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13. ABSTRACT *(Maximum 200 words)*

The lack of a systematic methodology for the design of feedback laws capable of controlling complex dynamical systems has been a limiting factor in several current and emerging DoD missions. The research carried out by the principal investigators in this three year research effort has focused on analyzing and computing the steady-state behavior of controlled complex dynamical systems.

One of the fundamental discoveries made during this research effort concerns the steady-state behavior of a dynamical system which provides estimates of the current state. The analysis led to an unanticipated geometric discovery which led to the solution of an outstanding problem in linear systems theory with applications in speech synthesis, voice recognition and signal processing. These advances were supported by computational methods developed in this research effort and which are documented in this final report and in a patent application.

Another fundamental discovery made during this research effort was focused on controller design for a class of distributed parameter systems. Our results quantify properties of attractors and the steady-state behavior of solutions. The principal investigating team also discovered results concerning the asymptotic behavior of linear distributed parameter systems undergoing harmonic, or periodic, forcing.

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DEPSCOR FINAL REPORT

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ANALYTICAL AND COMPUTATIONAL METHODS FOR NONLINEAR FEEDBACK DESIGN

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1 Executive Summary

The lack of a systematic methodology for the design of feedback laws capable of controlling complex dynamical systems has been a limiting factor in several current and emerging DoD missions. The research carried out by the principal investigator in this three year research effort has focused on analyzing and computing the steady-state behavior of controlled complex dynamical systems. The control of such systems typically involves both the design of feedback laws which use the current value of the state and the design of dynamical systems which produce estimates of the current state.

One of the fundamental discoveries made during this research effort concerns the steady-state behavior of a dynamical system which provides estimates of the current state, propagating smaller amounts of data than required by the standard Kalman filter. The analysis of this estimation scheme led to an unanticipated discovery about the geometry of certain classes of linear systems. The geometric properties which were discovered in this way led to the solution of an outstanding problem in linear systems theory with applications in speech synthesis, voice recognition and signal processing. These advances were supported by computational methods developed in this research effort and which are documented in this final report and in a patent application.

Another fundamental discovery made during this research effort was focused on the first part of controller design discussed above, for a class of distributed parameter systems. The principle results obtained quantify properties of attractors and the steady-state behavior of solutions for a controlled Burgers' equation, which is often used as a simple model for turbulence. The principal investigating team also discovered results concerning the asymptotic behavior of linear distributed parameter systems undergoing harmonic, or periodic, forcing.

1.1 The Rational Covariance Extension Problem and Speech Synthesis

In the course of our studying the design of nonlinear observers we discovered a description of the complete phase portrait of the Kalman filter, viewed as a nonlinear dynamical system on the space of positive real functions. This discovery in turn lead to the solution of an important problem in signal processing. In particular, in [7] we were able to give a complete parameterization of all shaping filters which match a given finite window of correlation coefficients. This was an important open problem in signal processing, first formulated for speech synthesis (and intensively studied) by researchers at the Phillips Laboratories in Europe. In [14, 15], T. T. Georgiou proved that to each choice of partial covariance sequence and numerator polynomial of a modeling filter there exists a rational covariance extension yielding a pole polynomial for the modeling filter, and he conjectured that this extension was unique so that it provides a complete parameterization of all rational covariance extensions. We note that this problem has a long history, with antecedents going back to potential theory in the work of Carathéodory, Toeplitz and Schur [10, 11, 32, 31], and continuing in the work of Kalman, Georgiou, Kimura and others [19, 15, 22]. This problem has been of more recent interest due to its significant interface with problems in signal processing and speech processing [12, 9, 26, 21] and in stochastic realization theory and system identification [2, 33, 23]. Our recent work [7], extends the result of Georgiou and answers his conjecture in the affirmative. This work has shed new light on the stochastic (partial) realization problem [6] through the development of an associated Riccati-type equation, whose unique positive semi-definite solution has as its rank the minimum dimension of a stochastic linear realization of the given rational covariance extension. However, our proof was not nonconstructive.In [8], we have been able to give a constructive proof of Georgiou's conjecture, which provides an algorithm for solving the problem of determining the unique pole polynomial corresponding to the given partial covariance sequence and the desired zeros. In this work, which was motivated by the effectiveness of interior point methods for solving nonlinear convex optimization problems, we first recast the fundamental problem as an optimization problem.

Historically, the only solution to rational covariance extension problem for which there has been simple computational procedures is the so called *maximum entropy* solution, which is the particular solution that maximizes the entropy gain. During this research period, we were able to demonstrate that the infinite-dimensional optimization problem for determining this solution has a simple finite-dimensional dual. This motivated the introduction of a nonlinear, strictly convex functional defined on a closed convex set naturally related to the covariance extension problem. We were able to show that any solution of the rational covariance extension problem lies in the interior of this convex set and that, conversely, an interior minimum of this convex functional will correspond to the unique solution of the covariance extension problem. Our interest in this convex optimization problem is, therefore, twofold: as a starting point for the computation of an explicit solution, and as a means of providing an alternative proof of the rational covariance extension theorem.

Concerning the existence of a minimum, we showed that this functional is proper and bounded below, i.e., that the sublevel sets of this functional are compact. From this, it follows that there exists a minimum. Since uniqueness follows from strict convexity of the functional, the central issue which needed to be addressed in order to solve the rational covariance extension problem was whether, in fact, this minimum is an interior point. Indeed, our formulation of the convex functional, which contains a barrier-like term, was inspired by interior point methods. However, in contrast to interior point methods, the barrier function we introduced did not become infinite on the boundary of our closed convex set. Nonetheless, we were able to show that the gradient, rather than the value, of the convex functional became infinite on the boundary. The existence of an interior point which minimizes the functional then follows from this observation.

Some Numerical Examples

Given an arbitrary partial covariance sequence $c_0, c_1, \dots, c_n$ and an arbitrary zero polynomial $\sigma(z)$, the constructive proof of Georgiou's conjecture provides algorithmic procedures for computing the corresponding unique modeling filter, which are based on the convex optimization problem to minimize the functional $\varphi : \mathbb{R}^{n+1} \to \mathbb{R}$, defined by

$$\varphi(q_0, q_1, \dots, q_n) = c_0 q_0 + c_1 q_1 + \cdots + c_n q_n - \frac{1}{2\pi}\int_{-\pi}^{\pi} \log Q(e^{i\theta}) |\sigma(e^{i\theta})|^2 d\theta, \tag{1.1.1}$$

over all $q_0, q_1, \dots, q_n$ such that

$$Q(e^{i\theta}) = q_0 + q_1 \cos\theta + q_2 \cos 2\theta + \cdots + q_n \cos n\theta > 0 \quad \text{for all } \theta. \tag{1.1.2}$$

over all $q_0, q_1, \dots, q_n$ such that (1.1.2) holds.

In general such procedures will be based on the gradient of the cost functional φ, which, is given by

$$\frac{\partial \varphi}{\partial q_k}(q_0, q_1, \dots, q_n) = c_k - \bar{c}_k \tag{1.1.3}$$

where

$$\bar{c}_k = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{ik\theta} \frac{|\sigma(e^{i\theta})|^2}{Q(e^{i\theta})} d\theta \quad \text{for } k = 0, 1, 2, \dots, n \tag{1.1.4}$$

are the covariances corresponding to a process with spectral density

$$\frac{|\sigma(e^{i\theta})|^2}{Q(e^{i\theta})} = \bar{c}_0 + 2\sum_{k=1}^{\infty} \bar{c}_k \cos(k\theta). \tag{1.1.5}$$

The gradient is thus the difference between the given partial covariance sequence $c_0, c_1, \dots, c_n$ and the partial covariance sequence corresponding to the choice of variables $q_0, q_1, \dots, q_n$ at which the gradient is calculated. The minimum is attained when this difference is zero.

