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**THESIS**

**PERFORMANCE ANALYSIS OF A SFH/NCBFSK  
COMMUNICATION SYSTEM WITH RATE 1/2  
CONVOLUTIONAL CODING AND SOFT DECISION  
VITERBI DETECTION OVER A RICEAN FADING  
CHANNEL WITH PARTIAL-BAND NOISE JAMMING**

by

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March 1998

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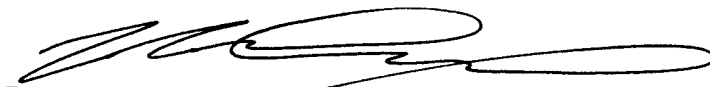
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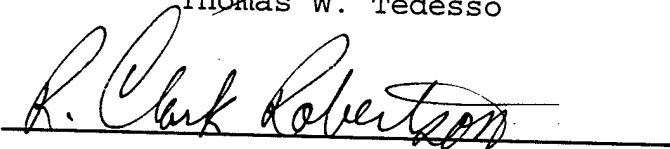
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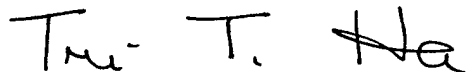


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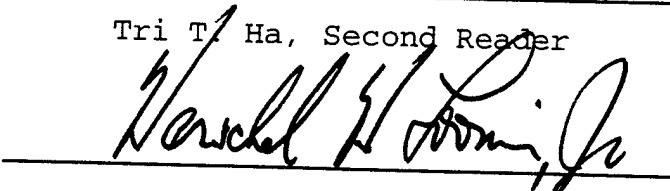
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## ABSTRACT

A performance analysis of a slow frequency-hopped, noncoherent binary frequency-shift keying (SFH/NCBFSK) communication system with rate  $\frac{1}{2}$  convolutional coding and soft decision Viterbi detection in the presence of partial-band noise jamming is performed. The effect of additive white Gaussian noise is also considered. The analysis is performed for both a non-fading channel and a Ricean fading channel. The system's performance is severely degraded by partial-band noise jamming. By way of comparison the analysis is also performed when the system utilizes hard decision Viterbi detection and for a system utilizing noise-normalized combining with soft decision Viterbi detection. In both cases a significant increase in the system's immunity to the effects of partial-band noise jamming is achieved.



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## I. INTRODUCTION

In the design of a digital communication system, it is desirable to obtain the minimum probability of bit error ( $P_b$ ) with minimum transmission power, minimum bandwidth, and minimum transmitter and receiver complexity. In some applications, particularly military applications, there are several considerations which may supersede more conventional ones. Military applications are typically concerned with a system's ability to reject hostile interference, to decrease the ability of a hostile observer to detect that communications are taking place, and/or to decrease the ability of a hostile observer to "listen in" when communications are taking place. To achieve these goals, both transmitter and receiver complexity and transmission bandwidth are frequently sacrificed. The general class of communications systems which require significantly more complex transmitter and receiver circuitry and significantly more transmission bandwidth than is required to transmit a particular signal are called spread spectrum communications systems.

The two types of spread spectrum techniques most commonly employed are direct sequence (DS) spread spectrum and frequency-hopped (FH) spread spectrum. In a direct

sequence spread spectrum system, the signal is multiplied by a high rate pseudo-random binary sequence, with the result that the transmitted signal's power spectrum is spread over a significantly larger bandwidth. This lowers the magnitude of the transmitted signal's spectrum and makes it appear more noise-like to an observer. [Ref. 1]

By making the transmitted signal's spectrum appear more noise-like, it is more difficult for a hostile observer to intercept and detect the signal as compared to a conventional, non-spread spectrum signal. Therefore, a direct sequence spread spectrum system may be characterized as both a low probability of detection (LPD) and a low probability of intercept (LPI) system as compared with conventional communication systems. DS spread spectrum can also be used as a multiple access technique which is referred to as code division multiple access (CDMA).

Another application of DS spread spectrum is ranging, and DS spread spectrum is the basis of the Global Positioning System (GPS).

In a frequency-hopped spread spectrum system, rather than transmit all the symbols with the same carrier frequency, the carrier frequency of the signal is changed (hopped) in a pseudo-random fashion over a large band of frequencies. Since a potential adversary does not know how

the carrier frequency is varied, it may be able to detect the signal; however, it will be more difficult to intercept than conventional signals. Therefore, a FH spread spectrum system may be considered to be a LPI system as compared to conventional communication systems. A FH spread spectrum system may be a fast frequency-hopped (FFH) system or a slow frequency-hopped (SFH) system. In a slow frequency-hopped system, more than one symbol is transmitted per frequency hop. In a fast frequency-hopped system, one symbol is transmitted for one or more consecutive hops. FH spread spectrum systems are typically used in military applications where anti-jam protection and low probability of interception is required. [Ref. 1] FH can also be used for multiple access applications. In this thesis only the performance of a FH spread spectrum system will be analyzed.

Communication system performance can also be improved significantly by utilizing some form of error control coding. There are two basic error control strategies, automatic repeat request (ARQ) and forward error correction (FEC) coding. ARQ systems require data to be transmitted in packets. If no errors are detected in a received packet, the receiver sends a positive acknowledgement. If errors are detected in a received packet, the receiver

sends a negative acknowledgement to the transmitter and the transmitter retransmits the incorrectly transmitted packet. All ARQ systems ideally require a noise-free feedback channel from the receiver to the transmitter and are generally easier to implement than FEC coding. [Ref. 2] ARQ systems are extensively used in computer network applications.

Forward error correction coding consists of adding a certain number of parity check bits to the actual data bits such that recovery of the actual data bits is enhanced. [Ref. 2] There are primarily two types of errors that are introduced by a transmission channel, random errors and burst errors. Random errors are introduced by additive white gaussian noise (AWGN). Burst errors, which are a cluster of errors occurring in a relative short period of time, are caused when the received signal-to-noise ratio is subject to large fluctuations. Signal fading or a hostile pulse noise-jamming signal may cause these fluctuations. The majority of FEC codes are random error correcting codes; however, these codes can also be used to correct burst errors by a process called interleaving. The two fundamental types of FEC codes are block codes and convolutional codes. Block codes are characterized as  $(n,k)$  codes where  $n$  is the number of message symbols, and  $k$

is the number of data symbols. [Ref. 2] An example of types of block codes are BCH (Bose, Chaudhuri and Hocquenghem) codes and Reed-Solomon codes. Block codes will not be studied in this thesis.

Convolutional codes are characterized as a  $(n,k,m)$  code where  $n$  and  $k$  have an analogous meaning as in block codes, and  $m$  refers to the memory of the code. Convolutional codes are often characterized as having a code rate,  $r$ , and a constraint length  $v$ . The code rate is equal to  $k/n$  for long data sequences. The most common definition for constraint length is the maximum number of shifts over which a single information bit can affect the encoder output. Thus, according to this definition the constraint length  $v$  is equal to  $(m + 1)$ . Convolutional codes are typically binary codes, although, non-binary convolutional codes exist. Convolutional codes may be either systematic or nonsystematic codes. A convolutional code may encounter a catastrophic error, which is an event where a finite number of code symbol errors cause an infinite number of decoded data bit errors. [Ref. 2] Systematic codes are guaranteed to be non-catastrophic. Non-systematic convolutional codes, however, have more coding gain than systematic convolutional codes of the same constraint length and generally are preferred with Viterbi

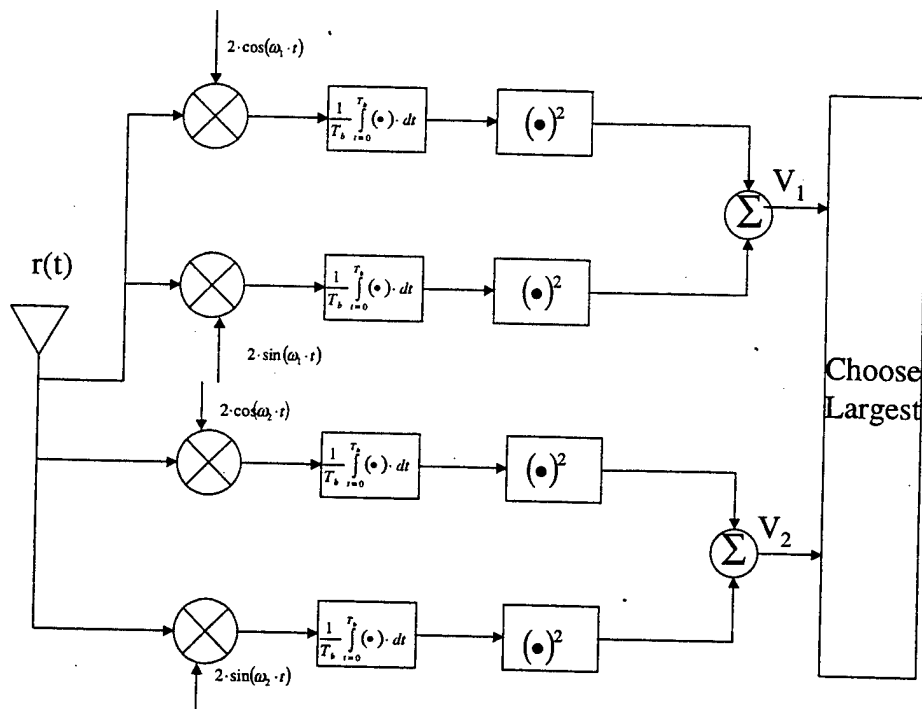
decoders. The Viterbi decoder utilizes a maximum-likelihood decoding algorithm and is relatively simple to implement. [Ref. 2] Typical convolutional codes are of rate between  $7/8$  and  $1/4$  and of constraint length from 2 to 9.

In this thesis we will examine the performance of a slow frequency-hopped, non-coherent binary frequency-shift keying (SFH/NCBFSK) communication system with rate  $1/2$  convolutional coding and soft decision Viterbi decoding. The performance will be analyzed for both a non-fading channel and a slowly fading, frequency non-selective channel modeled as a Ricean fading channel. [Refs. 3 and 4] The system's performance will be examined when it is subjected to partial-band noise jamming for both the non-fading channel and the Ricean fading channel cases. The analysis of this system is of interest because similar communication systems are currently in use. The resistance of a communication system to jamming is of extreme importance to military communication systems. Since partial-band noise jamming can be a particularly effective jamming scheme, the system's ability to resist the effects of partial-band noise jamming is of particular interest.

## II. BACKGROUND

### A. NCBFSK

A conventional noncoherent binary frequency-shift keying (NCBFSK) receiver is shown in Figure 1.



**Figure 1: NCBFSK Receiver**

In the presence of AWGN, the probability density functions (pdfs) of the decision variables  $V_1$  and  $V_2$  are [Ref. 3]

$$f_{V_1}(v_1) = \frac{1}{2\sigma^2} \cdot \exp\left(-\frac{v_1 + 2A_c^2}{2\sigma^2}\right) \cdot I_0\left(\frac{A_c\sqrt{2 \cdot v_1}}{\sigma}\right) \quad (2.1)$$

and

$$f_{V_2}(v_2) = \frac{1}{2\sigma^2} \cdot \exp\left(\frac{-v_2}{2\sigma^2}\right) \quad (2.2)$$

where  $\sigma^2 = N_0/T_b$ . The probability of bit error is obtained by evaluating [Ref. 4]

$$P_b = 1 - \int_0^{\infty} f_{V_1}(v_1) \cdot \left[ \int_0^{v_1} f_{V_2}(v_2) \cdot dv_2 \right] \cdot dv_1. \quad (2.3)$$

From Equation (2.3), we obtain an expression for the probability of bit error as [Ref. 3]

$$P_b = \frac{1}{2} \cdot \exp\left(-\frac{E_b}{2N_0}\right). \quad (2.4)$$

In the presence of a barrage noise jamming signal, a jamming signal with power spectral density  $N_J/2$  over the entire bandwidth of the conventional NCBFSK signal, the expressions for the pdfs of the decision variables  $V_1$  and  $V_2$  must be modified such that  $\sigma^2 = (N_0 + N_J)/T_b$ . Thus, the probability of bit error becomes [Ref. 1]

$$P_b = \frac{1}{2} \cdot \exp\left(-\frac{E_b}{2 \cdot (N_0 + N_J)}\right). \quad (2.5)$$

## B. FH/NCBFSK

A frequency-hopped spread spectrum system may either be fast frequency-hopped (FFH) or slow frequency-hopped (SFH). Fast frequency-hopping implies that the carrier

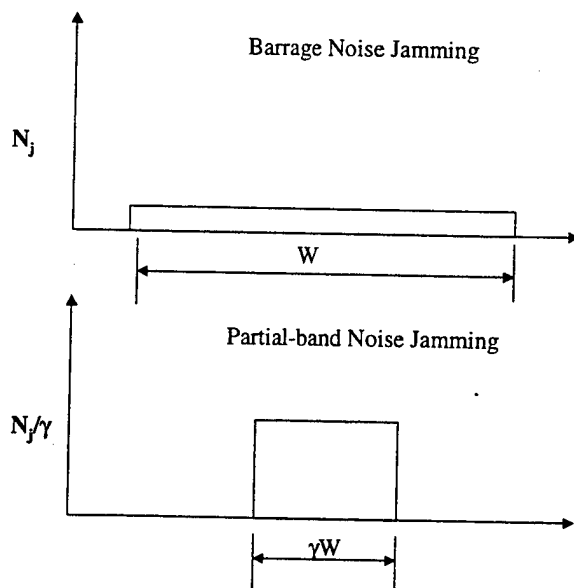
frequency changes one or more times for each bit that is transmitted. For slow frequency-hopping, multiple bits are transmitted per hop.

A FH/NCBFSK system utilizes  $N$  different carrier frequencies, resulting in  $N$  possible frequency bands, or bins, which may contain the spectrum of the NCBFSK signal at a given time. For a SFH/NCBFSK system, the level of diversity,  $L$ , is equal to one. For perfect synchronization, the performance of a SFH/NCBFSK system is the same as that of conventional NCBFSK as expressed in Equation (2.4) for AWGN. However, if a SFH/NCBFSK system is subjected to barrage noise jamming signal with power spectral density  $N_J/2$ , where the jamming signal's power is equal to that of the jamming signal for the conventional NCBFSK receiver, then  $\sigma^2 = (N_0 + N_J)/T_b$ , where  $N_J = N'_J/N$ . The probability of bit error is equal to [Ref. 1]

$$P_b = \frac{1}{2} \cdot \exp\left(-\frac{E_b}{2 \cdot (N_0 + N_J)}\right). \quad (2.6)$$

Hence, for  $N_J \gg N_0$ , SFH/NCBFSK results in a processing gain of approximately  $10 \cdot \log(N)$  dB when compared to conventional NCBFSK.

Partial-band noise jamming is when a hostile jams only a fraction of the spread-spectrum signal's bandwidth with a noise-like signal. For comparison purposes, we assume the total jammer power is the same regardless of the fraction of bandwidth jammed. Thus, the power spectral density of the partial-band jamming signal is  $N_J/2\gamma$ , where  $\gamma$  is the fraction of the total spread spectrum bandwidth,  $W$ , being jammed and  $1/W \leq \gamma \leq 1$ . This is shown in Figure 2.



**Figure 2: Power Spectrum of Barrage Noise Jamming and Partial-band Noise Jamming Signals**

If a SFH/NCBFSK system is subjected to partial-band noise jamming, the probability of bit error is [Ref. 4]

$$P_b = \frac{\gamma}{2} \cdot \exp\left(-\frac{E_b}{2 \cdot (N_0 + N_J/\gamma)}\right) + (1-\gamma) \cdot \frac{1}{2} \cdot \exp\left(-\frac{E_b}{2N_0}\right). \quad (2.7)$$

### C. FFH/NCBFSK

A fast frequency-hopped spread spectrum system employs frequency diversity by transmitting the same bit  $L$  times over separate frequencies. In a FFH/NCBFSK system, the  $L$  diversity receptions are summed at the receiver. There are a number of ways of combining the diversity receptions. A FFH/NCBFSK receiver with diversity is shown in Figure 3 that utilizes linear, soft decision combining. [Ref. 5]

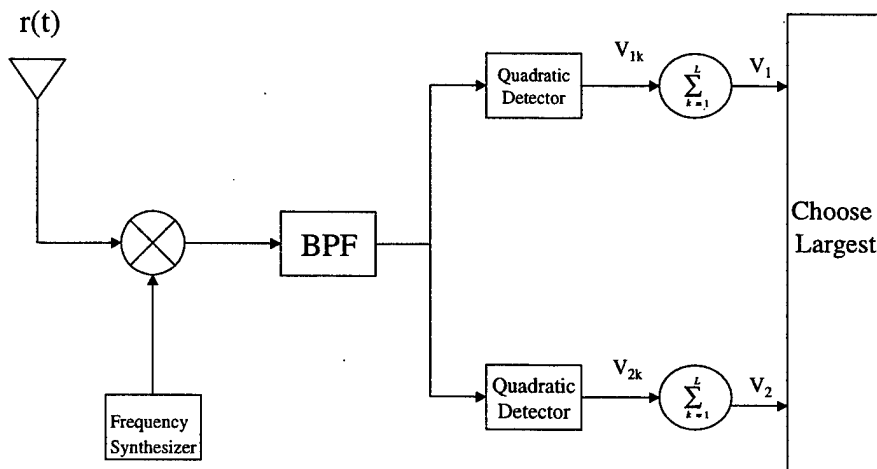


Figure 3: FFH/NCBFSK Receiver

If we assume independent diversity receptions and AWGN, for the receiver of Figure 3 we obtain the pdfs of the decision variables  $V_1$  and  $V_2$  from the L-fold convolution of the pdfs of  $V_{1k}$  and  $V_{2k}$ . Hence,

$$f_{V_1}(v_1) = [f_{V_{1k}}(v_{1k})]^{\otimes L}, \quad (2.8)$$

and

$$f_{V_2}(v_2) = [f_{V_{2k}}(v_{2k})]^{\otimes L}. \quad (2.9)$$

The pdfs of the decision variables for the independent diversity receptions are

$$f_{V_{1k}}(v_{1k}) = \frac{1}{2 \cdot \sigma_k^2} \cdot \exp\left[-\frac{(v_{1k} + 2 \cdot A_c^2)}{2 \cdot \sigma_k^2}\right] \cdot I_0\left(\frac{A_c \cdot \sqrt{2 \cdot v_{1k}}}{\sigma_k^2}\right) \quad (2.10)$$

and

$$f_{V_{2k}}(v_{2k}) = \frac{1}{2 \cdot \sigma_k^2} \cdot \exp\left(\frac{-v_{2k}^2}{2 \cdot \sigma_k^2}\right). \quad (2.11)$$

If the receiver is subjected to a barrage noise-jamming signal, then  $\sigma_k^2 = (N_0 + N_j) / T_c$ , where  $T_c$  is the hop duration. By taking the Laplace transform of the pdfs in Equations (2.10) and (2.11), we obtain [Ref. 5]

$$F_{V_{1k}}(s) = \frac{1}{2 \cdot \sigma_k^2} \cdot \left(\frac{1}{s + 1/2\sigma_k^2}\right) \cdot \exp\left(\frac{-A_c^2}{\sigma_k^2} \cdot \left(\frac{s}{s + 1/2\sigma_k^2}\right)\right) \quad (2.12)$$

and

$$F_{V_{2k}}(s) = \frac{1}{2 \cdot \sigma_k^2} \cdot \left(\frac{1}{s + 1/2\sigma_k^2}\right). \quad (2.13)$$

