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Detection Threshold Modelling Explained

Ross L. Dawe

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ABSTRACT

This document presents expressions for modelling the detection threshold for narrowband and broadband passive sonars using either power or amplitude detection, cross correlation sonars, CW and FM active sonars with or without replica correlation, as well as intercept sonars. Explanations of various terms are given, along with appropriate ROC curves and recommended default values for those cases where insufficient information is available to the modeller. Intended users of this document are sonar modellers and operations research modellers who want quick and credible answers without going into the fine detail of the sonar system's operation.

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Executive Summary

This document presents expressions for estimating detection thresholds for various sonars for use in operations research studies, or for comparison with experimental measurements of sonar performance and testing. It is intended as a guide for modellers who want quick and credible answers without having to go into great detail regarding the sonar system.

Detailed expressions are given for modelling the following types of sonars: narrowband and broadband passive sonars using either power or amplitude detection, cross correlation sonars, CW and FM active sonars with or without replica correlation and intercept sonars using either power or amplitude detection. Explanations of each variation of sonar are given, along with instructions to model it properly including the most appropriate receiver operating characteristic (ROC) curve.

The two ways to define detection threshold for a sonar system currently in use are explained and this report gives the alternative versions for each sonar, along with a means of directly converting results between the two cases. Extra guidance for the modeller includes the recommended definition to use for each sonar, along with recommended default values and choices of equations when insufficient information is available.

An extensive discussion of correction terms to account for flaws in the simplified model and for practical sonar operation is also given, with many of these corrections being collected into an 'operational processor loss'. The effects of using a human observer rather than a theoretical ideal observer in the model are also discussed.

Contents

Table of Figures

List of Tables

1. Introduction

This report is a guide describing how to carry out simple but believable modelling of the detection threshold (DT) for various sonar systems. It is intended for use by workers carrying out operations research modelling on the effectiveness of sonar systems, and is especially designed for modellers who need to understand how sonars work but who don't need to pick through large numbers of references and sonar system wiring diagrams. This report consists of summarised explanations of how various sonar systems work and are modelled, while avoiding going into excessive detail.

This work was carried out under the auspices of Task ADA 95/158: "Airborne Sonar Research" sponsored by the Director, Aerospace Combat Development. It has direct application to both RAAF and RAN sonar processing systems, including passive and active sonobuoys, towed arrays, helicopter dipping sonars, and hull mounted passive and active sonar systems on ships and submarines.

Chapter 2 briefly describes the generic sonar types, while chapter 3 explains how detection threshold (DT) modelling fits into the 'sonar equation': this equation describes how well the sonar is performing in detecting a given target in a given set of environmental conditions. The detection threshold is the decision level at which the observer decides 'yes' or 'no' as to whether a signal is present, based upon the criteria of the probability of making a correct detection and the probability of having a false alarm. Two rigorous definitions of DT appear in Chapter 4, along with an explanation of each choice of definition. Whichever definition is selected by the modeller, it provides an effective measure of the performance of a sonar system and allows systematic comparisons to be made under a variety of conditions. To assist in providing numerical values for DT, Chapter 5 shows how DT can be calculated in terms of sonar system parameters such as the detection index, frequency bandwidth and integration time, all of which are controllable by the sonar operator.

Detailed comments on the various scaling factors and correction terms are given in Chapter 6 for the different types of sonars being modelled. Chapter 7 explains some common misconceptions in terminology and shows how other concepts such as the minimum discernible signal (MDS), recognition differential (RD) and classification threshold are related to the detection threshold. In Chapter 8 suggestions are made for modelling display effects, applicable when a human observer rather than a theoretical 'ideal' observer is being modelled as part of the sonar system.

A summary of all the different detection threshold cases is given in Chapter 9, along with a summary of instructions for the use of each equation, such as which type of statistics to use and which definition of DT is most applicable. For those modellers working with insufficient information, recommendations for default equations and numbers are given in Section 9.2.

Useful documents on detection threshold modelling include the Sonar Modelling Handbook (abbreviated to 'SMH') (Ref. 1), Urick (Ref. 2), Dawe (Ref. 3), Conley (Ref. 4) and Peterson *et al.* (Ref. 5). Relevant unclassified material has been taken from some classified references where appropriate. As will be seen later, not all references are consistent with one another. An extensive list of references appears at the end of this report. For those modellers who do not have access to some of these references, necessary items such as suitable receiver operating characteristic curves have been reproduced in this report. No details of specific sonar systems are discussed here, as such details are usually classified. The modeller should learn details of the sonar system being modelled, such as frequency bandwidth and integration time, then follow the guidelines given here.

2. Generic Sonar Types

Consider a generic sonar receiving system, as shown in Figure 1. A hydrophone array with beamformer is used to obtain measurements of a sound field in water. These data are then transmitted to a receiver, processed and then displayed visually or aurally so an observer may make a decision as to whether a target is present or absent.

Here the observer, although often implicitly assumed to be a human, could instead be an electronic integrator or an automated computer algorithm. The 'observer' is in effect merely a device for making simple yes/no decisions based on comparing the input with a threshold level. Thus 'human factors' effects are not considered as being part of DT in the initial calculations (see Chapters 7 and 8).

2.1 Active and Passive Sonars

There are two general types of sonar: 'passive' and 'active'.

Passive sonar: in this case the observer is trying to detect signals sent out from the target(s). For example, the sonar is detecting sounds unintentionally emitted from a vessel located some distance away. The advantage of a passive sonar is that the listener can remain hidden during the observations.

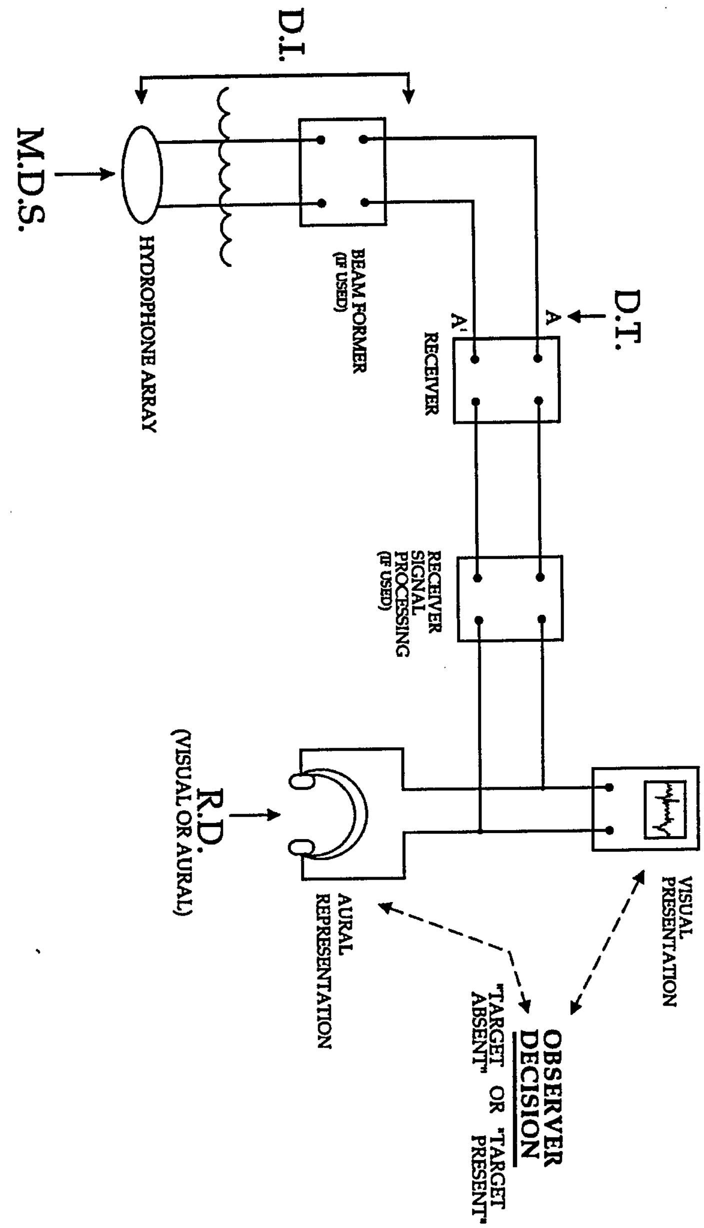

Figure 1. A diagram showing the components of the detection process and indicating where the signal to noise ratio for the minimum discernible signal, detection threshold and recognition differential are measured. Also shown is the receiving array and beamformer, which together comprise the part of the system related to the directivity index and array gain. Figure is adapted from Urick (Ref. 2, Figure 12.1).

Active sonar: in this case a sonar transducer has sent out a known signal and now the sonar operator is trying to detect reflections of the signal which struck an object and bounced towards the receiving array. For example, an active sonar 'ping' bounces off a submarine and the echo heads back to the receiver. While an active sonar ping gives a better idea of the location of the target object, it also gives away the location of the pinger.

The receivers used for sonar applications are hydrophones which respond to the sound pressure level in the water, or velocity gradient sensors (dipoles) which respond to the change in sound pressure levels in the water. A group of receiving elements forms an array, which receives and discriminates sounds arriving from different directions in a manner analogous to the way a radio frequency antenna behaves.

A sonar detector which has detection decisions based on the input pressure levels is an *amplitude detector*: examples of this include many towed arrays. A sonar detector which has detection decisions based on the square of the input pressure levels is a *power detector*: this is the most common type of passive sonar detector. An *energy detector* is equivalent to a power detector and is treated as being the same throughout all the modelling.

Passive, or 'listen only', receiving arrays are used on submarines, ships and sonobuoys deployed by aircraft. A monostatic active sonar has a receiving array in the same location as the sonar transducer ('pinger'): examples include hull mounted active sonars on ships and submarines, some types of sonobuoys and dipping sonars carried by helicopters. A bistatic active sonar system uses an active sound source such as a pinger or explosive charge at one location and the receiving array used to listen for the echo off the target is at another location.

2.2 Passive Sonar Subtypes

There are two main subtypes of passive sonar: 'Narrowband' (NB) and 'Broadband' (BB). From a theoretical standpoint there is essentially no difference between the narrowband and broadband cases, but there are practical differences in implementation. All that matters in each case is the ratio of the total signal to total noise (scaled appropriately) in the receiver bandwidth, and the probability distributions of the signal and the noise. For a passive sonar the incoming signal to be detected is completely unknown.

In narrowband detection, the signal is effectively a 'spike' in the frequency domain which is to be distinguished in some manner from the spikes caused by random noise fluctuations as measured at the detector. In practice all narrowband signals have some finite frequency width due to processes such as scattering.

When the processing chain contains a Fourier transform, the width of the frequency bin is set so that it is equal to, or larger than, the expected narrowband signal frequency bandwidth. Best processing performance can be achieved when the frequency bin width is matched to the signal bandwidth, once allowance has been made for the appropriate window function effects. The decision as to whether a signal is present or absent is made for each frequency bin. As there is typically a series of bins in a joined sequence spanning a frequency band, there is a commensurate series of independent decisions as to whether a signal is present or absent. Hence narrowband detection can be thought of as splitting a given frequency band into a series of smaller bins and examining each of the smaller bins for the presence or absence of a signal tone.

Now consider what would happen if the frequency band is not split into smaller bins, but the decision as to whether a signal is present or absent is made based on information received over the whole frequency band. While this is just the narrowband case for a single bin if the frequency bandwidth is small, when the frequency band becomes large such that the variation in signal and noise levels with frequency becomes significant then the problem becomes one of 'broadband detection'.

While a narrowband signal can be thought of as a 'spike' occurring in a frequency bin, a broadband signal is instead a general increase in the received power across part, or all, of the frequency band (and may have narrowband 'spikes' superimposed on it at various frequencies). A broadband signal which increases the received power over only part of the frequency band is sometimes referred to as a 'swathe'. Note that broadband detection is usually made by comparing the output of different sonar beams, rather than examining the content of each beam as is done for narrowband detection.

A variation of the passive sonar is a 'cross correlation' technique. This takes the output from one half of an array and then cross correlates it with the output from the other half of the array. While the signal remains at least partially correlated, the background noise is uncorrelated and so the signal to noise ratio is improved.

The effects of whether a sonar uses a conventional beamformer or an adaptive beamformer[1] are modelled as being part of the array gain (see Chapter 3). However, the choice of beamformer does have some consequences on modelling DT which will be discussed in Section 6.13.

1 For a conventional beamformer the weights on all the hydrophone inputs are fixed, whereas for an adaptive beamformer the weight on each of the hydrophone inputs changes as the ambient noise conditions change, so as to adaptively cancel interfering noise sources.

2.3 Active Sonar Subtypes

The most common subtypes of an active sonar are known as 'Continuous Wave' (CW) and 'Frequency Modulated' (FM), based on the type of sonar pulse emitted by the transducer. For an active sonar the original shape of the signal is known and so the receiver signal processing can be optimised to look for that known signal shape in the return echo. Modern active sonars all use this form of 'matched filter' processing. The drawback with an active sonar is that the background against which the signal is to be detected contains both ambient noise (just as for a passive sonar) and also reverberation, which is unwanted echoes from scatterers in the water or from its boundaries.

Continuous Wave (CW) has a constant frequency and is of duration t seconds. According to the Sonar Modelling Handbook (Ref. 1), for a rectangular or constant amplitude CW pulse the signal bandwidth is taken to be $1/t$ Hz. (A correction for shaded CW pulses can be found in Section 6.18.) This is also the bandwidth of the matched filter used in the detector. The input bandwidth of the sonar will usually be much wider to encompass Doppler shifts[2], and feeds a comb of contiguous matched filters.

Frequency Modulated (FM) has a varying frequency of bandwidth w over a duration of t seconds. For FM pulses the signal bandwidth of w Hz corresponds to a time resolution of $1/w$ seconds. The reciprocal of the bandwidth is referred to as the 'resolved pulse length'.

CW and FM pulses have both advantages and disadvantages. A CW pulse has good Doppler resolution but relatively poor range resolution, while an FM pulse typically has good range resolution but relatively poor Doppler resolution (Ref. 1).

There are other less common subtypes of active sonars, based on the various shapes of transmitted waveforms such as hyperbolic and exponential shapes. The design of active sonar pulse shapes for specific applications is an ongoing area of research and is driven by the need to optimise both range and Doppler information for the prevailing environmental conditions. In the absence of detailed information, it is recommended that these other waveforms be treated as being equivalent to an FM sonar with an appropriate effective noise bandwidth (see Section 5.1.3) for the purpose of estimating DT.

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[2] The Doppler shift is a change in the received frequency arising from the relative motion of the sound source, or target, and the receiver.

2.4 Intercept Sonars

An intercept sonar is used to detect active sonar 'pings' originating from other vessels. It is similar to a bistatic active sonar in having the receiver in a different location to the sound source, but in this case the detailed shape of the original signal is unknown and so matched filters cannot be used. As this case is similar to the detection of an unknown signal against a background of noise (which here has both ambient noise and reverberation), then an intercept sonar detection threshold is modelled in a similar way to a passive sonar detection threshold but with a correction term in the background noise level to account for reverberation.

3. Detection Threshold as Part of the 'Big Picture'

3.1 The Sonar Equation

The detection threshold (DT) is one of the fundamental terms in the **SONAR EQUATION** (Refs. 2, 6):

$$SE = SL - PL - NL + AG - DT - DF_0 . \tag{1}$$

SE is the signal excess and is related to the probability of detecting the target. In typical applications such as sonar performance calculations it is assumed that SE = 0.0 corresponds to a probability of detection of 50%. It is the aim of sonar system designers to maximise SE for given combinations of system parameters and environmental conditions. SE is typically measured in units of decibels (dB).

SL is the signal source level. This may also include the target strength (TS) for an active sonar echo. SL has units of dB/$\sqrt{\text{Hz}}$ re 1 μPa at 1 m from the source for consistency with DT definition choice 1 (see Section 4.1), or units of dB re 1 μPa at 1 m from the source for consistency with DT definition choice 2 (see Section 4.2).

PL (dB) is the total propagation loss as the signal travels from the source to the receiver. This term may be one path for a passive sonar or intercept sonar, or have two paths for an active sonar. This term is also known as the transmission loss TL. Care should be taken when consulting the active sonar literature as to whether the author refers to PL or TL as being a one-way loss or a two-way loss.

The sonar system 'Figure of Merit' (FoM) is the propagation loss corresponding to a signal excess of zero in the sonar equation (Ref. 1). This is used to estimate the 'range of the day' for a particular system operating in a particular environment when trying to detect a particular target. A high signal excess effectively means that setting SE = 0.0

dB in the sonar equation corresponds to a relatively large detection range for the prevailing conditions in which the sonar is being used.

NL is the background noise level at the receiver against which the observer is trying to detect the signal coming from the target. This term includes reverberation for an active sonar. NL has units of dB/$\sqrt{}$Hz re 1 μPa in a 1 Hz band for consistency with DT definition choice 1 (see Section 4.1), or units of dB re 1 μPa for consistency with DT definition choice 2 (see Section 4.2) (Ref. 6). This term is also known as the 'ambient noise' (AN) in the literature, or simply as the 'noise' (N), or if reverberation is present the Sonar Modelling Handbook (Ref. 1) calls it 'reverberation noise' (RN).

