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13. ABSTRACT (Maximum 200 words)

Discrete manufacturing process design optimization is a challenging problem for the Air Force, due to the large number of manufacturing process design sequences that exist for a given part. This has forced researchers to develop heuristic strategies to address such design problems. This report summarizes the work done in developing a new general heuristic search strategy for discrete manufacturing process design optimization, called generalized hill climbing algorithms. Generalized hill climbing algorithms provide a unifying approach for addressing such problems, in particular, and intractable discrete optimization problems, in general. Heuristic strategies such as simulated annealing, threshold accepting, Monte Carlo search, local search, and tabu search (among others) are all formulated as particular generalized hill climbing algorithms. Computational results are reported with various generalized hill climbing algorithms applied to computer simulation models of discrete manufacturing process designs under study at the Materials Process Design Branch of Wright Laboratory, Wright Patterson Air Force Base (Dayton, Ohio, USA). In particular, the design of an optimal manufacturing process for an integrated blade rotor part is studied, and computational results using various generalized hill climbing algorithm, applied to the manufacturing and production of the part, are reported.

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EXECUTIVE SUMMARY

The major technical accomplishments achieved during this grant were

- i)* the introduction and development of the generalized hill climbing algorithm paradigm as a unifying approach to address intractable discrete optimization problems,
- ii)* the description of numerous new search strategies for intractable discrete optimization problems using the generalized hill climbing algorithm framework,
- iii)* the formulation of sufficient convergence conditions for generalized hill climbing algorithms.
- iv)* the application of several generalized hill climbing algorithms to an integrated blade rotor manufacturing design optimization problem.

In addition, several other problems were studied during the grant period, including estimation procedures for discrete event simulation problems, access control security system issues, and flexible assembly design and scheduling problems.

All the accomplishments are documented in several archival journal articles and conference proceedings. In addition, many of the results have been presented at national and international conferences, and have won awards for their contribution.

One Ph.D. dissertation was completed during the period of the grant. Major Alan W. Johnson, USAF, successfully defended his Ph.D. dissertation "Generalized Hill Climbing Algorithms for Discrete Optimization Problems" in October 1996. Major Johnson also received an honorable mention prize in the 1997 George E. Nicholson Student Paper Competition, a national competition sponsored by the Institute for Operations Research and the Management Sciences (INFORMS).

1. Generalized Hill Climbing Algorithms

Generalized hill climbing algorithms provide a unifying framework to address intractable discrete optimization problems. To formally introduce and describe generalized hill climbing algorithms, several definitions are needed.

Define the *solution space*, Ω , to be the set of all solution for a discrete optimization problem. Define an *objective function* $f: \Omega \rightarrow R$ that assigns a real value to each element of the solution space. Define a *neighborhood* structure $n: \Omega \rightarrow 2^\Omega$, where $n(\omega) \subset \Omega$ for all $\omega \in \Omega$. The neighborhood structure provides connections between solutions in the solution space, hence allows the solution space to be traversed or searched by moving between solutions. The goal is to identify the globally optimal solution ω^* (i.e., $f(\omega^*) \leq f(\omega)$ for all $\omega \in \Omega$), or more realistically, a near-optimal solution.

1.1 Algorithm Description

The components of generalized hill climbing algorithms are summarized in pseudo-code form.

Select an initial solution $\omega_0 \in \Omega$

Define an objective function $f: \Omega \rightarrow R$

Define a neighborhood structure $n: \Omega \rightarrow 2^\Omega$

Set the iteration index $k=0$

Define the hill climbing (random) variables $R_k: \Omega \times \Omega \rightarrow R \cup \{-\infty, +\infty\}$

Repeat

 Generate a solution $\omega' \in n(\omega_k)$ in the neighborhood of ω_k

 Set $\Delta(\omega_k, \omega') = f(\omega') - f(\omega_k)$

 If $\Delta(\omega_k, \omega') < 0$, then $\omega_{k+1} \leftarrow \omega'$ (objective function decreases)

 If $\Delta(\omega_k, \omega') \geq 0$ and $R_k(\omega_k, \omega') \geq \Delta(\omega_k, \omega')$,

 then $\omega_{k+1} \leftarrow \omega'$ (accept hill climbing move)

 If $\Delta(\omega_k, \omega') \geq 0$ and $R_k(\omega_k, \omega') < \Delta(\omega_k, \omega')$,

 then $\omega_{k+1} \leftarrow \omega_k$ (reject hill climbing move)

$k \leftarrow k+1$

 Until stopping criterion is met

Generalized hill climbing algorithms allow inferior solutions to be visited enroute to optimal solutions. This hill climbing capability is the basis for the search strategy's name.

1.2 Examples

Numerous search strategies can be described using the generalized hill climbing algorithm framework. Several such strategies are described that illustrate the breadth of algorithms that fall within the generalized hill climbing algorithm framework, as well as demonstrate the unifying nature of the generalized climbing algorithm paradigm. All these search strategy variations are captured purely through a change in the definitions of the hill climbing random variable and the neighborhood structure.