The Laplace transforms of the decision variables  $V_1$  and  $V_2$  are

$$F_{V_1}(s) = [F_{V_{1k}}(s)]^L \quad (2.14)$$

and

$$F_{V_2}(s) = [F_{V_{2k}}(s)]^L. \quad (2.15)$$

If we substitute Equation (2.12) and Equation (2.13) into Equation (2.14) and Equation (2.15), respectively, take the inverse Laplace transform of Equation (2.14) and Equation (2.15) and perform the integration in Equation (2.3), the probability of bit error is found to be [Ref. 4]

$$P_b = \frac{1}{2^{2L-1}} \cdot \exp\left[\frac{-E_b}{2 \cdot (N_0 + N_J)}\right] \cdot \sum_{n=0}^{L-1} c_n \cdot \left[\frac{E_b}{2 \cdot (N_0 + N_J)}\right]^n \quad (2.16)$$

where

$$c_n = \frac{1}{n!} \cdot \sum_{m=0}^{L-1-n} \binom{2L-1}{m}. \quad (2.17)$$

For partial-band jamming of a FFH/NCBFSK system, where  $\gamma$  is the fraction of  $N$  frequency bins that are jammed and  $\frac{1}{N} \leq \gamma < 1$ , the analysis is significantly more complicated. In this case, the probability that a hop is jammed is  $\gamma$ , and the probability that a hop is not jammed is  $(1-\gamma)$ . Since whether or not a hop is jammed is independent of other hops being jammed or not, the probability that  $i$  of  $L$  hops are jammed is  $\gamma^i \cdot (1-\gamma)^{L-i}$ . The number of ways that  $i$  of  $L$  hops

can be jammed is an example of Bernoulli trials and is given by the binomial coefficient. The probability of bit error, therefore, equals [Refs. 5, 6, and 7]

$$P_b = \sum_{i=0}^L \binom{L}{i} \cdot \gamma^i \cdot (1-\gamma)^{L-i} \cdot P_d(i), \quad (2.18)$$

where  $i$  is the number of the  $L$  diversity receptions which are jammed. The pdfs of the decision variables conditioned on  $i$  hops being jammed can be determined by convolving the pdfs of the random variables that represent the branch outputs prior to diversity combining to obtain [Refs. 5 and 6]

$$f_{v_1}(v_1) = [f_{v_{1k}}^{(1)}(v_{1k})]^{\otimes i} \otimes [f_{v_{1k}}^{(2)}(v_{1k})]^{\otimes L-i} \quad (2.19)$$

and

$$f_{v_2}(v_2) = [f_{v_{2k}}^{(1)}(v_{2k})]^{\otimes i} \otimes [f_{v_{2k}}^{(2)}(v_{2k})]^{\otimes L-i} \quad (2.20)$$

where the superscripts (1) and (2) refer to hops which are jammed and not jammed, respectively. No simple analytical solution exists for the two previous pdfs. If we assume perfect side information, disregard all jammed hops when  $i$  is less than  $L$ , and assume that thermal noise is negligible, then the probability of bit error is [Ref. 4]

$$P_b = \frac{\gamma^L}{2^{2L-1}} \cdot \exp\left[\frac{-E_b}{2 \cdot (N_0 + N_j/\gamma)}\right] \cdot \sum_{n=0}^{L-1} c_n \cdot \left[\frac{E_b}{2 \cdot (N_0 + N_j/\gamma)}\right]^n \quad (2.21)$$

where

$$c_n = \frac{1}{n!} \cdot \sum_{m=0}^{L-1-n} \binom{2L-1}{m}. \quad (2.22)$$

#### D. CODING

In this thesis, a SFH/NCBFSK system with convolutional coding is analyzed. There are many types of forward error correcting codes, block codes and convolutional codes being the most common. Error correction coding is employed to achieve increased system performance up to or near the Shannon limit. The Shannon capacity theorem states that channel capacity is equal to [Ref. 3]

$$C = W \cdot \log_2 \left( 1 + \frac{E_b \cdot R_b}{N_0 \cdot W} \right) \quad (2.23)$$

where  $C$  is channel capacity,  $W$  is the noise equivalent bandwidth, and  $R_b$  is the bit rate. Equation (2.23) can be rearranged and the theoretical minimum signal-to-noise ratio may be calculated, resulting in  $E_b/N_0 = -1.6$  dB. In principle, we should be able to design a communication system that operates at or near the Shannon limit.

A convolutional code is classified as a  $(n,k,m)$  code, where  $n$  coded bits are generated for every  $k$  data bits, and  $m$  is the memory of the coder. A convolutional code produces  $n$  coded bits from  $k$  data bits where each set of  $n$  coded bits is determined by the  $k$  data bits and between  $v-1$

and  $k(v-1)$  of the preceding data bits. The parameter  $v$  is the constraint length, previously defined in chapter I.

Convolutional codes are commonly decoded using the Viterbi algorithm. The Viterbi algorithm may be implemented using hard or soft decision decoding. For hard decision decoding, the Viterbi algorithm is a minimum Hamming distance decoder and decodes a convolutional code by choosing a path through the code trellis which yields a code word that differs from the received code word in the fewest possible places. For soft decision decoding, the maximum-likelihood detector is optimal, and for a binary symmetric channel, maximizing the log-likelihood function is equivalent to minimizing the Hamming distance. Thus, soft decision decoding utilizes the analog inputs of the demodulator's matched filters and corresponds to a discrete memoryless channel with infinite quantization. [Ref. 2]

In a system employing convolutional coding with Viterbi decoding, an upper bound on the probability of bit error is [Ref. 2]

$$P_b < \frac{1}{k} \sum_{d=d_{free}}^{\infty} B_d \cdot P_d \quad (2.24)$$

where  $B_d$  is the total number of nonzero information bits on all weight  $d$  paths and  $P_d$  is the probability of selecting a code word that is a Hamming distance  $d$  from the correct

code word. The best maximum weight structure for a  $\frac{1}{2}$  rate convolutional code is shown in Table 1 [Ref. 8].

**Table 1: Best (maximum free distance) rate 1/2, convolutional code weight structure**

$v$	$d_{free}$	$B_{d_{free}}$	$B_{d_{free}+1}$	$B_{d_{free}+2}$	$B_{d_{free}+3}$	$B_{d_{free}+4}$
3	5	1	4	12	32	80
4	6	2	7	18	49	130
5	7	4	12	20	72	225
6	8	2	36	32	62	332
7	10	36	0	211	0	1404
8	10	2	22	60	148	340
9	12	33	0	281	0	2179

It can be shown for a NCBFSK system utilizing soft decision Viterbi decoding that  $P_d$  is equivalent to the probability of bit error for NCBFSK signaling with  $d_{th}$  order diversity [2]. Thus,  $P_d$  is [Ref. 4]

$$P_d = \frac{1}{2^{2d-1}} \cdot \exp\left[\frac{-rE_b}{2N_0}\right] \cdot \sum_{n=0}^{d-1} c_n \cdot \left[\frac{rE_b}{2N_0}\right]^n \quad (2.25)$$

where

$$c_n = \frac{1}{n!} \cdot \sum_{m=0}^{d-1-n} \binom{2d-1}{m} \quad (2.26)$$

In this chapter we presented the background material for both FH/NCBFSK systems and systems using convolutional

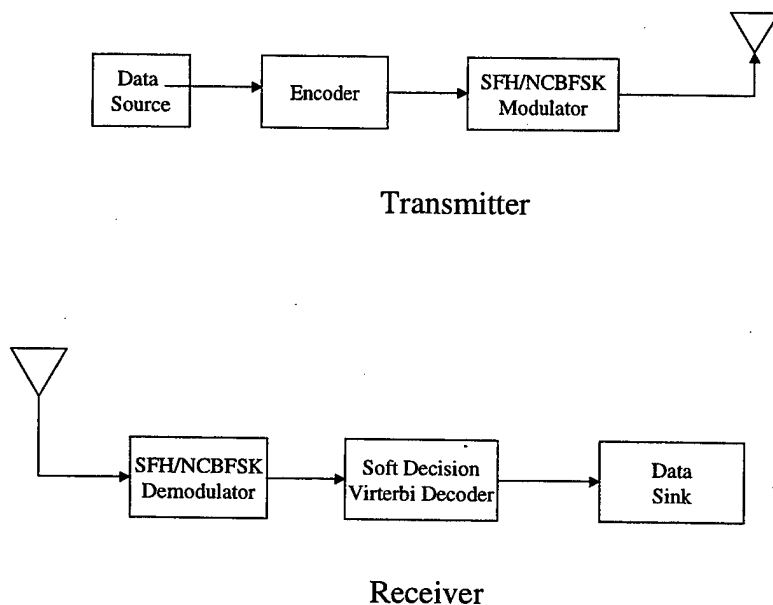
coding. These basic results will be applied in the following chapter to determine expressions for the probability of bit error for a SFH/NCBFSK communication system with rate  $1/2$  convolutional coding and soft decision Viterbi decoding in the presence of partial-band noise jamming for both non-fading and Ricean fading channels. By way of comparison, a receiver utilizing noise-normalized combining and soft decision Viterbi decoding is analyzed as well as a receiver using hard decision Viterbi decoding.

### III. PERFORMANCE ANALYSIS

#### A. SOFT DECISION DETECTION

##### 1. Without Fading

A SFH/NCBFSK communication system with rate 1/2 convolutional coding and soft decision Viterbi decoding is shown in Figure 4.



**Figure 4: SFH/NCBFSK Communication System with rate 1/2 convolutional coding and soft decision Viterbi detection**

For a communication system that employs convolutional coding with Viterbi decoding, the probability of bit error is upper bounded by

$$P_b < \sum_{d=d_{free}}^{\infty} B_d \cdot P_d . \quad (3.1)$$

A NCBFSK system employing convolutional coding and soft decision Viterbi decoding is equivalent to a system with  $d_{th}$  order diversity. For a SFH/NCBFSK communications system with rate 1/2 convolutional coding and soft decision Viterbi decoding in the presence of partial-band noise jamming,  $P_a$  is [Refs. 5, 6, and 7]

$$P_d = \sum_{i=0}^d \binom{d}{i} \gamma^i \cdot (1-\gamma)^{d-i} \cdot P_d(i) \quad (3.2)$$

where  $\gamma$  is equal to the fraction of the spread spectrum bandwidth being jammed, and  $P_d(i)$  is the probability of selecting a code word a Hamming distance  $d$  from the correct code word given that  $i$  of  $d$  diversity receptions are jammed.  $P_d(i)$  is given by [Refs. 5 and 6]

$$P_d(i) = \int_0^{\infty} f_{v_1}(v_1 | i) \cdot \left[ 1 - \int_0^{v_1} f_{v_2}(v_2 | i) dv_2 \right] dv_1 . \quad (3.3)$$

The probability density functions for the decision variables  $V_{1k}$  and  $V_{2k}$  are [Ref. 5]

$$f_{V_{1k}}(v_{1k}) = \frac{1}{2 \cdot \sigma_k^2} \cdot \exp \left[ -\frac{(v_{1k} + 2 \cdot A_c^2)}{2 \cdot \sigma_k^2} \right] \cdot I_0 \left( \frac{A_c \cdot \sqrt{2 \cdot v_1}}{\sigma_k} \right) \quad (3.4)$$

and

$$f_{V_{2k}}(v_{2k}) = \frac{1}{2 \cdot \sigma_k^2} \cdot \exp \left( \frac{-v_{2k}}{2 \cdot \sigma_k^2} \right) . \quad (3.5)$$

If we indicate jammed bits and bits which are not jammed by the superscripts (1) and (2), respectively, the probability density functions for the decision variables  $V_1$  and  $V_2$  are [Ref. 5]

$$f_{V_{1i}}(v_1 | i) = [f_{V_{1k}}^{(1)}(v_{1k})]^{\otimes i} \otimes [f_{V_{1k}}^{(2)}(v_{1k})]^{\otimes (d-i)} \quad (3.6)$$

$$f_{V_{2i}}(v_2 | i) = [f_{V_{2k}}^{(1)}(v_{2k})]^{\otimes i} \otimes [f_{V_{2k}}^{(2)}(v_{2k})]^{\otimes (d-i)} \quad (3.7)$$

where  $\otimes$  indicates convolution. We designate the noise power  $\sigma_k^2$  for jammed bits as  $\sigma_1^2$  and non-jammed bits as  $\sigma_2^2$ , which equal

$$\sigma_1^2 = \frac{(N_0 + \frac{N_j}{r})}{r \cdot T_b} \quad (3.8)$$

and

$$\sigma_2^2 = \frac{N_0}{r \cdot T_b} \quad (3.9)$$

where  $r$  is the code rate and  $T_b$  is the period of a bit.

Substituting for the pdfs of  $V_{1k}$  and  $V_{2k}$  and taking the Laplace transforms of Equations (3.7) and (3.8), we obtain [Ref. 5]

$$F_{V_{1i}}(s) = \left[ \frac{1}{2 \cdot \sigma_1^2} \cdot \frac{1}{s + \frac{1}{2\sigma_1^2}} \cdot \exp\left( \frac{-A_c^2}{\sigma_1^2} \cdot \frac{s}{s + \frac{1}{2\sigma_1^2}} \right) \right]^i \cdot \left[ \frac{1}{2 \cdot \sigma_2^2} \cdot \frac{1}{s + \frac{1}{2\sigma_2^2}} \cdot \exp\left( \frac{-A_c^2}{\sigma_2^2} \cdot \frac{s}{s + \frac{1}{2\sigma_2^2}} \right) \right]^{d-i} \quad (3.10)$$

and

$$F_{V_{2i}}(s) = \left[ \frac{1}{2 \cdot \sigma_1^2 \cdot s + 1} \right]^i \cdot \left[ \frac{1}{2 \cdot \sigma_2^2 \cdot s + 1} \right]^{d-i} \quad (3.11)$$

Since no simple analytic solutions exist for the inverse Laplace transforms of the above expressions which are easily obtainable except for the cases where  $i=0$  and  $i=d$ , the inverse Laplace transforms are determined numerically.

[Ref. 5]

For the special case of barrage noise jamming where  $\gamma=1.0$ , the following analytical expression exists for  $P_d$

[Ref. 4]

$$P_d = \frac{1}{2^{2d-1}} \cdot \exp\left(\frac{-d \cdot r \cdot E_b}{2 \cdot (N_0 + N_j)}\right) \cdot \sum_{n=0}^{d-1} c_n \cdot \left(\frac{d \cdot r \cdot E_b}{2 \cdot (N_0 + N_j)}\right)^n \quad (3.12)$$

where

$$c_n = \frac{1}{n!} \cdot \sum_{m=0}^{d-1-n} \binom{2d-1}{m} \quad (3.13)$$

## 2. Ricean Fading Channel

If the channel is modeled as a slowly fading, frequency non-selective Ricean fading channel, and the signaling tone only is subject to fading, the results obtained for the probability density functions obtained for the ideal non-fading channel must be modified. In this case, the signaling tone's amplitude is modeled as a Ricean

random variable. The probability density function of the amplitude is [Ref. 4]

$$f_{A_k}(a_k) = \frac{a_k}{\sigma^2} \cdot \exp\left(-\frac{a_k^2 + \alpha_k^2}{2 \cdot \sigma^2}\right) \cdot I_0\left(\frac{a_k \cdot \alpha_k}{\sigma^2}\right) \cdot u(a_k) \quad (3.14)$$

where  $\alpha_k^2$  is the direct signal component and  $2\sigma^2$  is the diffuse signal component. The total average signal energy of bit k is assumed to be constant from bit to bit and is equal to

$$\overline{a_k^2} = \alpha^2 + 2 \cdot \sigma^2. \quad (3.15)$$

The probability density function of the decision variable for the receiver branch corresponding to the signaling tone for each bit conditioned on  $a_k$  is equal to [Refs. 5, 6, and 9]

$$f_{V_{1k}|a_k}(v_{1k} | a_k) = \frac{1}{2 \cdot \sigma_k^2} \cdot \exp\left(-\frac{v_{1k} + 2 \cdot a_k^2}{2 \cdot \sigma_k^2}\right) \cdot I_0\left(\frac{a_k \cdot \sqrt{2 \cdot v_{1k}}}{\sigma_k^2}\right) \cdot u(a_k). \quad (3.16)$$

We can determine the pdf of  $V_{1k}$  by evaluating [Ref. 9]

$$f_{V_{1k}}(v_{1k}) = \int_0^{\infty} f_{V_{1k}|a_k}(v_{1k} | a_k) \cdot f_{A_k}(a_k) \cdot da_k, \quad (3.17)$$

to obtain [Refs. 5 and 9]

$$f_{V_{1k}}(v_{1k}) = \frac{1}{2 \cdot (\sigma_k^2 + 2 \cdot \sigma^2)} \cdot \exp\left(-\frac{1}{2} \cdot \frac{v_{1k} + 2 \cdot \alpha^2}{\sigma_k^2 + 2 \cdot \sigma^2}\right) \cdot I_0\left(\frac{\alpha \cdot \sqrt{2 \cdot v_{1k}}}{\sigma_k^2 + 2 \cdot \sigma^2}\right). \quad (3.18)$$

The probability density function of  $V_{2k}$  is not conditioned upon  $a_k$ , and is therefore equal to Equation (3.5), repeated below

$$f_{V_{2k}}(v_{2k}) = \frac{1}{2 \cdot \sigma_k^2} \cdot \exp\left(\frac{-v_{2k}}{2 \cdot \sigma_k^2}\right). \quad (3.5)$$

Following the same approach as for the non-fading channel case, we get the pdfs conditioned on  $i$  as [Ref. 5]

$$f_{V_{1i}}(v_1 | i) = [f_{V_{1k}}^{(1)}(v_{1k})]^{\otimes i} \otimes [f_{V_{1k}}^{(2)}(v_{1k})]^{\otimes (d-i)} \quad (3.19)$$

$$f_{V_{2i}}(v_2 | i) = [f_{V_{2k}}^{(1)}(v_{2k})]^{\otimes i} \otimes [f_{V_{2k}}^{(2)}(v_{2k})]^{\otimes (d-i)}. \quad (3.20)$$

The noise powers of the jammed and non-jammed receptions are

$$\sigma_1^2 = \frac{(N_0 + \frac{N_j}{\gamma})}{r \cdot T_b} \quad (3.21)$$

and

$$\sigma_2^2 = \frac{N_0}{r \cdot T_b}, \quad (3.22)$$

respectively. Substituting Equations (3.18) and (3.5) into Equations (3.19) and (3.20), respectively, and taking the Laplace transform, we obtain [Ref. 5]

$$F_{V_{1i}}(s) = \left[ \frac{\beta_1}{s + \beta_1} \cdot \exp(-2 \cdot \sigma_1^2 \cdot \rho_1) \cdot \exp\left(\frac{2 \cdot \sigma_1^2 \cdot \beta_1^2 \cdot \rho_1}{s + \beta_1}\right) \right]^i \cdot \left[ \frac{\beta_2}{s + \beta_2} \cdot \exp(-2 \cdot \sigma_2^2 \cdot \rho_2) \cdot \exp\left(\frac{2 \cdot \sigma_2^2 \cdot \beta_2^2 \cdot \rho_2}{s + \beta_2}\right) \right]^{d-i} \quad (3.23)$$

and

$$F_{V_{2n}}(s) = \left[ \frac{1}{2 \cdot \sigma_1^2 \cdot s + 1} \right]^i \cdot \left[ \frac{1}{2 \cdot \sigma_2^2 \cdot s + 1} \right]^{d-i} \quad (3.24)$$

where

$$\beta_n = \frac{1}{2 \cdot \sigma_n^2 \cdot (1 + \xi_n)}, \quad (3.25)$$

$$\rho_n = \frac{\alpha^2}{\sigma_n^2}, \quad (3.26)$$

and

$$\xi_n = \frac{2 \cdot \sigma^2}{\sigma_n^2} \quad (3.27)$$

with  $n=1,2$  denoting jammed and non-jammed bits, respectively. The inverse Laplace transforms of Equation (3.23) and Equation (3.24) are found numerically since analytic results are not easily obtainable for the general case.

