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West Point, New York 10996

An Enlistment Bonus Distribution Model

DEPARTMENT OF SYSTEMS ENGINEERING AND OPERATIONS RESEARCH CENTER TECHNICAL REPORT

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The US Army Recruiting Command (USAREC) recently completed a survey of potential recruits, using a market research approach known as "choice-based conjoint analysis." One outcome of this work was a set of utilities that could be used to estimate relative proportions of the target population that would choose each of certain offered incentive packages. The incentive package that were considered included a range of bonus awards, lengths of commitment, military occupational specialties (MOS) and payment of college loans. This report presents the results of a study sponsored by USAREC aimed at exploiting utilities from conjoint analysis to assist Army decision makers in allocating recruiting incentive funds. We discuss background of the problem, illustrate how estimates of market share for various incentive packages can be extracted from conjoint analyses, and demonstrate use of integer programming techniques to determine optimal bonus packages for a set of MOS categories. We propose an optimization model and present examples that illustrate its feasibility. We discuss strengths and weakness of the model and software selected for its implementation.

Specific numerical values obtained in the example illustrations are not presently valid for application to actual enlistment bonus allocations by the Army. However, with more extensive conjoint assessments and carefully determined category importance weights as inputs to this model, we believe useful allocation programs can be obtained.

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1 Executive Summary

The U.S. Army Recruiting Command (USAREC) recently completed a survey of potential recruits, using a market research approach known as "choice-based conjoint analysis." One outcome of this work was a set of utilities that could be used to estimate relative proportions of the target population that would choose each of certain offered incentive packages. The incentive packages that were considered included a range of bonus awards, lengths of commitment, military occupational specialties (MOS) and payments of college loans. This report presents the results of a study sponsored by USAREC aimed at exploiting utilities from conjoint analysis to assist Army decision makers in allocating recruiting incentive funds. We discuss background of the problem, illustrate how estimates of market share for various incentive packages can be extracted from conjoint analyses, and demonstrate use of integer programming techniques to determine optimal bonus packages for a set of MOS categories. We propose an optimization model and present examples that illustrate its feasibility. We discuss strengths and weaknesses of the model and software selected for its implementation.

Specific numerical values obtained in the example illustrations are not presently valid for application to actual enlistment bonus allocations by the Army. However, with more extensive conjoint assessments and carefully determined category importance weights as inputs to this model, we believe useful bonus allocation programs can be obtained.

2 Introduction

2.1 Problem Background

The U.S. Army offers certain enlistment bonuses in order to induce potential and new recruits to make career field selections that help shape its personnel inventory. The intent is to use monetary incentives to attract these recruits, and to channel them into the Military Occupational Specialties (MOS) that might not be filled through other, less costly means. This program is governed by DoD Directive Number 1304.21 [DoD].

Figure 1 shows the Army's Enlistment Bonus (EB) budgets during the present decade [Rae]. This data shows the budget for enlistment incentives decreased

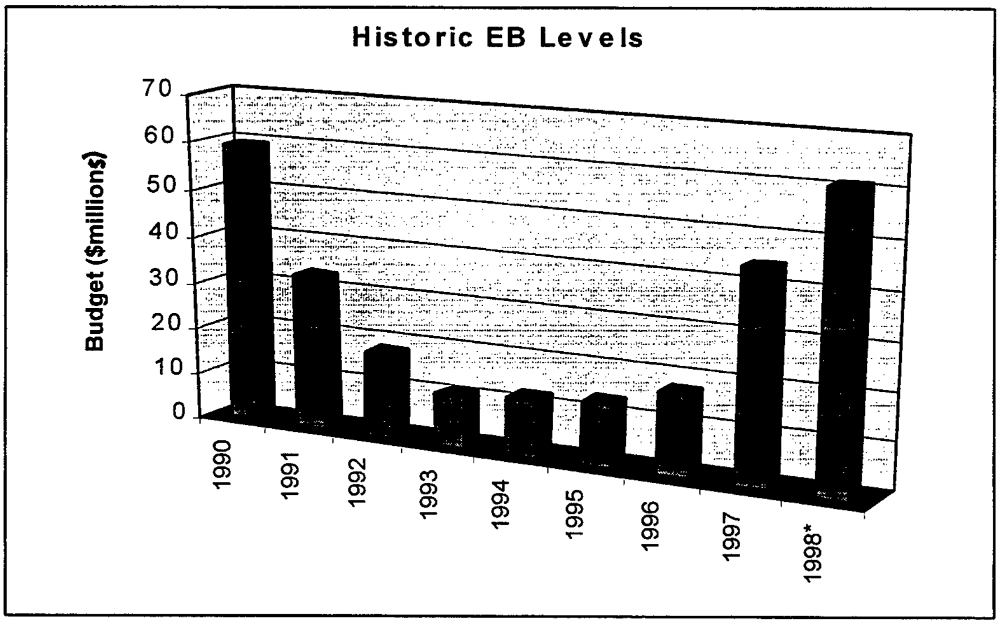

Figure 1

significantly during the drawdown of the early 1990s, but is now increasing sharply. The cost for 1998 is projected to be $61 million. However, in spite of the fact that the Army spends large amounts of money on these types of incentives, it appears there is currently no analytical method for allocating this money. Instead, the bonus structure is determined by an Incentives Review Board comprised of representatives from USAREC, the Office of the Deputy Chief of Staff for Personnel, and the Army Personnel Command. The Board's decisions are chiefly based on whether or not a particular MOS is projected to meet its required fill of high-quality recruits. As such, bonus levels are subject to frequent adjustment, but it has been difficult to capture the actual effects of these changes.

As one of the key players on the Incentives Review Board, the United States Army Recruiting Command (USAREC), determined that there is a need for an analytical model that could guide decision making on the efficient and effective allocation of the Army's enlisted bonus budget. Several previous research efforts looked into this bonus allocation problem[1]. However, those studies were based on modeling historic EB production data. The problem with such approaches is that the historic data are biased by the numbers and types of MOSs offered at a given time, recruiters' efforts to fill the various MOSs, and by exogenous factors such as the state of the U.S. economy and the public's attitude toward military service.

In late 1996 USAREC sponsored a pilot market study to determine the effects various packages of incentives would have on potential recruits, or "customers".[2] The study used a marketing tool known as choice-based conjoint analysis, a survey and analysis technique that helps evaluate how a defined target population will respond to various attributes of products offered. The results of that study were published in April of 1997 [Gale et al.].

USAREC recognized that there was potential to use results of such conjoint analyses to optimize enlistment bonus allocations. Ideally, such an approach would include a method of generating optimal mixes of incentives. USAREC sponsored the present study, aimed at developing an optimization approach and evaluating its utility to the Army. The research was conducted by the Department of Systems Engineering and the Operations Research Center (ORCEN) at the United States Military Academy, West

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[1] Harold Larson, Naval Postgraduate School, 1995 – Used linear splines to analyze 1988-1993 production data; Richard Morey, C.A. Lovell, and Lisa Wood, Sep 89 – Developed a regression based allocation model; RAND Enlistment Supply Study, 1994, 96 – Historically based supply elasticities.

[2] The "customers" in this study were 17-22 year olds, who earned mostly As or Bs in high school, and who had no members of their immediate family serving in the armed forces.

Point, NY, under the terms of a Memorandum of Agreement dated 2 July 1997 [Kays, Kaylor].

2.2 Model Purpose

The objective of this effort is **to provide decision-makers with a tool that can assist in efficient and effective allocation of EB incentives**. It uses a mixed-integer programming model with estimated market shares based on data from the actual customers as a basis for allocation decisions. The program finds a bonus distribution plan that maximizes attainment of recruiting and channeling effects goals, while meeting specific recruiting, legal, logical, and budgeting constraints. In addition, this prototype allowed us to run sample calculations in order to evaluate the legitimacy and usefulness of the modeling approach, and to recommend future studies that would be required in order to expand and refine the application.

2.3 Project Goals

During the initial background investigation for this project, we recognized that several important things were needed to ensure the model we were developing would meet USAREC's needs. These enabling events became our major project goals:

- Understand the current bonus allocation process.
- Determine the key stakeholders and their needs.
- Understand and assess USAREC's pilot conjoint analysis study.
- Develop a methodology for extracting customer preference distributions from the conjoint analysis data.
- Formulate a mixed integer program (MIP) that uses the customer preference distributions to produce an optimal bonus distribution plan.
- Evaluate the appropriateness of this modeling approach and the usefulness of the model.
- Recommend follow-on work that could enhance the model.

2.4 Methodology

We conducted a Needs-Analysis to gain a thorough understanding of the problem and to provide a means to ensure proposed solutions would satisfy USAREC's needs. We discussed enlistment incentives with representatives from USAREC, O/DCSPER, and PERSCOM, attended an Incentives Review Board (IRB) meeting, reviewed the legal and regulatory guidelines for EBs, and studied historic IRB and USAREC data. It quickly became evident that there are many different organizations and systems that impact the EB incentives program. Figure 2 shows some of the key stakeholders and the context within which the EBs are determined and implemented.

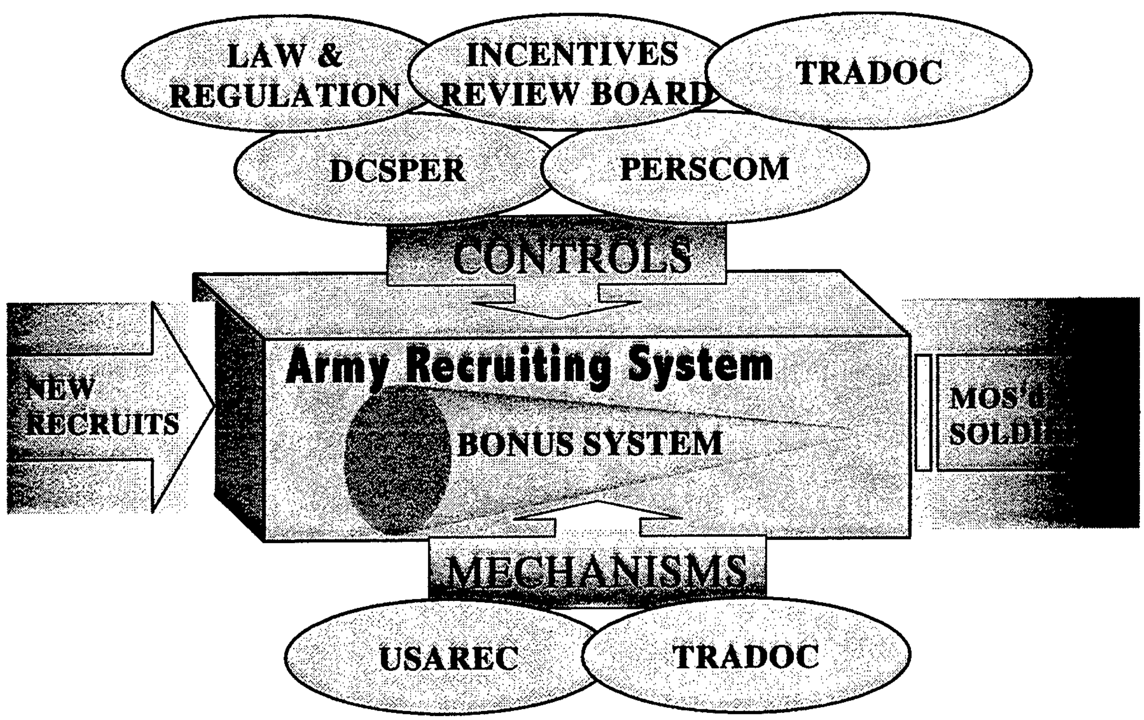

Figure 2

Based on the Needs-Analysis, we developed a list of model requirements that an ideal EB allocation model should be able to fulfill.[3] We determined the effective need for this project is as follows:

[3] A summary of stakeholders is given in Appendix A.

