NUWC-NPT Technical Report 10,712
2 February 1998

Vorticity Transport by Electromagnetic Forces

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PREFACE

This report was prepared under the "Synergistic Drag Reduction Project," principal investigator Richard B. Philips (Code 8233), sponsored by a Code 80 Special Initiative.

The technical reviewer for this report was John R. Grant (Code 8233).

Reviewed and Approved: 2 February 1998

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE 2 February 1998	3. REPORT TYPE AND DATES COVERED

4. TITLE AND SUBTITLE

Vorticity Transport by Electromagnetic Forces

5. FUNDING NUMBERS

6. AUTHOR(S)

Peter J. Hendricks

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Naval Undersea Warfare Center Division
1176 Howell Street
Newport, RI 02841-1708

8. PERFORMING ORGANIZATION REPORT NUMBER

TR 10,712

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

The principal focus of this work is the use of the Lorentz force to alter the turbulence in the flow or, more generally, the vorticity. An analysis of the Lorentz force in a conducting fluid flowing over an array of magnets and electrodes predicts the amplitude of the vorticity transport in the boundary layer due to the Lorentz force field. These predictions are used to estimate the performance of an electromagnetic array for sea water flows at several meters per second.

This investigation demonstrates that an analysis of the Lorentz force distribution in a flow above a flat plate shows that many of the features of the fields and their effect on the flow can be deduced without solving the governing equations. Moreover, many fundamental conclusions can be derived from order of magnitude scaling analysis and symmetry arguments. The effect of the Lorentz force on the flow is evaluated by equating the curl of the Lorentz force to the transport of vorticity.

14. SUBJECT TERMS

Electromagnetics   Electromagnetic Flow Control   Magnetohydrodynamics
Turbulent Boundary Layers   Lorentz Force

15. NUMBER OF PAGES
22

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	SAR

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INTRODUCTION

An electric charge moving in a magnetic field is subjected to a force when the velocity of the electric charge has a component that is normal to the magnetic field lines. This force, known as the Lorentz force, is written as the vector cross-product

$$\vec{L} = \vec{j} \times \vec{B}, \tag{1}$$

where $\vec{L}$ is the Lorentz force vector, $\vec{j}$ is the electric current density, and $\vec{B}$ is the magnetic induction, with the arrow overbar indicating a vector quantity. In a conducting fluid, the Lorentz force can alter the condition of the flow at specific positions and in specific directions. This force can affect the bulk properties of the flow (e.g., producing thrust by establishing a pressure gradient) or small features of the flow (e.g., producing velocity fluctuations).

GOVERNING EQUATIONS

In a viscous, Newtonian conducting fluid, the momentum equation is given by

$$\rho \frac{D\vec{u}}{Dt} = -\nabla p + \mu \nabla^2 \vec{u} + \rho\left(\vec{j} \times \vec{B}\right), \tag{2}$$

where ρ is the fluid density, $\vec{u}$ is the velocity, p is the pressure, and μ is the dynamic viscosity. Vector notation is followed:

$$\frac{D\vec{u}}{Dt} = \frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u}. \tag{3}$$

$\nabla \vec{u}$ and ∇p are gradients, and

$$\nabla^2 \vec{u} = \frac{\partial^2 \vec{u}}{\partial x^2} + \frac{\partial^2 \vec{u}}{\partial y^2} + \frac{\partial^2 \vec{u}}{\partial z^2} \tag{4}$$

is the Laplacian of $\vec{u}$.

The principal focus of the present work is the use of the Lorentz force to alter the turbulence in the flow or, more generally, the vorticity. The relationship between the Lorentz force and the vorticity is explicit in the vorticity equation, which is the curl of the momentum equation, viz.,

$$\frac{D\bar{\omega}}{Dt} = \bar{\omega}\cdot\nabla\bar{u} + \nu\nabla^2\bar{\omega} + \frac{1}{\rho}\nabla\times\left(\bar{j}\times\bar{B}\right), \tag{5}$$

where $\bar{\omega}$ is the vorticity, $\nabla\times\nabla p = 0$ (since the curl of a potential field is identically zero), and $\nu = \mu/\rho$ is the kinematic viscosity.

In general, the current density is given by

$$\bar{j} = \sigma\left(\bar{E} + \bar{u}\times\bar{B}\right), \tag{6}$$

where σ is the specific electric conductivity, but in sea water magnetohydrodynamic (MHD) flows, the imposed electric field is normally much greater than the induced electric field $(\bar{E} >> \bar{u}\times\bar{B})$ at locations where the electric field is significant, and the Lorentz force term can be approximated as

$$\begin{aligned}\nabla\times\left(\bar{j}\times\bar{B}\right) &= \nabla\times\left(\sigma\bar{E}\times\bar{B}\right), \\ &= \sigma\left[\left(\nabla\cdot\bar{B}\right)\bar{E} - \left(\nabla\cdot\bar{E}\right)\bar{B} + \left(\bar{B}\cdot\nabla\right)\bar{E} - \left(\bar{E}\cdot\nabla\right)\bar{B}\right].\end{aligned} \tag{7}$$

The first term is zero because the magnetic field lines are closed, and the second term is zero in the case where there is no net charge in the conductor. The resulting vorticity equation is

$$\frac{D\bar{\omega}}{Dt} = \bar{\omega}\cdot\nabla\bar{u} + \nu\nabla^2\bar{\omega} + \frac{\sigma}{\rho}\left[\left(\bar{B}\cdot\nabla\right)\bar{E} - \left(\bar{E}\cdot\nabla\right)\bar{B}\right]. \tag{8}$$

APPLICATION

The creation and transport of vorticity in the boundary layer is responsible for the distribution of turbulence and the viscous drag on the boundary surface. According to equation (8), it is possible to affect the distribution of vorticity in the flow using the Lorentz force and thereby change the distribution of turbulence and drag.