AG (dB) is the array gain and measures the ability of the receiving array to pick up and discriminate incoming sounds in a background of nonisotropic noise. In operational implementations of a sonar system the array beamformer is usually included in the sonar processor. However, for the purpose of simplifying the system modelling, the effect of the beamformer (conventional type or adaptive type) is combined with that of the receiving hydrophone array to give a performance measure called the 'directivity index' (DI) when the ambient noise is isotropic, or the array gain when the ambient noise is nonisotropic. The DI is a function of the number of hydrophones used, the array geometry and the frequency, with AG also accounting for the statistical properties of the input signal and ambient noise. In the Sonar Modelling Handbook (Ref. 1, Section 2) the array gain is subsumed into the noise term.

DT is the detection threshold and describes the decision level at which the observer decides 'yes' or 'no' as to whether a signal is present. In the context of the sonar equation, DT can be considered to be a measure of the quality of the sonar receiver and display system. DT has units of dB/$\sqrt{}$Hz re 1 μPa for definition choice 1 (see Section 4.1), or units of dB re 1 μPa for definition choice 2 (see Section 4.2).

DF_0 (dB) is an operational degradation term and is a catch-all for several cumulative imperfections in modelling all the other terms in the sonar equation, except for imperfections in the model of DT. The losses associated with DT have their own cumulative loss term OL (or 'DF_P' in Ref. 1): see Section 6.8. Note that DF_0 varies according to the sonar being modelled: recommended values for various sonars can be found in the Sonar Modelling Handbook (Ref. 1, Section 2.2.3). DF_0 is modelled as being 4 dB for passive narrowband sonars and all active sonars, 8 dB for all passive broadband sonars and 10 dB for an intercept sonar. These are, however, arbitrary values: if the modeller has a reliable estimate of DF_0 obtained from measurements on a similar type of generic sonar system then this should be used in preference.

Note that the notation for the same quantities varies between references. The Sonar Modelling Handbook (Ref. 1) uses '*T*' for the '*t*' as used here and '*B*' for the '*w*' as used here. The propagation loss 'PL' in the Sonar Modelling Handbook is equivalent to the transmission loss 'TL' in other texts such as Urick (Ref. 2). The modeller should always define the notation being used so as to avoid confusion, especially when dealing with subscripts.

3.2 Units for Quantities in the Sonar Equation

The abbreviation 'dB' is short for 'decibels', which is the commonly accepted logarithmic base 10 scale for the comparison of power levels in underwater acoustics. Thus a quantity X in linear units would be expressed in decibels as $10\log_{10}(X)$ (dB). Decibels as a unit are dimensionless, in that they merely indicate the power in one quantity relative to the power in another quantity. Thus a 'signal to noise ratio' (SNR) is measured in decibels and is dimensionless, but if either the signal or the noise are also measured relative to a particular standard then additional units are included based on that standard.

Units frequently appearing in this context are 'dB re 1 μPa at 1 m' or 'dB/√Hz re 1 μPa'. This means the decibel quantities are measured relative to a standard reference pressure of 1 μPa at a fixed distance of 1 metre and/or a frequency band of 1 Hertz. The reference to a fixed distance of 1 metre is designed to circumvent practical difficulties in making the measurements, especially in the case of the source level (SL) for an active sonar.

A shorthand notation sometimes used is to replace the 'relative to' with 're' (as is done in the previous section), or to replace it with a double slash '//'. Some documents also neglect to mention the quantity is being measured relative to a reference distance of 1 metre, to give either 'dB re 1 μPa' or 'dB//μPa' as the written version. The stipulation of 'relative to 1μPa' within the definition of each quantity carries no units: it is there merely to provide a baseline reference for a numerical scale (Ref. 2). Some references, e.g. Jones (Ref. 6), replace the notation 'dB/√Hz re 1 μPa' with the notation 'dB re 1 μPa/√Hz', but this is functionally equivalent once it is understood that the 'relative to 1μPa' component carries no dimensions. Modellers are encouraged to check the notation used by their sources and to maintain consistency of notation.

Quantities such as those appearing in the sonar equation can be measured either in terms of amplitude or power, with the amplitude being the square root of the power. Raw data at sonar sensors such as hydrophones are often measured as pressure amplitudes. The decibel scale as used in underwater acoustics is designed to work with powers, so that if Y is an amplitude in linear units and $X=Y^2$ is its corresponding power in linear units, then in the decibel scale it becomes $10\log_{10}(X) = 10\log_{10}(Y^2) = 20\log_{10}(Y)$ (dB). Thus quantities described as a power in decibels relative to a standard frequency band of 1 Hz, such as for one of the definitions of detection threshold (see the next section), correspond to a pressure amplitude in decibels per unit of √Hz.

When the modeller chooses the definition of DT to be used, care should be taken to make the rest of the terms in the sonar equation have consistent units. This will make the calculated signal excess work out correctly, regardless of which definition is used. For example, SL, NL and DT can be defined according to the bandwidth w of the receiver, but they are often quoted as being referenced to a 1 Hz bandwidth (Ref. 6). In practice, the bandwidth w is the bin width for narrowband fast Fourier transform (FFT) spectra, or the total spectral width (the bandwidth) in broadband analysis. In this case the values should be reduced by $10\log_{10}(w)$ to refer them to a 1 Hz bandwidth. (This is explained further in Section 4.3.) Whichever definition is used, SL and NL are required to have the same units for consistency in the calculations.

This report recommends the use of metric units for all quantities, but old military texts and texts from the United States use quantities such as 'yards' in place of 'metres' (Ref. 2). To convert between the two systems, remember that 1 metre = 1.0936 yards. Thus to convert terms within the sonar equation from being measured relative to 1 yard to being measured relative to 1 metre, subtract $20\log_{10}(1 \text{ metre/yard}) = 0.78$ dB. Conversely, add 0.78 dB when converting the reference from 1 metre to 1 yard. Archaic pressure units and their conversion to dB re 1 μPa are discussed by Urick (Ref. 2, Section 1.6).

4. Definitions of 'Detection Threshold'

The concept of a 'detection threshold' (DT) arises as it is necessary to set a threshold such that when it is exceeded, the decision is made by the observer of the form 'something other than background noise is present'. For a given decision criterion, a lower threshold indicates an improvement in the system. All losses raise the detection threshold. Over the years two ways of modelling the detection threshold have developed in the literature: each of these definitions is explained below.

The choice of definition of detection threshold is made by the modeller and should be stated clearly with each set of calculations. A great deal of confusion has arisen because the definitions of DT have different dimensions: Section 4.3 explains how to convert calculated values between the definitions in a consistent manner. Expressions for DT using each definition will be given below for each type of sonar system.

4.1 Definition Choice 1

According to Urick (Ref. 2, Section 12.1), the detection threshold is defined as the ratio, in decibel units, of the signal power (or mean-squared voltage) in the receiver frequency bandwidth to the noise power (or mean-squared voltage) in a 1 Hz frequency band, measured at the receiver input terminals, required for detection at some preassigned level of correctness of the detection decision. Here Urick's 'receiver' is referring to a receiver at the beginning of a signal processing and display system (excluding the beamformer). The receiver input, at which DT is measured, corresponds to A-A' in Figure 1. In this definition DT has units of dB/$\sqrt{}$Hz re 1 µPa, as does the units of SL and NL in the sonar equation (Section 3.1).

It is recommended that this definition is always used when modelling a narrowband passive sonar. It is the definition used by Urick (Ref. 2), Burdic (Ref. 7) and the Sonar Modelling Handbook (Ref. 1) for modelling a narrowband passive sonar, regardless of different notation and mistakes in notation made by some authors such as Urick. Consider an intercept sonar to be a narrowband passive sonar for the purpose of modelling.

This definition should also be used to model a broadband passive sonar, although this varies amongst some references such as the Sonar Modelling Handbook. Some authors such as Burdic imply that definition choice 1 also applies to an active sonar, while Urick neglects to mention that his definition specifically applies to a passive sonar.

4.2 Definition Choice 2

According to the Sonar Modelling Handbook (Ref. 1, Section 3), the detection threshold is defined as the ratio, in decibel units, of the signal power (or mean-squared voltage) in the receiver frequency bandwidth to the noise power (or mean-squared voltage) in the receiver frequency bandwidth, measured at the receiver input terminals, required for detection at some preassigned level of correctness of the detection decision. Here again 'receiver' is referring to a receiver at the beginning of a signal processing and display system (excluding the beamformer). The receiver input, at which DT is measured, corresponds to A-A' in Figure 1. In this definition DT has units of dB re 1 µPa, as does the units of SL and NL in the sonar equation (section 3.1).

It is recommended that this definition is always used when modelling any type of active sonar. It is the definition used by the Sonar Modelling Handbook for modelling an active sonar, and is also used in that same reference to model a passive broadband sonar. However, it should be noted that definition choice 2 is not always used in modelling an active sonar, e.g. by Burdic (Ref. 7).

4.3 DT Definitions in Mathematical Form

Let S be the signal power in the receiver bandwidth measured at the receiver input terminals (A-A' in Figure 1). Let N be the noise power in the receiver bandwidth w also measured at the receiver input terminals. Let N_0 be the noise power in a 1 Hz band also measured at the receiver input terminals. The detection threshold is given by:

Choice 1: $DT_0 = 10\log_{10}(S/N_0)\,,$ (2a)

Choice 2: $DT_w = 10\log_{10}(S/N)\,,$ (2b)

where the decision is made under the criteria of a specified probability of detection PD and a specified probability of false alarm PFA. The subscripts '0' and 'w' are used to distinguish between the two definitions of detection threshold throughout the text. Where no subscript appears on DT, the discussion applies to both definitions.

The convention of referring the noise to a 1 Hz bandwidth when calculating passive sonar DT_0 provides a common basis for both narrowband and broadband signals. It also reflects the common practice of expressing noise levels as spectrum levels, i.e. in bands 1 Hz wide.

To convert between the definitions of detection threshold use:

$$N = wN_0\,, \tag{3}$$

where w is the effective noise bandwidth of the receiver. Hence the relationship between the two detection thresholds is:

$$DT_0 = DT_w + 10\log_{10}(w)\,, \tag{4}$$

and this applies regardless of the type of detector. This conversion between definitions of DT is used throughout the rest of this report. For large values of w the numerical values of DT_0 and DT_w will be considerably different, clearly demonstrating why modellers should always take care to note which definition of DT was used in the calculations. Note also that the choice of definition for DT needs to be consistent with the choice of units for the source level SL and noise level NL in the sonar equation (Chapter 3).

Some terms which appear in the discussion of detection threshold warrant explanation. 'Probability of detection' (PD) is the probability that a signal, when present, will be detected. 'Probability of false alarm' (PFA) is the probability that a threshold crossing is caused by noise alone, i.e. an increase in the noise will exceed the threshold level and appear as a signal to the observer. Hence a 'false alarm' occurs each time the noise exceeds the threshold for a sufficient length of time that the observer considers it to be a signal. In the literature the probability of false alarm is sometimes also denoted as *p(FA)*, *Pfa*, or simply as *p*, while the probability of detection PD is sometimes also denoted as *p(D)* or *Pd.*

The values of PD and PFA taken together do not sum to one in most applications. PD applies when a signal is present, and 1 - PD is the probability that the signal will not be detected if it is present. PFA applies when the signal is absent, and 1 - PFA is the probability that the observer will correctly decide there is no signal present.

The definitions of DT given above effectively state that when a value of DT is quoted, it must be accompanied by a value of the probability of detection (PD) and a probability of false alarm (PFA) to have any significance: this then provides an effective performance measure of the sonar processing (and display) system. The value of PD most commonly associated with detection thresholds is PD = 0.5, with this often being implicitly assumed as a default value. Another value of PD sometimes used is PD = 0.9 (Ref. 2, Section 12.4). The values of PFA most often quoted with DT are PFA = 10^{-3} and PFA = 10^{-4}, although sometimes PFA = 10^{-5} and PFA = 10^{-2} are also used.

A combination of PD = 0.5 and PFA = 10^{-4} is a reasonable approximation to typical performance by an operational sonar system and so is often taken as the baseline for comparing values of DT. Instructions for converting DT to apply for other combinations of PD and PFA are given in Section 5.2.

The signal cannot be completely separated out from the noise in the actual measurements because noise is always present in any measurement, regardless of whether the signal is present or absent. As the signal distribution is superimposed on the noise distribution, it is actually the 'signal-plus-noise' distribution which is being measured and not a pure 'signal' distribution. It is this intrinsic intermixing of signal and noise which prevents the detection decision from being easy for an observer when the signal is weak compared to the noise. In such a situation there is a significant probability of the incorrect decisions being made of either 'signal absent' when the signal is really present (missed detection), or 'signal present' when the signal is really absent (false alarm). Thus *the concept of a detection threshold is a statistical quantity, and must be defined in that manner to have any valid meaning*. One consequence of this feature of DT is that it must be measured as a statistical quantity: this is an important consideration in testing sonar processor performance.

5. Detection Threshold Described by Parameters

5.1 DT as a Function of Detection Index, Bandwidth and Time

5.1.1 Basic Expressions for DT

The expression obtained below for DT applies for a square law (power) detector, which is the most efficient form of detector for unknown weak signals (Ref. 5): consequently, it is also the most commonly used form of passive detector. Amplitude (pressure) detectors are also known, e.g. some towed arrays. The most efficient form of detector is actually a cross correlator, but in this case it is necessary to know the exact waveform of the signal being detected, so effective cross correlation is presently limited to active sonar applications.

It is assumed that the incoming signal is completely unknown but has a steady mean power level and is in a background of Gaussian noise. Under the conditions of small signal to noise ratios and a large sample size it has been shown by Peterson *et al.* (Ref. 5) that the detection index d is given by:

$$d = wt(S/N)^2 \qquad (5)$$

where w is the effective noise bandwidth (Hz), S is the signal power level for a spectral line of width $<w$ and being entirely within the bandwidth w, N is the noise power in bandwidth w and t is the integration time (sec). The condition of small SNR (signal to noise ratio) is consistent with the calculation of the detection threshold. The derivation by Peterson *et al.* (Ref. 5) which led to equation 5 applies for any effective analysis bandwidth w, regardless of whether it is a single bin or a wide frequency band.

Combining equations 3 and 5 gives:

$$S/N_0 = Sw/N = (dw/t)^{1/2} \ , \qquad (6a)$$

$$S/N = (d/wt)^{1/2} \ . \qquad (6b)$$

Substituting back into equation 2 gives the basic expression for the detection threshold (depending upon the choice of definition) of a power detector for an unknown steady signal in a background of Gaussian noise as:

$$DT_0 = 10\log_{10}(S/N_0) = 5\log_{10}(dw/t) \ , \qquad (7a)$$

$$DT_w = 10\log_{10}(S/N) = 5\log_{10}(d/wt) \ . \qquad (7b)$$

Equation *7b* applies to active sonars using incoherent summation. It needs to be modified to account for modern active sonars which use coherent summation: see Section 6.2.

In practical applications the background noise is, in general, not Gaussian as was assumed by Peterson *et al.* (Ref. 5). This will change both the detection index and, for an amplitude detector, the DT equations. Choosing the correct detection index requires the modeller to know (i) the type of sonar detector in use, and (ii) the type of statistics most appropriate to that type of detector. Section 5.2 discusses the various cases of most use to the modeller, along with recommendations for each type of sonar.

5.1.2 Time

The integration time t is the length of time history used by the observer to make a detection decision. For a passive sonar detector t is equivalent to the time for the processor to perform successive updates on the observer's display, multiplied by the number of display updates. For example, if the processor takes 5 seconds to put a new row of data on the display and the observer is basing the detection decision on 6 rows of display data, then the net time used in the calculation of DT is 30 seconds. For an active sonar t is the initial pulse length. More detailed effects of the time history in influencing the human observer's detection decision are discussed in Section 8.1. In mathematics this is expressed as

$$t = T.(ndl)\,, \tag{8}$$

where T is the length of time for one display update and *ndl* is the number of display updates[3] containing a signal which are used by the observer to make the detection decision (Ref. 1). The factor *ndl* implicitly assumes that the observer, whether human or electronic, is part of the detection process. Thus any variation in measured DT due to the use of human observers to make the detection decision will manifest itself as a function of *ndl* (Section 8.1).

The electronic integration factor, IF, is the number of successive lines of data which are being combined within the processor, prior to display, in deriving the spectrum used by the observer to make the detection decision (Ref. 1). Here the spectral values in corresponding frequency bins of successive time steps are being added together by an electronic integrator. The effect of this incoherent electronic integration is to multiply the means of the signal plus noise and noise distributions by a factor of IF, while the

[3] Here *ndl* is the same as the number of display lines, or rows of data, on the time history of a LOFAR (time versus frequency) display or a time versus azimuth display.

standard deviations of the signal plus noise and noise are multiplied by a factor of $(IF)^{1/2}$.

