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Forecasting Market Index Performance

Using Population Demographics

THESIS

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FORECASTING MARKET INDEX PERFORMANCE USING POPULATION DEMOGRAPHICS

THESIS

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Abstract

This study attempted to verify claims made by forecaster Harry S. Dent, Jr. It utilized regression analysis in order to determine the correlation between the number of births and the closings on a market index with a specified lag between the input and output variables. While the research was able to develop a model with the factor Dent considers the most important, the predictions based on this model did not completely coincide with the forecasts Dent makes. Furthermore, the research could not confirm the accentuation Dent places on a 46-year lag between predictor and response variables. As an extension, scenarios for Japan, the United Kingdom, and Germany were examined. This analysis could not confirm the hypothesized extension of Dent's theory to other countries.

FORECASTING MARKET INDEX PERFORMANCE USING POPULATION DEMOGRAPHICS

I. Introduction

1.1 Background

In any game where money is involved, all involved parties look for some means to gain an advantage. The investment game is no different. In today's world, thousands of mutual funds fight for the resources necessary to thrive as a business. In order to maintain satisfaction among his customers, the fund manager needs a method to provide adequate returns on their capital. While extensive research about both specific industries and companies within that industry may prove successful, many managers develop models to support their market-timing strategies. Ideally, the goal of market-timing is to realize all the gains in a market without absorbing the losses. The makeup of a portfolio usually mimics a market index for comparative purposes. Managers publicize how well their strategies stack up against a passive strategy, one which remains fully invested all the time.

While many forecasters willingly broadcast the results of their strategies, they like to keep the variables included in their model a secret. On the other extreme, Harry S. Dent, Jr. has announced openly what factors he uses in his predictions, and he makes no apology for its simplicity (5). Dent believes market performance is solely determined by consumption, so he concentrates on identifying periods when consumption should increase. Based on this principal, he pronounces the leading predictor variable for market performance is the number of 46-year old people in a society.

Throughout his writings, he continues to expound upon this initial declaration, providing further evidence as support to his theory. However, the claims made in his book, *The Great Boom Ahead*, are not mathematically justified. In fact, this lack of computational support provided the basis for this research.

1.2 Research Objectives

1.2.1 Construct a Model to Capture Population Demographic Theory. With only one predictor variable, the theory ascribed by Dent can be analyzed by regression-based modeling techniques. In an attempt to accurately replicate Dent's model, the model does not include current population estimates of the number of people in a specific age group. Instead, the input variable is the number of births in any given year, and the response is an inflation-adjusted Dow Jones Industrial Average (DJIA). Initially, the goal of the regression analysis is to determine the validity of Dent's assessment; simultaneously, the analysis provides a means of evaluating other possible age groups as predictor variables for the model. After constructing an appropriate model, it can be used to critique the conclusions Dent arrives at through his work.

1.2.2 Apply Modeling Techniques to Other Markets. Although Dent alludes to the fact that the same technique can be used for foreign countries, he only graphically illustrates the example of the United States with respect to its two major indexes, the DJIA and the Standard & Poor's (S&P) 500. This hypothesis leads to the next goal of this research, testing his hypothesis for other countries. After the case of the United States is addressed, scenarios for other leading economic countries, the United Kingdom (UK), Japan, and Germany, are examined. In the same manner as the United States, the analysis can lead to predictions about long-term trends in each country's market. Hopefully, as a result, one distinguishable predictor variable presents itself as the most appropriate across the board. However, with cultural differences, every country is not likely to have identical age groups as the defining factor for their markets. A key factor in Dent's theory is the "Baby Boom" resulting from World War II. This cultural effect varied from nation to nation and from victors to vanquished. The objective, though, is to apply the method in other instances to evaluate the overall effectiveness of the theory. After completion of this thesis, recommendations will be made about what other paths might provide additional insight.

1.3 Summary

Chapter 2 provides a discussion of background material relevant to the research issues. It outlines the process for building regression-based models and documents the basis of Dent's argument. Chapter 3 describes the research methodology used both to prepare the data for analysis and to estimate the regression model for the United States. The chapter documents the steps used to build the model and comments on the existence of other equally feasible models. Chapter 4 examines each country on a case by case scenario. It includes the results from the model generated in Chapter 3 along with the regression models created for the other countries. It provides graphical representation of the estimated models compared to actual index performances. Finally, Chapter 5 summarizes the research, presents conclusions, and makes recommendations for further study in this area.

II. Literature Review

The primary research objective of this thesis is to build a regression-based model which captures the economic theory of Harry S. Dent, Jr. With this in mind, this chapter provides a description of linear regression followed by the background Dent uses to support his theory.

2.1 Linear Models and Linear Regression.

2.1.1 Simple Linear Regression Models. A statistical model is a model which utilizes mathematical techniques to relate system output to a set of input variables. In regression analysis, a statistical relation is created between input variables, referred to as predictors, and the output, called the response. In a statistical relation, as opposed to a functional relation, the observations do not necessarily correspond to points along a function or curve used to describe the relationship. This deviation from the curve is variation which cannot be explained by the predictor variables. Such variation is typically considered random in nature. Under observation, it is possible for identical sets of predictor variables to coincide with different responses. A simple linear regression model can be stated as in Equation 1 (12:10):

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i \qquad (i = 1,2,\ldots,n) \tag{1}$$

where

Y_i = value of the response variable in the *i*th trial

β_0 , β_1 = model coefficients, or parameters

X_i = value of the predictor variable in the *i*th trial

ε_i = model error term in the *i*th trial, representing the residual of the system data points from the underlying model

n = the number of trials

A linear regression model with only one predictor variable is said to be a simple regression model. In order for a model to be linear, it must be linear with respect to its parameters. Notice, in Equation 1, no parameters are multiplied with another parameter or appear as an exponent. Notice also that no predictor variables are multiplied by another predictor or appear as an exponent, so the model is also deemed "linear in the predictor variable". A model with these linear characteristics is called a first-order model (12:10).

From Equation 1, the meaning of the parameters β_0 and β_1 needs to be clearly identified. β_1, the slope of the regression line, represents how a unit change in X will affect the mean of the probability distribution about Y. The value of β_0 only has meaning when the scope of the problem covers X = 0. As discussed by Neter, Kutner, Nachtsheim, and Wasserman, scope refers to the interval of coverage for the predictor variables used in the model. They suggest using the range of the data. As the Y-intercept, β_0 identifies the mean of the probability distribution at X = 0. This meaning is lost for problems which do not include X = 0 in its scope (12:8-12).

The classical assumptions for regression analysis are (15:95):

1.) The model is linear with respect to the coefficients and error terms.

2.) The mean of the error term is zero.

3.) The predictor variables and error term are uncorrelated.

4.) The covariance between error terms for each observation is zero.

5.) The variance of the error terms is constant.

6.) The predictor variables are linearly independent of one another.

7.) The error terms are distributed normally.

The remainder of this section discusses both the conclusions based on these assumptions and the ramifications when they are violated.

2.1.2 The Method of Least Squares. The method of least squares is one technique used to estimate the parameters, β_0 and β_1. As previously stated, data points may not fall directly on the regression. According to the classical model, the error, the distance an observation lies from the regression line or the amount unexplained by the predictors, is assumed to be an independent random variable with a mean of zero. It follows that the expected value of the response (denoted E(Y)) will equal $\beta_0 + \beta_1 X$. For each observation, (X_i, Y_i), an error term , ε_i exists such that (12:17):

$$\varepsilon_i = Y_i - (\beta_0 + \beta_1 X_i) \tag{2}$$

Least squares minimizes the sum of the squared deviations for the *n* observations. An analytical approach applying calculus techniques can demonstrate the point estimators, b_0 and b_1, minimize this sum by solving the simultaneous equations below (12:19).

$$\Sigma Y_i = nb_0 + b_1 \Sigma X_i \tag{3}$$

$$\Sigma X_i Y_i = b_0 \Sigma X_i + b_1 \Sigma X_i^2 \tag{4}$$

If the assumptions stated in the previous section hold, the Gauss - Markov Theorem asserts "the least squares estimators, b_0 and b_1, are unbiased and of all unbiased linear estimators have the minimum variance" (12:20). Thus, Equation 5 estimates the regression function and can be used as a means of predicting a response, $\hat{Y}$, for given levels of the predictors (12:23).

$$\hat{Y} = b_0 + b_1 X \tag{5}$$

2.1.3 Significance Tests. After obtaining estimates for the β's, it is important to test whether each β_k is significantly different from zero; basically, the regression coefficient has a statistically significant effect on the model. For each β_k , the null and alternate hypotheses are formulated.

$$H_0 : \beta_k = 0$$

$$H_a : \beta_k \neq 0$$

As previously mentioned, each ε is assumed to be an independent random variable with a mean of zero. With the added assumptions that each ε has a normal distribution with a constant variance σ^2

($\varepsilon \sim N(0,\sigma^2)$), Neter, Kutner, Nachtsheim, and Wasserman prove both Y and the b's also follow normal distributions (12:45-48). Therefore, inferences can be made concerning the β's with a t-distribution. Namely, the test statistic, $t_0 = b_k / s(b_k)$, is computed for comparison to a table value (t^*), where $s(b_k)$ is the estimated standard error of β_k. The test statistic is applied to the following decision rule.

$$\text{If } |t_0| \leq t^*_{(1-\alpha/2;\, n-2)}, \text{ accept } H_0$$

$$\text{If } |t_0| > t^*_{(1-\alpha/2;\, n-2)}, \text{ reject } H_0$$

Above, α represents a predetermined Type I error, the probability H_0 is rejected when it is true. In addition, n is the number of sample points used for the regression analysis. The degrees of freedom (df) is n-2 in this case since two parameters need to be estimated in the simple linear regression model (12:49). Degrees of freedom, in general, refers to the number of opportunities a variable is free to vary for a given set of data (12:72).

The regression results can also be tested through the analysis of variance. First, several equations need to be introduced (12:70-72).

$$SSTO = \Sigma (Y_i - \bar{Y})^2 \tag{6}$$

$$SSE = \Sigma (Y_i - \hat{Y}_i)^2 \tag{7}$$

$$SSR = \Sigma (\hat{Y}_i - \bar{Y})^2 \tag{8}$$

where

SSTO	=	total sum of squares
SSE	=	sum of squares for error
SSR	=	sum of squares for regression
i	=	observation number
n	=	total number of observations
Y_i	=	the response for ith observation
$\hat{Y}_i$	=	the estimated response for ith observation
$\bar{Y}$	=	the average response

After computing the sums of squares, mean squares can be calculated. The mean squares for regression (MSR) and the mean squares for error (MSE) are computed by dividing the associated sums of squares by their corresponding *df*. With the equation for SSTO, it is easier to demonstrate the concept of *df*. Although there are *n* observations in the sample, the sum of the deviations from the average must, by definition, sum to zero. Therefore, *n*-1 observations are free to vary, but the last observation will be known. For SSE, there are *n*-2 *df* since two parameters are estimated. Each estimated parameter results in the loss of one degree of freedom. SSR, in the simple linear regression model, has 1 *df*. So, the equations for MSR and MSE are as follows:

$$MSR = SSR \tag{9}$$

$$MSE = SSE / (n-2) \tag{10}$$

Having computed the mean squares, another significance test can be conducted. Neter, Kutner, Nachtsheim, and Wasserman show that $F_0 = MSR / MSE$ follows an F-distribution when $\beta_1 = 0$ (12:76-77). Similar to the *t*-test, the null and alternative hypotheses for the test are:

$$H_0 : \beta_1 = 0$$

$$H_a : \beta_1 \neq 0$$

The decision rule for this test with the risk of Type I error equal to α is:

$$F_0 \leq F^*(1\text{-a}; 1, n\text{-}2), \text{ accept } H_0$$

$$F_0 > F^*(1\text{-a}; 1, n\text{-}2), \text{ reject } H_0$$

Both tests attempt to accomplish the same goal, verifying a hypothesized relationship between the predictor variables and the response. If H_0 is accepted, the input variable cannot significantly explain the observed deviations of the response variable from its mean. Rejecting H_0 supports the notion that the two variables are related. In essence, the level of the predictor acts as an additional measure when attempting to explain the level of the output.

2.1.4 Goodness of Fit. In linear regression, it is also important to assess how well a model fits the data. This "goodness of fit" is measured by the statistic R^2, the coefficient of determination. R^2 is interpreted as the proportion of variance between the predicted and actual responses explained by the model. Equation 11 shows how R^2 is calculated. Recall Equations 6, 7, 8:

$$SSTO = \Sigma (Y_i - \bar{Y})^2$$

$$SSE = \Sigma (Y_i - \hat{Y}_i)^2$$

$$SSR = \Sigma (\hat{Y}_i - \bar{Y})^2$$

Then,

$$R^2 = SSR / SSTO = 1 - (SSE / SSTO) \tag{11}$$

Based on the equations for R^2 and its components, it is apparent R^2 falls between zero and one. A higher R^2 value indicates a better empirical fit of the data; thus, the model may provide more meaningful prediction results. Anderson provides a less technical explanation of R^2, and his explanation translates the equations into words. First, the total sum of squares, SSTO, represents the cumulative squared deviation from the average response. Without the benefit of an explanatory variable, the average is the best guess for any observation. The regression line reduces the variation between an observed response and its predicted value based on the levels of the input variables. This variation is unexplained by the model, and it is captured in the sum of squares for error, SSE. Therefore, the sum of squares of regression, SSR, calculates the amount of variation explained by the model. In mathematical terms, total variation equals the sum of explained and unexplained variation, and R^2 is the ratio of the explained portion to the total (1:16).

Although R^2 is a useful measure for the goodness of fit, the statistic may conceal the truth when comparing the effectiveness of models. Whenever an independent variable is added to the regression model, the value of R^2 increases. In comparison to the original model without the additional variable, the new model appears to create a better fit. However, an additional variable introduces a new parameter; hence, the *df* reduces by one. An adjusted - R^2 statistic takes into account the different *df*. The equation for adjusted- R^2 (R^2_a) has a similar form to (11) for R^2. The statistic is corrected by dividing both SSE and SSTO on the right-hand side by their respective *df*. The result establishes the following relation where p is the number of parameters in the model (15:49-50):

$$R^2_a = 1 - ((1 - R^2) * (n - 1) / (n - p)) \tag{12}$$

2.1.5 Residual Analysis. Recall, the basis for the statistical tests is the assumption that the error terms are independent random variables distributed $N(0,\sigma^2)$. To maintain this assumption, analysis of the residuals is conducted to determine if the error terms uphold the initial assumptions (Residuals are usually denoted e_i). A plot of the residuals against either the predictor variable or the estimated response values can assist in the determination of a model's ability to capture the system. This graph identifies any correlation between the residuals and the variable they are plotted against. Figure 1 provides an example of a residual plot.

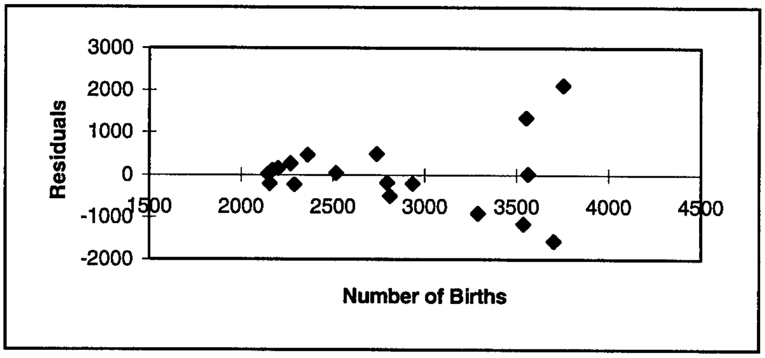

Figure 1. Example of Residual Plot

The concept of the graph is to ensure the assumption of constant variance exists over the different levels of the variable. Such constancy of variance is called homoskedasticity; heteroskedasticity is the term used when the assumption is violated. Furthermore, heteroskedasticity is categorized as either pure or impure. Pure heteroskedasticity occurs when an equation can describe how the variance behaves for different levels of the input variable. Notice, in Figure 1, the magnitude of the residuals increases as the input variable increases, so it appears the variance of the distribution increases as the level of the predictor variable increases. The impure form of heteroskedasticity occurs when there is an error of specification, usually the result of an omitted variable (15:365-371).

Heteroskedasticity has three major consequences on the regression parameters estimated by the method of least squares. First, the coefficient estimates remain unbiased ($E(b_k) = \beta_k$) when pure heteroskedasticity exists. The other two consequences are related and apply to both forms of heteroskedasticity. The variances of the distributions of the estimated parameters increases; however, the method of least squares tends to underestimate these standard errors. Without correction, the statistics used in the significance tests are less reliable. Since the variances are underestimated, the null hypothesis may be falsely rejected (15:374-375).

Like most problems which may arise with regression, heteroskedasticity has tests to detect its presence. Studenmund identifies the Goldfeld-Quandt test as the most commonly used test for heteroskedasticity. In the Goldfeld-Quandt test, the data is reordered according to size of the predictor variable. Using the regression function obtained by least squares, calculate the sum of squares for error (Equation 7) for the first and last third of the observations, labeled SSE_1 and SSE_3 respectively. The ratio of SSE_3 to SSE_1 is then compared to a critical F-value to determine if the null hypothesis of homoskedasticity is accepted or rejected. The value used for the F test is $F^*(1-\alpha; 1, n-2)$ where α is the level of Type I error tolerated (Studenmund: 381).