The following simulations have been done by Per Enqvist, using Newton's method (see, e.g., [24, 27]), which of course also requires computing the Hessian (second-derivative matrix) in each iteration. An straight-forward calculation shows that the Hessian is the sum of a Toeplitz and a Hankel matrix. More precisely,

$$H_{ij}(q_0, q_1, \ldots, q_n) = \tfrac{1}{2}(d_{i+j} + d_{i-j}) \quad i, j = 0, 1, 2, \ldots, n, \tag{1.1.6}$$

where

$$d_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{ik\theta} \frac{|\sigma(e^{i\theta})|^2}{Q(e^{i\theta})^2} d\theta \quad \text{for } k = 0, 1, 2, \ldots, 2n \tag{1.1.7}$$

and $d_{-k} = d_k$. Moreover, $d_0, d_1, d_2, \ldots, d_{2n}$ are the $2n+1$ first Fourier coefficients of the spectral representation

$$\frac{|\sigma(e^{i\theta})|^2}{Q(e^{i\theta})^2} = d_0 + 2\sum_{k=1}^{\infty} d_k \cos(k\theta). \tag{1.1.8}$$

The gradient and the Hessian can be determined from (1.1.3) and (1.1.6) respectively by applying the inverse Levinson algorithm (see, e.g., [28]) to the the appropriate polynomial spectral factors of $Q(z)$ and $Q(z)^2$ respectively and then solving the resulting linear equations for $\bar{c}_0, \bar{c}_1, \ldots, \bar{c}_n$ and $d_0, d_1, d_2, \ldots, d_{2n}$; see [13] for details.

To illustrate the procedure, let us again consider the sixth order spectral envelopes of Figure 1 and 2 together with the corresponding zeros and poles.

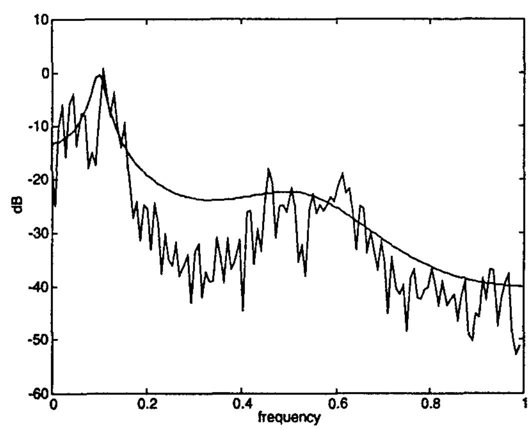

Figure 1: Spectral envelope of a maximum entropy solution.

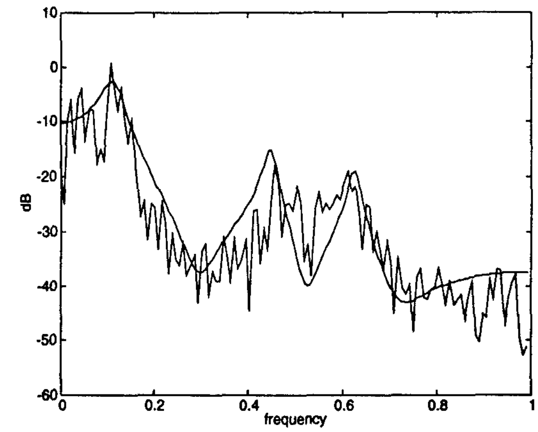

Figure 2: Spectral envelope obtained with appropriate choice of zeros.

Hence Figure 3 illustrates the periodogram for a section of speech data together with the corresponding sixth order maximum entropy spectrum, which, since it lacks finite zeros, becomes rather "flat". The location of the corresponding poles (marked by $\times$) in the unit circle is shown next to it. The zeros (marked by $\circ$) of course all lie at the origin.

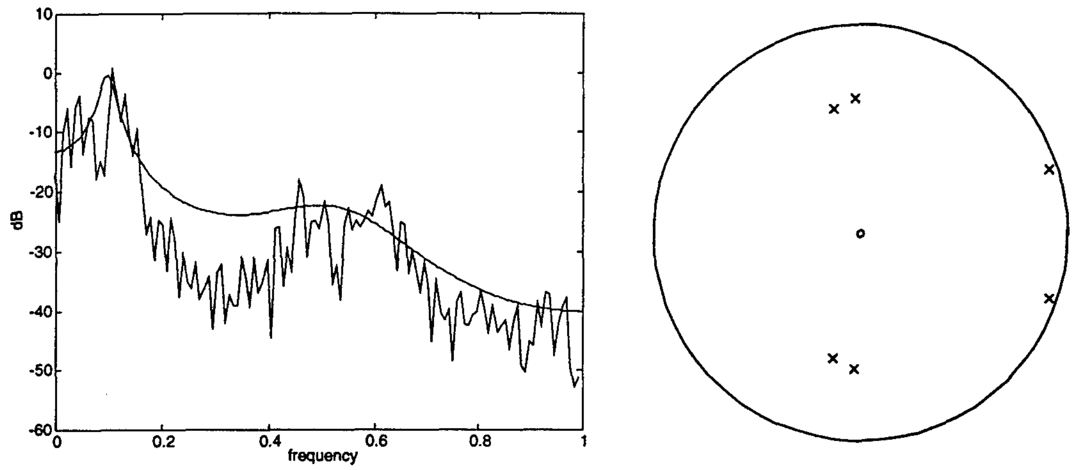

Figure 3

Now selecting the zeros appropriately as indicated to the right in Figure 4, we obtain the poles as marked, and the corresponding sixth order modeling filter produces the spectral envelope to the right in Figure 4. We see that the second solution has a spectral density that is less flat and provides a better approximation, reflecting the fact that the filter is designed to have transmission zeros near the minima of the periodogram.

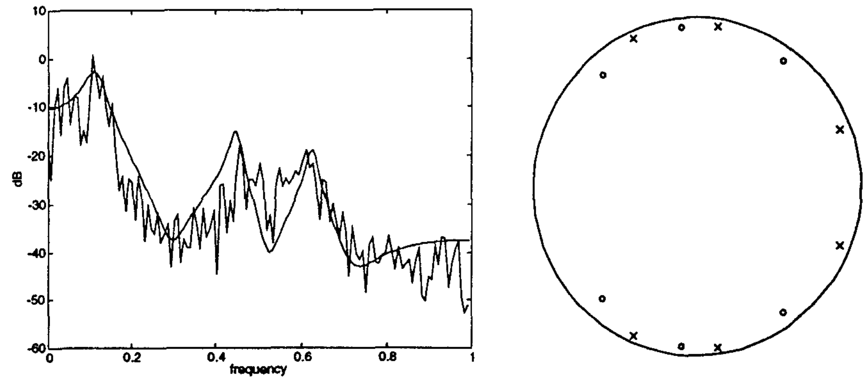

Figure 4

1.2 Structure of Attractor for a Boundary Controlled Burgers' Equation

Motivated in part by problems of flow control and combustion control, where nonlinear effects actually can improve mixing, there has been considerable attention given in the literature to the study of asymptotic or steady state properties of solutions of nonlinear distributed parameter systems, such as Navier-Stokes equations, which contain both nonlinear convective terms and diffusive terms. To this end, part of our research effort concerns the effect of boundary control on the structure of attractors for Burgers' equation. In the, soon to be published, work [123], we give a rigorous mathematical justification of the fact that there can be multiple stationary points contained in the global attractor. Partial numerical concerning this possibility were first results announced in [95]. The analysis in this paper is based on an extension of the classical analysis of Sturm-Louiville boundary value problems using the so-called Prüfer transformations or polar coordinates. We should comment that various partial results were obtained earlier using a variety of approaches but none of these approaches provided the more complete picture presented here.

In our work [123] we show that for a special class of forcing terms as the gain parameters in the boundary feedback control are increased from small positive values to large positive values, the number of stationary solutions vary from three to one. This is in keeping with our results on conergence of attractors, discussed in the last subsection, where it is shown that the zero dynamics systems has a single, global, asymptotically stable, equilibrium.