The two extreme search strategy variations, *Monte Carlo search* and *local search*, can be formulated as generalized hill climbing algorithms. Monte Carlo search can be formulated as a generalized hill climbing algorithm by setting $R_k(\omega_k, \omega') = +\infty$, $\omega_k \in \Omega$, $\omega' \in n(\omega_k)$, with $n(\omega_k) = \Omega$. Local search can be formulated as a generalized hill climbing algorithm by setting $R_k(\omega_k, \omega') = 0$, $\omega_k \in \Omega$, $\omega' \in n(\omega_k)$.

Simulated annealing can be formulated as a generalized hill climbing algorithm by setting $R_k(\omega_k, \omega') = -t_k \ln(u)$ for all $\omega_k \in \Omega$, $\omega' \in n(\omega_k)$, where t_k is a temperature parameter, and u is a $U(0,1)$ random variable. Note that $R_k(\omega_k, \omega')$ is an exponential random variable with mean t_k (from the inversion method for generating exponential random variables). Moreover, for $\Delta(\omega_k, \omega') = f(\omega') - f(\omega_k) \geq 0$, the acceptance probability can be written as

$$\begin{aligned} P\{R_k(\omega_k, \omega') \geq f(\omega') - f(\omega_k)\} &= P\{-t_k \ln(u) \geq f(\omega') - f(\omega_k)\} \\ &= P\{u \leq e^{-[f(\omega') - f(\omega_k)]/t_k}\} = e^{-[f(\omega') - f(\omega_k)]/t_k}, \end{aligned}$$

which is how simulated annealing is commonly described. Therefore, simulated annealing is a particular type of generalized hill climbing algorithm.

Threshold accepting can be formulated as a generalized hill climbing algorithm by setting $R_k(\omega_k, \omega') = Q_k$ for all $\omega_k \in \Omega$, $\omega' \in n(\omega_k)$ where Q_k is a non-negative real constant. Therefore, for $\Delta(\omega_k, \omega') = f(\omega') - f(\omega_k) \geq 0$, the acceptance probability can be written as

$$P\{R_k(\omega_k, \omega') \geq f(\omega') - f(\omega_k)\} = P\{Q_k \geq f(\omega') - f(\omega_k)\} \quad (1)$$

which is how threshold accepting is commonly described. Therefore, threshold accepting is a particular type of generalized hill climbing algorithm.

Ordinal optimization (Ho 1995) is a search strategy that focuses on finding good solutions, rather than finding the very best solution. In doing this, it reduces the search for an optimal solution from sampling over a very large solution space to sampling over a smaller, more manageable, set of good solutions. The ordinal optimization approach can be incorporated into the generalized hill climbing algorithm framework. The resulting algorithm, termed *ordinal hill climbing* (OHC), is a particular

generalized hill climbing algorithm, with the hill climbing component based on ordinal information. The OHC algorithm is summarized in pseudo-code form.

Select a set of M initial solutions $D(0) \subset \Omega$

Define an objective function $f: \Omega \rightarrow R$

Define a neighborhood structure $n: \Omega \rightarrow 2^\Omega$

Set the iteration index $k=0$

Define the hill climbing (random) variables $R_k: \Omega^M \rightarrow Z^+$, where

$$i) \sum_{m=1}^M P\{R_k = m\} = 1 \text{ for all } k \in Z^+$$

ii) $R_0 = 1$ with probability one,

iii) $R_k \rightarrow M$ with probability one as $k \rightarrow +\infty$

Repeat

Order the solutions in $D(k)$ from smallest to largest objective function values

Generate R_k

Keep the best (smallest objective function values) R_k solutions from the set $D(k)$.

Call this set E_1 .

Generate one or more neighbors of each of these R_k , to obtain $M-R$ new solutions.

Call this set E_2 .

Set $D(k+1) \leftarrow E_1 \cup E_2$.

$k \leftarrow k+1$

Until stopping criterion is met

Tabu search with short term memory can be formulated as a generalized hill climbing algorithm by setting

$$R_k(\omega_k, \omega') = \begin{cases} +\infty & \text{for } \omega' \notin L \\ -\infty & \text{for } \omega' \in L \end{cases} = I\{\omega' \notin L\} / [1 - I\{\omega' \notin L\}] - I\{\omega' \in L\} / [1 - I\{\omega' \in L\}] \quad (2)$$

for all $\omega_k \in \Omega \setminus \{\omega'\} \in n(\omega_k)$, where L is a tabu list of solutions (Glover 1995) and $I\{\cdot\}$ is a 0-1 indicator function.

Erlang accepting can be formulated as a generalized hill climbing algorithm by setting

$$R_k(\omega_k, \omega') = -t_k [\ln(u_1) + \ln(u_2)] \quad (3)$$

for all $\omega_k \in \Omega$, $\omega' \in n(\omega_k)$, where t_k is a temperature parameter, and u_1 and u_2 are independent and identically distributed $U(0,1)$ random variables. The resulting generalized hill climbing algorithm is called Erlang accepting since R_k is distributed Erlang with mean $2t_k$.

