The Applicability of Scaling Laws to Underwater Shock Tests

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ABSTRACT

The applicability and limitations of classical scaling relationships are reviewed in relation to an AMRL investigation into the structural response of air-backed plates to far-field underwater explosions. It is intended that this investigation will determine the effects of charge size and disposition and a range of structural parameters on the structural response of submerged, air-backed plates.

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Executive Summary

AMRL has embarked on a program to investigate the response of surface and submerged vessels to far-field underwater explosions, such as those caused by depth charges and mines. This program is supported by ongoing experimental tests using small-scale structures, primarily for cost reasons. A reduction in the scale of an experiment is inevitably accompanied by questions as to the experiment's applicability to a full-scale test.

The AMRL program investigates, numerically and experimentally, the structural response of stiffened and unstiffened air-backed plates to far-field underwater explosions and the relationships between small and large scale structural tests. The procedure investigates systematically, the effects of charge size and disposition, a range of structural parameters and the mechanical properties of the materials involved.

In recent times, an increasing reliance has been placed on numerical techniques, which have evolved as powerful analytical tools, primarily due to the accompanying increase in computational power. However, many of the numerical techniques available have not been rigorously validated. As a result, experimental testing has shifted from being a principal analytical tool for predicting structural response to a means of validation. The shift in emphasis towards numerical techniques has gained further prominence as questions of reliability of small-scale test methodologies arise and the opportunities to conduct full-scale tests are limited by cost and environmental constraints.

Notwithstanding the comments above, small-scale tests are still conducted as a prediction tool and consequently it is valuable to review their applications and limitations for underwater shock studies. The issue of whether small-scale testing can be used as an exclusive analytical procedure for underwater shock analysis or should only be used as a means of validating numerical techniques is the reason for the work at AMRL.

There is a paucity of information in the open literature addressing the issue of scaling applied to underwater shock testing. This paper also attempts to go some way in reviewing this literature and that for analogous programs such as impact loading. Based on this review, this report examines the applicability and limitations of classical scaling relationships for use in predicting the structural response of steel plates to a shock wave produced by a far-field underwater explosion.

It is concluded that classical scaling methods may not be appropriate for non-linear underwater shock response problems for the case of highly strain rate sensitive materials. Scaling methods have limited applicability for low shock level, linear-elastic structural responses where strain-rates are not significant. Also, structures that

structural responses where strain-rates are not significant. Also, structures that experience material failure, such as cracking, shearing or tearing, appear not to be scalable.

The report concludes that, with regards to future AMRL underwater shock experiments, small scale testing should not be used as the sole means of predicting non-linear structural response, particularly for materials that are highly strain-rate sensitive. However, such testing will provide useful data for validating the performance of numerical prediction techniques. Also, far-field underwater shock experiments must be designed such that gravitationally influenced phenomena (e.g. gas-bubble dynamics) are minimised, since they do not scale in the same manner as compressibility phenomena (e.g. shock wave propagation). This can be satisfactorily achieved for far-field geometries, where the explosive-target standoff distance may be of sufficient magnitude to minimise gas-bubble effects.

Contents

1. Introduction

Following the detonation of a high explosive underwater, a detonation wave passes from the detonation point to the surface of the charge, converting the solid material into gaseous reaction products (Cole, 1948). At the surface of the charge a sudden pressure increase is transmitted to the surrounding water leading to the rapid formation of an intense shock wave. The shock wave propagates outwards in a manner that rapidly becomes spherical for most charge shapes after a distance of only several charge radii. Initially, the shock wave travels at several times the speed of sound, with the pressure decaying more quickly than exponentially. As the speed of the shock wave slows to acoustic speeds, the pressure decay follows an exponential relationship (ie. with constant decay rate).

Once the shock wave has become approximately acoustic, its physical nature is predictable using similitude equations. Physical parameters such as peak pressure, impulse, shock wave energy and pressure pulse duration are easily determined using a scaled distance parameter (Swisdak, 1978). In addition to the shock wave, a pulsating gas bubble is also formed in the wake of the explosion which produces intermittent, but gradually weakening, pressure pulses at relatively much longer times after the initial pressure pulse.

Naval vessels have the potential to be subjected to both air-blast and underwater shock loading, although it is only the latter case which is being considered here. The response of an air-backed plate to an underwater shock loading is complicated by fluid-structure coupling effects caused by the deforming plate interacting with the incident pressure load. The plate surface reflects a significant portion of the incident pressure wave back into the water, interacting with the remainder of the incoming pressure wave. This complex coupling problem can cause cavitation of the water at close proximity to the plate surface, which in itself, will eventually collapse and possibly reload the plate. In addition, pulsations of the gas bubble produced by the explosion will produce pressure pulses which may also interact with the plate.

