alternative must not exist. Assuming there exists more than one firm, there must therefore be some common link which keeps workers within firms even though they are less than perfectly satisfied. My contention is that this link is a hierarchical labor

³²Again, see Rosen (1982).

promotion system. Firms exist because there must be a boss. Left to their own devices, individuals would rather be the boss than an underling. However, this is not the most efficient setting. Firms provide a hierarchical structure within which labor is allocated according to ability within the individual positions. The questions remain as to why a firm would differentiate individuals by ability across position and why, given that firms differentiate in this manner, would individuals desire one particular position over another.

If individual abilities enter the production function in an additive manner and firms pay a wage which is based on units of ability (however it is measured), then it is clear that firms are indifferent between hiring a very high ability worker and several low ability workers whose collective ability adds up to the same quantity as our higher ability individual. Thus, in order that a firm prefer higher over lower quality individuals, the wage must be relatively lower for higher quality individuals, or there must be some technological constraint which causes the marginal productivity of workers within a position to decline with the number of workers in the position.

The structural explanation seems plausible in that higher level laborers make decisions which affect a larger number of employees. In other words, the productive inputs of upper level employees provide an externality to the marginal products of other workers. Decisions made at top levels effect each area of the firm, whereas, lower level decisions affect only those laborers lower in the hierarchy on that particular branch of the hierarchy. Given that decisions are most important at higher levels, the firm chooses to place the highest ability individuals into these upper level positions. Presumably, the return is lower the higher the number of individuals in the position.

Consider the case of an industry whose technology consist of having individuals make a series of decisions chosen from a large number of possible alternative actions, each of which, if correct, adds to the successful production of output. It is possible that there may be more than one successful alternative. In that case, let a successful decision be the choice of an alternative from the set of all possible correct alternatives. Let the outcome of the decision be a binary random variable, x . The probability that $x=1$ is given by az , the product of exogenous heterogeneous ability, a , and endogenous effort, z , both defined over $[0,1]$, as in Chapter II. I will call the ability-effort product skill. Thus, x is a Bernoulli random variable with parameter az . The expected value of x is az . Individuals have incomplete information. They know their own ability and effort level and the distribution of ability in the labor force of the firm. They do not observe the ability of other workers. The firm itself can only observe the decision outcome variable, x . If individuals are paid the wage, w_1 , only upon a successful decision and the cost of effort, $d(z)$ is as defined in Chapter II, the optimal effort is implicitly defined by equation (35), which assumes firms and laborers are identically risk averse. To the extent that the actions of firms are determined by their managers, who have utility functions similar to those of other workers, the risk aversion assumption holds. To the extent that decreasing absolute risk aversion occurs, however, managers may be relatively less risk averse than lower paid workers. Nevertheless, I will maintain this assumption throughout.

$$(35) \quad w_1 a = d'(z).$$

Since $d(z)$ has positive first and second derivatives, it follows that effort is an increasing function of both the wage and ability.

Now, consider a situation in which one individual may be placed in a position over several (n) underlings. If the leader has total control of those below him, he may direct each worker toward a particular decision. Therefore, we have $n+1$ individuals taking the same action. If we know the skill of the supervisor, $a_s z_s$, the expected value of the decision becomes $(n+1)a_s z_s$. If the supervisor's skill is above the average individual level of the group, the expected value of the group's output has strictly increased. Thus, the optimal choice for supervisor would be the individual with the highest skill. Even if the supervisor's decision were taken out of the results, it is possible for the bi-level firm outlined above to have a higher output than the sum of all individual outputs if equation (36) holds. Yet, how is the wage for the subordinates determined and how is the best individual determined? There is no need for the underlings to exert the effort of making an individual decision, since the supervisor exerts effort for the group. The wage can be no lower than what workers would have earned based on the effort level implicit in equation (35) minus the corresponding cost of effort, otherwise they would quit the firm. This amount must be strictly positive, otherwise the laborer would have found it to his advantage to stop work and avoid the fixed cost of zero effort. Hence, workers are compensated for effort they have not exerted.

$$(36) \quad na_s z_s > \sum_{i=1}^n a_i z_i .$$

There are two problems with this particular model, however. First, individuals have no incentive to exert effort when they are subordinates. Their effort provides no added value to the firm. To be sure, there does exist the fixed cost of employment at $d(0)$, which takes into account the effort of minimally remaining with a job. This can be interpreted as the minimum effort required by the firm for continued employment and may actually be greater than zero. However, the firm also has no apparent method for selecting the upper level individuals. The lower level workers all exert the same level of effort and achieve the same results. There is no internal method for differentiating amongst workers. But, if leaders were selected randomly, there would not be the expected increase in productivity derived from the firm's structure.

A more realistic setup would include the fact that firms cannot directly observe ability. They only have information regarding the successful past outcomes from workers' decisions. From this, they choose whom to promote to the next higher level. As individuals are promoted, retire, or otherwise depart the firm, more employees are hired at the lowest level to attempt to work their ways through the ranks. Consequently, it may be the case that some workers in lower ranks are of higher ability than their "superiors". These higher ability, upwardly mobile underlings thus take the advice of their supervisors with a grain of salt, particularly since individuals can only observe their own ability and can only estimate the skill of their bosses based upon their position and any observed successful outcomes. The subsequent choices of the individuals become weighted averages based upon their own individual ability/effort decision and the expected quality of advice from on high. If promotions are based upon relative performance, it may be in the

individual's best interest to exert some effort in order to possibly arrive at a more correct decision.

An added complication might come from the dilution of advice as the number of subordinates increases. The rationale behind this is that the supervisor's time is limited. As more and more individuals request decision inputs, the ability of the supervisor to fully analyze situations and recommend appropriate actions diminishes. For instance, instead of being involved in every minute aspect of daily decision-making, supervisors resort to guidelines, rules-of-thumb, regulations, etc. These techniques may cover most ordinary situations but, when something out of the ordinary occurs, may not adequately solve the problem.

Consider a model in which individuals know only their own ability, a_i , and the distribution from which it is drawn, $f(a)$. The individuals choose a level of effort based upon the person's endowment of ability and any incentive induced by the wage or promotion structure. Firms do not observe individual ability or effort directly, but do observe the results of successful decisions. The decision points arrive in a Poisson process with parameter λ . The success of each decision depends upon the ability and level of effort being exerted when the decision opportunity arrives. The outcome of a decision is x and the expected value of a decision is $a_i z_i$, where z_i is the instantaneous effort measured at the point of the decision.

In order to examine the benefit of having a hierarchical structure with this firm setup, I will examine only a two level situation. An individual at either level gets rewarded according to the results of his successful decision-making, $w_i(x)$. The rewards to the

lower level will later be assumed to implicitly include the possibility of promotion.

First, let's examine the nature of effort in such a setup. Even though the decision points are discrete, since they arrive at random times it is in each individual's best interest to exert a continuous and constant amount of effort. If effort occurred at discrete times, the probability of effort being exerted when a decision needed to be made would be zero and any effort expended would be wasted. Hence, if effort occurs, it must be continuous. We can now assume that individuals who exert effort do so taking into account the arrival probability of the decisions. The expected value of an individual's decision will now be $\lambda a_i z_i$.

Now, consider a two-level firm. Upon arrival of a decision opportunity, the lower level (level 1) individual receives input from the supervisor (level 2) on the decision. The level 1 individual is paid $w_1(x)$ for each successful decision he makes. The level 2 supervisor is paid $w_2(x)$ for each successful decision she is involved in. For simplicity, let $w_1(x) = \delta x$ and let $w_2(x) = \gamma x$. At the time of the decisions, the level 1 individuals must decide whether to trust their own judgment, or that of the supervisor based upon the knowledge of their own effort and ability, $a_i z_i$, and an estimate of the supervisor's ability and associated equilibrium effort level adjusted for the diluting effect of the number of subordinates, $E(a_s z_s | n)^{33}$. For $u(w_1(\lambda a_i z_i)) - d(z_i) > u(w_1(E(a_s z_s | n)))$, the individuals follow their own individual decision-making advice. If the inequality is reversed, the individual follows the advice of the supervisor. Since the cost of effort is always strictly positive, some workers will not exert effort even though they expect that they are of higher ability

than their supervisor. The probability of an exact skill equality is zero. Let's define the subordinate skill level implied when the equality occurs as S^* . Again, we have a dichotomous situation involving subordinate effort. Workers below a certain equilibrium skill threshold will exert no effort and will merely mirror the decisions of their supervisor. When individuals are above the threshold they will act independently of their supervisor's advice.

This result has an interesting implication for supervisory effort. Since the supervisor's decisions affect the actions of less than 100% of subordinates, the supervisor takes this into account in the effort decision, and the effort of the leader is reduced accordingly. If $F(a_s z_s)$ is the cdf of the distribution of subordinate skill, the expected number of individuals a supervisor actually effects with his decisions is $nF(S^*)$. Let's define this number as n_e , for the number of subordinates effected by the supervisory decision. The expected wage flow of the supervisor with ability endowment α is thus given by equation (37). Let $s_i(a) = az(a)$ be the skill of an i -level worker with ability endowment a .

$$(37) \quad E(w_2(\alpha)|n) = \lambda[(n - n_e)\gamma\left(\int_{F^{-1}(S^*)}^1 tf(s_1^{-1}(t))\left|\frac{\partial s_1^{-1}(t)}{\partial t}\right|dt\right) + n_e\gamma(s_2(\alpha))].$$

The first portion on the rhs of equation (37) is assumed independent of the decision made by each individual supervisor, although it reflects the results of all possible decisions. The

$$^{33} \frac{\partial E(a_s z_s | n)}{\partial n} < 0 \text{ by assumption.}$$

skill decision facing a supervisor with ability endowment α is described in equation (38).

$$(38) \quad \max_z u[\lambda[(n - n_e)\gamma \left(\int_{F^{-1}(S^*)}^1 \text{tf}(s_1^{-1}(t)) \left| \frac{\partial s_1^{-1}(t)}{\partial t} \right| dt \right) + n_e \gamma(s_2(\alpha))]] - d(z).$$

While this may look horrendous, the vast majority of the utility argument is independent of the z chosen by the supervisor. Thus, the First Order Condition is relatively simple, as shown in equation (39).

$$(39) \quad u'(E(w_2(\alpha)|n)\lambda n_e \alpha \gamma) = d'(z).$$

With a linear wage function, as I have assumed, there is no guarantee that effort is increasing with ability, although sufficiently low risk aversion does ensure this result. However, given a sufficiently concave wage function, effort can be made strictly increasing in ability in all cases. Since the firm determines the wage function, this is definitely within the realm of the possible. Similarly, we may determine the optimal effort of the level 1 workers who choose to exert effort. The expected wage flow of a level 1 laborer with ability endowment β is given in equation (40).

$$(40) \quad E(w_1(\beta)|\beta > F^{-1}(S^*)) = \lambda \delta(s_1(\beta)).$$

The optimization problem of the highly skilled level 1 employee is given in equation (41).

$$(41) \quad \max_z u(\lambda\delta(s_1(\beta)) - d(z)).$$

This implies the First Order Condition given in equation (42).

$$(42) \quad u'(\lambda\delta(s_1(\beta)))\lambda\beta\delta = d'(z).$$

The level of effort of lower level workers who in fact exert effort is independent of the distribution of upper level workers, although the average skill of effort-exerting lower level workers depends upon S^* . The results of comparative statics concerning the relationship between effort and ability are similar to those found with the higher level workers. Upon comparing equations (39) and (42), we see that effort is greater in supervisors relative to subordinates of equal skill if and only if equation (43) holds true.

$$(43) \quad u'(\lambda[(n - n_e)\gamma\left(\int_{F^{-1}(S^*)}^1 tf(s_1^{-1}(t))\left|\frac{\partial s_1^{-1}(t)}{\partial t}\right|dt\right) + n_e\gamma(s_2(\alpha))])n_e\gamma > u'(\lambda\delta(s_1(\beta)))\delta.$$

The condition in equation (43) holds if the skill of the individual is relatively high compared to the average of workers who exert effort. The likelihood of this occurring increases when the per unit wage is relatively higher for the supervisor compared to the subordinate. Since firms are more profitable the higher the skill of upper level workers, it seems likely that firms will structure their wages so as to ensure a greater level of effort for higher level workers. Also, it is relatively more efficient if higher ability workers exert

the greatest effort at any level, which is the case in this model.

The purpose of this entire exercise was to build a conceptual framework around the idea that firms attempt to promote the most able employees into higher areas of responsibility. These individuals are responsible for creating productivity externalities which enhance the marginal products of their subordinates. Thus, there is a return to promotion which makes upper levels in a firm more desirable for the individual laborers. It is probable that employers will utilize a concave wage function in order to entice workers to exert increasing effort within each successive level of the firm's hierarchical structure as indicated by equation (43).

The phenomenon of disproportionately increasing wages for higher level employees has been studied extensively, particularly by Rosen. The emerging explanation for this occurrence was due to the promotion incentive structure. However, this alternative explanation does not even require a competitive promotion process to justify a higher wage for upper level workers. It is in the firm's best interest to pay a proportionately higher wage to employees further up the hierarchy since this induces greater productive externalities on increasing numbers of subordinates.

Having demonstrated this point, I will hereinafter assume a concave wage function. Firms pay higher levels in the hierarchy progressively higher wages in order to extract greater effort to provide greater productive externalities for lower level workers. Workers of equal ability, therefore, are individually better off in higher levels of the hierarchy. The firm, however, prefers placing higher ability workers in the upper echelons. This dichotomy is covered in the next section which examines whether a competitive promotion

system successfully identifies and promotes the best workers into the upper levels of the hierarchy.