Geometric accepting can be formulated as a generalized hill climbing algorithm by setting

$$R_k(\omega_k, \omega') = \lceil \ln(1-u) / \ln(1-p_k) \rceil \quad (4)$$

for all $\omega_k \in \Omega$, $\omega' \in \mathcal{N}(\omega_k)$, where $p_k \in (0,1)$ is a sampling parameter, u is a $U(0,1)$ random variable, and $\lceil \cdot \rceil$ denotes the ceiling function. The resulting generalized hill climbing algorithm is called geometric accepting since R_k is distributed geometric with mean $1/p_k$.

Weibull accepting can be formulated as a generalized hill climbing algorithm by setting $R_k(\omega_k, \omega') = t_k(-\ln(u))^{1/\alpha}$ for all $\omega_k \in \Omega$, $\omega' \in \mathcal{N}(\omega_k)$, where t_k is a temperature parameter, u is a $U(0,1)$ random variable, and α is a shape parameter. The resulting generalized hill climbing algorithm is called Weibull accepting since R_k is distributed Weibull with mean $(t_k/\alpha)\Gamma(1/\alpha)$.

The examples presented here are just a sampling of the unlimited number of new (and existing) search strategies that can be created (and described) using the generalized hill climbing paradigm. These examples serve to illustrate the breadth, scope, and variation of algorithms that lie within the generalized hill climbing framework.

1.3 Convergence Results

Convergence conditions provide a theoretical understanding of generalized hill climbing algorithms. Two sufficient conditions for convergence of generalized hill climbing algorithms were derived. The first result focuses on a particular class of generalized hill climbing algorithms. The complete details of this result are reported in Johnson and Jacobson (1996) as well as the Ph.D. dissertation of Major A.W. Johnson, USAF.

Theorem 1: Given a discrete optimization problem, with solution space Ω , objective function f , and neighborhood structure \mathcal{N} , suppose that ω^* denotes a global optimum, with objective function value $f(\omega^*)$. Consider the class of generalized hill climbing algorithms with hill climbing (random) variable $R_k(\omega_k, \omega')$, $\omega_k \in \Omega$, $\omega' \in \mathcal{N}(\omega_k)$, defined with acceptance probability

$$P\{R_k(\omega_k, \omega') \geq \Delta(\omega_k, \omega')\} = \min[1, (P\{\tilde{R}_k(\omega^*, \omega') \geq \Delta(\omega^*, \omega')\} / P\{\tilde{R}_k(\omega^*, \omega_k) \geq \Delta(\omega^*, \omega_k)\})], \quad (5)$$

where $\Delta(\omega, \omega'') = f(\omega'') - f(\omega)$ and \tilde{R}_k is any arbitrary non-negative random variable. Then the resulting generalized hill climbing algorithm converges in probability to the set of globally optimal solutions if

- i) $P\{R_k(\omega_k, \omega') \geq \Delta(\omega_k, \omega')\} > 0$ for all $k=0,1,\dots$,
- ii) $P\{R_k(\omega_k, \omega') \geq \Delta(\omega_k, \omega')\} \rightarrow 0$ as $k \rightarrow +\infty$.

Theorem 1 provides sufficient convergence conditions for a particular class of generalized hill climbing algorithms, where the acceptance probability is defined by (5). This class of algorithms include,

for example, simulated annealing. A large body of convergent generalized hill climbing algorithms can be created from (5), provided *i*) and *ii*) are satisfied.

The second result focuses on a general sufficient convergence condition for generalized hill climbing algorithms. The complete details of this result are reported in Johnson and Jacobson (1997) as well as the Ph.D. dissertation of Major A.W. Johnson, USAF. To describe this result, a number of definitions are needed.

For a particular discrete optimization problem, with a solution space, neighborhood structure, and objective function, decompose the solution space into the set of global optima, $G \subset \Omega$, the set of local optima, $L \subset \Omega$, and all other solution space elements, $H = \Omega \setminus (G \cup L)$. Define a *path* from i to j ($i \rightarrow j$), $i, j \in G \cup L$, as a sequence of solutions $\lambda_0 \lambda_1, \dots, \lambda_d \in \Omega$, with $\lambda_0 = i$, $\lambda_d = j$, $\lambda_1, \lambda_2, \dots, \lambda_{d-1} \in H$, and $\lambda_r \in n(\lambda_{r-1})$ $r=1, 2, \dots, d$. Therefore, paths are sequences of solutions between local and global optima. Two paths from i to j are *equivalent* if all the solutions along both paths are identical, and the order in which each solution is visited along both paths is identical. Suppose that all the paths from i to j are labelled, hence the n^{th} path from i to j is denoted by $i \rightarrow_n j$. For each distinct path from i to j , the probability of the n^{th} path for $k \in Z^+$, $P_k(i \rightarrow_n j)$, is the product of the probabilities associated with each solution transition. Therefore, for $k \in Z^+$, the probability of a path from i to j is

$$P_k(i \rightarrow j) = \sum_n P_k(i \rightarrow_n j) \text{ for all } i, j \in L \cup G, i \neq j. \quad (6)$$

If $i=j$, then

$$P_k(i \rightarrow i) = 1 - \sum_{s \in (L \cup G) \setminus \{i\}} P_k(i \rightarrow s). \quad (7)$$

For $k \in Z^+$, the *path of minimum positive probability from L to $L \cup G$* is

$$P_k(\text{Min_Path}) = \min\{P_k(j \rightarrow i) \mid j \in L, i \in L \cup G, \text{ with } P_k(j \rightarrow i) > 0\}. \quad (8)$$

The path of minimum positive probability defines the path with the smallest probability over all paths from local optima to local or global optima. Therefore, this probability is associated with the most difficult path from a local optima to a local or global optima.

