

Random Fields Governed by Stochastic Partial Differential
Equations and Their Applications to Oceanography

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13. ABSTRACT (Maximum 200 words)

An effective method of estimating transport parameters from tracer observations was developed based on the maximum likelihood approach in stochastic partial differential equations. Different classes of statistical estimators for near surface velocities and turbulence parameters have been studied. A complete error analysis for the method of moments was performed. Hamiltonian theory of wave turbulence was extended to the oceanic Rossby waves.

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Random Fields Governed by Stochastic Partial Differential Equations and Their Applications to Oceanography

PI: B. Rozovskii, Co-PI: L. Piterbarg

Final Report, March, 1998

1. Main Scientific Results

The basic goal of the research was to develop adequate mathematical techniques for solving important oceanographic problems such as extracting transport and diffusivity parameters from satellite tracer data, estimating turbulence parameters given Lagrangian observations, parameterization of mesoscale turbulent diffusion, and studying non-linear interactions of Rossby waves.

L. Piterbarg, B. Rozovskii and graduate students in cooperation with scientists from the Institute of Applied Mechanics (Kyushu University, Japan) developed an effective method of estimating transport parameters from tracer observations. The method based on asymptotic theory of maximum likelihood estimation in stochastic partial differential equations (SPDE) was successfully tested on various simulated data and was applied to computing zonal and meridional diffusivities in the North Pacific. The obtained results demonstrate an essential anisotropy of the oceanic large-scale turbulence. An alternative approach based on auto-regressive modeling was proposed by L. Piterbarg and A. Ostrovskii (Kyushu University). It gives better estimates of velocities, but works only for long time series of tracer observations, while the maximum likelihood method does not require big amount of data.

L. Piterbarg and B. Rozovskii together with researchers from Rosenstiel School of Marine and Atmospheric Research (Miami University) have studied different classes of statistical estimators for near surface velocities and turbulence parameters. In particular, a complete error analysis for the method of moments has been done. L. Piterbarg carried out a theoretical comparison of mean flow estimators based on Eulerian and Lagrangian observations respectively. The least square and maximum likelihood estimators were considered. It turns out that the Lagrangian estimators are asymptotically more exact than Eulerian estimators for incompressible fluctuating flows, while for potential flows the opposite relation holds.

L. Piterbarg has given a complete dimensional analysis of short-correlated approximation in the theory of turbulent analysis. It was shown that the resulting equation for the passive scalar mean field strongly depends on the order of a "hidden" time scale appearing in this problem. This time scale, called the turnover time, is defined as the ratio of the correlation radius to the mean square velocity fluctuation. As a consequence, different physical effects for different separation scales were found. Also some new rigorous results for the regime of superdiffusion were obtained.

Our findings in Hamiltonian theory of 2D hydrodynamics and Rossby waves comprise an effective method of introducing canonical variables into the Charney-Hasegawa-Mima equation and reducing the 2D hydrodynamics Poisson bracket to the Gardner-Zakharov-Faddeev bracket (L. Piterbarg). The canonical variables provide a power tool for deriving and analyzing the energy transform equation.

The progress reported above would not have been possible without substantial development of the Wiener chaos theory, the theory of stochastic integration and SPDE's. During the reporting

period we were able to make substantial progress in these three crucial areas. Mikulevicius and Rozovskii developed a Wiener chaos approach to parabolic SPDE's. It turned out that this approach is useful not only for numerical purposes (this feature was used in WONF), but also analytically. Specifically it allows one to study equations of nonlinear filtering under absolutely minimal assumptions on regularity of the coefficients and the free forces. Mikulevicius and Rozovskii developed a new general concept of stochastic integration, which includes the Ito integral, integration with respect to a stochastic flow, orthogonal martingale measures, etc.

In the reporting period we also continued our research on parameter estimation for stochastic differential equations. Huebner and Rozovskii investigated asymptotic properties of the maximum likelihood estimators (MLE's) for unknown parameters occurring in the coefficients of SPDE's. The investigators introduced a spectral method of computing MLE's based on Galerkin approximation of the original equation. Conditions are established to guarantee the consistency and asymptotic normality of the estimator as the dimension of the approximation tends to infinity.

2. Conferences

The results of the research supported by this grant were presented by B. Rozovskii and L. Piterbarg at the following major conferences.

1. AMS Summer Institute, Stochastic Analysis (Ithaca, NY), 1993.
2. Southern California Annual Conference in Probability and Statistics (Los Angeles), 1993.
3. Sixth Annual Copper Mountain Conference on Multigrid Methods (Copper Mountain, Colorado), 1993.
4. ONR Workshop on Random Fields for Oceanographic Modeling (Los Angeles), 1993.
5. USC-Hughes Workshop on Stochastic Modeling and Simulation (Los Angeles), 1993.
6. International conference "Stochastic Partial Differential Equations and Random Media" (Marseille, France), 1994.
7. Fourth Eugene Lukas Symposium (Bowling Green, Ohio), 1994.
8. Southern California Conference on PDE's and Analysis (Los Angeles), 1994.
9. U.S.-Japan Bilateral Seminar on Stochastic Analysis in Infinite Dimensional Spaces (University of Louisiana), 1994.
10. Workshop in Nonlinear Filtering (Chapel Hill, NC), 1994.
11. World Congress of the Bernoulli Society (Chapel Hill, NC), 1994.
12. 1994 SIAM Annual Meeting (San Diego, CA), 1994.
13. 1994 ONR Workshop on Random Fields (Santa Barbara).
14. University of Minnesota, 1995.
15. American Mathematical Society--Israel Mathematical Union, Joint Meeting, 1995.
16. Third IEEE Mediterranean Symposium on New Directions in Control and Automation (Limassol, Cyprus), 1995.
17. Joint Meeting of Southern California Sections of MAA and SIAM (San Diego, CA), 1996.
18. Fourth World Congress of Bernoulli Society (Vienna, Austria), 1996.
19. International Workshop on Computational and Statistical Issues for Stochastic Processes (Cremona, Italy), 1996.
20. Conference on Stochastic Analysis, Random Fields and Applications (Ascona, Switzerland), 1996.
21. 1996 SIAM Annual Meeting (Kansas City), 1996.
22. Workshop on Stochastic Control and Nonlinear Filtering, North Carolina State University, 1996.
23. 36th IEEE Conference on Decision and Control (Kobe, Japan), 1996
24. Some Problems of Stochastic Analysis, Workshop, Michigan State University, 1996.