The response of maritime vessel structures in the vicinity of an underwater explosion is of particular concern. This concern arises primarily in relation to the dynamic structural response and the subsequent transmitted response to equipment and systems within the vessel. An ability to predict structural and equipment response to an underwater explosion is highly desirable and of value to naval architects and to the designers of onboard equipment.

The physics of gas bubble expansion and the transmission of the shock pulse through the fluid medium are understood to a level which has allowed the development of computer codes to predict the loading experienced by submersed structures and (to a lesser degree) for buoyant structures. Analytical and hydrodynamic codes have been developed for this purpose and are described elsewhere (Baron & Daddazio, 1995; Norwood, Kumar & Palmeter, 1990; DeRuntz, 1989 and Marco & Saunders, 1996).

Structural performance and associated equipment behaviour is ultimately determined in a full-scale shock test using explosive charge sizes and standoff distances which are considered representative of the perceived threat scenario for the vessel. Most shock testing programs are for "first of class" vessels, such as the COLLINS Class Submarine and the Minehunter coastal (MHC), but other trials may involve the assessment of the level of "shock hardening" retained in older vessels.

While full scale shock testing programs allow detailed assessment of structural and equipment response, these type of tests are becoming increasingly more difficult to undertake, not only for reasons of cost but for the sensitivity of environmental issues in relation to the level of acoustic pressures attained in the ocean as well as damage to natural resources such as fishing zones, coral reefs and vegetated regions. For these, and many other reasons, full scale shock testing may not be possible in the near future and validation of naval designs may prove more difficult.

While reliance on numerical codes will increase, it is still considered desirable that some level of physical testing be undertaken, and in this regard it is considered that the testing of small-scale structures will become increasingly important. This work has been the subject of considerable investigation over the past 20 years, mainly in relation to the response of equipment within the structures (Cunniff and O'Hara, 1992; Cunniff and O'Hara, 1994). Structural aspects of underwater explosion damage have been studied less systematically, but the studies fall in the general area of scaling of structural response.

When studying the behaviour of structures to underwater explosions, the usual method is to assess the behaviour in relation to shock factor[1] as similar shock factors are considered to produce similar damage on similar structures. However, since high explosives produce spherical shock waves when detonated in water, it is likely that the spatial relationships between the charge and the structure must be considered as an influential factor in the ultimate response of and resultant damage to the structure.

The following paper will review the applicability of classical scaling methodologies to the effects of underwater explosions on air-backed structures and the known scaling relationships which would allow the prediction of structural response from small-scale models.

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[1] The Hull Shock Factor (HSF) and Keel Shock Factor (KSF) equations for submerged and floating vessels, respectively, use the charge weight and standoff distance as a measure of potential damage (Keil, 1961).

2. Scaling Theories

2.1 Background

The prediction of structural behaviour to either static or dynamic loads has, in many cases, provided useful design information (Baker et al, 1991). Scale modelling has provided a useful insight into structural response for a variety of engineering applications (Baker, 1963), including predicting:

- the structural response to blast loading which produces large transient elastic and plastic deformations (neglecting the effects of gravity, strain rate, heat conductivity and viscosity).
- elastic vibrational response characteristics of structures.
- fluid-structure coupling involving rigid bodies (e.g. wind tunnel aerodynamics studies).
- fluid-structure coupling involving elastic bodies (e.g. fuel sloshing in a rocket propellant tank with flexible tank structure, simulation of wave impact loads on naval vessels).

Scale modelling is particularly useful when it is not possible to test a full scale prototype for reasons of time, cost or availability of prototypes. Instead, a scale model can be built and subjected to similar loading conditions as the prototype. The following discussion on scaling will be limited to the most general and commonly used procedures, involving geometrically scaled models.

2.2 Similitude Requirements

A scale model is typically a physically scaled down version of a full-scale prototype, that is, all linear dimensions are scaled by a constant amount, known as the scaling factor, which is typically less than unity. Additionally, all external loads act at homologous[2] points at homologous times on the scale model with respect to the full-size prototype. The scale model will, in general, be made from the same material as the prototype, thus having the same modulus of elasticity (E), the same mass density (ρ) and the same Poisson's ratio (γ).

There are exceptions where a scale model can be larger than the prototype. This generally occurs for prototype structures that are inherently very small and difficult to test experimentally. Another deviation from the above description is that certain physical features may only add to the complexity of the scaled model construction, but will not influence the results of the structural response or phenomenon being investigated. In such cases, it is up to the discretion of the modeller to simplify the

[2] The word homologous means "at corresponding, but not necessary equal, values of a variable".

model design accordingly. Modelling of dynamic structural response requires that geometric, kinematic and dynamic similitude requirements are met (Snay, 1961).