For $k \in Z^+$, the *path of maximum probability from G to L* is

$$P_k(\text{Max_Path}) = \max\{P_k(i \rightarrow j) \mid i \in G, j \in L\}. \quad (9)$$

The path of maximum probability defines the path with the largest probability over all paths from global optima to local optima. Therefore, this probability is associated with the easiest path from a global optima to a local optima.

For $k \in Z^+$ the *maximal product of locally optimal solution equilibrium probabilities and their associated path probabilities to other local optima* is

$$P_k(\text{Max_Prod}) = \max\{\delta_j(k)P_k(j \rightarrow q) \mid j, q \in L, q \neq j, q \notin n(j)\}, \quad (10)$$

where $\delta_j(k) = \pi_j(k) / \sum_{i \in L \cup G} \pi_i(k)$ and $\pi_i(k)$, $i \in \Omega$ are the equilibrium probabilities for all the solutions in Ω .

Using these definitions, Theorem 2 gives sufficient conditions for the convergence of generalized hill climbing algorithms.

Theorem 2: Given a discrete optimization problem, with solution space Ω , objective function f , and neighborhood structure n , if

$$i) \sum_{k=1}^{+\infty} P_k(\text{Min_Path}) = +\infty,$$

$$ii) \sum_{k=1}^{+\infty} P_k(\text{Max_Path}) < +\infty,$$

$$iii) \sum_{k=1}^{+\infty} P_k(\text{Max_Prod}) < +\infty,$$

then $\sum_{i \in G} \pi_i(k) \rightarrow 1$ as $k \rightarrow +\infty$.

Theorem 2 provides sufficient convergence conditions for generalized hill climbing algorithms, by showing that if conditions *i*), *ii*), and *iii*) hold, then the equilibrium probability of the global optima will converge to one as k approaches infinity. Informally, these conditions state that the probability of escaping from a local optimum must approach zero slower than the probability of escaping from a global optimum. These conditions are based on the path construction described above. These conditions are the least restrictive convergence conditions available in the literature for any type of hill climbing algorithm.

2. Applications

This section describes applications of generalized hill climbing algorithms to manufacturing process design optimization problems of interest to the Air Force. This effort has been interdisciplinary, involving researchers from the Materials Process Design Branch of Wright Laboratory (Wright Patterson Air Force Base), Ohio University, and Harvard University.

2.1 Discrete Manufacturing Process Design Optimization Problems

The Materials Process Design Branch of Wright Laboratory at Wright Patterson Air Force Base (Dayton, Ohio), is faced with the challenge of identifying robust manufacturing process designs, where the finished unit meets certain geometric and microstructural specifications, and is produced at minimum cost. To date, the very expensive and time intensive approach of trial and error (on the shop floor) has been used to identify feasible manufacturing process designs. Wright laboratory, in conjunction with researchers at Ohio University (Athens, Ohio), have developed computer simulation models of manufacturing processes, such as forging and machining. Each such process affects the geometry and/or microstructure of the manufactured unit. Associated with each process are input (controllable and uncontrollable) and output

parameters. An exhaustive search through all possible process sequences and controllable input parameters can be undertaken, though such a search, especially using computer simulation models, would take a prohibitive amount of time, hence is infeasible. Therefore, it is necessary to construct efficient and effective optimization algorithms to identify optimal/near-optimal designs for manufacturers utilizing manufacturing process design computer simulation models in their manufacturing process design planning operations.

To describe the discrete manufacturing process design optimization problem, and how generalized hill climbing algorithms can be used to address the problem, a number of definitions are needed. Let the *manufacturing processes* be denoted by P_1, P_2, \dots, P_n . Associated with each process are (continuous or discrete) *controllable input parameters*, *uncontrollable input parameters*, and *output parameters* \square . The output parameters for a particular process may serve as the uncontrollable input parameters for a subsequent process. A sequence of processes, together with a particular set of input parameters constitutes a *manufacturing process design*. Label such designs D_1, D_2, \dots, D_N . Note that if any one controllable input parameter is continuous, then $N = +\infty$. Otherwise, $N < +\infty$. Without loss of generality, assume that all the controllable input parameters are discrete since any continuous controllable input parameter can be discretized over an arbitrarily fine grid. Under this assumption, the number of manufacturing process designs is finite, though potentially very large.