A MODEL OF HIERARCHICAL PROMOTIONS

Using the results of the previous section, I now envision a model which involves a firm with a hierarchical labor structure in which individuals are promoted to the next higher level based upon some observed proxy for output. For individuals of any given ability, positions higher up the corporate ladder are more lucrative than those at lower levels. Hence, individuals have an incentive to seek promotion irrespective of ability. However, the firm desires that the higher positions be filled by those with correspondingly higher skill.

For now, I consider a one period tournament, where any future return to current period effort is incorporated into the prize structure. Single period output is defined as in equation (1) with the time argument suppressed. In order that individuals may be ordered by ability, output must satisfy the following condition.

$$(44) \quad y'(a) = az'(a) + z(a) > 0.$$

This merely indicates that effort does not react too negatively when an individual is endowed with a high ability. There is a tendency for higher ability individuals to ease up in effort, since they are still virtually assured of receiving the prize. This condition just states that the incentive to sluff off is does not overcome the increase in output due to the higher

ability. The condition can be re-stated in terms of the ability elasticity of effort.

$$(45) \quad \varepsilon_a \equiv \frac{\partial \ln(z)}{\partial \ln(a)} = \frac{az'(a)}{z(a)} > -1.$$

As an alternative to employment in the tournament sponsoring firm, individuals may be paid a strict output-based wage, w_y . Individuals are faced with the following optimization problem.

$$(46) \quad \max_z w_y y - d(z) \Big|_a.$$

The first order condition implies equation (47). Since w_y is exogenously fixed, higher ability individuals must have higher marginal cost of effort, which implies a positive relationship between ability and effort.

$$(47) \quad w_y = \frac{d'(z)}{a} \Rightarrow \frac{\partial z}{\partial a} > 0.$$

In the tournament firm, there are n_h individuals in level h , all of whom are drawn from the same continuous ability distribution given by the pdf, $f(a)$. For now, I will suppress the level subscript and just consider a representative level. Of the n individuals in the h level, j will be promoted. For now, I assume that no individual effects the output of others. Individuals who are promoted in the tournament are given wage w , while those

who fail to be promoted are given the default wage, which I normalize to zero. Although the lower prize is zero, it implicitly contains the minimum payment needed to insure participation in the tournament. I assume that individual utility is of the form below. The subscript indicates the individual.

$$(48) \quad U_i = (\text{probability that individual } i \text{ is promoted})w - d(z_i) .$$

If the output were directly observable by the firm, there would be no reason for the tournament. As a matter of fact, a tournament can add no information when output is perfectly observed and can do no better than the alternative employment situation.³⁴ However, I assume that output is not directly observable, but is perfectly orderable. The order function is used as the proxy for output. This seems to be a fairly common occurrence. For instance, in team production, the direct output of individuals is seldom directly observable, yet the ordering of individual contributions can be and often is ordered in the determination of wages, bonuses, promotions, etc.

Given this assumption and the assumption that equation (44) holds, ordering by output and ordering by ability are equivalent. They are merely positive monotone transformations of each other. Hence, $F(a) = F(y)$. Therefore, for the representative individual, the optimization looks like equation (49) (suppressing the subscripts.)³⁵

³⁴ See Holmstrom (1982).

³⁵ See Appendix two for the full derivations of the first and second order conditions.

$$(49) \quad \max_z w[1 - F(j|y)] - d(z).$$

By maximizing with respect to effort, individuals are essentially choosing an output with which to compete against other individuals' outputs for the promotion. The probability of winning is one minus the proportion of individuals who fall below the promotion cutoff. The first order condition is given in equation (50). $\beta(j, n-j)$ indicates the beta function.³⁶

$$(50) \quad \frac{w[F(y)]^{j-1} [1 - F(y)]^{n-1-j} f(y)}{\beta(j, n-j)} - d'(z) = 0.$$

The second order condition described in equation (51) is problematic in that it holds only under certain parameter settings and depends upon the individual position in the underlying distribution, $F(y)$. If we assume a unimodal distribution in a tournament in which only the top performer is being promoted, the second order condition is guaranteed for high quality individuals. However, it is possible for the second order condition to fail depending upon the number being promoted and the shape of the underlying distribution of ability.

³⁶ $\beta(a, b) \equiv \frac{(a-1)!(b-1)!}{(a+b-1)!}.$

$$(51) \quad ad'(z) \left(\frac{f'(y)}{f(y)} + \frac{f(y)}{F(y)[1 - F(y)]} [(j-1) - (n-2)F(y)] \right) - d''(z) < 0.$$

Discrete comparative statics can be used to determine the effect of a change in j , the number of promotion opportunities in the current level, upon the optimal level of effort, assuming that the ordering of individual does not change in equilibrium. This leads to Proposition 3, the proof of which is in Appendix 2.

Proposition 3: The individual from a given ability distribution $F(a)$ who exerts the greatest effort when confronted with a promotion opportunity of j out of n individuals, has ability,

$$a = F^{-1} \left(\frac{j}{n-1} \right).$$

This means that the individual exerting the highest level of effort is the one most likely to be closest to the cutoff between being promoted and not being promoted. By a similar line of reasoning, we can determine Proposition 4, the proof of which is also given in Appendix 2.

Proposition 4: When all contestants in a contest are of equal ability, the optimal number of promotees is $j = (n - 1)$. In addition, the number of contestants, n , which gives the greatest individual effort is $n = 2$.

Without much further modification, this line of reasoning also leads to Proposition 5, the proof of which is also in Appendix 2.

Proposition 5: The number of contestants, n , which gives the greatest individual effort is $n = 2$.

This result is similar to that of Fullerton and McAfee³⁷ where they examine research tournaments and find the optimal number of contestants to be two. The setting and the proofs, however, are significantly different. Reexamining the second order condition in equation (51) gives the result that one-on-one situation in which there is a symmetrical ability distribution ensures that the condition holds with certainty.

Thus, the greatest amount of effort will be exerted if a promotion system can be subdivided into separate one-on-one contests, given such a scheme is feasible. However, the lower the number of contestants, the less likely is the assumption of private information particularly in a promotion setting where two individuals are competing for the same position from within the same firm. Obviously, though, with identical individuals, this would not pose a problem, since there is no private information.

With heterogeneous individuals, a lower number of participants in a contest would significantly raise the possibility of collusion and lowered effort. Firms would need to take actions to minimize this collusion (whether explicit or tacit) such as by randomly transferring individuals between groups, thus increasing the number of individuals who

³⁷ See Fullerton and McAfee, 1996.

must be involved in the collusive behavior.

CONCLUSION

Leaders provide direction to the firm. The actions or decisions of superiors effect the outcomes of subordinate actions. Leader selection is therefore extremely important, since leaders provide an externality to the output of followers. The better the leader, the more effective the firm at maximizing profits. For situations in which firms must observe worker behavior prior to worker promotion, a tournament structure is unavoidable, particularly when workers' actions potentially affect the information available to the firm about worker ability. In order to ensure that effort is increasing in ability, workers in higher positions must be compensated relatively more for the same amount of effort than those workers in lower positions. When promotions benefit individuals, they will attempt to modify their behavior to maximize their promotion opportunities. Each worker always prefers a higher position to a lower one, *ceteris paribus*. Knowing this, the firm may be structured in such a way as to maximize the effort of individual workers, taking full advantage of the incentives inherent in the ubiquitous tournament structure.

In examining the tournament structure, the firm may now determine the proportion of individuals promoted so as to locate the greatest amount of effort in the area of the ability distribution with the highest probability density, thus ensuring the highest expected effort by the greatest expected number of laborers. In the case of a unimodal ability distribution, the promotion point would most probably not be located at a position to the left of the mode, since the second order condition is most likely to hold when the density

function is decreasing. Also, given the results of the firm structure analysis, the lower level employees are the most likely to exert zero effort and free-ride on the effort of their superiors in making their decisions. In the case of a symmetrical ability distribution, no individuals will exert zero effort when their ability is located at a point higher than the median. Depending upon the magnitude of the curvature of the disutility of effort, the zero effort ability threshold relatively further to the left of the median.

In addition, the individuals being considered for promotion need to be subdivided into groups as small as possible, preferably into a one-on-one situation. This needs to be tempered both by the shape of the underlying ability distribution, which determines where the promotion cutoff will be located, and the need to avoid the possibility of collusion between individuals for the purpose of decreasing their effort. One way to avoid this last problem may be to transfer individuals, if possible, between groups, thus breaking up any possible collusive efforts.

The subdivision of workers into separate promotion-competing groups can both be done intertemporally and cross-sectionally. Thus, individuals in given groups may only be competing for promotion versus others in their group with similar tenure. This gives firms considerably more latitude in observing the performance of individuals prior to making the promotion decision.

It is also important to note that the firm is not necessarily limited in its ability to promote individuals based upon the number of upper level positions available. This can represent a lower bound on the number of promotions available. However, lower level employees may still be promoted into an artificially created hierarchy in order to better

take advantage of the incentives inherent in the promotion process. Hence, the promotion process may be separated from the hierarchical positioning process which gave rise to the initial structure. In this manner a firm may more nearly optimize over the level of effort provided by laborers depending upon the underlying ability distribution.

CHAPTER IV

EFFORT, PROMOTION AND RETIREMENT

INTRODUCTION

The standard reasoning for the retirement decision is that retirement is due to changes in productivity, effort, or utility. These explanations are all essentially the same. Workers no longer value the results of employment as highly in later years and therefore choose to work more during their earlier years when the relative return is highest. I have shown that if worker marginal revenue product does not decrease over time, retirement is due to the increase in worker opportunity cost relative to marginal revenue product over time.

The time an individual chooses for retirement affects not only his own utility, but that of the others in his organization, even if there is no correlation between workers' productivity. By choosing to retire earlier or later, a worker in a higher position changes the promotion probability of those in positions below. The higher level departure opens up a new slot for promotion. As the arrival rate of promotion opportunity varies, so does the incentive to exert effort. Thus, the higher level retirement decision creates an externality for the lower level workers which affects the productivity of the firm. The firm thus has an incentive to modify the behavior of the upper level workers.

Combining the results of Chapters II and III, I now examine a situation in which workers determine endogenous effort and retirement within the context of a hierarchical internal labor market with exogenous heterogeneous ability.

The effort decision once again stems from the utility of consumption vs. disutility

of effort at every time t . This is represented by equation (2) from Chapter II. However, there is a subtle change in the capital accumulation (or savings) equation. Upon retirement, the retiring employee retains an interest in the company. This is fairly common in industry, particularly among upper level management. However, in order to keep the future managers exerting effort, they must be given some amount of residual claimancy on the profits of the firm. In order to simplify things, I make the only current residual claimant the present top manager. It turns out that, if the new manager has no residual interest in the firm, no effort will be expended by the new manager and the firm becomes essentially worthless. However, if any amount of residual is owned by the new manager, the amount of effort is constant. Hence, the residual earnings of the firm are completely determined by the actions of the new manager, which are binarily determined by whether any residual is owned by the manager. Now, the new manager could be required to purchase the firm prior to assuming control, but would have no incentive to do so if she had to pay full market price, which presumably would represent the full present value of the firm. Hence, the situation is open to negotiation. The old manager and the new manager must negotiate a share of the present value of this residual in some manner. The purpose of this study is not to examine the negotiations, so I will merely designate the amount of the firm's value eventually acquired in excess of payment by the new manager as Ω . This represents an increase in the new manager's savings, a portion of which will, in turn, has to be partly relinquished when the new manager retires and chooses a successor.

The one-time acquisition of a part of the residual of the firm acts as an increase in the individual's lifetime wealth. This acts to shorten the time until retirement for the

individual. If retirement entails some transaction (or transition) cost to the firm, this shortening of the length of time until retirement needs to be taken into account when determining the share of the residual allotted to the new manager. The marginal benefit of increasing the quality of managers retained needs to be weighed against the marginal cost of replacing retirees.

A MODEL OF PROMOTION, RETIREMENT AND EFFORT

I will deal with only a two-level hierarchy in this model, although the results should be generalizable to a multiple-level structure. The current upper level manager selects the replacement manager according to a ranking system like that elucidated in Chapter III. Given that the next manager has been selected (and her ability estimated), the old manager must determine his retirement date and the two individuals must negotiate the value of the amount of the firm which the new manager receives below the current market value of the firm. This acts as a lump sum signing bonus, Ω , which I will assume for simplicity occurs at the time of the previous manager's retirement, but which may occur at any time following the new manager's selection for the promotion. Again for simplicity, I assume that the retiring manager does not retain any interest in the firm upon retirement, but sells his share on the open market, where the price is determined by the present value of expected future profits. As a matter of fact, the manager cannot fully divest ownership in the firm until retirement, since that would eliminate any effort incentive for the manager, which would eliminate any value added by the manager and drive the firm's price to zero. This payment thus acts in accordance with the necessary wage structure for inducing

effort in the Chapter III hierarchical promotion system.