It is important to distinguish between IF and *ndl* here. The quantity IF represents integration of the data within the processor and the result of the integration is output onto the display as one row of data, whereas *ndl* is the number of rows of data on the display. It is recommended that the modeller assume that IF = 1, unless the sonar system specifications indicate otherwise. The Sonar Modelling Handbook (Ref. 1) makes this distinction between IF and *ndl* in both its text and its glossary, but then proceeds to equate IF with the length of the time history *t* (Ref. 1, Section 3). This follows on from the source work by Paterson (Ref. 8) which also equates IF with *t*. To clarify this situation the equations given in this report will replace IF with *t* where necessary when sourcing material from References 1 and 8.

Using equation 8 in equation 7 gives the detection threshold for a power detector as:

$$DT_0 = 5\log_{10}(dw/T) - 5\log_{10}(ndl) \ , \qquad (9a)$$

$$DT_w = 5\log_{10}(d/wT) - 5\log_{10}(ndl) \ . \qquad (9b)$$

The term $5\log_{10}(ndl)$ is known as the 'visual integration gain' (VIG) and represents an improvement in detection threshold performance based on the incoherent summation of a time history of data across a display by the observer. If the observer making the detection decision is a human rather than an 'ideal' observer, then there is a correction to be applied to the VIG term: see Section 8.1.

The term $5\log_{10}(ndl)$ applies for a passive sonar. Its analogue for an active sonar is $5\log_{10}(NP)$, where *NP* is the number of pings of the sonar across which the observer is making an incoherent summation to make a detection decision: see Section 6.2.3.

5.1.3 Bandwidth

A hydrophone array has a response which depends on the direction in which it is steered. The directions of higher sensitivity of the array other than the intended look direction are sidelobes, and any signals coming through these sidelobes can degrade the accuracy of estimates of the true signal level and its direction. Frequency window functions are designed to minimise the effect of unwanted signals through these sidelobes while maintaining a relatively strong response in the intended look direction of the array.

The effective noise bandwidth *w* is "the width of a rectangle filter with the same peak power gain that would accumulate the same noise power" (Ref. 9). The effective noise bandwidth, also known as the equivalent noise bandwidth (ENBW), is the frequency binwidth (narrowband) or bandwidth (broadband) multiplied by an appropriate window scaling factor.

An extensive selection of window functions and corresponding effective noise bandwidths has been compiled by Harris (Ref. 9): the relevant table has been reproduced here as Figure 2. For example, an active sonar with a 2.0 Hz wide pulse and 'Hanning shading' on the pulse would have an equivalent noise bandwidth *w* of 2.0 x 1.50 = 3 Hz. For a passive narrowband sonar replace 'pulse' with 'bin' in the discussion, so that a frequency bin of width 2.0 Hz with Hanning shading has an effective noise bandwidth *w* of 2.0 x 1.50 = 3 Hz.

It is recommended that the modeller always makes this correction from binwidth to equivalent noise bandwidth. However, an alternative method for an active sonar exists wherein the time domain equivalent known as 'waveform shading' is used: this is explained further in Section 6.18.

For a passive broadband detector the receiving bandwidth is so wide that the effects of the signal and noise changing across the receiving bandwidth cannot be ignored as in the narrowband case. Instructions for calculating the effective source level, effective propagation loss and effective noise level for a passive broadband sonar are all given in Section 2 of the Sonar Modelling Handbook (Ref. 1), based on work by Paterson (Ref. 10). These effects include not only the distribution of the signal as well as the noise across the bandwidth, but also include the frequency dependence of the propagation loss and the sonar equalisation[4]. Note that these effects are not modelled in the detection threshold, but are modelled in the other terms of the sonar equation (Ref. 1).

When signal and noise variation across the receiving bandwidth is significant, e.g. a broadband detector is being used, then the effective noise bandwidth is modified (Refs. 1, 10) according to:

$$B_{eff} = \frac{\left(\int noise_{1Hz}(f)df\right)^2}{\int\left(noise_{1Hz}(f)\right)^2 df} \quad (10)$$

[4] The sonar equalisation is the response of the sonar electronics at a particular frequency. In order to make the noise effectively flat (i.e. white noise), a preprocessing filter is normally used which has a response with inverse shape to the expected average noise spectrum over a selected frequency range. This helps to prevent overloading the dynamic range in the receiver system, especially at low frequencies where ambient noise is relatively high. However, in practice such a preprocessing filter does not always completely flatten the ambient noise spectrum as conditions vary over time and from place to place.

WINDOW	HIGHEST SIDE-LOBE LEVEL (dB)	SIDE-LOBE FALL-OFF (dB/OCT)	COHERENT GAIN	EQUIV. NOISE BW (BINS)	3.0-dB BW (BINS)	SCALLOP LOSS (dB)	WORST CASE PROCESS LOSS (dB)	6.0-dB BW (BINS)	OVERLAP CORRELATION (PCNT)	
									75% OL	50% OL
RECTANGLE	−13	−6	1.00	1.00	0.89	3.92	3.92	1.21	75.0	50.0
TRIANGLE	−27	−12	0.50	1.33	1.28	1.82	3.07	1.78	71.9	25.0
$\cos^{\alpha}(x)$ HANNING $\alpha = 1.0$	−23	−12	0.64	1.23	1.20	2.10	3.01	1.65	75.5	31.8
$\alpha = 2.0$	−32	−18	0.50	1.50	1.44	1.42	3.18	2.00	65.9	16.7
$\alpha = 3.0$	−39	−24	0.42	1.73	1.66	1.08	3.47	2.32	56.7	8.5
$\alpha = 4.0$	−47	−30	0.38	1.94	1.86	0.86	3.75	2.59	48.6	4.3
HAMMING	−43	−6	0.54	1.36	1.30	1.78	3.10	1.81	70.7	23.5
RIESZ	−21	−12	0.67	1.20	1.16	2.22	3.01	1.59	76.5	34.4
RIEMANN	−26	−12	0.59	1.30	1.26	1.89	3.03	1.74	73.4	27.4
DE LA VALLE-POUSSIN	−53	−24	0.38	1.92	1.82	0.90	3.72	2.55	49.3	5.0
TUKEY $\alpha = 0.25$	−14	−18	0.88	1.10	1.01	2.96	3.39	1.38	74.1	44.4
$\alpha = 0.50$	−15	−18	0.75	1.22	1.15	2.24	3.11	1.57	72.7	36.4
$\alpha = 0.75$	−19	−18	0.63	1.36	1.31	1.73	3.07	1.80	70.5	25.1
BOHMAN	−46	−24	0.41	1.79	1.71	1.02	3.54	2.38	54.5	7.4
POISSON $\alpha = 2.0$	−19	−6	0.44	1.30	1.21	2.09	3.23	1.69	69.9	27.8
$\alpha = 3.0$	−24	−6	0.32	1.65	1.45	1.46	3.64	2.08	54.8	15.1
$\alpha = 4.0$	−31	−6	0.25	2.08	1.75	1.03	4.21	2.58	40.4	7.4
HANNING-POISSON $\alpha = 0.5$	−35	−18	0.43	1.61	1.54	1.26	3.33	2.14	61.3	12.6
$\alpha = 1.0$	−39	−18	0.38	1.73	1.64	1.11	3.50	2.30	56.0	9.2
$\alpha = 2.0$	NONE	−18	0.29	2.02	1.87	0.87	3.94	2.65	44.6	4.7
CAUCHY $\alpha = 3.0$	−31	−6	0.42	1.48	1.34	1.71	3.40	1.90	61.6	20.2
$\alpha = 4.0$	−35	−6	0.33	1.76	1.50	1.36	3.83	2.20	48.8	13.2
$\alpha = 5.0$	−30	−6	0.28	2.06	1.68	1.13	4.28	2.53	38.3	9.0
GAUSSIAN $\alpha = 2.5$	−42	−6	0.51	1.39	1.33	1.69	3.14	1.86	67.7	20.0
$\alpha = 3.0$	−55	−6	0.43	1.64	1.55	1.25	3.40	2.18	57.5	10.6
$\alpha = 3.5$	−69	−6	0.37	1.90	1.79	0.94	3.73	2.52	47.2	4.9
DOLPH-CHEBYSHEV $\alpha = 2.5$	−50	0	0.53	1.39	1.33	1.70	3.12	1.85	69.6	22.3
$\alpha = 3.0$	−60	0	0.48	1.51	1.44	1.44	3.23	2.01	64.7	16.3
$\alpha = 3.5$	−70	0	0.45	1.62	1.55	1.25	3.35	2.17	60.2	11.9
$\alpha = 4.0$	−80	0	0.42	1.73	1.65	1.10	3.48	2.31	55.9	8.7
KAISER-BESSEL $\alpha = 2.0$	−46	−6	0.49	1.50	1.43	1.46	3.20	1.99	65.7	16.9
$\alpha = 2.5$	−57	−6	0.44	1.65	1.57	1.20	3.38	2.20	59.5	11.2
$\alpha = 3.0$	−69	−6	0.40	1.80	1.71	1.02	3.56	2.39	53.9	7.4
$\alpha = 3.5$	−82	−6	0.37	1.93	1.83	0.89	3.74	2.57	48.8	4.8
BARCILON-TEMES $\alpha = 3.0$	−53	−6	0.47	1.56	1.49	1.34	3.27	2.07	63.0	14.2
$\alpha = 3.5$	−58	−6	0.43	1.67	1.59	1.18	3.40	2.23	58.6	10.4
$\alpha = 4.0$	−68	−6	0.41	1.77	1.69	1.05	3.52	2.36	54.4	7.6
EXACT BLACKMAN	−51	−6	0.46	1.57	1.52	1.33	3.29	2.13	62.7	14.0
BLACKMAN	−58	−18	0.42	1.73	1.68	1.10	3.47	2.35	56.7	9.0
MINIMUM 3-SAMPLE BLACKMAN-HARRIS	−67	−6	0.42	1.71	1.66	1.13	3.45	1.81	57.2	9.6
MINIMUM 4-SAMPLE BLACKMAN-HARRIS	−92	−6	0.36	2.00	1.90	0.83	3.85	2.72	46.0	3.8
61 dB 3-SAMPLE BLACKMAN-HARRIS	−61	−6	0.45	1.61	1.56	1.27	3.34	2.19	61.0	12.6
74 dB 4-SAMPLE BLACKMAN-HARRIS	−74	−6	0.40	1.79	1.74	1.03	3.56	2.44	53.9	7.4
4-SAMPLE $\alpha = 3.0$ KAISER-BESSEL	−69	−6	0.40	1.80	1.74	1.02	3.56	2.44	53.9	7.4

Figure 2. A list of window functions and their figures of merit, reproduced from Harris (Ref. 9, Table 1).

Here B_{eff} is the effective bandwidth (Hz) and of course must include any windowing effects (Ref. 9), $noise_{1Hz}(f)$ is the noise power in a 1 Hz band at frequency f and all integrals are over the full bandwidth of the sonar. Applying this correction gives B_{eff} as being the same as the effective noise bandwidth w in the rest of these notes.

This correction should be accounted for in the detection threshold model. Failure to apply this correction will, according to Reference 1, produce an error no larger than 2 dB, but this varies with the noise spectra used on a case by case basis.

5.2 Detection Index and ROC Curves

5.2.1 General Comments

The detection index is a function of the mean signal plus noise level, the mean noise level and the noise variance, with an assumption that the signal is steady over the integration time. The mathematical definition of the detection index is

$$d = \frac{\left[Mean(signal + noise) - Mean(noise)\right]^2}{Variance(noise)} . \tag{11}$$

Thus d gives an indication of how 'easy' it is to see the signal in the noise. Values of d can be determined for specific probability density functions (pdfs) for a given probability of detection (PD) and a given probability of false alarm (PFA).

Plots of PD as a function of PFA, with d as a parameter, are known as receiver-operating-characteristic (ROC) curves. One example of a set of ROC curves is shown as Figure 3, which is taken from Urick (Ref. 2). Note that the curves depend on the assumed probability density function of the noise field. Noise statistics are often assumed to be Gaussian[5], e.g. as in Figure 3, as this is the easiest distribution to work with. ROC curves are typically given as a family of curves so the modeller may vary the combination of PD and PFA as necessary. From Figure 3 it is clear that holding PD constant and varying PFA changes the required d, and also holding PFA constant while varying PD changes the required d.

[5] This is a reasonable assumption if a large number of samples are used. This is explained by the Central Limit Theorem, in which the distribution of the sum of a large number of uncorrelated samples is Gaussian, regardless of the actual distribution of the population from which the samples were drawn, so long as the population variance is finite.

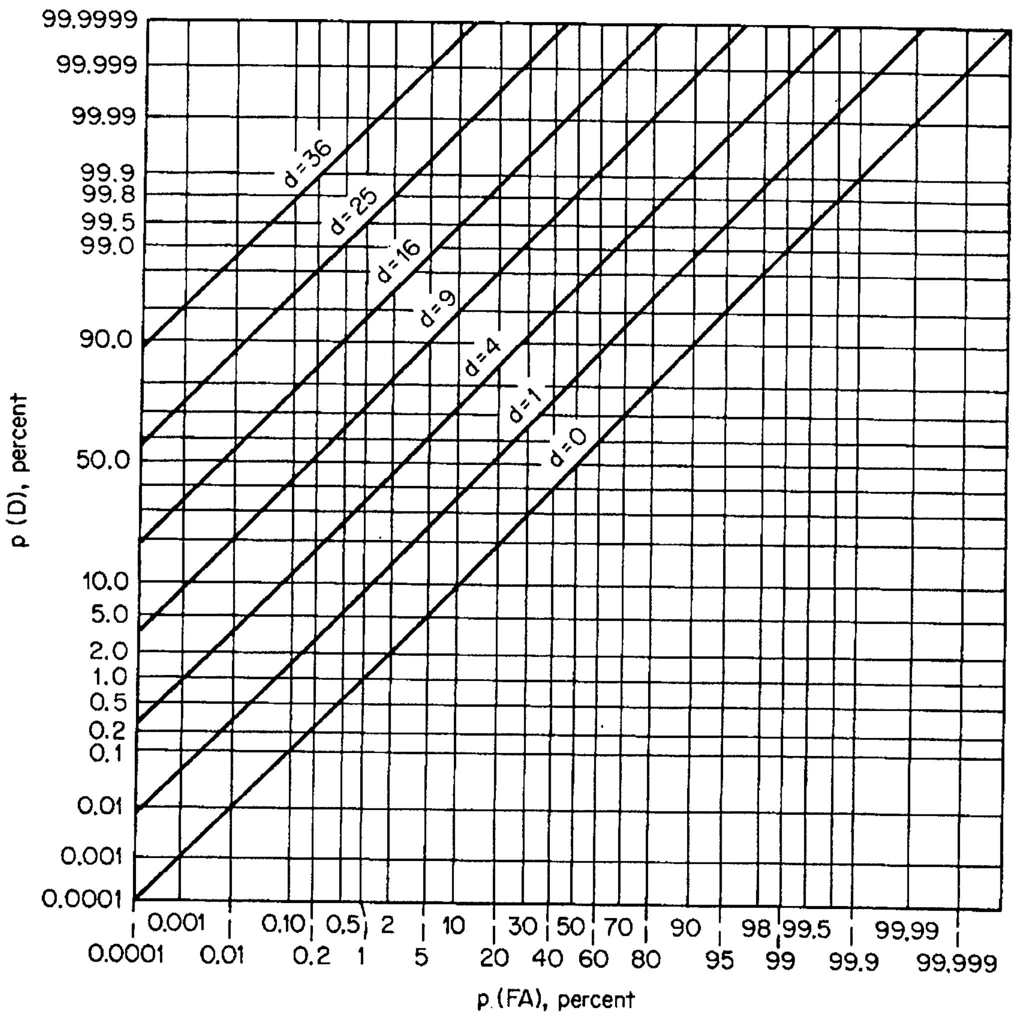

Figure 3. Receiver operating characteristic (ROC) curves for both the signal plus noise and noise as Gaussian probability density functions, with d being the detection index. Figure is reproduced from Urick (Ref. 2, Figure 12.7) and is © R. J. Urick. Note the difference in notation: here Urick's p(D) corresponds to the probability of detection PD, while p(FA) corresponds to the probability of false alarm PFA.

The 'bandwidth-time' product wt is the sample size being used in the calculations. For large values of wt, for example $wt > 100$, the modeller may safely use a Gaussian approximation to simplify the calculations. Gaussian statistics are often used because the properties of the Gaussian probability density function (pdf) are well known, amenable to analysis and the distribution is fully tabulated in many reference texts. There is however, a penalty incurred in using Gaussian statistics when the sample size wt is not very large, e.g. $wt << 100$ and especially when $wt \rightarrow 1$. The distributions discussed in the following sections all tend towards Gaussianity as an asymptotic limit when the sample size becomes extremely large. As different distributions produce different detection indices for the same PD and PFA (compare the values of d in Table 1 in Section 5.2.2 and Table 2 in Section 5.2.3 for the same value of IF), a correction factor to DT is needed to account for using a value of d derived from Gaussian statistics rather than the correct chi-squared or Rician statistics: this will be discussed in Section 6.3.