Determination of the probability of bit error then proceeds as for the non-fading channel case, as set forth in Equations (3.1) through (3.4).

#### B. HARD DECISION DETECTION

If hard decision Viterbi detection is utilized, the probability of bit error can be calculated using Equation (3.1); however, for hard decision detection the calculation of  $P_d$  is simpler. In this case,  $P_d$  is equal to [Ref. 8]

$$P_d = \sum_{i=\frac{d+1}{2}}^d \binom{d}{i} \cdot p^i \cdot (1-p)^{d-i} \quad (3.28)$$

when  $d$  is odd, and

$$P_d = \frac{1}{2} \cdot \binom{\frac{d}{2}}{\frac{d}{2}} \cdot p^{\frac{d}{2}} \cdot (1-p)^{\frac{d}{2}} + \sum_{i=\frac{d}{2}+1}^d \binom{d}{i} \cdot p^i \cdot (1-p)^{d-i} \quad (3.29)$$

when  $d$  is even. For a channel with no fading,  $p$  in Equations (3.28) and (3.29) is equal to the probability of bit error for a SFH/NCBFSK signal subjected to partial-band noise jamming given in Equation (2.7), where  $E_b$  is multiplied by the code rate  $r$  to yield

$$p = \frac{\gamma}{2} \cdot \exp\left(-\frac{r \cdot E_b}{2 \cdot (N_0 + \frac{N_J}{\gamma})}\right) + \frac{(1-\gamma)}{2} \cdot \exp\left(-\frac{r \cdot E_b}{2 \cdot N_0}\right). \quad (3.30)$$

If the channel is a Ricean fading channel,  $p$  is equal to [Ref. 10]

$$p = \frac{\gamma}{2} \cdot \frac{(1+\psi)}{2 \cdot (1+\psi) + \overline{\zeta}_{b1}} \cdot \exp\left(\frac{-\psi \cdot \overline{\zeta}_{b1}}{2 \cdot (1+\psi) + \overline{\zeta}_{b1}}\right) + \frac{1-\gamma}{2} \cdot \frac{(1+\psi)}{2 \cdot (1+\psi) + \overline{\zeta}_{b2}} \cdot \exp\left(\frac{-\psi \cdot \overline{\zeta}_{b2}}{2 \cdot (1+\psi) + \overline{\zeta}_{b2}}\right) \quad (3.31)$$

where

$$\overline{\zeta}_{b1} = \frac{r \cdot (\alpha_1^2 + 2 \cdot \sigma_1^2) \cdot T_b}{(N_0 + \frac{N_J}{\gamma})} \quad (3.32)$$

and

$$\overline{\zeta}_{b2} = \frac{r \cdot (\alpha_2^2 + 2 \cdot \sigma_2^2) \cdot T_b}{N_0}. \quad (3.33)$$

### C. NOISE-NORMALIZED RECEIVER

If a noise-normalized receiver [Ref. 6], a type of receiver utilizing side information, is utilized with soft decision Viterbi decoding in a channel without fading, the analysis follows as set forth in Equations (3.1) through (3.4). The pdfs of the decision variables  $Z_{1k}$  and  $Z_{2k}$  are [Ref. 6]

$$f_{Z_{1k}}(z_{1k}) = \frac{1}{2} \cdot \exp\left(-\frac{z_{1k} + 2 \cdot \frac{A_c^2}{\sigma_k^2}}{2}\right) \cdot I_0\left(\frac{A_c \cdot \sqrt{2 \cdot z_{1k}}}{\sigma_k}\right) \quad (3.34)$$

and

$$f_{Z_{2k}}(z_{2k}) = \frac{1}{2} \cdot \exp\left(\frac{-z_{2k}}{2}\right). \quad (3.35)$$

By following the procedure outlined in Equations (3.6) and (3.7), we can determine the pdfs of the decision variables  $Z_1$  and  $Z_2$ . Substituting the resulting pdfs and integrating in accordance with Equation (3.3), we obtain [Ref. 11]

$$P_d(i) = \frac{1}{2^{2 \cdot d - 1}} \cdot \exp\left(-\frac{r \cdot E_b}{2} \cdot \left(\frac{i}{N_0 + \frac{N_f}{\gamma}} + \frac{d-i}{N_0}\right)\right) \cdot \sum_{n=0}^{d-1} c_n \cdot \left(\frac{r \cdot E_b}{2} \cdot \left(\frac{i}{N_0 + \frac{N_f}{\gamma}} + \frac{d-i}{N_0}\right)\right)^n, \quad (3.36)$$

where  $c_n$  is defined by Equation (3.13).

In this chapter we determined the expressions required to determine the probability of bit error for a SFH/NCBFSK

communication system with rate 1/2 convolutional coding and soft decision Viterbi detection in the presence of partial-band jamming. This analysis was performed for both a non-fading channel and a Ricean fading channel. By way of comparison, the expressions required to determine the probability of bit error for a SFH/NCBFSK receiver using noise-normalized combining and soft decision Viterbi detection was presented for non-fading channels. Also, the expressions for a SFH/NCBFSK receiver utilizing hard decision Viterbi detection were presented for both non-fading channels and Ricean fading channels. In the next chapter, we utilize the results of this chapter to calculate the probability of bit error for these receivers and present our results for various cases of partial-band jamming and Ricean fading channels.

## IV. NUMERICAL ANALYSIS AND RESULTS

### A. NON-FADING CHANNEL

#### 1. Soft Decision Detection, Conventional Receiver

The probability of bit error was calculated numerically using Equations (3.1) through (3.3). The pdf of the decision variable  $V_1$  was calculated by taking the inverse Laplace transform of Equation (3.10) numerically. The integral of the pdf of  $V_2$  contained in the bracketed term of Equation (3.3) is calculated by taking the inverse Laplace transform of Equation (3.11) multiplied by  $1/s$ ; i.e.,

$$\frac{1}{s} \cdot \left[ \frac{1}{2 \cdot \sigma_1^2 \cdot s + 1} \right]^i \cdot \left[ \frac{1}{2 \cdot \sigma_2^2 \cdot s + 1} \right]^{d-i}$$

$P_d(i)$  was then calculated using Equation (3.3) using a Simpson's rule numerical integration. [Ref. 12]

The analysis was performed for two separate cases:  $v=3$  and  $v=7$ . For these two separate constraint lengths, four separate cases of partial-band jamming were examined. The four cases examined are  $\gamma = 1.0$ ,  $\gamma = 0.1$ ,  $\gamma = 0.01$ , and  $\gamma = 0.001$ . The performance of the system when there is no channel fading and soft decision detection is used is shown in Figures 5 and 6 for  $v = 3$  and  $v = 7$ , respectively. From

the results shown in Figure 5 and Figure 6, it is evident that this receiver is very susceptible to partial-band noise jamming. The  $E_b/N_J$  required to achieve a probability of bit error of  $10^{-5}$  is tabulated in Table 2. In order to achieve a probability of bit error of  $10^{-5}$ , an increase of over 20 dB in  $E_b/N_J$  is required if  $\gamma$  is decreased from 1.0 to 0.001. Thus, partial-band noise jamming is extremely effective at defeating this receiver.

**Table 2:  $E_b/N_J$  required for a probability of bit error of  $10^{-5}$  for a conventional SFH/NCBFSK receiver with rate 1/2 convolutional coding and soft decision Viterbi decoding in the presence of partial-band noise jamming**

$P_B=10^{-5}$	$\gamma = 1.0$	$\gamma = 0.1$	$\gamma = 0.01$	$\gamma = 0.001$
$v = 3$	15	20	28	36
$v = 7$	12.5	18	26	34

The poor performance of this receiver with respect to its ability to resist partial-band jamming is due to the lack of the use of side information. By using linear combining and soft decision decoding, the jammed receptions dominate the non-jammed receptions in the overall decision statistic, resulting in degradation of the system's performance. If side information is used the system's performance will dramatically improve with regard to its ability to resist the effects of partial-band noise

jamming. To demonstrate this, the performance of a noise-normalized SFH/NCBFSK receiver with rate 1/2 convolutional coding and soft decision Viterbi decoding is also analyzed.

In addition, we hypothesize that soft decision decoding in combination with linear combining actually results in performance degradation when partial-band noise jamming is present. This is because with soft decision decoding a single jammed hop can completely dominate the decision statistic.

Since soft decision detection is most commonly implemented using multi-bit quantization, we speculate that the performance of a system utilizing quantization will have improved performance in defeating partial-band jamming as compared to the system analyzed in this thesis. This is because the quantization limits the values that the output variables can take prior to combining. This should result in increased system robustness with regards to defeating partial-band jamming. Consequently, we expect hard decision decoding to actually outperform soft decision decoding when partial-band noise jamming is present since with hard decision decoding the effect of jammed hops is limited. This hypothesis is examined by analyzing the performance of a SFH/NCBFSK communication system using rate

1/2 convolutional coding and hard decision Viterbi decoding.

## 2. Soft Decision Detection, Noise-Normalized Receiver

The probability of bit error was also calculated for the noise-normalized receiver. The analysis was performed by implementing Equations (3.1) and (3.2). In this case the analytical expression for  $P_d(i)$  in Equation (3.36) was used to calculate  $P_d(i)$  and substituted into Equation (3.2). The analysis was performed for the same values of  $v$  and  $\gamma$  as the conventional receiver. The results are displayed in Figure 7 and Figure 8, for  $v = 3$  and  $v = 7$ , respectively.

With the use of noise-normalization, we achieved a significant increase in the system's ability to limit the effects of partial-band noise jamming. This increased resistance to partial-band jamming forces a potential hostile jammer to adopt barrage noise jamming, causing the hostile jammer to spread the energy of the jamming source over the entire bandwidth of the communication system. This effectively increases  $E_b/N_J$ . The maximum probability of bit error for  $v = 3$  and  $v = 7$  are tabulated in Table 3 for various values of  $\gamma$ .

**Table 3: Maximum Probability of Bit Error for a Noise-Normalized SFH/NCBFSK receiver with rate 1/2 convolutional coding and soft decision Viterbi decoding in the presence of partial-band noise jamming**

	$\gamma = 0.1$	$\gamma = 0.01$	$\gamma = 0.001$
$v = 3$	$2.1 \times 10^{-5}$	$9.7 \times 10^{-9}$	$5 \times 10^{-10}$
$v = 7$	$2.8 \times 10^{-8}$	$2.6 \times 10^{-14}$	$\times 10^{-16}$

### 3. Hard Decision Detection

The probability of bit error was also calculated when hard decision Viterbi decoding was used. The calculations of  $P_b$  were performed by implementing Equation (3.1).  $P_d$  was calculated by using Equation (3.28) when  $d$  was odd and Equation (3.29) when  $d$  was even. Equation (3.30) was used to calculate the probability of bit error for independent diversity receptions.

The results for  $v = 3$  and  $v = 7$  are shown in Figure 9 and Figure 10, respectively. The results obtained using hard decision Viterbi detection are plotted with the results obtained using soft decision Viterbi detection for  $v = 3$  and  $v = 7$  in Figure 11 and Figure 12, respectively. The use of hard decision Viterbi decoding demonstrates a significant improvement in its ability to withstand partial-band noise jamming as compared to the conventional receiver utilizing soft decision Viterbi decoding.

However, the lower asymptotic limit for  $P_b$  when the hard decision Viterbi detection is used is greater than the lower asymptotic limit for  $P_b$  when soft decision Viterbi detection is used. Also, for the case of barrage noise jamming, soft decision Viterbi decoding has approximately 3 dB better performance than that for hard decision Viterbi decoding. This result is expected. [Refs. 2 and 3] As in the previous cases examined, the receiver utilizing the constraint length seven code outperformed the constraint length three code.

## **B. RICEAN FADING CHANNEL**

### **1. Soft Decision Detection**

The analysis was also performed for the same values of  $v$  and  $\gamma$  as the non-fading channel case for a Ricean fading channel, with  $\psi = 100$ ,  $\psi = 10$ ,  $\psi = 1$ , and  $\psi = 0.01$ . The numerical analysis procedure was the same as for the non-fading channel case; however, Equations (3.23) and (3.24) were used in place of Equations (3.10) and (3.11). The results for  $v = 3$  are shown in Figure 13 through Figure 16. The results for  $v = 7$  are shown in Figure 17 through Figure 20. Examining the results in Figure 13 through Figure 20, we see results similar to those obtained for the non-fading channel case; however, the lower asymptotic limit for the

probability of bit error is increased as the direct-to-diffuse signal power ratio,  $\psi$ , decreases. The lower asymptotic limit for  $P_b$  is tabulated in Table 4 for several values of  $\psi$  and  $v$ . We see the same susceptibility to partial-band jamming and that the constraint length seven code provides better performance than the constraint length three code with regard to channel fading.

**Table 4: Minimum Probability of bit error for SFH/NCBFSK communication system with rate 1/2 convolutional coding and soft decision Viterbi decoding in the presence of partial-band noise jamming**

	$\psi = 100$	$\psi = 10$	$\psi = 1$	$\psi = 0.01$
$v = 3$	$< 10^{-9}$	$2.9 \times 10^{-7}$	$4.2 \times 10^{-4}$	$1.1 \times 10^{-3}$
$v = 7$	$< 10^{-9}$	$< 10^{-9}$	$4.9 \times 10^{-6}$	$2.5 \times 10^{-5}$

## 2. Hard Decision Detection

The analysis for a SFH/NCBFSK receiver with rate 1/2 convolutional coding and hard decision Viterbi decoding was also performed for the same values of  $v$  and  $\gamma$  as the non-fading channel case for a Ricean fading channel with  $\psi = 100$ ,  $\psi = 10$ ,  $\psi = 1$ , and  $\psi = 0.01$ . The results for  $v = 3$  are shown in Figure 21 through Figure 24, and the results for  $v = 7$  are shown in Figure 25 through Figure 28. As in the non-fading channel case, hard decision detection

provided decreased susceptibility to partial band jamming as compared to soft decision detection. Also, the lower asymptotic limit for the probability of bit error for hard decision detection was higher than that for hard decision detection for a given value of  $\psi$  and  $v$ . The lower asymptotic limit for the probability of bit error is tabulated below in Table 5.

**Table 5: Minimum Probability of bit error for SFH/NCBFSK communication system with rate 1/2 convolutional coding and hard decision Viterbi decoding in the presence of partial-band noise jamming**

	$\psi = 100$	$\psi = 10$	$\psi = 1$	$\psi = 0.01$
$v = 3$	$1.4 \times 10^{-6}$	$1.3 \times 10^{-4}$	$4.6 \times 10^{-2}$	$9.1 \times 10^{-2}$
$v = 7$	$1.1 \times 10^{-9}$	$1.8 \times 10^{-6}$	$1.8 \times 10^{-2}$	$5.4 \times 10^{-2}$

The results in Table 5 illustrate that this system utilizing hard decision detection is susceptible to channel fading and if the channel is severely faded performance will be unsatisfactory.

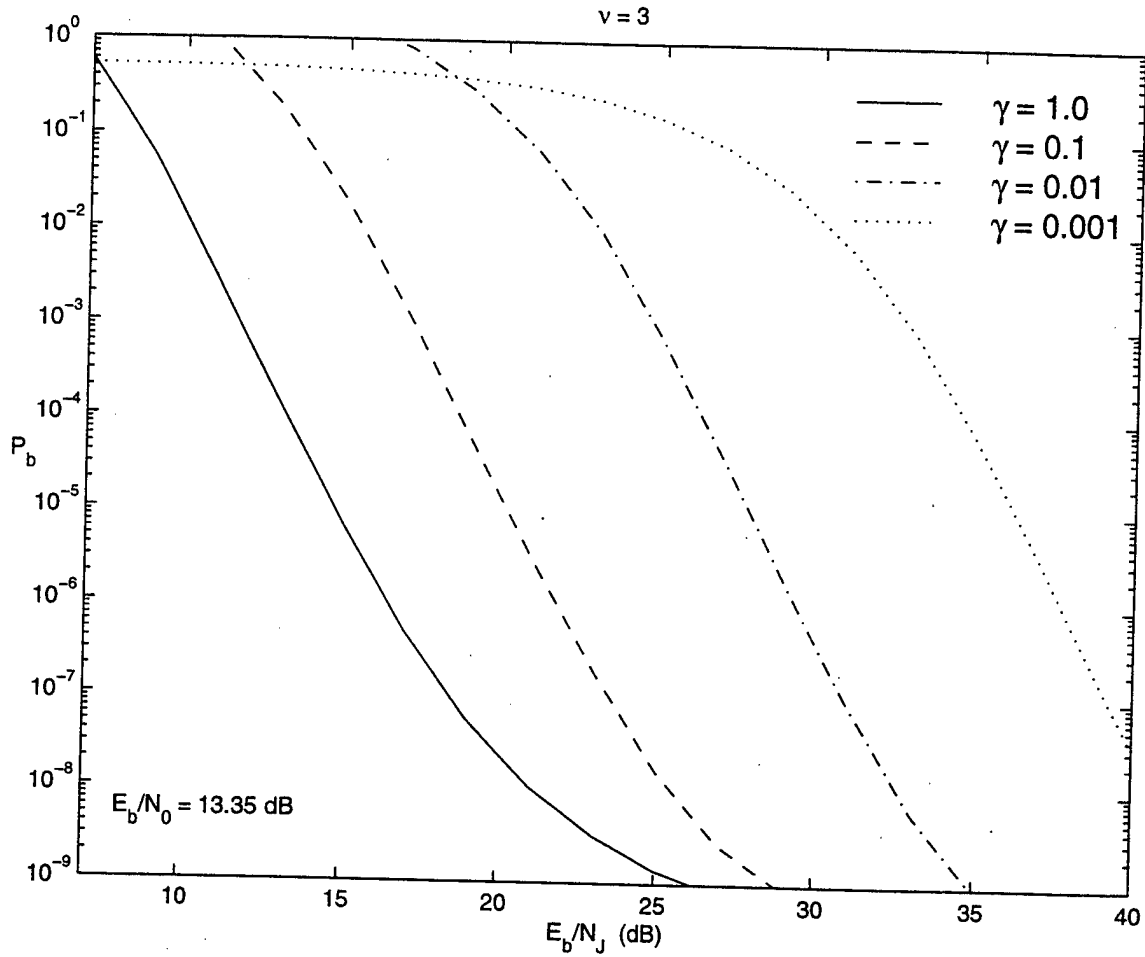


Figure 5: Performance for conventional SFH/NCBFSK receiver with rate 1/2 convolutional coding and soft decision Viterbi decoding in the presence of partial-band noise jamming for  $v = 3$ ,  $E_b/N_0 = 13.35 \text{ dB}$

