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13. ABSTRACT *(Maximum 200 words)*

The objectives of this research were threefold. First, to develop and apply a theory on breaking the computational intractability of continuous problems in many variables. Second, to examine the problem of high-dimensional integration. Finally, to build and test software for heterogeneous clusters of workstations to test theoretical predictions associated with objectives one and two.

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FINAL REPORT

March 4, 1998

BREAKING INTRACTABILITY, SOFTWARE, AND IMPLEMENTATION TESTING

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AFOSR PROJECT F49620-94-1-0216

CONTENTS

2. Objectives

2.1. Solve Model of Contaminant Transport at Air Force Bases

This problem was brought to our attention by Marc Jacobs. At his suggestion we contacted Ben Fitzpatrick. The mathematical model is a path integral with an integrand which involved the solution of a stochastic differential equation. Rather than using our high dimensional integration software on a finite dimensional approximation to the path integral, we decided to attack the numerical solution of the continuous model directly.

Our initial objective is to develop algorithms for solving a model problem in one space and one time dimension and to embed them in software. We will compare the new algorithms with the existing Monte Carlo software. If the results are promising, we will attack the higher dimensional transport equation.

2.2. Breaking Intractability

Continue to develop and apply a theory on breaking the computational intractability of continuous problems in many variables.

2.3. High Dimensional Integration

In particular, study the important problem of high dimensional integration. Theoretical results we obtained earlier suggest that low discrepancy methods may be superior to Monte Carlo for at least some applications of high dimensional integration..

2.4. Software

Build and test software running on a heterogeneous cluster of workstations to test the theoretical predictions of Section 2.3.

2.5. Implementation Testing of Continuous Problems

Develop a theory of implementation testing. That is, given a mathematical problem and a program which purports to compute its solution, test whether the program does indeed solve the problem.

3. Status of Effort

I'll use the same numbering as in Section 2.

3.1. Solve Model of Contaminant Transport at Air Force Bases

We have programmed a new algorithm for solving this problem and compared it with the standard Monte Carlo approach. The new algorithm is superior with respect to both accuracy and speed.

3.2. Breaking Intractability

We've made good progress on new algorithms for the numerical computation of path integrals. The first paper has been published in the physics literature. A second paper, on Feynman-Kac type integrals, is in preparation A new algorithm has been discovered for Feynman-Kac integrals. It has two

major advantages over Monte Carlo: It provides us with a worst case guarantee rather than only a stochastic guarantee and it has a better rate of convergence.

3.2.1 Bellman Equation

We have started to investigate the numerical solution of the Bellman equation. Again, we believe that the new algorithm will be superior to Monte Carlo in providing a worst case guarantee with a better rate of convergence.

3.3. High Dimensional Integration

We stated in our last progress report that "our results on low discrepancy deterministic methods have stirred considerable interest and led a number of researchers to write papers trying to explain our surprising results. We've decided to tackle this question ourselves. A paper is in progress which we believe explains not only our success in very high dimensional integration but can also be used to solve Bellman's equation and other important applications.

3.4. Software for High Dimensional Integration

We have continued on testing; the results are reported in Section 4.

3.5. Implementation Testing of Continuous Problems

This work continues jointly with David Lee, Lucent. Three papers have appeared.

4. Accomplishments/New Findings

We describe our accomplishments in two areas.

4.1. Breaking Intractability

A new algorithm has been created for Feynman-Kac integrals. It has two major advantages over Monte Carlo: It provides us with a worst case guarantee rather than only a stochastic one and it has a better rate of convergence. A paper describing this work is in progress.

4.2. Software for High Dimensional Integration

We tested the Generalized Faure Sequence on a 360 degree dimensional model problem from mathematical finance and found that we beat Monte Carlo by a wide margin. We and others believed that this was due to the non-isotropic nature of the model problem.

We decided to test a hard isotropic problem that had been published in "Computers in Physics" in 1996. We again found the Generalized Faure Sequence beat Monte Carlo by orders of magnitude. Our results appeared in "Computers in Physics" in November, 1997.

These results are empirical. We need a theory which tells us for which class of integrals low discrepancy methods beat Monte Carlo. We believe we have found the explanation for non-isotropic integrals; a paper is in progress. The explanation for the new empirical results on isotropic integrals is open.

5. Personnel Supported

Anargyros Papageorgiou (post-doc)

Joseph F. Traub (faculty)

Henryk Wozniakowski (faculty)

6. Publications Submitted and/or Accepted

Lee, D., and Wozniakowski, H., "Testing Linear Operators", BIT 35, 1995, 335-351.

Lee, D., and Wozniakowski, H., "Testing Nonlinear Operators", Numerical Algorithms 9, 1995, 319-342.

Novak, E., Ritter, K. and Wozniakowski, H., "Average case Optimality of a Hybrid Secant-Bisection method", Math. Computation 64, 1995, 1517-1539.

Paskov, S., and Traub, J. F., "Faster Valuation of Financial Derivatives", Journal of Portfolio Management 22, 1995, 113-120.

Wasilkowski, G., and Wozniakowski, H., "Explicit Cost Bounds of Algorithms for Multivariate Tensor Product Problems", Journal of Complexity 11, 1995, 1-56.