In the case of a boundary layer on a flat plate, reference 1 has postulated a mechanism for controlling the growth of vortex structures near the wall to suppress the creation of turbulence there. References 1 and 2 suggest a grid of magnets and electrodes that can produce a pattern of Lorentz force that will inhibit the production of turbulence on the plate and lead to lower viscous drag. Because the proposed mechanism is aimed at controlling the vorticity associated with turbulence production, it is natural to analyze this process in terms of the vorticity equation.

Two basic arrangements of magnets and electrodes seem to be the most viable in terms of manufacturability, although any number of configurations are imaginable. These two configurations are illustrated in figure 1, where they are designated "streamwise magnets" and "spanwise magnets." In both cases the mean flow (streamwise) is in the x-direction, the outward flow (normal from the wall) is in the positive y-direction, and the spanwise flow is in the z-direction, forming a right-hand coordinate system. The magnets are arranged in parallel strips with alternating polarity, and the spaces between the magnets have segments of electrodes. Because the magnets are permanent ones, the magnetic fields are always present, but the electrodes are energized inpairs with voltages of opposite polarity, which causes an electric current to flow through the conducting fluid from the positive electrode (anode) to the negative electrode (cathode).

STEADY LORENTZ FORCE

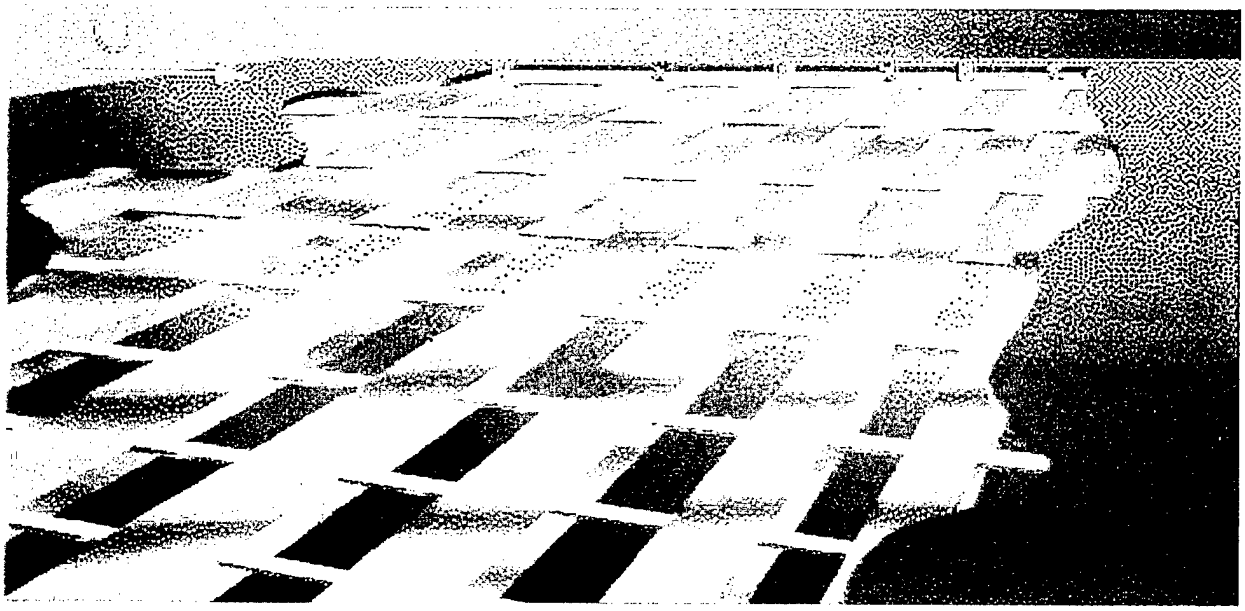
To evaluate the principal terms that contribute to the steady vorticity transport from the electromagnetic array, consider an array where the magnets are aligned parallel to the z-direction as shown in figure 2. Let x be in the plane of the plate perpendicular to z, and y is normal to the surface of the plate (and will often be referred to as "vertical"), positive up from the plate. Only the electromagnetic terms are being considered, so the analysis applies to either spanwise or streamwise magnet orientations. Away from the ends of long magnets, the magnetic field is approximately two-dimensional and

$$|B_z| << |B_x|,|B_y| \text{ and } \frac{\partial B}{\partial z} \cong 0. \tag{9}$$