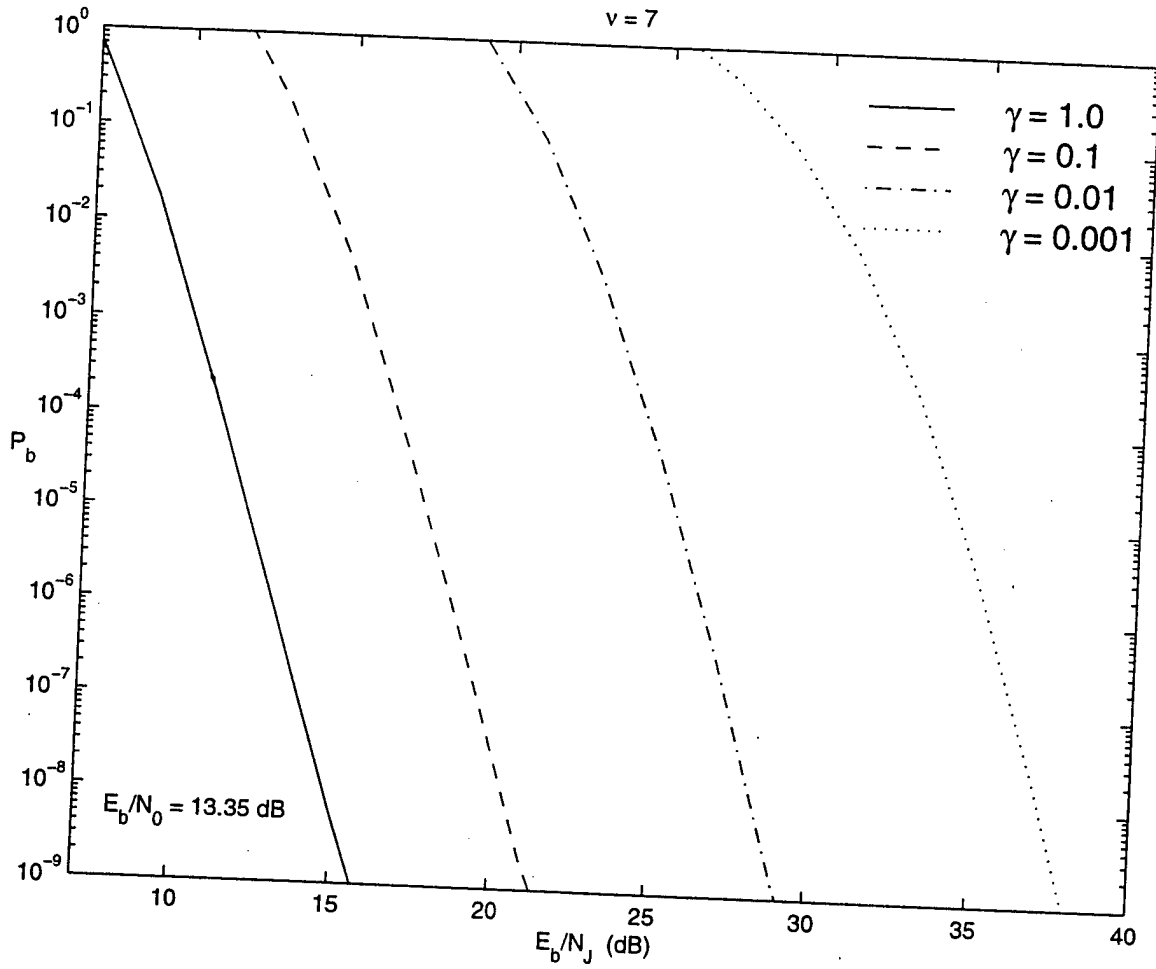


Figure 6: Performance of a conventional SFH/NCBFSK receiver with rate 1/2 convolutional coding and soft decision Viterbi decoding in the presence of partial-band noise jamming for  $v = 7$  and  $E_b/N_0 = 13.35 \text{ dB}$

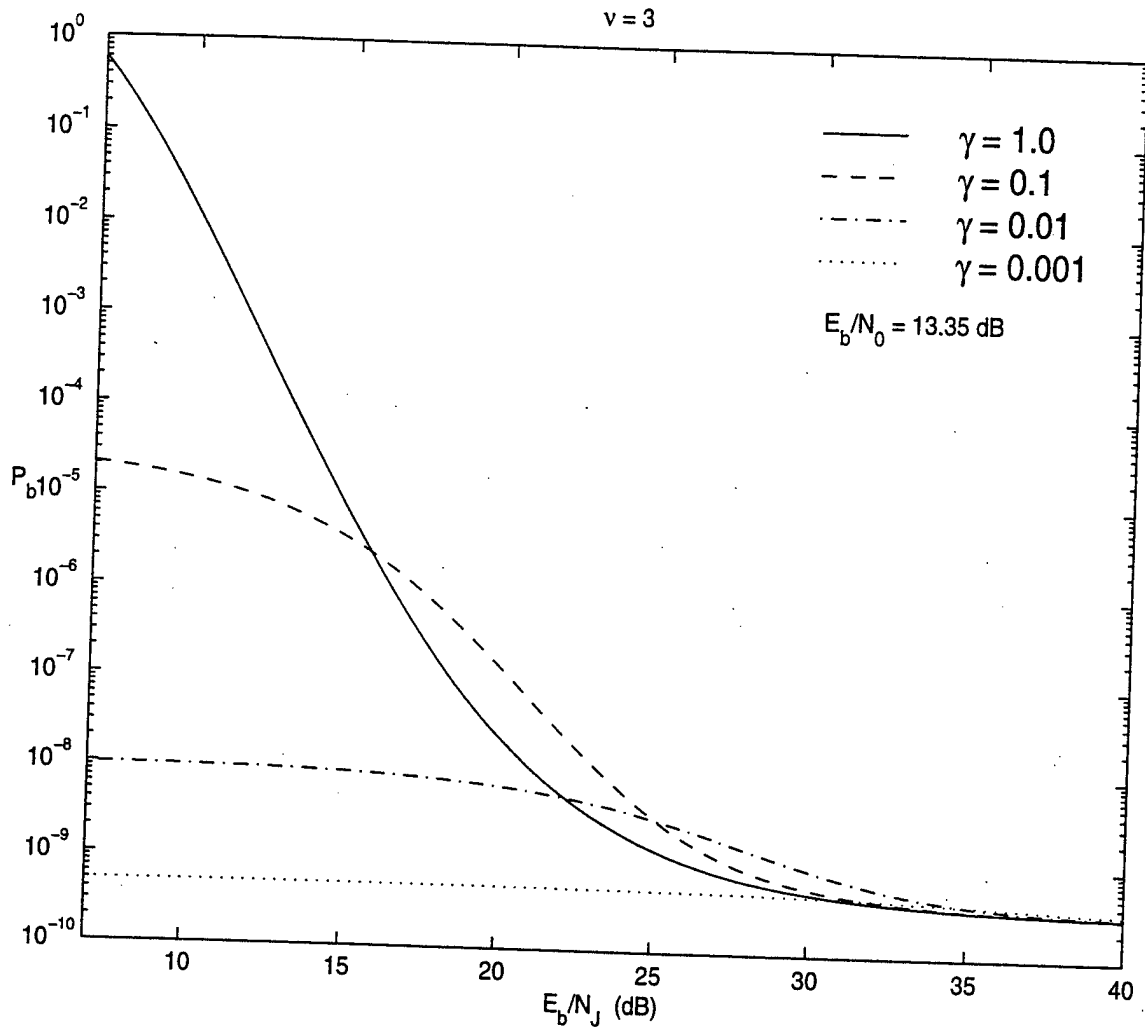


Figure 7: Performance of a Noise-Normalized SFH/NCBFSK receiver with rate 1/2 convolutional coding and soft decision Viterbi decoding in the presence of partial-band noise jamming for  $v = 3$  and  $E_b/N_0 = 13.35 \text{ dB}$

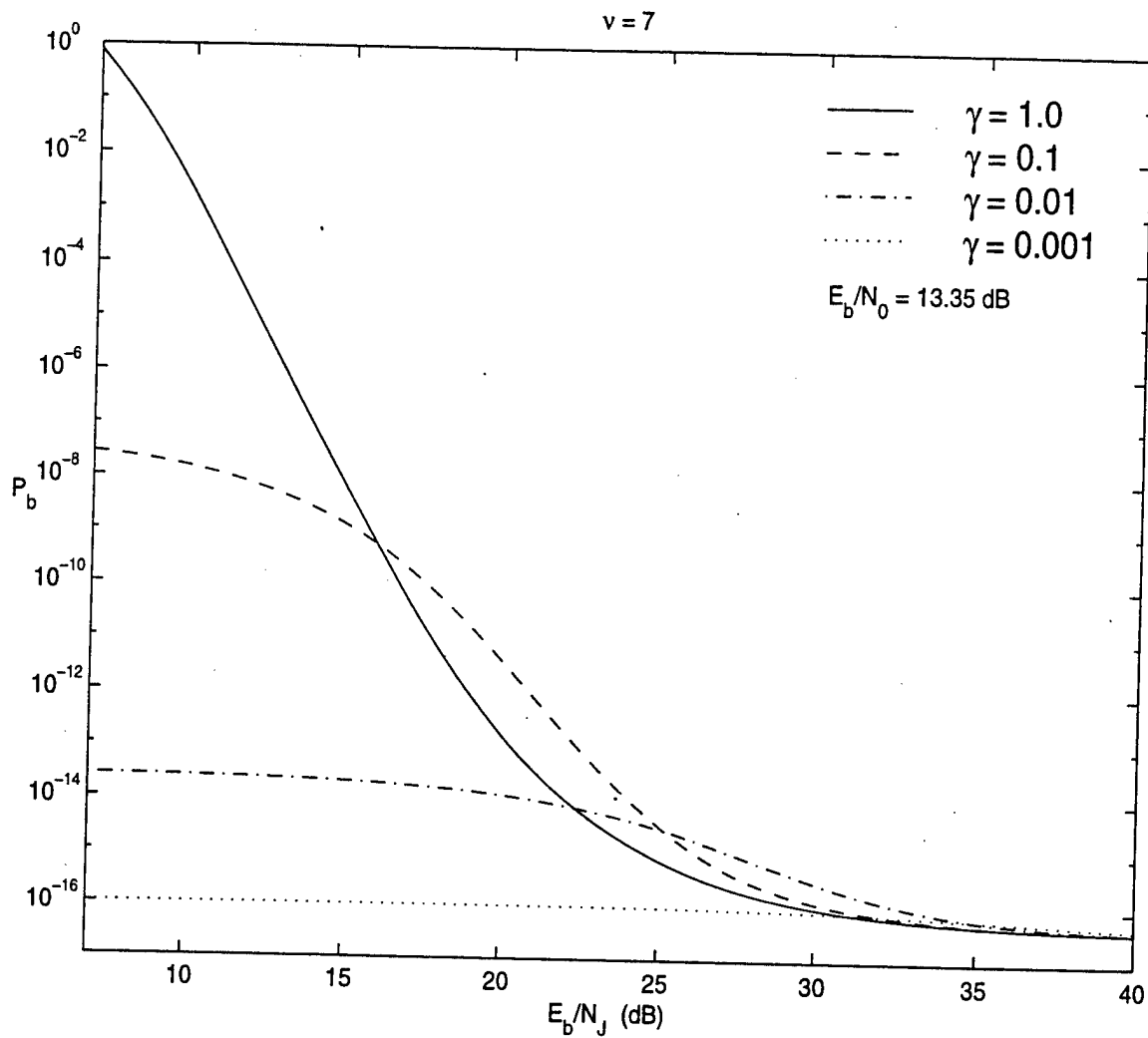
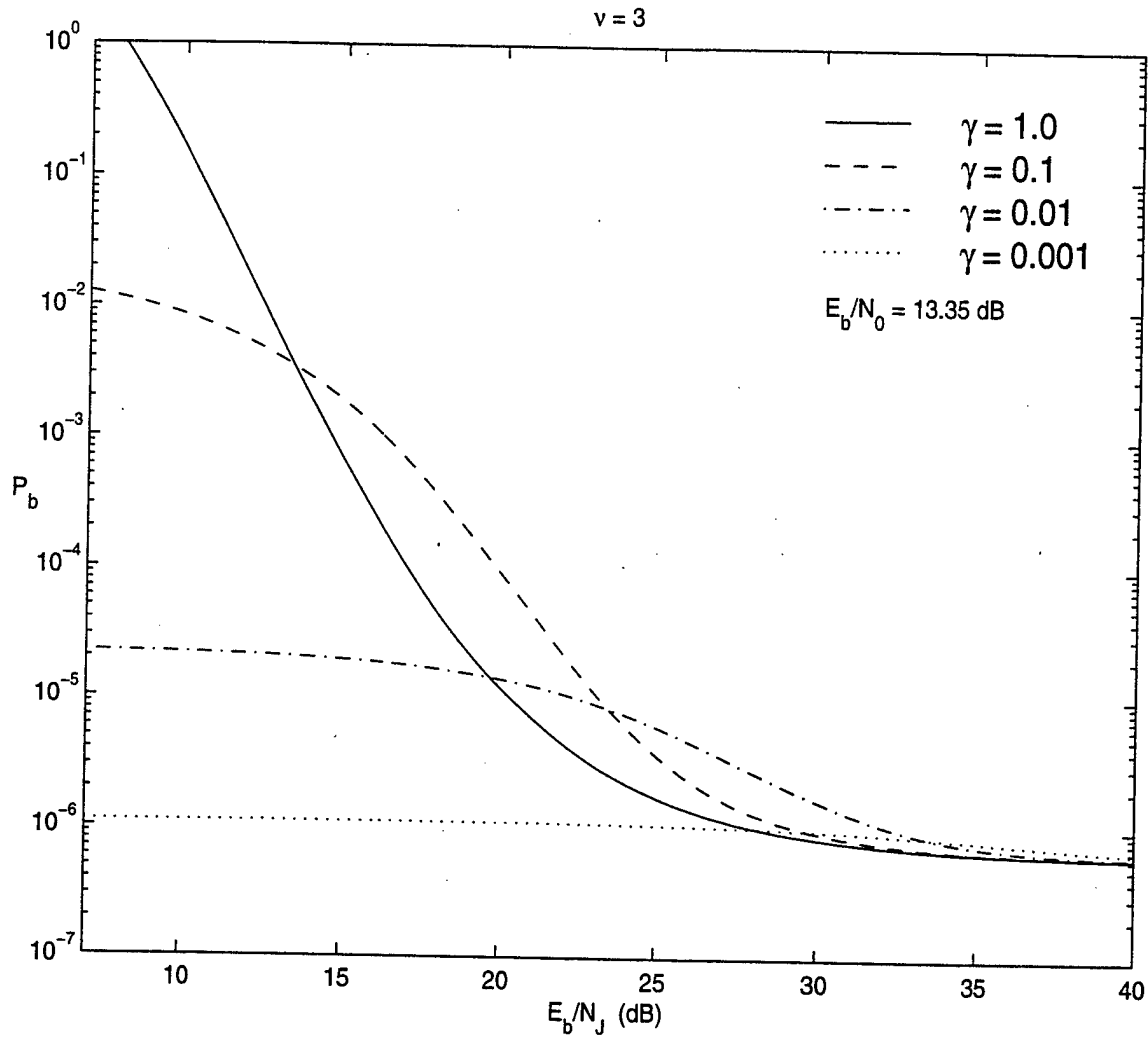


Figure 8: Performance of a Noise-Normalized SFH/NCBFSK receiver with rate 1/2 convolutional coding and soft decision Viterbi decoding in the presence of partial-band noise jamming for  $v = 7$  and  $E_b/N_0 = 13.35$  dB



**Figure 9: Performance of SFH/NCBFSK receiver with rate 1/2 convolutional coding and hard decision Viterbi decoding in the presence of partial-band noise jamming for  $v = 3$  and  $E_b/N_0 = 13.35 \text{ dB}$**

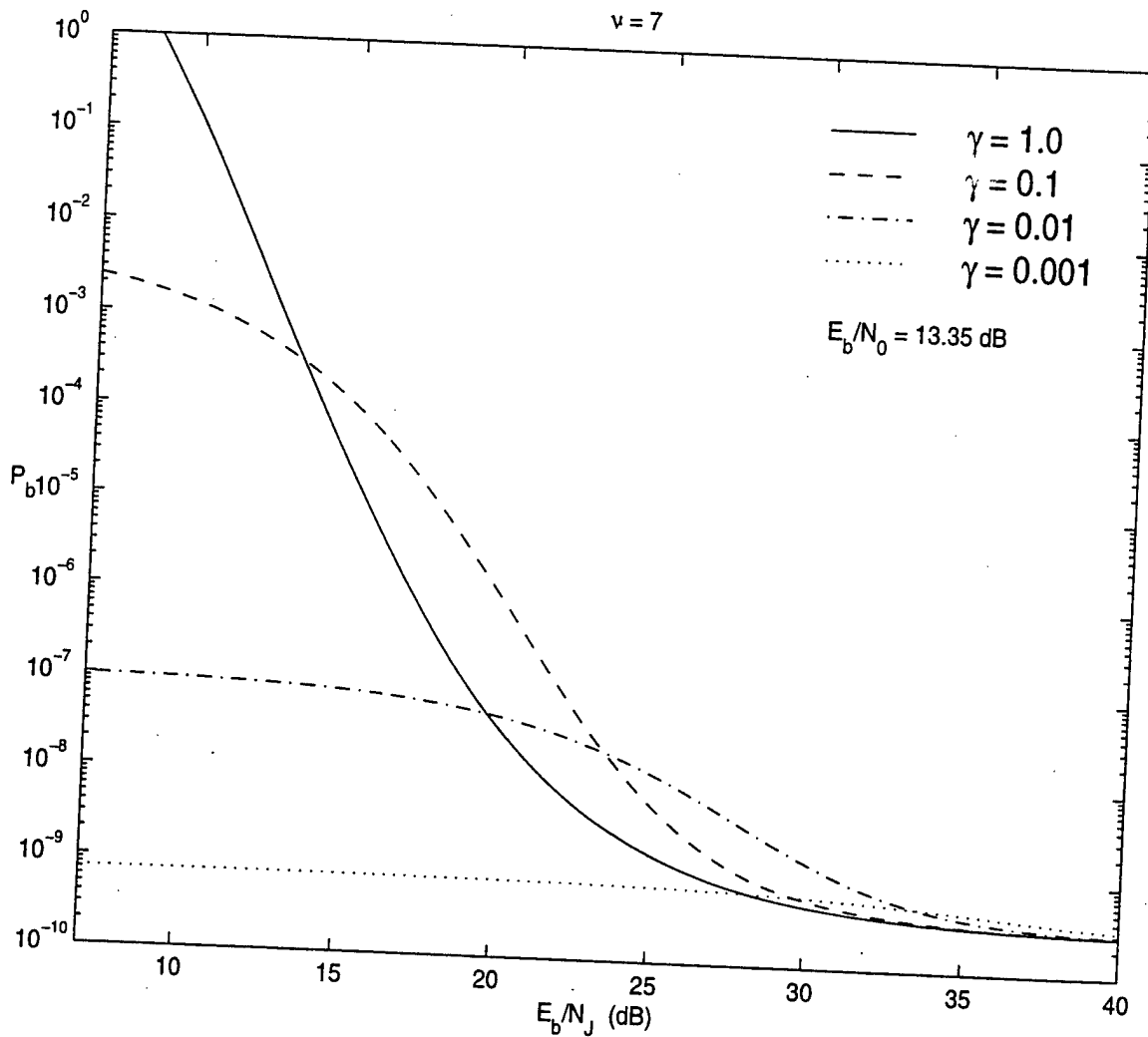


Figure 10: Performance of SFH/NCBFSK receiver with rate 1/2 convolutional coding and hard decision Viterbi decoding in the presence of partial-band noise jamming for  $v = 7$  and  $E_b/N_0 = 13.35 \text{ dB}$