Consider the controlled viscous Burgers' system

$$\begin{aligned} &w_t - \epsilon w_{xx} + w w_x = f(x), \\ &w = w(x,t), \ x \in (0,1), \ t \geq 0, \\ &-w_x(0,t) = u_0(t), \ \ w_x(1,t) = u_1(t), \\ &w(x,0) = \phi(x), \\ &y_0(t) = w(0,t), \ \ y_1(t) = w(1,t), \end{aligned} \tag{1.2.1}$$

where $u_0(t)$, $u_1(t)$ are boundary inputs, $y_0(t)$, $y_1(t)$ are boundary outputs, $f \in L^2(0,1)$ is an external forcing term modeling an unknown disturbance.

Formally introducing proportional error feedback, with $k_0 = k_1 = k$

$$u_0 = -ky_0, \quad u_1 = -ky_1, \tag{1.2.2}$$

with feedback gain $k > 0$ we obtain the closed loop Burgers' system

$$\begin{aligned}
&w_t - \epsilon w_{xx} + w w_x = f, \\
&x \in (0,1), \ t \geq 0, \\
&-w_x(0,t) + kw(0,t) = 0, \\
&w_x(1,t) + kw(1,t) = 0, \\
&w(x,0) = \phi(x).
\end{aligned}$$

In our work we are interested in forcing terms $f(x)$ possessing a certain symmetry property and an additional definiteness property. These assumptions considerably simplify the analysis of the resulting stationary problem.

Assumption 1.2.1. *We assume that f is an odd function about $x = 1/2$ in the interval $[0,1]$, i.e.,*

$$f(x) = -f(1-x) \quad \textit{for} \ \ x \in [0,1],$$

and we introduce the terminology 'antisymmetric about $1/2$" or simply "antisymmetric" to describe such a function.

We will also assume that

$$f(x) > 0 \textit{ for } x \in [0, 1/2).$$

An important fallout of the antisymmetry condition is that it is preserved by solutions of (1.2.3).

The stationary Burgers' system associated with (1.2.3) is

$$\begin{aligned}
&-\epsilon w_{xx}(x) + w(x) w_x(x) = f(x), \\
&w_x(0) - kw(0) = 0, \\
&w_x(1) + kw(1) = 0.
\end{aligned} \tag{1.2.3}$$

Our approach proceeds as follows. We first integrate the stationary Burgers' equation and then introduce the Riccati (or Hopf-Cole) transformation to reduce the stationary Burgers' equation to a second order linear equation containing a spectral type parameter. Unfortunately, as mentioned above, the boundary conditions are transformed into nonlinear boundary conditions. In the stationary case these boundary conditions factor into products of boundary conditions which appear to be of Sturmian type.

Integrating the differential equation (1.2.3) over the interval $[0,x]$, we arrive at

$$-\epsilon w'(x) + \frac{w(x)^2}{2} = \int_0^x f(s)\, ds + c, \tag{1.2.4}$$

where, c is a constant of integration. Using the boundary conditions, we see that

$$c = -\epsilon k w(0) + w(0)^2/2 \tag{1.2.5}$$

is actually not arbitrary. The parameter c plays an important part in our analysis.

Equation (1.2.4) is a Riccati ordinary differential equation and one classical approach to solving this equation is to introduce the so called Riccati transformation, i.e., we seek a solution in the form

$$w(x) = -2\epsilon \frac{v'(x)}{v(x)}. \tag{1.2.6}$$

Using (1.2.6), the differential equation (1.2.4) is transformed into

$$v''(x) - (F(x) + \lambda)\, v(x) = 0 \tag{1.2.7}$$

where

$$F(x) = \frac{1}{2\epsilon^2}\left(\int_0^x f(s)\, ds\right), \tag{1.2.8}$$

and

$$\lambda = \frac{c}{2\epsilon^2}. \tag{1.2.9}$$

The boundary conditions are transformed into nonlinear boundary conditions, which, after using the equations (1.2.7) and (1.2.8), can be written as

$$\begin{aligned} -cv(0)^2 + 2v'(0)^2\epsilon^2 + 2kv'(0)v(0)\epsilon^2 &= 0, \\ -cv(1)^2 + 2v'(1)^2\epsilon^2 - 2kv'(1)v(1)\epsilon^2 &= 0. \end{aligned} \tag{1.2.10}$$

These conditions can be factored into products of conditions that appear to be of "Sturm-Liouville" type:

$$\begin{aligned} \left(v'(0) - l^+v(0)\right)\left(v'(0) - l^-v(0)\right) &= 0 \\ \left(v'(1) + l^+v(1)\right)\left(v'(1) + l^-v(1)\right) &= 0. \end{aligned} \tag{1.2.11}$$

(1.2.12)

Unfortunately, the parameters $\ell^\pm$ depend on the "spectral" parameter λ:

$$l^\pm = l^\pm(k,\lambda) = -\frac{k}{2} \pm \sqrt{\left(\frac{k}{2}\right)^2 + \lambda}. \tag{1.2.13}$$

Considering the various possibilities we arrive at four systems that can deliver stationary solutions

$$v''(x) - (F(x) + \lambda)\, v(x) = 0 \tag{1.2.14}$$

$$
\begin{array}{ll}
v'(0) - l^+ v(0) = 0, & v'(1) + l^+ v(1) = 0, \\
v'(0) - l^+ v(0) = 0, & v'(1) + l^- v(1) = 0, \\
v'(0) - l^- v(0) = 0, & v'(1) + l^+ v(1) = 0, \\
v'(0) - l^- v(0) = 0, & v'(1) + l^- v(1) = 0.
\end{array} \tag{1.2.15}
$$

We are interested in non-vanishing solutions, because solutions of the stationary Burgers' equation are given by $-2\epsilon\frac{v'(x)}{v(x)}$, and, therefore, the existence of a zero of $v(x)$ together with uniqueness of solutions of the initial value problem would imply a blow-up solution.

We next introduce the classical Prüfer transformation for (1.2.14)-(1.2.15), namely,

$$v(x) = r(x)\cos\phi(x), \tag{1.2.16}$$

$$v'(x) = r(x)\sin\phi(x). \tag{1.2.17}$$

Differentiating (1.2.16) yields

$$v'(x) = r'(x)\ \cos\phi(x) - r(x)\ \sin\phi(x)\ \phi'(x). \tag{1.2.18}$$

Now combine (1.2.17) and (1.2.18), to get

$$r(x)\sin\phi(x) = r'(x)\ \cos\phi(x) - r(x)\ \sin\phi(x)\ \phi'(x). \tag{1.2.19}$$

From (1.2.14) we have

$$v''(x) = (F(x) + \lambda)r(x)\ \cos\phi(x), \tag{1.2.20}$$

and differentiating (1.2.17) gives

$$v''(x) = r'(x)\ \sin\phi(x) + r(x)\ \cos\phi(x)\ \phi'(x). \tag{1.2.21}$$

Combining (1.2.20) and (1.2.21) gives

$$(F(x) + \lambda)r(x)\ \cos\phi(x) = r'(x)\ \sin\phi(x) + r(x)\ \cos\phi(x)\ \phi'(x). \tag{1.2.22}$$

Multiplying (1.2.19) by $\cos\phi(x)$ and adding to (1.2.22) times $\sin\phi(x)$ gives

$$r'(x) = (1 + F(x) + \lambda)r(x)\sin\phi(x)\cos\phi(x). \tag{1.2.23}$$

Equation (1.2.23) can be solved in terms of ϕ as

$$r(x) = r(0)\exp\Big(\int_0^x (1 + F(s) + \lambda)\sin\phi(s)\cos\phi(s)\ ds\Big). \tag{1.2.24}$$

Multiplying (1.2.22) by $\cos\phi(x)$ and subtracting (1.2.19) times $\sin\phi(x)$ gives

$$r(x)\phi'(x) = (F(x) + \lambda)r(x)\cos^2\phi(x) - r(x)\sin^2\phi(x). \tag{1.2.25}$$

Since we are looking for solutions $v(x)$ which have no zeros on $[0,1]$ and also, since (1.2.24)-(1.2.25) are linear in $r(x)$, we may assume, without loss of generality, that $r(x) > 0$ for $x \in [0,1]$.
Using this assumption, we can divide by $r(x)$ in (1.2.25) to obtain

$$\phi'(x) = (F(x) + \lambda)\cos^2\phi(x) - \sin^2\phi(x). \tag{1.2.26}$$

In what follows, equation (1.2.26) is the main equation used in our analysis. On one hand it is independent of $r(x)$, on the other hand $r(x)$ is completely determined by $r(0)$ and $\phi(x)$ in (1.2.24).