For smaller sample sizes, ROC curves can be calculated for other probability distributions which more closely resemble measurements of the background noise field by the type of detector being used. Some useful examples are given in the next three sections. It is recommended that the modeller should make the utmost effort to determine the correct statistics for the signal plus noise and the noise distributions for the sonar processor being modelled. A useful suggestion is to consult the appropriate sonar design specifications if they are available.

5.2.2 Passive Sonar Power Detectors

If the passive sonar detector measures power (equivalent to an energy detector) then the noise follows an exponential distribution (Refs. 1, 8) for a small number of independent samples. A sample ROC curve from Reference 1 for a generic power detector is reproduced here as Figure 4. Paterson (Ref. 8) has shown that the detection index for exponential noise statistics with an integration factor of 1 is given by:

$$d = \left(\frac{\log_e(PFA)}{\log_e(PD)} - 1 \right)^2 . \tag{12}$$

This expression for d is consistent with the original derivation of an expression for DT by Peterson *et al.* (Ref. 5), once allowance has been made for the different statistics. Hence it may be used with equation 7 as long as similar assumptions are applied, i.e. the signal is unknown and the signal to noise ratio is small.

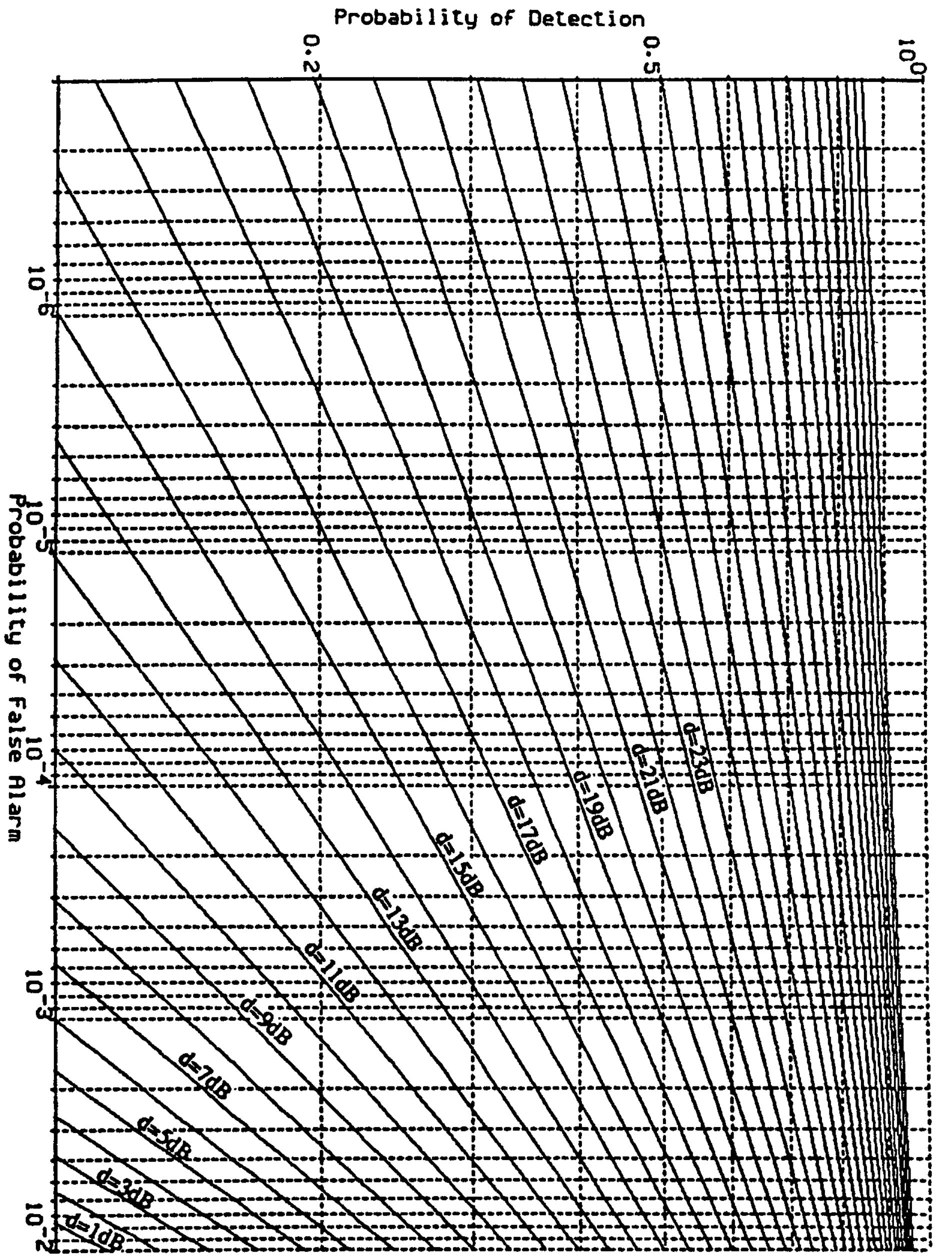

Figure 4. Receiver operating characteristic curves for a generic power detector, where both the signal plus noise and noise pdfs are exponential distributions. Figure is reproduced from the Sonar Modelling Handbook (Ref. 1, Figure 3.5). Values are $10\log_{10}(d)$.

For higher values of the integration factor IF the detection index no longer has a closed form solution and must be calculated numerically. Paterson (Ref. 8) has calculated the detection index for selected values of the integration factor for PD = 0.5 and PFA = 10^{-4} as shown in Table 1 below. The values of detection threshold in Table 1 are for w = 1 and IF = t in equation 7.

For IF >1 the noise distribution for a power detector follows a chi-squared pdf. For IF = 1 the chi-squared distribution becomes an exponential distribution as a special case. Sample ROC curves for a power detector with IF = 1, 2, 4 and 8, corresponding respectively to chi-squared distributions with order 1, 2, 4 and 8 statistics for the signal plus noise and noise distributions, can be found in chapter 3 of the Sonar Modelling Handbook (Ref. 1).

The recommended expression for modelling the detection threshold is equation 9*a* for DT_0 and equation 9*b* for DT_w, suitably modified for correction terms: see Sections 5.1.2 and 8.1 as well as Chapter 6.

If the modeller is uncertain as to the exact value of *IF*, make the substitution *IF* = *T* in equation 9 where *T* is the length of time for one display update. The effect of the number of display rows *ndl* is still separately accounted for by the visual integration term in equation 9. If both *IF* and *ndl* are unknown, use *IF* = *t* in the model.

Integration Factor *IF*	1	2	4	8
Detection Index *d*	161	70.7	44.6	25.1
Detection Threshold (dB)	11.0	7.7	5.2	2.5

Table 1: Values for detection index *d* and detection threshold for a power detector for different integration factors when PD = 0.5 and PFA = 10^{-4}, taken from Paterson (Ref. 8). Detection threshold values are for w = 1 and *IF* = *t* in equation 7.

5.2.3 Passive Sonar Amplitude Detectors

If the sonar system uses an amplitude spectrum as the basis for detection decisions the noise follows a Rayleigh pdf (Refs. 1, 8) for small numbers of samples: examples include pressure sensors in some types of towed arrays. A set of ROC curves for a generic amplitude detector, taken from Reference 1, has been reproduced here as Figure 5.

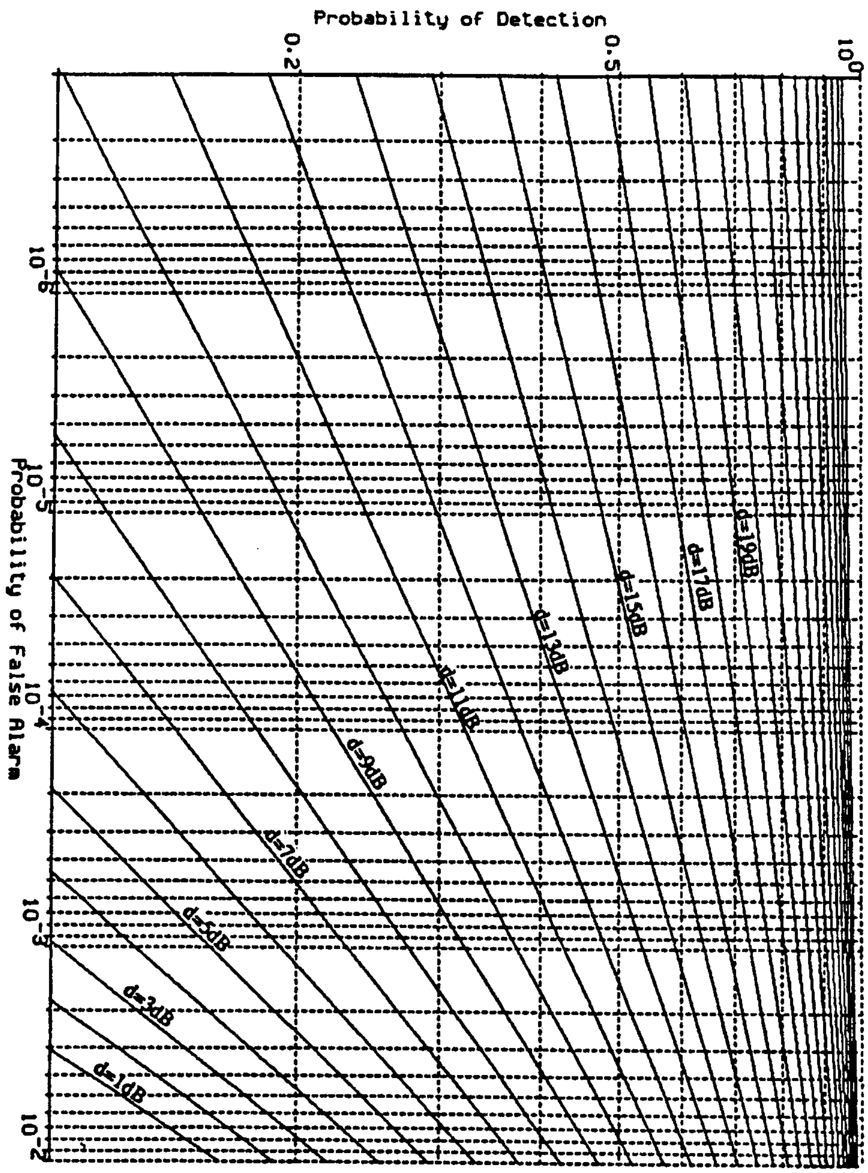

Figure 5. Receiver operating characteristic curves for a generic amplitude detector, where both the signal plus noise and noise pdfs are Rayleigh distributions. Figure is reproduced from the Sonar Modelling Handbook (Ref. 1, Figure 3.9). Values are $10log_{10}(d)$.

When modelling an amplitude detector the ambient noise values follow a Rayleigh pdf rather than the exponential pdf for a power detector, or the Gaussian pdf for a large number of samples with a power detector. Paterson (Ref. 8) has studied the detection threshold for an amplitude detector using the correct (i.e. Rayleigh) noise statistics and shown that

$$DT_0 = 10\log_{10}\left(\frac{0.273d}{t} + 1.045\sqrt{\frac{d}{t}}\right) + 10\log_{10}(w) \ . \tag{13a}$$

Here d is the detection index, t is the length of the time history being used to make the detection decision and w is the effective noise bandwidth. The corresponding expression for the alternative choice of definition for detection threshold is:

$$DT_w = 10\log_{10}\left(\frac{0.273d}{t} + 1.045\sqrt{\frac{d}{t}}\right) \ . \tag{13b}$$

Neither of these expressions include correction terms (see Chapter 6) and should be modified accordingly.

Paterson (Ref. 8) has shown that the detection index for Rayleigh noise statistics with an integration factor of 1 is given by:

$$d = \left(\frac{\pi}{4-\pi}\right)\left(\sqrt{\frac{\log_e(PFA)}{\log_e(PD)}} - 1\right)^2 \ . \tag{14}$$

This expression for d should be used with equation 13 and must NOT be used with equation 7, as the derivation by Peterson et al. (Ref. 5) which led to equation 7 used a Gaussian distribution and not a Rayleigh noise distribution. For larger values of the integration factor IF there is no closed form solution for d analogous to equation 14 and so the calculation of the detection index must be done numerically. For the values PD = 0.5 and PFA = 10^{-4} Paterson (Ref. 8) has calculated d for higher values of *IF*, as shown in Table 2. Values of DT in Table 2 are for $w = 1$ and IF = t in equation 13.

For IF >1 the noise distribution for an amplitude detector follows a Rician pdf. For IF = 1 the Rician distribution becomes a Rayleigh distribution as a special case. The details of modelling a Rician distribution for a narrowband amplitude detector can be found in Nielsen (Ref. 11), who in turn quotes Whalen (Ref. 12) as the reference source.

Integration Factor *IF*	1	2	4	8
Detection Index *d*	25.6	22.5	19.7	17.8
Detection Threshold (dB)	10.9	8.2	5.6	3.4

Table 2: Values for detection index d and detection threshold for Rician noise statistics for different integration factors when PD = 0.5 and PFA = 10^{-4}, taken from Paterson (Ref. 8). Detection threshold values are for $w = 1$ and $IF = t$ in equation 13.

Comparison of Tables 1 and 2 shows that there is a small difference in DT for corresponding values of IF. This is due to the different probability density functions applicable to each detector, with the lower values for the power detector showing why it is generally preferred. Note that for IF = 1 the difference in the detection thresholds in Tables 1 and 2 is just rounding error: DT should otherwise be the same in this particular case. Further comparison of Tables 1 and 2 shows that the typical range of d can vary greatly between types of detectors. The modeller must therefore ensure that the value of d being used in the calculations is appropriate to the type of detector being modelled.

5.2.4 Active Sonars

For an active sonar, it is recommended that the modeller use the ROC curve given by Figure 3-9-1 in the Sonar Modelling Handbook (Ref. 1) and reproduced here as Figure 6. This figure uses the correct statistics for a single sonar 'ping' in a background of noise: in this case Rician statistics for the signal plus noise distribution and exponential statistics ($IF = 1$) for the noise distribution. The values shown in Figure 6 are $10\log_{10}(d)$, so it will be necessary to halve these values to $5\log_{10}(d)$ when using them in equation 7.

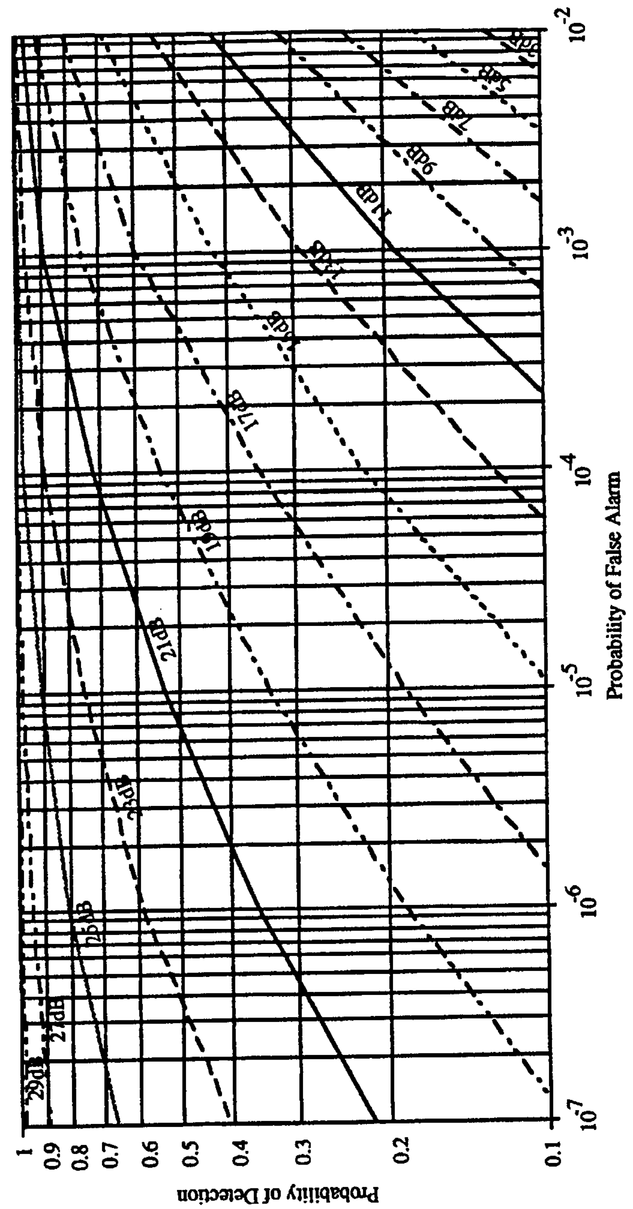

Figure 6. Receiver operating characteristic curves for an active sonar (CW or FM), with Rician statistics for the signal plus noise pdf and exponential statistics for the noise pdf. Figure is reproduced from the Sonar Modelling Handbook (Ref. 1, Figure 3-9-1). Values are $10\log_{10}(d)$.