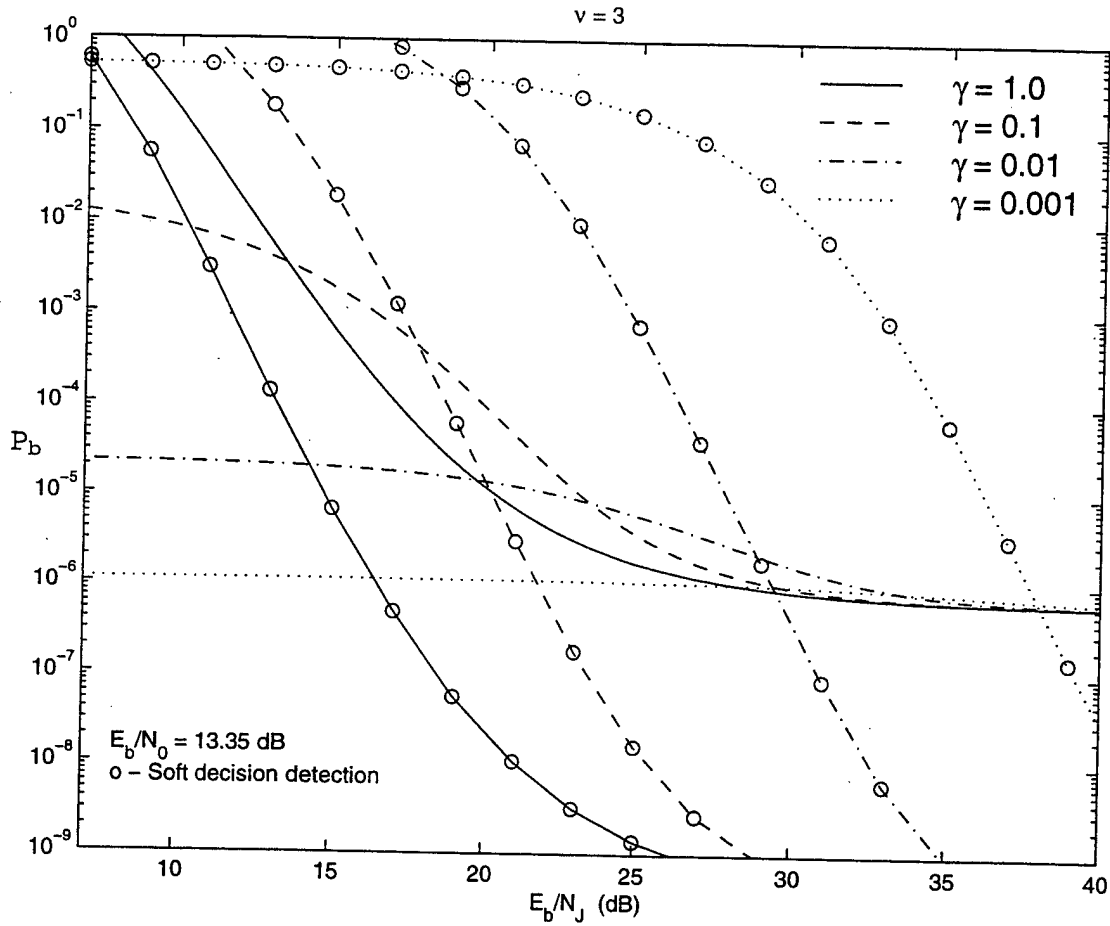


Figure 11: Comparison of the performance of SFH/NCBFSK receiver with rate 1/2 convolutional coding and hard decision Viterbi decoding to the performance of a SFH/NCBFSK with rate 1/2 convolutional coding and soft decision Viterbi decoding in the presence of partial-band noise jamming for  $v = 3$  and  $E_b/N_0 = 13.35$  dB

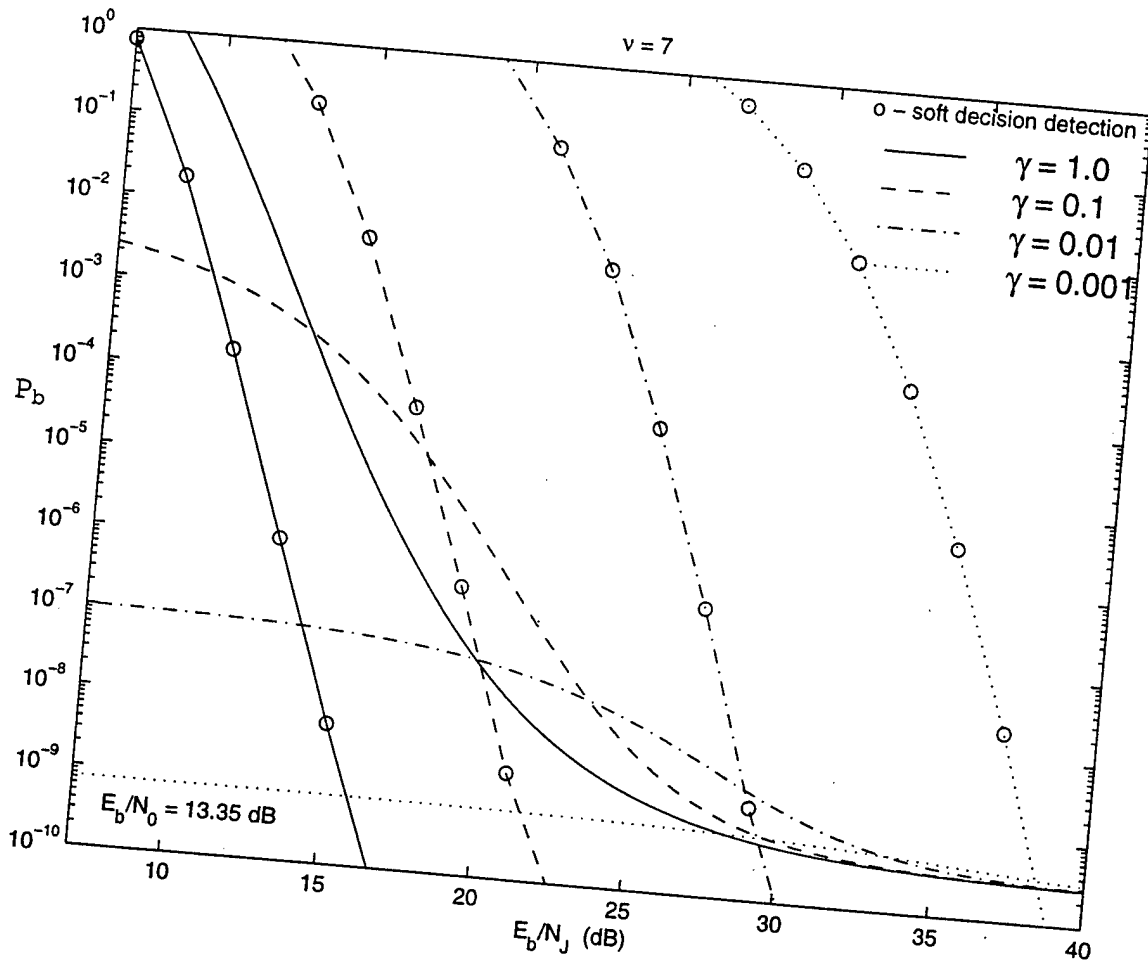


Figure 12: Comparison of the performance of SFH/NCBFSK receiver with rate 1/2 convolutional coding and hard decision Viterbi decoding to the performance of a SFH/NCBFSK with rate 1/2 convolutional coding and soft decision Viterbi decoding in the presence of partial-band noise jamming for  $v = 7$  and  $E_b/N_0 = 13.35 \text{ dB}$

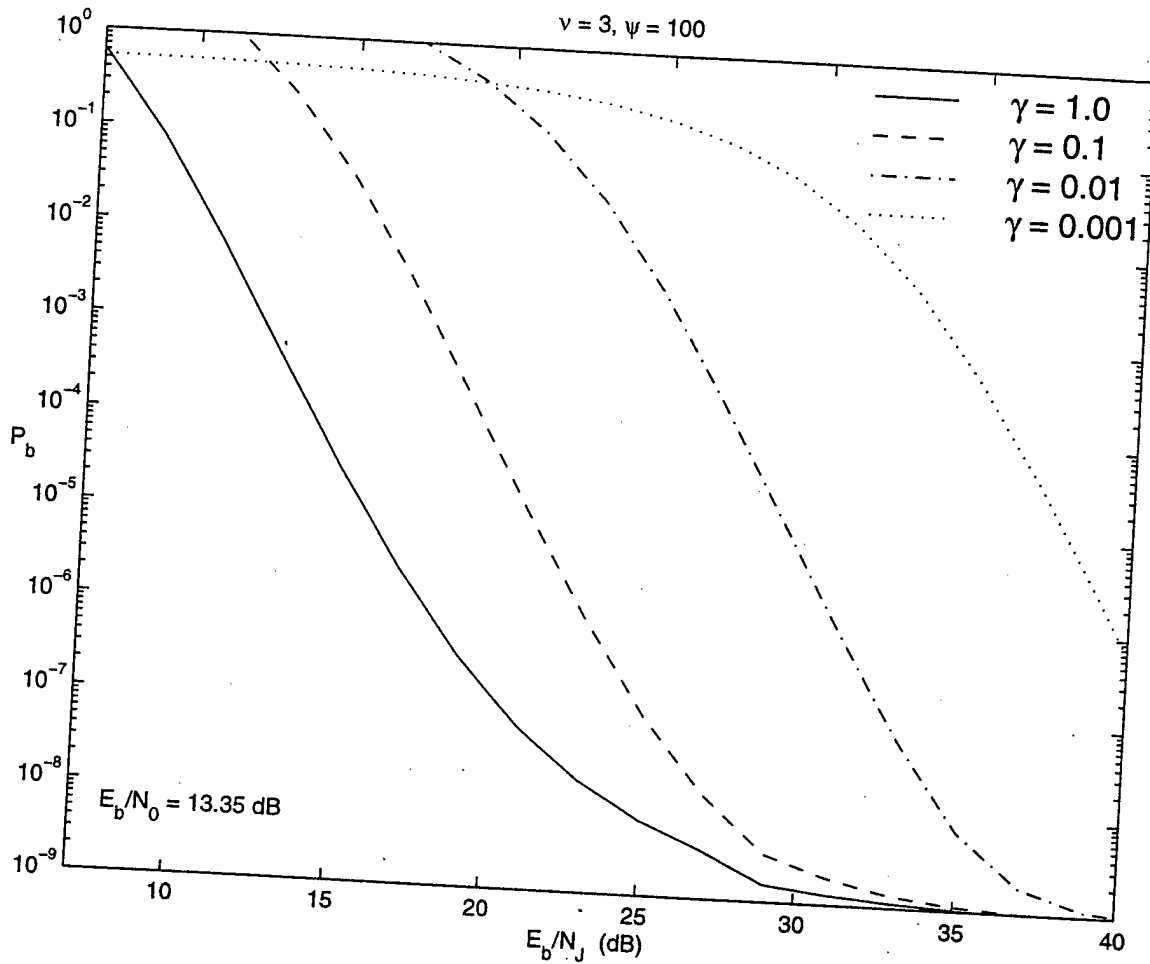
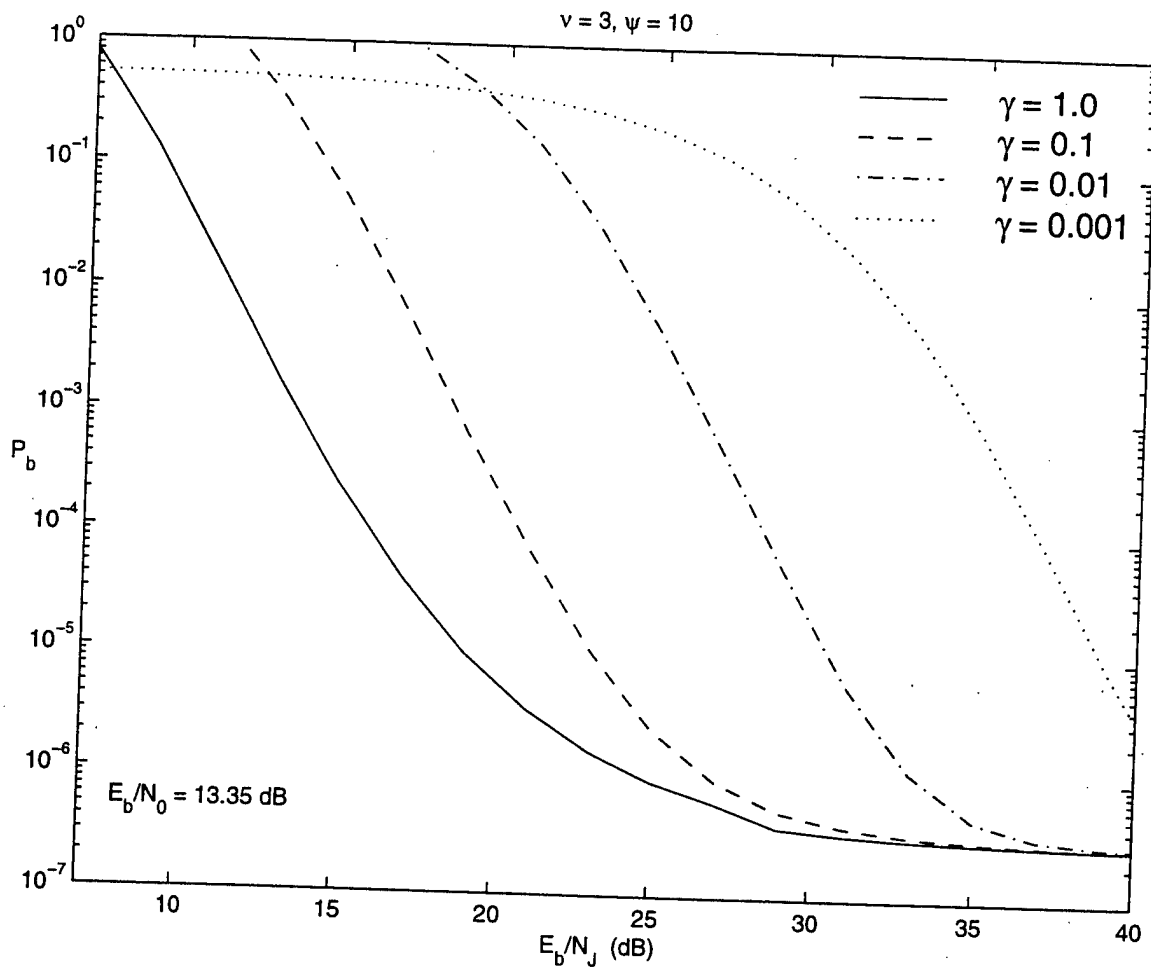


Figure 13: Performance of a conventional SFH/NCBFSK receiver with rate 1/2 convolutional coding and soft decision Viterbi decoding in the presence of partial-band noise jamming and a Ricean fading channel with  $\nu = 3$ ,  $E_b/N_0 = 13.35\text{dB}$ , and  $\psi = 100$



**Figure 14: Performance of a conventional SFH/NCBFSK receiver with rate 1/2 convolutional coding and soft decision Viterbi decoding in the presence of partial-band noise jamming and a Ricean fading channel with  $v = 3$ ,  $E_b/N_0 = 13.35 \text{ dB}$ , and  $\psi = 10$**