2.2.1 Geometric Similitude

The most common form of scaling uses principles of *geometric similitude*, in which the *model* and *prototype* are related by a characteristic length scale factor, λ. Physical dimensions of the model can be related by a constant, λ, to the equivalent physical dimensions in the prototype, then the homologous points, R_m and R_p, in a model and prototype, respectively, are related by λ according to:

$$\lambda = \frac{R_m}{R_p}$$

where $R_m = (x_m, y_m, z_m)$ and $R_p = (x_p, y_p, z_p)$.

Relationships between model and prototype of blast and structural response parameters are related by terms involving λ. For example, when modelling underwater shock response, geometric scaling could be applied to the explosive charge (see section 4.0), to the physical dimensions of the structure subjected to the shock loading and to the standoff distance from the explosive. If $\lambda = 0.5$, then the characteristic dimensions of the model would be half those of the full scale prototype. This would mean, for example, that for an unstiffened plate mounted in a shock rig, physical dimensions such as the width, breadth and thickness of the plate would be reduced by λ. However, it would be expected that whilst certain boundary condition criteria are maintained for the *responding* part of the structure, between the model and prototype structures, it would not be necessary for geometric similitude conditions to be applied to all dimensions of the shock rig itself.

2.2.2 Kinematic Similitude

Kinematic similitude is the similitude of motions, which is largely dependent on the time scale factor, τ, where at homologous times, t_m and t_p:

$$\tau = \frac{t_m}{t_p}$$

Kinematic similitude requirements can be summarised in terms of the velocity scale factor, φ, and the acceleration scale factor, α, where:

$$\varphi = \frac{\lambda}{\tau} \qquad \text{and} \qquad a = \frac{\lambda}{\tau^2}$$

Care should be taken in the interpretation of these scale factors. For example, the velocity scale factor is the distance scale factor, λ, divided by the time scale factor, τ.

The velocity scale factor should not be interpreted as distance divided by time, nor does it imply that velocity is constant with distance or time. The velocity scale factor requirement is that the ratio of the distance and time scale factors remain invariant. A similar approach should likewise be extended to all definitions of scale factors.

2.2.3 Dynamic Similitude

Dynamic similitude is the requirement for similitude between external and inertial forces between model and prototype and its requirements can be summarised in terms of the pressure scale factor, π, and the energy scale factor, ε, where:

$$\pi = \rho\varphi^2 \qquad \text{and} \qquad \varepsilon = \pi\lambda^3$$

where the density scale factor, ρ, is defined as: $$\rho = \frac{\rho_m}{\rho_p}$$

In underwater shock *field tests*, $\rho=1$, since water is used for both the model and prototype.

2.3 Development of Scaling laws using the Buckingham-π Theorem

In many cases the equations of state that govern a dynamics problem, such as in the structural response to an underwater explosion, are not completely known. Dimensional analysis procedures, such as the Buckingham-π theorem (Buckingham, 1914), can be used to generate a series of dimensionless terms that can subsequently be used to predict the behaviour of important physical parameters in a dynamic system (eg. spatial and temporal values of stress, strain and displacement) without prior knowledge of the differential equations which govern a system's behaviour.

In any set of physical quantities involving products of physical parameters, there are a limited number that can be shown to be dimensionless, or dimensionally independent (Sabnis et al., 1983). These dimensionless, independent products are referred to as π-terms, after the Buckingham-π Theorem (Buckingham, 1914). π-terms are generated from all known significant physical parameters that influence the structural response using the Buckingham-π theorem. If phenomena are to be successfully scaled, the π-terms must be invariant between a small scale model and a full scale prototype, although the parameter values within the π-terms may be varied. The shortcoming of the Buckingham-π theorem is that it is not able to predict the relative *significance* of each physical parameter.

2.3.1 Significant π-terms

As stated previously, the π-terms calculated by dimensional analysis must be invariant for both the model and the prototype in a scaling experiment. When scaling analysis is restricted to a particular field, e.g. gravitational effects, fluid mechanics, air-blast or underwater shock effects, one finds that the same group of π-terms arise. Several such π-terms of significance have been given the names of famous scientists; some of which are listed in Table 1.

