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13. ABSTRACT *(Maximum 200 words)*
The research conducted over the past four years of this AFOSR grant has focused on one of the fundamental factors influencing the accuracy of floating-point computations, namely, the condition or sensitivity of the particular problem being solved. Many fundamental calculations in scientific and engineering computation in general, and control engineering in particular, can be viewed as functions that map a set of input values to a set of output values. Assuming the algorithm used in the computation is numerically stable, the sensitivity of this input-output map determines how accurately the output values can be estimated. The methods used in our research rely on the fundamental results in the PIs' 1994 *SIAM J. Sci. Comp.* paper wherein it was shown that significant information about large linear operators (which arise naturally as the derivative of some nonlinear function of interest) can be obtained from their action on smaller-dimensional subspaces. Such information can be determined reliably and efficiently using the small-sample statistical method. In addition to further advancing our basic theory, we have explored applications of statistical techniques to a wide variety of problems in control theory and image processing. Our research has been reported in over 40 scholarly articles.

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Small-Sample Statistical Condition Estimation

FINAL TECHNICAL REPORT

Air Force Office of Scientific Research

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1 Summary of Principal Accomplishments

The research conducted over the past four years of this AFOSR grant has focused on one of the fundamental factors influencing the accuracy of floating-point computations, namely, the condition or sensitivity of the particular problem being solved. Many fundamental calculations in scientific and engineering computation in general, and control engineering in particular, can be viewed as functions that map a set of input values to a set of output values. Assuming the algorithm used in the computation is numerically stable, the sensitivity of this input-output map determines how accurately the output values can be estimated. The methods used in our research rely on the fundamental results in [1] and [19] wherein it is shown that significant information about large linear operators (which arise naturally as the derivative of some nonlinear function of interest) can be obtained from their action on smaller-dimensional subspaces. Such information can be determined reliably and efficiently using the small-sample statistical method. In addition to further advancing our basic theory, we have explored applications of statistical techniques to a wide variety of problems in control theory and image processing. Our research has been reported in over 40 scholarly articles.

1.1 Sensitivity in Linear Algebra and Control Computations

Standard approaches to measuring the condition of various problems, such as solving a system of linear equations, compress all sensitivity information into a single "condition number." Thus, a loss of information can occur in situations in which this standard condition number does not accurately reflect the actual sensitivity of a solution or particular entries of a solution. Our new method overcomes these and other common deficiencies by measuring the effects on the solution of small random changes in the input data. By properly scaling the results, we can obtain condition estimates for each entry of a computed solution. This approach, which is referred to as small-sample statistical condition estimation (SCE), applies to both linear and nonlinear problems. In the former case, when an explicit Fréchet derivative of the computed quantity in question is available, the method is especially efficient, costing no more than standard normwise or componentwise estimates. Moreover, SCE has the advantage of considerable flexibility. For example, it easily accommodates restrictions on, or structure associated with, allowable perturbations. The method has a rigorous statistical theory available for the probability of accuracy of the condition estimates. We have applied SCE to several basic problems in numerical linear algebra and several papers have been published or are in press in the leading journals, including papers on solving linear equations [40], linear least squares problems [41], and eigenvalue/eigenvector problems [37]. We have also made considerable progress in applying SCE to various computational problems in control, especially the numerical solution of algebraic Riccati equations [11], [15], [30], [31].

A recent thrust in our research program has been devoted to new algorithms for evaluating certain key matrix functions of interest to control engineers. Perhaps surprisingly to some, much remains to be done before algorithms for such matrix functions as the exponential and logarithm, as implemented in state-of-the-art codes in MATLAB's `expm` and `logm` for example, can be said to be truly reliable. We have explored a so-called Schur-

Fréchet method [42] of evaluating matrix functions which consists of putting the matrix in upper triangular form. computing the scalar function values along the main diagonal. and then using the Fréchet derivative of the function to evaluate the upper diagonals. This approach requires a reliable method of computing the Fréchet derivative accurately. For the logarithm this can be done by using repeated square roots and a hyperbolic tangent form of the logarithmic Fréchet derivative. Padé approximations of the hyperbolic tangent lead to a Schur-Fréchet algorithm for the logarithm that avoids problems associated with the standard "inverse scaling and squaring" method. Inverting the order of evaluation in the logarithmic Fréchet derivative gives a method of evaluating the derivative of the exponential. The resulting Schur-Fréchet algorithm for the exponential gives superior results compared to standard methods on a set of test problems from the literature.