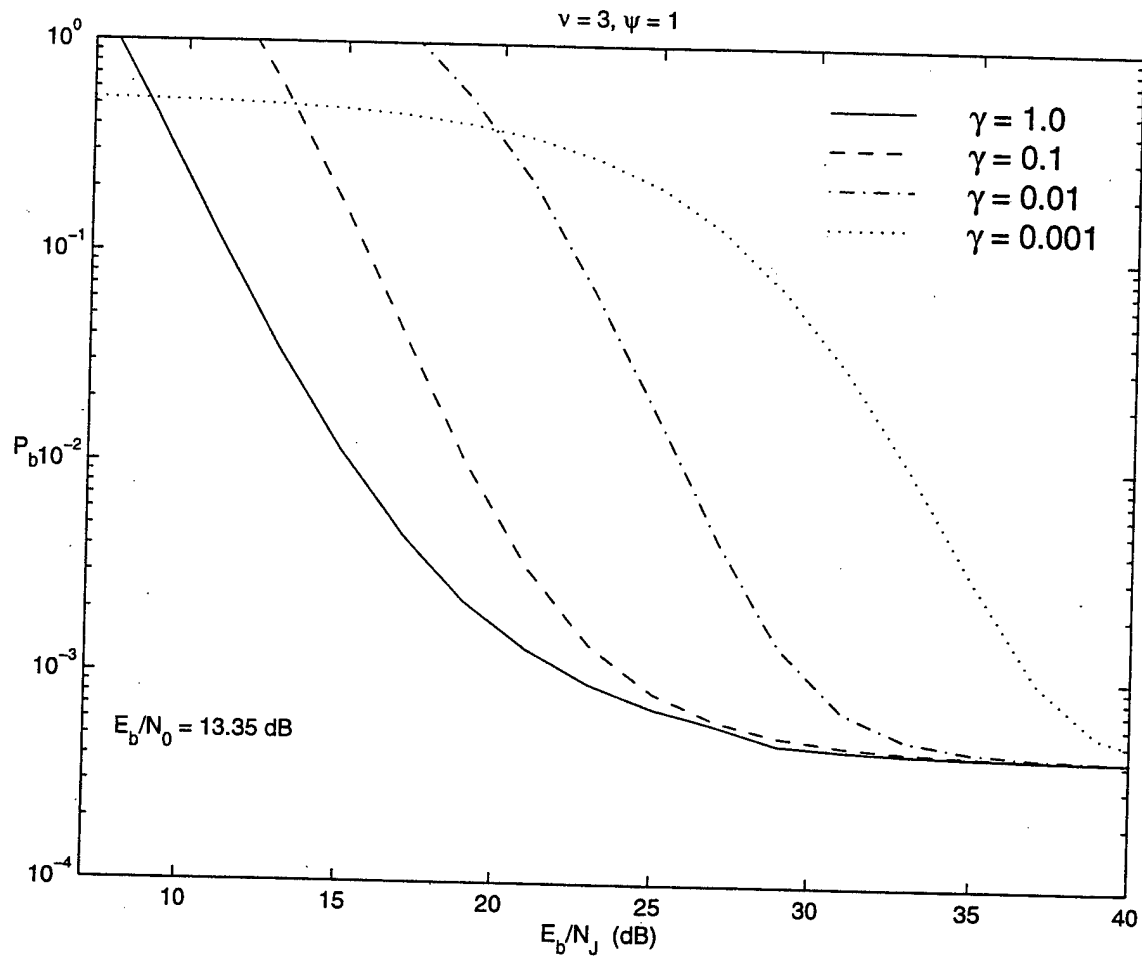
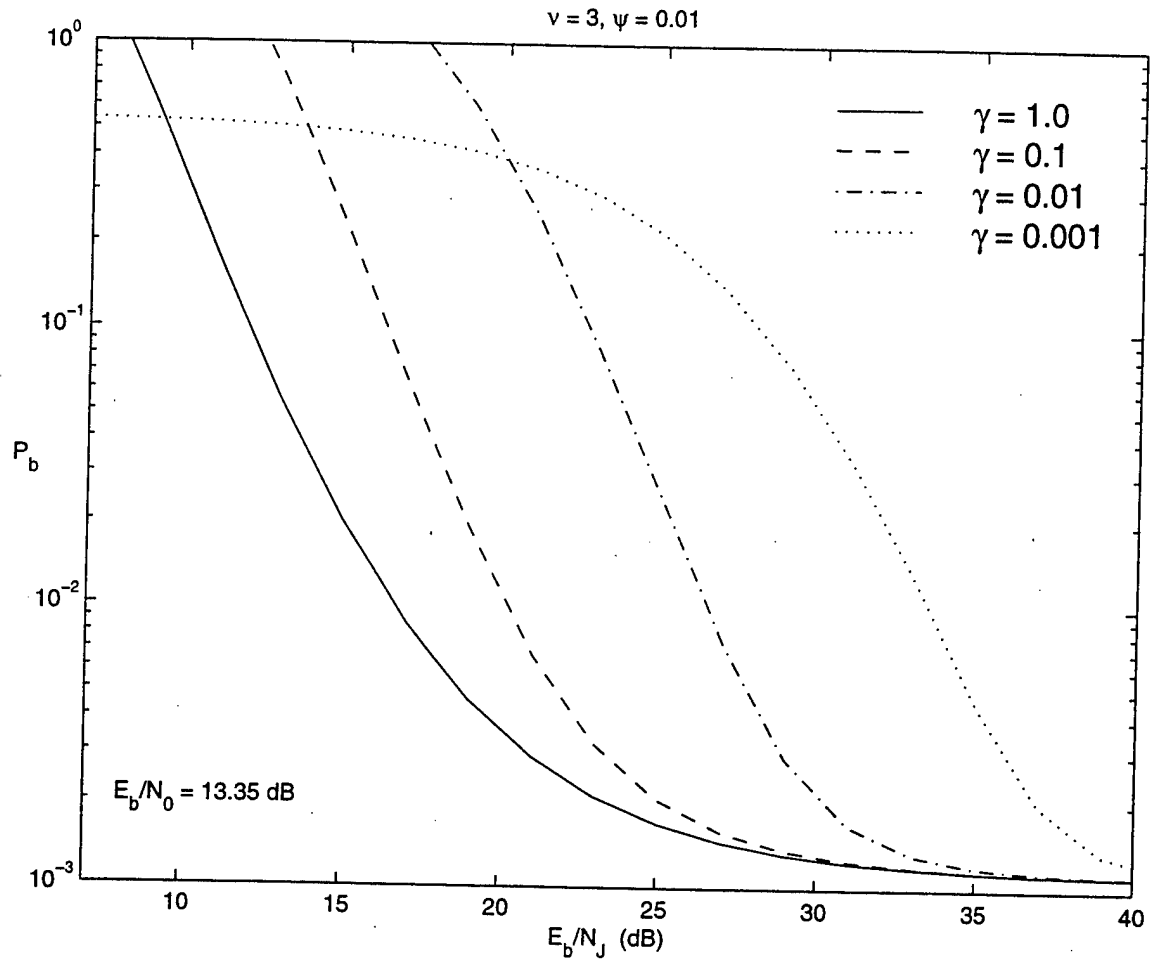


Figure 15: Performance of a conventional SFH/NCBFSK receiver with rate 1/2 convolutional coding and soft decision Viterbi decoding in the presence of partial-band noise jamming and a Ricean fading channel with  $v = 3$ ,  $E_b/N_0 = 13.35 \text{ dB}$  and  $\psi = 1$



**Figure 16: Performance of a conventional SFH/NCBFSK receiver with rate 1/2 convolutional coding and soft decision Viterbi decoding in the presence of partial-band noise jamming and a Ricean fading channel with  $\nu = 3$ ,  $E_b/N_0 = 13.35 \text{ dB}$ , and  $\psi = 0.01$**

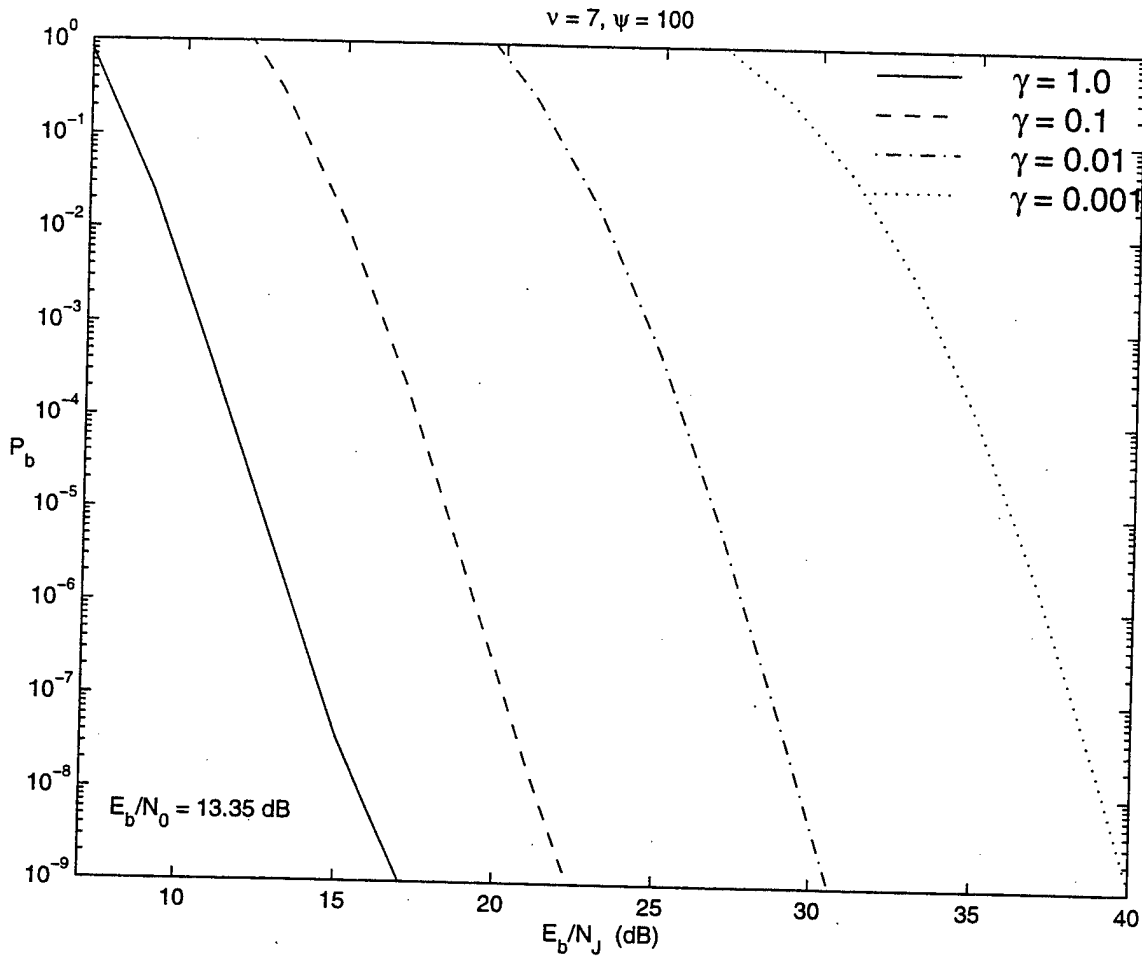


Figure 17: Performance of a conventional SFH/NCBFSK receiver with rate 1/2 convolutional coding and soft decision Viterbi decoding in the presence of partial-band noise jamming and a Ricean fading channel with  $v = 7$ ,  $E_b/N_0 = 13.35 \text{ dB}$ , and  $\psi = 100$

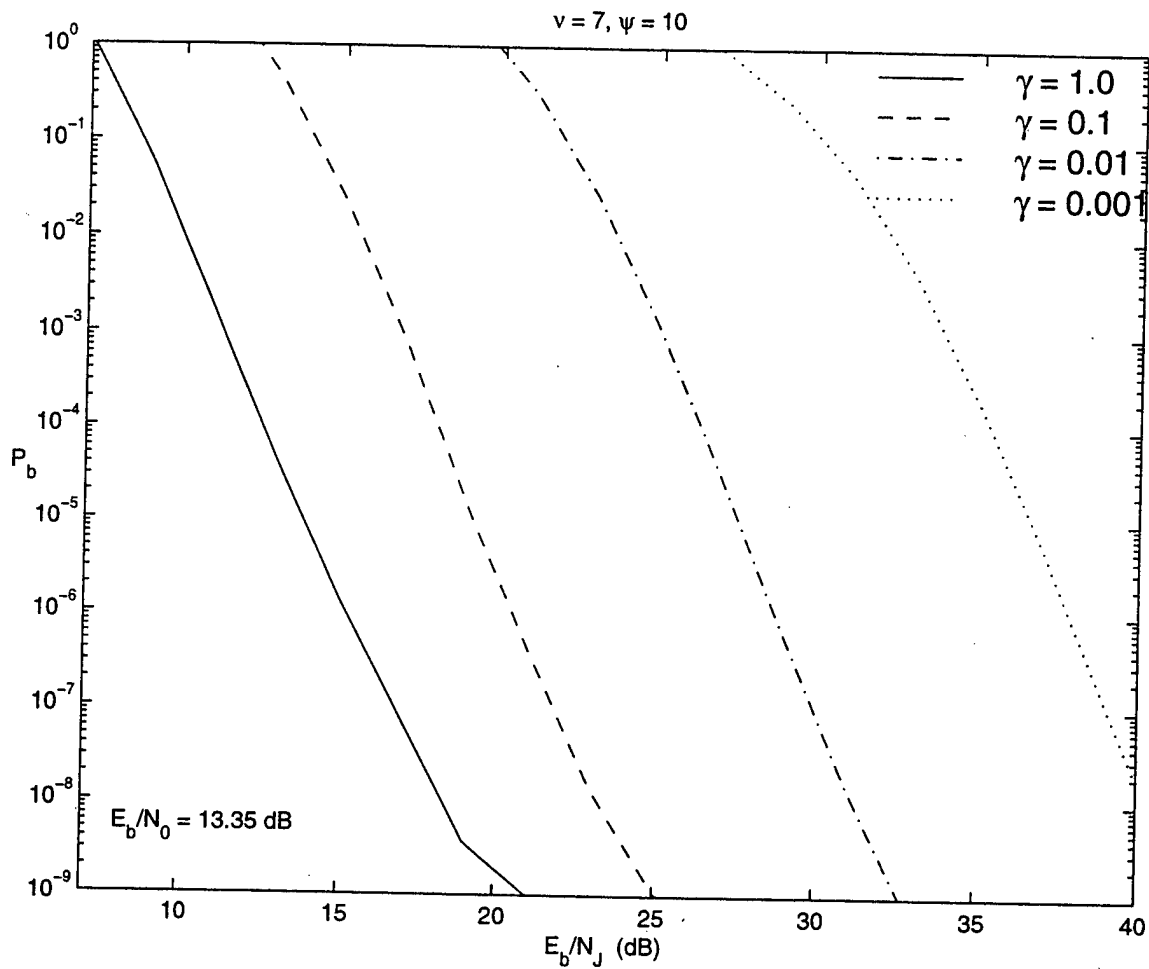


Figure 18: Performance of a conventional SFH/NCBFSK receiver with rate 1/2 convolutional coding and soft decision Viterbi decoding in the presence of partial-band noise jamming and a Ricean fading channel with  $v = 7$ ,  $E_b/N_0 = 13.35$  dB, and  $\psi = 10$

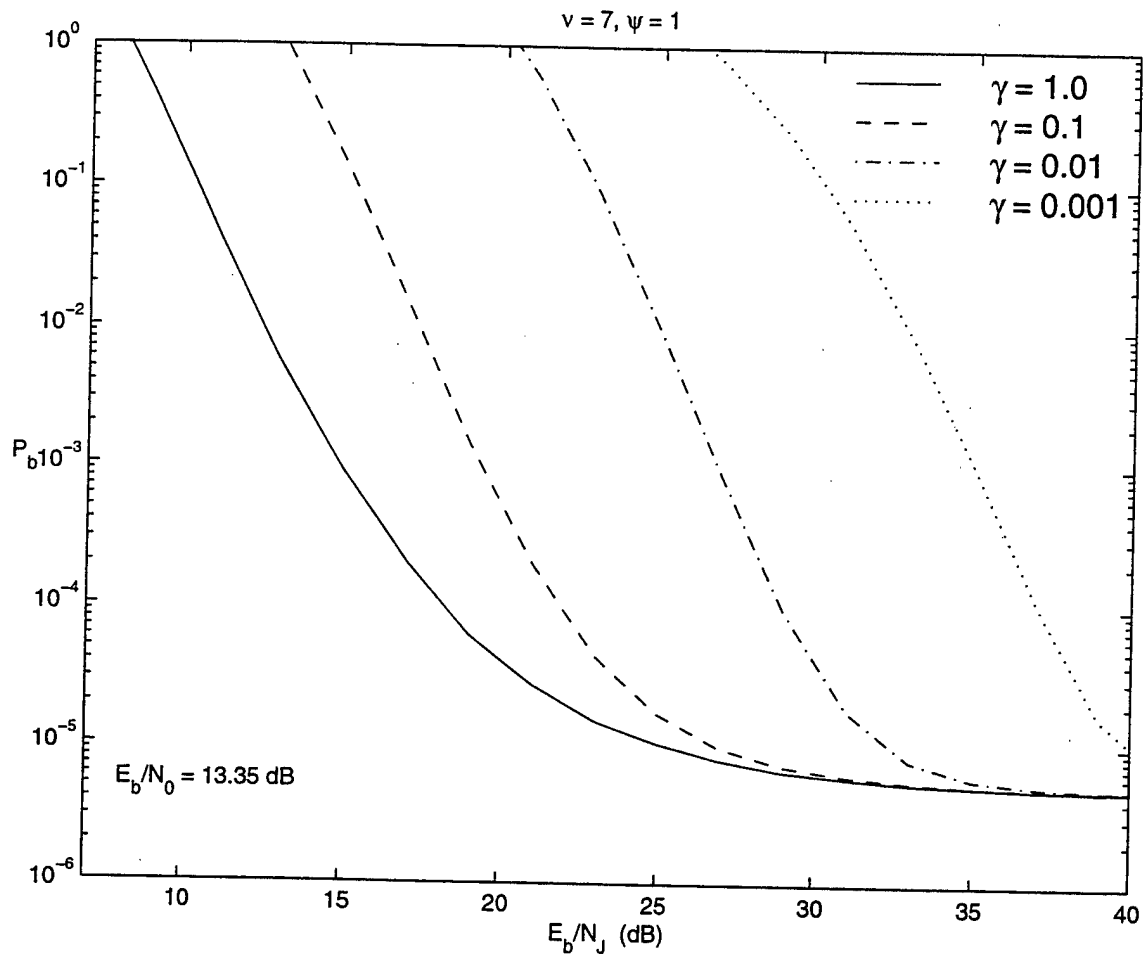
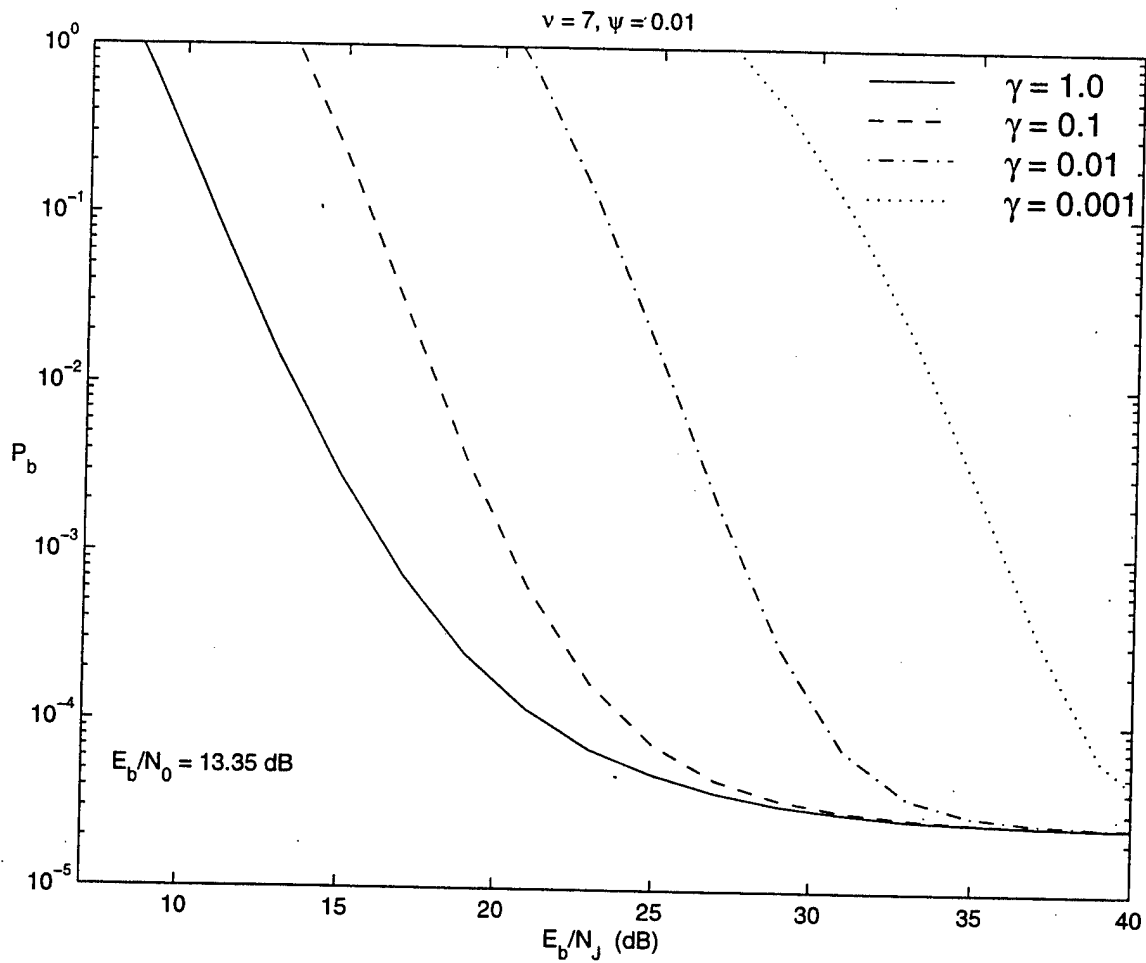
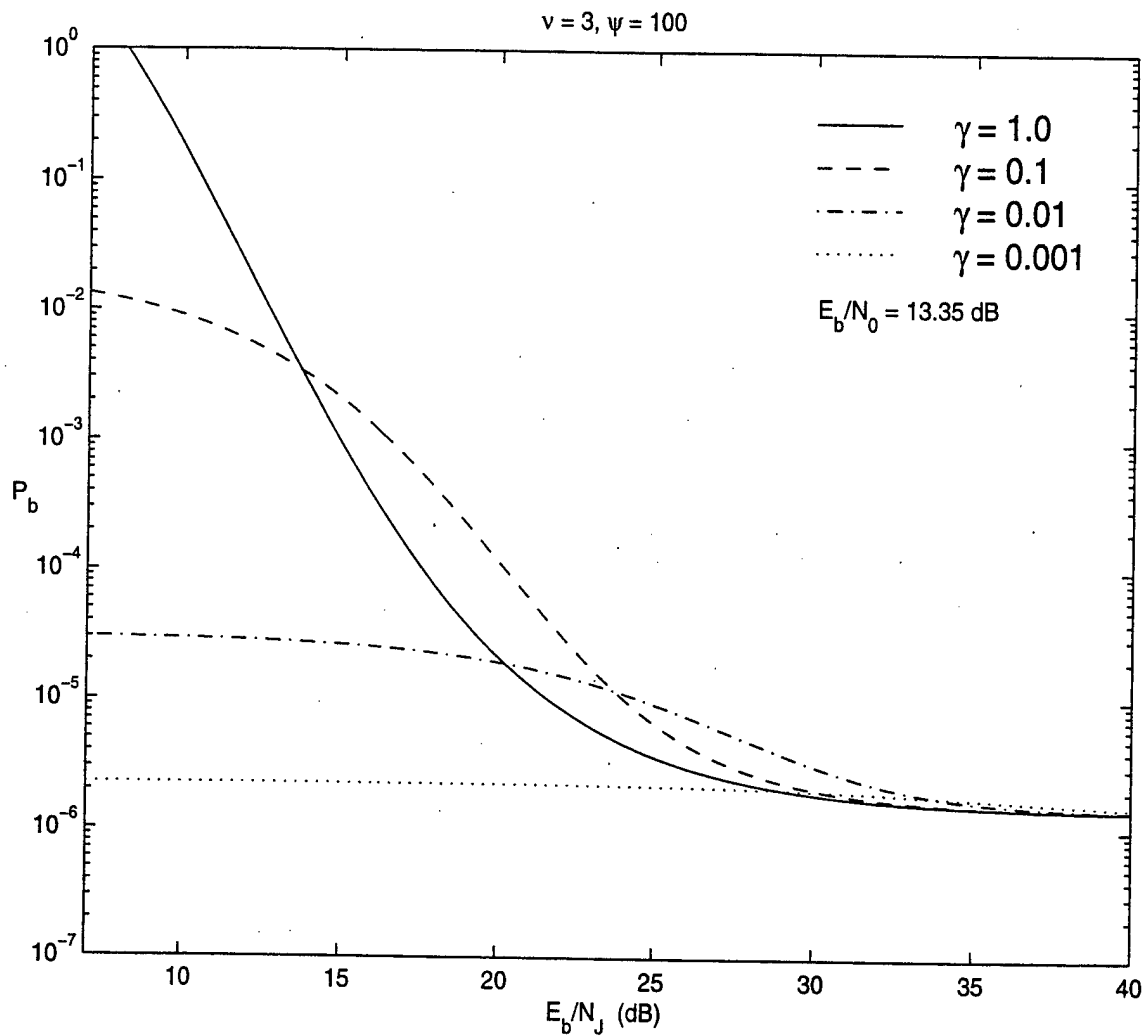