Residual analysis may demonstrate that the initial assumption of constant variance does not hold for the estimated regression model. Often, a transformation of the response or predictor variables (or both) adjusts the model appropriately. Neter, Kutner, Nachtsheim, and Wasserman give a discussion of transformations for different types of regression patterns (12:126-133). Taking the natural log (*ln*) of the response variable can perform one such transformation. The result of the operation is then used as the output variable.

2.1.5 Econometrics. The field of econometrics applies estimation techniques to establish relationships based on economic theory. The process provides two related objectives, empirical testing of the hypotheses and substantiation for predictions. The most crucial test of any theory is the ability to make pertinent forecasts. Econometric models are categorized as either cross-sectional or time series. Cross-sectional observations examine specific characteristics, such as household income, as a determining factor for some economic tendency. Time series problems, the class of models this thesis falls under, focus on a variable of interest observed at equally spaced time intervals. The variable of interest in this study is the annual level of a market index.

The added dimension of time invokes questions concerning the model's economic interpretation, especially the consideration of dynamic relationships. The full effect of a dynamic element is not realized immediately; instead, the contribution of the variable is partitioned until a specified time period has elapsed. Even though the current subject is an appropriate extension of this work, further discussion of this subject exceeds the level of this thesis, and the reader is referred to Harvey's work (8:1-8).

2.2 The Economic Philosophy of Harry Dent.

Harry Dent believes a nation's economy is highly predictable, and he contends the single, greatest factor in determining an economy's position is consumption. Since markets are theoretically designed to be indicators of the economy, they should also fluctuate based on consumption levels. Dent uses a unique method to identify when a country is at its peak spending level. In Figure 2, the median income level is given for each age group for the year 1996. Notice, the peak of the curve occurs between the ages of 45 and 54. If a constant percentage of income used for spending is assumed across the age groups, then the 45 to 54 age group has the largest impact on a per person basis. On this basis, Dent argues that a relationship exists between market indexes and age demographics.

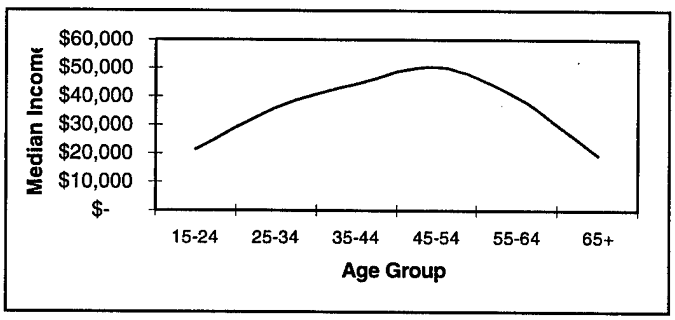

Figure 2. Median Income by Age Group, 1996
Source: U.S. Bureau of the Census

As more people enter this largest spending group, consumption must go up, and this increase should also be reflected in market indices; Dent terms this the Spending Wave. Therefore, the population of the age group can be used as a predictor of the market index. Furthermore, he concludes that the 46-year old age group is the best predictor for this phenomenon (5:21-43). This thesis attempts to verify this claim.

Over the two decades from 1945-1964, the number of births in the United States increased by fifty percent over the first ten years, and this level was sustained over the next ten years. Termed the Baby Boom, this generation produced by the largest number of births in the country's history is currently entering this economically influential age group. Applying Dent's suggestion, Figure 3 projects the total number of births for a given year plus 46 years into the future. Therefore, those born in 1934 entered the influential age group of 45-54 in the year 1980. Figure 2 shows that the number of people entering the influential age group continues to increase from 1994 until 2007 when a steady decline sets in for a 15-year span before another upturn takes place around 2022.

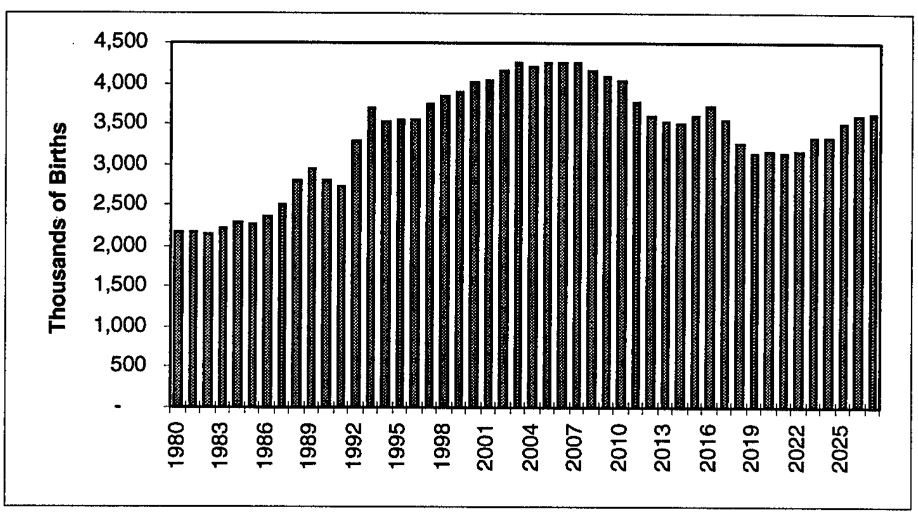

Figure 3. U.S. Births Lagged 46 Years

One upsetting feature of Dent's work overlays two graphs with different scales (4:71)(5:32), the projection for number of births similar to Figure 3 and an inflation-adjusted market index, without any mathematical justification. Neither scale uses zero as a lowest value, and no indication of statistical relation is stated. As Dent's work is proprietary, one of the first steps will be to outline a statistical model to confirm his conclusions while establishing the mathematical support for reproducing his graphical results. Using the Consumer Price Index as a means to calculate inflation, Figure 4 and Figure 5 chart the inflation-adjusted DJIA and S&P 500 respectively. Both graphs possess the same general tendencies. In addition, the indexes experience significant increases over the last twenty years similar to Figure 3.

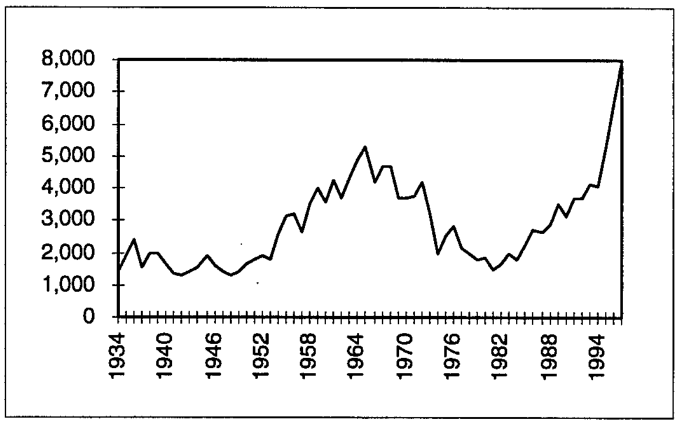

Figure 4. Inflation-Adjusted Dow Jones Industrial Average

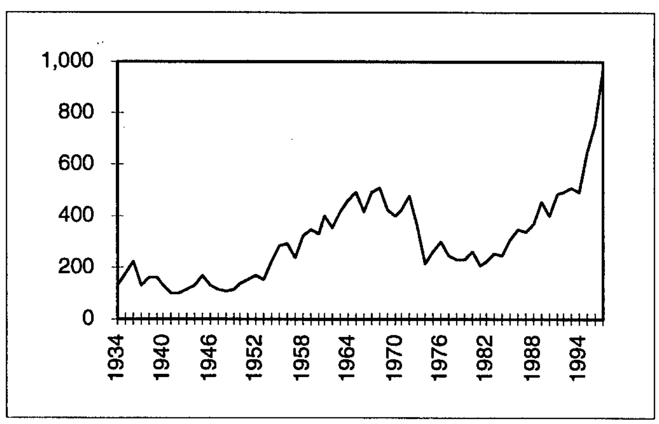

Figure 5. Inflation-Adjusted Standard & Poor's 500

Another factor determining economic growth is capital accumulation. The majority of capital is provided by consumer savings and corporate profits, with the prior being the largest sector. Similar to spending, saving fluctuates along the lines of age demographics. In fact, Dent says saving rates peak shortly after the peak of spending waves - occurring for U.S. citizens in their late forties to earlier fifties. One way to measure the relative savings rate in a society is to take a ratio of the number of older people to the number of younger people (3:72). Figure 6 charts the "Old / Young" Ratio, with the elderly reflected by the ages 45-54 and young people by the ages 25-34. A value above one indicates an older society while a younger population will correspond to a value below one. Notice, the curve's low occurs around 1986, and the ratio reaches its zenith in 2007, similar to the peak obtained in Figure 3. As more people save, companies are able to obtain more capital at a less expensive price in the form of equity (stocks) and debt markets; however, prices for capital accrue greater profit margins, and larger corporate profits allow businesses to finance other projects internally (3:72-73). These are two signs of a growing economy. The ratio in Figure 6 is another indication of the predicted economic boom over the next decade.

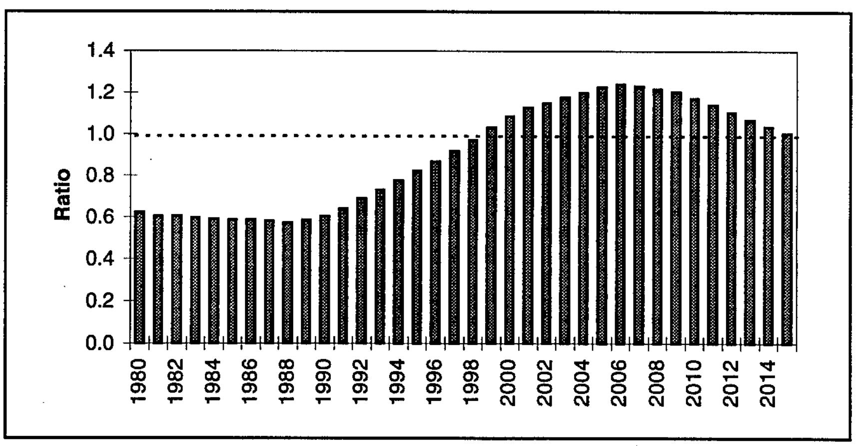

Figure 6. Old / Young Ratio for the United States

Dent believes this kind of analysis can be accomplished for any country. Few people have commented to the validity of his assertions; however, what has been written is supportive in nature. Hokenson, an economist with the same philosophy as Dent, places more emphasis on the younger generation leaving their parents' home and creating a new household. He uses Japan in the early 1990's as an example. With an aging culture, Japan's increased savings resulted in a current account surplus and bolstered the yen with respect to other currencies. He also concludes the United States will experience this type of economic strengthening until 2010 when the labor force size will actually begin to decrease (9:18-21).

2.3 Assessing the Model

Neter, Kutner, Nachtsheim, and Wasserman identify three primary purposes for regression analysis (12:9). These are description, control, and prediction. In practice, it is possible for a study to involve all three. In a regression model used for description, the model parameters attempt to describe why certain responses result from a given set of predictor variables. When control over the predictor variables exists, the regression model is used to identify what levels are necessary to maintain a specific response. Finally, regression models can be used to forecast system responses based on known levels of predictor variables. This thesis focuses on this last aspect since it involves verifying a method for predicting future market indices based on given birth figures.

The scope of a given study is critical in determining a useful model. To reiterate, scope defines an interval of effectiveness for the model. As part of this research, the United States' birth levels used as forecast data lie outside the range of birth levels used to construct the model. Since no evidence has been considered in the study outside these bounds, some doubt must prevail in the model's prediction capability.

Anderson provides a method to evaluate a market-timing strategy based on a forecasting process. For starters, the forecast needs to be compared to a baseline model of investing passively (1:119). Dent decides on investing based on the difference between his forecast and the actual value of the index. For example, when the "real value" of an index falls below his forecast, Dent recommends investing.

Another goal of this thesis is to explore market-timing strategies for the models constructed during the regression analysis phase. Maturi and Plummer provide numerous examples of forecasting theories developed over time (11)(13); however, neither book examines a theory based on population waves.

Data for regression analysis is obtained in one of two fashions, either experimental or non-experimental. The discussion will focus on non-experimental data since this thesis focuses on historical observations from several countries. Observational data has limitations associated with deriving cause-and-effect relationships, especially when the purpose of the regression model is description. Other variables may be overlooked which have a more direct explanation of the relationship (12:14).

2.4 Stock Indexes

Charles H. Dow, the first editor of the *Wall Street Journal*, theorized that a select group of stocks could be used to determine the trend of overall stock prices. With the assistance of S.A. Wilson, Dow constructed the first market index, the Dow Jones Industrial Average (DJIA). The list of companies on the DJIA is periodically changed in order for the index to be more representative of the market as a whole. Reasons for adjustments to the list include stock splits, takeovers, and bankruptcy. First, the average is price-weighted, placing more weight on higher priced stocks. In addition, the DJIA is composed generally of giant industrial and service companies, so representation of the multitude of securities for smaller companies seems to be excluded. As a result of the dissatisfaction with the DJIA, other indexes gained acceptance as market indicators. Today, the Standard & Poor's (S&P) 500 serves as a benchmark for other indexes and mutual fund performance. It consists of 400 industrial, 40 financial, 40 utility, and 20 transportation companies (11:13-16). The Nikkei 225, FTSE-100, and DAX are the major indexes respectively for Japan, the United Kingdom, and Germany.

III. Methodology

Throughout this chapter, the outlined method will refer to the model constructed in order to fulfill the primary objective of this thesis, capturing Dent's theory. The discussion will focus on the model for the United States with the number of births lagged 46 years as the predictor variable for the adjusted Dow Jones Industrial Average (DJIA(A)); however, commentary will be provided on the overall acceptability of this model. In Chapter 4, the forecasts based on this model will be presented, and the methodology will be applied to other countries and their respective indexes.

3.1 Data

The following pieces of data and sources were used to build a regression model with the number of births as an independent variable (X) and the DJIA(A) as the response (Y):

1.) The number of births for each country for the years 1934 - 1994 from the United Nations Demographic Yearbook (14).

2.) The closings on the DJIA from *The Dow Jones Averages: 1885-1990* (6) and *Irwin Investor's Handbook* (7). The values listed for a given year correspond to the recorded fourth quarter closings, the last day of operation in any year.

3.) The measures of the Consumer Price Index (CPI) from the Bureau of the Census website (2).

For the DJIA and the CPI, the listings in Table 1 on page 20 are the values recorded for December of that year.

3.2 Procedure

3.2.1 Discounting Closings for a Common Year. Before regression analysis was performed, the closings on each index were converted into terms of a common year. Like any other product, the DJIA must be adjusted for inflation in order to determine the amount of "real" growth that has occurred. The indices in this work are adjusted to the earliest recorded entry. For the United States, the DJIA was translated into a base year of 1997.

First, the CPI for each year was used to obtain inflation rates. For example, the CPI for the United States was 469.9 in 1996 and 480.8 in 1997. Thus, an inflation rate of 2.32% was calculated for the United States in the year 1997 according to Equation 13.

$$\text{Inflation}_{n} = (\text{CPI}_n - \text{CPI}_{n-1}) / \text{CPI}_{n-1} \tag{13}$$

In the same manner, growth rates for the DJIA were calculated for each year by Equation 14.

$$\text{Growth rate}_{n} = (\text{DJIA}_n - \text{DJIA}_{n-1}) / \text{DJIA}_{n-1} \tag{14}$$

Since both the CPI and DJIA closings were measured on the last day of the year, a "real" growth rate for each year could be calculated by taking the difference between the growth rate and the inflation rate of the same year, shown in Equation 15.

$$\text{"Real" Growth Rate}_{n} = \text{Growth Rate}_{n} - \text{Inflation}_{n} \tag{15}$$

Once the "real" growth rates for each year were calculated, the index was discounted appropriately. With 1997 as the base year, the DJIA closing for this year remains unchanged at 7,915.97. For the preceding years, the 1997 value of the DJIA was discounted using the "real" growth rate. Algebraic manipulation and substitution of the "real" growth rate for the growth rate in Equation 14 provided the necessary computations.

$$\text{DJIA(A)}_{n-1} = \text{DJIA(A)}_n \;/\; (1 + \text{"Real" Growth rate}_{n})$$

The values calculated for the series DJIA(A) are also shown in Table 1 on the next page.