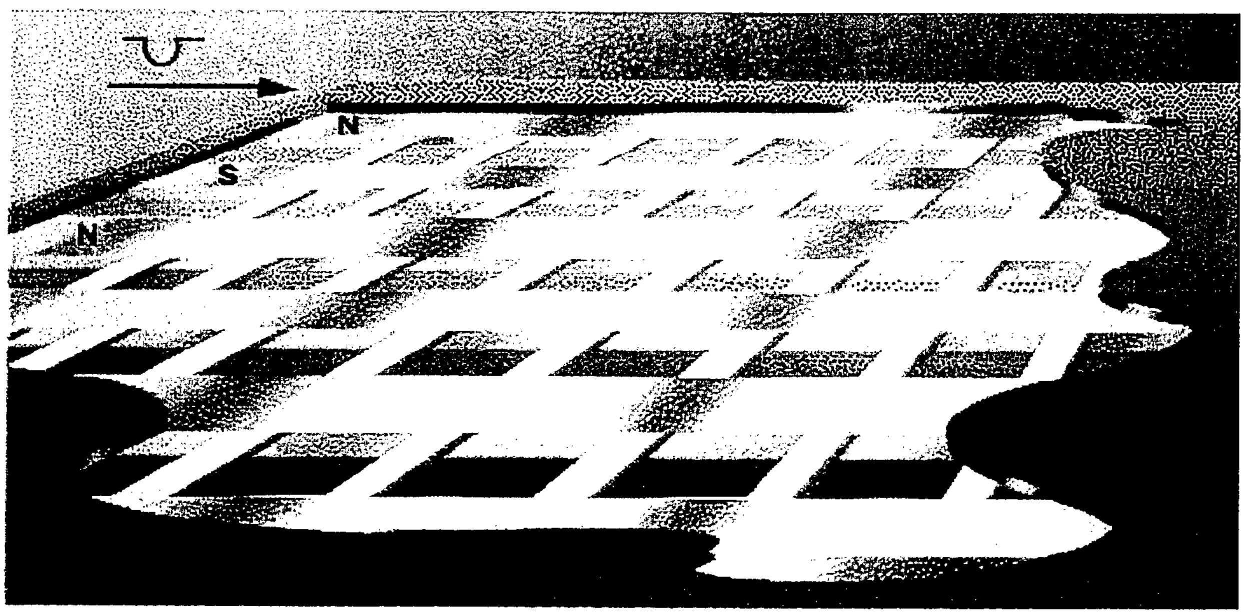
Using these assumptions, the remaining terms in the curl of the Lorentz force are

$$\begin{aligned}\frac{\rho}{\sigma}(\nabla x\bar{L}) = &\left(B_x\frac{\partial E_x}{\partial x} + B_y\frac{\partial E_x}{\partial y} - E_x\frac{\partial B_x}{\partial x} - E_y\frac{\partial B_x}{\partial y}\right)\hat{i} \\ &+\left(B_x\frac{\partial E_y}{\partial x} + B_y\frac{\partial E_y}{\partial y} - E_x\frac{\partial B_y}{\partial x} - E_y\frac{\partial B_y}{\partial y}\right)\hat{j} + \left(B_x\frac{\partial E_z}{\partial x} + B_y\frac{\partial E_z}{\partial y}\right)\hat{k},\end{aligned} \tag{10}$$

where $(\hat{i},\hat{j},\hat{k})$ are the unit vectors in the x-, y-, and z-directions, respectively. In this configuration, the fringe component of the electric field E_x is small compared to the other



SPANWISE MAGNETS



STREAMWISE MAGNETS

Figure 1. Streamwise and Spanwise Magnet Configurations

components, and terms with E_x and its derivatives can be neglected, leaving the simplified Lorentz vorticity transport equation:

$$\frac{\rho}{\sigma}\left(\nabla x\bar{L}\right)=\left(-E_y\frac{\partial B_x}{\partial y}\right)\hat{i}+\left(B_x\frac{\partial E_y}{\partial x}+B_y\frac{\partial E_y}{\partial y}-E_y\frac{\partial B_y}{\partial y}\right)\hat{j}+\left(B_x\frac{\partial E_z}{\partial x}+B_y\frac{\partial E_z}{\partial y}\right)\hat{k}. \tag{11}$$

In figure 2, in the area bounded by the magnets and the electrodes, with the electrode and magnet polarity shown in the figure, $B_x > 0$, $E_z > 0$, and $B_z = 0$, there is a normal component of Lorentz force

$$\rho L_y = \sigma E_z B_x - \sigma E_x B_z > 0 \tag{12}$$

pointing upward from the plate.