Figure 19: Performance of a conventional SFH/NCBFSK receiver with rate 1/2 convolutional coding and soft decision Viterbi decoding in the presence of partial-band noise jamming and a Ricean fading channel with  $\nu = 7$ ,  $E_b/N_0 = 13.35 \text{ dB}$ , and  $\psi = 1$



**Figure 20: Performance of a conventional SFH/NCBFSK receiver with rate 1/2 convolutional coding and soft decision Viterbi decoding in the presence of partial-band noise jamming and a Ricean fading channel with  $\nu = 7$ ,  $E_b/N_0 = 13.35 \text{ dB}$ , and  $\psi = 0.01$**



**Figure 21: Performance of a conventional SFH/NCBFSK receiver with rate 1/2 convolutional coding and hard decision Viterbi decoding in the presence of partial-band noise jamming and a Ricean fading channel with  $\nu = 3$ ,  $E_b/N_0 = 13.35 \text{ dB}$ , and  $\psi = 100$**

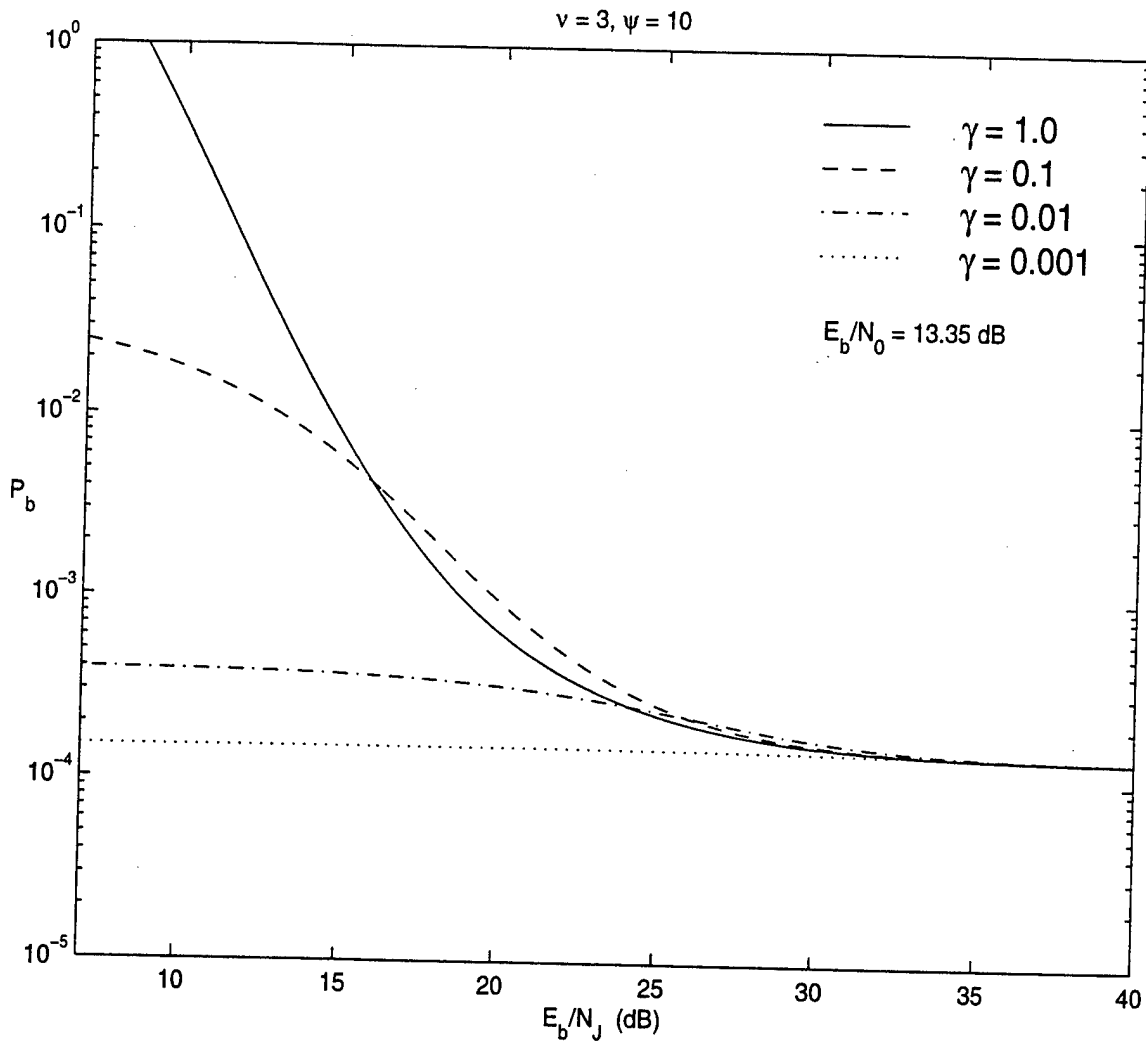
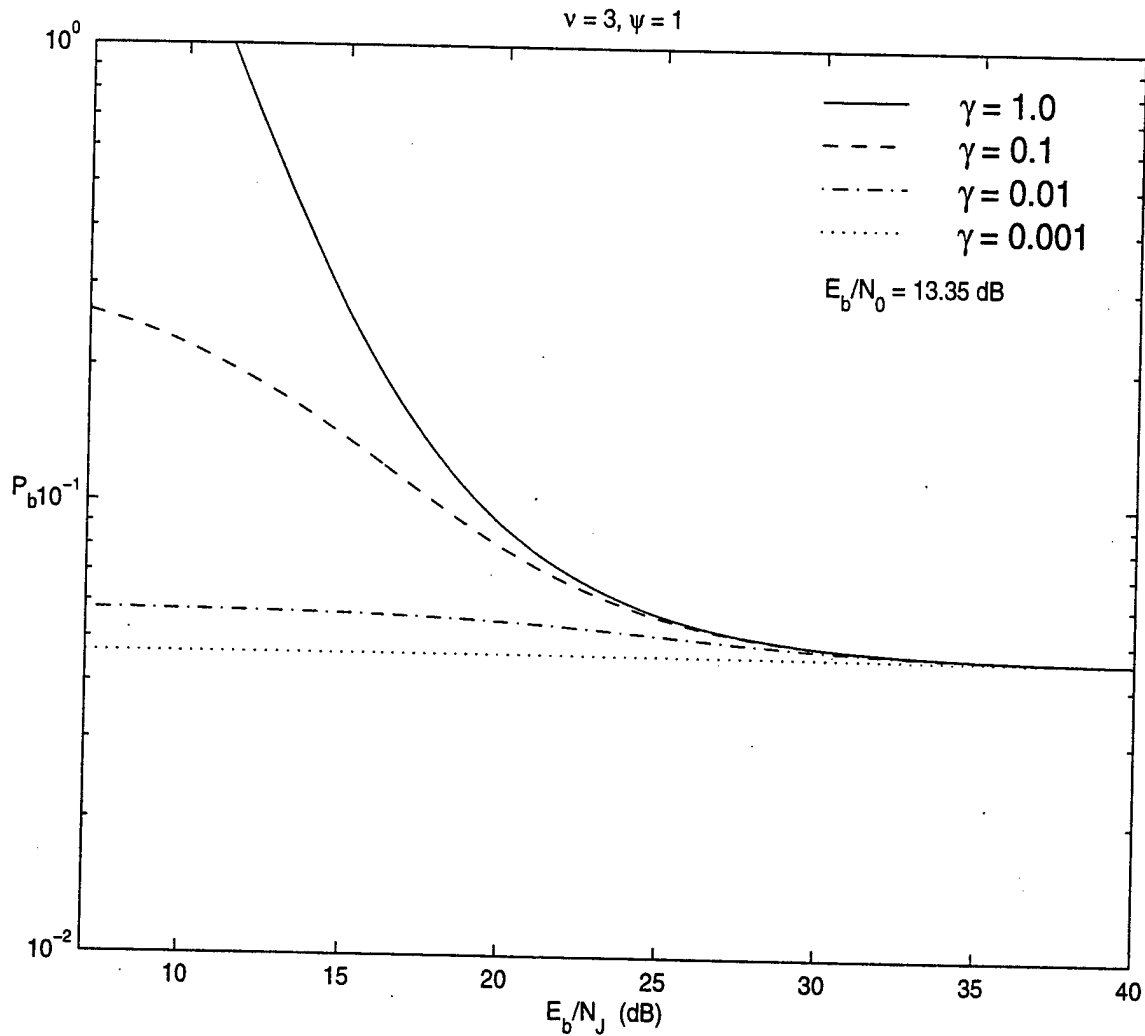
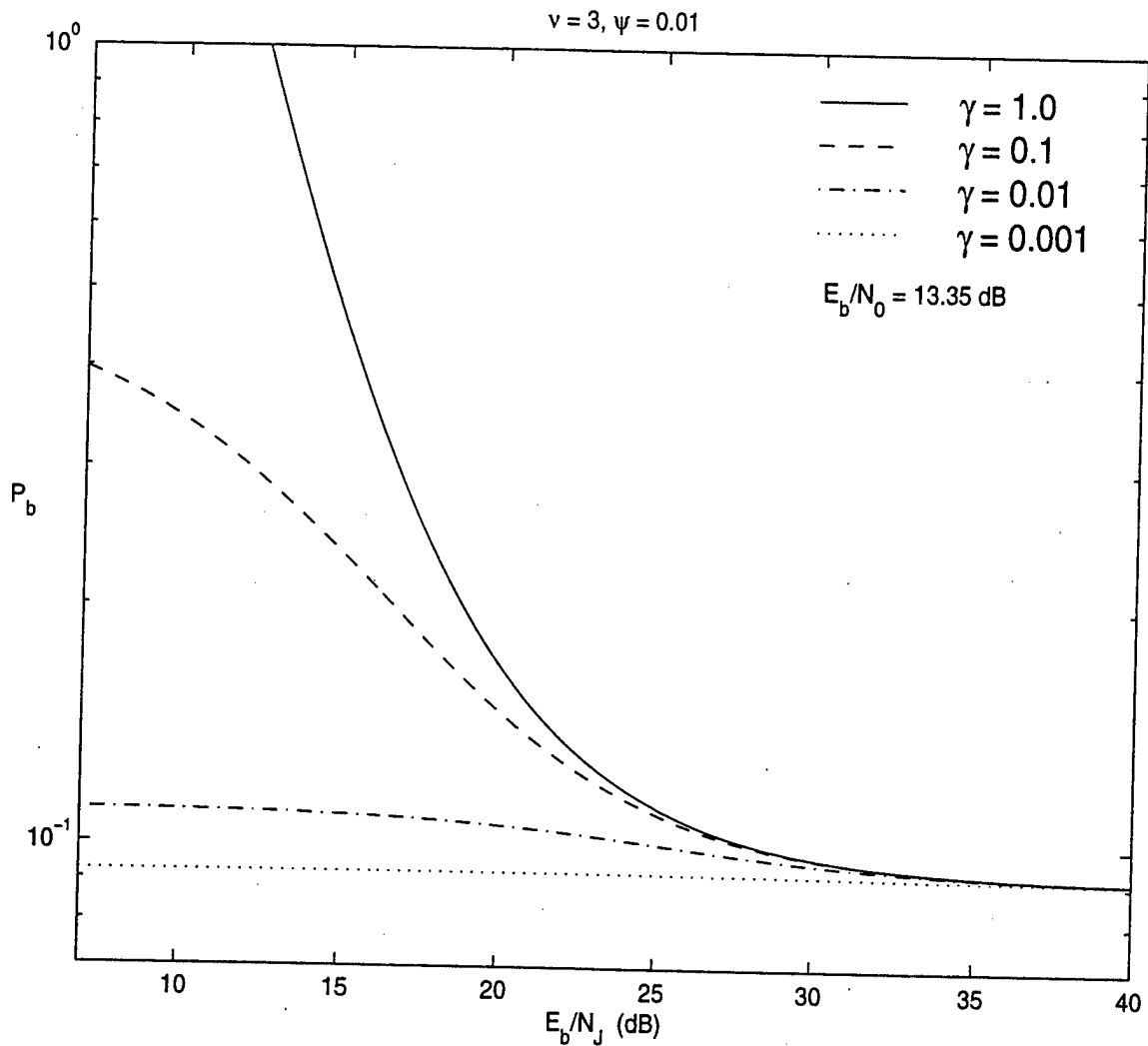


Figure 22: Performance of a conventional SFH/NCBFSK receiver with rate 1/2 convolutional coding and hard decision Viterbi decoding in the presence of partial-band noise jamming and a Ricean fading channel with  $v = 3$ ,  $E_b/N_0 = 13.35 \text{ dB}$ , and  $\psi = 10$



**Figure 23: Performance of a conventional SFH/NCBFSK receiver with rate 1/2 convolutional coding and hard decision Viterbi decoding in the presence of partial-band noise jamming and a Ricean fading channel with  $v = 3$ ,  $E_b/N_0 = 13.35 \text{ dB}$ , and  $\psi = 1$**



**Figure 24: Performance of a conventional SFH/NCBFSK receiver with rate 1/2 convolutional coding and hard decision Viterbi decoding in the presence of partial-band noise jamming and a Ricean fading channel with  $v = 3$ ,  $E_b/N_0 = 13.35 \text{ dB}$ , and  $\psi = 0.01$**