6. When and What Correction Factors to Use

The expression for detection threshold given by equation 7 applies to generic sonar detection systems. In practice sonars come in a variety of implementations and so the detection threshold equation needs to be modified to suit each particular combination of equipment and circumstances: this has already been demonstrated in Section 5.2.3 when modelling an amplitude detector. The material in this Chapter describes various scaling and correction terms and indicates when they should be applied in the DT model. The scaling factors in Sections 6.1 and 6.2 can all be rigorously derived, but such derivations are not given here for brevity. Some of the loss terms described in later sections are not correction terms to DT but apply instead to other terms in the sonar equation (see Chapter 3): they have been included here so the modeller can distinguish between them and assign them to their correct places in the overall model of sonar performance.

Much of the discussion in Sections 6.1 and 6.2 is adapted from Chapter 3 of the Sonar Modelling Handbook (Ref. 1), while material in later sections is sourced from various other references including Urick (Ref. 2), Conley (Ref. 4) and Harris (Ref. 9).

6.1 Scaling Factors for Passive Sonars

6.1.1 Passive Sonar Amplitude Detection

When modelling a narrowband passive sonar system, the modeller can use either the simple expressions given by equation 7 depending upon the choice of definition for DT, or in the case of an amplitude detector use the more accurate expressions given by equation 13.

The derivations leading to equation 7 applied for a power detection system (equivalent to an energy detection system), where it is the mean squared voltage which is effectively being measured. This is reflected in the definitions of DT given in Chapter 4. Power spectra are calculated as the sum of the squares of the real and imaginary components of the complex spectrum, whereas an amplitude spectrum is the square root of a power spectrum and so is measuring the root mean squared voltage. This means that in any derivation or experimental measurements, the modeller should be aware of which spectrum is in use and should check whether the measurements are in linear units or have been converted to the logarithmic decibel scale.

6.1.2 Cross Correlation

The technique of cross correlation is used with passive broadband detectors. The effect of cross correlation is to compare the signal plus noise received in one array with the signal plus noise received in another array at a known distance from the first. A cross-correlation array compares the input signal to noise ratio between two separately beamformed halves of a single array, but while this improves DT (see below) it lowers the array gain by 3 dB as only half the number of hydrophones are being used. Cross-correlation between sonobuoys should, in theory, use the array gain of a single buoy while retaining the improved DT.

The effect of cross-correlation on DT is to double the number of independent noise samples being combined in the detector: wt is effectively replaced by $2wt$ in modelling equations. The recommended means of modelling the effect on detection threshold for n arrays being cross correlated is to lower DT by a factor of $5\log_{10}(n)$. Here the n arrays can be either two halves of a split array ($n = 2$) or a field of n sonobuoys.

6.2 Scaling Factors for Active Sonars

6.2.1 Choice of Definition of Detection Threshold

The Sonar Modelling Handbook (Ref. 1) uses a definition consistent with DT_w for modelling active sonars in both ambient noise and reverberation noise limited cases. As this reference is regularly updated based on the latest defence research (indeed, most of the references in the Sonar Modelling Handbook are classified), it is recommended that this definition also be followed by the modeller. The alternative method of having separate detection thresholds for reverberation noise limited and ambient noise limited cases will be discussed below in Section 6.2.4.

It is important to note that Urick (Ref. 2) and Burdic (Ref. 7) use the same expression for the DT of an active sonar as they used for a passive sonar: they are effectively using the same definition of DT with noise measured relative to a 1 Hz bandwidth for all sonars.

6.2.2 Active Sonars with One Ping

CW processing is similar to the processing employed in passive narrowband (NB) sonars, in that after beamforming the data is passed through a fast Fourier transform (FFT) and then displayed to the observer. Hence some of the correction factors discussed in later sections for an active CW sonar will also apply to a passive NB sonar and vice versa. Note that the term 'ping' is used interchangeably with 'pulse' when referring to an active sonar.

For the special case of a single active sonar pulse in a background of Gaussian noise, the technique of cross correlation of the received signal plus noise with an exactly known signal gives the modified detection threshold equation as:

$$DT_w = 10\log_{10}(d/2wt)\ . \qquad (15)$$

This expression was derived by Peterson *et al.* (Ref. 5) and is known as a matched filter. It applies when incoherent summation is used.

For FM processing the FFT calculation is replaced by a *replica correlation* with the shape of the transmitted pulse. According to Reference 1, the processing gain using FM is the same irrespective of the type of frequency modulation. Moreover, the CW case is merely a trivial form of the FM case.

FM replica correlation operates by multiplying the received data with the shape of the transmitted pulse: this gives a measure of the correlation between the received time series data and the transmitted pulse. The multiplications and summations in the replica correlation process are all carried out on complex bandshifted data and so they are coherent operations which take account of phase as well as amplitude information in the received data. The received noise, regardless of whether it is ambient noise or reverberation, is completely uncorrelated with the transmitted pulse.

The noise power after replica correlation is described by a random variable following an exponential distribution: see the ROC curve Figure 3-9-1 in the Sonar Modelling Handbook (Ref. 1), reproduced here as Figure 6. The statistics do not become Gaussian even though FM pulses generally have a large bandwidth time product wt. This is because Gaussian statistics only arise from a large wt product if the summations across time and frequency are carried out incoherently (no phase information), whereas replica correlation carries out the summations coherently (phase information retained) and this does not alter the statistical form of the noise.

The replica correlation technique gives a gain equal to the number of independent samples combined, whereas an incoherent summation (e.g. *old* active sonars and passive broadband sonars) gives a gain equal to the square root of the number of independent samples being combined. Thus the wt term in equation *7b* becomes $(wt)^2$ and so the expression for DT arising from replica correlation is:

$$DT_w = 5\log_{10}(d) - 10\log_{10}(wt)\ . \qquad (16)$$

Here DT_w is defined such that the signal and noise are both expressed relative to the same receiver bandwidth. The detection index d, the effective noise bandwidth w and the integration time t all have the same meaning as before. As modern FM active sonars all use replica correlation, it is recommended that modellers use equation 16 for DT_w. Care should be exercised when considering terminology: replica correlation, as given by equation 16, is sometimes also known as matched filter processing (Ref. 1).

Nielsen (Ref. 10, Figure 3.13) gives an indication of how using phase information for coherent detection can improve detection performance compared with using incoherent detection, where the phase is unknown. For the specific case considered by Nielsen, where the background statistics are given by the Marcum Q-function[6], the improvement in DT by using phase information for PD = 0.5 and PFA = 10^{-4} is about 1.1 dB. This improvement decreases as PFA decreases for fixed PD (0.6 dB for PD = 0.5 and PFA = 10^{-8}), and decreases as PD increases for fixed PFA (0.8 dB for PD = 0.9 and PFA = 10^{-4}).

6.2.3 Active Sonars with Multiple Pings

For an active sonar (both CW and FM) it is possible to improve the performance still further by incoherently combining the data from successive pings. This process adds the power values in corresponding range bins from successive pings, with the power values being combined after replica correlation for FM. This process reduces the noise variability relative to the noise mean and so increases signal detectability. The improvement for incoherent summation goes as the square root of the number of samples, so the improvement is $5\log_{10}(NP)$ where NP is the number of active sonar pings in either CW or FM mode.

Hence an active sonar detection threshold (Ref. 1):

$$DT_w = 5\log_{10}(d) - 10\log_{10}(wt) - 5\log_{10}(NP)\,, \tag{17}$$

where t is the time length of a single pulse. Here CW is now considered to be a limiting case of an FM sonar for the purpose of calculations. Note that for many active sonars $wt = 1$ and so whether there is a factor of $5\log_{10}(wt)$ or $10\log_{10}(wt)$ doesn't matter. Equation 17 applies in both ambient noise and reverberation noise limited cases. Here the extra term $5\log_{10}(NP)$ can be considered to be the active sonar analogue to the visual integration gain term $5\log_{10}(ndl)$ discussed in Section 5.1.2 for passive sonars.

For the purpose of looking up which ROC curve to use (Figure 3-9-1 in the SMH, or Figure 6 here), in the absence of any incoherent combination of successive pulses **FM sonars should be treated as having a bandwidth time product *wt* of unity. This**

[6] The Marcum Q-function is one minus the cumulative Rician probability density function (Ref. 12). A detailed series of ROC curves for the Marcum Q-function can be found in Meyer and Mayer (Ref. 13).

provides a correct estimate of the detection index d. However, this does NOT apply to the numerical calculation of the $10\log_{10}(wt)$ term in equation 17, for which the true values of w and t should be used.

6.2.4 Reverberation Noise Limited and Ambient Noise Limited DT

A common practice by some active sonar modellers is to quote two detection thresholds: one each for 'reverberation noise limited' and 'ambient noise limited' cases: see for example Urick (Ref. 2, Chapter 2). Superceded versions of the Sonar Modelling Handbook (Ref. 1, Changes 1 and 2) also followed the practice of two separate noise limited cases for active sonar.

The ambient noise limited threshold assumes that the signal strength of the active sonar is not high enough to fully insonify the water. This means that the ambient noise is the same as what would be expected for a passive sonar. The reverberation limited threshold assumes that the signal strength of the active sonar is high enough to fully insonify the water. This means that there are now unwanted echoes in the water from the ocean volume, ocean surface and ocean bottom. The background noise now consists of both ambient noise and the reverberation noise.

The expression for the noise limited active sonar detection threshold in these earlier models is:

$$DT_{AN} = 5\log_{10}(d) - 10\log_{10}(t) - 5\log_{10}(NP)\,, \qquad (18a)$$

and for the reverberation limited active sonar detection threshold in these earlier models is:

$$DT_{RV} = 5\log_{10}(d) - 10\log_{10}(wt) - 5\log_{10}(NP)\,. \qquad (18b)$$

Making the replacements DT_0 for DT_{AN} and DT_w for DT_{RV} shows that the difference between the ambient noise limited and reverberation limited detection thresholds is actually a result of the difference in basic definitions of detection threshold, as shown in Chapter 4. Indeed, the standard conversion between the two definitions applies here.

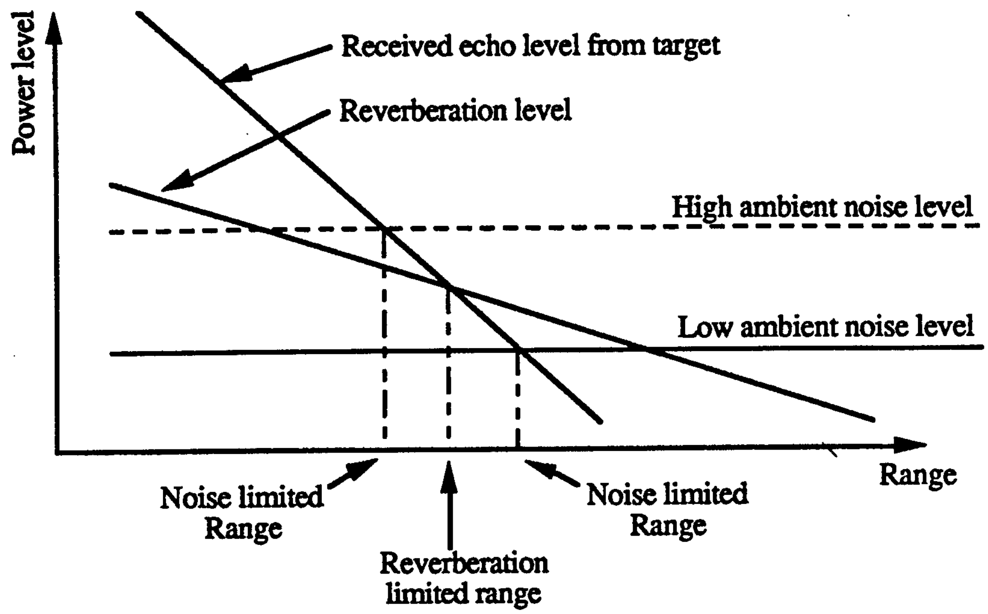

Figure 7. An illustration of how echo level, reverberation and ambient noise behave as a function of range. The ambient noise and reverberation need not be straight lines as a function of range as depicted here. Figure is reproduced from the Sonar Modelling Handbook (Ref. 1, Figure 2.1).

Figure 7, which is reproduced from Figure 2-1 of Reference 1, illustrates the simplified behaviour of echo level, reverberation and ambient noise as a function of target range. In Figure 7 the sonar is said to be 'noise limited' for the high ambient noise level line, and 'reverberation limited' for the low ambient noise level line.

The difference in numerical values given by the two detection threshold equations 18*a* and 18*b* has no effect on calculating the overall signal excess, as the supposed performance improvement in the reverberation case is offset by an increase in the noise term in the sonar equation (see Chapter 3). It is common practice when modelling active sonar DT for separate ambient noise and reverberation limited cases that the worst of the two numerical values is typically chosen for the scenario being studied, although it is then necessary to ensure that the dimensions of the terms in the sonar equation are consistent.

6.3 Correcting for Using Gaussian Noise for the ROC Curve

Under operational conditions the product of the bandwidth w and the integration time t (or signal pulse length t) is often not extremely large (i.e. >100), as was assumed in Peterson *et al.'s* (Ref. 5) derivation of equation 5. With small wt products (here <100), the noise at the output of the processor is not completely Gaussian as there are not enough samples for the Central Limit Theorem to be fully effective (Ref. 2). This causes a decrease in signal detectability as there are more 'spikes' due to the noise and leads to a correction factor $C(wt)$ which always increases DT. Figure 8 is a plot of $C(wt)$ against $wt+1$, and has been taken from Figure 12.11 in Urick (Ref. 2), which in turn is based on work by Nuttall and Magaraci (Ref. 14) and Nuttall and Hyde (Ref. 15). The figure shows that, for various combinations of PD and PFA, the correction is significant at very small values of wt where the deviation of the output noise from Gaussianity is greatest, and tapers off to relative insignificance for $wt > 100$.

The horizontal scale in Figure 8 gives $wt + 1$ as the number of independent samples. This is because the number of samples is wt, plus there is a sample of background noise with which to make a comparison for the purpose of deciding whether or not a signal is present.

To see how the correction term for Gaussian statistics fits into the detection threshold model, consider the following worked example. Two ways to calculate the detection index term will be given and it will be seen that they each lead to the same result, to within rounding error.

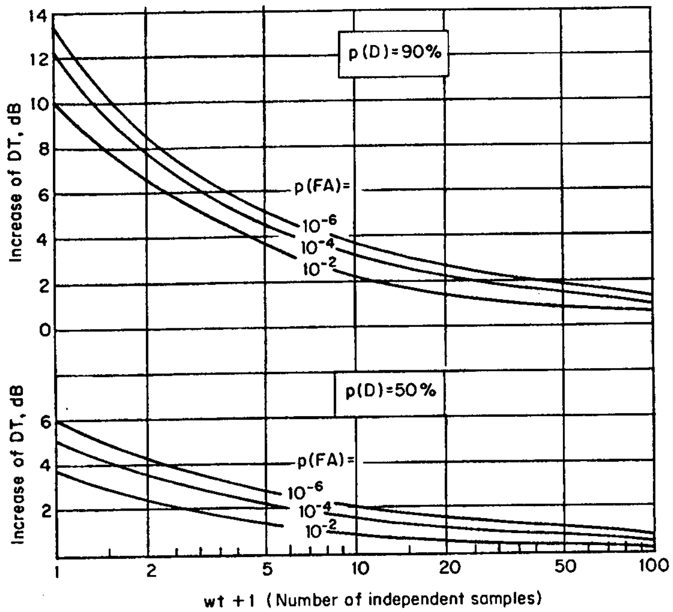

Figure 8. The increase C(wt) in detection threshold as a function of the bandwidth-time product for small values of wt in the cases of PD = 50% and PD = 90% for various values of PFA. Figure is reproduced from Urick (Ref. 2, Figure 12.11) and is © R. J. Urick. Note the difference in notation compared with that used in this report.

A Worked Example.

In this example an active sonar emitting a single pulse is to be modelled for PD = 0.5 and PFA = 10^{-4}.

The recommended method of solution is to use the correct signal plus noise and noise probability density functions. In this example the active sonar uses the ROC curve shown here as Figure 6, with Rician statistics for the signal plus noise and exponential statistics for the noise (Ref. 1). Reading off PD = 0.5 and PFA =10^{-4} in the figure gives $10\log_{10}(d)$ = 18.6 dB, so that $5\log_{10}(d)$ = 9.3 dB.