Denote the *design space* by Ω , the set of all manufacturing process designs (i.e., $\Omega = \{D_1, D_2, \dots, D_N\}$), where a subset of these designs are feasible. Infeasible designs violate prespecified constraints on the manufacturing process and the unit being manufactured, including geometric and microstructural properties, and constraint violations on the output parameters (e.g., the required forging press pressure may not exceed its upper bound limitations). Define a *cost function* $f: \Omega \rightarrow [0, +\infty)$ that assigns a non-negative value to each element of the design space, where cost includes monetary costs as well as measures for how well the finished unit meets prespecified geometric and microstructural properties, penalties for constraint violations on the output parameters, and measures that ensure a robust manufacturing design (i.e., the manufacturing process is stable) (note that the design space corresponds to the solution space, and the cost function corresponds to the objective function, as defined in Section 1). Define a *neighborhood structure* $n: \Omega \rightarrow 2^\Omega$, where $n(D) \subset \Omega$ for all $D \in \Omega$. The neighborhood structure establishes connections between the designs in the design space, hence allows the design space to be traversed or searched by moving between designs. For each (fixed) manufacturing process design sequence, the neighborhood rule allows movement between controllable input parameter values. The goal is to identify the globally optimal manufacturing process design D^* (i.e., $f(D^*) \leq f(D)$ for all $D \in \Omega$), or more realistically, near-optimal designs.

2.2 Integrated Blade Rotor

The Materials Process Design Branch of Wright Laboratory identified an integrated blade rotor as a part of interest to the Air Force for developing optimal manufacturing process designs. Therefore, a manufacturing process design is needed to transform a billet into the integrated blade rotor geometric shape (Gunasekera et al. 1996). Three manufacturing process sequences have been identified that can achieve this transformation. Define the following notation for the six processes that make up these three designs:

P_1 is the cast ingot process	P_2 is the upset process
P_3 is the machine preform process	P_4 is the blocker forge process
P_5 is the rough machining process	P_6 is the finished shape process

The three possible manufacturing process design sequences, provided by researchers at Wright Laboratory and Ohio University, are

$$P_1P_2P_5P_6 \quad P_1P_2P_4P_6 \quad P_1P_2P_3P_4P_6$$

Associated with each manufacturing process are uncontrollable and controllable input parameters, and its output parameters. For example, for process P_1 , there are two controllable input parameters (the radius of the billet and the height of the billet), zero uncontrollable input parameters, and two output parameters, which are just the controllable input parameter values. See Appendix 1 for a list of the controllable input parameters for each of the six processes. Gunasekera et al. 1996 and Fischer et al. 1997 present complete details on all six processes and their parameters.

Associated with each controllable input parameter is a discrete (naturally, or discretized from a continuous domain) set of feasible values (Appendix 1 contains a listing of these values for all the controllable input parameters for the six processes). The cost function quantifies the cost associated with not meeting certain geometric and microstructural properties of the finished product, the monetary cost in producing the finished product, and cost penalties for constraint violations. The goal is to identify a manufacturing process sequence, together with values for all the controllable input parameters for the processes, that result in a feasible manufacturing process design that produces the integrated blade rotor part at total minimum cost.

Computer simulation models of the manufacturing processes described above have been developed (Gunasekera et al. 1996, Fischer et al. 1997). This moves the search for an optimal manufacturing process design from the shop floor (where trial and error has typically been applied, using actual materials) to a computer platform. However, even using high speed computing resources, the search for an optimal

manufacturing process design can take a prohibitive amount of time. To circumvent this problem, generalized hill climbing algorithms are used with the computer simulation models to identify optimal/near-optimal manufacturing process designs.

Computational results for the integrated blade rotor manufacturing process designs are reported with GHC algorithms applied to the manufacturing process computer simulation models. The complete details of these experiments are available in Jacobson, Sullivan and Johnson (1997) (note that this paper received the best paper award in the Industrial Simulation track at the 1997 European Simulation Multiconference, held June 1-4, 1997, in Istanbul, Turkey). Five different GHC formulations (simulated annealing, threshold accepting, Monte Carlo search, local search, and Weibull accepting), together with three different neighborhood rules, were applied to the three computer simulation manufacturing process design sequences. The objective in running these experiments is to assess the performance of these GHC formulations and neighborhood rules, as well as identify optimal/near optimal manufacturing process design sequences and input parameters using computer simulation models.

For both simulated annealing and Weibull accepting, t_k is updated by multiplying the previous temperature parameter by the increment multiplier, β_1 , where $0 \leq \beta_1 \leq 1$ (i.e., $t_k = \beta_1 t_{k-1}$). The initial temperature parameter is $t_0 = 10,000$, with $\beta_1 = .96$. For Weibull accepting, the shape parameter is set to $\alpha = 2.0$. For threshold accepting, the initial threshold is $Q_0 = 10,000$, with $Q_k = \beta_2 Q_{k-1}$ and $\beta_2 = .96$. For Monte Carlo search, note that since $R_k(D, D') = +\infty$, $D, D' \in \Omega$, for all $k \in Z^+$ then all uphill moves are accepted. Conversely, for local search, since $R_k(D, D') = 0$, $D, D' \in \Omega$, for all $k \in Z^+$ then all uphill moves are rejected.