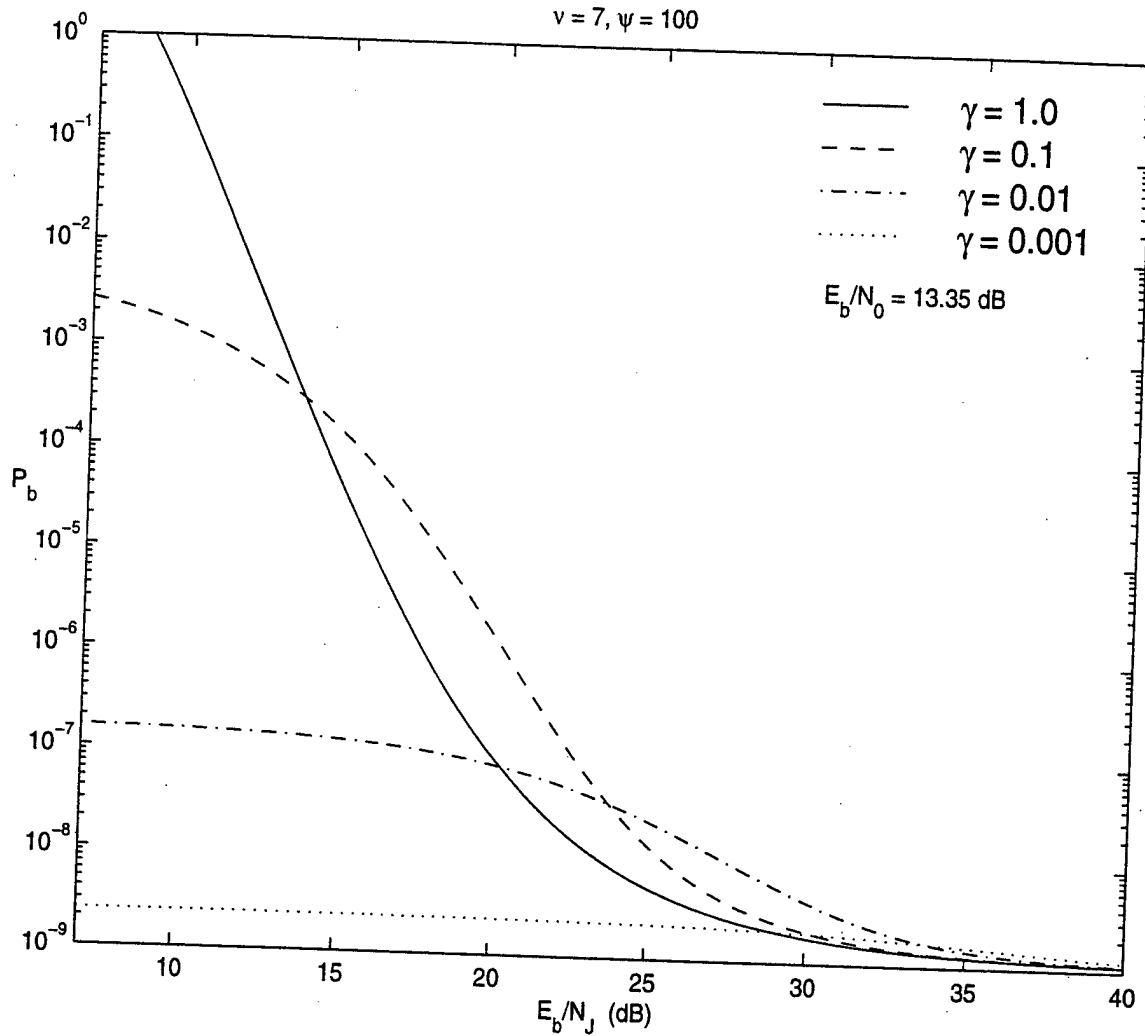


Figure 25: Performance of a conventional SFH/NCBFSK receiver with rate 1/2 convolutional coding and hard decision Viterbi decoding in the presence of partial-band noise jamming and a Ricean fading channel with  $v = 7$ ,  $E_b/N_0 = 13.35 \text{ dB}$ , and  $\psi = 100$

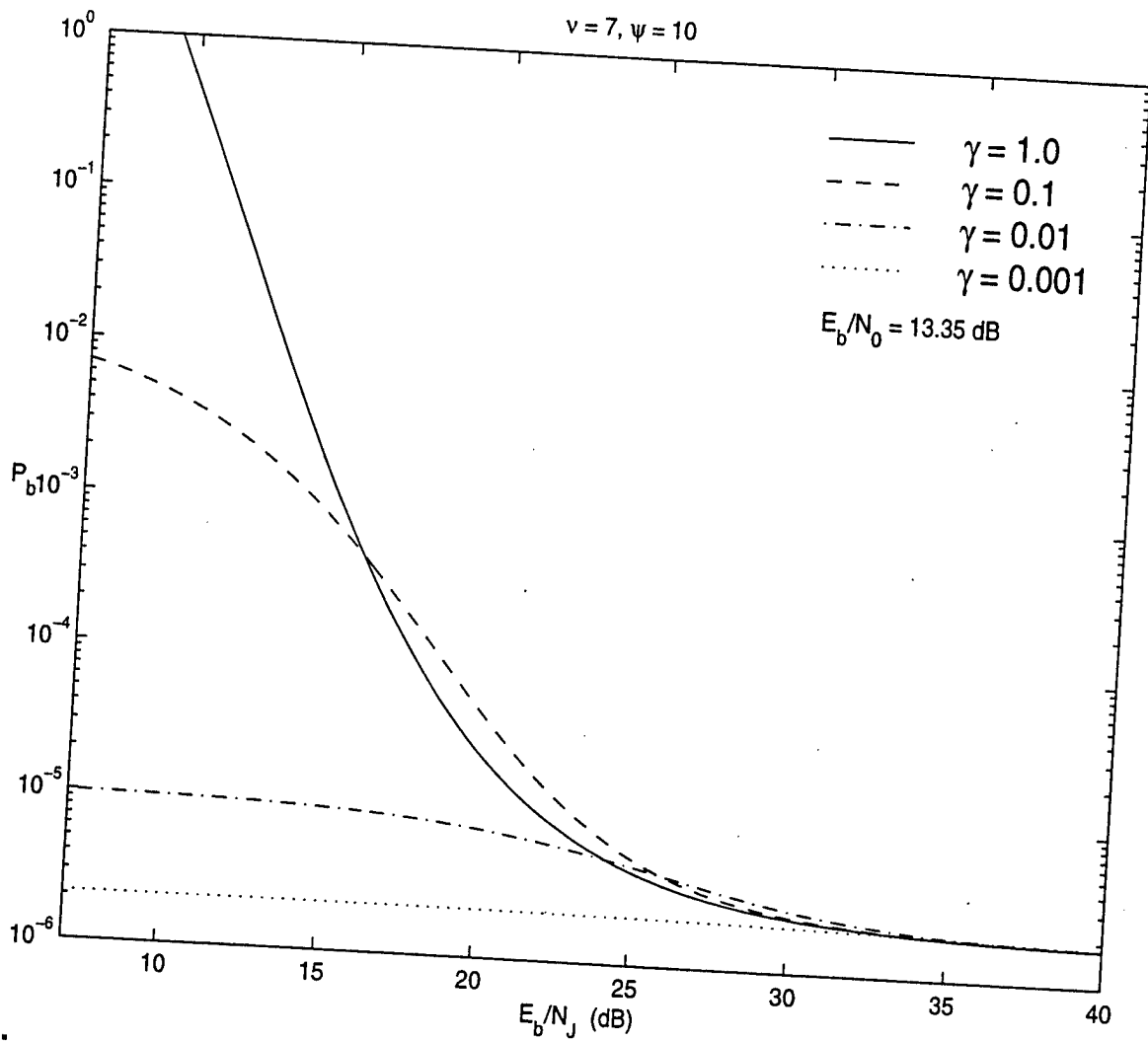
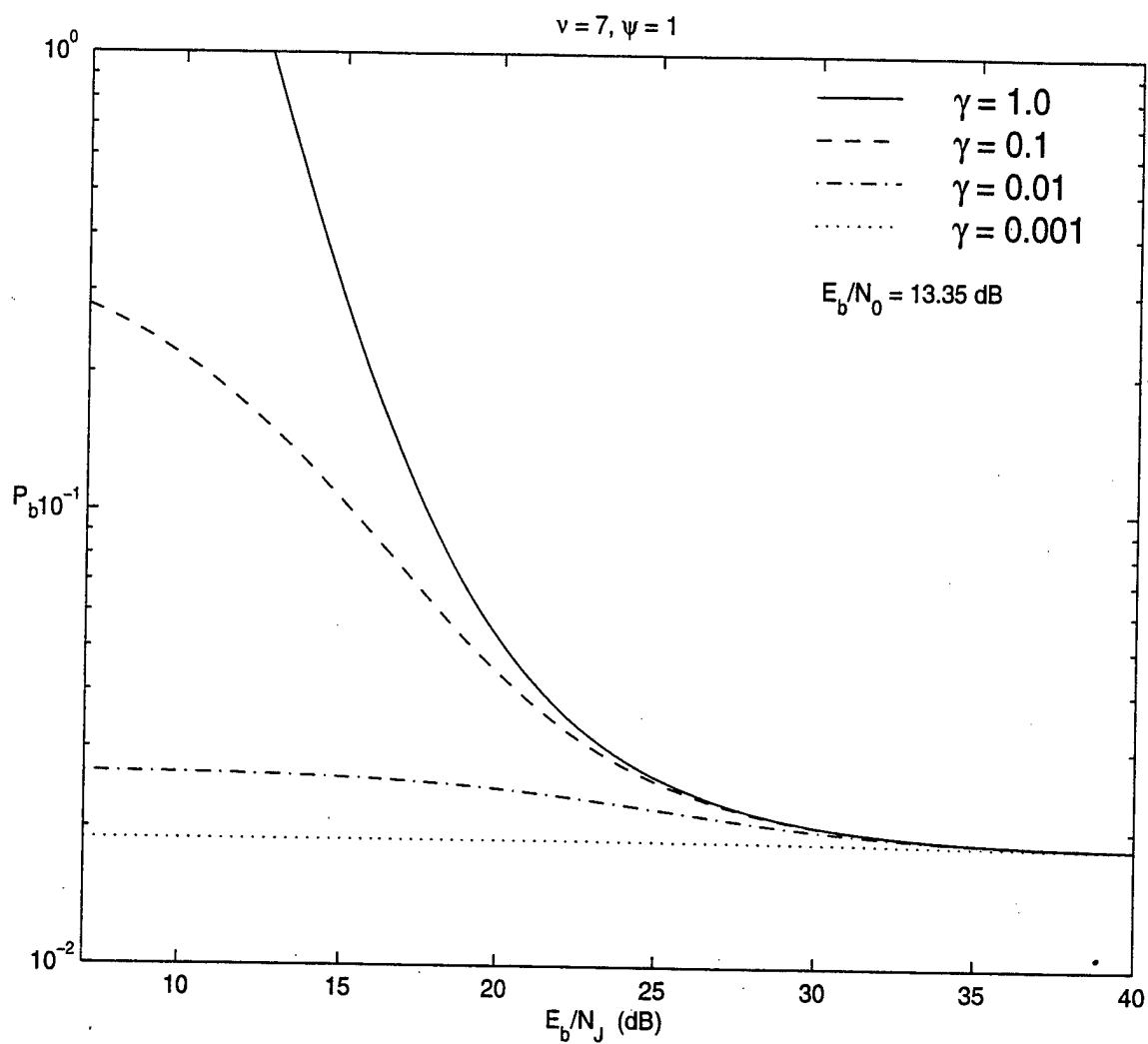


Figure 26: Performance of a conventional SFH/NCBFSK receiver with rate 1/2 convolutional coding and hard decision Viterbi decoding in the presence of partial-band noise jamming and a Ricean fading channel with  $v = 7$ ,  $E_b/N_0 = 13.35 \text{ dB}$ , and  $\psi = 10$



**Figure 27: Performance of a conventional SFH/NCBFSK receiver with rate 1/2 convolutional coding and soft decision Viterbi decoding in the presence of partial-band noise jamming and a Ricean fading channel with  $\nu = 7$ ,  $E_b/N_0 = 13.35 \text{ dB}$ , and  $\psi = 1$**

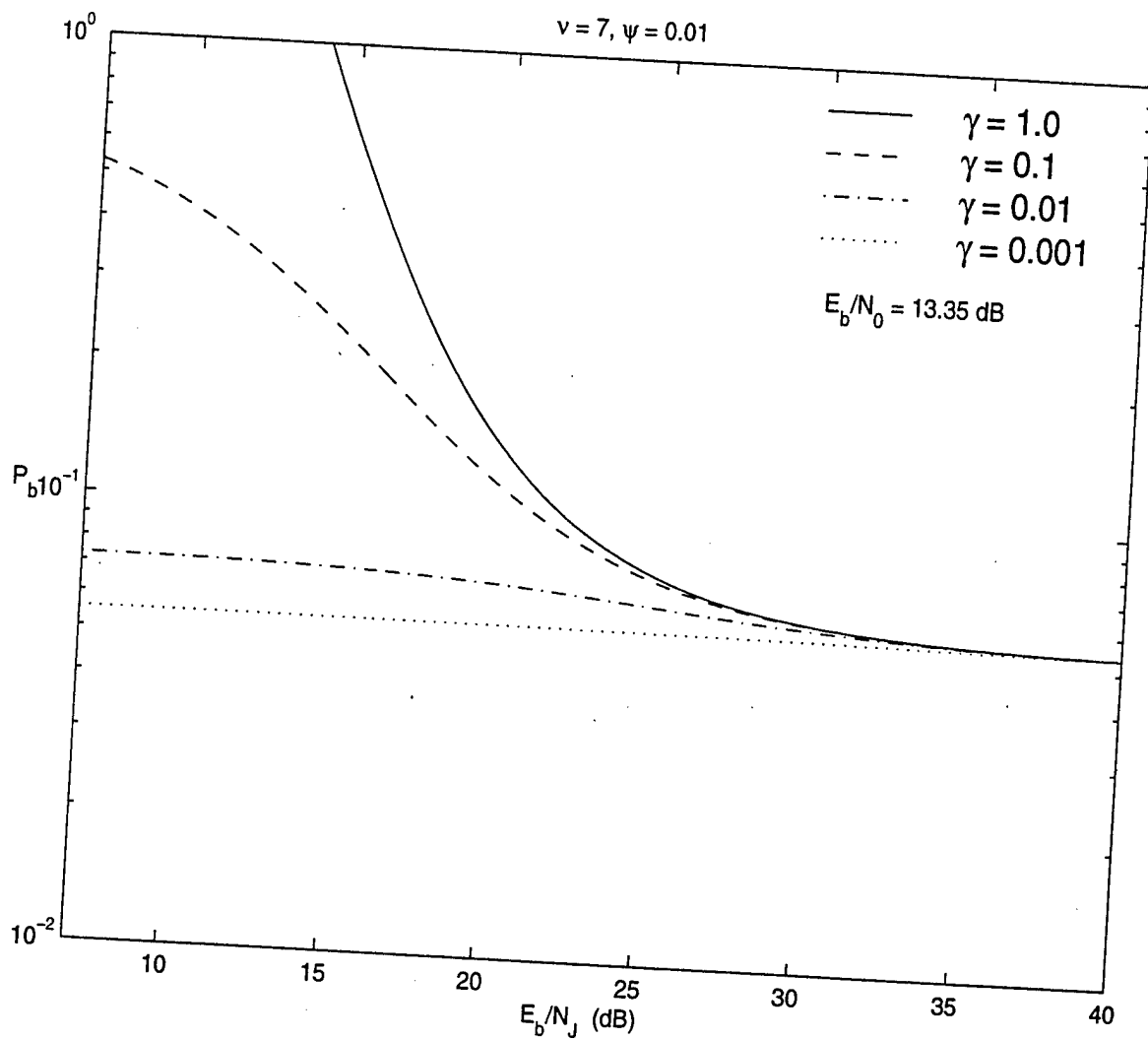


Figure 28: Performance of a conventional SFH/NCBFSK receiver with rate 1/2 convolutional coding and hard decision Viterbi decoding in the presence of partial-band noise jamming and a Ricean fading channel with  $\nu = 7$ ,  $E_b/N_0 = 13.35 \text{ dB}$ , and  $\psi = 0.01$

## V. CONCLUSIONS

The results of the numerical analysis illustrate that a SFH/NCBFSK communications system employing rate 1/2 convolutional coding and soft decision Viterbi decoding is easily defeated by partial-band noise jamming. The two constraint lengths examined showed that although the stronger code provided a lower asymptotic limit for the probability of bit error, it did not provide a significant increase in resistance to the effects of partial-band jamming. This susceptibility to partial-band noise jamming is believed to be due to the linear combining of the jammed and non-jammed receptions in the decoder. The linear combining resulted in the jammed receptions having the same weight as non-jammed receptions, allowing the jammed receptions to dominate the decision statistic. In order to examine this hypothesis, the analysis was also performed for both hard decision Viterbi detection and a noise-normalized receiver with soft decision Viterbi detection. These results were compared to the results obtained using soft decision Viterbi detection with linear combining.

For a channel not subject to fading, the results obtained using ideal soft decision Viterbi decoding were compared to the results obtained using hard decision

Viterbi decoding and the results obtained when using noise-normalized combining. The noise-normalized receiver provided improved performance in its ability to withstand partial-band noise jamming. It also provided the same lower asymptotic limit as the conventional receiver using soft decision Viterbi detection. For the receiver with hard decision Viterbi detection, an increased resistance to the effects of partial-band noise jamming was realized. When either hard decision Viterbi decoding is utilized or noise-normalized combining is used with soft decision Viterbi decoding, a potential jammer would be forced to adopt a barrage noise jamming scheme in order to be most effective in jamming the signal.

The performance of the communication system was also analyzed when subjected to a Ricean fading channel and partial-band noise jamming. When soft decision Viterbi decoding was used, the system was extremely susceptible to the effects of partial-band noise jamming. The effects of the Ricean fading channel were shown to increase the asymptotic lower limit on the probability of bit error. The analysis was also performed when hard decision Viterbi decoding was used. The results showed that hard decision Viterbi decoding was effective in mitigating the degradation due to partial-band noise jamming as it was in

the non-fading channel case. The lower asymptotic limit for the probability of bit error also increased due to the effects of channel fading for a receiver using hard decision detection. When the constraint length of the code was equal to three and the channel was severely faded, the communication system's performance was unsatisfactory with hard decision detection.

The analysis was not performed for the noise-normalized receiver for a Ricean fading channel. However, from the results for a non-faded channel and similar analysis for a Ricean fading channel [Refs. 5, 6, and 7], we can conclude that the use of a noise-normalized combining would nullify the effects of partial-band noise jamming. We can also conclude that a noise-normalized receiver with rate  $1/2$  convolutional coding and soft decision Viterbi detection would have the same lower asymptotic limit for the probability of bit error as the conventional receiver using the same coding and decoding scheme.

The reason for the poor performance of the system utilizing ideal soft decision Viterbi detection is hypothesized to be that the jammed receptions overwhelm the system since their weighting is equal to non-jammed receptions. The use of side information to de-emphasize

the jammed hops was illustrated to significantly increase the system's resistance to partial-band jamming. Also, hard decision detection was also shown to significantly improve the system's resistance to partial-band jamming because the effect of an error due to a jammed reception was limited to a single hop. For instance, if we assume that when  $E_b/N_J$  is small the jammed receptions are in error, and when  $E_b/N_0$  is large the non-jammed receptions are received error free, then as the fraction of the bandwidth that is being jammed is reduced; the maximum value  $P_b$  is reduced.

Most often, ideal soft decision detection is not utilized in real world systems, but multi-level quantization is used. We hypothesize that the use of three-bit quantization would improve the performance of the system analyzed in this thesis. By using three-bit quantization, we could achieve greater resistance to partial-band jamming analogous to that exhibited by the system using hard decision detection. Also, the use of multi-level quantization should allow for the lower asymptotic limit to be at or near the asymptotic limit when ideal soft decision detection was used as well as provide improved performance in the case of barrage noise jamming.

Future research should be performed to validate this hypothesis.



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