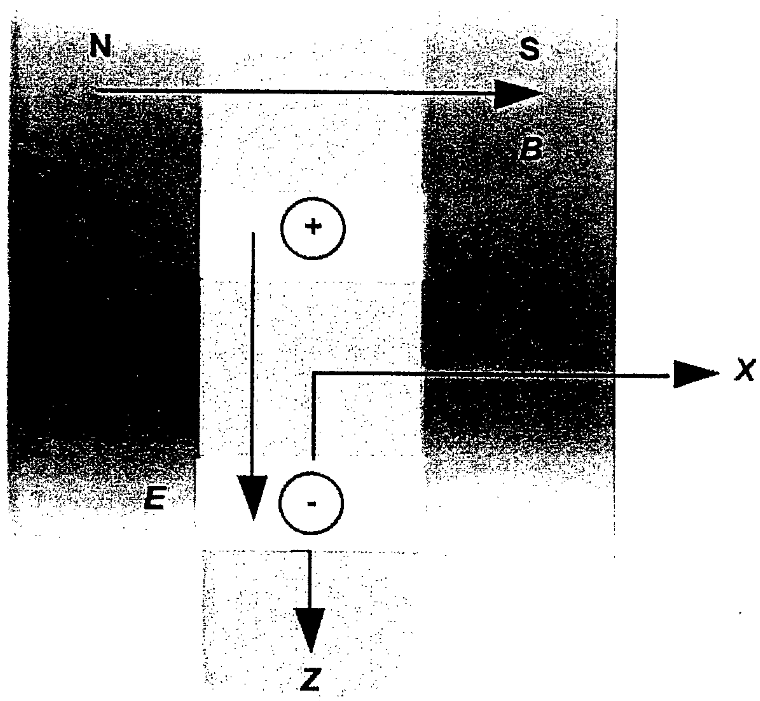


Figure 2. Plan View of Electromagnetic Element

Although the vorticity transport distribution is very complicated and inherently three-dimensional, even for relatively simple geometries, idealized distributions for the electric and magnetic fields and their derivatives may be used to evaluate the dominant components in the vorticity transport equation. In some cases it may be possible to estimate the absolute magnitude of individual components, but in other cases, particularly those that involve horizontal derivatives of the electric field, only the relative magnitude may be determined from these simple models.

First consider the distributions for the idealized case of long magnets and electrodes where the fields are two-dimensional. According to reference 3, the magnetic field is of the form

$$\vec{B} = B_0 \cos\left(\frac{\pi x}{2b}\right) e^{-\frac{\pi y}{2b}} \vec{i} + B_0 \sin\left(\frac{\pi x}{2b}\right) e^{-\frac{\pi y}{2b}} \vec{j} \;, \tag{13}$$

where the magnets are b wide, spaced b apart, and aligned with the z-axis, while B_0 is the magnetic induction at $x = 0$, $y = 0$. In the arrays considered here, the magnets are assumed to be long compared to their spacing, but the electrodes have lengths the same order of magnitude as their width and spacing. Although there will be a significant fringe field that is not accounted for with the two-dimensional (long electrode) approximation, a similar solution for the electric field is of order

$$\vec{E} = E_0 \cos\left(\frac{\pi z}{2a}\right) e^{-\frac{\pi y}{2a}} \vec{k} + E_0 \sin\left(\frac{\pi z}{2a}\right) e^{-\frac{\pi y}{2a}} \vec{j} \;, \tag{14}$$

where E_0 is the magnitude of the electric field at the middle of a long electrode parallel to the x-direction with width and spacing a. These expressions will be used to provide order of magnitude estimates for the magnetic and electric fields, their derivatives, and spatial distributions.

To identify sites in the flow where there is vorticity transport and to estimate the magnitude and direction of the vorticity components, consider a cross section through the center of the array parallel to the x-direction as indicated by section A in figure 3. Schematic distributions of the nonzero components of electric and magnetic field are shown below the sketch of the tile, assuming a level slightly above the surface of the plate. The largest component of electric field is E_z, which is nearly uniform in the area between the electrodes, but there will also be a small fringe field E_x near the edges of the electrodes, which has been neglected. The vertical component $E_y = 0$ by symmetry on this cross section. Because each of the vorticity production terms contains a spatial derivative of an electric or magnetic field, these derivatives must also be considered to estimate the vorticity production. Sketches of the significant derivatives on the cross section are also shown in figure 3. Equations (13) and (14) have been used to compute the form of the derivatives, and two or more terms may be represented by the same curve when they have the same shape. Considering the terms in equation (11) for the distributions in figure 3, the dominant term is

$$\rho \frac{D\omega_z}{Dt} = \sigma B_x \frac{\partial E_z}{\partial x} \,, \tag{15}$$