Our SCE method can be applied to estimate the sensitivity of each entry in a matrix function. For example. suppose that we are interested in the sensitivity of the (i, j) entry of a matrix function $F = F(A)$. This sensitivity can be measured by the norm of the gradient of the map $A \mapsto F_{ij}(A)$. Using SCE. we determine sensitivity by perturbing the argument (A) in a random fashion and looking at the resulting effect on the function value. In general. a random perturbation is not going to point in the direction of the gradient (i.e.. in the matrix direction that produces the maximal perturbation in $F_{ij}(A)$). Because of this. a scaling factor ω_m. also called the "Wallis" factor. must be introduced. The Wallis factor depends only on the number m of arguments being perturbed. In the case considered here. $m = n^2$ where n is the order of the matrix A. For computational purposes the approximation $\omega_m \approx \sqrt{2/(\pi(m - 0.5))}$ is sufficiently accurate.

This condition estimation procedure has a particularly simple form when expressed in terms of a known Fréchet derivative and. in the case of the matrix logarithm and matrix exponential. such a derivative is known explicitly. This random perturbation method of condition estimation is very flexible and can easily be adapted to problems in which the perturbation must preserve some structure of the matrix such as symmetry.

1.2 Sensitivity in Image Processing

During the past few years we have worked with Dr. Gary Hewer at NAWC in China Lake. CA and Prof. B.S. Manjunath (UCSB) on sensitivity issues in image processing. especially sensitivity of computed results for segmentation and recognition of small targets in images. registration of objects in sequences of images, and estimates of optical flow velocity fields. The goal has been to devise procedures such as the SCE method for automatically detecting sensitivity combined with efforts to develop algorithms for feature extraction that are inherently less sensitive. A general framework for variational approaches to image segmentation has been developed that includes most of the commonly used methods such as Mumford-Shah and Geman type functionals. The main advantage of our new framework is that the boundary function associated with the segmentation is given explicitly in terms of the approximation function. This reduces the computational burden of finding the segmentation as well as allowing direct analytical comparisons with other methods with regard to sensitivity to noise in the input. A description of the variational boundary approach is given in [32], [43].

We have also developed a second approach to segmentation and edge detection based on the idea of "peer group" processing. In this approach, at each pixel in the image a peer group is identified based on nearness of intensity. The peer group can also be based on other measures such as nearness in texture space or (for optical flow) nearness in velocity space. This method avoids the edge-blurring difficulties associated with gradient-based partial differential equation approaches that do not distinguish between pixel information for different regions near image boundaries. An interesting connection between the shock front image processing method of Rudin and Osher and the peer group image processing method was pointed out by Stan Osher at the September 1996 ONR workshop at UCLA. This connection allows one to analyze the sensitivity of the peer group method with respect to additive noise and to conclude that the total variation of the original image is preserved. This work is reported in [29], [33], [39].

1.3 Related Topics

The Publications list below includes numerous other AFOSR-supported papers on closely related research topics. These are listed below and were discussed in detail in previous years' Annual Progress Reports.

2 Personnel Supported

Faculty: Alan J. Laub

Research Faculty: Charles S. Kenney

Postdocs: Thorkell T. Gudmundsson

Grad Students: (all but Gudmundsson are U.S. citizens)

Mark Erickson

Ali Ghavimi

Thorkell Gudmundsson

John Hench

Ed Li

Scott Miller

Michael Reese

Keith Schubert

3 Publications (refereed only, since 1994)

[1] Kenney, C., and A.J. Laub, "Small-Sample Statistical Condition Estimates for General Matrix Functions," *SIAM J. Sci. Comp.*, 15(1994), 36–61.