The utility function of all individuals is modeled exactly as in equation (2) and is the sum of the utility of consumption and the disutility of effort. The output flow of each individual is multiplicative in exogenous ability, a_i , and endogenous effort, z_i , as given in equation (1). Each individual also accumulates savings according to flow equation (52). This is similar to equation (3), but includes the fact that a portion of the firm's value is given to the new manager as a lump sum at the time she assumes control.

$$(52) \quad K'(t) = r(K(t) + \Omega) + az(t) - c(t).$$

I assume that the new manager becomes the sole residual claimant. Ω represents that amount less than the full discounted present value of future profits which the new manager has to pay for the ownership of the firm. It is equivalent to a lump sum payment - a signing bonus if you will. The fact that the firm has an expected future income stream implicitly assumes that the new manager and future managers will continue to exert effort. Allowing the new manager to purchase the firm for the market value minus Ω provides an incentive for her to accomplish the transaction. Otherwise, she would be indifferent between owning the firm and not. However, residual claimancy is required for her to perform the optimal level of effort. There are at least two other effects arising from the transfer, however. First, it decreases the incentive for the previous manager to give up the reigns, since the lump sum transfer comes from him. This may have the adverse effect of causing managers to remain too long at the helm. In addition, the amount of the transfer

which is retained increases the incentive for managers to retire. This effect was discussed in Chapter II. Ω increases the stock of accumulated savings and thus directly decreases the time until retirement. This works in the opposite direction to the previous effect. Including these effects into equation (52) gives equation (53), which applies to the old managers savings accumulation. The 1 subscripts indicate variables dealing with the old manager. The 2 subscript deals with the new manager. The 0 subscript indicates the manager prior to the old manager. The R_i indicates the time of retirement for manager i . This equation shows that the individual's savings accumulation changes upon retirement.

$$(53) \quad K'(t) = \left(r(K(t) + \Omega_1) + a_1 z(t) \right) \Big|_{R_0}^{R_1} + \left(r(K(t) + (\Omega_1 - \Omega_2)) \right) \Big|_{R_1}^{R_\infty} - c(t).$$

In analyzing the equilibrium decision, the results of Chapter II still hold true. The equilibrium level of effort is independent of the individual ability level. Thus, the assumption that the manager owns 100% of the company is innocuous with respect to effort, since having a lower level of ownership would be equivalent to the individual having a lower ability.

Why does residual claimancy matter at all, then? An infinitely small level of ownership would still induce the same optimal level of effort. When ownership is zero, the retirement decision is the one being affected. Essentially, retirement occurs immediately. While in Chapter II the retirement decision depended upon the level of risk aversion, this effect is eliminated in this extreme situation, since the equation goes to negative infinity when ability approaches zero, as shown in equation (54).

$$(54) \quad \frac{\partial R}{\partial a} = \frac{u'(c)}{u''(c)a^2 z r e^{-rR}} + \frac{(e^{rR} - 1)}{ar}.$$

Since the effort results are identical to those of Chapter II, I won't belabor the point here. However, the retirement decision changes due to the two opposing reasons mentioned above. This deserves some additional consideration.

The current manager of a firm experiences a constant disutility of effort throughout his management tenure. This creates the incentive for retirement. The receipt of the lump sum payment Ω moves the retirement decision closer to the present in exactly the same way as the variable κ in Chapter II. We can thus replace the κ in equation (19) with Ω giving equation (55).

$$(55) \quad (R - T) = -\frac{\ln\left(\frac{r\Omega}{az} + e^{-rR}\right)}{r}.$$

Dealing with the initial lump sum payment of Ω_1 , we have an unambiguous lowering of the retirement decision. This is demonstrated by the unambiguously negative relationship in equation (56).

$$(56) \quad \frac{\partial R}{\partial \Omega} = -\frac{1}{r\Omega + aze^{-rR}(1 - az)}.$$

The relationship is further garbled, however, by the reality that the current manager must

give a proportion of the firm to the new manager in order to continue the firm's existence. This occurs at the end of the current manager's tenure, however, and the current manager is able to accrue earnings on the full Ω until the divestiture occurs.

The retirement decision arrived at in Chapter II can be thought of as the baseline efficient solution, since this exactly balances the marginal utility of additional consumption as a result of additional time at work against the marginal disutility of effort. Since the payment of Ω is only an enticement to accept the manager position, it adds nothing to the productivity of the firm. Hence, the resulting earlier retirement caused by this lump sum payment must be inefficient relative to the Chapter II solution. The payment made by the new manager to the following manager acts to alleviate this inefficiency, but even a case in which the entire lump sum Ω is given to each manager in turn fails to completely alleviate this discrepancy. This leads to Proposition 6, the proof of which follows.

Proposition 6: The case in which identical lump sum payments Ω are given to each successive manager leads to an inefficiently early retirement by each manager when compared to the baseline case.

Proof of Proposition 6: This situation is identical to that of Chapter II, with the exception of the Ω payment. In addition, equation (53) becomes equation (57) when the two lump sum payments, Ω_1 and Ω_2 , are equated and we start at time 0, only considering one manager at a time.

$$(57) \quad K'(t) = \left(r(K(t) + \Omega_1) + a_1 z(t) \right) \Big|_0^R + \left(r(K(t)) \right) \Big|_R^\infty - c(t).$$

The situation is now even more similar to Chapter II, where the value function in equation (4) was maximized. The only difference is that Ω_1 is accumulating earnings during the period from time 0 to R , the retirement date for the manager in question. The amount of accumulated earnings is given by Q in equation (58).

$$(58) \quad Q = \int_0^R \Omega e^{rt} dt.$$

When the lump sum payment is turned over to the next manager, the accumulated earnings still remain in the possession of the previous manager. Thus, it acts in exactly the same manner as the κ in equation (19). This acts to reduce the individual's choice of R (the retirement point) in accordance with the relationship in equation (56). Thus, the retirement is inefficiently early relative to the base line, which completes the proof.

Taking the analysis to the next logical step, it is natural to ask what condition must hold such that the efficient retirement outcome is chosen. This leads to Proposition 7.

Proposition 7: The efficient retirement decision for manager i is reached when the lump sum payment paid by manager i to manager $i+1$ is given by $\Omega_{i+1} = \Omega_i + Q_i$, where Q_i is defined by the following equation.

$$(59) \quad Q_i = \int_{R_{i-1}}^{R_i} \Omega_t e^{rt} dt.$$

Proof of Proposition 7: The manager deviates from the efficient level of retirement whenever the lump sum payment and any income derived from it is positive. This causes a shortening of the time to retirement as shown by the relationship in equation (56) and this is what drives the result in Proposition 6. Whenever Q_i is positive, it acts as though it was a lump sum payment. Hence, one way to eliminate the effect of the lump sum payment is to incorporate both the original lump sum and any income derived from it into the next manager's lump sum payment. Thus we have no net gain to the manager from the lump sum and no distortion of the retirement decision.

Of course, there are many other ways to alleviate the inefficient retirement decision. After all, using the method in Proposition 7 makes the job of manager less attractive and there is no reason to believe that this would be the outcome of the negotiation process. However, another attempt to retrieve the desired retirement decision could be to allow the manager less than total ownership of the firm. This seems reasonable since, if a firm increases in size over time, it becomes increasingly difficult for one individual to purchase it at market value. Also, having a firm trade on the market makes the price determination significantly easier.

The effect of the less than total ownership of the firm by the current manager is identical to the effect if ability were to decrease. The new savings accumulation equation

is shown in equation (60). The variable α^{38} is the proportion of the firm owned by the manager. Any residual claimancy remaining is relegated to the $K(t)$ term in retirement, since the manager can no longer effect the firm's output.

$$(60) \quad K'(t) = (r(K(t) + \Omega_1) + \alpha a_i z(t)) \mathbb{R}_0^i + (r(K(t) + (\Omega_1 - \Omega_2)) \mathbb{R}_1^i - c(t).$$

Individual i (the manager) acts as though her ability is now αa_i and her retirement decision is now effected accordingly. Let $\beta_i \equiv \alpha a_i$ be the combined effect of ability and partial ownership. Using this, we can examine the effects of a_i on the retirement decision by first examining the effects of b_i and the relationship $\partial \beta / \partial a = \alpha$. Thus, all derivatives with respect to a are identical, with the exception of being multiplied by an α . Since α is strictly positive, the relationships do not change in any meaningful way other than being slightly smaller in magnitude.

Using this result, the retirement decision can now be modified by using variation in manager firm ownership proportion. For instance, as demonstrated in equation (26), when the Arrow-Pratt measure of absolute risk aversion is fairly low (i.e. the manager is not very risk averse), decreasing the amount of the firm owned by the manager will delay her retirement decision. The relationship determining this is given by equation (61), which is a modified version of equation (24) taking the existence of α into account.

³⁸ $\alpha \in (0,1]$.

$$(61) \quad \frac{\partial R}{\partial \alpha} = \frac{u'(c) + u''(c)c}{u''(c)azre^{-rR}}.$$

Examining equation (61) leads to the relationship expressed in equation (62), which shows how a manager's degree of risk aversion effects her response to changes in ownership percentage. This shows the identical relationship between retirement as a function of ownership percentage relative to risk aversion and that of ability.

$$(62) \quad A_R \equiv -\frac{u''(c)c}{u'(c)} > 1 \Leftrightarrow R_\alpha > 0.$$

Although it seems more likely to me that managers will be relatively less risk averse, simply knowing the characteristics of the manager in question will allow her retirement decision to be modified in order to return to the optimal retirement decision. Managers with relatively low risk aversion require low levels of ownership to move back their retirement date. Managers who are relatively more risk averse require higher percentages of ownership for the same result.

This result fits in nicely with the use of reduced (below market) price stock options to management. The reduced price acts as the Ω . The stock options give the manager a residual claimancy in the firm. The degree of claimancy can be varied to offset the inefficiency effects of the lump sum payment. The converse is also true.

The discussion thus far in this chapter has assumed complete information on the part of the participants. What if this were not the case? I show in Chapter III that the

expected wage or prize effects the effort exerted by the promotion candidate. In the current model, this prize can cause excessively early retirement. These effects can be mitigated by the judicious use of assignment of residual claimancy toward the manager. Yet, too much residual claimancy may have external effects on lower level employees.

If a manager is enticed to remain too long at a firm, the effort decisions of lower level employees are effected. The expected present value of the reward decreases the further it is pushed into the future. Thus, the reward either needs to be increased, or the current manager must be enticed to retire earlier. The machinations are similar to the situation caused, in Chapter III, if the likelihood of promotion decreases. For instance, if the number of competitors is greater than optimal, effort decreases across the board. If a manager takes too long to retire, in this case, he causes a bottleneck in the hierarchy, which decreases the present value of promotion for all workers below him. In the extreme, some workers may choose to retire who otherwise might have been promoted and add to the value of the firm.

The incentive to retire earlier can be accomplished in a myriad of ways, but three methods immediately spring to mind. Mandatory retirement is one. This can be modified in the case of a multi-level hierarchy into an up-or-out policy. If a worker has not been promoted after a certain amount of tenure he would be required to leave in order to make room for qualified lower level workers to move up. Another method is the judicious use of residual claimancy discussed previously. For managers with relatively low risk aversion, for instance, the retirement decision can be expedited by increasing the worker's ownership of the firm, thus increasing his wealth and incentive to retire and live on his

investments. The retirement decision is further expedited by the use of the lump sum payments which can be used as promotion bonuses. These three methods also increase wealth and speed up retirement.

CONCLUSION

When the top manager of a firm retires, she must be replaced. Without a high quality new manager, the value of the firm is lost. In addition, the manager must be induced to exert effort. The manner in which this is accomplished is by making the new manager a residual claimant on the results of his effort. Although not specifically modeled, the transfer of control entails a certain amount of negotiation between the departing and arriving individuals. In any event, the incoming manager receives some portion of the available prize, which increases wealth, thus affecting his retirement decision. This may hasten the retirement suboptimally even if the lump sum payment must eventually be paid again to the next incoming manager.

The inefficiency in retirement can be alleviated by the circumspect use of partial residual claimancy on the part of the manager. The exact nature of this tradeoff was not discussed, since it depends upon the degree of risk aversion of the incoming manager. Nevertheless, the optimal solution, if known, can be achieved.

These instruments may also be used to correct any inefficiencies resulting from external effects of retirement decisions. By this I mean the effects of upper level managers on the actions of those below him. Managers need to retire expeditiously in order to provide room for those employees below to be able to move up. This increases the

incentive they have to exert effort in order to enhance their promotion opportunity.

Delaying the promotion opportunity decreases the value of the promotion and may cause inefficient levels of effort, particularly among those whom success is measured in terms of output and whose output then determines their promotability. The tools used to affect the managers retirement decision can also be those mentioned above.

The retirement decision is a complicated one, made more so by many possible mitigating factors. The promotion, wage, and bonus system in addition to the allocation of residual claimancy strongly affect the efficient outcome. These relationships may be exploited in order to ensure that the desired results are achieved.

CHAPTER V

**PROMOTION AND RETIREMENT IN AN INTERNAL LABOR MARKET:
INCENTIVES FOR AIR FORCE OFFICERS**

INTRODUCTION

With the recent downsizing of the military, there have been many questions regarding the quality of the force remaining. While it is possible to eliminate the lowest quality of people in the military in one fell swoop, I wonder if it is advisable to do so. While eliminating all lower ability people will obviously create a military which is more able, will it create a military which is more effective; which does more? This is not merely an intellectual exercise or hypothetical question. Further reductions are planned, perhaps through the year 2003.³⁹

Ability and effort affect individual promotion opportunities. The retirement decisions of superiors affect the promotion possibilities of subordinates. The ability of upper level management externally affects the output of lower level laborers. Looking at the military as a firm, the structure of the firm affects the incentive structure of all levels in a hierarchy. This structure is the technology which enables the firm to increase output beyond what would merely be the case with a collection of individuals. It is into this complicated situation that I now jump with both feet.