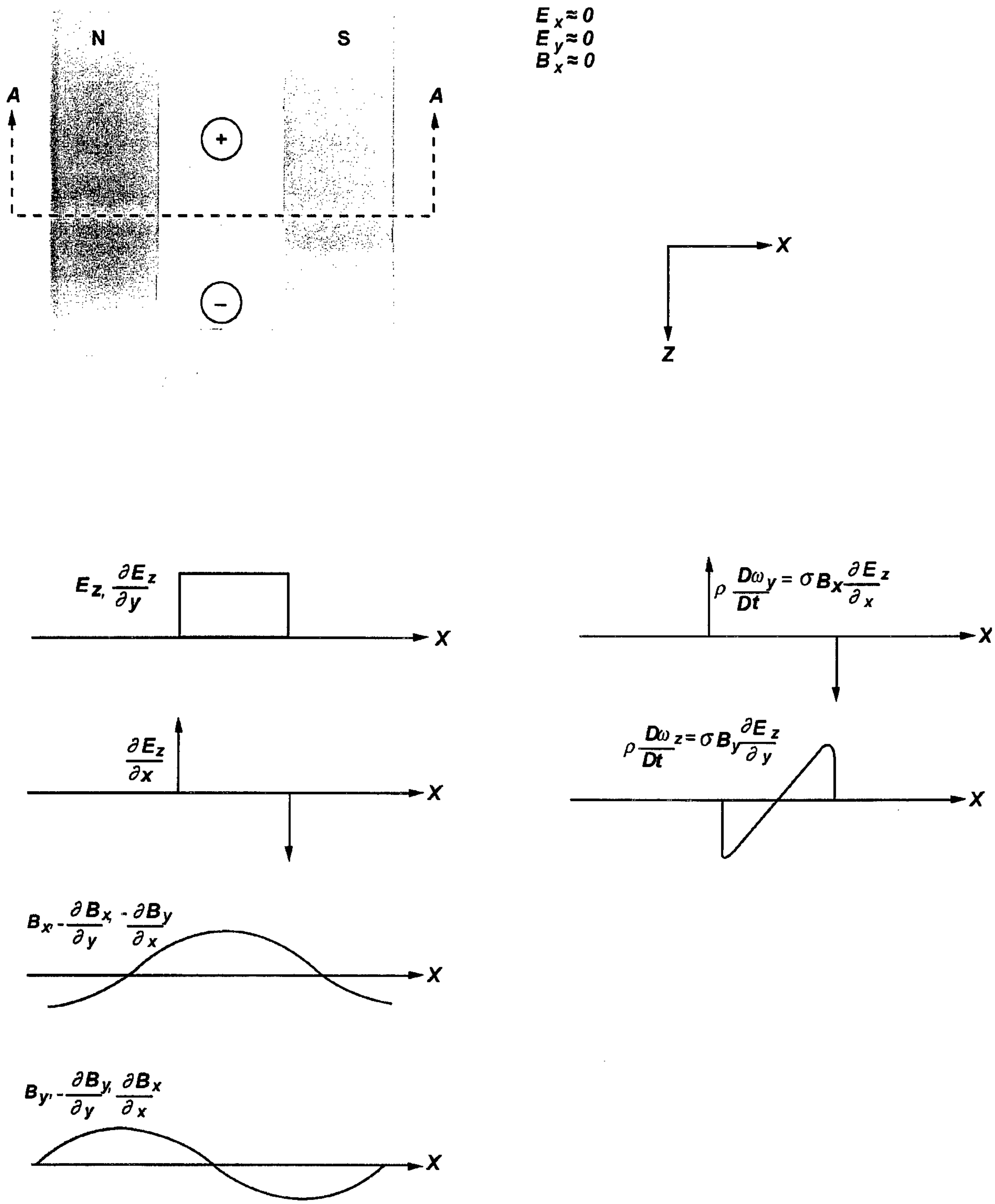


Figure 3. Schematic of Electric and Magnetic Fields and Vorticity Distributions for Section A-A

which is also shown in figure 3. Note that this vorticity component is horizontal and parallel to the magnets. It appears as two sharp spikes of opposite sign located in line with the edges of the electrodes, and, from equations (13) and (14), the amplitude decreases exponentially with distance above the plate. Furthermore, this vortex pair will be found in all parallel cross sections between the electrodes forming a pair of continuous vortex lines connecting the edges of opposing electrodes.

A similar analysis for a cross section through the center of an electrode is depicted in figure 4. Over the electrode, the electric field is very strong and purely normal, but the strength drops abruptly at the edge of the electrode. In this cross section, the most significant term in equation (11) is

$$\rho \frac{D\omega_y}{Dt} = \sigma B_x \frac{\partial E_y}{\partial x}. \tag{16}$$

This term is very similar to equation (15) in that it is the product of the horizontal magnetic field with the horizontal derivative of the electric field, but here it is the <u>normal</u> component of the electric field producing a <u>normal</u> component of vorticity, versus <u>horizontal</u> components of electric field <u>and</u> vorticity in equation (15). Other cross sections through the electrode have similar vorticity profiles, forming line source of wall-normal vorticity along each side of the electrode.