Year	Births	DJIA	Return	CPI	Inflation	DJIA(A)	Real Return
1934	2,168	104.04		40.1		1418.10	
1935	2,155	144.13	38.53%	41.1	2.49%	1929.17	36.04%
1936	2,145	179.90	24.82%	41.5	0.97%	2389.17	23.84%
1937	2,203	120.85	-32.82%	43.0	3.61%	1518.60	-36.44%
1938	2,287	154.76	28.06%	42.2	-1.86%	1972.97	29.92%
1939	2,266	150.24	-2.92%	41.6	-1.42%	1943.39	-1.50%
1940	2,360	131.13	-12.72%	42.0	0.96%	1677.52	-13.68%
1941	2,513	110.96	-15.38%	44.1	5.00%	1335.61	-20.38%
1942	2,809	119.40	7.61%	48.8	10.66%	1294.86	-3.05%
1943	2,935	135.89	13.81%	51.8	6.15%	1394.08	7.66%
1944	2,795	152.32	12.09%	52.7	1.74%	1538.42	10.35%
1945	2,735	192.91	26.65%	53.9	2.28%	1913.34	24.37%
1946	3,289	177.20	-8.14%	58.5	8.53%	1594.23	-16.68%
1947	3,700	181.16	2.23%	66.9	14.36%	1400.95	-12.12%
1948	3,535	177.30	-2.13%	72.1	7.77%	1262.20	-9.90%
1949	3,560	200.13	12.88%	71.4	-0.97%	1436.98	13.85%
1950	3,554	235.41	17.63%	72.1	0.98%	1676.22	16.65%
1951	3,751	269.23	14.37%	77.8	7.91%	1784.51	6.46%
1952	3,847	291.90	8.42%	79.5	2.19%	1895.78	6.24%
1953	3,902	280.90	-3.77%	80.1	0.75%	1810.03	-4.52%
1954	4,017	404.39	43.96%	80.5	0.50%	2596.72	43.46%
1955	4,047	488.40	20.77%	80.2	-0.37%	3145.86	21.15%
1956	4,163	499.47	2.27%	81.4	1.50%	3170.09	0.77%
1957	4,255	435.69	-12.77%	84.3	3.56%	2652.34	-16.33%
1958	4,204	583.65	33.96%	86.6	2.73%	3480.71	31.23%
1959	4,245	679.36	16.40%	87.3	0.81%	4023.36	15.59%
1960	4,258	615.89	-9.34%	88.7	1.60%	3582.95	-10.95%
1961	4,268	731.13	18.71%	89.6	1.01%	4217.01	17.70%
1962	4,167	652.10	-10.81%	90.6	1.12%	3714.12	-11.93%
1963	4,098	762.95	17.00%	91.7	1.21%	4300.38	15.78%
1964	4,027	874.13	14.57%	92.9	1.31%	4870.78	13.26%
1965	3,760	969.26	10.88%	94.5	1.72%	5316.97	9.16%
1966	3,606	785.69	-18.94%	97.2	2.86%	4158.06	-21.80%
1967	3,521	905.11	15.20%	100.0	2.88%	4670.28	12.32%
1968	3,502	943.75	4.27%	104.2	4.20%	4673.51	0.07%
1969	3,600	800.36	-15.19%	109.8	5.37%	3712.27	-20.57%
1970	3,731	838.92	4.82%	116.3	5.92%	3671.36	-1.10%
1971	3,556	890.20	6.11%	121.3	4.30%	3737.93	1.81%
1972	3,258	1020.02	14.58%	125.3	3.30%	4159.78	11.29%
1973	3,137	850.86	-16.58%	133.1	6.23%	3210.97	-22.81%
1974	3,160	616.24	-27.57%	147.7	10.97%	1973.35	-38.54%
1975	3,144	852.41	38.32%	161.2	9.14%	2549.25	29.18%
1976	3,168	1004.65	17.86%	170.5	5.77%	2857.48	12.09%
1977	3,327	831.17	-17.27%	181.5	6.45%	2179.70	-23.72%
1978	3,333	805.01	-3.15%	195.4	7.66%	1944.17	-10.81%
1979	3,494	838.74	4.19%	217.4	11.26%	1806.74	-7.07%
1980	3,612	963.33	14.85%	246.8	13.52%	1830.78	1.33%
1981	3,629	875.00	-9.17%	272.4	10.37%	1473.01	-19.54%
1982	3,681	1046.54	19.60%	289.1	6.13%	1671.48	13.47%
1983	3,639	1258.94	20.30%	298.4	3.22%	1956.95	17.08%
1984	3,669	1211.57	-3.76%	311.1	4.26%	1800.03	-8.02%
1985	3,761	1546.67	27.66%	322.2	3.57%	2233.66	24.09%
1986	3,757	1895.95	22.58%	328.4	1.92%	2695.10	20.66%
1987	3,809	1938.83	2.26%	340.4	3.65%	2657.57	-1.39%
1988	3,910	2168.57	11.85%	354.3	4.08%	2863.96	7.77%
1989	4,041	2753.20	26.96%	371.3	4.80%	3498.64	22.16%
1990	4,158	2633.66	-4.34%	391.4	5.41%	3157.34	-9.76%
1991	4,111	3168.83	20.32%	408.0	4.24%	3665.01	16.08%
1992	4,084	3301.11	4.17%	420.3	3.01%	3707.52	1.16%
1993	4,000	3754.09	13.72%	432.7	2.95%	4106.88	10.77%
1994	3,979	3834.44	2.14%	444.0	2.61%	4087.53	-0.47%
1995		5095.80	32.90%	456.5	2.82%	5317.07	30.08%
1996		6509.78	27.75%	469.9	2.94%	6636.37	24.81%
1997		7915.97	21.60%	480.8	2.32%	7915.97	19.28%

Table 1. Data Series for United States and DJIA

3.2.2 Building Linear Regression Models. Regression analysis was conducted with the data obtained in 3.2.1. Attempting to verify Dent's claim, simple linear regression models were built with the number of births as the input variable, and the DJIA(A) as the output. In each instance, a lag period was identified. Equation 16 shows the linear relationship between the predictor variables and the response (X represents the number of births for the corresponding year). For example, in the 46-year lag model:

$$DJIA(A)_n = \beta_0 + \beta_1 * X_{n-46} \qquad (16)$$

In this instance, the number of births in 1934 and the DJIA(A) for 1980 were combined to form a data point. The regression analysis was performed, and Table 2 illustrates the results obtained using the corresponding lag between input and output. For the models built with lags from forty to forty-six, eighteen data points were used. When the lag exceeds forty-six, a data point is lost for each additional year added to the lag. As mentioned in Chapter 2, the heading R^2_a reports the adjusted - R^2. The reported *t*-statistic is associated with b_1 since a relationship was trying to be established for the predictor variable. GQ refers to the statistic calculated by performing the Goldfeld-Quandt test (2.1.5).

Lag	R^2_a	b_0	b_1	t-stat	F-stat	GQ
40	0.702	-5125.45	2.49	6.40	40.97	5.61
41	0.686	-4475.52	2.37	6.18	38.21	6.45
42	0.715	-4276.58	2.39	6.61	43.66	11.04
43	0.732	-4030.03	2.39	6.89	47.42	15.47
44	0.737	-3843.05	2.41	6.98	48.72	25.78
45	0.756	-3817.05	2.48	7.33	53.75	71.79
46	0.758	-3863.34	2.58	7.38	54.45	54.79
47	0.741	-3964.81	2.70	6.84	46.83	20.57
48	0.802	-4154.78	2.86	7.86	61.84	23.51
49	0.884	-4585.30	3.14	10.38	107.79	9.36
50	0.880	-4927.18	3.39	9.85	96.97	1.93

Table 2. Data for Simple Linear Regression Models with Corresponding Lag

For each model, the following hypotheses were formed.

$$H_0: \beta_1 = 0$$

$$H_a: \beta_1 \neq 0$$

The *t*-statistic and F-statistic were both used either to accept or reject the null hypothesis, H_0. When the model was based on eighteen data points, the *t*-statistic has sixteen degrees of freedom (*df*), and the F-statistic has 1*df* in the numerator and sixteen *df* in the denominator. The critical values, t* and F*, for a Type I Error level $\alpha = .05$ and those *df* are:

$$t^*(.975, 16) = 2.12$$

$$F^*(.95, 1, 16) = 4.49$$

All the models rejected the null hypothesis for both tests, implying a statistically significant relationship does exist. Again, for each data point lost, the *df* for the *t*-statistic and the denominator of the F-statistic were reduced by one. Therefore, the critical values were different when making a decision rule for H_0; however, in all cases, H_0 was rejected.

After the two tests were conducted, analysis of the residuals was performed. Focusing on Dent's claim, Figures 7 shows the fitted regression line and residual plot when the number of births were lagged by forty-six years. The deviation of the points from the line increased as the predictor variable increased, an indication of possible heteroskedasticity.

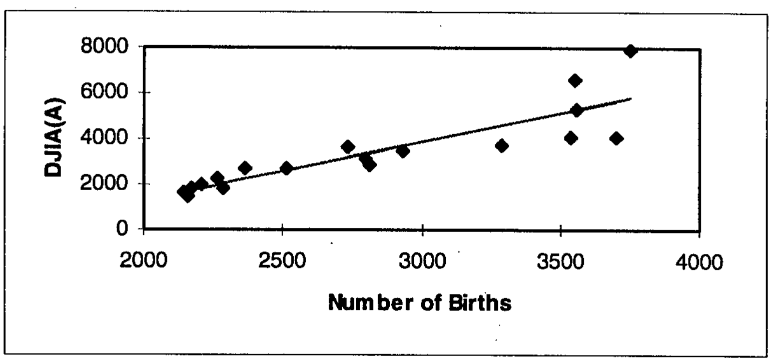

Figure 7. Scatterplot of 46-year Lag with Fitted Simple Linear Regression Model

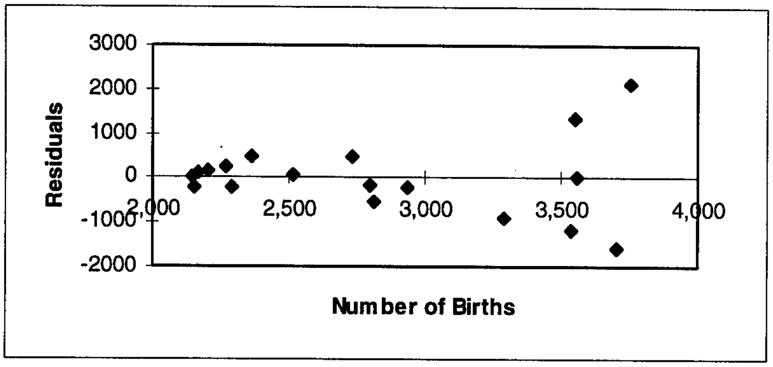

Figure 8. Residual Plot of 46-year Lag

The residual plot in Figure 8 reiterated the existence of heteroskedasticity, and similar plots existed when other lags were used. As a means of verification, the Goldfeld-Quandt test was performed on all the models. The hypotheses for this test are:

$$H_0: \text{ Homoskedasticity exists, } E(\varepsilon_i) = \sigma^2$$

$$H_a: \text{ Homoskedasticity does not exist, } E(\varepsilon_i) \neq \sigma^2$$

As stated in the literature, the predictors were ordered by increasing value because there is evidence that higher levels of the predictor produce larger variances. Dividing the series into thirds, the residuals corresponding to the data points in the first and last third were squared and then summed. When the number of data points was not divisible by three, an effort was made to keep the same number of data points in the first and last third. For example, sixteen data points were broken into sets of five, six, and, five. When the lag ranged from forty to forty-seven, the two groups each contained six data points, and the GQ-statistic follows an F distribution with one *df* in the numerator and four in the denominator. The other lags produced groups of five data points, so the *df* in the denominator was reduced to three. Referring back to Table 2, the GQ-statistic was compared to the following critical values for $\alpha = .05$:

$$F^*(.95, 1, 4) = 7.71$$

$$F^*(.95, 1, 3) = 10.13$$

The test rejected H_0 for lags from forty-two to forty-eight. The GQ was especially low for the fifty year lag, but the number of data points comprising the model was also low compared to the others. Nevertheless, further examination of this model was conducted, and the results are in Chapter 4.

3.2.3 Transformation. Continuing pursuit of Dent's theory, a transformation was necessary to potentially eliminate heteroskedasticity. As suggested by Neter, Kutner, Nachtsheim, and Wasserman an exponential transformation was performed on the response(8:130). In this instance, a natural log (*ln*) transformation of the output was suitable. Equation 17 demonstrates the transformation for a forty-six year lag.

$$ln\,(\mathrm{DJIA(A)}_n) = \beta_0 + \beta_1 * X_{n-46} \qquad (17)$$

Among the simple linear regression models which required a transformation, the forty-six year yielded the highest correlation before the transformation. Table 3 gives the regression results for the transformed models. Again, the *t*-test and F-test were used to assess the hypotheses:

$$H_0: \ \beta_1 = 0$$

$$H_a: \ \beta_1 \neq 0$$

Similarly, all models reject H_0, $\beta_1 = 0$, for both tests based on the critical values.

$$t^*(.975, 16) = 2.12$$

$$F^*(.95, 1, 16) = 4.49$$

In addition, all models accept H_0, homoskedasticity exists, for the GQ-test with the same critical values.

$$F^*(.95, 1, 4) = 7.71$$

$$F^*(.95, 1, 3) = 10.13$$

Lag	R^2_a	b_0	b_1	t-stat	F-stat	GQ
42	0.868	5.74	7.09E-04	10.61	112.52	2.51
43	0.873	5.83	7.03E-04	10.84	117.44	3.38
44	0.865	5.90	7.04E-04	10.5	110.27	4.45
45	0.867	5.93	7.17E-04	10.59	112.22	5.96
46	0.848	5.94	7.37E-04	9.83	96.54	3.45
47	0.809	5.97	7.54E-04	8.31	69.15	1.88
48	0.839	6.07	7.49E-04	8.91	79.32	1.43

Table 3. Data for Transformed Regression Models with Corresponding Lag

After the transformation, a feasible model was created to capture Dent's prognostication; however, a feasible model could have been created for any of the lags. Even though the forty-three year lag produced the highest correlation, the model with a forty-six year lag will be used in Chapter 4 to assess the conclusions Dent formulates based on this lag. The figures below document the fitted regression line and residuals for the forty-six year case. Notice, the residuals suggest homoskedasticity as the test verified.

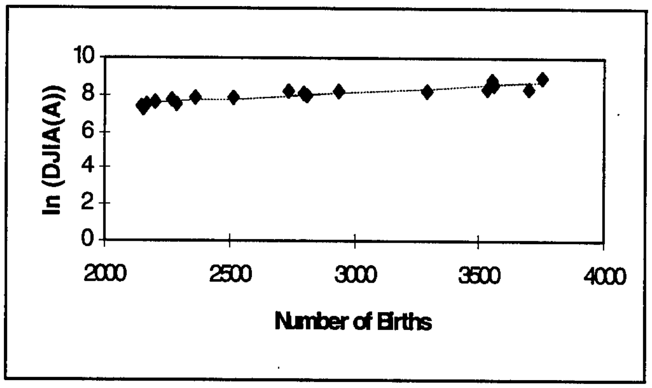

Figure 9. Scatterplot with Fitted Regression Line for Transformed 46-year Lag

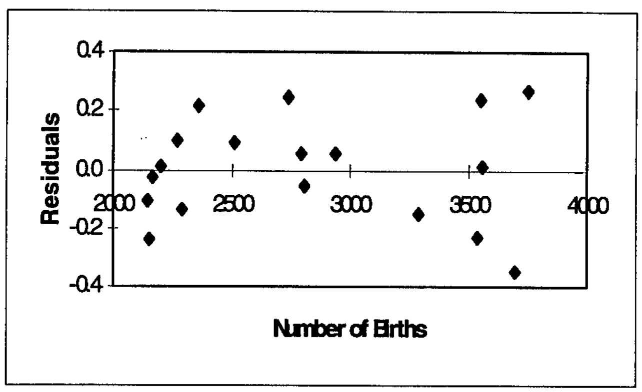

Figure 10. Residual Plot for Transformed Regression Function with 46-year Lag

Also, a residual plot with respect to time indicated the error terms were not serially correlated, a problem in time series analysis.

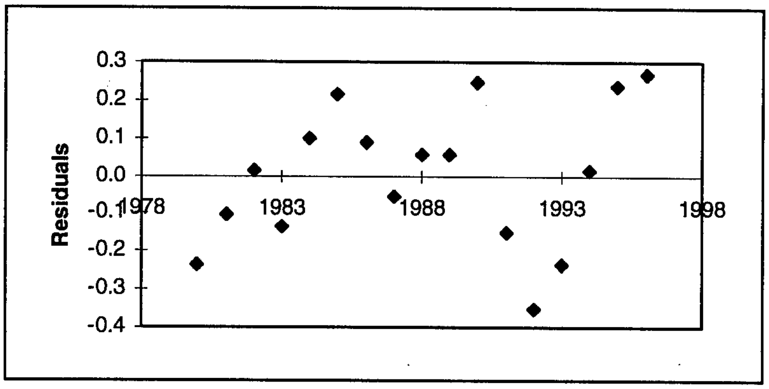

Figure 11. Residual Plot With Respect to Time

3.3 Application

A similar procedure as the one outlined in the previous section was carried out for the following countries and their respective indices, Japan with the Nikkei 225, the United Kingdom with the FTSE-100, Germany with the DAX, and the United States with both the DJIA and the S&P 500. The details for each scenario are presented in Chapter 4.

IV. Results

In Chapter 3, the methodology showed a feasible model could be created for any lag between forty and fifty for the United States' births lagged by forty-six years and the adjusted Dow Jones Industrial Average (DJIA(A)). This chapter will make projections based on the model with a forty-six year lag and compare these forecasts with Mr. Dent's. In addition, the results of the best simple linear regression model, a fifty year lag, will be presented. Finally, application of the procedure to other major indexes will be performed, and prognostications for these indexes will also be shown if a proper model can be formulated. As for the data, located in Appendix A, all country's births were obtained from *The United Nations' Demographic Yearbook* (14), and Brian Taylor of *Global Financial Data* (16) supplied the Consumer Price Index and index closings for the foreign countries.

4.1 The United States

4.1.1 The Dow Jones Industrial Average. From Chapter 3, both the forty-six year transformed and the fifty year linear model are feasible. In either case, skepticism should exist simply because of the size of the data sets available to build the models. Notwithstanding, forecasts will be made for both models in comparison to Dent's theories.