- **to determine what effect various enlistment incentives have on the target population, then,**
- **to develop a flexible, easy-to-use, strategic level model that uses these effects to:**

  (a) assist the Incentives Review Board in "optimally" allocating the EB budget;

  (b) maximize the accession of high quality recruits into priority MOSs.

  (c) provide insights concerning how changes in EB budget or its allocation will impact MOS fill rates.

In parallel with the Needs-Analysis, we also began to take a close look at the data from the conjoint analysis study. We determined it would be possible to extract customer preference information from such data, and this information could be used to model the probable responses of potential recruits to various incentive packages. We also realized, however, that the initial conjoint analysis study was fairly limited in scope, therefore our optimization model would have to be limited to the small number of categories that were used in that survey. We determined that numerical results with this initial input would serve to illustrate our approach and allow us to provide "proof of concept", but would not be appropriate for application.

We evaluated several modeling alternatives, and decided that a mixed integer programming (MIP) model would be the best way to represent this problem and solve it in a reasonable amount of time. We selected AMPL[4] as our algebraic modeling language and CPLEX as the solver. We formulated the problem as a goal program, where the goal is to come as close a possible to the targets for each MOS category while ensuring the solution meets the necessary legal, budgeting, and logical constraints.

We were interested in exercising the functioning model to assist in investigating several issues. First, we needed to demonstrate the approach was valid. We checked this by making sure the numerical results made logical sense, and by setting various model

[4] AMPL: A Modeling Language for Mathematical Programming, by Compass Modeling Solutions, Inc.

parameters to values for which we could predict the optimal solution, and verifying that the model did indeed produce predicted results. We also wanted to gain insight into how sensitive the model was to some its parameters; in particular we were concerned about sensitivity to the "customer preference data" from the conjoint analysis. Finally, we wanted to assess the usefulness of the model. We were interested in evaluating its ease-of-use, flexibility, and expandability. More details of these considerations are given in the following sections of this report. Results from these assessments led to specific recommendations concerning how the prototype model could be expanded and improved in the future.

3 Choice-Based Conjoint Analysis

One of the goals of this project was to evaluate the choice-based conjoint analysis approach to determine how the data it generates might be used to determine the customer preference distributions that would be required for an optimization model. This section reviews our findings and outlines an approach to using conjoint analysis results as inputs to the model we propose.

3.1 Description of Choice-Based Conjoint Analysis (CBC)

Conjoint analysis is a marketing research tool that permits the user to analyze customer preferences among competing products. It allows marketers to determine what features a new product should have, and how it should be priced [Curry, 1]. In the choice-based version of conjoint analysis, subjects are asked to repeatedly select "the best" product from short lists, where the attributes of the offerings vary on each list. This process is repeated with many different potential customers. The information is used to estimate customers' value system with respect to the product. Using logistic regression, marketers can predict customer responses to potential new product offerings.

3.2 USAREC Sponsored CBC Study

In July of 1996, USAREC contracted the Urban Studies Institute at the University of Louisville to conduct a conjoint analysis study in order to "better understand the relationship of a mix of attributes in recruitment packages" [Gale et al., preface]. For reasons reviewed below, we refer to this as the "pilot study." The Urban Studies Institute subcontracted with malls in San Diego, CA; Dallas, TX; Baltimore, MD; Chicago, IL; and Orlando, FL to conduct mall-intercept surveys. The subjects of these surveys were 17-22 year olds who reported having mostly As and Bs in high school and who had no immediate family members serving in the military.

Each mall conducted 100 interviews in which the targeted subjects were each asked to respond to 20 conjoint analysis tasks. A task consisted of selecting the best of four possible options that included three MOS/Term-of-Service/Incentive alternatives and a "None" option. The surveys were administered electronically, and the data were sent to the Urban Studies Institute.

Attributes included in the conjoint analysis study are shown in Figure 3. An enlistment option was formed by combining one of the MOS choices with one of the terms of service and one incentive.

MOS Categories	Terms of Service	Incentives
▪ Military/Counter Intelligence	▪ Two Years	▪ $60,000 Army College Fund (ACF)
▪ Administrative or Professional	▪ Three Years	▪ $40,000 ACF
▪ Medical	▪ Four Years	▪ $20,000 ACF
▪ Electronic Systems Operation & Maintenance	▪ Five Years	▪ $16,000 Enlistment Bonus (EB)
▪ Engineering/Chemical Operations	▪ Six Years	▪ $10,000 EB
▪ Mechanical and Aircraft Maintenance		▪ $4,000 EB
▪ Infantry, Artillery or Other Combat Arms		▪ Get student loans paid off
		▪ Choice of unit or location

Figure 3

3.3 Limitations of the Pilot Study

Several aspects of the pilot study suggest the data are best considered to be illustrative of the CBC methodology and the types of output that could be expected in a larger-scale study of the same type. Some of these are listed below:

- Only 81 "highly propensed" individuals were included in the study.
- It is assumed there were no interactions among attributes, based on findings of "no significant interactions" in statistical tests based on the data collected.
- It is not clear subjects (or even survey administrators) understood the various alternatives.
- Non-feasible incentives were offered and analyzed in the study (e.g., $16K for two years).
- Illogical inferences were drawn from the data collected (e.g., the analysis suggests that, for an equivalent incentives package and MOS, recruits would favor a 5-year term-of-service over a 4-year term-of-service).
- There is a high level of aggregation in the customer preference data. Since only seven MOS categories were used in the pilot study, the optimization model treats the entire incoming population as if they face only seven MOS choices.

3.4 Extracting Customer Preference Distributions

We used preference data for propensed respondents, reported in [Gale et al.], to estimate proportions of this population that would choose each of several competing recruiting "products." Since no significant interactions were found in the pilot study, we assume there are no interactions. This makes it possible to estimate joint proportions by multiplying marginal values for each attribute of a recruiting product, using either frequency data or utilities estimated from logistic regression. For example, using frequency data for a choice involving Medical jobs, 2-year enlistment and $20,000 Army

College Fund, the joint "utility" is estimated by the product (.266 / 1.674) (.240 / 1.195) (.269 / 2.150). Each term in this product is the relative fraction of times the attribute was chosen (called "frequency data" in [Gale et al.]). For example, the Medical jobs MOS was chosen .266 of the times it was offered; the relative fraction, among all seven MOS's in the study, was .266 / 1.674.

If a set of products is offered to a population, the proportion choosing each is estimated by "normalizing" the utilities over the products in the offered set. For example, if three products are offered, with respective utilities .220, .411 and .745, the fractions choosing each of these products is estimated to be, respectively, .220 / (.220 + .411 + .745) = .220 / 1.376 = .16, .411 / 1.376 = .30, and .745 / 1.376 = .54.

Details of these computations, together with discussion of the use of logistic regression to estimate the preference distributions, are given in Appendix B.

4 Mathematical Model

This section provides an overview of the model development, implementation, and results. A detailed discussion of the model, including the documented code, is given in Appendix C.

4.1 Modeling Alternatives Considered

When we began to develop the optimization model, we considered four candidate modeling techniques:

- exhaustive search techniques
- Monte Carlo simulation
- genetic algorithms (GA)
- linear programming (LP)

Each of these had the potential to determine either a "good" or an "optimal" allocation of enlistment bonus packages.

We used Microsoft Excel to create data, constraints, and implement logic that represented an instance of the problem, then evaluated the ability of the different modeling techniques to solve the problem in a reasonable amount of time. We quickly dismissed the feasibility of the exhaustive search and Monte Carlo simulation methods. Both techniques proved too time consuming (seven hours of run time with no acceptable answers). The GA was created using Evolver, an add-in program for Excel from Altae [*Evolver: The Genetic Algorithm Problem Solver*]. This method *did* provide reasonably good solutions within reasonable time periods (10 to 20 minutes) and appears to be a viable alternative if future applications become too complex to solve with mathematical programming techniques. The Mixed Integer Program (MIP) was able to produce good or even optimal feasible solutions within a short period of time (5 to 30 minutes) on a moderately fast PC. Based on this initial investigation, we selected the integer programming method due to its ability to find a good solution quickly, and with much longer run-times, an optimal enlistment bonus allocation.

4.2 Model Description

We developed this integer-programming model as a goal program[5]. This extension of linear programming can be used to solve problems in which one seeks to simultaneously optimize multiple objectives [Winston, 728]. In the present application, the USAREC enlistment targets for each MOS serve as the goals, and these goals are weighted according to their relative importance of satisfaction.[6] The remaining requirements constitute constraints within the problem. The majority of the coefficients for the constraints in this model are derived from the conjoint analysis conducted in the pilot study. Other constraints enforce budgetary and legal requirements. There also are "logical" constraints that can be used to cause the incentives program to allow or not allow various incentive combinations.

The utilities output from the conjoint analysis can be used to estimate the relative market share each bonus package can be expected to capture. These market shares, when

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[5] Also know as multi-objective programming.

[6] These weights must be determined in advance, based on the preferences of the decision-makers, and express the relative importance of meeting the recruiting targets for the different MOS categories.

paired with the decision variables, determine, up to a multiplicative constant, the expected number of enlistees that would be attracted by each bonus option.

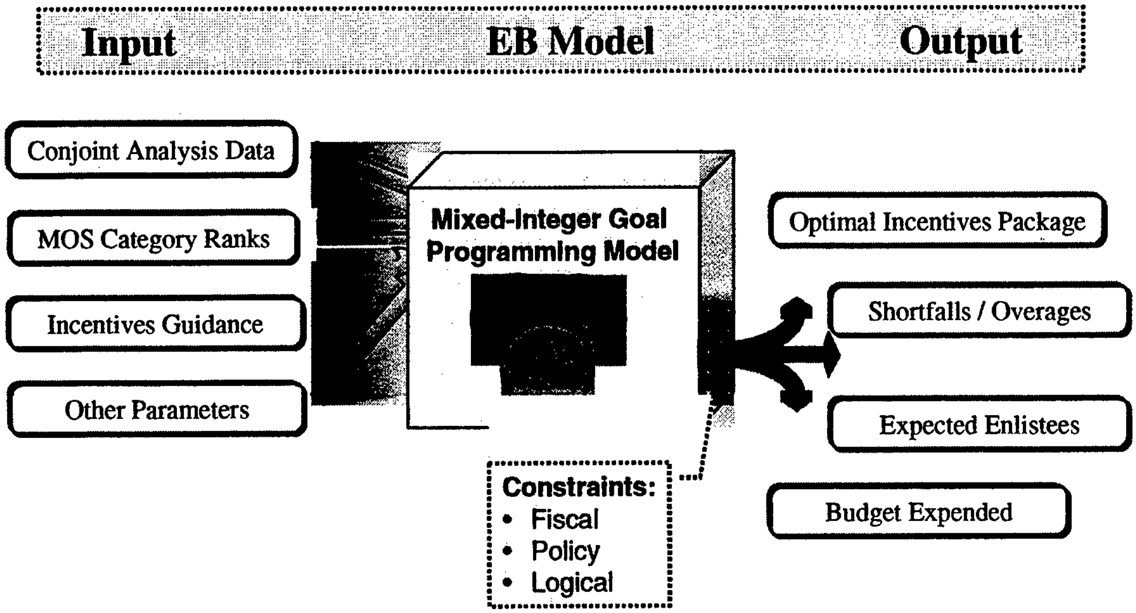

Figure 4

Figure 4 is a graphical depiction of the model. The inputs for this model include the USAREC targets for each MOS; importance weights for each MOS; market share estimates from the CBC; the budget allocated for each bonus program; and the penalties associated with falling short of targets or exceeding targets. The outputs of the model are a set of bonus packages that meets all the constraints and minimizes the deviations from the targets; the number of enlistments attributed to each bonus option; the short-fall or excess for each MOS; and money used from each bonus program budget.