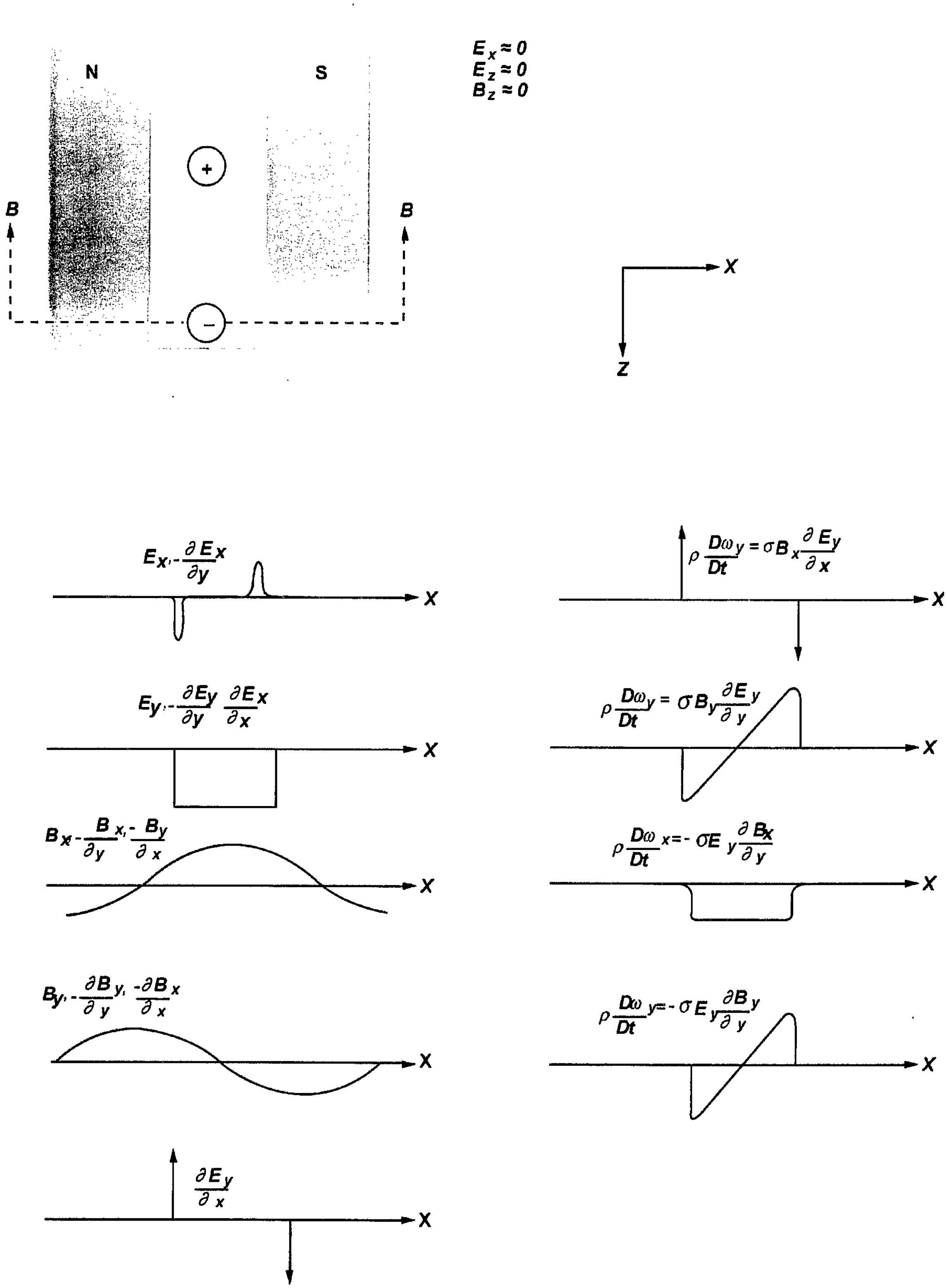


Figure 4. Schematic of Electric and Magnetic Fields and Vorticity Distributions for Section B-B

Helmholtz' vorticity theorems state that (1) the strength of a vortex filament is constant along its length and (2) a vortex filament cannot end in a fluid; it must extend to the boundaries of the fluid or form a closed path. In light of these constraints, a vortex structure emerges where the normal component of vorticity in equation (16) emerges from the plate at the edges of the electrodes, bends over near the inside corners of the electrodes, and connects to the horizontal vorticity in equation (15). This configuration forms a pair of continuous vortex structures, one on either side of the electromagnetic cell with vorticity of opposite sign. Because of the exponential decrease in electric and magnetic fields (and their derivatives), this vorticity is concentrated near the surface. By the symmetry of the vortex system, with equal and opposite vortices on opposing sides of the cell, there is no net vorticity production in any component direction, but there are regions of local vorticity transports.

Now consider the cross section in the *x-y* plane, taking a cut through the middle of the electrodes as shown as section C in figure 5. In this case, the principal vorticity transport component is located over the electrodes where it has the value

$$\rho \frac{D\omega_x}{Dt} = -\sigma E_y \frac{\partial B_x}{\partial y} \ . \tag{17}$$

In this case, the vorticity transport is due to the correlation between the normal component of the electric field and the normal derivative of the horizontal magnetic field. The large spikes in the horizontal derivative of the normal electric field at the edges do not alter the vorticity because they are multiplied by the B_Z component of the magnetic field, which is zero for long magnets. Although it is not obvious from this simplified model, these components of vorticity also enter the wall at the edges of the electrodes beside the magnets to comply with Helmholtz' theorems.

Including these vortices over the electrodes from figure 5, the overall picture is a set of four vortex arcs surrounding the electromagnetic element. The vorticity vector in each arc is predominantly horizontal and close to the wall, but it arches over to meet the wall normal to the wall at the edges of the electrodes. Each vortex line is paired with an identical line with opposite sign on the opposite side of the element. To first order, there is no vorticity transfer in the interior of the element.

Until this point the discussion has focused on the Lorentz term in the vorticity equation without reference to an external flow field. Now consider the addition of a mean flow over the plate. The two fundamental cases have been introduced as streamwise and spanwise magnets.

In both cases, the basic laminar or time-mean flow is essentially two-dimensional, and the mean vorticity is in the spanwise direction and always normal to the plane of the flow, so that $\vec{\omega} \cdot \nabla \vec{u} = 0$ in equation (8). Because of the antisymmetry of the vortex system produced by the cell, there will be no net vorticity change on the fluid. In Lagrangian terms, a fluid particle approaching an active element sees its vorticity increase (or decrease) over the upstream half of the element and then decrease (or increase) by an equal amount over the downstream half of the element.

Thus, it appears that the most accurate description of the net vorticity transport in the opposed magnet and electrode pair array (checkerboard) is a system of paired vortices over each active cell with exponentially decreasing strength with distance from the plate. To first order, each vortex pair imparts no net vorticity change to the flow. Any model that describes the effect of the Lorentz force on turbulence production, or any other effect on the flow, must be consistent with this conceptual model.