4.1.1.1 Log Transformation- Forty-six Year Lag Model. Recall from Chapter 3, the equation (where X represents the number of births) computed to fit the DJIA(A) with a forty-six year lag is:

$$ln\,(\text{DJIA(A)}_n) = 5.94 + 7.37*10^{-4} * X_{n-46} \qquad (18)$$

Since the model was constructed with the births from 1934-1951, the remainder of the years can be used as forecast data. To project the DJIA(A), insert these values into the regression equation, and take the exponential of this number to obtain a forecast of the "real" Dow Index. The results are graphed in Figure 12 on top of the next page.

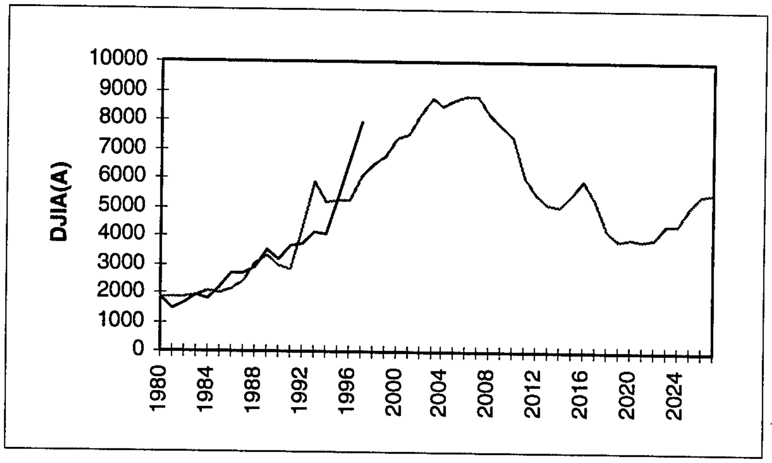

Figure 12. DJIA(A) Forecast Using Transformation Model 46-Year Lag

Several statements can be made about the graph. First, the actual outcomes and model predictions remain relatively close until 1992 when the forecast tends to overestimate the actual level before the market makes a recovery in 1995. For the most recent year, 1997, the model indicates the market is overvalued. In his newsletter, Dent has been predicting a correction in the market toward 7500; however, this model would suggest an even harsher setback in the neighborhood of 7000. As a result of the forty-six year lag, the curve peaks in 2007 before a downward trend over the next fifteen years.

Since a base year of 1997 was used to discount the index, the prediction for 1997 is expressed in comparable terms to the actual closing for that year. At the same time, all predictions cast into the future are void of inflation. Therefore, inflation was incorporated into the forecasts in the following manner. First, the predicted value for 1997 remains the same. Then, similar to the discounting procedure, the growth rate between observations is augmented with an inflation rate. The result is:

$$DJIA_{n+1} = DJIA_n * (1 + \text{Growth rate}_n + \text{Inflation}_n) \quad (19)$$

Tables with varying inflation rates are located in Appendix C. The graph on the next page exemplifies the procedure with three percent inflation.

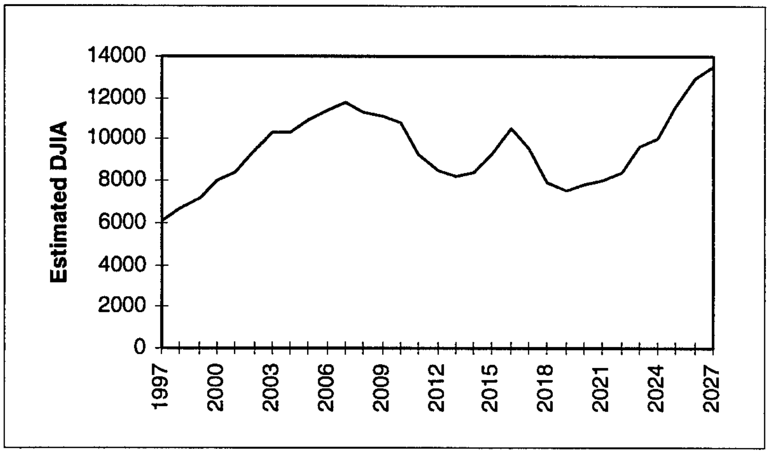

Figure 13. Estimated DJIA with 3% Inflation Rate and 46-year Lag

With inflation, the downturns do not appear quite as drastic as before. In this instance, the DJIA tops 12,000 around 2007, nowhere near the forecast of 20,000 by Dent. Of course, the gaps between the peaks and valleys still depend on the forty-six year lag.

4.1.1.2 Simple Linear Regression Model With Fifty Year Lag. While attempting to model Dent's theory, a fifty year lag produced a particularly low GQ-statistic for homoskedasticity even though only fourteen data points were used. It was difficult to avoid analysis of such a highly correlated model with a prediction very close to the true 1997 close. The regression function is:

$$DJIA(A)_n = -4927.18 + 3.39 * X_{n-50} \quad (20)$$

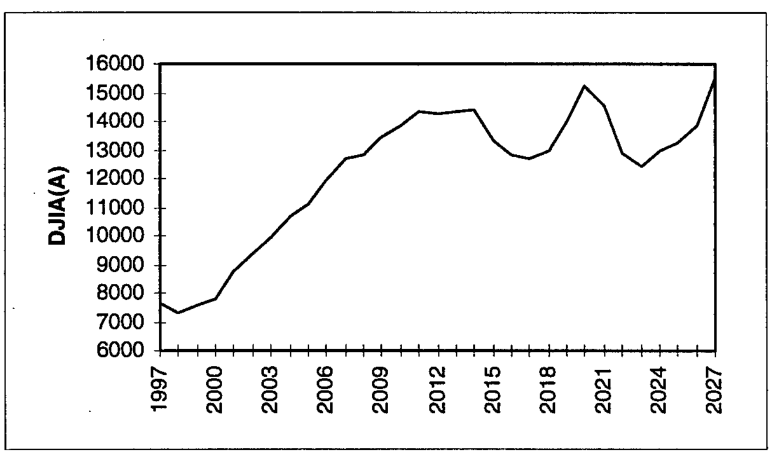

Figure 14. Estimated DJIA with 3% Inflation Rate and 46-year Lag

This model generates a plateau slightly over 14,000 before a downturn around 2015. In direct response to the lag, the local extremum have shifted four years to the right. One interesting feature of this model is the predicted ability of the market to regain its highs in a shorter time frame.

4.1.2 The Standard & Poor's (S&P) 500. As pictured in Chapter 2, the shape of the adjusted S&P 500 (S&P(A)) is very similar to the contour of the DJIA(A). With that said, it should not be a surprise that the same problems encountered for the DJIA(A) were experienced in the S&P(A). Tables of the regression results can be found in Appendix B. All models rejected the hypothesis $\beta_1 = 0$ for both the *t*-test and the F-test; however, lags ranging from forty-two to forty-eight also failed to indicate homoskedasticity based on the GQ-test. Again, a natural log (*ln*) of the response was used to transform the data, and feasible models could have been constructed for all lags. After the transformation, a forty-five year lag yielded the highest $R^2_a = .867$, but the simple linear regression model with a fifty year lag furnished an $R^2_a = .883$. Remember, this model was based on only fourteen data points as opposed to eighteen for the majority of the others. The two regression functions are:

$$ln\,(S\&P(A)_n) = 4.10 + 6.43*10^{-4} * X_{n-45} \qquad (21)$$

$$S\&P(A)_n = -514.25 + .385 * X_{n-50} \qquad (22)$$

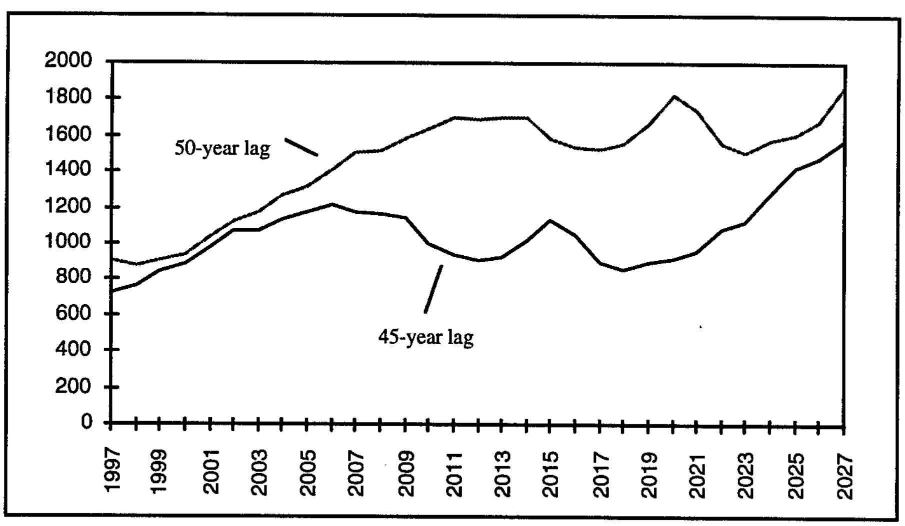

Figure 15. Estimated S&P with 3% Inflation for Two Models

For observation's sake, the ratio of the coefficient estimators for the S&P(A) in Equation 22 and the DJIA(A) in Equation 20 are roughly one ninth, the same relative proportion of the index values. Conclusions can be drawn parallel to the ones made for the DJIA(A). The forty-six year model suggests the actual S&P is overvalued and predicts a cap slightly over 1,200. On the other hand, the fifty year model indicates the S&P will vault close to 1,700. Again, three percent inflation is assumed for these predictions, and models could have been created for any lag. The presentation has focused on the models with the highest correlation.

4.2 Japan and the Nikkei 225.

The next objective of this research is to apply the methodology to other countries and their respective indexes in order to determine if similar correlation exists. Even though the indexes do not have a historical base as long as the DJIA, each of the countries examined have well-established economies, and the nations consider their indexes a good measure for their economy.

It is well known that Japan was on the forefront when it came to the development of technological products during the 1980s. As evidence, refer to the graph of the Nikkei for the last fifty years, and the substantial increase from 1979 to 1989 really stands out. In fact, the index nearly multiplied its worth by five times over that decade. The question arising from this analysis is whether the increase has any relation to a growing population of domestic consumers.

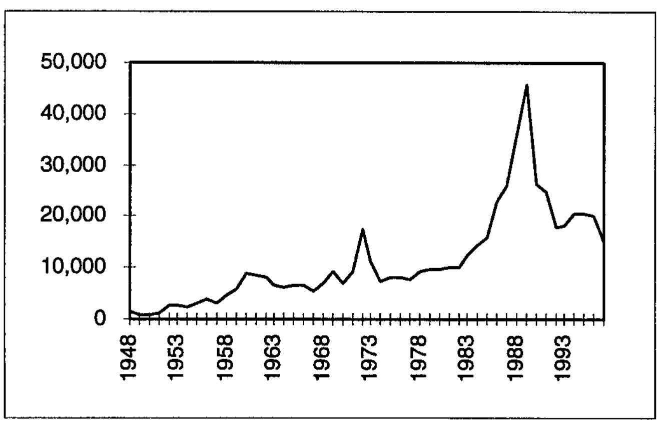

Figure 16. Inflation-Adjusted Nikkei 225 with Base Year of 1997

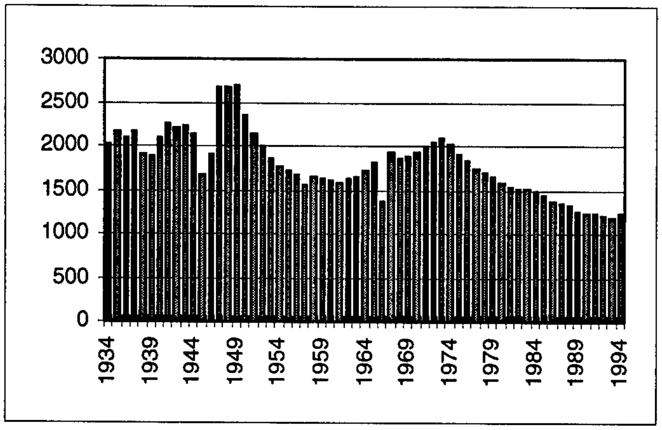

Figure 17. Number of Births in Japan

Just looking at Figure 17, a potential relationship presents itself in the form of the prominence around the year 1948, approximately a forty year differential to the height of the Nikkei. When regression analysis was performed (results in Appendix B), the forty year lag produced an $R^2_a = 0.238$. The best model, however, was the thirty year lag which only returned an $R^2_a = 0.312$. In this case, a negative relationship was established, $b_1 = -23.91$, and both the *t*-test and F-test rejected the null hypothesis of $\beta_1=0$.

The residual plot revealed heteroskedasticity toward the lower levels of the predictor variables. In this case, the GQ-test needed to be altered in some fashion. The test states the data should be ordered according to the value of a proportionality factor. For the United States, an increase in the number of births

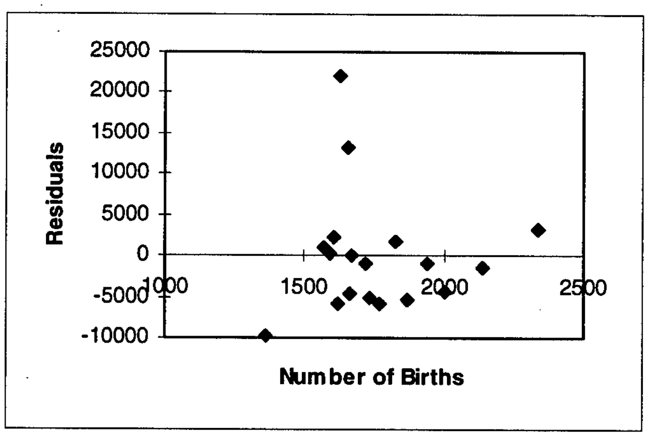

Figure 18. Residual Plot of Simple Linear Regression Model with 30-Year Lag (Japan)

was associated with larger residuals, so the data was ordered in ascending order of the number of births. For Japan, the data needed to be ordered in descending order of the input variable. The test also could have been altered by dividing SS1 / SS3, the reciprocal of the statistic for the United States. In either case, the thirty year lag model had a GQ = 10.08, rejecting the assumption of homoskedasticity.

In order to correct for this type of heteroskedasticity, a transformation was conducted by taking the inverse of the response variable. The regression equation for this transformation was:

$$(\text{Nikkei(A)}_n)^{-1} = -9.49*10^{-5} + 8.71*10^{-8} * X_{n-30} \qquad (23)$$

The regression function produced the following residual plot and forecast. Even though the model produced an $R^2_a = 0.669$, it made some very poor forecasts. It failed to capture the boom of the 1980s, and it predicted an upturn that did not occur.

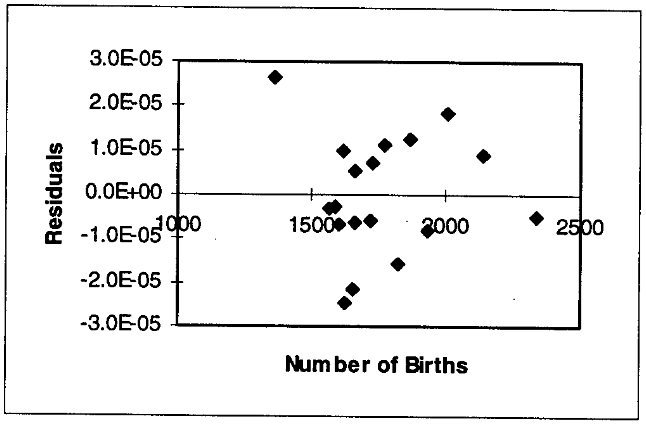

Figure 19. Residual Plot for Transformed Regression Function with 30-Year Lag (Japan)

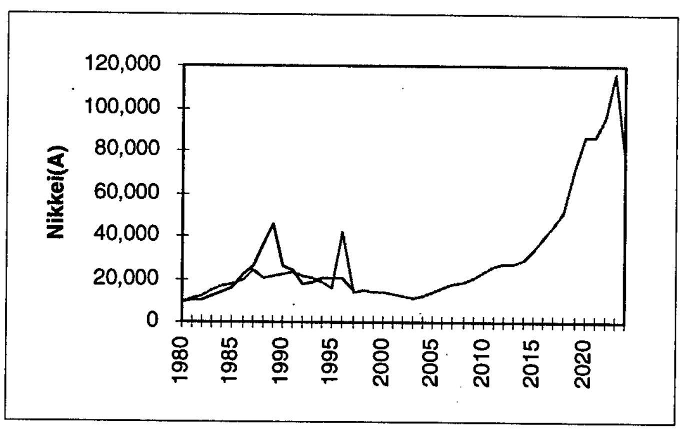

Figure 20. Nikkei(A) Forecast Using Transformation Model with 30-Year Lag

Overall, an acceptable model based solely of national birth rates that depicted the Japanese economy accurately was difficult to create. The model that showed the most promise violated the premise of the theory. Recall, the concept of the theory is to identify periods of higher consumption within a country. This model did not incorporate that belief. The most obvious explanation for these discrepancies is the fact that Japan has maintained a high trade surplus with most countries, so their economy seems more dependent on worldwide demand than the consumption within its borders.