Papageorgiou, A.. and Traub, J.F., "Beating Monte Carlo", Risk 9, 1996, 63-65.

Traub, J. F., "On Reality and Models", in Boundaries and Barriers: On the Limits to Scientific Knowledge, (J. Casti and A. Karlqvist, Eds.), Addison-Wesley, 1996, 238-251.

Wasilkowski, G.W. and Wozniakowski, H., "On Tractability of Path Integration", Journal of Mathematical Physics 37, 1996, 2071-2088.

Lee, D. and Wozniakowski, H. , "Testing Linear Operators: An Average Case Analysis", BIT, 37, 1997, 355-376.

Papageorgiou, A. and Traub, J.F., "Faster Evaluation of Multidimensional Integrals", in Computers in Physics 11, 1997, 573-578.

Traub, J.F., "From Infoware to Infowar", in Defining a Decade: Envisioning CSTB's Second 10 Years, National Academy of Sciences, 1997, 1-7.

Traub, J.F., "Do Negative Results from Formal Systems Limit Scientific Knowledge?", Complexity 3:1, 1997, 29-31.

Wozniakowski, H., "Computational Complexity of Continuous Problems", in Nonlinear Dynamics, Chaotic and Complex Systems, E. Infeld, R. Zelazny and A. Galkowski, eds., Cambridge University Press, 1997, 283-295.

Wozniakowski, H., "Strong Tractability of Weighted Tensor Products", Annals of Numerical Mathematics 4, 1997, 607-622.

Traub, J. F., "Non-Computability and Intractability: Does it Matter to Physics?", Santa Fe Institute Working Paper, 1998.

Traub, J.F., and Werschulz, A.G., "Complexity and Information", Cambridge University Press, 1998.

7A. Talks

Traub - FANNIE MAE, Washington, DC, October, 1995

Traub - Courant Institute of Mathematical Sciences, New York University, New York, NY, November, 1995

Traub - Computer Science Department, University of Toronto, Toronto, Canada, November, 1995

Traub - Computer Science Department, University of Waterloo, Ontario, Canada, November, 1995

Traub - Fields Institute, Toronto, Canada, November, 1995

Traub - Address to Chief Scientists and technical Officers, U.S.A.F., Santa Fe, NM, July, 1996

Traub - Workshop on Fundamental Limits to Knowledge in Economics, Santa Fe Institute, Santa Fe, NM, August, 1996

Traub - After dinner address, Century Association, New York, NY, October, 1996

Wozniakowski - Foundation of Computational Mathematics, co-organizer and speaker of the workshop, "Information-based Complexity", Rio de Janeiro, January, 1997

Wozniakowski - City University of Hong Kong, Hong Kong, January, 1997

Wozniakowski - Workshop, "Computability and Complexity in Analysis", Dagstuhl, Germany, April, 1997

Wozniakowski - International Computer Science Institute, Berkeley, CA, July, 1997

Wozniakowski, Los Alamos National Laboratory, Los Alamos, NM, August, 1997

7B Advisory Functions to Air Force, DOD, and Other Laboratories

7B.1. On July 24, 1996 I was invited to attend a meeting of the Air Force Chief Scientists Group in Santa Fe. The group consisted of chief scientists and chief technical officers from many Air Force Bases and Laboratories. The meeting was organized by Edward Feigenbaum, Chief Scientist of the Air Force. Participants included Joseph Janni and Helmut Hellwig.

7B.2. On August 12, 1996 I participated in a meeting of the Highlands Group in Santa Fe at the invitation of Frank B. Horton, III, Principal Deputy Assistant Secretary of Defense (Command, Control, Communications, and Intelligence) and Andrew W. Marshall, Director, Net Assessment, Office of the Secretary of Defense. We worked on scenarios regarding the impact of information technologies on the military for a time horizon of up to twenty years.

7B.3. In November, 1996 I participated at a meeting of the Highlands Forum at the Naval Academy, Annapolis, to discuss information warfare. We worked with members of the Presidential Commission on Protecting the National Information Infrastructure.

7C. Transitions

We have assigned the rights to license our high dimensional integration software

system FINDER to Columbia University. Columbia has licensed the software to FANNIE MAE. The University is actively seeking other licensees.

8. New Discoveries

We have applied for a patent on our software.

9. Honors/Awards

Traub, Joseph F. (Lifetime Achievements)

Fellow, American Association for the Advancement of Science, 1971
Member, National Academy of Engineering, 1985
Founding Chair, Computer Science and Telecommunications Board, National Academy of Sciences, 1986
Emanuel R. Piore Gold Medal, IEEE, 1991
Sherman Fairchild Distinguished Scholar, California Institute of Technology, 1991-1992
Distinguished Senior Scientist Award, Alexander von Humboldt Foundation, 1992
Lezione Lincee, Accademia Nazionale dei Lincei, 1993
Fellow, ACM, 1994

Wozniakowski, Henryk (Lifetime Achievements)

Prize of the First Degree, Ministry of Sciences and Higher Education of Poland, 1980
Stanislaw Mazur Award, Polish Mathematical Society, 1988