Three neighborhood rules were implemented. The first neighborhood rule consists of randomly selecting one process in a given design sequence, and randomly changing (using the discrete uniform over the set of possible values, as listed in Appendix 1) one of the controllable input parameters associated with this process. The second neighborhood rule is similar to the first, but the algorithm randomly changes one controllable input parameter in each of the processes of the design sequence (rather than in just one particular process). Similar to the second neighborhood rule, the third neighborhood rule considers every process in the design sequence. However, the key difference is that each controllable input parameter in all the processes is changed with a prespecified probability ($p=.3$).

The cost function evaluates the cost associated with the simulated manufacturing process design, in US dollars. The initial cost is the cost of the initial billet, that depends on the dimensions of the billet and the specific metal being processed. The costs for the forging processes include:

- i) set up costs,*
- ii) post-inspection costs,*
- iii) die wear costs,*
- iv) press run costs, and*
- v) the cost of possible strain-induced-porosity damage in the workpiece.*

Penalties are incurred with the forging processes when

- i) the press capacity is exceeded,*
- ii) the aspect ratio of the workpiece is too large, and*
- iii) the geometry of the workpiece conflicts with the die geometry.*

The cost to machine the workpiece is the cost of the material removed from the workpiece, where a penalty cost is incurred when the geometry of the workpiece conflicts with the desired final geometry of the workpiece after machining. After the workpiece is processed, a mandatory ultrasonic non-destructive evaluation cost and, if necessary, a cost of heat treatment is accrued. In addition, the final microstructure of the workpiece is evaluated; if the microstructure violates predetermined specifications, a penalty cost is incurred. All penalties are translated into US dollars in the cost function.

Computational results with the GHC algorithms and the neighborhood rules are reported for the three manufacturing process design sequences. All the GHC algorithms were executed with $K=250$ and $M=40$, resulting in 10,000 total iterations. Thirty replications of each GHC formulation were made, each initialized with a different initial design and a different seed for the random number generator. Common initial designs were used across the five GHC formulations (when the process design sequences and neighborhood rules were fixed), for all thirty replications. For the first replication, a feasible controllable input parameters setting was used, set by the user. The initial input parameters settings for the remaining twenty-nine replications were obtained by randomly selecting a neighbor of the first replication's feasible controllable input parameters setting. The mean (μ) and standard deviation (σ), as well as the minimum and maximum cost function values, were computed from these thirty replications. All computational experiments were executed on a SUN ULTRA-1 workstation (128 Mb RAM). Each set of thirty replications took approximately 30 CPU minutes. A listing of the computer code (in C) for the generalized hill climbing algorithms used to run these experiments is available from the principal investigator.

Table 1

GHC Algorithm Computational Results with Neighborhood Rule One

Sequence	GHC Formulation	μ	σ	minimum	maximum
$P_1P_2P_5P_6$	Simulated Annealing	2239.3	250.8	1927.0	2487.3
	Threshold Accepting	2305.1	238.7	1932.1	2518.7
	Monte Carlo	3056.9	108.1	2484.3	3076.6
	Local Search	2474.2	550.2	1919.3	3179.5
	Weibull Accepting	2224.5	247.7	1941.0	2532.7
$P_1P_2P_4P_6$	Simulated Annealing	2285.3	6.3	2277.9	2293.0
	Threshold Accepting	2286.9	16.3	2275.7	2330.6
	Monte Carlo	2245.3	0.0	2245.3	2245.3
	Local Search	2450.0	286.9	2285.4	3321.9
	Weibull Accepting	2284.8	8.3	2277.9	2312.4
$P_1P_2P_3P_4P_6$	Simulated Annealing	2415.7	208.0	2245.5	2824.6
	Threshold Accepting	2505.6	239.7	2245.5	2827.7
	Monte Carlo	2353.2	0.0	2353.2	2353.2
	Local Search	2685.4	279.1	2277.1	2975.1
	Weibull Accepting	2498.6	238.8	2245.5	2824.6

Table 2

GHC Algorithm Computational Results with Neighborhood Rule Two

Sequence	GHC Formulation	μ	σ	minimum	maximum
$P_1P_2P_5P_6$	Simulated Annealing	1921.8	7.0	1919.3	1957.4
	Threshold Accepting	1921.7	2.2	1919.3	1927.0
	Monte Carlo	2395.7	85.3	1943.9	2411.3
	Local Search	2097.2	255.9	1919.3	2458.0
	Weibull Accepting	1920.3	2.8	1919.3	1932.4
$P_1P_2P_4P_6$	Simulated Annealing	2238.8	1.5	2238.4	2244.2
	Threshold Accepting	2240.4	3.9	2238.4	2350.0
	Monte Carlo	2250.0	1.2	2248.0	2254.6
	Local Search	2439.0	2.5	2238.4	2248.1
	Weibull Accepting	2238.6	1.1	2238.4	2244.2
$P_1P_2P_3P_4P_6$	Simulated Annealing	2264.3	19.1	2245.5	2282.7
	Threshold Accepting	2282.8	21.4	2245.5	2301.6
	Monte Carlo	2874.0	0.0	2874.0	2874.0
	Local Search	2406.2	327.0	2245.5	3543.1
	Weibull Accepting	2263.7	21.9	2245.5	2308.6