4.3 Model Assumptions

In the process of developing this model, we made the following assumptions.

- *All incentives are paid out within one fiscal year.* We recognize that payment frequently crosses fiscal years, but we assumed that these payments could be represented as a single "present value figure". This eliminated the need to project and optimize against out-year budgets, a process that would add a great deal of uncertainty to the model inputs.

- *There is a 100% payout on bonuses.* That is, all enlistees who sign up for a particular bonus will be paid that bonus. Attrition should already be reflected in the target figures for each MOS category.

- *An equivalent lump-sum value can be calculated for each of the "other" non-EB, non-college fund incentives.* The "other" category includes non-monetary incentives such as Unit of Choice, or Station of Choice. Like the EB, however, these incentives must be constrained. For the purposes of this model, we've estimated monetary values for these programs, and then used an estimated budget figure to cap the number of recruits who could chose these options.

- *Proportionality holds for the objective function variables and coefficients.* The penalty for overfilling or underfilling a particular MOS category is a *linear* function based on the deviation from the target. Other penalty functions could be developed and incorporated if necessary.

- *The MOS categories are homogenous.* Each MOS category contains many different MOSs. Our prototype model treats all the MOSs within a given category *exactly the same.* So, if a four-year enlistment in the Infantry gets a $16K EB, so do all the Combat Engineers, Aviators, Field Artillery, etc. Also, the penalties for each category apply equally to all MOSs within the category. These limitations were necessary in order to use the data from the pilot conjoint analysis study.

- *The influence of the recruiter and guidance counselor is not represented.* The model only considers the effect of the possible incentives. Other factors that could influence MOS selection are not represented.

4.4 Implementation Software

To implement the model, we chose AMPL [Fourer] as the algebraic modeling language. This allowed us to specify the linear program in a compact algebraic form. This model, along with a data file, serves as the input to the CPLEX [CPLEX, ILOG Inc.] solver that actually finds the optimal incentives package. More detailed discussions of the model and data files are given in Appendices C and D.

4.5 Model Results

The final solution produced by the model is a package of incentives for which predicted accessions are as close as possible to the targets, that does not exceed the budget allocation for the three bonus programs, and which meets all the legal and logical constraints. The selection of bonus packages also follows a logical process: higher bonus packages are offered for longer lengths of service. Infeasible combinations, such as multiple cash bonuses for a given MOS and a length of service, are never considered as candidate solutions.

5 Assessment

This section discusses the legitimacy and usefulness of this modeling approach. It shows that linking the output from a conjoint analysis into a mixed-integer programming model can produce results that allow decision makers to make informed bonus allocation decisions. However, there are inherent limitations to this approach, including the size of problem that can be solved in a reasonable amount of time, and the level of resolution that reasonably can be built into the model.

5.1 Customer Preference Data

The market share estimates from USAREC's pilot study don't always reflect the sort of choices a rational individual would make.[7] In spite of this, we found that the MIP *would* successfully find an optimal solution with the pilot study inputs; unrealistic probabilities just resulted in solutions that didn't always make logical sense until they were traced back to the source probability distributions.

[7] For example, the probabilities from the conjoint analysis would predict that a potential recruit would rather serve 5 years in the Infantry for a $10,000 bonus that serve 4 years in the same MOS for the same bonus, and that they would prefer to serve in combat arms MOSs than administrative jobs.

5.2 Problem Size and Solution Times

The prototype MIP model currently has 350 binary variables, 14 continuous variables, and approximately 1500 constraints. The total number of possible combinations for the binary variables is:

$$\binom{4}{1}^{35} * \binom{4}{1}^{35} * \binom{4}{1}^{35} = 1.65 * 10^{63}$$

where we select from one of four possible levels (including "none") for EB, ACF, and Other Incentives, and must perform this selection for each of the 35 possible combinations of MOS/TOS.

This is a large solution space, but the CPLEX solver, employing a branch-and-bound strategy, proved able to find optimal or good solutions in a reasonable amount of time. Running on a P-90 computer, the solution times varied from under two minutes up to over three hours, depending on the initial parameter values for a run. By carefully setting the stopping conditions[8], we found we could usually find a near-optimal solution in under 30 minutes.

If the size of the model is increased to include multiple time periods, more MOS categories, and other incentive effects, the solution times can be expected to increase exponentially. It is important, then, to ensure that any variables added to the problem can be expected to contribute to obtaining significantly better solutions, from an operational point of view. Likewise, it would be helpful to identify MOSs that require no incentives (those that traditionally have been easy to fill) and to consider these as candidates to be removed from the model. If, however, solution times still become excessive, it may be possible to employ other solution techniques such as tabu-search or genetic algorithms.

5.3 Modeling Environment

The AMPL modeling environment has both advantages and disadvantages. It allows good separation of model from problem data. The algebraic model is stored in one file

[8] See Appendix D for a discussion of stopping conditions.

while the data that represents various instances of the problem are stored in separate text files. This is advantageous because multiple data files can be easily created and modified without changing the model, and likewise, it is possible to run different versions of the model on the same data files.

Another advantage of AMPL is that, since it is an algebraic modeling language, it compactly represents large mathematical programs. An entire "family of constraints" can be generated by one algebraic statement. In this problem, for example, the AMPL statement:

```
subject to Turn_Off_EB {M in MOS, T in TOS, B_EB in bonus_EB}: Offer_EB[M,T,B_EB] <=
Shut_Down_Package_EB[M,T,B_EB];
```

actually represents 140 constraints that check to see if each MOS/TOS/EB combination is turned on (allowed) or turned off (not allowed).

Several different solvers can be used with AMPL. Our implementation uses CPLEX, one of the leading high-end solvers for linear problems. If, however, the problem is revised so that it becomes non-linear, a solver that handles non-linear might be used.

There are also several *disadvantages* to AMPL. First and foremost, it is very syntax intensive, so the user must have a good grasp of the language before they can successfully understand or write any of the modeling code. Related to this, we found the AMPL documentation to be inadequate. There is an AMPL student textbook [Fourer] that explains the syntax of the language, and steps through developing relatively simple models, but very little documentation comes with the application, and we had to make multiple calls to the vendor to help us solve problems that ought to have been discussed in a user's manual or reference guide. The installation procedures were also troublesome and required many calls to the vendor before we could get it up and running.

Another of AMPL's limitations is that it is difficult to create a good, customizable user interface. The default output can either be viewed on the screen or else directed to a file, but parsing out the pertinent data from the solution and presenting it to the user in a concise, easy to understand format is difficult. We ended up having to write a significant amount of Visual Basic code in order to generate summary reports in MS Excel.

In addition to the other drawbacks, AMPL and CPLEX are also fairly expensive. The academic/research version cost $1,250 for AMPL and $650 for the CPLEX solver [Compass Modeling Solutions]. The production version, which will be required if the prototype model is developed and deployed as a working model, costs $8,500 per copy.

5.4 Level of Resolution

The MOS categories in our prototype model are large and may exclude combinations that could result in better answers. For example if one needed 100 19D's, one would have to offer all enlistees selecting the combat arms a bonus. Based on limited budgets, this is not a feasible option. By breaking down MOS categories to actual MOS series (11, 12, 13, 19, etc.) and possibly the actual MOS's (11B, 11M, 11C) the model might find a better solution that comes closer to meeting USAREC targets. This refinement of MOS visibility will increase the number of variables from 250+ to well over 1000. Certainly this will increase the effort required in the conjoint analysis and the amount of time required to find an optimal solution, but the amount of detail gained could be well worth the additional solution time. Once again, eliminating MOSs that do not require incentives could help offset this breakout of the priority MOSs.

It would be helpful to provide a higher level of resolution by using more categories in the conjoint analysis. In particular, the conjoint analysis should solicit preferences concerning at least the larger priority MOSs. Although this would increase the size of both the conjoint analysis study and the optimization model, the propensities of USAREC's customers could be modeled more realistically. In addition, a higher resolution model would allow more flexibility in forming the incentive packages, so is likely to determine a solution that is better than one produced by the aggregated model.

5.5 Sensitivity Analysis

The binary nature of the decision variables makes it more difficult to perform sensitivity analysis testing. However, one way to generate sensitivity analyses is to solve the problem numerous times with different values for the model parameters or input values. We selected key model parameters, raised and lowered their values slightly, and compared the resulting solutions. Our findings are summarized in the following table:

Parameter	**Percent Change**	**Results**
MOS Category Weights	± 5%	In general, the model is fairly robust against changes in the MOS category weights. In most cases, the 5% change in Overweight or Underweight values resulted in only small changes in the resulting bonus packages. (Stopping time was 30 minutes). The exception to this was under conditions where the baseline case very quickly found an excellent integer solution. In these instances, the 5% change resulted in a significantly different (inferior) bonus recommendations. This is evidence that "good" solutions are path dependent.

6 Future Research

There are several areas of future research that could grow out of this problem. They include:

- Studying the costs and benefits of including finer divisions in the MOS categories.
- Alternative modeling approaches might be considered, including:
  - use of optimization capabilities in large-scale spreadsheets [*Excel Very Large Scale Solver*];
  - genetic algorithms
- The interface between the user, AMPL, and CPLEX is unfriendly. Given their ease of use and general familiarity with spreadsheets (especially Excel), consideration should be given to formulating this model in Excel developing an interface to a large-scale solver that could solve the integer program. This action could result in a user-friendly model that would be able to generate sensitivity analysis reports.

- It would be useful to examine alternatives to AMPL for optimization. We currently have a cadet group evaluating a limited set of alternatives, as part of a selected topics course at USMA.

7 Conclusions

Market share estimates can be extracted from conjoint analysis data. The prototype mixed integer programming model demonstrates that these market share predictions can be used in an optimization model to determine a set of bonuses that will best contribute to meeting recruiting, budget, and legal constraints. Since the optimization model is limited by the scope and organization of the conjoint analysis, a large-scale study that extends the results of the pilot conjoint study is needed. Future conjoint analyses must be carefully planned, because the optimization model will only have the same level of resolution as the CBC.

Our exercises of the prototype model demonstrate that, depending of the number of MOS categories included, the model can find a near-optimal solution within a few minutes on a modest PC. We believe these results demonstrate the approach we propose is sound, and future extensions are warranted.