Rewriting the boundary conditions (1.2.15), taking into account that $r(0)$ and $r(1)$ are not zero, and after some simplification using (1.2.26), we obtain the following four sets of boundary value problems

$$\phi'(x) = (F(x) + \lambda)\cos^2\phi(x) - \sin^2\phi(x), \tag{1.2.27}$$

$$\phi(0) = \arctan(l^{s_1}), \tag{1.2.28}$$

$$\phi(1) = -\arctan(l^{s_2}), \tag{1.2.29}$$

where $s_1,\ s_2 \in \{+,-\}$.

Our main result is the following theorem.

Theorem 1.2.1. *The following statements hold:*

a) for any $k > 0$ there exists an antisymmetric solution of the stationary Burgers' system (1.2.3). This solution is the unique solution of (1.2.3) for sufficiently large k;

b) for sufficiently small k, in addition to the antisymmetric stationary solution, there also exist at least two non-antisymmetric stationary solutions;

c) the antisymmetric stationary solution is asymptotically stable for sufficiently large k.

1.3 Harmonic Forcing for Linear Distributed Parameter Systems

In general, the ability to systematically control or influence nonlinear effects would make a substantial contribution to existing and emerging commercial and defense research and development programs. Notable examples, widely appreciated within the aerospace industry, include the development of flight controllers for high angle-of-attack or high agility aircraft. Indeed, the importance of including the nonlinear behavior of aerodynamic parameters, such as the coefficient of lift, as a function of the angle-of-attack has long been recognized since at high angles-of-attack, wind angle moments also exhibit nonlinear effects which cannot be ignored. Another area of interest is the control of flutter, which can shorten the life cycle of aircraft and aircraft parts. Indeed, one example of the potential impact of nonlinear control in problems of flow control is in the control of instabilities in the unsteady separated shear layer, which has been experimentally shown to greatly influence stall and lift behavior at high angles of attack.

It is worth noting, however, that the active control in experiments, such as Batill and Mueller [100], is based on a priori harmonic forcing, while in nonlinear systems with resonance it is known that simple harmonic forcing will not necessarily produce the desired response (see e.g. [41] for an analysis of the steady state response of nonlinear systems by center manifold methods). This point was also illustrated theoretically by Keefe [59], who showed that the success of a priori control for the Ginzburg-Landau equation was dependent on the initial state of the system, which of course may not be controlled or even known, and that therefore undesirable responses are to be expected in the nonlinear regime. Moreover, the computation by Fuglsang and Cain [38] of flow over an open cavity suggests that harmonic forcing at non-resonant harmonic frequencies can produce a limit cycle or chaotic response that is far more severe than the natural harmonic resonance.

We consider a special class of Single Input Single Output (SISO) linear distributed parameter control systems in the form

$$\dot{z} = Az + bu, \tag{1.3.1}$$

$$z(0) = z_0, \tag{1.3.2}$$

$$y = cz \tag{1.3.3}$$

where A is the infinitesimal generator of a C_0 semigroup in a Hilbert space Z and $b \in \mathcal{L}(\mathbb{R}, Z)$, $c \in \mathcal{L}(Z, \mathbb{R})$. Here $\mathcal{L}(X, Y)$ denotes the space of bounded operators from X to Y.

We assume that the input u is given, in feedback form, as the output of a harmonic oscillator with frequency α:

$$\dot{w} = Sw, \quad S = \begin{bmatrix} 0 & \alpha \\ -\alpha & 0 \end{bmatrix},$$

$$w(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad u = \Gamma w, \tag{1.3.4}$$

where Γ is a given 1×2 matrix: $\Gamma = [\gamma_1, \gamma_2]$. Thus u represents a periodic function of period $T = 2\pi/\alpha$ as a linear combination of $\sin(\alpha t)$ and $\cos(\alpha t)$, namely,

$$u(t) = \gamma_1 \sin(\alpha t) + \gamma_2 \cos(\alpha t). \tag{1.3.5}$$

Problem 1.3.1. *Suppose that we are given the input u in (1.3.5), find an initial condition z_0 in (1.3.2) so that the output y in (1.3.3) is a nontrivial periodic function of period $T = 2\pi/\alpha$.*

It is well known in finite dimensional linear control theory that if a system is driven by a periodic input for which the complex frequency $i\alpha$ is a transmission zero of the system, then the output of the system is zero for all time. Therefore, we should also state the following more general problem.

Problem 1.3.2. *Find conditions on (A, b, c) guaranteeing there is a nontrivial periodic output with the desired period for all α and arbitrary γ_1, γ_2 with $\gamma_1^2 + \gamma_2^2 \neq 0$, i.e., that the system will support a periodic output of arbitrary period.*

In order that the solution to (1.3.1) be periodic we would at least need that $z(T) = z(0)$. By the variation of parameters formula we have

$$z(t) = e^{At} z_0 + \int_0^t e^{A(t-\tau)} bu(\tau)\, d\tau. \tag{1.3.6}$$

In order that $z(\cdot)$ satisfy $z(T) = z(0) = z_0$ we need

$$\left(I - e^{AT}\right) z_0 = \int_0^T e^{A(T-\tau)} bu(\tau)\, d\tau. \tag{1.3.7}$$

Assumption 1.3.1. *In this paper we will avoid the various technical difficulties and make the assumption that A is a discrete Riesz spectral operator with simple eigenvalues (multiplicity one) $\{\lambda_j\}_{j=1}^{\infty}$ and eigenvectors $\{\psi_j\}_{j=1}^{\infty}$. These eigenvectors form a Riesz basis in Z (i.e., a linear isomorphic image of an orthonormal basis). In this case the adjoint A^* is also a discrete Riesz spectral operator whose eigenvectors $\{\psi_j^*\}_{j=1}^{\infty}$ form a biorthogonal Riesz basis, i.e., $\langle \psi_j, \psi_k^* \rangle = \delta_{jk}$.*

Note that due to our assumption that b and c are bounded rank one operators, we have a well defined transfer function given by $g(s) = c(sI - A)^{-1}b$.

Assumption 1.3.2. *A natural assumption on our system is that the transfer function is real, i.e.,*

$$g(\overline{s}) = \overline{g(s)}. \tag{1.3.8}$$

For systems governed by differential equations with real coefficients this condition is automatic.

Definition 1.3.1. *A complex number s_0 is a transmission zero if $g(s_0) = 0$.*

Assumption 1.3.3. *Our final assumption is that there are no pole zero cancellations. That is, we assume that if s_0 is a transmission zero, then $s_0 \in \rho(A)$, the resolvent set of A.*

Our main results are stated in the following theorem.

Theorem 1.3.1. *Let the operator A in (1.3.1) be a discrete Riesz spectral operator with $\sigma(A) = \{\lambda_j\}_{j=1}^{\infty}$, the input u is given by (1.3.5) with $\gamma_1^2 + \gamma_2^2 \neq 0$ and let (A, b, c) satisfy Assumptions 1.1, 1.2 and 1.3. Then we have the following results.*

1. *There exists an initial condition z_0 so that the solution z to (1.3.1) is periodic with period $T = 2\pi/\alpha$ provided*

   $$\text{distance}(\sigma(A), \{k\alpha i \mid k = 0, \pm 1, \pm 2, \cdots\}) > 0. \tag{1.3.9}$$

   Furthermore, the system supports all positive periods T (i.e., we can find a periodic solution for all possible frequencies α) if

   $$\text{distance}(\sigma(A), \mathbb{C}^0) > 0$$

   where $\mathbb{C}^0 = \{\lambda \in \mathbb{C} : \operatorname{Re} \lambda = 0\}$ denotes the imaginary axis.

