



Final Technical Report  
**Explicit integration schemes for the hyperbolized  
Navier-Stokes equations**

Grant No. F49620-93-1-0417

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## 1 Objectives

This project considered the application of local preconditioning to a stiff hyperbolic system describing viscous conducting flow with a finite relaxation time. It was funded by an Augmentation Grant tacked on to the larger project "Local preconditioning of the Euler equations and its numerical applications," funded under Air Force Grant No. F49620-92-J-0158-DEF. During the second funding year, part of the effort in the augmentation project was diverted to the larger subject of preconditioning for the Euler and Navier-Stokes equations, in order to help solve the problem of stagnation-point instability, which was halting any progress in the application of local preconditioning. This situation persisted during the third year and following no-cost extension period (7/1-12/31/97), particularly because another preconditioning instability, one for very low cell Reynolds-numbers, was encountered. The reformulated goals for the augmentation project therefore were:

1. To develop accurate, fast and robust integrators for hyperbolized dissipative systems, but without the use of local preconditioning;
2. To develop a local preconditioning for the Euler equations with non-degenerate eigenvector system for the entire Mach-number range (the degeneracy causes the stagnation-point instability);
3. To develop a local preconditioning for the original (non-hyperbolized) Navier-Stokes equations that is effective and stable for all combinations of Mach and cell Reynolds-numbers.

## 2 Final status vs. goals

**Goal 1:** The development and testing of integration schemes for hyperbolized dissipative systems with stiff source terms was rounded off with the production of the doctoral thesis of Mohit Arora, defended in February 1996. The systems studied were:

- (a) 1-D hyperbolized heat equation (2 eqs.);
- (b) hyperbolized Burgers equation (2 eqs.);
- (c) 1-D Broadwell gas (3 eqs.);
- (d) 1-D Euler equations with heat conduction (3 eqs.);
- (e) 1-D 10-moment equations, equivalent to N-S without heat conduction (4 eqs.).

**Goal 2:** A matrix developed previously (under the parent grant) by Dohyung Lee was successfully applied to a standard stagnation-point test flow. Convergence acceleration was obtained where the Van Leer-Lee-Roe and Turkel matrices produce a rapidly growing instability. This is very promising, since the "fix" proposed by Darmofal and Schmid (1995) requires the introduction of a rather arbitrary lower limit for the Mach number, which significantly complicates the algebra in designing the corresponding artificial-viscosity matrix. The Lee matrix has a non-degenerate (non-parallel) eigenvector structure for  $M \downarrow 0$ , but not for  $M \uparrow 1$ ; the Van Leer matrix has it precisely the other way. This suggests a blend between these two matrices as a function of the Mach number. A new matrix was found that is close to Lee's matrix for low  $M$ , is positive definite (Lee's is not) and approaches Van Leer's matrix for  $M$  close to 1, with improved eigenvector structure for the entire range of  $M$ . This was the last theoretical result obtained in the project, and has not yet been tested in practice. A full presentation of these results can be found in D. Lee's thesis, defended in November 1996.

**Goal 3:** Following a suggestion by S. Venkateswaran (NASA ARC, 1995, private communication) we used the results of a simplified Navier-Stokes dispersion analysis to put Reynolds-number dependence into the Van Leer preconditioner. This does equalize the various time scales in the N-S equations, but, unfortunately, produces an unstable mode for very low cell Reynolds-numbers. D. Lee found that the preconditioning can be stabilized by adding to it the matrix coefficients of the leading dissipation terms that would also arise in point-Jacobi relaxation. This preconditioning was successfully implemented in the computation of a flat-plate laminar boundary-layer, with the cell-aspect-ratio going up to 1700 at the plate, and a turbulent boundary layer. These results are contained in D. Lee's thesis.

### 3 Project accomplishments/highlights

The research effort spent in the 3.5 years of this augmentation project has produced a significant improvement of local preconditioning for the standard Navier-Stokes equations. A matrix was developed that equalizes the time-scales implied in the equations for all Mach and cell-Reynolds numbers, and leads to stable calculations with strongly accelerated convergence. Such a matrix produces not only a single-grid benefit but also an independent multi-grid benefit. This is highly desirable, because current Navier-Stokes codes are notorious for their slow convergence. Robustness is still a point of concern; more testing is needed.

In the area of Euler preconditioning we may have found a matrix with a sufficiently "healthy" effect on eigenvector structure so as to yield stable calculations even in the presence of a stagnation point. More testing is still needed; the matrix will eventually be modified to accomodate Reynolds-number dependence, for use in Navier-Stokes calculations.

In the area of hyperbolized dissipative systems, the integration method developed previously by Arora was tested on a wide variety of 1-D systems of equations and was found highly satisfactory. It is accurate and fast, and extremely robust. In the test calculations it handled relaxation-time-scales ranging over a factor  $10^{10}$ . The main challenge is to extend the approach to multidimensional systems. The benefit of the hyperbolized approach is that infinite signal speeds (as in the standard diffusion equation) are avoided, relieving the severe time-step restriction, and the need to go globally implicit. A locally implicit integration technique then takes care of the source-term restriction. This may be the way of the future of computing any type of more or less continuous flow, including chemically reacting flow, and flow at intermediate Knudsen numbers.