Appendix A: Stakeholder Needs

Key players on the Incentives Review Board, representing their respective agencies, articulated the following needs for the EB allocation model:

USAREC

- *A scientific approach for allocating the EB budget*
- *A tool for the efficient and effective allocation of EB incentives, particularly for the Priority MOSs*
- *A means of improving the channeling effect of the EB*
- *A business decision support tool for the Incentives Review Board*
- *A tool that will help us use Army money efficiently*
- *A joint understanding of EB options and trade-offs by USAREC, PERSCOM, and DCSPER*
- *A better understanding of the EB - Preference - Recruiting dynamics*

DCSPER:

- *Determine the appropriate EB budget for a given mission*
- *Get the max effect possible from whatever EB money is allocated*
- *Provide the capability for flexible "what-if" analysis*

PERSCOM:

- *Use the EB to help get the right soldiers into the right MOSs*
- *Be able to determine when bonuses are no longer needed*

Appendix B: Developing Customer Preference Distributions

In this appendix, we discuss several aspects of logistic regression, based on statistical textbooks on the subject [Agretsi; Hosmer and Lemeshow]. We also establish the basis of estimates of preference probabilities used in the optimization examples presented in this report, and give a brief description of the data involved.

Choice based conjoint analysis is based on polling data [Curry]. Each respondent is presented with numerous frames; in each, several product choices are presented. The respondent chooses which product is "best" in each frame. In the case of the US Recruiting Command poll [Gale et al., 8], the "products" consisted of Army enlistment choices defined in terms of three attributes: Military Occupational Specialty (MOS, at seven levels), incentives (at nine levels) and Length of Service (TOS, at five levels). Thus, for the USAREC study, up to 7 X 9 X 5 = 315 products were possible; each frame presented three different products, plus a "none of these" option. Each respondent made a choice of product in each of a total of 20 frames.

According to the University of Louisville report on the polling and analysis efforts for the USAREC study [Gale et al., 6], software provided by Sawtooth Software, Inc. was used with the data collected from approximately 500 respondents, contacted in shopping malls located in five different areas of the country. Three types of analytical results were extracted from the data: an analysis based on the relative frequency of times products with each attribute level was selected (called "frequency data"), logistic regression, and "market simulations." The frequency data estimate "the relative impact of each attribute level ... by counting 'wins.' The impact of each level can be assessed by counting the proportion of times concepts [products] including it are chosen" [*The CBC System...*]. The logistic regression, called "multinomial logit estimation" in *The CBC System*, applies a categorical analysis method involving multivariate regression, to estimate coefficients that are used in turn to estimate response probabilities, called "utilities." The "market simulations" use the fitted logistic regression model to predict the relative odds of choice among a set of competing products. This is similar to using a fitted regression model to predict responses with given values of the independent variables.

The precise details of Sawtooth's "multinomial logit" analysis and the exact form of the data input to the procedure are not made clear in the documents we reviewed. We surmise the general idea is as follows. Suppose values of the dependent variable, Y, denote whether or not an individual agrees to join the Army under specific conditions of MOS, incentive and TOS. Y=1 for an individual means the individual indicates he or she would join under the conditions given and Y=0 indicates he or she would not accept the "product" offered. Suppose, over a target population, the probability an individual will agree to join is p. The scale [0,1] for p is transformed to the scale $(-\infty,\infty)$, appropriate for a regression variable, using the "logit" transformation: $\text{logit}(p) = \log(p / (1 - p))$. The ratio p/(1-p) is known as the odds in favor of the positive response, Y = 1, so logit(p) is "log-odds" of a positive response. The probability of a positive response depends upon the product offered. That is, the value of p (and hence logit(p)) depends upon the levels of the attributes (MOS, incentive and TOS) offered; these attributes constitute independent variables, say x_1, x_2, and x_3. Strictly speaking, MOS and incentive are nominal-scale variables, so a system of dummy variables is used to generate levels of these variables. For example, the seven nominal levels of MOS can be represented by six dummy variables having values 0 and 1. TOS is a ratio-scale variable and can be entered into the model directly. We will ignore dummy variables in the present discussion, and proceed as if all the variables are ratio-scale. (See [Hosmer and Lemeshow] for details.)

It is assumed logit(p) depends on these independent variables through a linear relationship: $\text{logit}(p) = b_0 + b_1x_1 + b_2x_2 + b_3x_3 = \Sigma\, b_ix_i$, where $x_0=1$. In addition, two-way interaction terms of the form $b_sx_jx_k$ and similar terms for higher level interactions can be included in the linear model. The logistic regression process uses statistical methods with the polling data to estimate the coefficients, b_i. It is common practice to test whether each of the b_i are zero. If such a hypothesis cannot be rejected, the corresponding term is deleted from the model. Once the significant (i.e., non-zero) coefficients are estimated, any feasible set of independent variable values may be substituted into the equation, resulting in the corresponding estimate of log-odds of a positive response. The odds in favor of a positive response at these values of the x_i's is thus given by exponentiating the value obtained from the linear estimating equation. It appears these are the "utilities" described in the Sawtooth Software and University of Louisville documents we reviewed. However, in order to obtain the probability of positive response (for the given product), it is necessary to convert odds to probabilities. The resulting p(x) is given by,

$$p(x) = \frac{e^{\sum b_i x_i}}{1 + e^{\sum b_i x_i}}$$

To estimate the fraction of a population that would chose each of a set of competing products, assuming all products are available to each member of the population, the positive response probability (or "utility") of each is expressed as a percent of the sum of utilities over all the products [*The CBC System...*]. Thus if there were 3 products available, and logistic regression and use of the above equation gave utilities .460, .743, and .341, the estimated fractions of the population that would select the three products are, respectively, 30%, 48% and 22%. For example, .460 / (.460 + .743 + .341) = .300.[9]

The Sawtooth software documentation [*The CBC System* ...] claims essentially the same results as those derived from the "logit analysis" can be obtained from frequency data. For the latter application, the count fractions are also normalized and expressed as fractions of the total. This is especially easy to carry out when there are no significant interactions, as was the case for the USAREC study [Gale et al., 29-30]. Frequency data for the high propensity sample in the USAREC study are given in [Gale, *et. al.*, 50]. The seven MOS levels included in the study received frequency .266, .288, .207, .240, .220, .218, and .235. The sum of these fractions is 1.674, so the utility for the first MOS value is estimated by .266 / 1.674 = .159. In a similar way the marginal utilities for the remaining MOS levels are obtained, as well as those for the five levels of TOS and the nine levels of incentive. Since there are assumed to be no interactions, the joint utility for a given MOS, TOS and incentive is estimated by the product of the marginal values. For example, for a choice with MOS = Medical, TOS = 2 years and

[9] Examples shown in [*The CBC System*, page 15] appear to indicate Sawtooth Software uses log-odds as utilities. This is done by exponentiating $b_i x_i$. These values are then expressed as fractions of the sum

$20,000 Army College Fund, the joint utility is estimated by .159 x .201 x .125 = .004. Note that multiplying the frequencies corresponds to exponentiating the additive logistic regression model, that is,

$$e^{\sum b_i x_i} = \prod e^{b_i x_i}$$

If a set of alternative "products" are presented to the population in question (strictly speaking, the population sampled in the USAREC polling process), the fractions choosing each can be estimated by normalizing its estimated utility [Hu]. That is, sum the joint utilities over the products in question, then take the ratio of each product's utility to this total. This was the procedure followed, using the data in [Gale, *et. al.*, 50] for determining the fractions used as coefficients in the integer program described below.

If a logistic regression were used to estimate coefficients in the linear model, possibly including interaction terms, then a similar process could be followed. The utility, p, for each product in an offered set of products can be calculated as described above. Then the fraction of the population choosing each product, constrained to choices form the offered set, is estimated by the ratio of the utility to the total of utilities over the set. Note this process avoids multiplying marginal utilities, as was done with the frequency estimates, so it is easily applicable when there are significant interaction terms.

---

over all products presented for choice, as before. It appears to the authors the further transformation of log odds to positive response probabilities should be applied, as described above.

Appendix C: Mixed Integer Programming Model

This appendix describes, in a fair amount of detail, the prototype Mixed-Integer Goal Programming Model that we developed to study this problem. The model is kept as abstract as possible, so that it can be implemented in various modeling languages. The actual AMPL implementation is explained in Appendix D.

Model Description:

The Enlisted Bonus Distribution Model (EB Model) is a multi-objective (also known as goal programming) mixed integer programming problem. The majority of the decision variables are binary variables (350 of 364 variables). The remaining 14 variables are any non-negative real number. The model seeks to minimize the sum of weighted penalties associated with failing to meet USAREC recruiting targets for each MOS category while simultaneously ensuring that all problem constraints are satisfied. The constraints in this model (over 1000) seek to meet USAREC policy and force a "reasonable" sense of logic. We made several assumptions about the recruiting incentives in order to make this problem tractable. The primary assumptions affecting this model are:

- *All incentives are paid out within one fiscal year.*
- *There is a 100% payout on bonuses..*
- *An equivalent lump-sum value can be calculated for each of the "other" non-EB, non-college fund incentives.*
- *Proportionality holds for the objective function variables and coefficients*
- *The MOS categories are homogenous*
- *The influence of the recruiter and guidance counselor is not represented*
- *The incentive packages are offered for the entire fiscal year.*

Each assumption will be discussed in greater detail as it comes up in the body of this appendix. The AMPL implementation of the EB model can be found in Appendix D.

Decision Variables:

The binary decision variables are switches that represent whether an incentive package is offered or not. An incentive package is defined as a unique combination of an MOS category, a specific length of service, and a particular bonus. For example, the "*Combat Arms*" MOS category, for a four year length of service, with a $16,000 bonus option is one distinct incentive package, while the "*Combat Arms*" MOS category, for a four year length of service, with a $10,000 bonus is another potential incentive package. If the decision variable for an incentive package is equal to one (1), the incentive package should be offered, if it is equal to zero (0), the incentive package should not be offered. In this model, there are three types of incentives: a cash enlistment bonus, a college fund payment, and an "other" category. A list of specific bonuses within each incentive program is provided with the variable descriptions. The following binary decision variables were used in this prototype:

Cash Bonus Program

$$X_{i,j,k} = \begin{cases} 1 & \text{if cash bonus package for MOS } i\text{, length of service } j\text{,} \\ & \text{and cash bonus amount } k \text{ is offered} \\ 0 & \text{if cash bonus package for MOS } i\text{, length of service } j\text{,} \\ & \text{and cash bonus amount } k \text{ is not offered} \end{cases}$$

where $i = 1$ (Medical Jobs), 2 (Military Intelligence), 3 (Combat Arms), 4 (Administrative Jobs), 5 (Electronic Repair), 6 (Engineering), and 7 (Maintenance)

$j = 2$ (2 year length of service), 3 (3 year length of service), 4 (4 year length of service), 5 (5 year length of service), and 6 (6 year length of service)

$k = 0$ ($0 cash bonus), 4 ($4,000 cash bonus), 10 ($10,000 cash bonus), and 16 ($16,000 cash bonus)

In this formulation, there are 140 possible cash bonus alternatives.