4.3 The United Kingdom (UK) and the FTSE-100.

After little progress in confirming Dent's theory with the Japan scenario, the next case involved the United Kingdom (UK). Since the FTSE-100 has a relatively short history as the leading economic index for the UK, no cyclical nature could be extracted from its graph. During the efforts to create a model for the UK, the main problem that arose was similar to one encountered during construction of the model for the United States. Many different lags produced statistically significant models.

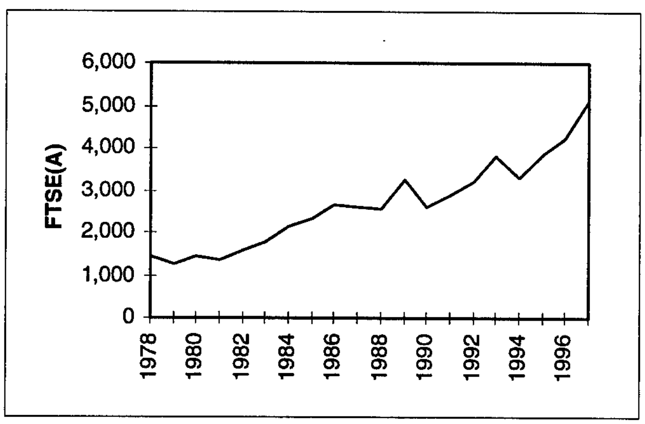

Figure 21. Inflation-adjusted FTSE-100 with Base Year of 1997

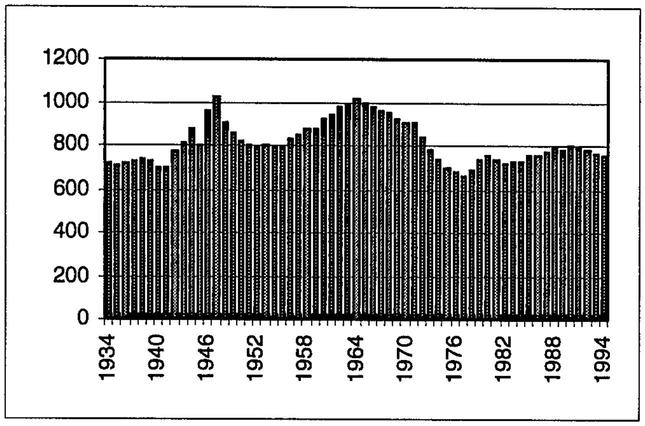

Figure 22. Number of Births in the UK

Observing the graph for the number of births, a possible generation wave was detected between the peaks in 1947 and 1964 even though Dent suggests a generation wave lasts approximately nineteen years. Regression analysis confirmed this presumption. The decision rules for the models did not begin rejecting H_0, $\beta_1 = 0$, until the lag was at least forty-five. Similar to the analysis of Japan, most of the models did not fit the data particularly well; however, lags of forty-nine and fifty years were suitable with an R^2_a over 0.65 in both instances. Intrigue created by the two cases for the United States and an $R^2_a = 0.783$ led to the selection of the fifty-year lag.

$$FTSE(A)_n = -851.70 + 2.44 * X_{n-50} \tag{24}$$

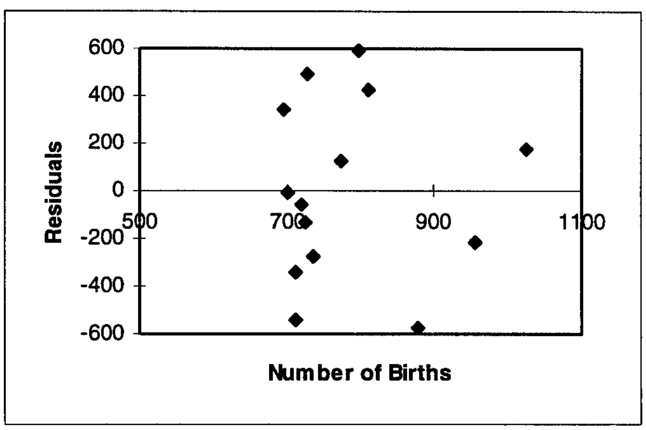

Figure 23. Residual Plot of Simple Linear Regression Model with 50-Year Lag (UK)

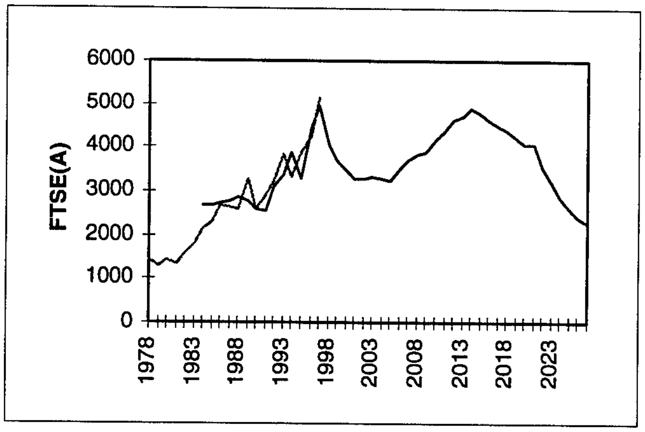

Figure 24. FTSE-(A) Forecast Using 50-Year Lag

Based on the fifty year lag, the predictions for the current future of the FTSE are not very promising. The index would expect to lose a third of its value over the next four years. When inflation was factored back into the equation (Appendix C), the total loss was reduced to one fourth of the current value, but that would be a major crash as well. Notice, however, a fifty year lag of births produced a significant correlation for both the United States and the United Kingdom's indexes.

4.4. Germany and the DAX.

Finally, Germany was the last country analyzed in this research. The DAX(A), unlike the DJIA and Nikkei, has not experienced severe fluctuations in its value. It maintained a relatively stable level over the first twenty years of its existence. Since then, it has steadily increased except in times of major political transformations in Europe, the crashing of the Berlin Wall and the breakup of the former Soviet Union.

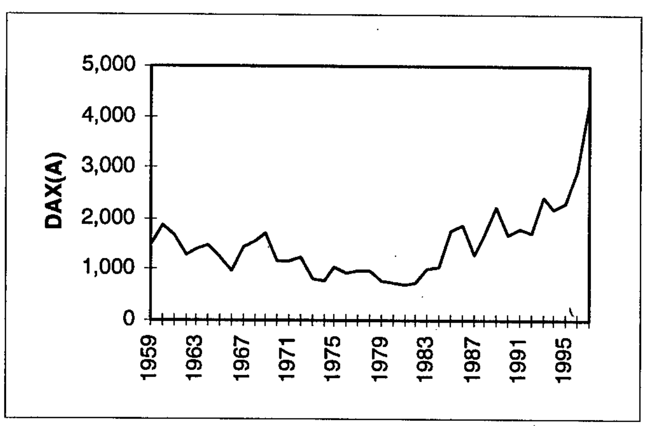

Figure 25. Inflation-Adjusted DAX with Base Year of 1997

The most difficult part in the analysis of Germany was in the data accumulation. The number of births attributed to Germany are sums of the number of births reported by both East and West Germany during their period of separation. Figures prior to World War II reflect a unified Germany. A similar contour to the one for the DAX(A) was viewed in the number of births for Germany; however, the relationship did not exist in the traditional upper forties for the other countries.

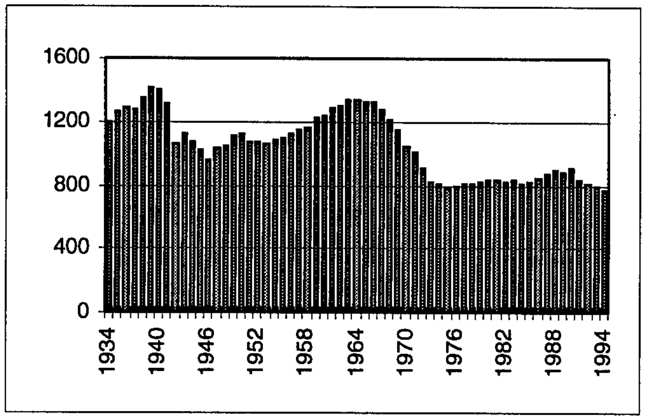

Figure 26. Number of Births in Germany

A suitable relationship between the DAX(A) and the predictors was not established for lags between forty and fifty years. First, the negative relationship seen in the Japan scenario persisted for Germany, and the correlation for these models was quite low; however, the forty-two year lag failed the GQ-test for homoskedasticity. An inverse transformation was performed, and $R^2_a = 0.669$ for this model. Still, the information from the graphs suggested models needed to be run for younger generations. As suspected, a thirty-six year lag became the best indicator with an $R^2_a = 0.791$. The equations for the two regression functions were:

$$(DAX(A)_n)^{-1} = -1.82*10^{-3} + 2.23 * X_{n-42} \quad (25)$$

$$DAX(A)_n = -8901.90 + 9.67 * X_{n-36} \quad (26)$$

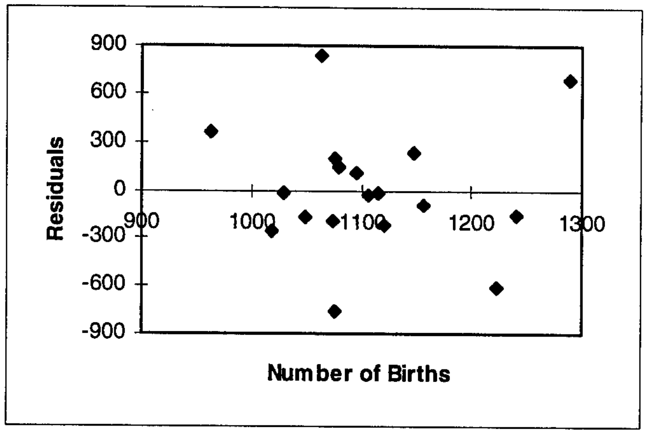

Figure 27. Residual Plot for Simple Linear Regression Model with 36-Year Lag (Germany)

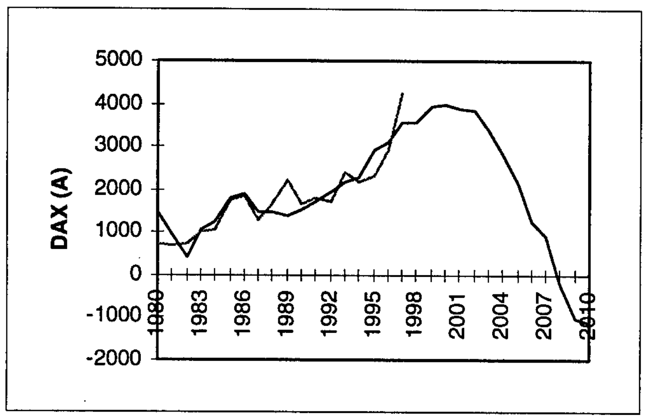

Figure 28. DAX(A) Forecast Using 36-Year Lag

The residual plot and forecast were made from the linear model. The major problem of regression analysis demonstrated in Figure 28 was also exhibited in the forecast for the transformed model. When forecast data does not fall within the range of data used to construct the model, it has exceeded the scope of the problem. This causes unrealistic predictions since the level of births in Germany experienced a serious decline over the decade from 1965-1975, most likely a compound result of the losses during World War II. Even when inflation was introduced, the model could not correct for this factor.

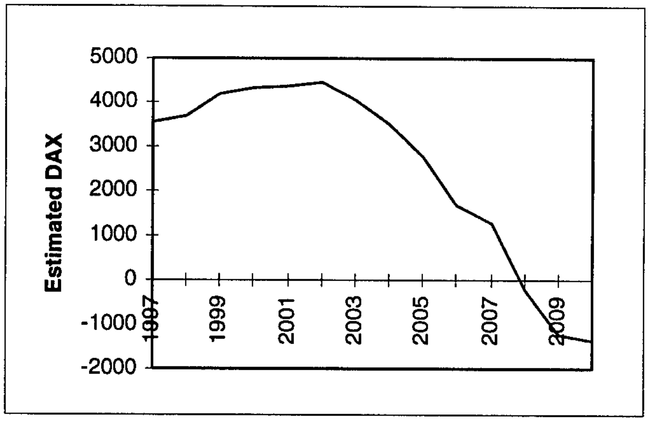

Figure 29. Estimated DAX with 3% Inflation Rate and 36-year Lag

4.5 Summary

In summation, a model that captured Dent's theory was constructed, but the model did not precisely agree with the conclusion he postulates. It predicted much lower levels of the DJIA even when a modest inflation factor was introduced. On the same token, it was statistically feasible to create a model for all the lags tested in the United States, so this research could not confirm the emphasis he places on the forty-six year lag. The major difference between this study and Dent's work is the amount of data. Apparently, Dent has been able to obtain figures for the number of births before 1934, the earliest year used in this research.

As for the foreign countries, the models had more distinction between age groups and index correlation. Specifically, certain age groups were eliminated because their parameter estimates were not statistically significant. Of the models selected, it is more difficult to substantiate these claims for foreign countries without the population demographics, such as income levels and savings rates, Dent used to develop his theory about the United States.

To recap, the ability to predict the level of the indexes had mixed results. The United States did a decent job of fitting the data; however, it was difficult to assess which attribute was the defining factor. The United Kingdom somewhat verified the technique by producing a model similar to that for the United States, and, as a result, the model indicated a major downturn for the FTSE. Japan showed little ability to manufacture an effective model, and Germany incurred incorrigible problems with respect to the scope of the problem.

V. Summary, Conclusions, and Recommendations

Attempting to capture Dent's economic theory, this research effort set out to reproduce his predictions via linear regression models. Recall Dent believes periods of increased consumption can be identified through analysis of birth waves. The research objectives, as presented in Chapter 1, were:

1.) Construct a Model to Capture Population Demographic Theory

2.) Apply Modeling Techniques to Other Markets

Data for the analysis was compiled from several sources. The predictor variable was the number of births, and the index closing was the response variable. The following sections provide a brief synopsis of the research methods used to meet these objectives, present the conclusions, and make recommendations for further research.

5.1 Summary and Conclusions

5.1.1 Construct a Model to Capture Population Demographic Theory. Once the data was obtained to perform the analysis of the United States with respect to the Dow Jones Industrial Average (DJIA), a series of regression models were constructed with different lags between the input and output variables. Several statistical tests were conducted to determine the feasibility of the model, and transformation techniques were employed

In the construction phase, two models were chosen for forecasting purposes. The first model, a transformed model with a forty-six year lag, was the best attempt at producing a model with the lag labeled the best indicator of market performance. The other model appeared to be the best simple linear regression model. While similar predictions could be made about the direction of market performance, the models failed to generate similar magnitude prognostications.

As a final thought, the background for Dent's theory needs to be addressed. Compared to five years ago, it is much easier to become involved in the market today. Mutual funds have seen an enormous amount of capital flow through their businesses. Computers allow investors to make split-second decisions, and these investors seem more committed to an apparent financial stability of the market. With these traits, the ability to use one age group as a determining factor for market performance diminishes.

5.1.2 Apply Modeling Techniques to Other Markets. After a methodology was devised to estimate possible models for the DJIA, the next step was to apply the procedure to other markets. Since the "real" values of the Standard & Poor's (S&P) 500 and DJIA have resembling shapes, the modeling approaches were comparable, and the forecasts were quite similar.

Once the two major indexes of the United State were analyzed, efforts were made to examine how universal Dent's theory was. Cases for Japan, the United Kingdom (UK), and Germany were also formulated. During this investigation, the theory did not appear as promising. The models for Japan and Germany would be characterized "weak" at best. They fit the data poorly, and the forecasting ability encountered significant troubles. It must be said, though, these countries have a broader consumer base than their own residents. Their economy is internationally dictated. The model for the UK, however, supported Dent's claims. It possessed the same lag, fifty years, as the simple linear model for the United States. The prognosis for the UK based on this model was quite detrimental to its index. In either case, several scenarios were examined, and the methodology demonstrated varying degrees of success.

5.2 Recommendations

The previous section provided a brief summary of the research objectives, methodology, and findings. It did not, however, discuss the additional research that can be extended from this work. First and foremost, collection of data for the years prior to 1934 can be undertaken, especially for the United States. Of course, refer to Dent (5) for corrections he makes due to shortened life expectancies of past generations. More data allows the regression function to be based over a longer time period. In the case of the United States, this may assist in eliminating age groups as feasible predictor variables, and better assessments of Dent's conclusions can be made.

The accumulation of more data serves a second purpose. One of the goals of this research was to validate how well the model performs as a market-timing strategy. Validation can be accomplished by applying the methodology to a portion of the data. Then, the remainder of the data can evaluate the accuracy of the model. For instance, if forty data points can be gathered, thirty observations could be used to generate a model, and the other ten points could assess its predictability.

As discovered, the Nikkei and the DAX may not rely as heavily on national consumption as the DJIA. Japan, as one of the world's largest exporters, seems more dependent on worldwide demand, and Germany acts in the same manner for Europe. Therefore, if population numbers for the continent of Europe and the world as a whole can be calculated, regression models could be run to validate these claims. The source (14) used for this research did not supply these cumulative totals. Each country was reported independently.

Another consideration for the discrepancies observed between countries may be the continual effects of World War II. Separating the victors from the defeated, analysis of the U.S. and the UK produced similar representations while the study of both Japan and Germany did not yield a dependable model. In addition, the United States was the only party to avoid mass physical destruction within its borders. Perhaps, better comprehension can be obtained through inclusion of death rates. As suggested earlier, demographic information for the foreign countries may provide a better indication of what age group acts as a leading indicator for each index.