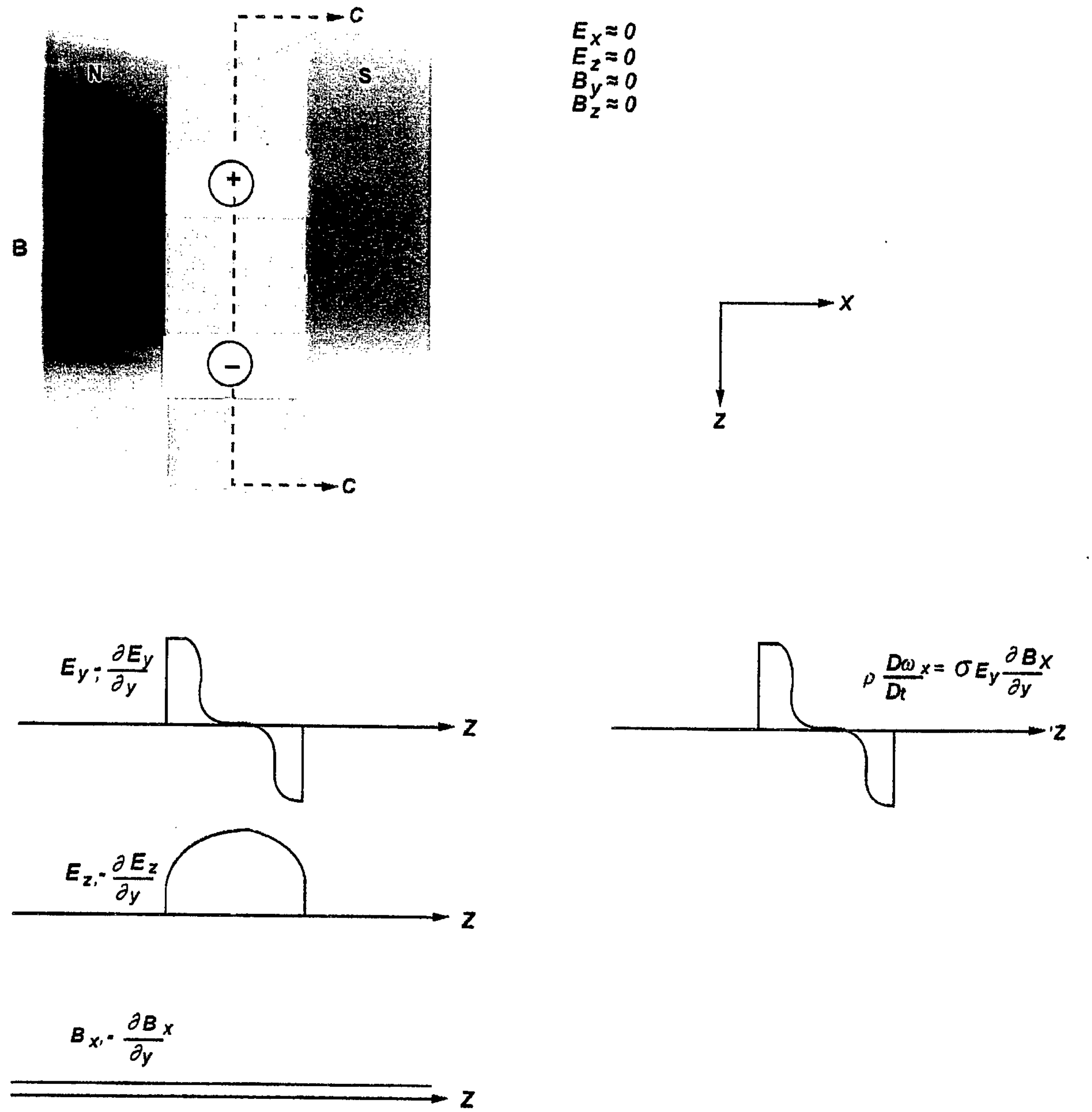


Figure 5. Schematic of Electric and Magnetic Fields and Vorticity Distributions for Section C-C

PULSED ELECTRODES

The preceding analysis has tacitly assumed a quasi-steady Lorentz force distribution by neglecting the time-dependent characteristics of the Lorentz force that arise when the electrodes in the array are pulsed, i.e., $\bar{L} = \bar{L}(t)$. When the Lorentz force changes with time while a fluid particle is traversing an active cell, this fluid particle can have a net change in vorticity.

Consider, for example, a fluid particle that is at the upstream edge at the center of the active cell with spanwise magnets when the voltage is applied to the cell and is convected to the center of the cell by the time the voltage is turned off. As shown in figure 3, the change in vorticity is proportional to the integrated product of $B_x > 0$ and $\frac{\partial E_z}{\partial x} > 0$, and there is a net vorticity change on the particle $\frac{D\omega}{Dt} > 0$. At the same time, however, a particle starting at the center of the cell moves to the downstream edge, and it experiences an equal and opposite vorticity change because $B_x > 0$ and $\frac{\partial E_z}{\partial y} < 0$; hence, $\frac{D\omega}{Dt} < 0$. Again, there is no global vorticity change, but there is a vortex pair separated in space by a distance that is of the same order as the streamwise dimension of the cell. In the case where the pulse duration Δt is short compared with the transit time across the cell, there is very little effect because the Lorentz force change between the time the current is turned on and the time the current is turned off is very small.

Conversely, when the pulse duration is very long compared with the transit time for the fluid particle across the array, the situation approaches the steady condition where the cell induces no net vorticity change. Hence, the pulse duration must be of the same order as the particle transit time to create a permanent vortex pair in the fluid with a pulsed Lorentz force.