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[12] Kenney, C.S., and A.J. Laub, "An Overview of the Matrix Sign Function," *Proc. Fifth SIAM Conference on Applied Linear Algebra*, p. 90, 1994 (abstract).

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[16] Erickson, M.A., and A.J. Laub, "An Algorithmic Test for Checking Stability of Feedback Spectral Systems," *Automatica*, 31(1995), 125–135.

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4 Interactions/Transitions

4.1 Participation/presentations at meetings, conferences, seminars, etc.

During the tenure of this grant, the P.I. has been invited to deliver three major plenary addresses at prestigious control theory, numerical linear algebra, and applied mathematics conferences. The first was at the Allerton Conference; the second was at the Householder Symposium, the premier venue for numerical linear algebra and held only every three years; and the third was at the recent SIAM National Meeting, held last year at Stanford University. Details follow.

1. *The Matrix Sign Function and Its Applications*, Invited **Plenary Lecture** for the Thirty-Third Annual Allerton Conference on Communications, Control, and Computing, Allerton, Illinois, Oct. 4–6, 1995.

2. *Statistical Condition Estimation in Numerical Linear Algebra*, Invited **Plenary Lecture** for the Thirteenth Householder Symposium on Numerical Linear Algebra, Pontresina, Switzerland, June 17–21, 1996.

3. *Algorithms for Computing Matrix Logarithms and Exponentials*, Invited **Plenary Lecture** for SIAM's 45th Anniversary Meeting, Stanford, California, July 14–18, 1997.

4.2 Transitions and other consultative and advisory functions

Prof. Laub consults regularly with colleagues at The MathWorks, Inc. in Natick, Mass. concerning various aspects of next-generation software and algorithms for control. Prof. Laub's expertise in the numerical solution of algebraic Riccati equations was also sought in May and June of 1996 by Mr. Kevin J. Shortelle (`sdi@afn.org`) of System Dynamics International, Inc. (9120 SW 46th Blvd., Gainesville, FL 32608). Shortelle was performing work under a six-month, $80K contract with the U.S. Air Force to investigate the benefits of innovative nonlinear estimation techniques for improving the accuracy of integrated navigation systems (e.g., integrated GPS, Doppler, and INS systems). He was assessing not only conventional nonlinear approaches (extended Kalman filter (EKF), iterated EKF, modified-gain EKF) but also a new technique referred to as the State Dependent Riccati Equation (SDRE). The SDRE techique is based on work originating from research at the U.S. Air Force's Wright Laboratory, at Eglin AFB, Florida. Dr. Jim Cloutier (a Wright Lab scientist) and LTC Curtis Mracek have authored several papers on the subject. One of Shortelle's software tasks is the efficient computation of solutions to the ARE based on certain system state matrices. He sought from Prof. Laub a public-domain Fortran source code available for such a computation and it was provided to him together with some explanation of how to use it and where to acquire related codes.

Dr. Kenney has had conversations and provided technical guidance concerning conditioning and statistical condition estimation for automatic target recognition and image processing (such as optical flow computations) with Jim Huang (`jim.huang@alphatech.com`) of Alphatech, Inc., (50 Ball Road, Burlington, MA 01803). In addition to discussing sensitivity of optical flow algorithms, Dr. Kenney sent him a copy of a MATLAB preprocessing routine for optical flow that eliminates a lot of image noise from saturated or dead pixels.

5 New Discoveries, Inventions, or Patent Disclosures

None.

6 Honors/Awards

Professional honors received during career

A.J. Laub:

- Elected IEEE Fellow in 1986 ("for contributions to algorithms, numerical analysis, and mathematical software for control and systems theory")
- Distinguished Member Award, IEEE Control Systems Society, 1991
- Outstanding Achievement in Teaching Award, School of Engineering, University of Southern California, May 1983
- Academic Senate Distinguished Teaching Award (Engineering), University of California, Santa Barbara, June 1993
- Control Systems Technology Award, IEEE Control Systems Society, 1993 ("for pioneering efforts and continuously advancing the state of the art in Computer-Aided Control System Design")