Because the variable of interest, market index values, is recorded each year, the field of econometrics was briefly mentioned in the literature review. The analysis of the United States demonstrated that many different lags produced statistically significant models. The graph for the median income level of the United States is categorized by age groups of ten years, so specifying a single year as the defining age group seems highly unlikely. If more age groups are added as predictor variables, then the concept of lagged variables (different from lag between predictor variables and response in this study) arises. Econometrics discusses appropriate techniques to properly analyze the results generated by these models.

Appendix A. Data Used for Regression Analysis

Year	Births	DJIA	Return	CPI	Inflation	S&P(A)	Real Return
1934	2,168	9.50		40.1		126.94	
1935	2,155	13.43	41.37%	41.1	2.49%	176.29	38.87%
1936	2,145	17.18	27.92%	41.5	0.97%	223.79	26.95%
1937	2,203	10.55	-38.59%	43.0	3.61%	129.34	-42.21%
1938	2,287	13.21	25.21%	42.2	-1.86%	164.36	27.07%
1939	2,266	12.49	-5.45%	41.6	-1.42%	157.73	-4.03%
1940	2,360	10.58	-15.29%	42.0	0.96%	132.10	-16.25%
1941	2,513	8.69	-17.86%	44.1	5.00%	101.89	-22.86%
1942	2,809	9.77	12.43%	48.8	10.66%	103.70	1.77%
1943	2,935	11.67	19.45%	51.8	6.15%	117.49	13.30%
1944	2,795	13.28	13.80%	52.7	1.74%	131.66	12.06%
1945	2,735	17.36	30.72%	53.9	2.28%	169.11	28.45%
1946	3,289	15.30	-11.87%	58.5	8.53%	134.61	-20.40%
1947	3,700	15.30	0.00%	66.9	14.36%	115.28	-14.36%
1948	3,535	15.20	-0.65%	72.1	7.77%	105.57	-8.43%
1949	3,560	16.76	10.26%	71.4	-0.97%	117.43	11.23%
1950	3,554	20.41	21.78%	72.1	0.98%	141.85	20.80%
1951	3,751	23.77	16.46%	77.8	7.91%	153.99	8.56%
1952	3,847	26.57	11.78%	79.5	2.19%	168.76	9.59%
1953	3,902	24.81	-6.62%	80.1	0.75%	156.31	-7.38%
1954	4,017	35.98	45.02%	80.5	0.50%	225.90	44.52%
1955	4,047	45.48	26.40%	80.2	-0.37%	286.39	26.78%
1956	4,163	46.67	2.62%	81.4	1.50%	289.60	1.12%
1957	4,255	39.99	-14.31%	84.3	3.56%	237.83	-17.88%
1958	4,204	55.21	38.06%	86.6	2.73%	321.86	35.33%
1959	4,245	59.89	8.48%	87.3	0.81%	346.54	7.67%
1960	4,258	58.11	-2.97%	88.7	1.60%	330.68	-4.58%
1961	4,268	71.55	23.13%	89.6	1.01%	403.81	22.11%
1962	4,167	63.10	-11.81%	90.6	1.12%	351.61	-12.93%
1963	4,098	75.02	18.89%	91.7	1.21%	413.76	17.68%
1964	4,027	84.75	12.97%	92.9	1.31%	462.01	11.66%
1965	3,769	92.43	9.06%	94.5	1.72%	495.92	7.34%
1966	3,606	80.33	-13.09%	97.2	2.86%	416.83	-15.95%
1967	3,521	96.47	20.09%	100.0	2.88%	488.58	17.21%
1968	3,502	103.86	7.66%	104.2	4.20%	505.48	3.46%
1969	3,600	92.06	-11.36%	109.8	5.37%	420.89	-16.74%
1970	3,731	92.15	0.10%	116.3	5.92%	396.38	-5.82%
1971	3,556	102.09	10.79%	121.3	4.30%	422.10	6.49%
1972	3,258	118.05	15.63%	125.3	3.30%	474.17	12.34%
1973	3,137	97.55	-17.37%	133.1	6.23%	362.31	-23.59%
1974	3,160	68.56	-29.72%	147.7	10.97%	214.89	-40.69%
1975	3,144	90.19	31.55%	161.2	9.14%	263.05	22.41%
1976	3,168	107.48	19.17%	170.5	5.77%	298.30	13.40%
1977	3,327	95.10	-11.52%	181.5	6.45%	244.70	-17.97%
1978	3,333	96.11	1.06%	195.4	7.66%	228.56	-6.60%
1979	3,494	107.94	12.31%	217.4	11.26%	230.96	1.05%
1980	3,612	135.76	25.77%	246.8	13.52%	259.25	12.25%
1981	3,629	122.55	-9.73%	272.4	10.37%	207.13	-20.10%
1982	3,681	140.64	14.76%	289.1	6.13%	225.01	8.63%
1983	3,639	164.93	17.27%	298.4	3.22%	256.63	14.05%
1984	3,669	167.24	1.40%	311.1	4.26%	249.30	-2.86%
1985	3,761	211.28	26.33%	322.2	3.57%	306.06	22.77%
1986	3,757	242.17	14.62%	328.4	1.92%	344.91	12.70%
1987	3,809	247.08	2.03%	340.4	3.65%	339.30	-1.63%
1988	3,910	277.72	12.40%	354.3	4.08%	367.53	8.32%
1989	4,041	353.40	27.25%	371.3	4.80%	450.04	22.45%
1990	4,158	330.22	-6.56%	391.4	5.41%	396.16	-11.97%
1991	4,111	417.09	26.31%	408.0	4.24%	483.58	22.07%
1992	4,084	435.71	4.46%	420.3	3.01%	490.59	1.45%
1993	4,000	466.45	7.06%	432.7	2.95%	510.72	4.10%
1994	3,979	459.27	-1.54%	444.0	2.61%	489.53	-4.15%
1995		616.71	34.28%	456.5	2.82%	643.56	31.47%
1996		740.47	20.07%	469.9	2.94%	753.81	17.13%
1997		970.43	31.06%	480.8	2.32%	970.43	28.74%

Table 4. Data Series for United States and S&P 500

Year	Births	Nikkei	Return	CPI	Inflation	Nikkei(A)	Real Return
1934	2,028						
1935	2,174						
1936	2,086						
1937	2,165						
1938	1,912						
1939	1,886						
1940	2,100						
1941	2,260						
1942	2,216						
1943	2,235						
1944	2,150						
1945	1,686						
1946	1,906						
1947	2,679						
1948	2,682	176.52		15.5		1364.06	
1949	2,697	109.91	-37.74%	14.5	-6.29%	935.18	-31.44%
1950	2,338	101.91	-7.28%	14.4	-0.75%	874.09	-6.53%
1951	2,138	166.06	62.95%	16.1	12.03%	1319.16	50.92%
1952	2,005	362.64	118.38%	16.2	0.67%	2871.92	117.71%
1953	1,868	377.95	4.22%	18.0	10.67%	2686.82	-6.45%
1954	1,770	356.09	-5.78%	18.2	1.20%	2499.05	-6.99%
1955	1,731	425.69	19.55%	18.0	-0.91%	3010.30	20.46%
1956	1,665	549.14	29.00%	18.5	2.65%	3803.43	26.35%
1957	1,567	474.55	-13.58%	18.8	1.81%	3218.07	-15.39%
1958	1,653	666.54	40.46%	18.9	0.32%	4509.84	40.14%
1959	1,626	874.88	31.26%	19.2	1.89%	5834.11	29.36%
1960	1,606	1356.71	55.07%	20.0	3.72%	8830.43	51.36%
1961	1,589	1432.60	5.59%	21.8	9.25%	8507.23	-3.66%
1962	1,619	1420.43	-0.85%	22.8	4.64%	8039.83	-5.49%
1963	1,660	1225.10	-13.75%	24.2	6.27%	6430.43	-20.02%
1964	1,717	1216.55	-0.70%	25.4	4.67%	6085.36	-5.37%
1965	1,824	1417.83	16.55%	27.0	6.57%	6692.21	9.97%
1966	1,361	1452.10	2.42%	28.2	4.40%	6559.17	-1.99%
1967	1,936	1283.47	-11.61%	29.8	5.70%	5423.84	-17.31%
1968	1,872	1714.89	33.61%	31.0	3.79%	7041.29	29.82%
1969	1,890	2358.96	37.56%	32.9	6.35%	9238.96	31.21%
1970	1,934	1987.10	-15.76%	35.6	8.07%	7036.98	-23.83%
1971	2,001	2713.70	36.57%	37.3	4.78%	9274.08	31.79%
1972	2,039	5207.90	91.91%	39.5	5.90%	17251.02	86.01%
1973	2,092	4306.80	-17.30%	46.7	18.23%	11121.66	-35.53%
1974	2,030	3817.22	-11.37%	56.5	20.98%	7523.52	-32.35%
1975	1,901	4358.43	14.18%	61.0	7.96%	7990.99	6.21%
1976	1,833	4990.80	14.51%	67.4	10.49%	8312.02	4.02%
1977	1,755	4865.60	-2.51%	70.8	5.04%	7684.20	-7.55%
1978	1,709	6001.90	23.35%	73.5	3.81%	9185.71	19.54%
1979	1,643	6569.73	9.46%	77.7	5.71%	9529.85	3.75%
1980	1,577	7116.38	8.32%	83.3	7.21%	9635.97	1.11%
1981	1,529	7681.80	7.95%	86.9	4.32%	9985.14	3.62%
1982	1,515	8016.70	4.36%	88.7	2.07%	10213.63	2.29%
1983	1,509	9893.82	23.42%	90.2	1.69%	12432.44	21.72%
1984	1,490	11542.60	16.66%	92.6	2.66%	14173.48	14.00%
1985	1,432	13113.30	13.61%	93.9	1.40%	15903.21	12.20%
1986	1,383	18701.30	42.61%	93.6	-0.32%	22730.88	42.93%
1987	1,347	21564.00	15.31%	94.4	0.85%	26016.12	14.45%
1988	1,314	30159.00	39.86%	95.3	0.95%	36137.64	38.90%
1989	1,247	38915.90	29.04%	97.8	2.62%	45682.49	26.41%
1990	1,222	23848.70	-38.72%	101.5	3.78%	26267.36	-42.50%
1991	1,223	22983.80	-3.63%	104.2	2.66%	24615.81	-6.29%
1992	1,209	16924.90	-26.36%	105.4	1.15%	17843.22	-27.51%
1993	1,188	17417.20	2.91%	106.5	1.04%	18176.03	1.87%
1994	1,238	19723.10	13.24%	107.2	0.66%	20462.94	12.58%
1995		19868.20	0.74%	106.9	-0.28%	20670.74	1.02%
1996		19369.00	-2.51%	107.4	0.46%	20056.44	-2.97%
1997		15258.70	-21.22%	110.3	2.70%	15258.70	-23.92%

Table 5. Data Series for Japan and the Nikkei 225

Year	Births	FTSE	Return	CPI	Inflation	FTSE(A)	Real Return
1934	712						
1935	711						
1936	720						
1937	724						
1938	736						
1939	727						
1940	702						
1941	696						
1942	772						
1943	811						
1944	878						
1945	796						
1946	955						
1947	1,025						
1948	905						
1949	855						
1950	818						
1951	797						
1952	793						
1953	804						
1954	795						
1955	789						
1956	825						
1957	851						
1958	870						
1959	879						
1960	918						
1961	944						
1962	976						
1963	990						
1964	1,015						
1965	997						
1966	980						
1967	962						
1968	947						
1969	920						
1970	904						
1971	902						
1972	834						
1973	780						
1974	737						
1975	698						
1976	676						
1977	656						
1978	687	484.2		41.1		1452.68	
1979	735	509.2	5.16%	48.1	17.18%	1278.09	-12.02%
1980	754	647.4	27.14%	55.4	15.16%	1431.26	11.98%
1981	731	684.3	5.70%	62.1	12.02%	1340.84	-6.32%
1982	719	834.3	21.92%	65.4	5.36%	1562.84	16.56%
1983	722	1000.0	19.86%	68.9	5.33%	1789.88	14.53%
1984	730	1232.2	23.22%	72.1	4.60%	2123.10	18.62%
1985	751	1412.6	14.64%	76.1	5.61%	2314.81	9.03%
1986	755	1679.0	18.86%	79.0	3.75%	2664.56	15.11%
1987	776	1713.9	2.08%	81.9	3.71%	2620.96	-1.64%
1988	788	1793.1	4.62%	87.5	6.78%	2564.47	-2.16%
1989	777	2422.7	35.11%	94.2	7.71%	3267.29	27.41%
1990	799	2143.5	-11.52%	103.0	9.34%	2585.48	-20.87%
1991	793	2493.1	16.31%	107.6	4.47%	2891.72	11.84%
1992	781	2846.5	14.18%	110.4	2.58%	3227.04	11.60%
1993	762	3418.4	20.09%	112.5	1.94%	3812.82	18.15%
1994	751	3065.5	-10.32%	115.8	2.89%	3309.03	-13.21%
1995		3689.3	20.35%	119.5	3.22%	3875.85	17.13%
1996		4118.5	11.63%	122.4	2.44%	4232.19	9.19%
1997		5135.5	24.69%	126.5	3.35%	5135.50	21.34%

Table 6. Data Series for United Kingdom and the FTSE-100

Year	Births	DAX	Return	CPI	Inflation	DAX(A)	Real Return
1934	1,198						
1935	1,264						
1936	1,279						
1937	1,277						
1938	1,349						
1939	1,413						
1940	1,402						
1941	1,308						
1942	1,056						
1943	1,125						
1944	1,075						
1945	1,019						
1946	962						
1947	1,030						
1948	1,049						
1949	1,107						
1950	1,116						
1951	1,074						
1952	1,075						
1953	1,064						
1954	1,079						
1955	1,096						
1956	1,120						
1957	1,148						
1958	1,157						
1959	1,223	417.80		35.1		1479.11	
1960	1,240	534.10	27.84%	35.4	0.85%	1878.28	26.99%
1961	1,290	489.80	-8.29%	36.4	2.74%	1671.08	-11.03%
1962	1,292	386.30	-21.13%	37.4	2.87%	1270.02	-24.00%
1963	1,330	438.90	13.62%	38.7	3.39%	1399.94	10.23%
1964	1,331	477.90	8.89%	39.5	2.12%	1494.67	6.77%
1965	1,325	422.40	-11.61%	41.1	3.96%	1261.87	-15.58%
1966	1,318	333.40	-21.07%	42.3	2.90%	959.35	-23.97%
1967	1,272	503.20	50.93%	42.5	0.53%	1442.86	50.40%
1968	1,215	555.60	10.41%	43.5	2.28%	1560.21	8.13%
1969	1,142	622.40	12.02%	44.4	2.06%	1715.68	9.96%
1970	1,048	443.90	-28.68%	46.1	4.03%	1154.43	-32.71%
1971	1,014	473.50	6.67%	48.7	5.49%	1168.00	1.18%
1972	902	536.40	13.28%	51.8	6.43%	1248.03	6.85%
1973	816	396.30	-26.12%	55.8	7.77%	825.09	-33.89%
1974	806	401.80	1.39%	59.0	5.74%	789.18	-4.35%
1975	782	563.20	40.17%	62.2	5.43%	1063.34	34.74%
1976	798	509.00	-9.62%	64.6	3.71%	921.53	-13.34%
1977	805	549.30	7.92%	66.8	3.46%	962.57	4.45%
1978	808	575.10	4.70%	68.4	2.46%	984.14	2.24%
1979	817	497.80	-13.44%	72.2	5.45%	798.26	-18.89%
1980	828	480.90	-3.39%	76.1	5.48%	727.45	-8.87%
1981	832	490.40	1.98%	81.2	6.76%	692.66	-4.78%
1982	822	552.80	12.72%	85.0	4.59%	749.03	8.14%
1983	827	774.00	40.01%	87.2	2.63%	1029.03	37.38%
1984	812	820.90	6.06%	88.9	1.97%	1071.16	4.09%
1985	814	1366.20	66.43%	90.4	1.70%	1764.46	64.72%
1986	848	1432.25	4.83%	89.5	-1.00%	1867.35	5.83%
1987	868	1000.00	-30.18%	90.4	1.01%	1284.99	-31.19%
1988	893	1327.87	32.79%	92.1	1.79%	1683.23	30.99%
1989	880	1790.37	34.83%	94.9	3.09%	2217.52	31.74%
1990	902	1398.23	-21.90%	97.5	2.74%	1671.07	-24.64%
1991	830	1577.98	12.86%	102.7	5.32%	1797.07	7.54%
1992	809	1545.05	-2.09%	106.1	3.31%	1700.07	-5.40%
1993	798	2266.68	46.71%	110.6	4.24%	2421.99	42.46%
1994	770	2106.58	-7.06%	113.4	2.53%	2189.60	-9.60%
1995	760	2253.88	6.99%	115.4	1.76%	2304.10	5.23%
1996		2888.69	28.17%	117.0	1.39%	2921.11	26.78%
1997		4249.69	47.11%	118.9	1.63%	4249.69	45.48%