Without a hierarchical promotion structure, worker effort is constant across workers and across time until retirement. However, this phenomenon is eliminated with the introduction of a hierarchical promotion structure. The interaction between ability and

effort must, therefore, be introduced when the probability of promotion is available. This effort increasing incentive cannot occur unless there is some added value to the firm from the upper level employees as compared to the lower positions.

This chapter therefore takes a look at a hierarchical promotion system wherein effort multiplicatively combined with ability increases the probability of advancement. Workers additionally experience disutility of effort and value consumption in their utility function which will be identical to that previously assumed in earlier chapters. Individual promotion probability is determined by each worker's flow of output. Workers endogenously determines effort and retirement based upon their ability and the number of competitors in the promotion system

The hierarchy in this case is required for organizational purposes.⁴⁰ The tournament structure is the result of the hierarchical structure of the firm. However, once the hierarchy exists, given that higher levels in the hierarchy provide a higher return for individuals, a tournament is necessarily introduced. The tournament serves two purposes. It provides information about the workers most deserving of promotion in terms of output and also serves an incentive to increase worker effort throughout the ranks.

In this chapter, I will further examine a promotion scheme with heterogeneous ability workers competing for promotion, providing a closer look at what factors may effect a firm's promotion structure. Within this context I then examine how the choice of retirement also affects the incentive structure, providing some additional insight into the structure of retirements. Following this, I make some observations concerning the military

³⁹ See GAO/NSIAD-97-78 Force Structure March 1997 and the Quadrennial Defense Review.

⁴⁰ Rosen (1982) also examines this idea in the context of a hierarchical control model.

hierarchy.⁴¹ I pay particular attention to the implications of the current force reductions and how they affect the military incentive and retirement structure. I focus on one particular area - that of Air Force officers. The internal military labor market makes this a reasonable division, since there is very little movement between services or from the enlisted to the officer corps other than at entry levels. These sections are then followed by a brief conclusion.

AIR FORCE OFFICER PROMOTION AND RETIREMENT SYSTEM

The military rank structure and retirement system is governed by Section 10 of the United States Code⁴². Within the confines of this system the individual services are given some leeway in their methods for deciding who gets promoted and when, as long as the method stays within the general parameters determined by Congress. I use the Air Force officer system as my primary example so that I may deal with specifics. However, the general setup is similar in the other services and also within the enlisted ranks.

My reason for using officer promotions is that the criteria for promotion are more nebulous than those for enlisted. Since enlisted personnel tend to be better characterized as technicians, their skill level and output are more easily measured. Since my modeling tends to deal with a managerial level in which output is orderable but not directly observable, the officer corps is a more appropriate forum for discussion.

I will first examine the promotion system of Air Force officers and then examine

⁴¹ Asch and Warner (1994) have developed a comprehensive model of military compensation, but mine is significantly simpler, uses a different type of worker effort, relies less on uncertainty to drive the model and adds a few new insights.

⁴² See 10 USCA 501-774, 1071-1090, 1161-1489.

the retirement system. Following that, I will outline some of the recent changes which have occurred during the drawdown.

Promotion

All services have a total of 10 possible officer ranks. Although there are different names for the ranks, all designate them alphanumerically as O-1 (lowest - Second Lieutenant or Ensign) through O-10 (highest - 4-star General or Admiral). For my purposes, I consider only what the Air Force calls "line officers," since medical officers, chaplains and lawyers are promoted separately. The number of officer positions available in each rank are determined by Congress and decrease with increasing rank. Pay increases with each rank achieved.

Promotion from O-1 to O-2 and then to O-3 is virtually automatic. Only those who have distinguished themselves in a negative fashion, such as by violating some part of the Uniform Code of Military Justice (UCMJ)⁴³, are not allowed to move to the next rank. The first truly competitive promotion occurs from O-3 to O-4. The performance of the officer throughout the career is considered when competing for the promotion. All following promotions are increasingly competitive as a relatively lower percentage is promoted to each higher position.

All officers are given several opportunities for promotion. After a certain number of years in each rank, the officer is considered to be "in-the-zone" for promotion. The officer may also be promoted early ("below-the-zone") or late ("above-the-zone".) There

⁴³ See 10 USCA 801-940.

are two possible promotion opportunities while an officer is in-the-zone. However, the vast majority of individuals promoted are promoted during their first opportunity in-the-zone. Above-the-zone promotions are rare and below-the-zone promotions are even rarer.⁴⁴

The system in the Air Force has one additional permutation. Prior to the actual promotion board, officers eligible for promotion consideration are ordered at the local level by their commanders and given a rating of definitely promote (DP), promote (P), or do not promote (DNP). For O-4, commanders are limited to giving 50% of their eligible individuals a DP. There is no limit on P or DNP ratings. The limit on DP ratings is 40% for O-5 and 20% for O-6. These ratings are given to the promotion board and have significant impact on the board's promotion decision.⁴⁵ Hence, the promotion determination is largely accomplished at the local level.

Once an officer has entered the above-the-zone promotion period without getting promoted, the number of years they are allowed to remain in the military is limited. For instance, those promoted to O-4 are allowed to stay until 18 years of service. Those promoted to O-5 are allowed to stay until 28 years of service while those promoted to O-6 and above are normally allowed to remain until 35 years. It has been Air Force policy to allow those who attain O-4 to stay until 20 years, but this is not mandatory. It is merely a show of good faith to allow these individuals to remain until vested in retirement.

⁴⁴ For example, from CY89-CY96 the Air Force average promotion rate to O-4 for those above-the-zone was 3.2% and below-the-zone was 1.9% according to the Air Force Personnel Center (AFPC) web site at <http://www.afpc.af.mil/analysis/promote/offprom.htm>.

⁴⁵ Again, according to the AFPC web site, on **average** 99.9% of those given a DP are promoted to O-4, whereas only 45% of those given a P are promoted. None of those given a DNP have ever been promoted.

Retirement

The military retirement is a cliff vesting system. An individual is not vested until 20 years of service. Once the 20 year point is reached, the individual is eligible to retire.⁴⁶ The military currently has people operating under three different retirement systems. Those who entered service prior to October 1980 receive 50% of their maximum achieved base pay plus 2.5% per year above 20, up to a maximum of 75% of base pay. Those who entered service after that date but prior to October 1986 receive the same basic retirement, but the pay is determined by averaging the top three years of base pay, thus lowering the overall payment. Those entering after October 1986 receive 40% of the average of the top three years of base pay plus 3.5% for every year over 20. While the lessening of the retirement payment may cause some individuals to leave the military prior to retirement eligibility, those who do remain will tend to remain for a longer period. Not only does this decrease the immediate costs of retirement, it delays the payment of the retirement until a later date. The extra year of service is one less year of retirement. The military is thus able to use the services of more experienced workers for a longer period.

While the changes to retirement pay structure may seem to be designed to lower the cost (and, hence, value) of retirement, they also have other effects. Using the top three years as the basis for retirement pay serves as an incentive for those who have recently been promoted to stay past the 20 year point so that their retirement pay will be calculated on a relatively higher basis. In addition, the most recent change offers a higher marginal benefit (3.5% additional per year vs. 2.5%) to those staying additional time in the

⁴⁶ See 10 USCA 1161-1489 for the details of the military retirement system.

military.

Recent Developments

The end of the cold war and reductions in the defense budgets have dictated a reduction in the size of the military. The necessity of military personnel downsizing has resulted in a multitude of programs designed to reduce the size of the military. Most of these have been instigated with the desire to reduce military personnel without having to resort to mass layoffs. The desire of Congress seemed to be to avoid the ill will such a move would create, particularly in the aftermath of the Persian Gulf victory. This viewpoint has not always been the case. The conclusion of most wars has usually resulted in a massive reduction of military forces. With the all volunteer force, this is no longer considered politically feasible or desirable.

When the military force drawdown first started in 1986 the Air Force, not fully comprehending the length and depth of the force reduction, attempted to take care of the matter merely by reducing accessions. While this could work temporarily, it soon became obvious that this would result in a severe shortage of officers at certain ranks over time as the large gap in available promotion candidates worked its way temporally. The Air Force had to ensure that it had an adequate pool of qualified promotion candidates at higher ranks in the future. The personnel managers began then to look at reducing the number of officers at middle and upper levels.

Rather than directly force individuals to leave the service, one approach that Congress came up with to entice personnel to voluntarily leave the military service was the

Voluntary Separation Incentive/Special Separation Benefit (VSI/SSB) plan from January 1992 to January 1995. The VSI was a series of payments which varied with rank and time in service and the SSB was a single lump sum payment also based upon the same factors. Both were designed to facilitate an increase in voluntary separation from the military. The implementation of these programs was conducted slightly differently between services. The Air Force had a policy in which they made it abundantly clear which individuals were in danger of being involuntarily separated. For these people, the bonuses were obviously a far better option. The VSI/SSB payments were not just open to those who were identified as substandard employees. Some career field in which there were shortages (such as pilots) were not eligible.

Another option which the Air Force resorted to was called the Selective Early Retirement Board (SERB). Individuals in the grades of O-5 and O-6 who had been passed over for promotion were reviewed and those deemed nonessential were required to retire prior to their usual 28 or 35 year time limit. The idea behind these boards was that it was less of an implied breach of contract to force those already vested in retirement to leave than to force those who had in good faith remained with the military for quite a while to depart prior to retirement vesting. When these steps still proved less than enough to cause the required number of individuals to leave, another step was taken.

Congress has already approved the military services to authorize early retirement from the military after the 15 year point through Fiscal Year 1998. In other words, the retirement vesting point has been moved to an earlier point for those eligible. The Air Force has again chosen to implement this program selectively, making only certain

individuals eligible.

Thus the drawdown has been accomplished by the Air Force using reduced accessions, the VSI/SSB program, the SERB and early retirement. I will examine a simplified model of the Air Force officer rank, promotion and retirement system and follow this model by an examination of the Air Force drawdown programs and the incentive effects they may have had on military personnel effort.

A SIMPLIFIED MODEL OF THE MILITARY PROMOTION SYSTEM

I combine the results of models discussed in the previous chapters in an attempt to get somewhat near the military retirement system. Consider a two level hierarchy. The first level offers a constant flow wage of w . The second level offers a return on effort as compensation of βaz , with $\beta > 1$. Unless otherwise noted, variables remain as previously defined. The two-stage hierarchy is obviously a simplification, so β implicitly contains the probability of future promotion. As such, it would probably be more accurate for me to use $\beta(a)$, but, since $\beta(a)$ is monotone increasing in a , this would only change the magnitude of the results and not the direction. Hence, I treat β as a constant without loss of generality.

An individual begins employment at time 0. At that time he chooses a level of effort z_1 to be expended during the entire first period, from time 0 to time τ , which is exogenously determined by the employer. At time τ , the promotion decision is made by the employer and the individual is promoted with probability $p(az_1, \pi)$. az_1 is the flow marginal product exerted by our hero during the first period, which I assume to be

constant and this turns out to be correct. This implicitly assumes no learning by the worker occurs concerning the exact level of ability of co-workers. This assumption may be a little tough to believe particularly when dealing with small groups. However, this earning can be reduced by frequently moving laborers between different groups. π is a parameter on probability determined exogenously by the employer and subject to change. The first derivatives of both arguments in the promotion probability function are positive. If promoted, the worker will receive the wage flow βaz_2^u . If not promoted, the worker is terminated from the firm and earns his opportunity wage az_2^l until he chooses to retire. The worker receives one shot at promotion. At the time of promotion, the worker chooses effort level z_2^u and retirement date $\tau + R^u$ if promoted or z_2^l and $\tau + R^l$ if not promoted. Initially, I will place no restrictions on the time of retirement chosen by the workers. This, however, will be examined more closely later. I assume a discount rate of r (both internal and external.)

$$(63) \quad V(0) = \max_{z_1, z_2^u, z_2^l, R^u, R^l} \left[\int_0^\tau (u(c_1) - d(z_1)) e^{-rt} dt \right] + \frac{e^{-r\tau}}{r} \left[p(az_1, \pi) (V(\tau^u) - V(\tau^l)) + V(\tau^l) \right].$$

The value functions after the promotion decision has been made are designed to mimic equation (17).

$$(64) \quad V(\tau^u) = \left(\int_0^{R^u} (u(c_2^u) - d(z_2^u)) e^{-rt} dt \right) + \frac{e^{-rR^u}}{r} u(c_2^u).$$

$$(65) \quad V(\tau^1) = \left(\int_0^{R^1} (u(c_2^1) - d(z_2^1)) e^{-\pi t} dt \right) + \frac{e^{-rR^1}}{r} u(c_2^1).$$

I will solve the worker's problem by backward induction. Choices are made at time zero based upon the optimal choices made later on. Hence, I will look at the post-promotion opportunity period first. Given these optimal choices, I will then examine the first period. In examining the second period, I assume a level of savings at time τ of $\kappa(a, \tau, B)$. B is the retirement benefit differential between those promoted and those not. It is similar to Ω from Chapter IV, but is not interpreted as the benefit of purchasing residual claimancy for a reduced amount. For simplicity, I assume that non-promotees receive 0 retirement from the employer. The expected present value of B must be, however, at least as great as the expected difference between the expected present value of the difference between the wage earned during the first employment period and that which could have been earned through an alternative employment source. Otherwise, it would be impossible to entice potential employees to accept employment. The savings accumulation depends upon B , since the amount of individual savings changes based upon a change in expected benefits in a manner similar to that discussed in Chapter IV. Also, τ changes savings as the time available to accumulate the savings increases. Ability effects savings accumulation both in earnings and in the probability of promotion. Thus, the time to retirement determined in both periods is always modified through the mechanism in equations (18) and (19). By inspection, it is obvious that $\frac{\partial(R - \tau)}{\partial \kappa} < 0$, the time to retirement decreases with an increase in accumulated savings. κ may be negative, which

merely indicates dissaving in the pre-promotion time period.