The alternative method of solution is to use an approximation of a Gaussian signal plus noise distribution. An appropriate Gaussian ROC curve for this approximation is shown here as Figure 3 (Ref. 2). Reading off PD = 0.5 and PFA = 10^{-4} in the figure gives $d \approx 14$. For better accuracy, the value of $\sqrt{d}$ can be read off tables of the normal (i.e. Gaussian) distribution in a mathematics text such as Kreyszig (Ref. 16, Table A9): this gives $\sqrt{d}$ = 3.719 so that d = 13.83. Hence $5\log_{10}(d)$ = 5.7 dB. The next step is to use Figure 8 to determine the correction for having made the assumption of Gaussian statistics. To calculate d for an active sonar we must use $wt = 1$, and since the results of this single ping are compared with a sample of background noise in order to make a detection decision, then the total number of independent samples here is $wt + 1 = 2$. Reading off Figure 8 for PD = 0.5 and PFA = 10^{-4} gives the correction $C(wt) \approx 3.6$ dB. This gives the net effect of the detection index term in the detection threshold model as 5.7 (Gaussian statistics) + 3.6 (correction for assumptions) for a total of 9.3 dB, the same as for the recommended method where the correct statistics were used. Of course, the $10\log_{10}(wt)$ term must be calculated using the correct values of w and t.

The moral of this worked example is clear. If the model contains simplifying assumptions then the modeller must always:

- ensure that appropriate correction factors are applied, as they were in the above example for the use of Gaussian statistics, or

- ensure that the simplification has negligible effect on the calculations.

Another point to be remembered when dealing with correction factors is that rounding errors will be common. For example, reading off the correction factor $C(wt)$ from Figure 8 incurs an error of, say, ± 0.1 dB or higher, so there would be a corresponding uncertainty in the final estimate of DT.

6.4 Smoothing Filter Mismatch

In the case of pulsed signals for an active sonar, the expression for DT given by equation 17 assumes that any postdetector averager, or smoothing filter, is perfectly matched to the signal duration (Ref. 2, p.385). If the filter is mismatched the detection threshold is increased by an extra term of $\left|5\log_{10}(\tau/t)\right|$ added into equation 17, where τ is the integration time (in seconds) of the smoothing filter and t is the integration time of the system input, i.e. t is the signal duration.

6.5 Echo Time Spreading

Conley (Ref. 4) has described 'echo time spreading' for an active sonar, wherein the received echo is spread in time beyond that of the transmitted signal with which it is being matched. This is also called 'energy splitting loss' (ESL). For the example given by Conley of a mid frequency FM pulse, the ESL correction which should be applied to an active sonar DT varies from 3.9 dB (in a convergence zone or surface duct) to 7.4 dB (shallow water) and up to 11.7 dB (bottom bounce). ESL applies to both noise and reverberation limited cases and so should always be considered when modelling an active sonar. Conley recommends that ESL be included in the sonar equation as a separate term.

Suppose the distribution of arrival times of the multiple paths is Gaussian. Let s be the standard deviation of the time spread and r be the resolution of the FM pulse (effectively the inverse of the effective noise bandwidth). The ESL is then given by the Bell-Jones relation (Ref. 4):

$$\begin{aligned} ESL &= 0 && \text{for } s/r < 0.1\,, \\ ESL &= 3.34 + 3.4\log_{10}(s/r) && \text{for } 0.1 < s/r < 1.82\,, \\ ESL &= 2.36 + 7.2\log_{10}(s/r) && \text{for } s/r > 1.82\,. \end{aligned} \tag{19}$$

Typical values for s are 15 ms for convergence zone and surface duct conditions and 50 ms in shallow water. For bottom bounce conditions s varies from 15 ms for MGS 1 to MGS 4, 30 ms for MGS 5, 50 ms for MGS 6 and 200 ms for MGS 7 or 8. Here 'MGS' is a measure of the bottom reflection of an acoustic pulse: it is a complicated function of the distribution of the particles in the seabed. MGS 1 is a highly reflective bottom, with increasing MGS corresponding to lower reflectivity and increased sound absorption. MGS 4 is considered to be quite absorptive of sound.

6.6 Noise Nonstationarity

The noise is called 'stationary' when the probability density function of the noise does not change with time (Refs. 17, 18). Whether or not the noise is stationary has little effect on the performance of a conventional beamformer. However, an adaptive beamforming algorithm requires the noise to be nearly stationary throughout the integration time, otherwise the mathematical computations often become unstable and erroneous solutions are obtained. Normally the modeller can ignore any losses coming from noise nonstationarity, unless the integration time is so long that the nature of the ambient noise field changes due to biological and weather effects, etc. See Frost (Ref. 19) for an idea of natural fluctuation timescales: as a guide short term effects last 3 - 15 minutes and medium term effects last 1 - 2 hours, while long term effects follow an approximately daily cycle.

6.7 Signal Variability

6.7.1 Variable Emitted Signal

There is an assumption inherent in deriving the detection index and conventional ROC curves, which is that the signal is steady throughout the integration time. This assumption is equivalent to saying that the occurrence of the signal does nothing more than shift the mean level of the noise, and that the standard deviation of the combined signal plus noise distribution is the same as the standard deviation of the pure noise distribution.

References 2, 20 and 21 discuss expressions for PD and PFA when Gaussian signal fluctuations are present. Signal variability is a subject of ongoing research (Ref. 22), but its effects can be approximated on modified ROC curves using a fluctuation index, or by use of transition curves which are plots of PD versus signal excess (SE). The signal excess is defined as the excess or deficiency of input SNR relative to that needed for PD = 0.5, for a given value of PFA (Ref. 2, Section 12.4). Sample modified ROC curves and transition curves can be found in References 2 and 20.

An example of a modified ROC curve is shown here as Figure 9, which is reproduced from Figure 3-4 of the Sonar Modelling Handbook (Ref. 1). The fluctuation index k is defined as:

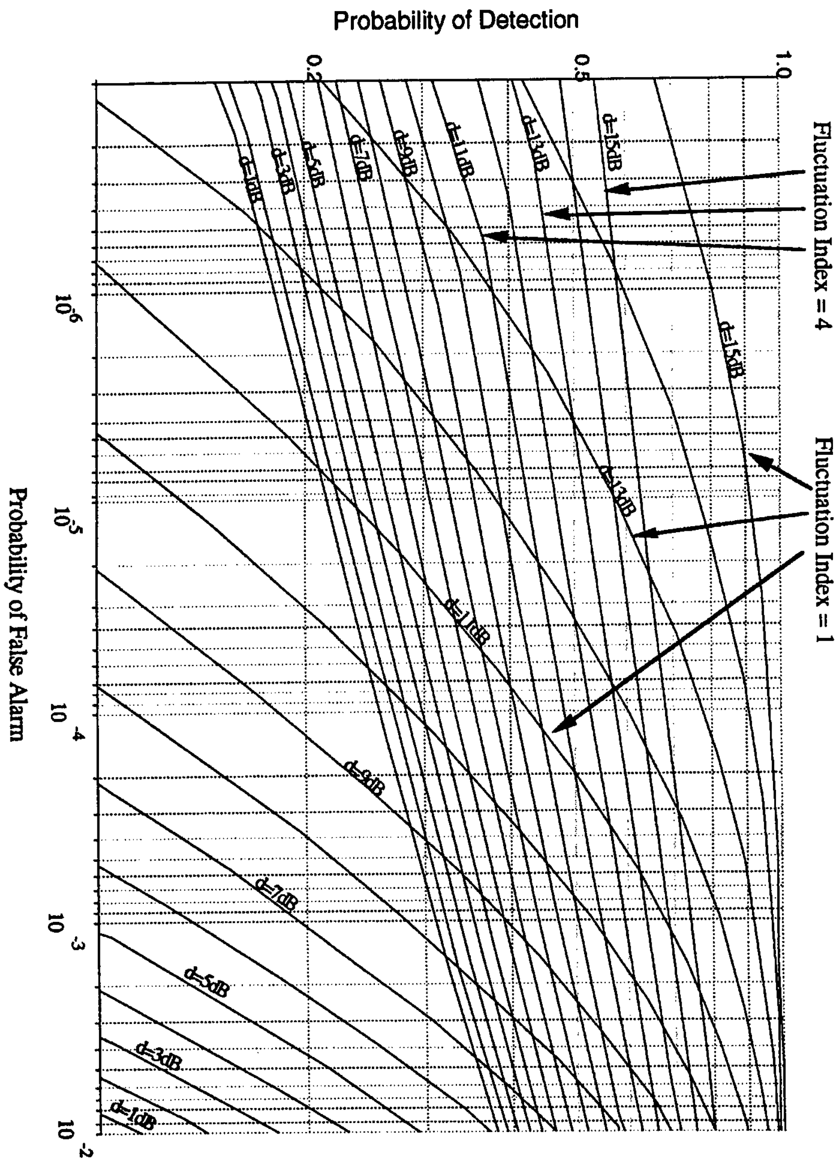

Figure 9. Receiver operating characteristic curves for both the signal plus noise and noise pdfs as Gaussian distributions, with d being the detection index. Separate sets of curves appear for the fluctuation index k = 1 and k = 4. The curves cross at PD = 0.5 for a given value of d. *Figure is reproduced from the Sonar Modelling Handbook (Ref. 1, Figure 3.4). Values are* $10\log_{10}(d)$.

$$k = \frac{\sigma_{s+n}}{\sigma_n}, \tag{20}$$

where σ_{s+n} is the standard deviation of the signal plus noise distribution and σ_n is the standard deviation of the noise distribution. The fluctuation index is only meaningful in the mathematical sense when Gaussian statistics are assumed, as is the case in Figure 9. It can be seen that for PD = 0.5 the fluctuations make no difference to the calculated value of d for a given value of PFA.

Figure 9 shows that for PD > 0.5 and a given value of PFA, having $k = 4$ gives a higher value for d than for $k = 1$. This in turn gives a higher value of DT, indicating that the detection performance has become worse due to the signal variability causing dropouts in the SNR. Now consider Figure 9 for PD < 0.5 and a given value of PFA: in this case having $k = 4$ gives a lower value for d than for $k = 1$. This in turn gives a lower value of DT, indicating that the detection performance has been improved due to the signal variability causing detectable spikes in the SNR.

While fluctuations have no effect on DT for PD = 0.5 and a given value of PFA, the further away from 0.5 is the selected value of PD, the greater the effect on the results. This is one reason why PD = 0.5 is often chosen for use in simple estimations of DT. If it is necessary to account for fluctuations in the model, it is recommended that either Figure 9 or the modified ROC curve in Urick (Ref. 2, Figure 12.9) be used to calculate the modified detection index d.

6.7.2 Loss of Signal Coherence Across the Array

For relatively large apertures there is a loss in signal coherence across the receiving array aperture due to propagation effects. According to Conley (Ref. 4) for a "mid frequency FM pulse" in an active sonar the loss varies from 0.0 dB for a convergence zone and half channel propagation conditions, 0.1 dB for shallow water and surface duct propagation conditions, and anywhere from 0.1 to 5.2 dB for a bottom bounce signal. However, this loss is not a sonar processor loss and is therefore not part of the detection threshold calculation: instead it should be modelled as part of the operational degradation factor DF_0 in the sonar equation. This coherence loss should not affect Difar sonobuoys, but may need to be accounted for when considering relatively large receiving arrays such as the Barra passive sonobuoy, towed arrays, flank arrays and large active sonar receivers.

6.8 Operational Processor Loss, or "Processor Degradation Factor"

When the detection threshold is used in simulation studies there is an additional correction term often described as an 'operational processor loss' (OL) or as a 'processing degradation factor' DF_P (Ref. 1). This is a catch-all for several small but cumulative losses which typically occur in an operational sonar system. These losses are typically due to deficiencies and approximations in the detection model, electronic noise ignored in the detection model, etc.

Some authors call this loss an 'operator loss', but this mistakenly implies the loss is due mainly to flawed use of an operational sonar processor by a human observer. Various other names for this same arbitrary loss factor will often appear in the literature: the modeller is advised to exercise caution in 'decoding' terminology when consulting references.

A typical value considered to represent the sum total of these modelling errors, which are otherwise variable to some degree and often difficult to specifically quantify, is to set the operational loss ('processing degradation factor' in the Sonar Modelling Handbook) at OL = 4 dB for the following types of sonar: passive narrowband, passive cross-correlation broadband, passive full array broadband, both CW and FM active sonars and intercept sonars. This value for OL is completely arbitrary and should be revised accordingly if the modeller has additional information on the performance of the particular sonar being studied. For example, some authors instead use OL = 3 dB for their sonar processors, while commercial sonar manufacturers at times will quote a degradation in the range of 1 to 3 dB. If the modeller expressly includes a correction for human observers and time history effects (Section 8.1) then OL should be reduced to approximately 1 dB.

All models of an operational sonar processor include an arbitrary loss factor in the detection threshold. The modeller should always mention the value of OL selected for use in the model. Some of the correction factors listed in the rest of this chapter are often included as part of the operational processor loss term in DT. The modeller has the responsibility of deciding which of the correction factors discussed below become part of OL and which are modelled separately. Note that the operational processor loss correction term OL (or DF_P in Ref. 1) is cumulative with the operational degradation factor DF_0 used in the sonar equation (Section 3.1).

6.9 Multiple Signals

The standard ROC curves assume that only one signal at a time is to be detected. Hence when there are multiple signals to be detected simultaneously, this also modifies the ROC curves (Refs. 2, 23). For example, for a fixed value of PFA the effect of multiple signals is to reduce PD for a given signal. However, for the purpose of obtaining a simple estimate of DT it can (conveniently) be assumed that only one signal is present with little loss in accuracy, as the correction to DT if a few signals are present is of the order of a fraction of 1 dB. This correction can be subsumed into OL.

6.10 Receiver Hydrophone Position Error

Conventional and adaptive beamformers normally operate on the assumption that the positions of the receiving array elements are known exactly. In practice the position of the receiving elements is only known approximately in many cases such as for towed arrays. Conventional beamformers can generally tolerate a positional error of up to about one quarter of the signal wavelength, while adaptive beamformers can generally only tolerate a positional error of about one tenth of the wavelength throughout the data integration period, before significant performance degradation will occur. For a rigid array this is not a significant problem, e.g. hull mounted active and passive sonars or Difar and Barra sonobuoys. Sonar processors for towed arrays have array shape estimation algorithms to try and compensate for the effects of array distortion.

For practical modelling purposes the effect of any hydrophone positional errors can either be subsumed into the operational loss term OL, or written off as being negligible. Alternatively, the array gain can be reduced by 0.2 dB if using a towed array as a receiver (Ref. 4).

6.11 Scalloping Losses

6.11.1 Receiving Beam

A potentially significant loss term is due to 'scalloping' of the sonar beams. For an array of hydrophones, the sonar processor forms beams at regular intervals in azimuth. These beams are rounded in shape, often ellipsoidal in cross section, so that where their edges overlap there is a small dip in the array response. If the target is not central to one of the beams, or is crossing from one beam to the next due to high relative transverse motion, there is a dip in gain as the target moves through the dip in the array response where the beams overlap. The scalloping loss varies with the type of window function used (Ref. 9), but it can often vary up to 3.0 dB or more at some frequencies if the beams are widely spaced (Ref. 4). For *closely spaced* beams modern signal processing techniques can generally render this scalloping loss negligible (Ref. 24).

The scalloping loss can be either subsumed into the operational loss factor OL or subtracted from the effective array gain of the system. This choice is at the modeller's discretion, but it is recommended that it be used in the array gain (or directivity index for isotropic noise) as scalloping in the receiving beam is a beamformer effect. This effect only applies for receiving arrays having more than one hydrophone.

Scalloping losses of 1.5 to 2.5 dB are also typically encountered in wide search beam patterns used with active sonar pings (Ref. 4), but this is more correctly modelled as a loss term in the "source level" in the sonar equation.

6.11.2 Frequency Filters

Frequency scalloping occurs because the filter response is reduced between the centre and the crossover points of adjacent filters, i.e. this effect is due to window weighting. The loss term is given by (Ref. 4):

$$LOSS = 1.11\left(\frac{\text{crossover frequency}}{\text{3dB down frequency}}\right)^2 . \tag{21}$$

The frequency separation between adjacent filters may also be used. If the modeller does not have the crossover frequency and 3 dB down frequency available, then this correction term should be subsumed into the OL term and otherwise ignored. While this loss is due to the weighting of the windows, a further loss due to overlapping of the windows is discussed in Section 6.12.

6.12 FFT Overlap

When a fast Fourier transform (FFT) is performed a series of windows is established to sample the incoming time series to obtain its spectra. In active sonar applications the window size is typically equal to the pulse width. To reduce sidelobes the window is weighted at the edges, and to compensate for the side effects of losing energy the FFT windows are overlapped in time. For typical overlaps the average loss of this process for an active sonar is 0.6 dB for a 50% overlap and 0.3 dB for a 75% overlap. This applies against both noise and reverberation cases for CW (active) processors and also affects passive narrowband receivers. It should be modelled as a component of OL in modelling DT. Note that the separate loss due to the weighting of each window is accounted for in the frequency scalloping loss: see Section 6.11.2.

6.13 Not Enough Integrations

Another correction factor is sometimes necessary if an adaptive beamformer is used with a passive sonar. The effect of an adaptive beamformer is to significantly increase the array gain at some frequencies, but at the price of an increased requirement in processor computational power and an increased noise variance which raises the detection threshold by a small amount. This increased noise variance results in an increase K in DT if the number of integrations used in the adaptive processing is small. Normally an adaptive beamformer will require the number of integrations to be equal to, or greater than, the number of hydrophones in the receiving array. An example is given by Gray (Ref. 25) for one specific case. For most applications K is of the order of 0.5 dB, decreasing to negligible size if a large number of integrations is used (Ref. 26). If in doubt or lacking sufficient information, the modeller may safely assume this is yet another small cumulative contribution to OL and otherwise ignore it.