Table 1 Important π-terms and the physical effects they govern (Snay, 1961).

physical effect	reference name:-	π-term	scale factor
compressibility	Mach number:-	$\frac{u}{c}$	$\varphi = \frac{c_m}{c_p}$
gravity	Froude number:-	$\frac{L}{gT^2}$ or $\frac{u^2}{gL}$	$\alpha = \frac{g_m}{g_p}$
pressure	Thomas number:-	$\frac{P}{P_v}$	$\pi = \frac{(P_v)_m}{(P_v)_p}$
surface tension	Weber number:-	$\frac{PL}{\nu}$	$\pi\lambda = \tilde{\nu} = \frac{\nu_m}{\nu_p}$
viscosity	Reynolds number:-	$\frac{uL}{\gamma}$	$\tilde{\gamma} = \frac{\gamma_m}{\gamma_p}$

u: particle velocity
L: characteristic length
T: response time
v: surface tension
c: acoustic velocity
g: acceleration due to gravity
P_v: vapour pressure
γ: kinematic viscosity

2.4 Development of Scaling Laws from System Equations

The most conventional method of developing model laws (ie. those pertaining to scaling relationships between a model and a prototype) is using dimensional analysis, and more specifically, the Buckingham-π Theorem (see section 2.3). Detailed knowledge of a system's governing differential equations is not essential for the development of model laws by dimensional analysis techniques. However, if a system's governing equations are obtainable, they can be used in a more rigorous fashion than otherwise would be the case with dimensional analysis techniques.

Solutions of differential equations are frequently difficult and sometimes impossible unless the differential equations are of a particular form. Obtaining model laws from differential equations is far less popular than the Buckingham-π method. Unlike the latter method, the differential equations for a particular problem must be known so that they can be used for the development of a model law. However, the π-terms can be obtained without necessarily obtaining solutions of the differential equations. Avoidance of the solution of the differential equations is achieved by making some arbitrary assumptions about the relationships between scale factors. These interrelations between scale factors may be determined partly through knowledge of the physics involved and partly through experience, although ultimately their choice will be verified, or otherwise, through substitution into the system's equations. If the equations are shown to be invariant under substitution, then the relationships are deemed acceptable. Failure of some or all of these assumed scale factor relationships will simply necessitate the choice of others.

There are other procedures involving the use of differential equations to obtain model laws. An alternative method involves the rearrangement by algebraic manipulation of the individual terms in the systems equations into dimensionless form. The dimensionless quantities generated are subsequently used to develop the model laws described by differential equations.

If solutions of these equations are obtainable, then a model law can also be generated by rendering each term in the solution dimensionless as before. To simplify the solution of the system's differential equations, non-linear effects such as gravity, friction and damping are generally not considered. The model laws are then derived from the dimensionless terms as in the previous procedure. Discussion of these three procedures involving the use of the system's differential equations is extensively covered by Baker (1991).

3. Scaling of Explosion Effects

Scaling is frequently used for the prediction of shock wave parameters, whether it be air-blast or underwater shock waves. In considering the scaling effects of shock waves, the Mach (compressibility) similitude requirement must be satisfied. Parameters such as peak pressure, time constant, impulse and energy are readily determined for a given charge weight (W) and standoff distance (R) scenario using simple similitude relationships (Swisdak, 1976). Each of these parameters can be quantified in terms of similitude constants employed in simple equations, all of which use the same scaled distance parameter, $W^{1/3}/R$. The charge weight is used instead of a characteristic charge dimension (which would render the term dimensionless) as it is typically a more convenient parameter.

3.1 Hopkinson Scaling

In shock wave studies it is more common to refer to Hopkinson scaling (Baker, 1991) than Mach's similitude, from which the former is derived. However, it is the condition of Mach's similitude which must be satisfied. To achieve Mach's similitude, all velocities should be scaled by φ (see Table 1), including the velocity of sound. If experiments are conducted in a *field test* environment (i.e. model test is conducted in the same medium as the full-scale prototype), then the following scale factors are constrained:

velocity scale factor: $\varphi = \frac{c_m}{c_p} = 1$ hence: $u_m = u_p$

and density scale factor: $\tilde{\rho} = \frac{\rho_m}{\rho_p} = 1$

$\Rightarrow$ pressure scale factor: $\pi = \tilde{\rho}\varphi^2 = 1$ hence: $p_m = p_p$

$\Rightarrow$ energy scale factor: $\varepsilon = \pi\lambda^3 = \lambda^3$

$\therefore$ time scale factor: $\tau = \frac{\lambda}{\varphi} = \lambda$

and as the energy of an explosive is proportional to the weight of explosive used:

$$\varepsilon = \lambda^3 \equiv \frac{W_m}{W_p}$$

$$\therefore \quad \lambda = \left(\frac{W_m}{W_p}\right)^{1/3} = \left(\frac{R_m}{R_p}\right)$$

As $\tau = \lambda$, for time similitude: $\tau = \lambda = \left(\frac{W_m}{W_p}\right)^{1/3}$

$$\therefore \quad t_m = \left(\frac{W_m}{W_p}\right)^{1/3} t_p$$

which is referred to as *Hopkinson scaling*, which is a direct consequence of *Mach's scaling*.