The value function in equation (64) is similar in setup to equation (17), so I will examine it first. Since consumption, effort and savings are constant throughout the period of work, the situation simplifies to equation (66), which is maximized subject to equation (67). This takes into account the fact that there is a constant flow of savings into the retirement account.

$$(66) \quad V(\tau^u) = \max_{z_2^u, R^u} \left(\frac{(u(c_2^u) - d(z_2^u))(1 - e^{-rR^u})}{r} \right).$$

$$(67) \quad c_2^u = \beta a z_2^u (1 - e^{-rR^u}).$$

The first order conditions are given in equations (68) and (69). Together with equation (67) they give the three equations required to determine c , z and R for the upper level position in the second time period.

$$(68) \quad V_z^u = \left(\frac{1}{r} \right) \left(u'(c_2^u) \beta a (1 - e^{-rR^u}) - d'(z_2^u) (1 - e^{-rR^u}) \right) = 0.$$

$$(69) \quad V_R^u = u'(c_2^u) \beta a z_2^u e^{-rR^u} - d(z_2^u) e^{-rR^u} = 0.$$

Manipulating these results gives equation (70), which is a slightly modified version of

equation (20) and in which the second equality comes from equation (69). Equation (21) holds exactly as it did before. Effort is still exerted at the point at which it imposes the minimum average disutility to the individual. The reason that these equations are so similar is that β merely acts as an ability multiplier. Individuals now act as though they have ability βa rather than merely a . In a sense, βa can be considered position augmented ability. The general relationships, however, still hold. Hence, the superscript on z is redundant in period 2.

$$(70) \quad u'(c_2^u) = \frac{d'(z_2^u)}{\beta a} = \frac{d(z_2^u)}{\beta a z_2^u}.$$

Rewriting the second equality in equation (70) and noting that consumption in retirement is equal to both the investment income and to the pre-retirement consumption gives us equation (71), which is a simplified version of equation (21). Retirement occurs when the flow disutility of effort per unit of output is equal to the marginal utility of consumption, keeping in mind that the level of consumption at retirement is also effected by κ , the level of accumulated savings at time τ .

$$(71) \quad d(z_2^u) = u'(c_2^u) \beta a z_2^u.$$

Rather than rehashing the relationships between ability and retirement or consumption, I will point out that the relationships in equations (26) and (27) are still valid. Ordering

workers by retirement or consumption is still not possible as long as there exists a possible distribution of risk aversion among the workers. This has interesting implications for firms which choose β . As β increases, employees with low risk aversion will retire earlier and those with high degrees of risk aversion will retire later. If β can be interpreted as including the likelihood of further promotion, raising the probability of promotion may decrease the long term retention of high ability workers who are relatively less risk averse. This creates a sorting process in two dimensions. Raising β will result in higher ability risk averse workers intermixed with lower ability workers who have relatively low risk aversion.

Another interesting item of note: the retirement decision is unaffected by a firm-sponsored pension plan unless the firm offers a pension that is somehow higher than what the individual would have been able to accumulate on her own, assuming equal risk between the pension plans. To see this, remember that individuals save a constant proportion of their income. If the employer withholds less than this proportion, the worker will privately save the balance. If the employer withholds more, but eventually pays it back in the form of a pension, the worker will borrow against her future pension. With no liquidity constraint, the results are a wash and worker behavior is unaffected. However, *ceteris paribus*, changing B still changes worker behavior when κ is held constant in the short run.

The employer is constrained to return the entire expected value of the earnings in order to attract employees. There is a little leeway, however, with $\beta > 1$ and alternative wages available at az (the marginal revenue product of the next best alternative).

Presumably, however, the individuals who get promoted in the military would do similarly well on the civilian job market.⁴⁷ To the extent that the size of β is job specific, there exists some negotiation power on the part of the employer both in β and in B .

The value function in equation (64) is for the workers who were not promoted. I assume that the workers not promoted depart the firm and earn their flow marginal product az_2 until their final retirement. Since z_2 is the same in either promotion position eventuality, I have dropped the superscript. Once workers have been passed over for promotion, there is no longer an incentive for the worker to exert effort and, thus, produce output. Unless the firm has an up-or-out policy, the worker will remain on the job until the marginal utility of added consumption flow⁴⁸ from the wage, w is balance by the disutility of having to show up to work but not exerting any effort, $d(0) = u'(c_2^1)w$. The military euphemistically calls officers in this category, ROAD officers (for "Retired On Active Duty"). In an effort to avoid this phenomenon, the employer may choose to monitor worker effort. If there is a cost of monitoring, $m(z)$ (with positive first and second derivatives), the optimal amount of monitoring from the employer perspective will be where $m'(z_m) = a$. Thus, optimal monitoring depends upon individual worker ability.

There are two problems with this. First, the optimal amount of effort will be induced only where equation (72) holds, which, given a continuous ability distribution, occurs with probability 0. Second, assuming workers are able to earn income equal to

⁴⁷ Some cross section analysis of this subject has been done. Military starting wages tend to be lower than what the individuals were earning in the military. However, this can be explained by the fact that military retirees choose to work less and also are willing to start at a lower wage in order to gain entry into the job market, progressing through the ranks as civilian job experience increases. Gotz and McCall, (1984).

⁴⁸ Keep in mind that consumption is also effected by the savings accumulation, κ , at time τ .

their potential output on the outside, increasing worker effort with ability will cause all workers above a cutoff ability, $\hat{a}$, to leave the firm and pursue other employment where a more efficient level of effort is exerted. This occurs because of the fact that even with a minimum amount of monitored effort, z_m , the retirement decision is based upon a consumption that is independent of ability. The only individuals who will choose to not remain are those who can eventually earn more on the outside - the higher ability workers. Alternatively, the effort level could be set based upon average worker ability, $\bar{a}$. Again, however, this is only efficient when equation (72) holds, which may or may not be the case. The adverse selection problem still remains, but is not as severe, particularly when considering that the workers who have not been promoted have already been sorted by ability. However, the firm still only retains the relatively low ability workers.

$$(72) \quad d(m'^{-1}(\bar{a})) = \frac{d'(m'^{-1}(\bar{a}))}{m'^{-1}(\bar{a})}.$$

There also remains the issue of whether or not the firm desires to retain the workers who are not promoted. These passed over employees cause several effects upon the first period employees who have yet to come up for promotion. If the firm is not growing, the more passed over employees retained, the longer employees must wait for a chance to be promoted (the greater is τ). In addition, retaining workers after they have not been promoted decreases the relative reward to promotion, which may have a feedback on period one worker effort. Also, it may cost more to retain passed over employees due to the cost of monitoring, particularly since output is monitored for first

period employees, whereas effort is monitored for passed over employees. A separate monitoring system must thus have to be put into place. In a sense, first period employees also provide more value to the company. Not only do they provide output, but they are a potential source of future upper level managers. Whereas, the passed over employees have already been sorted for quality and been found relatively lacking. These are all arguments favoring an up-or-out promotion system. I assume a mandatory retirement date, R_m , for those individuals who are passed over for promotion. This date may be determined by the point at which the marginal cost of monitoring is equal to expected ability, implicitly including the lowered benefits to using second period versus first period workers in this cost of monitoring. Further simplifying the situation, I assume $R_m = \tau$. When workers are not promoted, they are summarily dismissed.

The first period analysis contains the most complicated interaction. In order to simplify the analysis, I will attempt to find a simplified method for including the present value of the two possible second period outcomes. In addition, for those not promoted, I assume that the adverse selection and moral hazard problems are such that the employer has a strict up-or-out policy, where those not promoted are asked to leave at time τ . Those not promoted will then enter their next best alternative and earn their flow marginal product, az , until their eventual final retirement, R^i . The level of effort and consumption become what they would have been had az always been the wage and time to retirement, $R^i - \tau$, is determined by equation (19), where $i = u, l$ (for the upper level vs. non-promoted individuals). Since I know that consumption and effort are constant during the second period, the time to retirement is effected by κ only through equation (19) in either second

period case. I assume that promoted individuals remain employed in their current position until retirement, since $\beta az > az$. The level of effort is identical for all individuals after the promotion decision, since all individuals choose to exert effort where equation (21) holds, which is independent of worker ability. The choice at time zero can thus be reduced to pre- τ effort and pre- τ consumption conditional on the optimal choices once the promotion decision is made at time τ . Equation (63) can be simplified to equation (73).

$$(73) \quad V(0) = \max_{z_1, c_1} \left[(u(c_1) - d(z_1)) \left(\frac{1 - e^{-r\tau}}{r} \right) + (p(az_1, \pi)(V(\tau^u) - V(\tau^l)) + V(\tau^l)) \left(\frac{e^{-r\tau}}{r} \right) \right].$$

Using the results of equation (19), the value functions at time τ can also be restated.

$$(74) \quad V(\tau^u) = \max_{z_2, R^u} \left(\frac{u(c_2^u)}{r} - \frac{d(z_2)}{r} \left(1 - \frac{r(\kappa + B)}{\beta az_2} - e^{-rR^u} \right) \right).$$

$$(75) \quad V(\tau^l) = \max_{z_2, R^l} \left(\frac{u(c_2^l)}{r} - \frac{d(z_2)}{r} \left(1 - \frac{r\kappa}{az_2} - e^{-rR^l} \right) \right).$$

With constant consumption, the amount of savings accumulated by the individual worker on her own, κ , can be described by equation (76). It may be negative. This savings merely changes the time to retirement decision from τ onward. However, it does not change R^u or R^l , which can be interpreted as the length of time until retirement in the respective situation when there has been no saving or dissaving ($\kappa = 0$).

$$(76) \quad \kappa = (w - c_1) \left(\frac{(e^{r\tau} - 1)}{r} \right).$$

Equation (67) still holds when optimizing equation (74), while equation (77) is used with equation (75). Similar results hold.

$$(77) \quad c_2^1 = az_2(1 - e^{-rR^1}).$$

The first order conditions for the optimization problem in equation (73) give the relationships described in equations (78) and (79). Equation (78) comes from the derivative with respect to effort. The marginal disutility of effort exerted during the promotion attempt per ability-weighted change in promotion probability is determined by the difference in the values of being promoted versus not being promoted.

$$(78) \quad V_{z_1} = 0 \Rightarrow \frac{d'(z_1)}{ap_1(az_1, \pi)} = \frac{V(\tau^u) - V(\tau^l)}{(e^{r\tau} - 1)}.$$

Equation (79) comes from the condition with respect to consumption.

$$(79) \quad V_{c_1} = 0 \Rightarrow u'(c_1) = p(az_1, \pi) \frac{d(z_2)}{\beta az_2} + (1 - p(az_1, \pi)) \frac{d(z_2)}{az_2}.$$

Noting the relationships in equations (67) and (77) and combining them with the relationship in equation (79) gives equation (80). The marginal utility of consumption during the promotion attempt is a weighted average of the marginal utilities of consumption which will be chosen after the promotion decision has been made.

$$(80) \quad u'(c_1) = p(az_1, \pi)u'(c_2^u) + (1 - p(az_1, \pi))u'(c_2^l).$$

Using equations (67) and (77) also gives us the relationship that $u'(c_2^u)\beta = u'(c_2^l)$. Hence we can conclude that $c_2^u > c_2^l$. The level of consumption prior to the promotion decision must therefore be between that chosen if promoted and that chosen if not. The promotion decision therefore coincides with a discrete change in consumption for the individual workers, depending upon their success in being promoted.

Assuming the second order sufficiency condition holds, Cramer's Rule can be used to analyze the comparative statics. In particular, I would like to analyze the effects of ability (a), the augmentation to ability after promotion (β), the retirement bonus (B), an exogenous change in promotion probability (π), the promotion timing (τ) and the pre-promotion wage (w).

Like the one-period situation, the relationship between consumption and ability depends upon the underlying risk aversion of the individual. This comes from the feedback effects of ability on consumption and retirement after promotion and the curvature of the promotion probability function. Basically, a change in ability affects both the difference in the value functions after the promotion opportunity and the probability of

promotion. Consumption and effort must now be chosen by the individual taking both these feedbacks into account. These same feedback effects now make pre-promotion effort dependent upon ability. These relationships are extraordinarily complicated, however, and I can deduce no insight from them other than to point out that, once again, it is impossible to separate high ability individuals by effort, consumption or retirement decision. Also, since β acts merely as an ability multiplier, the relationship between pre-promotion consumption or effort and β mirrors that of effort. Luckily, the other relationships examined are more tractable.