Army College Fund Program

$$Y_{i,j,m} = \begin{cases} 1 & \text{if Army College Fund for MOS } i \text{, length of service } j, \\ & \text{and fund amount } m \text{ is offered} \\ 0 & \text{if Army College Fund for MOS } i \text{, length of service } j, \\ & \text{and fund amount } m \text{ is not offered} \end{cases}$$

where i = 1 (Medical Jobs), 2 (Military Intelligence), 3 (Combat Arms), 4 (Administrative Jobs), 5 (Electronic Repair), 6 (Engineering), and 7 (Maintenance)

j = 2 (2 year length of service), 3 (3 year length of service), 4 (4 year length of service), 5 (5 year length of service), and 6 (6 year length of service)

m = 20 ($20,000 Army college fund), 40 ($40,000 Army college fund), and 60 ($60,000 Army college fund)

In this formulation, there are 105 possible ACF alternatives.

"Other" Incentives Program

$$Z_{i,j,n} = \begin{cases} 1 & \text{if "Other" incentive for MOS } i \text{, length of service } j, \\ & \text{and incentive type } n \text{ is offered} \\ 0 & \text{if "Other" incentive for MOS } i \text{, length of service } j, \\ & \text{and incentive type } n \text{ is not offered} \end{cases}$$

where i = 1 (Medical Jobs), 2 (Military Intelligence), 3 (Combat Arms), 4 (Administrative Jobs), 5 (Electronic Repair), 6 (Engineering), and 7 (Maintenance)

j = 2 (2 year length of service), 3 (3 year length of service), 4 (4 year length of service), 5 (5 year length of service), and 6 (6 year length of service)

n = L (college loan paid), and U (Select a unit or location)

In this formulation, there are 70 possible ACF alternatives.

In addition to the binary variables, there are fourteen other variables. These are the only variables in the objective function and represent by how much a specific USAREC enlistment target was missed. They are represented by:

O_i = amount by which enlistment target for MOS i was exceeded

U_i = amount by which enlistment target for MOS i was short

where $i = 1$ (Medical Jobs), 2 (Military Intelligence), 3 (Combat Arms), 4 (Administrative Jobs), 5 (Electronic Repair), 6 (Engineering), and 7 (Maintenance)

Objective Function:

The objective function for this model seeks to minimize the sum of all penalties associated with exceeding or failing to meet the specified USAREC targets for each MOS. The penalty weights express the relative importance of the MOSs that are to be considered in the model. Their values must be determined in advance and become model parameters. The more critical it is to meet an MOS, the higher the associated penalty should be. Weights for underachieving or overachieving a target for a particular MOS do not have to be the same. For instance it is possible to associate a weight of 50 for falling short of the specified target for the Combat Arms MOS and a weight of 100 for exceeding the target in Combat Arms. The default value for the weights is 100. The objective function for the EB Model is:

$$\textbf{MIN} \quad U_1*UW_1 + O_1*OW_1 + U_2*UW_2 + O_2*OW_2 + U_3*UW_3 + O_3*OW_3 + U_4*UW_4 + O_4*OW_4 + U_5*UW_5 + O_5*OW_5 + U_6*UW_6 + O_6*OW_6 + U_7*UW_7 + O_7*OW_7$$

where $i = 1$ (Medical Jobs), 2 (Military Intelligence), 3 (Combat Arms), 4 (Administrative Jobs), 5 (Electronic Repair), 6 (Engineering), and 7 (Maintenance)

UW_i = the penalty associated with falling short of the target for MOS i

OW_i = the penalty associated with exceeding the target for MOS i

The primary assumption that affects the objective function is that proportionality constraints hold for the objective function. Specifically, the penalty for overfilling or underfilling a particular MOS category is a *linear* function based on the deviation from the target. The objective function could be modified to approximate a nonlinear penalty function, but in order for the problem to remain linear, it would have to be modified to be piecewise linear. This modification to the objective function would cause an increase to the number of constraints, decision variables, and solution time.

Set of Constraints:

There are six families of constraints in the EB model that define the feasible region. The coefficients for all constraints are stored in the data file that supports the EB model.

Target Constraints:

The first family of constraints determines how many enlistees will be assessed into each MOS category. These are actually soft constraints that represent the "goals" in this model. The constraints are formulated so that they allow a pre-specified margin of error before any penalty is applied. By default, this margin of error is set to ± 2% of the target for each MOS category. There are two constraints for each MOS, one for determining how far we fall short of the allowable margin of error, the other for determining how far we exceed the allowable margin of error. In most goal programming problems, both of these functions can be determined with one constraint, but the introduction of the 2% margin of error requires the use of two constraints. At any given time only one constraint will be binding in each of these pairs.

The pre-specified margin of error is called the "Leeway" in the model. The underlying principle is the EB model does not accrue a penalties as long as the numbers assessed remains close to the USAREC targets. For example, if Leeway = 2% and the target for the Combat Arms MOS Category is 8,315 enlistees, then the EB Model will not begin accruing a penalty unless the number of enlistees accessed into the Combat Arms is less than

$$8{,}513*(1-.02) = 8{,}342.74 \text{ enlistees}$$

or greater than

$$8{,}513*(1+.02) = 8{,}683.26 \text{ enlistees.}$$

While it is impossible to recruit a fraction of an enlistee, these real values were allowed in order to maintain linearity in the model. The impact on the overall solution is negligible since the penalty associated with a fraction of a person is tiny compared to the overall penalties that accrue across the MOS categories.

We developed the overachieving goal constraints for the Combat Arms MOS as follows:

- Let **A** = the *i* by *j* by *k* matrix of marginal market share captured by incentive packages that include enlistment bonuses[10]
- Let **B** = the *i* by *j* by *m* matrix of marginal market share captured by incentive packages that include the Army College Fund incentives
- Let **C** = the *i* by *j* by *n* matrix of marginal market share captured by incentive packages that include the "other" incentives
- Let **P** = the population of propensed enlistment candidates

A tentative formulation is:

(The Sum of Accessions due to the enlistment bonus program over all lengths of service)

$$\begin{array}{llll}
P*A_{3,2,0}*X_{3,2,0} & + P*A_{3,2,4}*X_{3,2,4} & + P*A_{3,2,10}*X_{3,2,10} & + P*A_{3,2,16}*X_{3,2,16} \\
+ P*A_{3,3,0}*X_{3,3,0} & + P*A_{3,3,4}*X_{3,3,4} & + P*A_{3,3,10}*X_{3,3,10} & + P*A_{3,3,16}*X_{3,3,16} \\
+ P*A_{3,4,0}*X_{3,4,0} & + P*A_{3,4,4}*X_{3,4,4} & + P*A_{3,4,10}*X_{3,4,10} & + P*A_{3,4,16}*X_{3,4,16} \\
+ P*A_{3,5,0}*X_{3,5,0} & + P*A_{3,5,4}*X_{3,5,4} & + P*A_{3,5,10}*X_{3,5,10} & + P*A_{3,5,16}*X_{3,5,16} \\
+ P*A_{3,6,0}*X_{3,6,0} & + P*A_{3,6,4}*X_{3,6,4} & + P*A_{3,6,10}*X_{3,6,10} & + P*A_{3,6,16}*X_{3,6,16}
\end{array}$$

(plus the Sum of Accessions due to the ACF program over all lengths of service)

$$\begin{array}{lll}
+ P*B_{3,2,20}*Y_{3,2,20} & + P*B_{3,2,40}*Y_{3,2,40} & + P*B_{3,2,60}*Y_{3,2,60} \\
+ P*B_{3,3,20}*Y_{3,3,20} & + P*B_{3,3,40}*Y_{3,3,40} & + P*B_{3,3,60}*Y_{3,3,60} \\
+ P*B_{3,4,20}*Y_{3,4,20} & + P*B_{3,4,40}*Y_{3,4,40} & + P*B_{3,4,60}*Y_{3,4,60} \\
+ P*B_{3,5,20}*Y_{3,5,20} & + P*B_{3,5,40}*Y_{3,5,40} & + P*B_{3,5,60}*Y_{3,5,60} \\
+ P*B_{3,6,20}*Y_{3,6,20} & + P*B_{3,6,40}*Y_{3,6,40} & + P*B_{3,6,60}*Y_{3,6,60}
\end{array}$$

(plus the Sum of Accessions due to the "Other" incentives program over all lengths of service)

$$+ P*C_{3,2,L}*Z_{3,2,L} \quad + P*C_{3,2,U}*Z_{3,2,U} \quad + P*C_{3,2,G}*Z_{3,2,G}$$

---

[10] The matrices are developed from the conjoint analysis data as demonstrated in Appendix B, and the predicted market shares can be seen in their entirety in Appendix D.

$$
\begin{array}{lll}
+ P^*C_{3,3,L}{}^*Z_{3,3,L} & + P^*C_{3,3,U}{}^*Z_{3,3,U} & + P^*C_{3,3,G}{}^*Z_{3,3,G} \\
+ P^*C_{3,4,L}{}^*Z_{3,4,L} & + P^*C_{3,4,U}{}^*Z_{3,4,U} & + P^*C_{3,4,G}{}^*Z_{3,4,G} \\
+ P^*C_{3,5,L}{}^*Z_{3,5,L} & + P^*C_{3,5,U}{}^*Z_{3,5,U} & + P^*C_{3,5,G}{}^*Z_{3,5,G} \\
+ P^*C_{3,6,L}{}^*Z_{3,6,L} & + P^*C_{3,6,U}{}^*Z_{3,6,U} & + P^*C_{3,6,G}{}^*Z_{3,6,G}
\end{array}
$$

is less than or equal to the upper bound on Combat arms enlistments

$$<= 8{,}513^*(1+\text{Leeway})$$

Although this formulation is a good start, only a subset of these probabilities will be active at any given time, so the probabilities that are "turned on" must be normalized [*The CBC System* ...]. Therefore each $A_{i,j,k}$, $B_{i,j,m}$, and $C_{i,j,n}$ term must be divided by the sum of all probabilities of *offered* incentive packages. This, however, would cause the constraint to be non-linear. But, if we multiply both the RHS and LHS of the constraint through by this same quantity, we remove the denominator from the LHS, introduce a factor to the RHS, and the problem remains linear.

- let Normalize = $\sum\sum\sum A_{i,j,k} X_{i,j,k} + \sum\sum\sum B_{i,j,m} Y_{i,j,m} + \sum\sum\sum C_{i,j,n} Z_{i,j,n}$

Note that the variable Normalize represents the sum of active probabilities across *all* MOSs.