Hopkinson's scaling states that if the dimensions of a charge are scaled by a value, λ, then at an equivalent distance, λR, from the scaled charge, the peak pressure will remain unchanged but the impulse and the pulse duration will both be scaled (ie. multiplied) by λ. Hopkinson scaling applies only to the shock wave itself and does not extend to structural effects. To utilise Hopkinson scaling, it is desirable, if not necessary, that geometric similitude occurs[3], making scaling between model and prototype a relatively simple matter. The simple, linear relationships of Hopkinson

[3] This ignores other factors such as welding residual stresses, heat affected zones, weld bead size and other size-related structural and material variations.

scaling are tabulated in Table 2 and illustrated in Figure 1. These relationships are considered to be reliable means of predicting the shock parameters in the case of a midwater explosion (i.e. considered to be unimpaired by boundaries).

Table 2 Hopkinson scaling relationships for the shock wave parameters shown in Figure 1.

Parameter	prototype	model
Geometrical Parameters (scaled by λ)		
charge diameter	d	λd
standoff distance	R	λR
characteristic length	L	λL
Shock Wave Parameters		
pulse duration	T	λT
impulse	I	λI
peak pressure	P_{max}	P_{max}

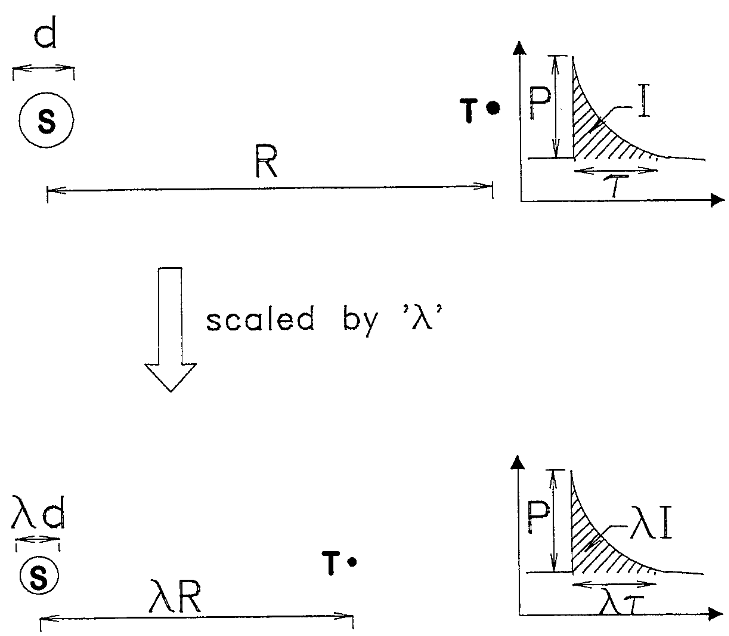

Figure 1 Hopkinson scaling applied to a spherical underwater shock wave emanating from a source, S, impinging on a target, T. Parameters are scaled according to the parameter, λ.

The detonation of a charge will lead to the formation of a spherical shock front, emanating in all directions away from the charge centre. At extreme scaled distances (either too close or too distant from the detonated charge), these relationships fail due to the effect of various physical phenomena. At very close distances (ie. within a couple of charge radii) the shock wave is still forming and is travelling at speeds in excess of the acoustic speed. At very large distances, the shock wave is travelling significantly slower than the acoustic speed and will be affected by temperature and viscosity effects.

3.2 Limitations of Scaling Laws in Underwater Shock Phenomena

One of the most common conflicts with π-terms occurs in field test situations, which use identical media for model and prototype. For example, in underwater shock field tests (where $\varphi = 1$, $\alpha = 1$), Mach's and Froude's similitude requirements cannot be met simultaneously, since $\varphi = 1$ necessitates that $u_m = u_p$ for Mach's similitude. However, if $u_m = u_p$ and $\alpha = 1$ then Froude's similitude cannot be met since $\lambda \neq 1$, as the model and the prototype are of different sizes. Gravitational similitude can only be achieved in a controlled environment, e.g. a gravitational tank. This is a serious drawback as it means ideal similitude can never be achieved in underwater shock tests. This doesn't preclude the use of scaling procedures for underwater shock tests although these procedures must be restricted to physical phenomena which are governed by only one of these conflicting effects. This may mean conducting two types of experiments where only one effect is significant in each (e.g. gravitational effects, such as in gas bubble phenomena, or compressibility effects, such as in shock wave phenomena).

4. Scaling of Structural Response to Underwater Shock

The structural response to underwater shock presents a case for application of the conventional scaling methodologies discussed above. There are, however, some limitations which will be discussed below.