2. *In this case, there is a nontrivial periodic output y if and only if $i\alpha$ is not a transmission zero, i.e., $g(i\alpha) \neq 0$.*

3. *Finally, let us denote the amplitude of the periodic input u by*

$$M_u \equiv \sup_{t\in[0,T]} |u(t)| = \sqrt{\gamma_1^2 + \gamma_2^2}.$$

Then the amplitude of the output y is a linear function of the amplitude of the input u. In particular, the output can be written in the forms

$$y(t) = \big[\mathrm{Re}\, g(i\alpha)\big] u(t) + \frac{1}{\alpha}\big[\mathrm{Im}\, g(i\alpha)\big] \frac{du}{dt}(t) \tag{1.3.10}$$

$$= M_u |g(i\alpha)| \big[\tilde{\gamma}_1 \sin(\alpha t) + \tilde{\gamma}_2 \cos(\alpha t)\big]$$

$$= M_u |g(i\alpha)| \sin(\alpha t + \phi) \tag{1.3.11}$$

where $\tilde{\gamma_1}^2 + \tilde{\gamma_2}^2 = 1$ and we can easily write explicit formulas for $\tilde{\gamma}_1$, $\tilde{\gamma}_2$ and ϕ in terms of γ_1, γ_2 and $g(i\alpha)$. Thus the amplitude M_y of y can be written as

$$M_y \equiv \sup_{t\in[0,T]} |y(t)| = M_u |g(i\alpha)|.$$

The proof is based on a functional calculus valid for the discrete Riesz spectral operators considered here. In particular, under the assumptions of the theorem, we can use spectral theory to obtain explicit representations for the initial data z_0 and the solution z. We show that

$$z_0 = -\big(\gamma_1\alpha + \gamma_2 A\big)\big(A^2 + \alpha^2\big)^{-1} b, \tag{1.3.12}$$

$$z(t) = \Big[\sin(\alpha t)\big(-\gamma_1 A + \gamma_2 \alpha\big) + \cos(\alpha t)\big(-\gamma_1 \alpha - \gamma_2 A\big)\Big] (A^2 + \alpha^2)^{-1} b. \tag{1.3.13}$$

Applying c to (1.3.13), using the resolvent identity and the fact that the transfer function is real (cf, (1.3.8)), we can write y as

$$y(t) = \sin(\alpha t)\big[\gamma_1 \,\mathrm{Re}\, g(i\alpha) - \gamma_2 \,\mathrm{Im}\, g(i\alpha)\big]$$

$$+ \cos(\alpha t)\big[\gamma_1 \,\mathrm{Im}\, g(i\alpha) + \gamma_2 \,\mathrm{Re}\, g(i\alpha)\big] \tag{1.3.14}$$

from which a straightforward calculation shows that y is nontrival if and only if

$$|g(i\alpha)|^2 = (\mathrm{Re}\, g(i\alpha))^2 + (\mathrm{Im}\, g(i\alpha))^2 \neq 0,$$

i.e., if and only if $i\alpha$ is not a transmission zero.

Part 3 of the theorem follows from formula (1.3.14). In particular, a direct calculation shows that the applitude M_y of y is related to the amplitude M_u of u by

$$M_y^2 = \big(\gamma_1^2 + \gamma_2^2\big)\big(\mathrm{Re}\, g(i\alpha)^2 + \mathrm{Im}\, g(i\alpha)^2\big)$$

$$= \big(\gamma_1^2 + \gamma_2^2\big)\, |g(i\alpha)|^2 = M_u^2 |g(i\alpha)|^2.$$

1.4 Existence of Numerical Equilibria

Another result obtained during this research effort was concerned with the effect of boundary control on the structure of attractors for Burgers' equation. We were able to provide a rigorous proof that the structure of the attractor is, in general, nontrivial. Indeed, for a special class of forcing terms we are able to show that as certain gain parameters in the boundary feedback control are increased from small positive values to large positive values, the number of stationary solutions vary from three to one. This result supports our conjecture that for large values of the gains, there is a single global asymptotically stable equilibrium. In particular, for zero external forcing term this would say that the resulting closed loop system is asymptotically stable. This conjecture is based on development of the notion of nonlinear zeros as a nonlinear enhancement of the classical concept of rootlocus. In particular, we have given a definition of the zero dynamics associated with the closed loop Bugers' system and shown that this zero dynamics systems has a single, global, asymptotically stable, equilibrium. One would therefore expect that as the gains are increased, trajectories of the closed loop system would approach the corresponding trajectories of the zero dynamics.

During this period, in joint work with Dr. John Burns (VT & State University), we also made an interesting discovery concerning the long time behavior of solutions to Burgers' equation on a finite interval with Neumann boundary conditions. It is easy to see that, for this problem, constants are equilibria, and for the related linearization about zero – the one dimensional heat equation with Neumann boundary conditions – it is well known that the steady state temperature is a constant. Namely, the steady state temperature distribution is the mean value of the initial temperature distribution. For the Burgers' equation with small initial data this same type of result holds as a consequence of the Center Manifold Theorem. That is, Burgers' equation with Neumann boundary conditions possesses a one dimensional center manifold (constants) and it can be shown that for a small initial conditions solutions converges exponentially to constant values. As we have shown in our earlier work, in contrast to the heat equation, the steady state constant is not simply the mean of the initial condition, but it also depends in some complicated way on both the viscosity parameter and the shape of the initial condition.

Since the Center Manifold Theorem is only a local result, a natural question is whether, for arbitrary initial data, the corresponding solution of Burgers' equation tends to a constant steady state. The answer to this question is still unresolved. Nevertheless, we were able to resolve in the affirmative an intermediate question. If, for a given initial condition, the solution approaches a time independent steady state, even in the L sense, then this steady state must be a constant.

In spite of this result, after considerable numerical testing, it was discovered by researchers at Virginia Tech University, that for moderately small viscosity and larger "antisymmetric" initial conditions, numerical solutions can approach a <u>nonconstant</u>, time independent steady state. We could only conclude that this is simply a numerical anomaly. Nevertheless, due to the relevance of hydrodynamic problems in applications, it was important to understand how these nonconstant numerical stationary solutions arise. Our discovery was that these numerical solutions, for a fixed mesh size or degree of approximation, approach explicit solutions of the equation that satisfy the boundary conditions only to within values that are approximately machine precision zero (or smaller). In particular, these numerical stationary solutions are given in terms of hyperbolic tangent functions whose derivatives at the end points behave like hyperbolic secant squared. For moderate initial conditions these derivatives are on the order of 10^{-30} which are well below machine precision zero.

This work sheds important light on the use of numerical proofs. In particular, numerical calculations have been used to "suggest" that Euler equations do not have unique solutions. The justification for this claim is that a "very fine mesh" is used in the calculation. The conclusion of our work is that "numerical based" proofs of non-existence must be done with extreme care.

Burgers' equation on the interval $(0,1)$ subject to Neumann Boundary Condition is given by the dynamical system

$$\begin{aligned} &w_t - \epsilon w_{xx} + w w_x = 0, \\ &x \in (0,1), \quad t > 0 \\ &w_x(0,t) = w_x(1,t) = 0 \\ &w(x,0) = \phi(x), \end{aligned} \tag{1.4.1}$$

The associated stationary Burgers' problem is

$$\begin{aligned} &-\epsilon v_{xx} + v v_x = 0, \\ &v_x(0) = v_x(1) = 0. \end{aligned} \tag{1.4.2}$$

It is not clear without further information that solutions of (1.4.1) should even exist for all time. The answer to this question is contained in the recent work [119] which, for the special case of (1.4.1), gives the following result.