So, the overachieving constraint for the combat arms becomes:

$$
\begin{array}{llll}
 P^*A_{3,2,0}{}^*X_{3,2,0} & + P^*A_{3,2,4}{}^*X_{3,2,4} & + P^*A_{3,2,10}{}^*X_{3,2,10} & + P^*A_{3,2,16}{}^*X_{3,2,16} \\
+ P^*A_{3,3,0}{}^*X_{3,3,0} & + P^*A_{3,3,4}{}^*X_{3,3,4} & + P^*A_{3,3,10}{}^*X_{3,3,10} & + P^*A_{3,3,16}{}^*X_{3,3,16} \\
+ P^*A_{3,4,0}{}^*X_{3,4,0} & + P^*A_{3,4,4}{}^*X_{3,4,4} & + P^*A_{3,4,10}{}^*X_{3,4,10} & + P^*A_{3,4,16}{}^*X_{3,4,16} \\
+ P^*A_{3,5,0}{}^*X_{3,5,0} & + P^*A_{3,5,4}{}^*X_{3,5,4} & + P^*A_{3,5,10}{}^*X_{3,5,10} & + P^*A_{3,5,16}{}^*X_{3,5,16} \\
+ P^*A_{3,6,0}{}^*X_{3,6,0} & + P^*A_{3,6,4}{}^*X_{3,6,4} & + P^*A_{3,6,10}{}^*X_{3,6,10} & + P^*A_{3,6,16}{}^*X_{3,6,16} \\
+ P^*B_{3,2,20}{}^*Y_{3,2,20} & + P^*B_{3,2,40}{}^*Y_{3,2,40} & + P^*B_{3,2,60}{}^*Y_{3,2,60} & \\
+ P^*B_{3,3,20}{}^*Y_{3,3,20} & + P^*B_{3,3,40}{}^*Y_{3,3,40} & + P^*B_{3,3,60}{}^*Y_{3,3,60} & \\
+ P^*B_{3,4,20}{}^*Y_{3,4,20} & + P^*B_{3,4,40}{}^*Y_{3,4,40} & + P^*B_{3,4,60}{}^*Y_{3,4,60} & \\
+ P^*B_{3,5,20}{}^*Y_{3,5,20} & + P^*B_{3,5,40}{}^*Y_{3,5,40} & + P^*B_{3,5,60}{}^*Y_{3,5,60} & \\
+ P^*B_{3,6,20}{}^*Y_{3,6,20} & + P^*B_{3,6,40}{}^*Y_{3,6,40} & + P^*B_{3,6,60}{}^*Y_{3,6,60} & \\
+ P^*C_{3,2,L}{}^*Z_{3,2,L} & + P^*C_{3,2,U}{}^*Z_{3,2,U} & + P^*C_{3,2,G}{}^*Z_{3,2,G} & \\
+ P^*C_{3,3,L}{}^*Z_{3,3,L} & + P^*C_{3,3,U}{}^*Z_{3,3,U} & + P^*C_{3,3,G}{}^*Z_{3,3,G} & \\
+ P^*C_{3,4,L}{}^*Z_{3,4,L} & + P^*C_{3,4,U}{}^*Z_{3,4,U} & + P^*C_{3,4,G}{}^*Z_{3,4,G} & \\
+ P^*C_{3,5,L}{}^*Z_{3,5,L} & + P^*C_{3,5,U}{}^*Z_{3,5,U} & + P^*C_{3,5,G}{}^*Z_{3,5,G} & \\
+ P^*C_{3,6,L}{}^*Z_{3,6,L} & + P^*C_{3,6,U}{}^*Z_{3,6,U} & + P^*C_{3,6,G}{}^*Z_{3,6,G} & \\
 & <= 8{,}513 * (1+\text{Leeway}) * \text{Normalize} & &
\end{array}
$$

We still must include the overachieving variable (O_3) for this problem in order to tie it in with the objective function. The final formulation of the overachieving constraint for the combat arms follows:

$$
\begin{array}{llll}
P*A_{3,2,0}*X_{3,2,0} & + P*A_{3,2,4}*X_{3,2,4} & + P*A_{3,2,10}*X_{3,2,10} & + P*A_{3,2,16}*X_{3,2,16} \\
+ P*A_{3,3,0}*X_{3,3,0} & + P*A_{3,3,4}*X_{3,3,4} & + P*A_{3,3,10}*X_{3,3,10} & + P*A_{3,3,16}*X_{3,3,16} \\
+ P*A_{3,4,0}*X_{3,4,0} & + P*A_{3,4,4}*X_{3,4,4} & + P*A_{3,4,10}*X_{3,4,10} & + P*A_{3,4,16}*X_{3,4,16} \\
+ P*A_{3,5,0}*X_{3,5,0} & + P*A_{3,5,4}*X_{3,5,4} & + P*A_{3,5,10}*X_{3,5,10} & + P*A_{3,5,16}*X_{3,5,16} \\
+ P*A_{3,6,0}*X_{3,6,0} & + P*A_{3,6,4}*X_{3,6,4} & + P*A_{3,6,10}*X_{3,6,10} & + P*A_{3,6,16}*X_{3,6,16} \\
+ P*B_{3,2,20}*Y_{3,2,20} & + P*B_{3,2,40}*Y_{3,2,40} & + P*B_{3,2,60}*Y_{3,2,60} & \\
+ P*B_{3,3,20}*Y_{3,3,20} & + P*B_{3,3,40}*Y_{3,3,40} & + P*B_{3,3,60}*Y_{3,3,60} & \\
+ P*B_{3,4,20}*Y_{3,4,20} & + P*B_{3,4,40}*Y_{3,4,40} & + P*B_{3,4,60}*Y_{3,4,60} & \\
+ P*B_{3,5,20}*Y_{3,5,20} & + P*B_{3,5,40}*Y_{3,5,40} & + P*B_{3,5,60}*Y_{3,5,60} & \\
+ P*B_{3,6,20}*Y_{3,6,20} & + P*B_{3,6,40}*Y_{3,6,40} & + P*B_{3,6,60}*Y_{3,6,60} & \\
+ P*C_{3,2,L}*Z_{3,2,L} & + P*C_{3,2,U}*Z_{3,2,U} & + P*C_{3,2,G}*Z_{3,2,G} & \\
+ P*C_{3,3,L}*Z_{3,3,L} & + P*C_{3,3,U}*Z_{3,3,U} & + P*C_{3,3,G}*Z_{3,3,G} & \\
+ P*C_{3,4,L}*Z_{3,4,L} & + P*C_{3,4,U}*Z_{3,4,U} & + P*C_{3,4,G}*Z_{3,4,G} & \\
+ P*C_{3,5,L}*Z_{3,5,L} & + P*C_{3,5,U}*Z_{3,5,U} & + P*C_{3,5,G}*Z_{3,5,G} & \\
+ P*C_{3,6,L}*Z_{3,6,L} & + P*C_{3,6,U}*Z_{3,6,U} & + P*C_{3,6,G}*Z_{3,6,G} & - O_3 \\
 & <= 8{,}513*(1+\text{Leeway})*\text{Normalize} & &
\end{array}
$$

Or, equivalently:

$$P * \sum_j \left(\sum_k A_{3,j,k} X_{3,j,k} + \sum_m B_{3,j,m} Y_{3,j,m} + \sum_n C_{3,j,n} Z_{3,j,n} \right) - O_3 \le 8{,}513 * (1 + Leeway) * Normalize$$

Note that O_3 is not multiplied by the scalar Normalize. This is because, as a scalar, it proportionally decreases all values of O_i and U_i in the objective function. The true value of the objective function could be calculated by dividing the optimal solution objective function value by Normalize. There is no need to do this, however, because we are not actually interested in the objective function value. It only represents the penalty applied for failing to meet the specified targets. We are really interested in which incentive packages to offer.

The underachieving constraint for the Combat Arms MOS is presented below and was formulated the same way as the overachieving variable:

$$
\begin{array}{llll}
P*A_{3,2,0}*X_{3,2,0} & + P*A_{3,2,4}*X_{3,2,4} & + P*A_{3,2,10}*X_{3,2,10} & + P*A_{3,2,16}*X_{3,2,16} \\
+ P*A_{3,3,0}*X_{3,3,0} & + P*A_{3,3,4}*X_{3,3,4} & + P*A_{3,3,10}*X_{3,3,10} & + P*A_{3,3,16}*X_{3,3,16} \\
+ P*A_{3,4,0}*X_{3,4,0} & + P*A_{3,4,4}*X_{3,4,4} & + P*A_{3,4,10}*X_{3,4,10} & + P*A_{3,4,16}*X_{3,4,16} \\
+ P*A_{3,5,0}*X_{3,5,0} & + P*A_{3,5,4}*X_{3,5,4} & + P*A_{3,5,10}*X_{3,5,10} & + P*A_{3,5,16}*X_{3,5,16} \\
+ P*A_{3,6,0}*X_{3,6,0} & + P*A_{3,6,4}*X_{3,6,4} & + P*A_{3,6,10}*X_{3,6,10} & + P*A_{3,6,16}*X_{3,6,16} \\
+ P*B_{3,2,20}*Y_{3,2,20} & + P*B_{3,2,40}*Y_{3,2,40} & + P*B_{3,2,60}*Y_{3,2,60} & \\
+ P*B_{3,3,20}*Y_{3,3,20} & + P*B_{3,3,40}*Y_{3,3,40} & + P*B_{3,3,60}*Y_{3,3,60} & \\
+ P*B_{3,4,20}*Y_{3,4,20} & + P*B_{3,4,40}*Y_{3,4,40} & + P*B_{3,4,60}*Y_{3,4,60} & \\
+ P*B_{3,5,20}*Y_{3,5,20} & + P*B_{3,5,40}*Y_{3,5,40} & + P*B_{3,5,60}*Y_{3,5,60} & \\
+ P*B_{3,6,20}*Y_{3,6,20} & + P*B_{3,6,40}*Y_{3,6,40} & + P*B_{3,6,60}*Y_{3,6,60} & \\
+ P*C_{3,2,L}*Z_{3,2,L} & + P*C_{3,2,U}*Z_{3,2,U} & + P*C_{3,2,G}*Z_{3,2,G} & \\
+ P*C_{3,3,L}*Z_{3,3,L} & + P*C_{3,3,U}*Z_{3,3,U} & + P*C_{3,3,G}*Z_{3,3,G} & \\
+ P*C_{3,4,L}*Z_{3,4,L} & + P*C_{3,4,U}*Z_{3,4,U} & + P*C_{3,4,G}*Z_{3,4,G} & \\
+ P*C_{3,5,L}*Z_{3,5,L} & + P*C_{3,5,U}*Z_{3,5,U} & + P*C_{3,5,G}*Z_{3,5,G} & \\
+ P*C_{3,6,L}*Z_{3,6,L} & + P*C_{3,6,U}*Z_{3,6,U} & + P*C_{3,6,G}*Z_{3,6,G} & + U3 \\
 & & \multicolumn{2}{r}{<= 8{,}513 * (1\text{-Leeway}) * \text{Normalize}}
\end{array}
$$

The six remaining underachieving constraints are similarly formulated.

Three major assumptions are invoked in the development of these constraints:

1. **MOS categories are homogenous**. That is to say, each MOS category contains many different MOSs. This model treats all the MOSs within a given category *exactly the same.* So, if a four-year enlistment in the Infantry gets a $16K EB, so do all the Combat Engineers, Aviators, Field Artillery, etc. Also, the penalties for each category apply equally to all MOSs within the category. These limitations were necessary in order to use the data from the conjoint analysis study. This assumption can be eliminated by developing probability distributions that considers each individual MOS. This will of course come at the expense of more constraints, many more variables, and much longer solution times.

2. **The influence of the recruiter and guidance counselor is not represented**. The probabilities expressed in the matrices A, B, and C reflect marginal market share potential based solely on individual preferences without pressure to join one MOS or another. Historically, however, recruiters and guidance counselors have proven they are very skilled at swaying enlistees into enlistment packages they may not have otherwise selected. This model only considers the effect of the

possible incentives. Other factors that could influence MOS selection are not represented.