4.1 Similitude in Structural Dynamics

If geometric similitude exists for a small-scale model and a large-scale prototype of the same material, then (Jones, 1986):

(i) the engineering strain and the associated stresses are the same in both cases. This applies for any type of structural material.

(ii) the pressures acting on the surfaces of both are the same, since the stresses normal to the structural boundary are equal and opposite in magnitude to any pressure acting on the surface.

(iii) the tensile or compressive wave(s) that propagate through the structural medium are the same.

(iv) the time sequence of events of the small-scale model is λ times that of the large-scale prototype, in accordance with the need for homology (see section 3.2).

(v) the structural velocities are identical, since velocity is the ratio of distance with time and both quantities are scaled by λ.

4.2 Small Scale Elastic Deflection Theory

Small scale elastic deflections are those characterised by infinitesimal strains and displacements and central deflections of a plate or beam less than half the plate thickness (Evans, 1939). The response of a plate structure subjected to an underwater shock is characterised by the shock loading parameters: peak pressure (P_{max}) and pulse duration (T); and the structural response parameters: period of response (τ_i,), displacement amplitude (x_i) and strain (e_i), for the ith response mode and the boundary conditions of the plate (Baker, 1960). For the case of small scale deflections, several assumptions are made:

(i) all loads and reactions are normal to the plane of the plate,
(ii) no point on the plate is stressed beyond the elastic limit,
(iii) the middle plane of the plate remains unstressed under deflection; the maximum *bending* stresses occurring on the external surfaces of the plates, and
(iv) strain rate and gravitational effects are assumed negligible.

The scaling relationships between parameters for two geometrically similar systems which undergo small-scale deflections are summarised in Table 3.

Table 3 Scaling relationships for linear-elastic small deflection structural response.

Structural Response Parameters (for vibrational modes: i = 1, 2,....n)		
vibrational period	τ_I	$\lambda\tau_I$
displacement amplitude	X_i	λX_i
strain amplitude	e_i	e_i

4.3 Large Scale Elastic Deflection Theory

The linear equations, which adequately describe the small scale deflections of a structure, are inappropriate in the case of large scale deflections when *membrane* stresses in the middle surface of a plate structure are no longer negligible. For large scale deflections, the load-deflection and load-stress relations are non-linear (Way, 1938).

The governing equations of motions of both fluid and solid media are invariant under systematic substitution of scaled parameters for the case of large deflection elastic response (Baker, 1960). However, it is again assumed that gravitational, viscosity and strain-rate effects are negligible and that there is negligible heat loss in the fluid. Baker predicts "that the geometrical modelling which applies to small-deflection response of elastic structures to blast loading describes the large-deflection response equally well, although the equations of motion are dissimilar."

Baker successfully illustrated the validity of scaling theory for large deflection structural response by conducting a series of relatively simple experiments which consisted of subjecting a series of thin, 6061-T6 (strain-rate insensitive) aluminium cantilevers of geometrically different scales to air blast loading from spherical pentolite explosive charges detonated in air. Strain-time histories were measured using strain gauges mounted near the base of the cantilevers and permanent tip deflections were recorded. The data from each set of cantilevers lie on a single curve of maximum strain versus scaled distance ($R/W^{1/3}$), as well as for the curve of dimensionless tip deflection (δ/L) versus scaled distance. These results indicate that the structural response scaling is possible and valid. Baker also showed that vibrational frequencies, strain amplitudes and deformed shapes scaled geometrically between the different sizes of cantilever. These experiments recorded large tip deflections of up to 30% of the original beam length.

The above example illustrates a successful application of scaling large-deflection structural response using a strain-rate insensitive material. In many other applications, such as in the air-blast or underwater shock loading of steel plates, the strain rate sensitivity of the target material cannot be considered negligible and in such cases, the scaling procedure would either have to be abandoned or alternatively, a *distorted model* would need to be considered. Distorted model tests can be performed so long as "...for every similitude requirement that cannot be satisfied in a model test, there must be provided a relationship, theoretical or empirical, between the dependent variable and that independent variable for which similitude could not be provided" (Ezra & Penning, 1962).

For example, if a model and a prototype have different stress-strain relationships, then the displacements in the model (which are a function of strain multiplied by length) would be dissimilar to those in the prototype (Sabnis et al., 1983). Thus, if the displacements are large, then there would be unacceptable deviations from similitude in structural behaviour and a relationship would need to be determined (empirically or theoretically) between displacements in the model and displacements in the prototype.