Table 3
GHC Algorithm Computational Results with Neighborhood Rule Three

Sequence	GHC Formulation	μ	σ	minimum	maximum
P ₁ P ₂ P ₅ P ₆	Simulated Annealing	2145.1	257.5	1927.0	2471.2
	Threshold Accepting	1966.0	13.3	1950.6	1977.3
	Monte Carlo	2421.6	0.0	2421.6	2421.6
	Local Search	2131.8	308.5	1919.3	3158.0
	Weibull Accepting	2042.4	202.3	1927.0	2473.2
P ₁ P ₂ P ₄ P ₆	Simulated Annealing	2247.4	6.8	2238.4	2254.9
	Threshold Accepting	2279.0	12.9	2248.1	2285.7
	Monte Carlo	2298.9	6.6	2263.7	2300.1
	Local Search	2275.6	14.0	2248.1	2297.9
	Weibull Accepting	2250.3	9.6	2238.4	2275.7
P ₁ P ₂ P ₃ P ₄ P ₆	Simulated Annealing	2291.3	43.0	2245.5	2403.3
	Threshold Accepting	2311.3	20.4	2266.4	2320.3
	Monte Carlo	2836.4	0.0	2836.4	2836.4
	Local Search	2619.1	461.8	2245.5	3838.1
	Weibull Accepting	2299.8	99.0	2245.5	2786.2

The results in Tables 1-3 illustrate the performance of the five GHC algorithms and three neighborhood rules on the three manufacturing process design sequences. In particular, simulated annealing, threshold accepting, and Weibull accepting all yielded comparable results that were superior to the results produced by Monte Carlo search and local search. Local search also tended to yield results with higher variance.

Comparing the three neighborhood rules, the results with the second rule were superior to the results with the first and third rules. The first rule is very myopic (conservative) in how it traverses the design space, while the third rule is more aggressive; the second rule provides an effective balance between the two.

Comparing the three design sequences, the first design sequence (P₁P₂P₅P₆) yielded the minimum cost to produce the integrated blade rotor part. This is consistent with manufacturing process practices, since machining is more cost effective if the material being worked is sufficiently inexpensive to purchase and easy to machine (i.e., soft versus hard). Overall, the computational results are consistent with what would have been obtained using trial and error on the shop floor. The advantage of using generalized hill climbing algorithms and computer simulation manufacturing processes is the speed and efficiency at which

these results can be obtained, at a fraction of the cost that would be spent if trial and error on the shop floor would be required. The computational results reported suggest that generalized hill climbing algorithms provide a practical tool for determining optimal/near-optimal manufacturing process designs for the integrated blade rotor part. The application of optimization algorithm to identify good manufacturing process designs using computer simulation is a new and significant advance in how designs can be cost-effectively identified. Work is in progress to apply this approach to identify good manufacturing process designs for other complex parts of interest and concern to the Air Force. The significant savings in both time and cost for identifying optimal manufacturing process designs has the potential to make this approach a valuable shop floor tool for real-time analysis. In addition, work is in progress to transfer this technology into the hands of manufacturing companies who can benefit from its application. Several manufacturing companies (Pratt & Whitney, Hughes Research Laboratories, Alcoa, and Wyman-Gordon) have expressed an interest in using optimization algorithms to address their computer simulation discrete manufacturing process design optimization problems. These companies are open to using algorithms that have the potential to globally optimize manufacturing process designs; generalized hill climbing algorithms can provide tools to fill this need. Dr. James C. Malas at the Materials Process Design Branch of Wright Laboratory (Wright Patterson Air Force Base) has been the point of contact with these companies.

3. Other Research Results

In addition to the results reported with generalized hill climbing algorithms, and discrete manufacturing process design optimization problems, several other results were obtained during the period of the grant. These results are briefly discussed here.

Discrete optimization and discrete event simulation are two disparate areas within the general field of operations research. Building a framework that facilitates the cross-fertilization of tools in these two areas has provided much of the motivation and basic research leading up to the creation of the generalized hill climbing algorithm paradigm. Seven search problem for discrete event simulation models have been formulated and proven to be NP-hard. These problems consider fundamental issues faced by simulation practitioners (e.g., can a state be accessed, does changing the order of events impact the states, does representing the same simulation model on a computer in two different ways result in different states when executed, can a simulation run stall, how do permutations of events impact the states and future events lists, or how events are cancelled, resulting in a loss of information). The first problem, termed ACCESSIBILITY, is a generalization of the discrete manufacturing design problems described in Section 2. Jacobson and Yucesan (1995, 1998) prove these problems to be NP-hard, and discuss their practical implications for simulation practitioners.

Fleischer and Jacobson (1996, 1997) consider how information theory can be used to assess the finite-time performance of the simulated annealing algorithm. These results provide insights into how simulated annealing can be executed to optimize its effectiveness. Jacobson and Schruben (1998) look at how harmonic analysis can be used to perform sensitivity analysis of steady state discrete event simulation models. Kumar et al. (1997) identify a design and scheduling optimization problem associated with flexible assembly systems, prove the problem to be NP-hard, identify a lower bound for the problem, and introduce an efficient deterministic heuristic to address the problem. Jacobson et al. (1997) also report on the effectiveness of simulated annealing and tabu search for this problem, and give a polynomial-time algorithm to compute the lower bound. Kobza and Jacobson (1996, 1997) provide a probabilistic analysis of access control security systems. They consider various security models, and show how design of such systems impact the performance of the security systems. Jacobson and Morrice (1996, 1998) study a combinatorial medical problem that evaluates the temporal association between health disorders and medical treatments.