3. **The incentive packages are offered for the entire fiscal year.** This model assumes once an incentive package is offered, it will be offered the entire year. This causes some of the overachieving variables in the constraints to become active. In reality, there would always be the option of turning off a particular incentive once the MOS had reached its target or is close to it. This would avoid the problem of being penalized for exceeding an MOS target. This assumption can be eliminated by quadrupling the number of binary decision variables so that they represent quarters instead of annual decisions. This would, of course, add many more constraints, four times the number of variables, and increased solution times.

Single Incentive Package per MOS/TOS Constraints

The second major family of constraints limits no more than one enlistment bonus per MOS category per length of service available. For instance, it will not allow a $4,000 bonus and a $10,000 bonus to be simultaneously offered for a 4 year enlistment in the combat arms. This family of constraints can also be used to prevent any particular enlistment bonus from being offered in a specific MOS category for a specific length of service. The effect of the constraint is a result of its RHS value.

- Let **D** = an *i* by *j* matrix of zeros (0's) and ones (1's) where any $D_{i,j}$ value set to <u>zero</u> means an incentive package for MOS *i* and length of service of *j* years <u>may not be offered</u>, and any $D_{i,j}$ value set to a <u>one</u> means an incentive package for MOS *i* and length of service of *j* years <u>may be offered</u>. The default value for any D term is 1.

The constraint controlling the enlistment bonuses for the Combat Arms MOS for a two year length of service is formulated as follows:

$$X_{3,2,0} + X_{3,2,4} + X_{3,2,10} + X_{3,2,16} = D_{3,2}$$

where $D_{3,2} = 1$ limits the choice to one bonus package and $D_{3,2} = 0$ limits the choice to no bonus packages. In the prototype, there are a total of 35 constraints in this family (one for each combination of MOSs and lengths of service for the enlistment bonus).

The constraint controlling the ACF packages for the Combat Arms MOS for a two year length of service is formulated as follows:

$$X_{3,2,20} + X_{3,2,40} + X_{3,2,60} \leq D_{3,2}$$

where $D_{3,2} = 1$ limits the choice to one ACF package and $D_{3,2} = 0$ limits the choice to no ACF packages. In the prototype there are a total of 35 constraints in this family (one for each combination of MOS and length of service for the ACF).

On/Off Constraints for Individual Bonus Packages

The third family of constraints allows the modeler to include or exclude specific bonus packages in the model. For instance, this constraint checks to see whether or not a \$16,000 enlistment bonus for a two year TOS in an Electronics MOS is an allowable incentive package.

- Let **F** = an *i* by *j* by *k* matrix of zeros (0's) and ones (1's) where any $\mathbf{F}_{i,j,k}$ value set to <u>zero</u> means enlistment bonus package *k* for MOS *i* and length of service of *j* years <u>may not be offered</u>, and any $\mathbf{F}_{i,jk}$ value set to a <u>one</u> means enlistment bonus package *k* for MOS *i* and length of service of *j* years <u>may be offered</u>. The default value for any **F** term is 1.
- Let **G** = an *i* by *j* by *m* matrix of zeros (0's) and ones (1's) where any $\mathbf{G}_{i,j,m}$ value set to <u>zero</u> means ACF package *m* for MOS *i* and length of service of *j* years <u>may not be offered</u>, and any $G_{i,j,m}$ value set to <u>one</u> means ACF package *m* for MOS *i* for length of service of *j* years <u>may be offered</u>. The default value for any **G** term is 1.
- Let **H** = an *i* by *j* by *n* matrix of zeros (0's) and ones (1's) where any $\mathbf{H}_{i,j,n}$ value set to <u>zero</u> means "Other" incentive package *n* for MOS *i* and length of service of *j* years <u>may not be offered</u>, and any $\mathbf{H}_{i,j,n}$ value set to <u>one</u> means "Other" incentive package *n* for MOS *i* for length of service of *j* years <u>may be offered</u>. The default value for any **H** term is 1.

The constraint controlling the specific bonus package of $4,000 for 2 years enlistment in the Combat arms is:

$$X_{3,2,4} \leq F_{3,2,4}$$

where $F_{3,2,4} = 1$ means that specific enlistment bonus package may be offered, and if $F_{3,2,4} = 0$ it means that specific enlistment bonus package may not be offered. The constraints for the ACF and the "Other" incentives are constructed the same way. In this prototype, there are a total of 350 constraints in this family (one for each combination of MOS, length of service, and incentive program).

Required Package Constraints

The fourth family of constraints checks to make sure no single bonus package is required to be offered. For instance, this constraint checks to see if the $16,000 enlistment bonus for a two year enlistment in electronics was mandated as an enlistment package.

- Let **Q** = an i by j by k matrix of zeros (0's) and ones (1's) where any $\mathbf{Q}_{i,j,k}$ value set to <u>zero</u> means enlistment bonus package k for MOS i and length of service of j years <u>may be offered</u>, and any $\mathbf{Q}_{i,jk}$ value set to a <u>one</u> means enlistment bonus package k for MOS i and length of service of j years <u>must be offered</u>. The default value for any **Q** term is 0.
- Let R = an i by j by m matrix of zeros (0's) and ones (1's) where any Ri,j,m value set to zero means ACF package m may be offered for that i MOS for a length of service of j years, and any Ri,j,m value set to a one means ACF package m for that i MOS for a length of service of j years must be offered. The default value for any R term is 0. Let **R** = an i by j by m matrix of zeros (0's) and ones (1's) where any $\mathbf{R}_{i,j,m}$ value set to <u>zero</u> means ACF package m for MOS i and length of service of j years <u>may be offered</u>, and any $\mathbf{R}_{i,j,m}$ value set to <u>one</u> means ACF package m for MOS i for length of service of j years <u>must be offered</u>. The default value for any **R** term is 0.

- The default value for any S term is 1. Let **S** = an i by j by n matrix of zeros (0's) and ones (1's) where any $\mathbf{S}_{i,j,n}$ value set to zero means "Other" incentive package n for MOS i and length of service of j years may be offered, and any $\mathbf{S}_{i,j,n}$ value set to one means "Other" incentive package n for MOS i for length of service of j years must be offered. The default value for any **S** term is 0.

For example, the constraint controlling the specific bonus package of a $4,000 EB for a 2 years enlistment in the Combat arms is:

$$X_{3,2,4} \geq Q_{3,2,4}$$

where $Q_{3,2,4} = 1$ means that specific enlistment bonus package *must* be offered, and if $Q_{3,2,4} = 0$ it means that this specific enlistment bonus package *may* be offered. The constraints for the ACF and the "Other" incentives are constructed the same way. In this prototype, there are a total of 350 constraints in this family (one for each combination of MOS, length of service, and incentive program).

Budget Constraints

The fifth family of constraints monitors the budget to ensure the total incentive packages offered do not exceed the budget allocated for each incentive program. There are three different budgets (one for each incentive program: enlistment bonus, ACF, and "Other" incentives).

- Let ME = [$0, $4,000, $10,000, $16,000] represent the amount of money expended for each enlistee expected to select a bonus package that includes the corresponding enlistment bonus.
- Let MA = [$10,000, $20,000, $30,000] represent the amount of money put away in the current fiscal year in order to meet future ACF payments for each recruit who selects this incentive. For the purpose of this prototype, these values are rough estimates and are not based on any net present worth formulas. The final MA matrix should probably be a j by m matrix derived using a uniform net present worth formula incorporating the standard government bond rate. Because ACF payments are not made until ETS,

expected payment may be a function of a Markov chain determining the probability of ETS for each MOS and length of service combination.

- Let MO = [$15,000, $2,500] represent the amount of money required for each enlistee allocated a bonus package that includes one of the "Other" incentives (in this prototype: college loan repayment, or unit or location of choice). For the purpose of this prototype, these values are rough estimates of the present worth of each of these incentives. An accurate estimate for each incentive in this category would have to be developed in any implementation of this prototype.
- Let the Enlistment Bonus Program budget = $61,000,000. (The value for this parameter was provided by USAREC for the FY 97 budget.)
- Let the ACF Program budget = $32,000,000. This is an artificial value used to constrict the model to check its feasibility. An accurate estimate for each incentive in this category would have to be developed in any implementation of this prototype.
- Let the "Other Incentives budget = $50,000,000. Again, this is an artificial value used to constrict the model to check its feasibility. An accurate estimate for each incentive in this category would have to be developed in any implementation of this prototype.

The structure for this family of constraints is:

$$\sum\sum\sum ME_k \cdot P \cdot A_{i,j,k} \cdot X_{i,j,k} \leq \$BudgetParameter$$

where $P^*A_{i,j,k}$ determines how many enlistees are expected to select a certain enlistment bonus option. Multiplying that value by ME_k determines how much will be spent for the particular bonus package. Multiply that cash value by $X_{i,j,k}$ only includes that amount of money if the bonus option is selected. By summing these values over all *i* MOSs, all *j* lengths of service, and all *k* enlistment bonus levels, the total expected cost for selected packages can be determined. The two other budget constraints are structured similarly.

Three major assumptions are invoked in the development of this family of constraints:

1. **All incentives are paid out within one fiscal year**. We recognize that incentive payment frequently crosses fiscal years, but assumed that these payments could be represented as a single "present value figure". This eliminated the need to project and optimize against out-year budgets, a process that would add a great deal of uncertainty to the model.

2. **There is a 100% payout on bonuses**. That is, all enlistees who sign up for a particular bonus will be paid that bonus. Attrition should already be reflected in the target figures for each MOS category.

3. **An equivalent lump-sum value can be calculated for each of the "other" non-EB, non-college fund incentives**. The "other" category includes non-monetary incentives such as Unit of Choice, Station of Choice, or college loan repayment. Like the EB, however, these incentives must be constrained. For the purposes of this model, we've estimated monetary values for these programs, and then used a pseudo budget figure to cap the number of recruits who could chose these options.

Reasonable Person Logic Constraints

The sixth family of constraints is comprised of 140 constraints designed to ensure that within any MOS category, the incentive package value for a given term of service is less than or equal to the incentive package value for the same MOS with a greater term of service. For example, we would not want to offer a 3 year length of service in the combat arms with a $16,000 enlistment bonus and a 6 year length of service for enlistment in the combat arms with a $4,000 enlistment bonus. Therefore we label this family as “reasonable” person logic constraints. These constraints take into consideration whether or not each MOS and length of service combinations were allowed (see the second family of constraints). In order to guarantee reasonable person logic, each MOS and bonus program requires ten (10) constraints. (Due to the non-monetary nature of the “Other” incentives we felt it was not necessary to include a set of reasonable logic constraints for this program. They could, however, be easily added).