Theorem 1.4.1. *[119] For arbitrary initial data $\varphi \in L^2(0,1)$ and $0 < T < \infty$,*

a) (1.4.1) has a unique weak solution

$$w \in L^\infty([0,T], L^2(0,1)) \cap L^2([0,T], H^1(0,1)),$$

b) On any cylinder $[0,1] \times [t_0, T]$ for any $0 < t_0 < T < \infty$, $w \in H^{2,1}([0,1] \times [t_0, T])$.

c) There is a globally defined dynamical system on the state space $L^2(0,1)$ given in terms of a nonlinear semigroup $\{T_t, t \geq 0\}$.

* *T_t is continuous in t and $\varphi \in L^2(0,1)$.*
* *T_t is compact for $t > 0$.*
* *The system is globally Lyapunov stable.*
* *There is a global, locally compact attractor.*

Even though we cannot, at this time, say that solutions of (1.4.1) approach a constant steady state, we prove the following intermediate result.

Theorem 1.4.2. *Fix $\epsilon > 0$ and $\phi \in L^2(0,1)$. Let $w(\cdot,t)$ be a weak solution of (1.4.1). If there is a function $h \in L^2(0,1)$ such that*

$$\lim_{t\to\infty} \|w(\cdot,t) - h(\cdot)\|_{L^2(0,1)} \to t \to \infty 0$$

Then $h(\cdot) = c_{\phi,\epsilon}$ for some constant $c_{\phi,\epsilon}$.

For fixed ϵ and for small initial data numerical approximation of the solutions to (1.4.1) supports the conclusion of the Center Manifold Theorem, namely, solutions tend to a constant as t tends to infinity. But for small ϵ and "certain" initial data (not to small), the numerical solution converges to a nonconstant function, cf. [94]. We are lead to conjecture the existence of some type of *Numerical Stationary Solutions* for the problem (1.4.1).

One class of initial data for which we obtain this anomaly are functions in the class $\mathcal{S}$ consisting of "antisymmetric" functions, that is,

$$\mathcal{S} = \left\{\phi \in L^2(0,1) : \phi(x) = -\phi(1-x)\right\}. \tag{1.4.3}$$

For initial data $\phi \in \mathcal{S}$, a straightforward consequence of Theorem 2.1, is that $w(\cdot,t) \in \mathcal{S}$ for all t and hence $w(1/2,t) = 0$ for all $t > 0$. Thus if $\lim_{t\to 0} w(x,t) = c_{\phi,\epsilon}$ exists then the constant $c_{\phi,\epsilon}$ must be zero.

The nonconstant solutions in $\mathcal{S}$ of the stationary Burgers' equation (not the boundary conditions) are given in terms of a one parameter family depending on the parameter c_0.

$$h(x) = \sqrt{2c_0}\tanh\left(\frac{\sqrt{2c_0}}{2\epsilon}(1/2 - x)\right),\ h \in \mathcal{S}. \tag{1.4.4}$$

It is easy to see that for suitable initial data and c_0 the functions (1.4.4) are actually *numerical stationary solutions* to (1.4.1), i.e., they satisfy the Burgers' equation and they approximately satisfy the boundary conditions.

Namely, the functions in (1.4.4) satisfy (1.4.2) and the boundary conditions

$$h'(0) = h'(1) = -\frac{c_0}{\epsilon}\operatorname{sech}^2\left(\frac{\sqrt{2c_0}}{4\epsilon}\right) = -\gamma, \tag{1.4.5}$$

where γ is an exponentially small positive number.

There is no reason to believe that numerical solutions to Burgers' equation should approach a function of the type (1.4.4), especially in light of Theorem 2.2 which suggests they should approach a constant. Nevertheless, this does happen for larger initial data and/or smaller ϵ.

For a small positive number γ there are exactly two solutions (1.4.4) satisfying (1.4.5), i.e., there exist $c_0^1 \approx 0$ and $c_0^2 \gg 0$ giving

$$h^j(x) = \sqrt{2c_0^j}\tanh\left(\frac{\sqrt{2c_0^j}}{2\epsilon}(1/2 - x)\right) \tag{1.4.6}$$

$$\frac{c_0^j}{\epsilon}\,\mathrm{sech}^2\left(\frac{\sqrt{2c_0^j}}{4\epsilon}\right) = \gamma. \tag{1.4.7}$$

The solution h^1 is very nearly the zero function, whereas the solution h^2 is not usually small.

2 References

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3 Participating Professionals

1. Principal Investigators
   - Christopher I. Byrnes
   - Alberto Isidori
2. Senior Personnel
   - David S. Gilliam

4 Scientific Publications

a. *Peer Reviewed Journal:*
Name of Journal: Automatica
Title of Article: "Passivity and Absolute Stabilization of a Class of Discrete-Time Nonlinear Systems"
Author: W. Lin and C.I. Byrnes
Publisher: Elsevier Science Ltd
Volume: 31, Page: 263-268, Year Published: 1995.

Name of Journal: Automatica
Title of Article: "Zero-State Observability and Stability of Discrete-Time Nonlinear Systems"
Author: W. Lin and C.I. Byrnes
Publisher: Elsevier Science Ltd
Volume: 31, Page: 269-274, Year Published: 1995.

Name of Journal: Automatica
Title of Article: "Discrete-Time Nonlinear H-infinity Control with Measurement Feedback"

Author: W. Lin and C.I. Byrnes
Publisher: Elsevier Science Ltd
Volume: 31, Page: 419-434, Year Published: 1995.

Name of Journal: Systems and Control Letters
Title of Article: "Remarks on Linearization of Discrete-Time Autonomous Systems and Nonlinear Observer Design"
Author: W. Lin and C.I. Byrnes
Publisher: Elsevier Science BV
Volume: 25, Page: 31-40, Year Published: 1995.

Name of Journal: IEEE Trans. Automat. Contr.
Title of Article: "H-infinity Control of Discrete-Time Nonlinear Systems"
Author: W. Lin and C.I. Byrnes
Publisher: IEEE
Volume: 41, no. 4 Page: 494–510, Year Published: 1996.

Name of Journal: Journal of Mathematical Systems, Estimation and Control
Title of Article: " H-infinity Control of Nonlinear Systems with Sampled Measurements"
Author(s): S.Suzuki, A.Isidori and T.J.Tarn
Publisher: Birkhauser
Volume: 5 , Page(s): 259-262,
Month Published: March, Year Published:1995

Name of Journal: IEEE Transactions on Automatic Control
Title of Article: "H-infinity control of discrete time nonlinear control systems via state and full information feedback"
Author: C.I.Byrnes and W. Lin
Publisher: IEEE Volume: , Page:
Year Published: 1996 (to appear).

Name of Journal: IEEE Transactions on Automatic Control
Title of Article: "Robust regulation for nonlinear systems with gain-bounded uncertainties"
Author: A.Isidori and T.J.Tarn
Publisher: IEEE
Volume: 40, Page: 1744-1754, Year Published: 1995.

Name of Journal: Automatica
Title of Article: "Structurally Stable Output Regulation of Nonlinear Systems"
Author: C.I.Byrnes, F. Delli Priscoli, A. Isidori and W.Kang
Publisher: Pergamon Press
Volume: 33 no. 3 Page: 369–385, Year Published: (1997).

Name of Journal: IEEE Control Systems Magazine
Title of Article: "Force regulation and contact transition control"
Author: T.J.Tarn, Y. Wu, N.Xi and A.Isidori
Publisher: IEEE
Volume: 16, Page: 32-40, Year Published: 1996.

Name of Journal: Systems and Control Letters
Title of Article: "A note on almost disturbance decoupling for nonlinear minimum phase systems"
Author: A.Isidori
Publisher: Elsevier Science BV Volume: 27, Page: 191-194,
Year Published: 1996.