4.4 Elasto-Plastic Deflection Theory

For small scale elastic deflections, the stress components are unique functions of the strain components and deformations are therefore reversible (i.e. they return by the same path). Conversely, for large scale deflections resulting in plastic deformations, the stress components associated with increasing strain differ from those associated with decreasing strain, thus they are not unique and the deformations are not reversible. It has been shown (Baker, 1960) that, while the strains are monotonically increasing in magnitude, the geometrical scaling of response to shock loading is valid and additionally, that the limiting strain magnitudes for the initial stress-strain law are the same in the prototype and model structures. However, when the strain starts decreasing in magnitude, the initial stress-strain law applied to both systems is no longer valid although the relationships which governed the initial stress-strain behaviour also govern the latter, with only minor revisions to coefficients in the law. The new stress-strain law for the geometrically scaled elasto-plastic model *should* be identical to that for the original prototype structure, since the maximum strains and the physical properties which determine the coefficients are identical. Hence, theoretically, the entire elastic-plastic response of a structure subjected to an underwater shock loading should scale geometrically.

By way of example, geometrically scaled-down models have been successfully used in explosive forming as a means of predicting plastic deformation in full-scale structures (Ezra & Penning, 1962). The experiments involved a submerged explosive suspended in a water-filled tank above a *blank* (a flat aluminium sheet). The blank is supported on an open-bottomed die with water pressure above and air pressure below. A series of experiments was conducted with a range of die sizes with similitude being achieved for the loading conditions using Hopkinson scaling. The same blank material was used in all cases. As is often the case, similitude could not be achieved for all of the independent variables. The explosive pressure imparted to the blank was not an independent variable since it is related to a number of other independent variables. Also, since the tests were conducted in a similar fluid, viscosity cannot be scaled. Likewise, gravity is invariant between different scales of the experiment. However, the invariance of gravity is not significant, since the explosive forces provide most of the actual hold-down pressure on the blank. Similarly, viscous effects were insignificant at this scale. Hence for this example, it was shown that a valid scale model is possible, even when similitude cannot be achieved for all independent variables. Strain rate problems were not encountered since aluminium, which is essentially strain rate insensitive, was used as a blank material.

4.5 Limitations in Scaling of Structural Response

Unfortunately it is difficult or impossible to apply similitude constraints to *all* physical parameters. Notable parameters, typically associated with non-linear response, which cannot be geometrically scaled are:

(i) *gravity*, since the acceleration due to gravity is effectively, a constant in field experiments. Luckily, gravitational forces are not significant compared to those acting in most structural dynamics problems, reducing the significance of its inherent lack of scalability.

(ii) *strain rate effects*, which vary between model and prototype since strain is invariant but time related phenomena are scaled by λ. Hence the strain rate in a small-scale model is $1/\lambda$ times larger than the full-scale prototype. As a consequence, the dynamic flow stresses in a small scale model are larger than those in the related prototype. Strain rate effects are of particular significance in underwater shock analysis where they can easily reach orders of magnitude of 10^2 and $10^3 s^{-1}$. Increasing strain rates are reflected in the stress-strain properties of materials in the following ways (Sabnis et al, 1983):

(a) the yield stress, σ_y, will increase to a dynamic value, σ_{yd}.
(b) the yield point strain, ε_y, will increase.
(c) the modulus of elasticity, E, will remain unchanged.
(d) the strain at which strain hardening begins, ε_{st}, will increase.
(e) the ultimate strength increases slightly.

In the context of the above, "material strain rate effects have not been modelled successfully in small-scale structural model tests" (Jones, 1986).

While this discussion is limited to the pre-failure (i.e. elastic-plastic) behaviour of structures, it is also instructive to briefly review the crack growth characteristics of structures. For example (Jones, 1986):

(i) crack propagation does not obey geometric similitude laws in linearly elastic structures.

(ii) brittle fracture in non-linearly elastic materials does not obey geometric similitude laws. It has been observed that brittle fracture can occur prior to yielding in some structures, but will produce yielding for a small-scale model of the same material. Crack initiation will occur at lower stresses in a prototype than in a small-scale model. This phenomenon has the potential to provide misleading results when using small-scale models[4]

[4] The problem of specimen size has been investigated extensively from a "classical" fracture mechanics viewpoint (e.g. Broek, D. 1986).

(iii) Yield stress of a material can increase with increasing strain rate, whereas fracture stresses are less sensitive to strain rate. Hence if a structure is loaded in such a way that it produces a yield stress smaller than the corresponding fracture stress, it will respond in a ductile fashion. Alternatively, if the fracture stress is smaller than the yield stress the structure may respond with a brittle fracture. (Fracture stress increases with decreasing body size).

In addition, "although the increase in deformation is linearly scalable for a constant applied force, for a structure that exhibits a decreasing load with increasing deflection, then the increase in deflection will be greater than the decrease in yield stress, resulting in a deviation from linear scalability" (Calladine, 1983). The degree of deviation from scalability is determined by the shape of the load-deflection characteristic.