6.14 Loss Due to Ship or Sonobuoy Motion

As the receiving array bobs around in the ocean, be it a sonobuoy or a hull mounted array, sometimes beam stabilisation (e.g. hull mounted active sonar) doesn't quite manage to keep the beam pointing in the proper direction. A sonobuoy will sometimes, in bad conditions, tilt too far and 'jam' its compass. The performance loss due to this effect should be modelled as part of the operational degradation factor DF_0 or as part of the source level term in the sonar equation and not in the OL term as part of the detection threshold.

6.15 Equipment Misalignments and Display Losses

Conley (Ref. 4) has described the various possible equipment misalignments for an active sonar which cause losses in performance. Loss mechanisms can occur in the filters (both active and passive sonars), FM replication (active sonar), ODN[7] circuits (active sonar), display controls (active and passive sonars), servos (active sonars), drifting of the transmitter drive (active sonar) and drifting of analog circuits (active and passive sonars).

According to Conley, measurements of existing active sonar systems give typical values for losses due to equipment misalignments as 5 dB for displays, 2 dB for receivers and 2 dB for transmitters. While the transmitter loss is properly modelled as being part of the source level or as part of DF_0, the receiver loss is part of the active sonar detection threshold term and applies in both noise and reverberation limited cases. Note that in the case of a passive sonar this loss would be smaller and would then be consistent with being a significant component of OL.

The relatively large display loss due to 'equipment misalignment' mentioned by Conley is quite possibly due to a cumulative contribution of human factors effects and is therefore properly modelled as being part of the recognition differential (see Chapters 7 and 8) and not detection threshold. However, if the system being modelled uses a human observer to make the detection decision, then human factors losses and display losses must be included in the model of DT.

For Conley's example of a mid frequency FM pulse (Ref. 4), the loss due to equipment misalignment is listed as 2.0 dB. This loss is consistent with the receiver loss mentioned above. If the modeller does not have measurements of the equipment

[7] 'Ownship Doppler Nullification': see Nielsen (Ref. 11, p.210) for a description of how this is used to make returns from ocean scatterers centre at a fixed frequency, regardless of the platform motion and the direction of the transmitted beam.

misalignment losses for the sonar being modelled, it may be assumed that these losses are a significant part of the arbitrary OL term.

There are also systematic losses at the display due to misalignment, as well as due to display implementation errors such as quantisation, thresholding and paging. For example, quantisation loss occurs in matching a smooth distribution of data samples to a finite number of greyscale steps[8] on the sonar display: some errors in rounding off occur regardless of what quantisation scheme is used. One example of quantisation loss is given by Dawe and Grigorakis (Ref. 27), but this loss varies between quantisation schemes and depends on the length of time history (passive sonar) used to make the detection decision. For an example of a mid frequency FM pulse, Conley gives the cumulative display loss as being 0.3 dB.

6.16 Doppler Mismatch

With FM pulse compression processing a loss is incurred when the frequency of the received signal is shifted off the reference frequency, thereby reducing the overlap of the signal and reference (Ref. 4). The loss is given by:

$$Loss = 20\log_{10}\left(1 - f/B\right) \,, \tag{22}$$

where f is the frequency shift and B is the original bandwidth of the waveform. This affects both noise and reverberation limited cases for FM waveforms processed by pulse compression techniques and should be included in the DT model. As with other losses, if there is insufficient information to use equation 22, assume this loss is part of OL in the model of DT.

[8] Monochromatic green on black sonar displays typically use 8 greyscale levels, this being consistent with the ability of the human eye to clearly distinguish relative differences in saturation in a single colour hue.

6.17 Echo Frequency Spreading

'Echo Frequency Spreading' occurs when the signal frequency spectrum extends beyond the limits of the receiving filter for Doppler processing of CW signals. It is due to the frequency spreading in the target echo arising from target motion. This loss can vary by up to several dB in extreme cases (Ref. 4), although for the great majority of cases this loss will be negligible. For the purpose of detailed modelling, it is essential that the modeller should check that the width of the receiving frequency filter can accommodate all the expected variations in target echo frequency.

6.18 Waveform Shading

Waveform shading is the application of a specific type of amplitude weighting to the envelope of a CW pulse. This is to reduce the spectral sidelobes of the reverberation and so better detect targets with Doppler shifts greater than the bandwidth of the mainlobe of the reverberation. For modelling purposes, the actual pulse length (PL) should be reduced to an equivalent pulse length (EPL). Examples from Conley (Ref. 4) are:

Hanning shading: EPL = 0.5*PL.

Hamming shading: EPL = 0.54*PL.

Half cycle sine shading: EPL = 0.64*PL.

These scaling factors to convert PL to EPL are the "coherent gain" factors from Harris (Ref. 9, Table 1), reproduced here as Figure 2, so the modeller can select the appropriate factor for whichever window function is used by the particular sonar being studied. Failure to use EPL in place of PL results in an error of 2 to 3 dB depending on the shading scheme used. Waveform shading is appropriate to being modelled as part of the detection threshold.

Note that this correction has the same effect as converting the frequency binwidth to the equivalent noise bandwidth in the detection threshold equation *7b* or equation 10 for an active sonar. However for passive sonars which are modelled using the alternative definition of DT, equation *7a*, the modeller should follow correct procedure and convert the binwidth to the equivalent noise bandwidth using Harris (Ref. 9), Figure 2, or equation 10 for complicated spectra.

6.19 Normalisation

Normalisation is used so that an active sonar system can maintain a constant false alarm rate (CFAR), even when background conditions are changing. Normalisation is implemented by sampling the background before and after a potential reception window: the more samples the better the estimate of CFAR and the smaller the "CFAR loss". Conley (Ref. 4) gives an example for a broadband Rayleigh background as:

$$Loss = \frac{35\left(0.88^{k}\right)}{n}, \tag{23}$$

where n is the number of noise samples and $k = 3.32\log_{10}(n/10)$. This loss affects both noise and reverberation calculations of active sonar DT. For an example of a mid frequency FM pulse Conley (Ref. 4) gives the normalisation loss or "CFAR loss" as 1.2 dB for both noise limited and reverberation limited cases.

7. Quantities Related to DT

Consider the information processing chain for a complete sonar system, as shown schematically in Figure 1. The concepts of minimum discernible signal (MDS), detection threshold (DT) and recognition differential (RD) are all related to each other and the concept of a level at which the signal can be detected, for a given combination of PD and PFA. The difference between MDS, DT and RD is simply the place in the information processing chain, as represented in Figure 1, at which the threshold signal to noise ratio (SNR) is effectively being measured.

The detection threshold is the SNR measured at the *receiver input* terminals (*A-A′* in Figure 1) required for detection at some preassigned level of correctness of the detection decision. (Remember the choice of definitions in Chapter 4.) Here any beamforming is modelled as occurring prior to the receiver input terminals, so that the value of DT excludes the beamformer effects. DT is a value which can be calculated theoretically in some cases and approximated in other cases (Ref. 5), and is amenable to measurement. Since many system loss terms can be combined into the generic 'operational processor loss' term, DT can therefore be used as a reasonable figure of merit to describe the sonar system performance.

The minimum discernible signal is the SNR measured at the *hydrophone input* required for detection at some preassigned level of correctness of the detection decision. The MDS applies before any beamforming or clipping due to finite word length is performed on the hydrophone data, so there is a factor of "array gain" difference between MDS and DT:

$$MDS = DT - DI \qquad \text{(isotropic noise)}, \tag{24a}$$

$$MDS = DT - AG \qquad \text{(nonisotropic noise)}, \tag{24b}$$

where the directivity index DI applies for isotropic noise and the array gain AG applies for directional noise. Note that MDS also has a choice of definitions similar to that for DT, where for a given PD and PFA the MDS can be defined relative to a 1 Hz bandwidth for the noise or relative to the receiver bandwidth for the noise.

In Figure 1 the recognition differential is the SNR measured at the *display* required for detection at some preassigned level of correctness of the detection decision. The RD applies after any extra signal processing (excluding beamforming) has been done, and also applies after the results have been suitably conditioned for display to the (human) observer by either visual or aural means. For an operational sonar system RD is complicated by human factors effects and so is inherently more difficult to accurately measure than DT. Note that RD also has a choice of definitions similar to that for DT.

The above definition of RD comes from the Sonar Modelling Handbook (Ref. 1) and should not be confused with the RD used in the older literature in relation to auditory detection, as mentioned by Urick (Ref. 2, Section 12.10). The two quantities are actually defined in different ways, with the auditory version of RD being obsolescent and inherently meaningless as it ignores the effects of false alarms.

There are several losses which can occur in the system as the signal makes its way to the output where the detection decision is made by the observer. First, there can be imperfections or even failures in hydrophones in the sensing array. There will be electronic noise throughout the sonobuoy and receiver systems degrading the SNR. In some systems, the signal level received from the array of hydrophones may be so high that the level is clipped[9] in order to transmit it to the receiver. With sonobuoys sending data to an aircraft, some losses may occur in inclement weather due to the sonobuoy antenna being temporarily immersed, or the radio link may be temporarily broken due to operational factors. The radio receiver in the aircraft may not have a perfect response to the incoming signal: this is often the case as the response at low frequencies is tapered off to prevent the large amount of noise measured at low frequencies from swamping the dynamic range of the receiver electronics.

There are also gains and losses introduced into the system as the signal undergoes any final processing and conditioning for display to the observer. Losses may again occur due to electronic noise, imperfect formatting of the displays and poor conditions in which observations are made (e.g. a noisy environment when listening to an aural presentation).

[9] This 'clipping loss' is properly modelled in the sonar equation as a loss accompanying the array gain term instead of being included in the calculation of DT.

A concept related somewhat to passive sonar detection threshold is that of 'classification threshold' (CT). This is effectively the SNR measured at the *receiver input* terminals required for classification of a target at some preassigned level of correctness of the classification decision. Obviously the process of classification requires there to have been a detection, so CT is related to DT. Normally CT will be higher than DT as a sonar operator will try to obtain a classification on two or three (or possibly more) narrowband lines or broadband features, not just one line or feature as is used for the initial detection. See Dawe (Ref. 3) for a suitable range of estimates for CT.

8. Observer and Display Effects

The initial calculation of DT considers the observer to be a device making simple yes/no decisions by comparing the input signal to noise ratio with a threshold level. In practice humans are often used as the observer, so there may be losses due to 'human factors' such as fatigue. Strictly speaking, human factors effects are associated with the display presentation and observation conditions and so they are more correctly modelled as being part of the 'recognition differential' (see Chapter 7). Inclusion of a 'human factors' degradation as part of DT is a choice for the modeller, and it is recommended by the author that the modeller includes this correction, *with the human now considered to be an intrinsic part of the detector.*

8.1 Time History Loss for Human Observers

Research by Mohindra and Smith (Ref. 28) and more recently by Dawe and Grigorakis (Ref. 27) and Dawe (Ref. 29) has shown that human observers using time history displays to make detections do not perform as well as theoretical predictions, once allowance has been made for correction factors such as display quantisation. This loss of performance is because the detection threshold theoretical model assumes the observer has all the numerical information concerning the data (Ref. 5), but human observers must instead make do with relative brightness levels and use pattern recognition techniques when making their detection decisions from sonar displays.

The theoretical time history component of detection threshold is given by equation 8 for a passive sonar. However, to account for the difference L in average performance between an 'ideal' observer using the numerical information within each display cell, and a human observer using only pattern recognition, the modeller can use an empirical expression:

$$L = 1.38\log_{10}(ndl) + 0.20(ndl)^{0.65} , \tag{25}$$

where *ndl* is the number of rows (i.e., time steps or display updates) of independent data used by the observer to make the detection decision. This expression was derived from the results of Dawe and Grigorakis (Ref. 27) from an experiment using a passive sonar lofargram. It agrees with the data trend of Mohindra and Smith (Ref. 28) and the results of Dawe (Ref. 29) to within the experimental uncertainty of ±0.5 dB for all values of *ndl*. This loss is added to the expression for DT, such as equations 7 or 9, and its associated uncertainty must of course carry through to any final number quoted for DT by the modeller.

Equation 25 is expected to apply to any passive sonar display based on accumulating a time history as a series of independent parallel display rows to aid in detection. Examples include a lofargram (frequency-time-intensity plot) for either an omnidirectional receiving array or a beam from a directional array, and also a T-θ display (bearing-time-intensity plot) which is also used with a directional receiving array.

It is important to note that this 'time history loss' *L* effectively includes a considerable part of the otherwise arbitrary operational processor loss OL (Section 6.8) for a passive sonar. In this case the modeller should decrease OL to approximately 1 dB.

8.2 'Human Losses'

Quantifying 'human losses' for the purpose of detailed modelling of a sonar system with a human observer is a difficult task. In laboratory experiments there is always the knowledge amongst the test subjects that, no matter how well briefed on making the results their best effort, the experiment is still inherently artificial. The only truly reliable measurements of human performance as sonar observers in making detections would be measurements made in true combat conditions, but for obvious reasons this is not a practical experiment.

Numerous factors can influence the performance of a human observer. These can include the following:

- the training level and concentration span of the observer,
- mission duration: a long mission induces fatigue in its later stages,
- the availability of stimulants such as coffee,
- fatigue causes which are pre-existing at the start of the mission, e.g. lack of sleep, worries from home life, hangovers, etc.
- stress levels in true combat environments, especially for militarily critical missions,

- observers being jolted around by the platform (particularly on low flying maritime patrol aircraft and helicopters),
- environmental conditions within the platform distracting the observer. For example, listening to acoustic information is difficult on a noisy platform such as a helicopter or maritime patrol aircraft, but it is a recommended method of detection by sonar observers on board a quiet platform such as submarine.

Detailed measurements of the effect on human performance of each of these items taken in isolation are generally not available. However, it should be noted that trained sonar observers are always tweaking the system and trying to eke out the best performance, especially in relatively realistic ocean exercises: this is simply a manifestation of professional pride and competence. If measurements attributed to 'human losses' are available from such exercises the modeller is highly recommended to incorporate these losses into either the operational processor loss OL if the loss is due to sonar system factors, or as part of the operational degradation factor DF_0 in the sonar equation. If these measurements are not available, then the modeller must assume they form part of the arbitrary loss values assigned to OL and DF_0.

8.3 Time Sharing of Sonobuoy Data

In an operational scenario, airborne sonar operators typically deploy and monitor a field of sonobuoys. The field of sonobuoys deployed is designed to cover the largest possible area consistent with the effective detection range in the ambient conditions. However, more sonobuoys may be deployed than can be effectively monitored at any one time: this could be due to either limited sonar processing capability, limited sonar display area, or too many buoys being displayed for the sonar observer to assess in real time. Switching observations back and forth between sonobuoys is known as 'time sharing', and this is properly modelled as part of the operational degradation factor DF_0 in the sonar equation.

Time sharing losses are not appropriate for the deployment of a single active sonobuoy, as the deployment of such a system typically attracts the full attention of the sonar observer.

8.4 Colour Displays

Many of the sonar displays currently in use are monochromatic 'greyscale' displays consisting of either green or yellow on a black background. There has been a recent upsurge of interest in using colour sonar displays, so some advice to the modeller is appropriate here.

While research in this area is still in progress, it seems that the most significant use of colour will be to enhance the bearing and classification information contained within a passive sonar display such as a lofargram[10] (Ref. 30). For example, display pixels can be colour coded red, orange, yellow, green, blue, etc. with each colour representing a sector of azimuth. Thus if two contacts with distinct narrowband spectra were present on the colour lofargram display, the observer could identify which signal tones were from which sector and identify each of the contacts separately. Active sonar displays also have ways for colours to represent useful information (Ref. 31). The danger associated with using multiple colours within a sonar display is that of information clutter, wherein the colours seem to merge and obscure the observer's ability to make a clear detection.

Research has shown that using colour coded information within sonar displays leads to a small but significant improvement in DT for human observers. Work by Buratti, Rio and Witlin (Ref. 31) for active sonar displays and by Dawe and Galbreath (Ref. 32) for passive sonar displays such as lofargrams indicates that DT can be improved by up to 1.5 dB with appropriate colour coding. This improvement of 1.5 dB is described as a 'colour gain' (CG) by Dawe and Galbreath. It seems that the human observers use the additional colour coding to make extra detections as an adjunct to their normal pattern recognition techniques used for spotting targets on the sonar display. More work in this area is required to determine the exact conditions in which this colour gain improvement in DT can be obtained. However, present indications are that it can be effectively modelled as an additional term in the DT model, and which partially counteracts the degradation term OL.

9. Summary of Equations for Different Sonars

This chapter is intended as a quick summary for those workers needing to chart their way through the maze of options to the appropriate detection threshold equation. Modellers should read Sections 9.1 and 9.2, then turn to the subsection in 9.3 most appropriate to the sonar system they are modelling.