Table 7. Data Series for Germany and the DAX

Lag	R^2_a	b_0	b_1	t-stat	F-stat	GQ
40	0.691	-535.17	0.281	6.24	38.97	5.01
41	0.681	-464.92	0.270	6.11	37.33	6.21
42	0.714	-444.89	0.272	6.59	43.48	12.10
43	0.731	-416.97	0.272	6.88	47.30	16.07
44	0.738	-396.23	0.275	6.99	48.86	22.85
45	0.760	-394.62	0.283	7.40	54.70	44.58
46	0.750	-393.94	0.292	7.22	52.05	42.46
47	0.711	-396.62	0.303	6.36	40.49	21.22
48	0.779	-421.42	0.323	7.34	53.90	21.45
49	0.864	-468.74	0.353	9.46	89.58	9.17
50	0.883	-514.25	0.385	9.96	99.11	4.92

Table 8. Regression Results - U.S. and the S&P 500

Lag	R^2_a	b_0	b_1	t-stat	F-stat	GQ
42	0.861	3.93	6.34E-04	10.33	106.70	3.10
43	0.866	4.02	6.28E-04	10.51	110.41	3.95
44	0.860	4.08	6.30E-04	10.26	405.20	3.95
45	0.867	4.10	6.43E-04	10.55	111.34	4.98
46	0.838	4.12	6.57E-04	9.43	88.97	3.58
47	0.783	4.14	6.71E-04	7.67	58.80	2.40
48	0.820	4.21	6.72E-04	8.34	69.55	1.72

Table 9. Transformation Regression Results - U.S. and the S&P 500

Lag	R^2_a	b_0	b_1	t-stat	F-stat	GQ
40	-0.003	316.26	3.03	0.97	0.95	
41	-0.004	437.87	2.91	0.97	0.94	
42	0.033	-124.01	3.62	1.26	1.59	
43	0.059	-384.85	3.95	1.43	2.06	
44	0.075	-482.49	4.09	1.54	2.39	
45	0.194	-1520.76	5.42	2.26	5.09	1.20
46	0.295	-2151.62	6.24	2.85	8.12	1.10
47	0.337	-2053.47	6.22	3.02	9.12	2.80
48	0.397	-1870.71	6.12	3.30	10.87	1.85
49	0.658	-2499.72	7.07	5.29	27.97	1.33
50	0.783	-2555.23	7.33	6.92	47.95	1.76

Table 10. Regression Results - UK and the FTSE-100

Lag	R^2_a	b_0	b_1	t-stat	F-stat	GQ
25	-0.059	25769.01	-3.12	0.24	0.06	
29	0.274	69772.24	-28.51	2.72	7.41	8.72
30	0.312	62417.15	-23.91	2.95	8.73	10.08
31	0.259	49271.31	-16.05	2.63	6.94	4.20
35	0.049	34849.15	-7.50	1.37	1.89	
39	0.092	524.46	9.58	1.65	2.72	
40	0.238	-8800.54	13.92	2.51	6.31	6.83
41	0.223	-10083.55	14.41	2.43	5.89	2.89
42	0.076	-1989.37	10.51	1.55	2.41	
43	-0.057	24879.28	-2.11	0.27	0.08	
44	-0.041	30201.18	-4.55	0.57	0.33	
45	-0.062	19419.47	0.43	0.05	0.00	
46	-0.045	10997.00	4.28	0.52	0.27	
47	-0.045	11209.34	4.46	0.55	0.30	
48	-0.071	22228.01	-0.26	0.03	0.00	
49	-0.017	38876.74	-7.67	0.87	0.76	
50	0.113	56517.36	-15.84	1.63	2.66	

Lag	R^2_a	b_0	b_1	t-stat	F-stat	GQ
29	0.499	1.08E-04	9.60E-08	4.23	17.90	1.23
30	0.668	9.49E-05	8.71E-08	5.94	35.32	1.63

Table 11. Regression Results - Japan and the Nikkei 225

Lag	R^2_a	b_0	b_1	t-stat	F-stat	GQ
35	0.731	-7453.32	8.27	6.87	47.23	3.32
36	0.791	-8901.90	9.67	8.08	65.37	1.03
37	0.559	-9078.32	9.91	4.75	22.53	4.60
38	0.424	-9298.74	10.20	3.68	13.56	2.55
39	-0.052	484.48	1.20	0.39	0.16	
40	-0.002	4070.18	-2.06	0.98	0.96	
41	0.139	5233.69	-3.17	1.93	3.74	
42	0.251	5778.63	-3.51	2.59	6.70	14.84
43	0.352	6294.78	-3.93	3.20	10.25	3.80
44	0.310	6021.83	-3.66	2.94	8.62	4.53
45	0.199	5366.20	-3.06	2.29	5.23	1.34
46	0.158	5131.38	-2.84	2.05	4.19	
47	0.155	5102.09	-2.75	1.98	3.93	
48	0.133	4877.63	-2.49	1.82	3.31	
49	0.199	5209.46	-2.69	2.11	4.47	
50	0.236	5446.12	-2.81	2.24	5.01	6.28

Lag	R^2_a	b_0	b_1	t-stat	F-stat	GQ
42	0.669	1.82E-03	2.23	5.95	35.41	4.32

Table 12. Regression Results - Germany and the DAX

	0.0%	1.0%	2.0%	3.0%	4.0%	5.0%	6.0%	7.0%	8.0%
1997	6,051.8	6,051.8	6,051.8	6,051.8	6,051.8	6,051.8	6,051.8	6,051.8	6,051.8
1998	6,495.6	6,556.1	6,616.6	6,677.1	6,737.7	6,798.2	6,858.7	6,919.2	6,979.7
1999	6,764.3	6,892.9	7,022.7	7,153.7	7,285.9	7,419.4	7,554.0	7,689.8	7,826.9
2000	7,362.7	7,571.6	7,784.4	8,001.2	8,221.9	8,446.7	8,675.5	8,908.4	9,145.5
2001	7,527.4	7,816.6	8,114.2	8,420.1	8,734.6	9,057.9	9,390.0	9,731.2	10,081.6
2002	8,199.3	8,592.6	9,000.8	9,424.4	9,863.7	10,319.3	10,791.6	11,281.0	11,788.1
2003	8,774.6	9,281.4	9,812.4	10,368.4	10,950.4	11,559.4	12,196.3	12,862.3	13,558.2
2004	8,450.9	9,031.8	9,646.6	10,296.9	10,984.4	11,710.9	12,478.1	13,288.1	14,142.7
2005	8,710.2	9,399.2	10,135.5	10,921.7	11,760.8	12,655.7	13,609.7	14,626.0	15,708.0
2006	8,794.1	9,583.7	10,435.8	11,354.5	12,344.5	13,410.4	14,557.3	15,790.6	17,115.9
2007	8,859.1	9,750.5	10,721.7	11,779.2	12,929.6	14,180.1	15,538.5	17,012.8	18,611.8
2008	8,223.5	9,148.4	10,166.9	11,287.5	12,519.1	13,871.8	15,356.0	16,983.1	18,765.4
2009	7,815.7	8,786.3	9,866.1	11,066.4	12,399.1	13,877.5	15,515.8	17,329.7	19,336.1
2010	7,417.2	8,426.1	9,560.4	10,834.1	12,262.8	13,863.7	15,655.6	17,659.2	19,897.1
2011	6,092.1	7,005.0	8,043.6	9,223.6	10,562.6	12,080.1	13,798.0	15,740.5	17,934.2
2012	5,438.4	6,323.4	7,341.3	8,510.5	9,851.6	11,387.8	13,145.3	15,153.2	17,444.4
2013	5,108.1	6,002.6	7,042.3	8,248.9	9,647.3	11,265.6	13,135.6	15,293.6	17,780.5
2014	5,037.0	5,979.1	7,085.2	8,381.7	9,899.1	11,672.2	13,741.1	16,151.5	18,955.7
2015	5,414.4	6,486.8	7,757.6	9,261.0	11,036.6	13,130.2	15,594.9	18,492.0	21,892.1
2016	5,963.3	7,209.3	8,699.2	10,477.7	12,596.9	15,117.8	18,111.6	21,661.1	25,862.8
2017	5,241.6	6,408.9	7,820.4	9,524.0	11,576.3	14,044.1	17,006.4	20,555.9	24,801.9
2018	4,207.9	5,209.1	6,434.6	7,931.5	9,756.4	11,976.7	14,673.0	17,941.0	21,894.9
2019	3,848.8	4,816.7	6,014.2	7,492.7	9,314.1	11,553.6	14,301.3	17,666.0	21,778.2
2020	3,914.6	4,947.2	6,237.3	7,845.6	9,846.0	12,328.8	15,403.9	19,204.7	23,892.9
2021	3,868.8	4,938.7	6,288.9	7,988.9	10,124.4	12,800.7	16,147.6	20,323.9	25,524.1
2022	3,937.8	5,076.2	6,527.0	8,371.2	10,710.0	13,669.2	17,404.6	22,109.3	28,021.6
2023	4,427.5	5,758.2	7,469.1	9,663.3	12,470.2	16,052.4	20,613.1	26,406.1	33,747.7
2024	4,447.1	5,841.3	7,651.6	9,996.0	13,024.3	16,926.1	21,941.2	28,371.6	36,597.1
2025	5,007.4	6,635.7	8,768.8	11,555.4	15,186.4	19,905.2	26,022.4	33,932.5	44,136.3
2026	5,462.5	7,305.1	9,741.0	12,952.2	17,173.9	22,709.3	29,948.5	39,391.4	51,678.1
2027	5,531.4	7,470.3	10,058.7	13,504.1	18,077.4	24,131.2	32,123.0	42,645.5	56,464.0

Table 13. Forecast - US-DJIA with 46-year Transformed Model

	0.0%	1.0%	2.0%	3.0%	4.0%	5.0%	6.0%	7.0%	8.0%
1997	7,622.2	7,622.2	7,622.2	7,622.2	7,622.2	7,622.2	7,622.2	7,622.2	7,622.2
1998	7,062.6	7,138.8	7,215.0	7,291.2	7,367.5	7,443.7	7,519.9	7,596.1	7,672.3
1999	7,147.4	7,295.9	7,445.9	7,597.5	7,750.6	7,905.2	8,061.4	8,219.1	8,378.3
2000	7,127.0	7,348.1	7,573.7	7,803.8	8,038.6	8,278.0	8,522.1	8,771.0	9,024.7
2001	7,795.2	8,110.5	8,435.2	8,769.5	9,113.7	9,468.0	9,832.4	10,207.3	10,592.7
2002	8,120.8	8,530.3	8,956.2	9,398.9	9,859.0	10,336.8	10,833.0	11,348.1	11,882.6
2003	8,307.3	8,811.6	9,341.1	9,896.8	10,479.8	11,091.1	11,731.9	12,403.2	13,106.1
2004	8,697.4	9,313.4	9,966.5	10,658.4	11,391.0	12,166.4	12,986.6	13,853.7	14,770.0
2005	8,799.1	9,515.5	10,282.4	11,102.8	11,980.0	12,917.1	13,917.8	14,985.6	16,124.4
2006	9,192.6	10,036.1	10,947.8	11,932.4	12,994.8	14,140.5	15,375.1	16,704.6	18,135.3
2007	9,504.6	10,477.2	11,538.4	12,695.4	13,955.7	15,327.5	16,819.5	18,441.0	20,201.7
2008	9,331.6	10,391.3	11,559.2	12,845.2	14,260.0	15,815.0	17,522.6	19,396.2	21,450.2
2009	9,470.7	10,650.0	11,962.6	13,422.0	15,042.9	16,841.4	18,835.1	21,043.0	23,485.9
2010	9,514.8	10,806.1	12,257.6	13,887.1	15,714.6	17,761.9	20,052.9	22,614.0	25,474.1
2011	9,548.7	10,952.7	12,546.4	14,353.2	16,399.2	18,713.3	21,327.5	24,277.6	27,602.9
2012	9,206.1	10,669.3	12,347.2	14,268.9	16,466.9	18,977.6	21,842.1	25,106.1	28,820.8
2013	8,972.1	10,504.8	12,280.3	14,334.2	16,706.9	19,444.1	22,597.3	26,225.3	30,393.8
2014	8,731.3	10,327.9	12,196.3	14,379.5	16,926.8	19,894.4	23,346.7	27,357.1	32,009.6
2015	7,825.7	9,360.0	11,175.2	13,319.5	15,848.2	18,825.7	22,326.0	26,434.7	31,250.4
2016	7,303.4	8,828.8	10,652.9	12,830.1	15,424.4	18,510.5	22,175.4	26,520.8	31,664.6
2017	7,015.1	8,568.6	10,445.4	12,708.5	15,432.5	18,705.3	22,630.6	27,330.3	32,947.8
2018	6,950.6	8,575.6	10,558.4	12,973.0	15,908.0	19,468.7	23,780.5	28,992.4	35,281.0
2019	7,283.0	9,071.4	11,274.4	13,982.6	17,305.1	21,373.2	26,344.6	32,408.3	39,790.6
2020	7,727.4	9,715.6	12,187.7	15,255.1	19,053.0	23,745.8	29,532.5	36,654.0	45,401.4
2021	7,133.8	9,066.5	11,495.3	14,541.0	18,351.7	23,109.1	29,036.0	36,404.3	45,546.1
2022	6,123.1	7,872.6	10,096.6	12,917.0	16,485.6	20,990.4	26,664.2	33,794.8	42,736.7
2023	5,712.7	7,423.6	9,621.8	12,438.8	16,040.1	20,633.0	26,476.9	33,895.3	43,291.3
2024	5,790.7	7,599.2	9,945.6	12,981.8	16,900.7	21,946.4	28,427.1	36,730.9	47,345.7
2025	5,736.4	7,604.0	10,051.3	13,249.6	17,418.4	22,838.1	29,866.3	38,957.8	50,689.7
2026	5,817.8	7,788.0	10,395.0	13,835.1	18,362.3	24,304.1	32,082.1	42,237.7	55,464.2
2027	6,357.1	8,587.7	11,566.4	15,532.6	20,798.9	27,772.1	36,980.9	49,109.5	65,042.6

Table 14. Forecast - US-DJIA with 50-year Linear Model

	0.0%	1.0%	2.0%	3.0%	4.0%	5.0%	6.0%	7.0%	8.0%
1997	717.0	717.0	717.0	717.0	717.0	717.0	717.0	717.0	717.0
1998	742.8	750.0	757.2	764.3	771.5	778.7	785.9	793.0	800.2
1999	799.9	815.1	830.4	846.0	861.6	877.4	893.3	909.4	925.6
2000	815.4	839.1	863.2	887.8	912.9	938.4	964.3	990.8	1,017.7
2001	878.6	912.5	947.4	983.2	1,020.1	1,058.0	1,096.9	1,136.9	1,178.0
2002	932.2	977.3	1,024.1	1,072.7	1,123.1	1,175.4	1,229.6	1,285.8	1,344.0
2003	902.1	955.5	1,011.5	1,070.2	1,131.7	1,196.2	1,263.7	1,334.3	1,408.2
2004	926.2	990.6	1,058.8	1,130.9	1,207.3	1,288.0	1,373.3	1,463.4	1,558.5
2005	934.0	1,008.8	1,088.8	1,174.3	1,265.7	1,363.2	1,467.2	1,578.1	1,696.2
2006	940.0	1,025.4	1,117.6	1,217.2	1,324.5	1,440.1	1,564.7	1,698.7	1,842.9
2007	880.9	971.2	1,069.7	1,177.1	1,294.2	1,421.6	1,560.2	1,710.8	1,874.4
2008	842.6	938.7	1,044.6	1,161.3	1,289.7	1,430.9	1,586.0	1,756.3	1,943.0
2009	805.0	906.2	1,018.9	1,144.3	1,283.8	1,438.6	1,610.4	1,800.8	2,011.7
2010	678.0	772.3	878.5	998.1	1,132.5	1,283.5	1,452.9	1,642.7	1,855.2
2011	614.1	707.2	813.2	933.9	1,071.0	1,226.7	1,403.1	1,602.8	1,828.6
2012	581.4	676.6	786.2	912.2	1,056.9	1,222.7	1,412.6	1,629.7	1,877.6
2013	574.3	675.2	792.4	928.5	1,086.3	1,269.0	1,480.2	1,724.0	2,005.0
2014	611.7	725.8	859.8	1,016.8	1,200.5	1,415.0	1,665.3	1,956.8	2,295.9
2015	665.5	796.9	952.6	1,136.7	1,354.0	1,610.2	1,911.6	2,265.8	2,681.4
2016	594.6	720.0	870.2	1,049.8	1,264.0	1,519.3	1,822.8	2,183.2	2,610.5
2017	490.9	601.6	735.8	898.2	1,094.1	1,330.2	1,614.3	1,955.3	2,364.0
2018	454.1	562.6	695.5	857.9	1,056.0	1,297.2	1,590.2	1,945.7	2,376.1
2019	460.9	576.6	719.7	896.4	1,113.9	1,381.3	1,709.4	2,110.9	2,601.6
2020	456.2	576.5	726.8	914.1	1,147.1	1,436.3	1,794.4	2,237.1	2,783.1
2021	463.3	591.2	752.6	955.7	1,210.8	1,530.4	1,930.0	2,428.5	3,049.0
2022	513.2	660.8	848.7	1,087.3	1,389.6	1,771.7	2,253.6	2,860.0	3,621.3
2023	515.2	670.0	869.0	1,124.1	1,450.6	1,867.2	2,397.6	3,071.2	3,925.0
2024	571.4	749.8	981.1	1,280.5	1,666.9	2,164.3	2,803.0	3,621.3	4,667.2
2025	616.4	816.4	1,078.1	1,419.9	1,865.0	2,443.1	3,192.2	4,160.3	5,408.6
2026	623.2	833.5	1,111.5	1,478.1	1,960.1	2,592.1	3,418.8	4,497.3	5,900.7
2027	644.4	870.2	1,171.6	1,572.7	2,105.2	2,809.9	3,740.2	4,965.1	6,573.5