5. Discussion

There is a paucity of experimental data reported in the literature, making it difficult to ascertain the viability of scaling procedures for structures subjected to underwater explosions and air blasts. Only one publication (Ezra & Penning, 1962) was located which referred directly to the scaling of structural response to underwater explosions. Although this work showed the scaling procedures to be feasible in such an application, the experimental data was limited to tests using aluminium blanks, which are strain-rate insensitive and as yet, "material strain rate effects have not been modelled successfully in small-scale structural model tests" (Jones, 1986). This statement does not preclude successful experiments where strain rate effects occur, but have not been accounted for.

The feasibility of using geometric similitude in scaling tests for dynamic, impact loading of steel plating will depend on (Booth et al., 1983):

(a) the *strain-rate sensitivity*. This will be dependent on the type of steel being used eg. the yield stress of mild steels show more marked increases than do stainless steels under equivalent strain rate conditions. Also, the relative significance of the strain rate is affected by the scale factor, which is inversely proportional to the strain rate, e.g. the strain rate would be four times greater in a quarter-scale model than in the full-scale prototype.

(b) the *impact velocity*. Impact velocities should be the same in both the model and the prototype.

(c) *gravity*. Gravitational forces should be relatively insignificant compared to other external forces.

These considerations will also apply to a shock loaded structure, with the exception that in this case the incident peak pressures, rather than the impact velocities, must be the same in both the model and the prototype (Baker, 1991). It has been shown by way of example that, although in many cases it is impossible to achieve similitude for all independent variables, the effect of some of these variables may prove to be relatively small on the overall result. It is the inherent weakness of the Buckingham-π technique that there is no indication as to the relative significance of each of the π-terms. To definitively determine whether a structure is feasible for scale modelling may ultimately require a set of experimental tests.

Geometric scaling procedures have often proven successful when applied to structures that are not strain-rate sensitive and undergoing a dynamic response that is completely ductile in nature. Several authors reported here (W. Baker, 1991; Ezra & Penning, 1962; Calladine & English, 1984; Wen & Jones, 1993) have shown that geometric scaling could be applied to elastic and plastic dynamic structural response, so long as the non-scalable effects such as gravity, viscosity and strain rate were negligible.

For problems involving the dynamic loading of steel plates through shock mechanisms, strain-rate effects are not necessarily negligible, hence scaling procedures may not be suitable for these strain-rate sensitive structures. In addition, metallic structures become less scalable when their behaviour is no longer ductile. It is also possible that there are other effects which may prove not to be scalable, eg. the behaviour of the fluid cavitation region near the plate surface, the interaction of the incident spherical waves and the diffracted shock waves with the reflected waves from the plate surface.

6. Conclusion

This paper has discussed the use of scaling procedures for the study of structural response to underwater explosions. It has been shown that in this case the scaling approach has limitations, but may be feasible if the study is limited to specific scaling relationships under particular conditions.

Hopkinson scaling is a well-accepted approach for the pressure-time characteristics of underwater and air blast explosions. For example, this has been embodied in the similitude equations of Swisdak and form the basis for determining the scenarios for first of class underwater shock testing.

Although limited evidence has been cited which points to scalable behaviour with respect to structural response, generality in this area has not been demonstrated. There is a conspicuous paucity of data in the area of underwater explosions. The limited evidence on scaling of structural response to dynamic loads suggests that strain rate effects (for strain rate sensitive materials) are likely to make assessment of structural response from small scale structures difficult because of the inverse scaling relationship of strain rate. The problem is complicated further by changes in material properties under dynamic conditions (strain rate).

It would seem that further investigation of strain-rate effects and structural scaling relationships are required since the testing of scaled structures is likely to assume greater importance as the cost of full scale structural tests escalates. A distorted model approach (Ezra & Penning, 1962) may be feasible for scaled structure testing programs, provided there is a demonstrated "relationship between the dependent variable and the independent variable for which similitude could not be provided".

The designs of future AMRL small-scale underwater shock experiments ought to take into account the limited range of shock levels over which scaling is likely to be successful. It should not be used as the sole means for predicting non-linear structural response, particularly where the materials being tested are highly strain-rate sensitive. Far-field small-scale underwater shock experiments are more likely to be representative of similar large scale tests since gravitational effects can be minimised with respect to shock wave phenomena. This can be satisfactorily achieved for far-field geometries because of the greater explosive-target standoff distance.

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19. ABSTRACT
The applicability and limitations of classical scaling relationships are reviewed in relation to an AMRL investigation into the structural response of air-backed plates to far-field underwater explosions. It is intended that this investigation will determine the effects of charge size and disposition and a range of structural parameters on the structural response of submerged, air-backed plates.