[10] A great deal of this work is commercially sensitive, as several companies are using colour enhancements on their latest generation of military sonar systems. This generally precludes quoting specific results in an unclassified guide such as the present report.

9.1 Summary of Common Terms

The expressions for the detection threshold (DT) given below in Section 9.3 are specific to types of sonars, as shown in each subsection heading. To help demonstrate the distinction between equations based on the alternative definitions of detection threshold, a subscript "0" on DT indicates the detection threshold applies for noise referred relative to a 1 Hz bandwidth, while a subscript "w" on DT indicates the detection threshold applies for noise referred relative to the receiver bandwidth. The more common usage of either DT_0 or DT_w is mentioned for each sonar.

In all the following sections d is the detection index. The modeller should use the recommended ROC curve to determine d for each sonar application. Sample ROC curves appear in this report as Figures 3, 4, 5, 6 and 9.

The effective noise bandwidth w should always be calculated via equation 10 using the measured background spectrum if that is available. Otherwise w can be determined by multiplying the frequency binwidth (for narrowband sonars) or frequency bandwidth (for passive broadband sonars and active sonars) by the appropriate windowing scaling factor. See the extensive list of equivalent noise bandwidths (ENBWs) in Figure 2, which is reproduced from Harris (Ref. 9). If windowing is not known to be in use, the modeller should assume a rectangular window is applicable. If windowing is in use but the exact type of window is unknown, a reasonable approximation can be obtained using a Hanning (a = 2.0) window in Figure 2.

The total length of time history used by the observer to make a detection decision is $t = T.(ndl)$, where T is the length of time for one display update, e.g. one row of data on a lofargram, and ndl is the number of display updates. When using human observers to make the detection decisions on passive sonar time history displays such as lofargrams and bearing-time history (T-θ) displays, there is a loss due to a flawed assumption within the detection theory (see section 8.1). The empirical estimate of this 'time history loss' L for passive sonars is:

$$L = 1.38\log_{10}(ndl) + 0.20(ndl)^{0.65} \quad , \tag{25}$$

with an uncertainty of ±0.5 dB for all ndl. This uncertainty should be carried through to the final estimate of DT.

The recommended catch-all for the many model imperfections is the 'operational processor loss' OL, also known as the processor degradation factor DF_p (Ref. 1). *The value of OL is typically set to the arbitrary value of 4 dB*, although this may be changed if the modeller has more precise operational details of the sonar being studied. If loss terms are to be modelled in more detail, then the modeller should read Sections 6.3 to

6.19 and note which sonar is affected by each loss. If the loss L from equation 25 is used, the modeller should reduce OL to approximately 1 dB.

A further improvement likely to be encountered with recent sonar displays is the use of colour-coded information as an aid to the human observer. This 'colour gain' (CG) improves DT by CG = 1.5 dB (Refs. 31, 32). If the type of display is unknown, the modeller should be conservative and assume CG = 0.

9.2 Recommended Default Models

For passive narrowband sonars it is recommended that DT_0 be used: this is consistent with standard references such as the Sonar Modelling Handbook (Ref. 1) and Urick (Ref. 2). For passive broadband sonars it is recommended that either DT_0 or DT_w be used: the literature is about evenly split on the preferred definition of DT in this case.

It is recommended for active sonar modelling that DT_w be used: this is consistent with the Sonar Modelling Handbook (Ref. 1). Other references such as Urick (Ref. 2) and Burdic (Ref. 7) use DT_0 for their active sonar detection threshold: this choice is, as always, at the discretion of the modeller.

Note that for active sonars replica correlation is used in modern equipment: this is equivalent to coherent summation when deciding which section below to consult for recommended equations. A CW active sonar is now modelled as a trivial form of an FM sonar.

Recommended 'default' models are suggested below for the modeller who is unsure of which case specifically applies to the system being modelled, or who has insufficient information to make a specific choice.

- Passive sonars: assume the system is a narrowband power detector and use DT_0 for the detection threshold in Section 9.3.1. This equation also applies to a passive broadband detector. Applicable display types are lofargrams and T-θ displays.

- Active sonars: assume the system is a replica correlation FM sonar and use DT_w for the detection threshold in Section 9.3.8. Applicable display types: any active sonar display, as long as the correction term checklist is followed.

- Intercept sonars: these should be modelled in the same way as for a passive sonar. Hence the recommended default model for an intercept sonar is to use the narrowband power detector described in Section 9.3.1, with DT_0 for the detection threshold.

Use PD = 0.5 and PFA = 10^{-4} as recommended default values for each type of sonar. This can be altered later using the appropriate ROC curves.

In older descriptions of active sonar modelling there is reference to 'ambient noise limited' and 'reverberation limited' sonars. The ambient noise limited case is found by making the substitution $DT_{AN} = DT_0$, while the reverberation limited case is found by making the substitution $DT_{RV} = DT_w$. Current modelling practice (Ref. 1) recommends to use DT_w for modelling the detection threshold for an active sonar.

If the modeller encounters a new type of sonar processing, approximate it as the nearest generic type of sonar and include a small correction (~ 1 dB) to estimate any additional processing gain. With several decades of research and development behind the present generation of sonars, improvements in sonar detection threshold are most likely to be incremental in size. An estimate of the likely effect of the improvements on other terms in the sonar equation (Section 3.1) is left to the modeller's discretion.

9.3 Specific Models

9.3.1 Passive Narrowband Power Detector

Details for modelling a passive narrowband sonar power detector can be found in Section 5.2.2. Note that an 'energy detector' is the same as a 'power detector' in the literature. The detection threshold is modelled as:

$$DT_0 = 5\log_{10}(d) + 5\log_{10}(w) - 5\log_{10}(t) + L + OL - CG\,, \tag{26a}$$

$$DT_w = 5\log_{10}(d) - 5\log_{10}(w) - 5\log_{10}(t) + L + OL - CG\,. \tag{26b}$$

The definition of DT usually used in this case is consistent with DT_0.

For an integration factor IF = 1 the signal plus noise and the noise follow an exponential probability density function (pdf), while for IF > 1 the signal plus noise and the noise follow chi-squared statistics. A sample ROC curve for a generic power detector with IF = 1 is given here as Figure 4. For IF > 1 sample values for d for PD = 0.5 and PFA = 10^{-4} can be found in Table 1 in Section 5.2.2. If uncertain about the correct value of IF, assume IF = 1 as a default value.

9.3.2 Passive Broadband Power Detector

A passive broadband power detector is modelled the same way as for a passive narrowband power detector. The same equations and recommendations which appear in Section 9.3.1 for a narrowband power detector also apply here, although the effective noise bandwidth should be calculated using equation 10 if the spectrum is available. The choice of DT definition is left to the modeller, with DT_w being the one more commonly used (Ref. 1).

9.3.3 Passive Narrowband Amplitude Detector

Details for modelling a passive narrowband sonar amplitude detector can be found in Section 5.2.3. Here there are two ways to model the detection threshold, with the recommended way to model DT being:

$$DT_0 = 10\log_{10}\left(\frac{0.273d}{t} + 1.045\sqrt{\frac{d}{t}}\right) + 10\log_{10}(w) + L + OL - CG \ , \tag{27a}$$

$$DT_w = 10\log_{10}\left(\frac{0.273d}{t} + 1.045\sqrt{\frac{d}{t}}\right) + L + OL - CG \ . \tag{27b}$$

For an integration factor IF = 1 the signal plus noise and the noise follow a Rayleigh pdf, while for IF > 1 the signal plus noise and the noise follow Rician statistics. See the sample ROC curve, Figure 5, for a generic amplitude detector with IF = 1. For IF > 1 sample values for d for PD = 0.5 and PFA = 10^{-4} can be found in Table 2 in Section 5.2.3. If uncertain about the correct value of IF, assume IF = 1 as a default value.

The more simplistic way to model DT for an amplitude detector assumes the signal plus noise and noise statistics are Gaussian:

$$DT_0 = 10\log_{10}(d) + 10\log_{10}(w) - 10\log_{10}(t) + L + OL - CG + C(wt) \ , \tag{28a}$$

$$DT_w = 10\log_{10}(d) - 10\log_{10}(t) + L + OL - CG + C(wt) \ . \tag{28b}$$

The definition of DT usually used in this case is consistent with DT_0. With Gaussian statistics and a ROC curve such as Figure 3 (or Figure 9 with the fluctuation index k = 1) being used, then the correction term for finite sample size discussed in Section 6.3 must be included in equation 28.

9.3.4 Passive Broadband Amplitude Detector

A passive broadband amplitude detector is modelled the same way as for a passive narrowband amplitude detector. The same equations and recommendations which appear in Section 9.3.3 for a narrowband amplitude detector also apply here, although the effective noise bandwidth should be calculated using equation 10 if the spectrum is available. The choice of DT definition is left to the modeller, although DT_w is possibly the one more commonly used.

9.3.5 Passive Cross Correlation Broadband Detector

Details for modelling a passive broadband sonar cross correlation detector can be found in Section 6.1.2. The cross correlation detector is a modified version of the broadband power detector and so its detection threshold is modelled as:

$$DT_0 = 5\log_{10}(d) + 5\log_{10}(w) - 5\log_{10}(2t) + L + OL - CG, \quad (29a)$$

$$DT_w = 5\log_{10}(d) - 5\log_{10}(w) - 5\log_{10}(2t) + L + OL - CG. \quad (29b)$$

The definition of DT usually used in this case is consistent with DT_w (Ref. 1). In the sonar equation the array gain term may also need to be modified to account for the cross correlation if a split array is being used.

For the cross correlation broadband detector, the signal plus noise and the noise follow an exponential pdf for an integration factor IF = 1, while for IF > 1 the signal plus noise and the noise follow chi-squared statistics. Figure 4 is a sample ROC curve for a generic power detector with IF = 1. For IF > 1 sample values for d for PD = 0.5 and PFA = 10^{-4} can be found in Table 1. If uncertain about the correct value of IF, assume IF = 1 as a default value.

9.3.6 Active Sonar with Cross Correlation of an Exactly Known Signal

For a single active sonar pulse being detected in a background of Gaussian noise, the detection threshold for an exactly known signal is:

$$DT_0 = 10\log_{10}(d) - 10\log_{10}(2t) + OL - CG. \quad (30a)$$

$$DT_w = 10\log_{10}(d) - 10\log_{10}(w) - 10\log_{10}(2t) + OL - CG. \quad (30b)$$

This expression for DT describes a cross correlation, or matched filter, with incoherent summation of the received power. The expression for DT_w is the recommended choice to use in this case. Further details can be found in Section 6.2.2. Equation 30a is the 'Case I' referred to by Urick (Ref. 2, p.384). Figure 3 gives the appropriate ROC curve, or use Figure 9 with the fluctuation index $k = 1$.

9.3.7 Active Sonar with Incoherent Summation of Power

The detection threshold for an active sonar with incoherent summation of power is modelled as:

$$DT_0 = 5\log_{10}(d) + 5\log_{10}(w) - 5\log_{10}(t) - 5\log_{10}(NP) + OL - CG\,. \qquad (31a)$$

$$DT_w = 5\log_{10}(d) - 5\log_{10}(w) - 5\log_{10}(t) - 5\log_{10}(NP) + OL - CG\,. \qquad (31b)$$

Here the number of sonar pulses NP are also incoherently summed. The more commonly used version of equation 31 is DT_w. The appropriate ROC curve is given by Figure 6, which has been reproduced from Figure 3-9-1 of the Sonar Modelling Handbook (Ref. 1). Rician statistics apply for the signal plus noise, while exponential statistics apply for the noise. It is important to note that equation 31 applies for old active sonars: most of these systems are no longer used. Instead a technique known as replica correlation is now used in operational sonars, wherein the system is based on a coherent summation of power: see Section 9.3.8.

If the modeller instead uses a ROC curve based on Gaussian statistics, e.g. Figure 3 (or Figure 9 with the fluctuation index k = 1), then a correction term $C(wt)$ is required, where the correction is for wt = 1: see Section 6.3 for more details.

9.3.8 Active Sonar with Coherent Summation (Replica Correlation)

The discussion below is the recommended model for a modern FM sonar. A CW active sonar is just a trivial case of the FM sonar. The detection threshold for an active sonar using replica correlation, i.e. coherent summation of power, is modelled as:

$$DT_0 = 5\log_{10}(d) - 10\log_{10}(t) - 5\log_{10}(NP) + OL - CG\,. \qquad (32a)$$

$$DT_w = 5\log_{10}(d) - 10\log_{10}(w) - 10\log_{10}(t) - 5\log_{10}(NP) + OL - CG\,. \qquad (32b)$$

Note that the number of pulses NP are still added incoherently. The more commonly used version of equation 32 is DT_w. The appropriate ROC curve is given by Figure 6, which has been reproduced from Figure 3-9-1 of the SMH (Ref. 1). Rician statistics apply for the signal plus noise, while exponential statistics apply for the noise. More details of this model can be found in Sections 6.2.2 and 6.2.3. Note that equation 32 applies regardless of whether it is power (mean squared voltage) or amplitude (root mean squared voltage) being coherently summed.

9.3.9 Intercept Sonars

Intercept sonars are intended to detect the active transmissions of other sonars. They are properly modelled in the same way as for a passive sonar, and so are effectively split into two cases: power and amplitude.

The model to be used for an intercept sonar based on a power detector is the same as for the narrowband (or broadband) passive power detector as described in Section 9.3.1:

$$DT_0 = 5\log_{10}(d) + 5\log_{10}(w) - 5\log_{10}(t) + L + OL - CG\,, \tag{26a}$$

$$DT_w = 5\log_{10}(d) - 5\log_{10}(w) - 5\log_{10}(t) + L + OL - CG\,. \tag{26b}$$

The definition of DT usually used in this case is consistent with DT_0. The appropriate ROC curve for IF = 1 is given here as Figure 4, where the signal plus noise and the noise follow an exponential pdf. For IF > 1 the signal plus noise and the noise follow chi-squared statistics. Sample values for d for PD = 0.5 and PFA = 10^{-4} for IF > 1 can be found in Table 1 in Section 5.2.2. If uncertain about the correct value of IF, assume IF = 1 as a default value.

If instead the intercept sonar is based on an amplitude detector, then the appropriate model is the same as that for the passive narrowband (or broadband) amplitude detector described in Section 9.3.3:

$$DT_0 = 10\log_{10}\left(\frac{0.273d}{t} + 1.045\sqrt{\frac{d}{t}}\right) + 10\log_{10}(w) + L + OL - CG\,, \tag{27a}$$

$$DT_w = 10\log_{10}\left(\frac{0.273d}{t} + 1.045\sqrt{\frac{d}{t}}\right) + L + OL - CG\,. \tag{27b}$$

The detection threshold most commonly used in this case is DT_0. For an integration factor IF = 1 the signal plus noise and the noise follow a Rayleigh pdf, while for IF > 1 the signal plus noise and the noise follow Rician statistics. See the sample ROC curve, Figure 5, for a generic amplitude detector with IF = 1. For IF > 1 sample values for d for PD = 0.5 and PFA = 10^{-4} can be found in Table 2 in Section 5.2.3. If uncertain about the correct value of IF, assume IF = 1 as a default value.

For an amplitude intercept sonar the other (not recommended) model is to use Gaussian statistics. This gives the detection threshold as:

$$DT_0 = 5\log_{10}(d) + 5\log_{10}(w) - 5\log_{10}(t) + L + OL - CG + C(wt)\,, \tag{28a}$$

$$DT_w = 5\log_{10}(d) - 5\log_{10}(w) - 5\log_{10}(t) + L + OL - CG + C(wt)\,. \qquad (28b)$$

The definition of DT usually used in this case is consistent with DT_0. With Gaussian statistics and a ROC curve such as Figure 3 (or Figure 9 with the fluctuation index k = 1) being used, then the correction term $C(wt)$ for finite sample size discussed in Section 6.3 must be included in equation 28.

10. Conclusion

The equations and discussion presented throughout this report have been designed to explain to the sonar modeller how to obtain a good estimate of the detection threshold of various sonar systems. Such estimates are crucial to the quality of predictions made by operations research models, which seek to predict via mathematical calculation the performance of sonar systems under a variety of conditions against various targets of military interest. These estimates of DT are also crucial in measuring the detection threshold of operational sonar systems, as they give the baseline prediction of what the system should theoretically be able to achieve for a variety of parameters.

The summary, Chapter 9, is especially recommended to those modellers who do not have the time to study the sonar system in great detail, but merely require quick and credible estimates. Extensive cross referencing has been provided for those cases where more detailed explanations are required by the reader.

11. Acknowledgments

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19. ABSTRACT

This document presents expressions for modelling the detection threshold for narrowband and broadband passive sonars using either power or amplitude detection, cross correlation sonars, CW and FM active sonars with or without replica correlation, as well as intercept sonars. Explanations of various terms are given, along with appropriate ROC curves and recommended default values for those cases where insufficient information is available to the modeller. Intended users of this document are sonar modellers and operations research modellers who want quick and credible answers without going into the fine detail of the sonar system's operation.