Table 15. Forecast - US-S&P with 45-year Transformed Model

	0.0%	1.0%	2.0%	3.0%	4.0%	5.0%	6.0%	7.0%	8.0%
1997	909.9	909.9	909.9	909.9	909.9	909.9	909.9	909.9	909.9
1998	846.4	855.5	864.6	873.7	882.8	891.9	901.0	910.1	919.2
1999	856.0	873.8	891.7	909.8	928.1	946.6	965.3	984.1	1,003.2
2000	853.7	880.1	907.1	934.7	962.7	991.4	1,020.6	1,050.4	1,080.7
2001	929.5	967.1	1,005.8	1,045.7	1,086.8	1,129.0	1,172.5	1,217.2	1,263.1
2002	966.5	1,015.2	1,065.9	1,118.7	1,173.4	1,230.3	1,289.4	1,350.8	1,414.4
2003	987.6	1,047.6	1,110.6	1,176.7	1,246.1	1,318.8	1,395.0	1,474.9	1,558.5
2004	1,031.9	1,105.0	1,182.6	1,264.8	1,351.8	1,443.8	1,541.3	1,644.2	1,753.1
2005	1,043.4	1,128.5	1,219.5	1,316.9	1,421.0	1,532.2	1,651.0	1,777.7	1,912.9
2006	1,088.1	1,188.0	1,296.0	1,412.7	1,538.6	1,674.4	1,820.7	1,978.2	2,147.8
2007	1,123.5	1,238.6	1,364.1	1,501.1	1,650.2	1,812.6	1,989.2	2,181.1	2,389.5
2008	1,103.9	1,229.3	1,367.6	1,519.9	1,687.4	1,871.5	2,073.8	2,295.7	2,539.0
2009	1,119.7	1,259.2	1,414.5	1,587.2	1,779.0	1,991.9	2,227.8	2,489.2	2,778.4
2010	1,124.7	1,277.4	1,449.1	1,641.9	1,858.1	2,100.4	2,371.5	2,674.6	3,013.1
2011	1,128.5	1,294.6	1,483.1	1,696.8	1,938.8	2,212.6	2,521.9	2,870.9	3,264.4
2012	1,089.6	1,262.9	1,461.6	1,689.2	1,949.6	2,247.0	2,586.3	2,973.0	3,413.1
2013	1,063.1	1,244.8	1,455.2	1,698.7	1,980.0	2,304.6	2,678.5	3,108.6	3,603.0
2014	1,035.7	1,225.2	1,446.9	1,706.0	2,008.3	2,360.5	2,770.3	3,246.3	3,798.6
2015	933.0	1,115.9	1,332.3	1,587.9	1,889.4	2,244.4	2,661.7	3,151.5	3,725.6
2016	873.7	1,056.2	1,274.3	1,534.7	1,844.9	2,214.0	2,652.2	3,171.9	3,786.9
2017	841.0	1,027.2	1,252.1	1,523.3	1,849.6	2,241.8	2,712.1	3,275.1	3,948.1
2018	833.7	1,028.5	1,266.2	1,555.7	1,907.5	2,334.4	2,851.2	3,475.9	4,229.6
2019	871.4	1,085.3	1,348.8	1,672.8	2,070.2	2,556.7	3,151.3	3,876.5	4,759.3
2020	921.8	1,159.0	1,453.9	1,819.7	2,272.8	2,832.5	3,522.7	4,372.1	5,415.5
2021	854.5	1,085.9	1,376.7	1,741.4	2,197.6	2,767.1	3,476.7	4,358.7	5,453.0
2022	739.8	951.0	1,219.4	1,559.9	1,990.5	2,534.0	3,218.6	4,078.7	5,157.2
2023	693.2	900.6	1,167.1	1,508.4	1,944.8	2,501.2	3,209.0	4,107.4	5,245.1
2024	702.0	921.1	1,205.3	1,573.0	2,047.4	2,658.2	3,442.6	4,447.4	5,731.7
2025	695.9	922.3	1,218.8	1,606.4	2,111.4	2,767.8	3,618.9	4,719.7	6,140.0
2026	705.1	943.7	1,259.4	1,675.9	2,223.8	2,942.9	3,884.1	5,112.8	6,712.7
2027	766.3	1,035.1	1,393.9	1,871.6	2,505.8	3,345.5	4,454.3	5,914.4	7,832.3

Table 16. Forecast - US-S&P with 50-year Linear Model

	0.0%	1.0%	2.0%	3.0%	4.0%	5.0%	6.0%	7.0%	8.0%
1997	13569.0	13569.0	13569.0	13569.0	13569.0	13569.0	13569.0	13569.0	13569.0
1998	14679.0	14814.7	14950.4	15086.0	15221.7	15357.4	15493.1	15628.8	15764.5
1999	14348.9	14629.6	14913.1	15199.3	15488.3	15779.9	16074.3	16371.3	16671.1
2000	13601.1	14013.6	14434.3	14863.3	15300.7	15746.6	16201.1	16664.2	17136.1
2001	12601.3	13123.5	13661.8	14216.5	14787.9	15376.4	15982.2	16605.7	17247.2
2002	12096.9	12729.5	13388.3	14074.0	14787.5	15529.7	16301.4	17103.4	17936.7
2003	11457.3	12183.7	12948.1	13752.1	14597.2	15485.1	16417.6	17396.3	18423.2
2004	12212.7	13108.8	14060.8	15071.3	16143.5	17280.3	18485.0	19761.0	21111.7
2005	14154.3	15324.1	16577.5	17919.6	19355.8	20891.7	22533.0	24286.0	26157.2
2006	15449.1	16879.1	18425.4	20096.4	21900.6	23847.3	25946.2	28207.6	30642.4
2007	17260.1	19026.5	20953.9	23055.1	25343.9	27835.1	30544.5	33488.8	36685.9
2008	18542.0	20629.8	22929.1	25459.0	28239.9	31294.1	34645.6	38320.1	42345.4
2009	20753.4	23296.6	26122.4	29259.1	32737.6	36591.2	40856.4	45572.8	50783.4
2010	23563.7	26684.3	30182.2	34099.1	38480.3	43375.8	48840.5	54934.3	61722.9
2011	26137.9	29866.2	34083.1	38847.2	44223.2	50283.1	57106.4	64780.9	73403.6
2012	26998.2	31147.8	35886.5	41291.1	47447.6	54452.2	62412.2	71447.6	81691.7
2013	27384.4	31904.9	37117.6	43120.6	50024.3	57953.8	67049.9	77471.1	89395.8
2014	28684.0	33738.0	39621.4	46460.5	54399.2	63601.7	74254.7	86570.4	100789.7
2015	33543.0	39790.6	47125.7	55724.7	65790.4	77555.9	91288.8	107295.4	125926.7
2016	39145.3	46834.2	55939.0	66703.4	79410.1	94386.9	112012.9	132726.2	157032.7
2017	44620.5	53853.2	64881.9	78034.2	93693.5	112308.0	134400.7	160581.3	191559.4
2018	51182.8	62311.9	75721.7	91851.7	111220.7	134440.5	162231.0	195438.6	235056.6
2019	72971.9	89461.9	109471.7	133709.5	163017.4	198395.3	241028.4	292319.7	353927.3
2020	86752.2	107250.9	132334.3	162971.1	200323.1	245781.0	301007.1	367985.1	449078.9
2021	86101.8	107519.3	133988.8	166638.5	206834.2	256227.4	316810.8	390985.2	481638.4
2022	96198.9	121203.2	152381.3	191179.1	239362.8	299086.3	372971.5	464204.6	576650.6
2023	116732.5	148286.0	187954.7	237721.6	300029.2	377880.5	474960.4	595783.3	745868.6
2024	77397.9	99801.8	128379.8	164749.6	210931.3	269442.4	343413.5	436730.6	554207.2

Table 17. Forecast - Japan -Nikkei with 30-year Transformed Model

	0.0%	1.0%	2.0%	3.0%	4.0%	5.0%	6.0%	7.0%	8.0%
1997	4,960.3	4,960.3	4,960.3	4,960.3	4,960.3	4,960.3	4,960.3	4,960.3	4,960.3
1998	4,080.4	4,130.1	4,179.7	4,229.3	4,278.9	4,328.5	4,378.1	4,427.7	4,477.3
1999	3,713.8	3,800.3	3,887.7	3,976.2	4,065.6	4,156.0	4,247.4	4,339.8	4,433.2
2000	3,442.5	3,560.7	3,681.5	3,805.0	3,931.2	4,060.2	4,192.0	4,326.6	4,464.0
2001	3,288.6	3,437.0	3,590.4	3,748.9	3,912.6	4,081.6	4,256.0	4,435.9	4,621.5
2002	3,259.2	3,440.7	3,630.2	3,828.0	4,034.2	4,249.3	4,473.4	4,706.9	4,950.0
2003	3,339.9	3,560.3	3,792.7	4,037.5	4,295.4	4,566.9	4,852.5	5,152.8	5,468.5
2004	3,273.9	3,525.6	3,793.6	4,078.9	4,382.4	4,705.0	5,047.8	5,411.7	5,797.9
2005	3,229.9	3,513.4	3,818.5	4,146.5	4,498.8	4,877.0	5,282.8	5,717.8	6,183.8
2006	3,493.9	3,835.7	4,206.9	4,609.7	5,046.4	5,519.5	6,031.5	6,585.3	7,183.9
2007	3,684.5	4,083.3	4,520.6	4,999.5	5,523.6	6,096.6	6,722.5	7,405.6	8,150.6
2008	3,823.8	4,278.6	4,781.9	5,338.5	5,953.4	6,631.9	7,380.0	8,204.0	9,110.8
2009	3,889.8	4,395.2	4,960.1	5,590.8	6,294.3	7,078.0	7,950.2	8,919.9	9,996.9
2010	4,175.8	4,762.3	5,423.9	6,169.6	7,008.8	7,952.2	9,011.7	10,200.0	11,531.5
2011	4,366.4	5,027.3	5,780.0	6,636.3	7,609.1	8,712.9	9,963.8	11,379.7	12,980.5
2012	4,601.0	5,347.7	6,206.2	7,192.0	8,322.3	9,616.7	11,097.0	12,787.8	14,716.5
2013	4,703.7	5,520.5	6,468.8	7,568.2	8,840.9	10,312.1	12,010.4	13,968.2	16,222.1
2014	4,887.0	5,790.8	6,850.3	8,090.2	9,539.1	11,229.6	13,199.1	15,490.3	18,152.1
2015	4,755.0	5,692.4	6,802.3	8,114.4	9,663.0	11,487.8	13,634.6	16,156.3	19,114.0
2016	4,630.4	5,600.1	6,760.0	8,145.2	9,796.3	11,761.0	14,095.2	16,863.7	20,142.1
2017	4,498.4	5,496.4	6,702.6	8,157.3	9,908.9	12,013.9	14,539.2	17,563.5	21,179.3
2018	4,388.4	5,417.0	6,672.7	8,202.6	10,063.0	12,320.8	15,056.1	18,363.6	22,355.9
2019	4,190.4	5,226.8	6,505.2	8,078.7	10,011.5	12,381.1	15,280.2	18,820.6	23,135.8
2020	4,073.1	5,132.8	6,453.1	8,094.9	10,131.7	12,653.5	15,769.3	19,611.1	24,339.0
2021	4,058.5	5,165.6	6,559.0	8,308.6	10,500.5	13,240.6	16,658.6	20,913.3	26,198.4
2022	3,559.9	4,582.7	5,884.4	7,537.1	9,630.5	12,276.0	15,611.6	19,808.0	25,075.8
2023	3,163.9	4,118.8	5,347.6	6,924.9	8,944.6	11,524.4	14,811.9	18,991.4	24,292.8
2024	2,848.6	3,749.5	4,921.6	6,442.6	8,411.0	10,952.2	14,224.6	18,428.3	23,815.4
2025	2,562.7	3,410.6	4,526.0	5,989.1	7,903.1	10,400.4	13,650.2	17,868.4	23,330.0
2026	2,401.4	3,230.1	4,331.6	5,791.8	7,721.8	10,265.8	13,609.9	17,994.4	23,727.9
2027	2,254.7	3,065.1	4,153.8	5,611.9	7,559.1	10,152.1	13,595.4	18,155.2	24,177.1

Table 18. Forecast - UK-FTSE with 50-year Linear Model

	0.0%	1.0%	2.0%	3.0%	4.0%	5.0%	6.0%	7.0%	8.0%
1997	3566.6	3566.6	3566.6	3566.6	3566.6	3566.6	3566.6	3566.6	3566.6
1998	3585.9	3621.6	3657.3	3692.9	3728.6	3764.3	3799.9	3835.6	3871.3
1999	3953.2	4028.8	4105.0	4182.0	4259.7	4338.0	4417.1	4497.0	4577.5
2000	3962.9	4078.9	4197.2	4317.7	4440.5	4565.6	4693.0	4822.7	4954.9
2001	3904.9	4060.0	4219.7	4384.0	4553.1	4727.0	4905.9	5089.8	5278.8
2002	3837.3	4030.3	4231.0	4439.6	4656.3	4881.5	5115.2	5357.9	5609.6
2003	3392.6	3603.6	3825.3	4058.4	4303.1	4559.9	4829.5	5112.1	5408.4
2004	2841.7	3054.4	3280.7	3521.1	3776.4	4047.4	4335.0	4639.8	4962.8
2005	2136.1	2326.6	2531.7	2752.4	2989.8	3244.9	3518.7	3812.5	4127.6
2006	1227.6	1360.3	1505.5	1664.3	1837.7	2027.0	2233.2	2457.8	2702.2
2007	898.9	1009.7	1132.6	1268.7	1419.3	1585.7	1769.4	1971.9	2195.0
2008	-183.6	-196.1	-208.7	-221.1	-233.1	-244.6	-255.2	-264.7	-272.7
2009	-1014.8	-1086.1	-1157.6	-1228.5	-1297.8	-1364.1	-1426.0	-1481.7	-1529.2
2010	-1111.5	-1200.4	-1291.0	-1382.4	-1473.3	-1562.2	-1647.4	-1726.6	-1797.2
2011	-1343.5	-1462.9	-1586.2	-1712.4	-1839.7	-1966.4	-2090.0	-2207.7	-2316.0
2012	-1188.8	-1309.1	-1435.4	-1566.6	-1701.5	-1838.4	-1974.9	-2108.2	-2234.7
2013	-1121.2	-1247.7	-1382.4	-1524.5	-1672.8	-1825.7	-1981.0	-2135.7	-2286.3
2014	-1092.2	-1227.9	-1374.3	-1530.8	-1696.4	-1869.7	-2048.6	-2230.0	-2410.1
2015	-1005.2	-1142.4	-1292.3	-1454.8	-1629.2	-1814.3	-2008.3	-2208.5	-2410.9
2016	-8901.9	-10128.5	-11470.7	-12927.2	-14493.1	-16158.3	-17906.4	-19713.2	-21544.3
2017	-8901.9	-10229.8	-11700.1	-13315.0	-15072.8	-16966.2	-18980.8	-21093.1	-23267.9
2018	-8901.9	-10332.1	-11934.1	-13714.5	-15675.7	-17814.5	-20119.7	-22569.7	-25129.3
2019	-908.5	-1157.8	-1456.7	-1811.1	-2226.9	-2708.8	-3260.6	-3883.3	-4575.0
2020	-1053.5	-1354.1	-1718.3	-2154.5	-2671.3	-3276.6	-3976.5	-4774.8	-5671.1
2021	-1034.2	-1342.8	-1721.1	-2179.6	-2729.2	-3380.3	-4142.2	-5021.5	-6020.7
2022	-705.5	-929.6	-1208.6	-1552.4	-1971.1	-2475.1	-3074.4	-3777.3	-4589.2
2023	-512.2	-684.2	-901.6	-1173.6	-1509.9	-1920.7	-2416.5	-3006.8	-3698.9
2024	-270.6	-368.3	-494.3	-655.2	-858.0	-1110.7	-1421.6	-1798.8	-2249.9
2025	-396.2	-542.9	-733.8	-979.1	-1290.7	-1682.0	-2167.0	-2760.1	-3474.7
2026	-183.6	-257.0	-354.7	-483.0	-649.7	-863.5	-1134.1	-1472.1	-1888.0
2027	-879.5	-1233.7	-1706.1	-2328.4	-3138.3	-4179.4	-5500.8	-7154.9	-9195.2

Table 18. Forecast - Germany-DAX with 36-year Linear Model

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Vita

Second Lieutenant Bradley J. Alden was born in Fort Lauderdale, FL on 1 May 1974, and he was raised in nearby Pompano Beach. He graduated from Ely High School in 1992 and attended the University of Miami in the fall of that year. He graduated from the University of Miami with a Bachelor of Science degree in Mathematics. While at college, he completed the requirements for the Reserve Officer Training Corps where he was one of the few to possess the title Cadet Wing Commander. Upon graduation, he received his commission in the United States Air Force. His Air Force career to this point consisted entirely of his stay at the Air Force Institute of Technology (AFIT), School of Engineering, to pursue a Master of Science degree in Operations Research. Following graduation from AFIT, he will be assigned to the Defense Office of Operations Research and Resource Analysis (DORRA).