### 4 Personnel supported

Bram van Leer (PI), professor,  
Philip L. Roe, professor,  
John F. Lynn, doctoral candidate (defense 17 May 1995)  
Dohyung Lee, doctoral candidate (defense November 20, 1996)  
Mohit Arora, doctoral candidate (defense February 8, 1996),  
all in the same department.

### 5 Publications

1. Mohit Arora and Philip L. Roe, "Characteristic-based numerical algorithms for stiff hyperbolic relaxation systems," presented at the 15th International

Conference on Numerical Methods in Fluid Dynamics, Monterey, CA, June 24-28, 1996. Proceedings to be published with Springer.

2. Mohit Arora, "Explicit characteristic-based high-resolution algorithms for hyperbolic conservation laws with stiff source terms," Ph.D. thesis, University of Michigan, 1996.
3. Mohit Arora, "A strategy for hyperbolic systems with stiff source terms," presented (by P. L. Roe) at the ICASE/LaRC workshop on Barriers and Challenges in CFD, Hampton, VA, August 5-7, 1996. Proceedings to be published with Kluwer.
4. J. F. Lynn and B. van Leer, "Multigrid Euler Solutions with Semi-Coarsening and Local Preconditioning", in: Proceedings of the 14th International Conference on Numerical Fluid Dynamics, Springer, 1995.
5. J. F. Lynn and B. van Leer, "A Semi-Coarsened Multigrid Solver for the Euler and Navier-Stokes Equations with Local Preconditioning", AIAA Paper 95-1667-CP, 1995.
6. John F. Lynn, Bram van Leer and Dohyung Lee, "Multigrid solution of the Euler equations with local preconditioning," presented at the 15th International Conference on Numerical Methods in Fluid Dynamics, Monterey, CA, June 24-28, 1996. Proceedings to be published with Springer Verlag.
7. D. Lee, "Local Preconditioning of the Euler and Navier-Stokes Equations," Ph.D. thesis, University of Michigan, 1996.

## 6 Interactions/transitions

### A. Participation/presentations, etc.

The Principal Investigator Bram van Leer gave numerous CFD seminars and other presentations (without ensuing publications) in which results on preconditioning were presented. Starting with the fall of 1995, such presentations included positive results obtained in the augmentation project. These are listed below.

- September 16, 1995, "Local preconditioning of the Euler and Navier-Stokes equations," invited lecture at the Conference on Numerical Methods for the Euler and Navier-Stokes Equations, Montréal, Québec, Canada.
- February 15, 1996, USAF Phillips Lab., Kirtland AFB, Albuquerque, NM, "CFD in the nineties and beyond"
- March 21, 1996, CFD Research Corp., Huntsville, AL, "Is CFD dead?"

- March 28, 1996, Texas A&M, College Station, TX, "Is CFD dead?"
- April 24, 1996, MacNeal-Schwendler BV, Gouda, Netherlands, "Is CFD dead?"
- April 29, 1996, "Does industry need fanciful CFD methods?", plenary lecture at the BRITE/EURAM Workshop on Multi-Dimensional and Sparse-Grid Methods, Von Kármán Institute, St. Genesius-Rode, Belgium.
- May 2, 1996, MacNeal-Schwendler BV, Gouda, Netherlands, "Local preconditioning for the Euler and Navier-Stokes equations."
- May 6, 1996, CWI (Center for Mathematics and Informatics), Amsterdam, Netherlands, "Numerical integration of stiff hyperbolized systems: why and how."
- August 7, 1996, "Towards the ultimate local preconditioning matrix: Is the end in sight?" Invited lecture at the ICASE/LaRC workshop on Barriers and Challenges in CFD, Hampton, VA, August 5-7, 1996.

#### B. Consultative and advisory functions, etc.

- Bram van Leer visited the USAF Phillips Lab., Kirtland AFB, Albuquerque, NM, on February 15. Discussions included the use and numerical treatment of hyperbolized dissipative systems.
- Bram van Leer is a regular consultant to ICASE (Institute for Computer Applications in Science and Industry), located at NASA Langley Research Center, and has advised NASA branches extensively on the uses and implementation of local preconditioning of the Euler and Navier-Stokes equations.

#### C. Transitions

Dr. Veer Vatsa (NASA Langley Research Center) has successfully implemented in the aerodynamics-analysis code TLNS3D a preconditioner of the family that includes the matrices of D. Lee and B. van Leer, which remain stable in the presence of a stagnation point. Implementation of these new matrices is scheduled to take place during a collaborative project funded under Advanced Subsonic Technology, during the summer of 1997.

## 7 New discoveries, etc.

None patentable.

## 8 Honors and awards

- Bram van Leer was made Fellow of the AIAA in May 1995.

- Bram van Leer received the UMI College of Engineering's Award for Excellence in Research, January 1996.
- Phil Roe was made Fellow of the AIAA in May 1996.