The employer has some leeway in choosing B , which can be considered the present value of retirement. In reality this may be increasing in ability. The retirement package value affects the amount of time until the worker chooses to retire. The higher the value of the package, the earlier the retirement, unambiguously. This package can be varied in many ways. Vesting is one way. In this setup, I assume that the worker is vested upon promotion. In the military, this is not quite the case. Upon reaching a certain rank, individuals are allowed to stay until retirement, but are not able to retire until having served a minimum of 20 years. After the 20 year cliff, the retirement pay increases with each year of service. Changing the time at which retirement is available obviously changes the present value of retirement. Changing the rate at which retirement increases once vested also effects the present value. Changing the present value of retirement effects both the pre-promotion consumption and effort. Before the promotion opportunity occurs, effort and consumption tend to vary together, although the magnitudes differ. In this situation, a change in B effects the value of promotion. This changes the amount of

effort exerted by the worker in an attempt to get promoted. This, in turn, changes the amount of consumption due to the change in probability of higher earnings after promotion. Hence, I am able to deduce the results in equation (81).

$$(81) \quad \text{sgn}\left[\frac{\partial z_1}{\partial B}\right] = \text{sgn}\left[\frac{\partial c_1}{\partial B}\right] > 0.$$

Changing the probability of promotion independent of worker effort has a slightly more complicated effect upon consumption and effort, but the intuition is straightforward. The relationship between π and effort is given in equation (82). $\Delta V(\tau)$ denotes the difference in value between equations (74) and (75). The subscripts on p denote the arguments with respect to which the partial derivatives have been taken. The first portion is unambiguously positive. This is the direct effect of increased promotion probability on effort. The second, however, depends upon the feedback effect on the return to worker effort caused by the exogenous change (p_{12}). Should the return to increasing effort decrease as a result of the change in π , there is a possibility that increasing promotion probability may actually decrease worker effort, acting like a probabilistic income effect. This is important, since the employer presumably has quite a bit of influence on the promotion probability structure. This is similar to the result in Proposition 1. Judiciously manipulating τ may help to mitigate this effect in terms of aggregate worker performance.

$$(82) \quad \text{sgn}\left[\frac{\partial z_1}{\partial \pi}\right] = \text{sgn}\left[\frac{p_1 p_2}{a} \left(\frac{d(z_2)}{z_2}\right)^2 \left(\frac{\beta-1}{\beta}\right)^2 (1 - e^{-\pi}) - (a p_{12} \Delta V(\tau) u''(c_1) e^{-\pi})\right].$$

The relationship between π and consumption is equally interesting and is shown in equation (83). Again, the first portion is positive and is the direct effect of the increase in promotion probability on consumption. The second term can be explained as the “why bother” effect. Since promotion probability has increased, the individual has less incentive to exert effort and, thus, decreases consumption in favor of reduced effort.

$$(83) \quad \text{sgn}\left[\frac{\partial c_1}{\partial \pi}\right] = \text{sgn}\left[\frac{p_2 d''(z_2)}{a} (e^{\pi} - 1) + (a \Delta V(\tau))(p_1 p_{12} - p_2 p_{11})\right].$$

Putting off the promotion opportunity until later reduces the present value of the promotion. Delaying promotion also increases the time during which savings (κ) accumulates, which helps to offset the delay in retirement caused by the delay in promotion. Thus, we would expect a delay in promotion to decrease effort, since exerting effort provides less return. However, if a large amount of dissaving occurs during the pre-promotion period ($w \ll c_1$), then effort may increase to keep eventual retirement from being delayed too long due to the lowered chance of promotion and a longer period of repaying accumulated debts. This can be seen in equation (84). Consumption is subject to the same effects as can be seen in equation (85).

$$(84) \quad \text{sgn} \left[\frac{\partial z_1}{\partial \tau} \right] = \text{sgn} \left[\left(\frac{p_1(\beta - 1)}{\beta} \right) \left(\frac{d(z_2)}{z_2} \right) \left(\frac{e^{-r\tau}}{r} \right) (w - c_1) + d'(z_1) + ap_1 \Delta V(\tau) \right].$$

$$(85) \quad \text{sgn} \left[\frac{\partial c_1}{\partial \tau} \right] = -\text{sgn} \left[\left(\frac{p_1(\beta - 1)}{\beta} \right) \left(\frac{d(z_2)}{z_2} \right) \left(\frac{e^{-r\tau}}{r} \right) (w - c_1) + d'(z_1) + ap_1 \Delta V(\tau) \right].$$

The first period wage only effects first period consumption and effort through the feedback effect of κ . The first period consumption is chosen as a weighted average of the two possible post-promotion consumptions. The value of second period situation depends in part upon the earlier savings decision. Increasing the first period wage increases savings through equation (76). This decreases the relative value of promotion, since the savings accrues irrespective of whether promotion occurs. Hence, an increase in first period wage leads to a decrease in effort. The argument for the effect of increased first period wages on first period consumption is similar.

APPLICATIONS TO MILITARY FORCE REDUCTIONS

With the demise of the cold war, it is only natural that the United States look toward reducing the size of the standing military. Yet, the end of the Cold War does not mean the end of threats to national security, as the war against Iraq poignantly illustrates. Thus, the reduction in force strength needs to be accomplished with circumspection, preferably avoiding the "hollow force" of the late 1970's and the unpreparedness prior to World War II, Korea and other less blatant instances. This all needs to be accomplished in the context of the hierarchical internal labor structure within which the military operates.

The basic compensation structure of the military seems sound. A hierarchical structure elicits effort on the part of lower level workers in the hopes that they are promoted. The multiple promotion levels ensure continued effort for those who are promoted. The up-or-out promotion system⁴⁹ ensures that individuals who are not promoted are asked to leave, avoiding a reduction in effort incentive often known in the military as the Retired On Active Duty (ROAD) syndrome. Those selected for promotion to the O-4⁵⁰ pay grade (the rank of major in the Air Force) are eligible to remain until retirement. The higher the rank achieved, the greater the time until mandatory retirement. This seems to fit the model I have proposed reasonably well.

While it would seem obvious that the military prefers to keep only higher quality individuals and weed out the lower quality individuals during a force draw down, this may not be possible, particularly using a voluntary separation system. With heterogeneous workers, I have shown that it is not possible to sort individuals by ability using their retirement decision as a metric. Risk averse workers tend to act in a similar manner to lower quality workers. Conversely, higher quality workers and relatively less risk averse individuals tend to behave similarly. In addition, removing only low quality from the ranks while reducing the number of individuals on active duty may pose other problems as well.

Assuming the military was at a stable equilibrium prior to the drawdown, a one-shot reduction in low quality individuals would have a detrimental effect on effort throughout the ranks. Whereas prior to the drawdown, an individual may have had an excellent chance at promotion, following the elimination of lower quality workers the

⁴⁹ See 10 USCA 611. Chapter 36 deals with military retirement, separation, etc.

⁵⁰ The "O" indicates an officer pay grade, the number indicates the level of rank achieved.

chance of promotion is much smaller. In keeping with the relationship in equation (82), the effort level for many workers will probably decrease, thus keeping higher quality workers who may exert less effort than previously. The entire distribution of effort across the ability spectrum will be changed. There will be a less deleterious effect on effort if reductions were made across the entire spectrum of ability. There is some evidence that this was the case throughout the drawdown.⁵¹ However, some mission elements were reduced to a greater extent than others.⁵² Hence, there probably has been some amount of changes in effort incentive in the Air Force and the rest of the military.

The reduced accession of the late 1980's undoubtedly will have some amount of shifting in the structure of effort incentive for those officers who entered service during that time period. Presumably the Air Force will have a requirement for a certain number of higher ranking officers which will have to be drawn from these under-sized cohort groups. Individuals of given abilities will have proportionately greater chances of promotion in these cohorts than in previous ones. Hence, in the manner of Proposition 3, the distribution of effort across the ability distribution will change, probably for the worse.

There are other changes which have changed at least the perceived probability of promotion. After all, perception is what counts. The Air Force's relatively localized promotion system appears to have made the troops less than perfectly satisfied.⁵³ If this leads to a lowered probability of promotion (or at least the perception thereof) effort will be reduced. This may not, however, necessarily be the case. The promotion system reduces the promotion competition more toward the local level, with commanders having

⁵¹ See GAO Report GAO/NSIAD-95-97 Military Personnel (1995).

⁵² See GAO Report GAO/NSIAD-97-78 Force Structure (1997).

a very large input into the promotion process. As I showed in Proposition 5, the lower the number of contestants, the better. Workers may dislike the promotion system because it makes them work more, not less. During the drawdown, however, if promotion opportunities are reduced, the level of effort will decrease.

Reducing the value of retirement will have an unambiguously bad effect on worker effort. This includes any attempt to reduce retirement benefits which were previously in effect, whether or not such benefits were legally guaranteed under law. When retirees have their benefits reduced, those still on active duty react to this, since it represents a decrease in the present value of their potential retirement (B). This can happen in insidious ways. For example, when bases are closed, the military retirees in the surrounding community no longer have access to the military health services. This reduces the value of retirement and has a ripple effect throughout the entire active duty force.⁵⁴ In addition, the reduction in military retirement at the 20 year point (from 50% of base pay to 40%) almost certainly had a negative effect on effort. Granted, this reduces the cost of retirement to the military. However, the marginal savings needs to be weighed against the marginal cost of the reduced effort.

One action which has probably had a positive effect on effort is early retirement. Congress has authorized the military to allow voluntary early retirement to certain individuals (the criteria are left up to the individual services to dictate). This has the effect of increasing B, the present value of retirement. To the extent that military personnel expect this to continue, effort is enhanced. What drives this increase in effort is not the

⁵³ See Callander (1995).

⁵⁴ See "Base Closure and Health Care", *Air Force Magazine* (1995).

increase in benefit itself, however. It is the increase in probability of receiving the benefit which drives the effort. Separation benefits, therefore, are another problem altogether.

The Air Force's implementation of the VSI/SSB program was an attempt to sort out the low ability. In this program, certain military members were offered the opportunity to accept a bonus payment in return for voluntarily separating from the military. It seems obvious that those who accepted the offer self-identified themselves as having a lower present value of remaining in the military than the value of the bonus. Obviously, some of these were lower quality, but I have shown that, in terms of retirement (or, equivalently in this case, separation), it is not possible to strictly order by ability. The military lost some high quality individuals who happened to be relatively more risk averse. While there may have been a large number of low quality individuals departing the service, particularly if the services were able to sort for eligibility, a large number of highly qualified individuals may also have left. This is not all bad, however, since it is better to have a reduction in force which spans the ability spectrum, thus minimizing adverse incentive effects.

Selective early retirement in which higher ranking individuals are chosen for retirement prior to voluntarily retiring appears to have desirable incentive effects. Holding a board which selects certain individuals to retire early acts like an additional promotion point, with all the increased effort that entails. This further reduces the detrimental effects of ROAD officers. In addition, reducing the length of time some officers are on active duty, allows more promotion opportunities for those in lower ranks. This increases the probability of promotion.

CONCLUSION

The military is experiencing a huge downsizing effort in the wake of the Cold War. In an effort to determine incentive effects for those remaining on active duty after the dust settles, I have created a simplified model of a hierarchical promotion system. In developing this model, I use some of the results and relationships developed in earlier chapters. The hierarchy is based upon the external effects which the upper level workers have on the lower level ones. Through superior decision making ability upper level workers provide an externality which increase the value of effort exerted by the lower level workers. Firms desire to have the most able individuals in the highest positions. In a hierarchical promotion system, when output determines the probability of promotion, the greatest effort is provided by those who expect to be near the promotion cutoff. Also, the smaller the number of individuals competing for promotion in a group, the greater the aggregate effort, since the likelihood of being near the cutoff point is greater. In determining a workers retirement and consumption decision, workers attempt to smooth consumption throughout life and save in order to accumulate enough wealth to avoid the effort of working after a point. Workers' retirement decisions are determined not only by their individual ability, but their degree of risk aversion, making it impossible to order workers by ability using the retirement decision.

Putting all these results together into a simplified model of the military promotion and retirement system, I show that worker's effort is dependent upon the probability of future promotion and the value attached to it. The more valuable the promotion and the retirement opportunity attached to it, the more effort is produced. Workers consume

based upon their expected lifetime income and retire when they've accumulated enough wealth to continue in the lifestyle they've become accustomed to.

I then take a look at the military promotion and retirement system and explore some possible implications of my model with respect to the recent military drawdown. Some of the Air Force's actions appear to have positive incentive effects and some negative. While it is not obvious anecdotally whether the net effect was good or bad, it is obvious that incentive effects were present in all of the drawdown programs.

Certainly some of the programs were budget driven, but the military must be cognizant of the incentive effects produced throughout the drawdown and actively pursue a policy which creates the relatively better outcome given the constraints under which they operate.

CHAPTER VI

CONCLUSION

SUMMARY

Workers of varying ability get jobs, exert effort, get paid, save, spend, consume and retire. How they go about doing this and what determines their incentives is an interesting question. Many of these subjects have been examined in the economic literature to some degree. I have tried to provide a more unified approach to modeling these questions. In tackling this problem I broke down the subject into its components and examined them separately, later bringing them together. I first examined endogenous effort and retirement choice with heterogeneous workers. Next, I examined the structure of a hierarchical firm and looked at the nature of a hierarchical promotion system with endogenous effort and heterogeneous workers. Following that, I looked at the retirement and effort choice of heterogeneous workers in a hierarchy. Finally, I attempted to apply the previous results to a more particular real world problem, that of the military in general and Air Force officers in particular. This chapter provides a brief synopsis of each model, an outline of the results and a brief discussion of a few of the possible implications.