SCALING

The electric and magnetic fields form a complex pattern, and the curl of their vector cross product is even more complex because it depends not only on the magnitude between the components but also on their correlation. However, a scaling analysis places bounds on the amplitudes of these effects for comparison with boundary layer parameters.

Assume that the magnets have a maximum magnetic induction at the surface of B_0 and that the potential between a neighboring pair of electrodes is V_0. From equations (13) and (14),

$$B(y) \approx B_0 \, e^{-\frac{\pi y}{2b}} ,$$

$$E(y) \approx \frac{V_0}{a}\, e^{-\frac{\pi y}{2a}}. \tag{18}$$

Using these expressions in the vorticity equation and assuming that horizontal derivatives are of the same order as normal derivatives gives an order-of-magnitude estimate :

$$\rho \frac{D\omega}{Dt} \approx \frac{\pi \sigma B_0 V_0}{2a^2} e^{-\frac{\pi(a+b)}{ab} y}, \tag{19}$$

or

$$\rho \frac{D\omega}{Dt} \approx \frac{\pi \sigma B_0 V_0}{2ab} e^{-\frac{\pi(a+b)}{ab} y}. \tag{20}$$

With the following typical numerical values,

$$B_0 \approx 0.5T,$$
$$V_0 \approx 10V,$$
$$a \approx b \approx 10^{-2} m,$$
$$\sigma \approx 4\ S/m,$$
$$\rho = 1000 \frac{kg}{m^3},$$

one gets

$$\frac{D\omega}{Dt} \approx 3x10^2 e^{-2\pi y} s^{-2}, \tag{21}$$

where y is in centimeters. Therefore, the maximum vorticity transport near the wall for the given conditions is

$$\left.\frac{D\omega}{Dt}\right)_{\max} \approx 3x10^2 s^{-2}, \tag{22}$$

and this factor increases as the *square* of the array length scale decreases, for a given voltage and magnetic field.

BOUNDARY LAYER PARAMETERS

For reasonable values of magnetic field and electrode voltage in a centimeter-scale array, the maximum vorticity transport that can be achieved is

$$\frac{D\omega}{Dt} \approx \frac{3x10^5}{a^2} s^{-1}, \tag{23}$$

where a is the magnet and electrode spacing in centimeters. In the outer part of a turbulent boundary layer (the inertial sublayer) on a flat plate, the mean velocity profile is

$$\frac{u}{u_*} = 2.5 \ \ln y_+ + 5, \tag{24}$$

where $u_* = \sqrt{\tau / \rho}$ is the friction velocity, τ being the shear stress at the wall, and $y_+ = \left(yu_* / \nu\right)$ is the dimensionless distance from the wall, scaled by inner variables. The mean vorticity in the spanwise direction is primarily due to the mean shear

$$\omega_z \approx \frac{du}{dy} = 2.5 \frac{u_*^2}{y_+ \nu}. \tag{25}$$

Most turbulent drag reduction is thought to take effect in the buffer layer, where $y_+ \approx 30$ and

$$\omega_z \approx 0.1 \frac{u_*^2}{\nu}. \tag{26}$$

Because $u_* \approx u_0/25$, where u_0 is the freestream velocity (assumed to be m/s), and $\nu = 10^{-6}$ m^2/s for water,

$$\omega_z \approx 160 u_0^2, \tag{27}$$

and the vorticity transport

$$\frac{D\omega_z}{Dt} \approx u \frac{\partial \omega_z}{\partial x} \approx 160 \frac{u_0^3}{L}, \tag{28}$$

where L is the streamwise scale of the array. For $L \approx 0.3$ m,

$$\frac{D\omega}{Dt} \approx 500 u_0^3, \tag{29}$$

and when $u_0 = 1$ m/s

$$\frac{D\omega}{Dt} \approx 5 \times 10^2 s^{-2}. \tag{30}$$

This vorticity transport is the same order of magnitude as the maximum electromagnetic vorticity transport, indicating that electromagnetic control has potential for modifying the vorticity in a turbulent boundary layer.

SUMMARY AND CONCLUSIONS

An analysis of the Lorentz force distribution in a flow above a flat plate shows that many of the features of the fields and their effect on the flow can be deduced without solving the governing equations. Many fundamental conclusions can be derived from order of magnitude scaling analysis and symmetry arguments. The effect of the Lorentz force on the flow is evaluated by equating the curl of the Lorentz force to the transport of vorticity. Analysis of a planar array of magnets and electrodes yields the following conclusions regarding the vorticity transport:

1. the amplitude is a maximum near the surface of the plate;

2. the amplitude is inversely proportional to the square of the linear dimension of the element spacing in the array;

3. the dominant terms in the vorticity transport equation are proportional to a horizontal derivative of the electric field; and

4. the amplitude decreases exponentially with distance from the wall with an *e*-folding distance that is twice the element spacing.

When there is a mean flow past the electromagnetic array, the following occurs:

1. there is no net change in spanwise vorticity for a fluid particle traversing a steady electromagnetic cell;

2. a pair of equal and opposite streamwise vortices are formed over an electrode pair with the separation of the vortex cores equal to the electrode spacing; and

3. switching a cell on and off produces a pair of equal and opposite spanwise vortices over the cell, where the strength of the pair is maximized when the switching interval is approximately equal to half the transit time for a fluid particle over the cell.

The magnitude of the vorticity transport appears to be the same order of magnitude as the mean spanwise vorticity in a turbulent boundary layer for the test conditions reported in reference 1.

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