Name of Journal: IEEE Trans. Automat. Contr.
Title of Article: "H-infinity Control of Discrete-Time Nonlinear Systems"
Author: W. Lin and C. I. Byrnes
Publisher: IEEE
Volume: 40, Page: 494-510, Year Published: 1996.

Name of Journal: Differential and Integral Equations
Title of Article: "Global Solvability for Damped Abstract Nonlinear Hyperbolic Systems"
Author: H.T. Banks, D.S. Gilliam, V.I. Shubov
Publisher:
Volume: , Page: , Year Published: 1996.

Name of Journal: Journal of Mathematical Systems, Estimation and Control
Title of Article: "High Gain Limits of Trajectories and Attractors for a Boundary Controlled Viscous Burgers' Equation"
Author: C.I. Byrnes, D.S. Gilliam, V.I. Shubov
Publisher:
Volume: , Page: , Year Published: 1996

Name of Journal: Journal of Dynamical and Control Systems
Title of Article: "On the Global Dynamics of a Controlled Viscous Burgers' Equation"
Author: C.I. Byrnes, D.S. Gilliam, V.I. Shubov
Publisher:
Volume: , Page: , Year Published: 1997.

Name of Journal: Automatica
Title of Article: "Structurally stable output regulation of nonlinear systems"
Author: C.I.Byrnes, F.Delli Priscoli, A. Isidori, W.Kang
Publisher: Elsevier Science BV

Volume: 33
Page: 369-385
Year Published: 1997.

Name of Journal: IEEE Trans. on Automatic Control
Title of Article: "A remark on the problem of semiglobal nonlinear output regulation"
Author: A. Isidori
Publisher: IEEE Press
Volume: 43
Page: to appear
Year Published: 1997.

Name of Journal: IEEE Trans. on Automatic Control
Title of Article: "On the partial stochastic realization problem"
Author: C.I. Byrnes and A. Lindquist
Publisher: IEEE Press
Volume: 42, No. 8
Page: 1049 - 1070
Year Published: 1997.

Name of Journal: SIAM J. Cont. and Opt.
Title of Article: "A convex optimization approach to the rational covariance extension problem"
Author: C.I. Byrnes, S.V. Gusev, and A. Lindquist
Publisher: SIAM Press
Status: To appear

Name of Journal: *Journal of Mathematical Systems, Estimation and Control*
Title of Article: "Stationary solutions for a boundary controlled viscous Burgers' equation"
Author: A. Balogh, D.S. Gilliam and V.I. Shubov
Status: Submitted

Name of Journal: *Journal of Mathematical Systems, Estimation and Control*
Title of Article: "Numerical stationary solutions for a viscous Burgers' equation"
Author: A. Balogh, J.A. Burns, D.S. Gilliam and V.I. Shubov
Publisher: Birkhäuser
Accepted: December 1996.

Name of Journal: *Journal of Mathematical Systems, Estimation and Control*
Title of Article: "Harmonic forcing for linear distributed parameter systems"
Author: C.I. Byrnes, D.S. Gilliam, I. Lauko and V.I. Shubov
Publisher: Birkhäuser

Accepted: December 1996.

Name of Journal: *Journal of Mathematical Systems, Estimation and Control*
Title of Article: "On the Riccati Partial Differential Equation for Nonlinear Bolza and Lagrange Problems"
Author: C.I. Byrnes
Publisher: Birkhäuser
Accepted: May 1997.

b. *Peer Reviewed Conference Proceedings:*

Name of Conference: IEEE Conference on Decision and Control
Title of Article: "High Gain Limit for Boundary Controlled Convective Reaction Diffusion Equations"
Author: C.I. Byrnes, D.S. Gilliam, N. Okasha, V.I. Shubov
Publisher: IEEE
Volume: 34th CDC, Page: , Year Published: 1995.

Name of Conference: IEEE Conference on Decision and Control
Title of Article: "Well-Posedness for Controlled Nonlinear Damped Membranes with Fixed Boundary"
Author: H.T. Banks, D.S. Gilliam, V.I. Shubov
Publisher: IEEE
Volume: 34th CDC, Page: , Year Published: 1995.

Name of Conference: 1995 American Control Conference
Title of Article: "Some New Results on Stability and Observability of Discrete-time Autonomous Systems"
Author(s): W. Lin and C.I. Byrnes
Publisher: IEEE
Volume: 6, Page(s): 4214-4218, Month Published: June,
Year Published:1995

Name of Conference: IFAC Nonlinear Control Systems Symposium
Title of Article: "On the Dynamics of Boundary Controlled Nonlinear Distributed Parameter Systems"
Author(s): C.I. Byrnes, D.S. Gilliam and Victor I. Shubov Publisher: Pergamon
Volume: 62 Page(s): 913-913, Month Published: June,
Year Published:1995

Name of Conference: 34th IEEE Conference on Decision and Control
Title of Article: "Well-Posedness for Controlled Nonlinear Damped Membranes with Fixed Boundary "

Author(s): H.T. Banks, D.S. Gilliam and Victor I. Shubov
Publisher: IEEE
Volume: Page(s): xxx-xxx, Month Published: December,
Year Published:1995

Name of Conference: 35rd IEEE Conf. Decision and Control (Kobe, Japan, December 1996)
Title of Article: "Global normal forms for MIMO nonlinear systems, with application to stabilization and disturbance attenuation"
Author: B.Schwartz, A. Isidori and T.J. Tarn
Page: 1041-1046
Year Published: 1996.

Name of Conference: 35rd IEEE Conf. Decision and Control (Kobe, Japan, December 1996)
Title of Article: "L_2 disturbance attenuation and performance bounds for linear non-minimum phase square invertible systems"
Author: B.Schwartz, A. Isidori and T.J. Tarn
Page: 227-228
Year Published: 1996.

Name of Conference: 4th European Control Conference (Brussels, Belgium, July 1997)
Title of Article: "Performance bounds for disturbance attenuation in nonlinear non-minimum-phase systems "
Author: B.Schwartz, A. Isidori and T.J. Tarn
Page: to appear
Year Published: 1997.

Name of Conference: 36th IEEE Conf. Decision and Control (San Diego, CA, December 1997)
Title of Article: "Output Regulation for Parabolic Distributed Parameter Systems: Set Point Control"
Author: C.I. Byrnes, D.S. Gilliam, I. Laukó and V.I. Shubov
Page: to appear
Year Published: 1997.

Name of Conference: 36th IEEE Conf. Decision and Control (San Diego, CA, December 1997)
Title of Article: "Global L_2-gain State Feedback Design for a Class of Nonlinear Systems"
Author: A.Isidori and W.Lin
Page: to appear
Year Published: 1997.

Books:
Title of Book: "Output Regulation of Uncertain Nonlinear Systems"
Author of Book: C.I.Byrnes, F.Delli Priscoli, A. Isidori Publisher: Birkhäuser (Boston)
Pages: 1-120
Year Published: 1997.

Title of Book: "Systems and Control in the Twenty-First Century"
Editors of Book: C.I.Byrnes, B.N. Datta, D.S. Gilliam, C.F. Martin
Publisher: Birkhäuser (Boston)
Pages: 1-434
Year Published: 1997.

Book Chapters:
Title of Book: "Colloquim on Automatic Control"
Editor of Book: C.Bonivento, G.Marri, G.Zanasi
Title of Chapter: "Semiglobal robust regulation of nonlinear systems",
Author of Chapter: A.Isidori
Publisher: Springer Verlag
Pages: 31-59, Year Published: 1996.