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CONTRIBUTING PERSONNEL

The principal investigator for this project, Dr. Sheldon H. Jacobson, has devoted 33% of his academic year time, and 33% of his summer, in each of the three years of this project. Major Alan W. Johnson, USAF, has worked on this project throughout the three years of the grant. Major Johnson successfully defended his Ph.D. dissertation, "Generalized Hill Climbing Algorithms for Discrete Optimization Problems," in October 1996. Ms. Kelly A. Sullivan and Mr. Zafar Ansari are two Ph.D. students of the principal investigator who have worked on various aspects of this project.

Three of the participants on this project have received national and international recognition for their work. Ms. Kelly A. Sullivan was the recipient of a Women and Minority award (sponsored by the INFORMS College on Simulation) to attend the 1996 Winter Simulation Conference in San Diego, California, December 8-11, 1996. The program is designed to educate women and minorities on simulation as a career track. Ms. Sullivan was one of only two student winners. The selection committee based their decision on the quality of the application document, including an essay on each applicant's interest in simulation and their academic record.

The paper, "Generalized Hill Climbing Algorithms for Discrete Manufacturing Process Design Problems," by Sheldon H. Jacobson, Kelly A. Sullivan, and Alan W. Johnson, was the recipient of the best paper award in the Industrial Simulation track at the European Simulation Multiconference, held in Istanbul, Turkey, June 1-4, 1997.

Major Alan W. Johnson, Ph.D., USAF, was awarded a Honorable Mention prize in the 1997 George E. Nicholson Student Paper Competition (sponsored by INFORMS). Dr. Johnson was one of six award winners among forty-three papers submitted. The research presented in the award winning paper, "A General Convergence Result for Hill Climbing Algorithms" is described in Section 1.3, Theorem 2.

Lastly, the principal investigator for this project hosted the AFOSR Electronic Prototyping Review Meeting at Virginia Tech, May 27-29, 1997. The meeting is used to foster interactions and exchanges between military, academic, and industrial researchers involved with discrete optimization problems. The meeting was well attended and very successful in meeting its goals and objectives.

PUBLICATIONS

The following is a list of all publications that have resulted from research on this grant.

Submitted but not yet Accepted

- Fleischer, M.A., Jacobson, S.H., 1996, "Information Theory and the Finite-Time Behavior of the Simulated Annealing Algorithm: Experimental Results."
- Johnson, A.W., Jacobson, S.H., 1996, "A Convergence Result for a Class of Generalized Hill Climbing Algorithms."
- Johnson, A.W., Jacobson, S.H., 1997, "A General Convergence Result for Hill Climbing Algorithms."

Published or Accepted

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APPENDIX 1: Input Parameters for the Integrated Blade Rotor

The following are the input parameters for the six processes:

<u>Process</u>	<u>Input Parameters</u>	<u>Possible Values</u>
P ₁	Billet Radius Billet Height	[2,2.5,3.0,3.5,4.0,4.5,5.0,5.5,6.0] [1.0,2.0,3.0]* Billet Radius
P ₂	Die Geometry: Height 1 Height 2 Height 3 Die Speed Die Friction Factor Die Temperature/Ambient Temperature [1562,1607,1652,1697,1742,1787,1832]	[1.5, 2.0, 2.5, 3.0,3.5] [1.5, 2.0, 2.5, 3.0,3.5] [1.5, 2.0, 2.5, 3.0,3.5] [.4,.6,.8,1.0,1.2] [.2,.4,.6,.8]
P ₃	Radius Height	[2.0,2.25,...,5.0] [1.5,2.0,2.5,3.0,3.5]
P ₄	Die Geometry: Radius 1 Height 1 Radius 2 Height 2 Die Speed Die Friction Factor Die Temperature/Ambient Temperature [1562,1607,1652,1697,1742,1787,1832]	[1.5, 1.75, ..., 3.5] [0.5, 0.625, 0.75, 0.875] 9.0 (fixed) [1.0,1.25,1.5,1.75,2.0] [0.4,0.6,0.8,1.0,1.2] [0.2,0.4,0.6,0.8]
P ₅	Radius 1 Height 1 Radius 2 Height 2 Radius 3 Height 3	[1.25, 1.50, ..., 3.25] [0.5,0.625,0.75,0.875] [1.5, 1.75, ..., 3.5] [0.5,0.75,1.0,1.25,1.5,1.75,2.0] [3.5, 3.75, ..., 5.0] [1.0,1.25,1.5,1.75,2.0]
P ₆	Radius 1 Height 1 Radius 2 Height 2 Radius 3 Height 3	3.0 (fixed) 0.5 (fixed) 3.0 (fixed) 1.5 (fixed) 4.0 (fixed) 1.5 (fixed)