25. Topics on Stochastic Control, Workshop, Osaka University (Osaka, Japan), 1996.
26. Instructional Conference on Stochastic Partial Differential Equations (Edinburgh, UK), 1997.
27. Ocean Waves Workshop, Tucson, Arizona, March 16-18, 1994.
28. 3dWorld Congress of the Bernoulli Society, Chapel Hill, North Carolina, June 20-25, 1994.
29. 2d International Symposium on Assimilation of Observations in Meteorology and Oceanography, Tokyo, Japan, March 13-17, 1995.
30. Dynamics of Ocean and Atmosphere, Moscow, November 22-25, 1995.
31. Inverse Problems: Geophysical Applications, Fish Camp, California, December 16-19, 1996.
32. AGU Fall Meeting, San Francisco, California, December 15-19, 1996.
33. Mathematical and Computational Issues in the Geosciences, Albuquerque, New Mexico, June 16-19, 1997.
34. IMS Annual Meeting, Park City, Utah, July 28-31, 1997.
35. International Probability Symposium, Park City, Utah, July 30-August 1, 1997.
36. Ocean Science Meeting, San Diego, February 9-13, 1996.

3. Cooperation

The research was carried out in close cooperation with

A. Griffa and A. Mariano, Rosenstiel School of Marine and Atmosphere Sciences, Department of Meteorology and Physical Oceanography, Miami University, Miami.

A. Ostrovskii, Research Institute for Applied Mechanics, Kyushu University, Kasuga, Japan.

K. Owens, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California.

4. List of Publications supported by ONR Grant N00014-91-J-1526

Monographs

[1]. R. Adler, P. Müller, B. Rozovskii (eds.). *Stochastic Modeling in Physical Oceanography*, Birkhäuser, Boston-Basel-Berlin, 1996.

[2]. L. Piterbarg and A. Ostrovskii, *Advection and Diffusion in Random Media: Implications for Sea Surface Temperature Anomalies*, Kluwer Academic Publishers, Dordrecht-Boston-London, 1997, 344pp.

Technical Papers

[1] B. Rozovskii, M. Hubner, "On parameter estimation for parabolic SPDE's. A ML approach," *Probab. Theory and Relat. Fields*, Vol. 103, No. 2, 1995.

[2] B. Rozovskii, R. Mikulevicius "On stochastic integrals in topological vector spaces," *Proceedings of Symposia in Pure Mathematics*, AMS, Vol. 57, 1995.

[3] B. Rozovskii, R. Mikulevicius "Soft solutions of linear parabolic SPDE's and the Wiener Chaos expansion" In H. Kunita and H. -H. Kuo, editors, *Stochastic Analysis on Infinite Dimensional Spaces*, Pitman Research Notes in Mathematics Series 310, Longman, 1994.

[4] B. Rozovskii, R. Mikulevicius "Uniqueness and absolute continuity of weak solutions for parabolic SPDE's", *Acta Applicanda Math* Vol. 35, no.1, 1994.

- [5] B. Rozovskii, P. Baxendale "Kinematic dynamo and intermittence in a turbulent flow", *Geophys., Astrophys., Fluid Dynamics, Vol.73*, 1993.
- [6] B. Rozovskii, R. Mikulevicius "Separation of observations and parameters in nonlinear filtering", *Proceedings of the 32nd IEEE Conference on Decision and Control*, vol. 2, IEEE Control Systems Society, 1993.
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- [14]. S. Molchanov, L. Piterbarg, "Heat transport in a random media", *Russ. J. Math. Phys.*, Vol.1, no. 3, 353-376, 1994.
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- [16]. L. Piterbarg, "Hamiltonian formalism for Rossby waves", *Amer. Math. Society Transl.*, Vol. 182, 131-165, 1998.
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- [18]. L. Piterbarg, B. Rozovskii, "Maximum likelihood estimators in the equations of physical oceanography", *Stochastic Modeling in Physical Oceanography*, eds. R. Adler, P. Müller, B. Rozovskii, Birkhäuser, Boston-Basel-Berlin, 397-422, 1996.

5. Ph.D. Students

Ph.D. Students Supported by the Grant

M. Huebner (USC, June 1993, Presently--Assistant Professor, Michigan State University).

K. Owens (USC, June 1994, Presently-Jet Propulsion Laboratory, NASA).

A. Fung (USC, July 1995, Presently, Engineer, Aegir Systems, Oxnard, CA).

S. Lototsky (USC, July 1996, Presently-Assistant Professor, MIT).