Title of Book: "Systems and Control in the Twenty-First Century"
Editor of Book: C.I.Byrnes, B.N. Datta, D.S. Gilliam, C.F. Martin
Author of Chapter: C.I. Byrnes and A. Lindquist
Title of Chapter: "On duality between filtering and interpolation"
Pages: 101-136 Publisher: Birkhäuser (Boston)

Title of Book: "Current and Future Directions in Applied Mathematics"
Editor of Book: M.Alber, B. Hu, J. Rosenthal
Author of Chapter: C.I. Byrnes, H.J. Landau, and A. Lindquist
Title of Chapter: "On the well-posedness of the rational covariance extension problem"
Pages: 83-108
Publisher: Birkhäuser (Boston)

5 Scientific Interactions

In addition to collaborative research with engineering research and development personnel at McDonnell-Douglas Aircraft Co., St. Louis, MO, and scientific interaction with AFOSR personnel at Bolling AFB, Ft. Eglin AFB and Wright Patterson AFB, we have presented many inivited lectures and colloquia nationally and internationally:

September 1994
"Feedback Stabilization About Attractors and Inertial Manifolds," Invited lecture at Royal Institute of Technology, Stockholm, lecture presented by Professor C.I. Byrnes

February 1995:
"Feedback Stabilization About Attractors and Inertial Manifolds" Universita di Roma - La Sapienza Presented by Professor C. I. Byrnes

June 1995:
"Nonlinear Control Systems," AFOSR Contractors Review Meeting Minneapolis-St. Paul, Presented by Professor C.I. Byrnes

"A Riccati Equation for Partial Stochastic Realization Theory" Royal Institute of Technology Stockholm, Sweden Presented by Professor C.I. Byrnes

"On the Dynamics of Boundary Controlled Nonlinear Distributed Parameter Systems" IFAC Nonlinear Control Systems Symposium, Tahoe City, Presented by Professor D. S. Gilliam

October 1995:
"Robust Regulation of Nonlinear Systems," Invited talk presented by Prof. C.I. Byrnes at VPI & State University.

"Robust Regulation of Nonlinear Systems," Plenary talk presented by Prof. C.I. Byrnes at NCSU Southeast Regional Differential Equations Conference.

"On the Dynamics of Boundary Controlled Convective Reaction Diffusion Equations," Invited talk presented by D.S. Gilliam in the minisymposium "Control and Computatuional Fluid Dynamics" at the 1995 SIAM Annual Meeting at Charlotte, North Carolina.

"Electromagnetic Waves Over a Three Dimensional Conducting Half Space Invited lecture presented by D.S. Gilliam in the Center For Research in Scientific Computation at North Carolina State University.

November 1995:
"Robust Regulation of Nonlinear Systems," Invited talk presented by Prof. C.I. Byrnes in the Departemnt of Electrical Engineering at the University of Hong Kong, Hong Kong.

"Output Regulation Revisited" Distinguished lectures series, Department of Electrical Engineering, IOWA State University, presented by Dr. Alberto Isidori.

December 1995:
"Disturbance Attenuation for a Class of Nonlinear Non-Minumum Phase Systems," Invited lecture presented by Prof. Alberto Isidori at the 34th Control and Decision Conference.

"High Gain Limit for Boundary Controlled Convective Reaction Diffusion Equations" lecture presented by D.S. Gilliam at the 34th Control and Decision Conference.

"Well-Posedness for Controlled Nonlinear Damped Membranes with Fixed Boundary" lecture presented by D.S. Gilliam at the 34th Control and Decision Conference.

January 1996:
"The Regulator Problem for Linear Distributed Parameter Systems," Invited lecture presented by D.S. Gilliam in the Department of Systems Science and Math at Washington University in St. Louis.

March 1996:
"Complete Parameterization of Solutions to the Rational Covariance Problem," Invited lecture presented by C.I. Byrnes at Ecole Normale Superieure, Paris.

April 1996:
"Complete Parameterization of Solutions to the Rational Covariance Problem," Plenary lecture presented by C.I. Byrnes at Conference on the Future Directions in Applied Mathematics at Notre Dame.

June 1996:
"The Structure of Attractors for a Boundary Controlled Viscous Burgers' Equation," Invited talk presented by Andras Balogh at the 96 Mathematical Theory of Networks and Systems (MTNS) Conference, St. Louis, MO.

"Recent Results on Boundary Control for a Class of Higher Dimensional Convection-Reaction Diffusion Equations," Invited talk presented by David S. Gilliam at the 96 Mathematical Theory of Networks and Systems (MTNS) Conference, St. Louis, MO.

"Robust Nonlinear Output Regulation," Invited talk presented by Prof. Alberto Isidori at the 96 Mathematical Theory of Networks and Systems (MTNS) Conference, St. Louis, MO.

August 1996:
"On the Rational Caratheodory Extension Problem," Invited lextrue presented by Prof. C.I. Byrnes at the Fifth Bozeman Conference on Computation and Control.

"On the Dynamics and Control of a Class of Higher Dimensional Convection Reaction Diffusion Equations," Invited lecture presented by David Gilliam at the Fifth Bozeman Conference on Computation and Control.

"Numerical Stationary Solutions for Burgers' Equation with Neumann Boundary Conditions," Invited lecture presented by David Gilliam at the Fifth Bozeman Conference on Computation and Control.

October 1996:
"Zeros and Zero Dynamics for Linear Distributed Parameter Systems," A series of lectures given by Dr. David Gilliam in the Texas Tech University Joint Civil Engineering, Mathematics and Mechanical Engineering Seminar.

December 1996:
"Global normal forms for MIMO nonlinear systems, with application to stabilization and disturbance attenuation" Lecture presented by Dr. Alberto Isidori at the 35rd IEEE Conf. Decision and Control (Kobe, Japan).

"L_2 disturbance attenuation and performance bounds for linear non-minimum phase square invertible systems" Lecture presented by Dr. Alberto Isidori at the 35rd IEEE Conf. Decision and Control (Kobe, Japan).

"Immersions and the internal model principle: tools for robust nonlinear control" Lecture presented by Dr. Christopher I. Byrnes at the 35rd IEEE Conf. Decision and Control (Kobe, Japan).

"The rational covariance extension problem with applications to speech synthesis" Invited lecture presented by Dr. Christopher I. Byrnes at The University of Tokyo.

"Robust nonlinear control" Invited lecture presented by Dr. Christopher I. Byrnes at Dewan Riset Nasional, Serpong, Indonesia.

January 1997:
"Global L_2-Gain State Feedback Design for a Class of Nonlinear Systems" Lecture presented by Dr. Alberto Isidori at COSY Workshop on Nonlinear Systems held at the ETH-Zurich, Switzerland.

February 1997:
"The geometry of positive real functions with applications to signal processing and to speech synthesis" Invited lecture presented by Dr. Christopher I. Byrnes at The Texas Tech University.

March 1997:
"The rational covariance extension problem with applications to speech synthesis" Invited lecture presented by Dr. Christopher I. Byrnes at Texas Tech University.

"Robust Output Regulation of Nonlinear Systems" Invited lecture presented by Dr. Alberto Isidori at the University of Aalborg, Denmark.

April 1997:
"On the Global Dynamics of a Class Boundary Controlled Hydrodynamic Systems in Higher Dimensions," Lecture presented by Dr. Victor I. Shubov at the Texas Partial Differential Equations Conference in Denton, TX.

May 1997:
"Nonlinear control systems" Invited lecture presented by Dr. Christopher I. Byrnes at the 1997 AFOSR Contractors Review Meeting, Wright Patterson AFB, Dayton, OH.

June 1997:
"On the covariance extension problem" 5th IEEE Mediterranian Conference, Paphos, Cyprus Invited lecture presented by Professor C. I. Byrnes

"Robust Output Regulation of Nonlinear Systems" Invited lecture presented by Dr. Alberto Isidori at the Imperial College, London, UK.

"Robust Nonlinear Control" Invited lecture presented by Dr. Christopher I. Byrnes at the 28th AIAA Fluid Dynamics Conference/ 4th AIAA Shear Flow Control Conference, Snowmass Village, CO

July 1997:
Dr. David S. Gilliam participated in 12th International Conference of the American Meterological Society on Boundary Layers and Turbulence, Vancouver, BC.

"On Duality between Filtering and Interpolation" University of Tokyo, Japan Invited lecture presented by Professor Anders Lindquist.