Retirement and Heterogeneous Ability

In Chapter II I analyze a model of workers who are heterogeneous in ability. They are paid their marginal value product and their marginal product is the product of ability and effort. Workers receive disutility from effort and utility from consumption. All prices are normalized to 1. Workers choose their level of effort and consumption endogenously.

The workers may retire at a time of their own choosing. Since there is no worker interaction, I can approach this situation from the perspective of the individual worker.

It turns out that, in order for workers to choose a finite length of employment, I must assume that the disutility of zero effort is positive. In other words, the worker must experience some disutility from employment even when no effort is expended. This actually seems plausible since, presumably, a worker would prefer to not have to get up out of bed, get dressed and go to work even if no effort was expended once arriving. Also, there is the opportunity cost of not being able to experience leisure while at work, although leisure is not explicitly modeled here.

With some reasonable simplifying assumptions, workers tend to consume at a constant level throughout their lifetime. They exert a constant level of effort throughout their employment. They earn an amount greater than their consumption while employed and save the difference. Once this savings has accumulated to the point at which the interest income on accumulated savings is enough to support the workers level of consumption, the worker chooses to retire.

The level of effort exerted by the worker is independent of ability. Each worker, regardless of ability, chooses to exert effort at the point where the average disutility of effort per effort exerted is at a minimum. In addition the retirement decision occurs where the disutility of effort is equal to the marginal utility of added consumption plus the added benefit from increasing the savings accumulation slightly.

Examining the comparative statics shows that the retirement and consumption decisions are not able to provide complete information concerning the ability of the

individual. Workers cannot be simultaneously ordered by ability and retirement age without further knowledge about worker risk aversion characteristics. If we allow worker disutility of effort to vary, those who have greatest disutility of effort will tend to work harder, but for a shorter period of time. They will also consume less. They attempt to get work over with so they can spend less time working and more in retirement, even if this means consuming less over their lifetime.

Some implications of this model are obvious. With only a minor assumption about disutility of effort, workers choose to retire of their own volition. Therefore, retirement plans of firms may have less of an effect than previously assumed, since workers would choose to retire even in their absence. Any change in worker behavior from a firm's retirement plan must be examined using the worker's retirement decision in the absence of a firm plan. Worker behavior may not be affected at all by some plans, particularly if the worker would have saved a greater amount on her own. In addition, firms are not able to directly impute the ability of workers using retirement as a metric. Since the retirement decision depends upon the structure of a worker's utility, in particular her degree of risk aversion, ability cannot be imputed from retirement unless the worker's utility is also known.

Heterogeneous Ability in Hierarchical Labor Promotions

In Chapter III I examine two models in order to better understand the hierarchical labor structure. First, I examine a model of decision-making in which managers give advice to those workers beneath them. Using this context, I try to establish a justification

for the existence of a hierarchical system in the first place. Then I examine a hierarchical competitive promotion model in which workers exert effort in an attempt to get promoted. I examine the nature of effort in the context of heterogeneous ability.

In the firm structure model, marginal product is once again the product of a worker's endogenous effort and exogenous heterogeneous ability. Output is the result of worker reaction to a series of randomly arriving decision opportunities. The manager gives advice concerning the decisions, but the workers may choose to ignore the advice. Workers are paid according to the successful outcomes from these decisions.

When effort is costly, workers of low ability choose to rely upon the advice of the manager while workers of higher ability choose to exert effort and rely on their own judgment. At the border of these two sections of the ability distribution lie a group of individuals with ability greater than that of the manager but who choose to rely on the manager's decision since it beats exerting effort on their own. Above this effort cutoff, the firm is able to order workers by ability using output as a measure if the wage function is sufficiently concave. In addition, managers exert more effort than subordinates of equal skill. Also, firms prefer higher skilled workers in upper level positions.

The implications of this portion of the chapter are intriguing. I have given a justification for the firm in terms of manager impact on lower level worker output. In addition, I demonstrate a justification for providing higher wages to upper level employees even if they are of the same skill level as lower level employees. This, then provides a reason why workers may prefer to be managers rather than lower level workers at any skill level. Hence, there will be competition for upper level jobs and a hierarchical labor

tournament is unavoidable. A labor promotion tournament may develop even if there is no need to use one in order to motivate worker effort. Hence, firms must take this into account when determining their wage structure. Firms may also be able to take advantage of this in inducing even greater worker effort.

I next consider a model in which workers exert effort in an attempt to be promoted. Only a certain number are promoted from a given group. The promotion is represented by a prize, the justification of which is provided by the previous model. Workers are aware of their own ability, but only know the distribution of ability from which other workers are drawn. The number of competitors and promotion slots are common knowledge.

Some interesting results arise from this setup. First, the individual who would exert the greatest amount of effort in this situation is the one with the ability drawn from the percentile roughly closest to the percentage of individuals not being promoted. In other words, the individual who is most likely at the cutoff between promotion and non-promotion will work the hardest. In addition, the optimal number of contestants from the viewpoint of inducing maximum worker effort is two. Although this stretches the limit on the believability of the private information assumption, it does imply that the lower the number of contestants, in the absence of collusion, the greater overall worker effort.

Implications of this model are also interesting. Firms now have an incentive to adjust the promotion cutoff such that the highest effort occurs at the densest location in the ability distribution. This would create the greatest incentive for worker effort at the point where the highest number of workers are expected to be located. In addition, firms

may structure promotion systems so as to have a relatively low number of individuals competing against each other for a particular promotion opportunity. For instance, a firm may create a multi-stage promotion system in order to limit the number of competitors in each stage even though the firm production structure may only require a single promotion step. The number of production levels in a firm is only the lower bound for the number of promotion levels.

Effort, Promotion and Retirement

In Chapter IV I combine the models of the previous chapters and jointly examine the effort, retirement and promotion environment. For simplicity, I examine a two-level hierarchy. Once a manager has been selected, he is made a residual claimant of the firm. This creates the incentive for the upper level of management to exert effort. Otherwise, the promotee would merely rest on his laurels and exert no effort. This residual claimancy is purchased by the new manager at a discount to the present value, which is represented by a lump sum bonus at the time of promotion.

Since, as shown in Chapter II, worker effort in this setup is independent of ability or the level of savings, the lump sum acts to merely decrease the time until the newly promoted manager retires. However, the new manager must eventually give up the residual claimancy to the following manager. Thus, the advantage of the lump sum payment only occurs during the tenure of the manager. Once the manager retires, all that is left is the earnings from the lump sum which accrued during the manager's tenure plus any other accumulated manager savings.

I show that the only way to achieve the efficient level of retirement is for the lump sum to be increasing with successive managers such that the benefits of the lump sum disappear. Making the manager less than 100% owner of the firm may help to alleviate this result. However, greater rewards still tend to affect the retirement decision.

Implications of this model are interesting. We may expect to see progressively larger rewards to successive managers of a firm. Also, over time, we would expect to see relatively lower percentage levels of ownership by firm management.

A Model of the Military Internal Labor Market

In Chapter V I combine the previous models in an attempt to give some insight into a more specific problem, that of corporate downsizing in general and that of the military drawdown in particular. I first describe the military officer promotion, compensation and retirement structure and outline some recent developments in the military drawdown. Then, I provide a relatively simple model using results from previous chapters. Finally, I examine the model in the context of a reduction in the number of employees, particularly in a situation where voluntary worker exit is preferred and discuss the possible effects of the recent drawdown actions.

The description of the military is fairly straightforward and comes mainly from Title 10 of the United States Code. The military officer structure contains 10 ranks with progressively fewer numbers in each. Wages increase with rank. There is a limited time frame in which to achieve promotion. The system is, basically, up-or-out. The retirement includes a cliff-vesting system. Currently vesting occurs at the 20 year point.

Recent attempts at reducing the size of the military include separation bonuses, early retirement opportunities, and reducing the number of individuals hired at the entry level. Some non-voluntary activities include lowering the percentage of workers promoted to given levels and selectively identifying individuals for retirement earlier than their choosing.

The model is, again, a two-level hierarchy. Those not promoted at the promotion point are required to resign. They then pursue their next best opportunity. Those promoted are placed in a position which gives them a relatively greater return on output than was previously available to them. In addition, they become vested in retirement.

Worker effort post-promotion is identical to that in Chapter II. Workers strictly prefer to be promoted due to the increase in return to effort, but their effort itself is independent of ability. The presence of a promotion opportunity, however, creates a distortion in worker effort similar to that in Chapter III. In addition, the retirement vesting creates a distortion similar to that in Chapter IV.

Once again, workers cannot be ordered by ability in the pre-promotion period, making the choice of those promoted problematic and further justifying the presence of a hierarchical promotion structure. Worker reaction to change in the compensation structure depends upon their level of risk aversion and the structure of changes in promotion probability.

In examining the model in the context of corporate downsizing or the military drawdown, the primary conclusion is that these changes effect the entire spectrum of workers across ability. For instance, changing the percentage of individuals promoted

changes the effort level of every individual. Also, providing separation bonuses causes individuals to leave across the entire ability spectrum. Using a single program to target specific individuals for removal may not be possible short of direct firing. Any drawdown should take into account these myriad effects.

REFERENCES

- Asch, Beth J. and John T. Warner.** *A Theory of Military Compensation and Personnel Policy*, Santa Monica, CA: RAND, MR-439-OSD 1994.
- Becker, Gary S.** *Human Capital: A Theoretical and Empirical Analysis, With Special Reference to Education*. 3d ed. Chicago: University of Chicago Press, 1992.
- Ben-Porath, Yoram.** "The Production of Human Capital and the Life Cycle of Earnings," *Journal of Political Economy*, (August 1967), 75(2), pp. 352-65.
- Bull, Clive, Andrew Schotter, and Keith Weingelt.** "Tournaments and Piece Rates: An Experimental Study," *Journal of Political Economy*, (February 1987), 95(1), pp. 1-33.
- Burrelli, David F. and Elizabeth A. Dunstan.** "Military Retiree Health Care: Base Closures and Realignments," *Congressional Research Service*, adapted for "Base Closure and Retiree Health Care," *Air Force Magazine*, July 1995, pp. 74-79.
- Callander, Bruce D.** "A New Shot at the Promotion System," *Air Force Magazine*, July 1995, pp. 70-72.
- Campbell, J.B.** *The Emperor and the Roman Army: 31 BC - AD 235*. Oxford, UK: Oxford University Press, 1984.
- Demski, Joel S., and Gerald A. Feltham.** "Economic Incentives in Budgetary Control Systems," *Accounting Review*, (April 1978), 53(2), pp. 336-359.
- Dye, Ronald A.** "The Trouble with Tournaments," *Economic Inquiry*, (January 1984), 22(1), pp. 147-149.
- Ehrenberg, Ronald G., and Michael Bognanno.** "Do Tournaments Have Incentive Effects?" *Journal of Political Economy*, (December 1990), 98(6), pp. 1307-1324.
- Ellwood, David T.** "Pensions and the Labor Market: A Starting Point (The Mouse Can Roar)" with comment by Zvi Bodie in David A. Wise. ed., *Pensions, Labor, and Individual Choice*. Chicago: University of Chicago Press, 1985 pp. 19-54.
- Frederickson, James R.** "Relative Performance Information: The Effects of Common Uncertainty and Contract Type on Agent Effort," *Accounting Review*, (October 1992), 67(4), pp. 647-669.

Fullerton, Richard L. and R. Preston McAfee. "Auctioning Entry Into Tournaments," *Journal of Political Economy*, forthcoming (1997).

Gotz, Glenn and John J. McCall. *A Dynamic Retention Model for Air Force Officers: Theory and Estimates*, Santa Monica, CA: RAND, R-3028-AF, 1984.

Green, Jerry R., and Nancy L. Stokey. "A Comparison of Tournaments and Contracts," *Journal of Political Economy*, (June 1983), 91(3), pp. 349-364.

Holmstrom, Bengt. "Moral Hazard in Teams," *Bell Journal of Economics*, (Autumn 1982), 13(3), pp. 324-340.

Jovanovic, Boyan. "Job Matching and the Theory of Labor Turnover," *Journal of Political Economy*, (October 1979), 87(5), pp. 972-990.

_____. "Matching, Turnover, and Unemployment," *Journal of Political Economy*, (February 1984), 92(1), pp. 108-122.

Klein, Benjamin, Robert G. Crawford, and Armen A. Alchian. "Vertical Integration, Appropriable Rents, and the Competitive Contracting Process," *Journal of Law and Economics*, (October 1978), 21(3), pp. 297-326.

Lazear, Edward P. "Why is There Mandatory Retirement?" *Journal of Political Economy*, (December 1979), 87(6), pp. 1261-1284.

_____. "Pensions as Severance Pay," with comment by David A. Wise in Zvi Bodie and John S. Shoven, eds., *Financial Aspects of the United States Pension System*. Chicago: University of Chicago Press, 1984, pp. 57-89.

_____. "Incentive Effects of Pensions," with comment by Roger H. Gordon in David A. Wise, ed., *Pensions, Labor, and Individual Choice*. Chicago: University of Chicago Press, 1985, pp. 253-282.

_____. "Social Security and Pensions," *Research in Labor Economics*, (1986), 7, pp. 1-30.

Lazear, Edward P., and Sherwin Rosen. "Rank-Order Tournaments as Optimum Labor Contracts," *Journal of Political Economy*, (October 1981), 89(5), pp. 841-864.

McAfee, R. Preston, and John McMillan. "Auctions and Bidding," *Journal of Economic Literature*, (June 1987), 25(3), pp. 699-737.

- McLaughlin, Kenneth J.** "Aspects of Tournament Models: A Survey," *Research in Labor Economics*, (1988), 9, pp. 225-255.
- Mortensen, Dale T.** "Specific Capital and Labor Turnover," *Bell Journal of Economics*, Summer 1978, 9(3), pp. 572-586.
- Murphy, Kevin M., and Finis Welch.** "Empirical Age-Earnings Profiles," *Journal of Labor Economics*, (April 1992), 8(2), pp. 202-229.
- Nalebuff, Barry J., and Joseph E. Stiglitz.** "Prizes and Incentives: Towards a General Theory of Compensation and Competition," *Bell Journal of Economics*, Spring 1983, 14(1), 21-43.
- Nalebuff, Barry J., and Richard J. Zeckhauser.** "Pensions and the Retirement Decision," in David A. Wise, ed., *Pensions, Labor, and Individual Choice*. Chicago: University of Chicago Press, 1985, pp. 283-316.
- O'Flaherty, Brendan, and Aloysius Siow.** "Up-or-Out Rules in the Market for Lawyers," *Journal of Labor Economics*, October 1995, 13(4), pp. 709-735.
- O'Keefe, Mary, W. Kip Viscusi, and Richard J. Zeckhauser.** "Economic Contests: Comparative Reward Schemes," *Journal of Labor Economics*, April 1984, 2(1), pp. 27-56.
- Posner, Richard A.** *Aging and Old Age*. Chicago: University of Chicago Press, 1995.
- Pratt, John W.** "Risk Aversion in the Small and in the Large, reprinted from *Econometrica* January-April 1964, 32(1-2) in Peter Diamond and Michael Rothschild, eds., *Uncertainty in Economics: Readings and Exercises*. San Diego: Academic Press, Inc., 1989, pp. 61-75.
- Rosen, Sherwin.** "Authority, Control, and the Distribution of Earnings," *Bell Journal of Economics*, Autumn 1982, 13(3), pp. 311-323.
- _____. "Prizes and Incentives in Elimination Tournaments," *American Economic Review*, September 1986, 76(4), pp. 701-715.
- _____. "The Military as an Internal Labor Market: Some Allocation, Productivity, and Incentive Problems," *Social Science Quarterly*, June 1992, 73(2), pp. 227-237.
- Stern, Steven.** "Promotion and Optimal Retirement," *Journal of Labor Economics*, October 1987, 5(4, part 2), pp. S107-S123.

_____. "Ability, Promotion, and Optimal Retirement," *Journal of Labor Economics*, January 1994, 12(1), pp. 119-137.

United States Code Annotated. *Title 10, Armed Forces, 1-3000*. St. Paul, MN: West Publishing Co., 1983, with supplementary pamphlet from 1997.

United States General Accounting Office. "Military Personnel: High Aggregate Personnel Levels Maintained Throughout Drawdown," Report to the Chairman, Subcommittee on National Security, House of Representatives, GAO/NSIAD-95-97, June 1995.

_____. "Force Structure: Potential Exists to Further Reduce Active Air Force Personnel," GAO/NSIAD-97-78, March 1997.

Williamson, Oliver E. "Credible Commitments: Using Hostages to Support Exchange," *American Economic Review*, September 1983, 73(4), pp. 519-540.

APPENDIX 1

Proof of Proposition 1:

Proposition 1 and equations (3) and (7) are repeated here for convenience.

Proposition 1: If $\rho = r$ and equations (7) and (3) hold, then $rK(R) = c(t) = \text{constant}$ for all $t > R$.

$$(3) \quad K'(t) = rK(t) + az(t) - c(t).$$

$$(7) \quad \frac{u''(c)c'(t)}{u'(c)} = (\rho - r).$$

Combining the equality between ρ and r with equation (7) leads to the conclusion that $c(t)$ is constant with respect to time. Noting that $z(t) = 0$ when $t > R$, rearranging equation (3), multiplying by e^{-rt} , and integrating from time R to infinity gives us the following equation.

$$(A1) \quad \int_R^{\infty} (K'(t) - rK(t))e^{-rt} dt = - \int_R^{\infty} c(t)e^{-rt} dt.$$

Solving the left-hand integral and using integration by parts on the right-hand integral gives the following.

$$(A2) \quad K(t)e^{-rt} \Big|_{t=R}^{\infty} = \frac{c(t)e^{-rt}}{r} \Big|_{t=R}^{\infty} + \int_R^{\infty} c'(t)e^{-rt} dt.$$

Since consumption is constant with respect to time, the remaining integral on the right-hand side of (A2) equals zero. Completing the solutions and multiplying both sides by r gives the following result.

$$(A3) \quad c(R) = rK(R) \equiv P, \quad t \geq R.$$

Thus, consumption after retirement is constant and exactly equals the interest income generated from accumulated savings, which completes the proof.

List of partial second derivatives of equation (17):

$$(A4) \quad V_{zz} = \frac{1}{\rho} \left(u''(c) a^2 (1 - e^{-rR})^2 - d''(z) (1 - e^{-\rho R}) \right).$$

$$(A5) \quad V_{zR} = V_{Rz} = \frac{1}{\rho} \left(u''(c) a^2 z r (1 - e^{-rR}) e^{-rR} \right).$$

$$(A6) \quad V_{RR} = \frac{1}{\rho} \left(u''(c) a^2 z^2 r^2 e^{-2rR} \right).$$

$$(A7) \quad V_{za} = \frac{1}{\rho} \left(u'(c)(1 - e^{-rR}) + u''(c)az(1 - e^{-rR})^2 \right).$$

$$(A8) \quad V_{Ra} = \frac{1}{\rho} \left(u'(c)zre^{-rR} + u''(c)az^2r(1 - e^{-rR}) \right).$$

$$(A9) \quad V_{z\delta} = 0.$$

$$(A10) \quad V_{R\delta} = -e^{-\rho R}.$$

APPENDIX 2

Deriving the First and Second Order Conditions of Equation (49):

$$(49) \quad \max_z w[1 - F(j|y)] - d(z).$$

Using the concept of order statistics gives us the following string of equations.

$$F(j|y) = 1 - \sum_{k=j}^{n-1} \binom{n-1}{k} [F(y)]^k [1 - F(y)]^{n-k-1} = 1 - \int_0^{F(y)} \frac{1}{\beta(j, n-j)} t^{j-1} (1-t)^{n-1-j} dt.$$

Applying Leibnitz's Rule gives $\frac{dF(j|y)}{dy} = -\frac{[F(y)]^{j-1} [1 - F(y)]^{n-1-j} f(y)}{\beta(j, n-j)}$. Using this fact in

the first order condition gives us equation (50).

$$(50) \quad \frac{wa[F(y)]^{j-1} [1 - F(y)]^{n-1-j} f(y)}{\beta(j, n-j)} - d'(z) = 0.$$

For the second order condition, we get

$$wa^2 \frac{[F(y)]^{j-1} [1 - F(y)]^{n-1-j} f(y)}{\beta(j, n-j)} \left(\frac{f'(y)}{f(y)} + \frac{f(y)}{F(y)[1 - F(y)]} [(j-1) - (n-2)F(y)] \right) - d''(z) < 0$$

Then, using the fact that, at the optimum, $\frac{wa[F(y)]^{j-1}[1-F(y)]^{n-1-j}f(y)}{\beta(j, n-j)} = d'(z)$ gives us equation (51).

$$(51) \quad ad'(z) \left(\frac{f'(y)}{f(y)} + \frac{f(y)}{F(y)[1-F(y)]} [(j-1) - (n-2)F(y)] \right) - d''(z) < 0.$$

Proof of Proposition 3:

Proposition 3: The individual from a given ability distribution $F(a)$ who exerts the greatest effort when confronted with a promotion opportunity of j out of n individuals, has ability,

$$a = F^{-1} \left(\frac{j}{n-1} \right).$$

In equilibrium, all individuals retain their position in the distribution, $F(a)$, following any change in parameters. Therefore, $\frac{dF(y)}{dj} = 0$. Also, since $d'(z) > 0$ and

$$d''(z) > 0 \text{ globally by assumption, } \text{sign} \left[\frac{d}{dj} d'(z) \right] = \text{sign} \left[d''(z) \frac{dz}{dj} \right] = \text{sign} \left[\frac{dz}{dj} \right]. \text{ Thus,}$$

applying discrete comparative statics to equation (50) results in the following relationship.

$$(A11) \quad \text{sign} \left[\frac{dz}{dj} \right] = \text{sign} \left[\frac{wa[F(y)]^j [1-F(y)]^{n-2-j} f(y)}{\beta(j+1, n-j-1)} - \frac{wa[F(y)]^{j-1} [1-F(y)]^{n-1-j} f(y)}{\beta(j, n-j)} \right].$$

Removing all the common factors simplifies the expression immensely.

$$(A12) \quad \text{sign} \left[\frac{dz}{dj} \right] = \text{sign} \left[\frac{1}{[1 - F(y)]j} - \frac{1}{F(y)(n - j - 1)} \right].$$

This implies that $\frac{dz}{dj} = 0$ at the point where $\frac{1}{[1 - F(y)]j} = \frac{1}{F(y)(n - j - 1)}$. This also

indicates what the optimal j must be in order to maximize an individual's effort, given their position in the ability distribution. Simplifying this expression gives $F(y) = \frac{j}{n - 1}$. Since,

in equilibrium, $F(y) = F(a)$, solving for the ability level at which effort is maximized gives

$$(A12) \quad a = F^{-1} \left(\frac{j}{n - 1} \right).$$

which is the equation given in the proposition. All that remains is to verify that it is indeed a maximum. The discrete second order condition is given by

$$(A13) \quad \frac{wa[F(y)]^{j+1}[1 - F(y)]^{n-3-j}f(y)}{\beta(j+2, n-j-2)} - \frac{wa[F(y)]^{j-1}[1 - F(y)]^{n-1-j}f(y)}{\beta(j, n-j)} < 0.$$

which holds, when evaluated at the point where $F(y) = \frac{j}{n - 1}$, provided $n > 1$. Since this

must be the case in order that the entire analysis has any meaning, the ability level given in equation (A12) must be the point of maximum effort, completing the proof.

Proof of Proposition 4:

Proposition 4: When all contestants in a contest are of equal ability, the optimal number of promotees is $j = (n - 1)$. In addition, the number of contestants, n , which gives the greatest individual effort is $n = 2$.

From proposition 3 we have the location in the distribution of the individual with the maximum effort defined by $F(a) = \frac{j}{n-1}$. When all individuals are of identical ability,

$F(a) = 1$. Hence, in order that the maximum amount of effort be exerted by all n individuals, the number of individuals promoted, j , must be set such that $j = n-1$. In addition, using discrete comparative statics in a manner similar to the previous proof, it can be shown that $\frac{dz}{dn} = 0$ when $n = j$. The second order condition is strictly positive for $n \geq j + 1$. Thus, $n = j$ implies minimum effort. (Which actually makes sense, since no effort is exerted when promotion is certain.) The problem now becomes one of getting j/n as small as possible while still keeping the optimal relationship between n and j , such that $j = n - 1$. In other words, the problem becomes $\min_n \frac{n-1}{n}$. Since this function is strictly increasing in n , the optimal n is the lowest integer possible consistent with the promotion setting. Thus, the optimal choice is $n = 2$, which completes the proof.

Proof of Proposition 5:

Proposition 5: The number of contestants, n , which gives the greatest individual effort is $n = 2$.

Using the results in the proof of Propositions 3 and 4, the individual ability level at which individuals exert the highest effort occurs where $j = F(a)(n - 1)$ and the minimum effort occurs where $j = nF(a)$. By the same reasoning as in the previous proof, the problem becomes $\min_n \frac{(n-1)F(a)}{nF(a)}$. This gives the same result as in Proposition 4, since the cdf's

cancel. Using discrete comparative statics, examine the effect on $\frac{dz}{dj}$ of a change in n .

$$(A14) \quad \text{sign} \left[\frac{d^2 z}{djdn} \right] = \text{sign} \left[\frac{n}{(n-j-1)j} - \frac{n(1-F(y))}{F(y)(n-j)(n-j-1)} - \frac{1}{(1-F(y))j} + \frac{1}{F(y)(n-j-1)} \right].$$

Evaluating this at $F(y) = \frac{j}{n-1}$ gives equation (A15).

$$(A15) \quad \text{sign} \left[\frac{d^2 z}{djdn} \right] = \text{sign} \left[\frac{1}{(n-j-1)} - \frac{1}{(n-j)} \right] > 0.$$

Thus, effort decreases faster around the maximum when n is large. Therefore, the optimal individual effort must occur in the situations where n is as small as possible, which occurs at $n = 2$, completing the proof.

VITA

Candidate: Gerald Edwin Sohan

Degree: Doctor of Philosophy, Texas A&M University, Economics

Address: 789 Sunridge Circle
P.O. Box 651
Palmer Lake, CO 80133

Education: B.S., United States Air Force Academy, 1982, Economics
M.S., Texas A&M University, 1991, Economics

Professional

Experience: United States Air Force Officer, 1982 - present.

United States Air Force Academy, Department of Economics and
Geography, Instructor of Economics, 1991-1993.

United States Air Force Academy, Department of Economics and
Geography, Assistant Professor of Economics, 1996 - present.