To demonstrate the development of the reasonable person logic constraints, we use the combat arms MOS. The first set of 4 constraints checks to make sure the enlistment bonus offered for a six year enlistment in the Combat arms MOS is greater than any enlistment bonus offered for 5, 4, 3, and 2 year lengths of service. The structure of these constraints is as follows:

$$\left(\sum ME_k \cdot X_{3,6,k}\right)-\left(\sum ME_k \cdot X_{3,5,k}\right) \geq -L\cdot\left(1-D_{3,6}\right)$$
$$\left(\sum ME_k \cdot X_{3,6,k}\right)-\left(\sum ME_k \cdot X_{3,4,k}\right) \geq -L\cdot\left(1-D_{3,6}\right)$$
$$\left(\sum ME_k \cdot X_{3,6,k}\right)-\left(\sum ME_k \cdot X_{3,3,k}\right) \geq -L\cdot\left(1-D_{3,6}\right)$$
$$\left(\sum ME_k \cdot X_{3,6,k}\right)-\left(\sum ME_k \cdot X_{3,2,k}\right) \geq -L\cdot\left(1-D_{3,6}\right)$$

where ME_k is the amount of money offered for each enlistment bonus option, $X_{3,6,k}$ is the decision variable that represents which of the bonus package is being offered in the 6 year length of service option, and $X_{3,5,k}$ is the decision variable that represents which bonus package is being offered in the five year length of service option. L is a very large number; to enforce the relationships, it must be greater than the largest bonus offered, which in this case is \$16,000. For the prototype we let L = \$20,000. Finally $D_{3,6}$ is a binary value that represents whether a six year length of service in the Combat arms MOS was allowed. In long form the constraint that checks the 6 year bonus against the 5 year bonus is (assuming the 6 year length of service option was allowed):

$$\begin{array}{llll}(\$0*X_{3,6,0} & +\$4{,}000*X_{3,6,4} & +\$10{,}000*X_{3,6,10} & +\$16{,}000*X_{3,6,16}) \\ -\ (\$0*X_{3,5,0} & +\$4{,}000*X_{3,5,4} & +\$10{,}000*X_{3,5,10} & +\$16{,}000*X_{3,5,16}) \\ & & & \geq -\$20{,}000*(1-1)\end{array}$$

From the long form above, it can be seen that if the 6 year length of service option for the Combat arms MOS is offered ($D_{3,6} = 1$), then the left hand side of the constraint must be non-negative. This means that the bonus offered for a 5 year TOS in the Combat Arms MOSs may not be greater than that offered for a 6 year TOS. If however the 6 year length of service option in the Combat Arms is not offered ($D_{3,6} = 0$), the left hand side of the constraint must be non-positive, so all bonus levels are available for a 5 year TOS.

The constraints which ensure the 5 year bonus package is greater than the 4 year, 3 year, and 2 year packages are developed the same way and presented in summation notation below:

$$\left(\sum ME_k \cdot X_{3,5,k}\right)-\left(\sum ME_k \cdot X_{3,4,k}\right) \geq -L\cdot\left(1-D_{3,5}\right)$$
$$\left(\sum ME_k \cdot X_{3,5,k}\right)-\left(\sum ME_k \cdot X_{3,3,k}\right) \geq -L\cdot\left(1-D_{3,5}\right)$$
$$\left(\sum ME_k \cdot X_{3,5,k}\right)-\left(\sum ME_k \cdot X_{3,2,k}\right) \geq -L\cdot\left(1-D_{3,5}\right)$$

The constraints which ensure the 4 year package is greater than the 3 year and 2 year packages are presented in summation notation below:

$$\left(\sum ME_k \cdot X_{3,4,k}\right)-\left(\sum ME_k \cdot X_{3,3,k}\right) \geq -L\cdot\left(1-D_{3,4}\right)$$
$$\left(\sum ME_k \cdot X_{3,4,k}\right)-\left(\sum ME_k \cdot X_{3,2,k}\right) \geq -L\cdot\left(1-D_{3,4}\right)$$

Finally the constraint that ensures the 3 year bonus package is greater than the 2 year bonus package is presented below:

$$\left(\sum ME_k \cdot X_{3,3,k}\right)-\left(\sum ME_k \cdot X_{3,2,k}\right) \geq -L\cdot\left(1-D_{3,3}\right)$$

The constraints for the 6 other MOSs are developed the same way as are the constrains for all MOS and length of service combinations for the ACF program.

Model Output and Results:

The output from this model includes the bonus packages offered broken down by MOS categories, the expected number of enlistees based on each offered package, the USAREC MOS targets, the expected total enlistees per MOS category, the expected shortfall or excess of enlistees by each MOS category and the expected budget expenditure by incentive program. Currently AMPL does not provide a user friendly and rapidly readable output format therefore we have developed the EB model so that it produces text files. We then use Excel to format the report into output discussed above. A sample Excel report for the EB model is shown in Appendix G.

Appendix D: AMPL Implementation

This appendix shows the AMPL implementation of the EB model. Lines that begin with the pound sign (#) are comments; all other lines are AMPL commands.

```
# The set of indices used in the EB Model. They represent the set of subscripts used
# throughout the model. MOS corresponds to i in all Subscript variables in Appendix C,
# TOS corresponds to j in all subscript variables in Appendix C, Bonus_EB corresponds
# to subscript k, Bonus_ACF corresponds to subscript m, and Bonus_Other corresponds
# to subscript n.

set Bonus_ACF;
set Bonus_EB;
set Bonus_Other;
set MOS;
set TOS;

# The Set of Coefficients for this model. In AMPL you must declare all of your
# coefficients ahead of time so they may be loaded into the solver. The coefficients
# themselves are not in the model part of the problem, instead they are in the data file that
# supports the EB model and can be seen in Appendix C-2.

# These are the coefficients used in the objective function

param Underweight{MOS}>=0;
param Overweight{MOS}>=0;

# These are the coefficients used in the constraints.

param Probability_ACF{MOS,TOS,Bonus_ACF}>=0;
param Probability_EB{MOS,TOS,Bonus_EB}>=0;
param Probability_Other{MOS,TOS,Bonus_Other}>=0;
param Money_ACF{Bonus_ACF}>=0;
param Money_EB{Bonus_EB}>=0;
param Money_Other{Bonus_Other}>=0;

#The set of constants used to generate RHS values for each of the constraints

param Population >= 0;
param Leeway >=0;
param Budget_ACF >=0;
param Budget_EB >=0;
param Budget_Other >=0;
param Target{MOS} >=0;
param Shut_Down_TOS{MOS,TOS}>=0;
param Shut_Down_Package_ACF{MOS,TOS,Bonus_ACF}>=0;
param Shut_Down_Package_EB{MOS,TOS,Bonus_EB}>=0;
```

param Shut_Down_Package_Other{MOS,TOS,Bonus_Other}>=0;
param Force_On_Package_ACF{MOS,TOS,Bonus_ACF}>=0;
param Force_On_Package_EB{MOS,TOS,Bonus_EB}>=0;
param Force_On_Package_Other{MOS,TOS,Bonus_Other}>=0;
param Big_M >=0;

#The set of variables for the EB Model. **Offer_EB** corresponds directly with $\mathbf{X}_{i,j,k}$,
Offer_ACF corresponds directly with $\mathbf{Y}_{i,j,m}$, **Offer_Other** corresponds directly with
$\mathbf{Z}_{i,j,n}$, **Undergoal** corresponds directly with $\mathbf{U}_i$, **Overgoal** corresponds directly with
$\mathbf{O}_i$, and Normalize corresponds directly to Normalize

var Offer_ACF {MOS,TOS,Bonus_ACF} binary;
var Offer_EB {MOS,TOS,Bonus_EB} binary;
var Offer_Other {MOS,TOS,Bonus_Other} binary;
var Undergoal {MOS} >= 0;
var Overgoal {MOS} >= 0;
var Normalize >=0;

#The Objective function - It minimizes the penalties associated with failing to meet targets

minimize Penalty: sum{M in MOS}Underweight[M]*Undergoal[M] +
sum{M in MOS}Overweight[M] * Overgoal[M];

#The set of constraints

This constraint does nothing but set the sum of all offered bonuses equal to a variable
called Normalize. This was done just to make the model easier to read and formulate.
Normalize is used in subsequent constraints in order to force the sum of all offered
probabilities equal to 1

subject to Normalizing_value: (sum{M in MOS, T in TOS,B_ACF in
Bonus_ACF}Probability_ACF[M,T,B_ACF]*Offer_ACF[M,T,B_ACF])
+ (sum{M in MOS,T in TOS, B_EB in Bonus_EB} Probability_EB[M,T,B_EB]
* Offer_EB[M,T,B_EB]) + (sum{M in MOS,T in TOS, B_Other in Bonus_Other}
Probability_Other[M,T,B_Other]*Offer_Other[M,T,B_Other])-Normalize=0;

This family of constraints makes sure the bonus packages offered comes as close as
possible without going too far OVER the target. These constraints correspond to the
first family of constraints.

 # The overachieving goals

subject to USAREC_Goals_Over {M in MOS}: Population*((sum{T in TOS, B_ACF in
Bonus_ACF} Probability_ACF[M,T,B_ACF]*Offer_ACF[M,T,B_ACF]) +
(sum{T in TOS, B_EB in Bonus_EB}Probability_EB[M,T,B_EB] *Offer_EB[M,T,B_EB]) +
(sum{T in TOS, B_Other in Bonus_Other} Probability_Other[M,T,B_Other] *
Offer_Other[M,T,B_Other])) - Overgoal[M]<= ((1 + Leeway)*Target[M])*Normalize;

 # The underachieving goals

subject to USAREC_Goals_Under {M in MOS}: Population*((sum{T in TOS, B_ACF in Bonus_ACF} Probability_ACF[M,T,B_ACF]*Offer_ACF[M,T,B_ACF])+ (sum{T in TOS, B_EB in Bonus_EB}Probability_EB[M,T,B_EB]*Offer_EB[M,T,B_EB])+(sum{T in TOS, B_Other in Bonus_Other} Probability_Other[M,T,B_Other] * Offer_Other[M,T,B_Other])) + Undergoal[M]>= ((1 - Leeway)*Target[M])*Normalize;

This family of constraints ensures only one Army College Fund Bonus Plan (ACF
family) is offered per MOS and TOS combination. It also checks to make sure a length
of service and MOS combination has not been turned off. These constraints correspond
to the second family of constraints in Appendix C.

 # The Army College Fund constraints

subject to One_Bonus_ACF{M in MOS, T in TOS}: sum{B_ACF in Bonus_ACF}Offer_ACF[M,T,B_ACF]<=Shut_Down_TOS[M,T];

 # The Enlistment Bonus constraints

subject to One_Bonus_EB{M in MOS, T in TOS}: sum{B_EB in Bonus_EB}Offer_EB[M,T,B_EB]=Shut_Down_TOS[M,T];

 # Other Incentive constraints

subject to Limit_Bonus_Other{M in MOS, T in TOS}: sum{B_Other in Bonus_Other}Offer_Other[M,T,B_Other]<=2*(Shut_Down_TOS[M,T]);

This family of constraints checks to see that no individual bonus option has been turned
off. These family of constraints correspond with the third family of constraints in
Appendix C.

The constraints affecting the Army College fund

subject to Turn_Off_ACF{M in MOS, T in TOS,B_ACF in Bonus_ACF}:Offer_ACF[M,T,B_ACF]<=Shut_Down_Package_ACF[M,T,B_ACF];

The constraints affecting the Enlistment Bonuses

subject to Turn_Off_EB{M in MOS, T in TOS,B_EB in Bonus_EB}:Offer_EB[M,T,B_EB]<=Shut_Down_Package_EB[M,T,B_EB];

The constraints affecting the Other Incentives

subject to Turn_Off_Other{M in MOS, T in TOS,B_Other in Bonus_Other}:Offer_Other[M,T,B_Other]<=Shut_Down_Package_Other[M,T,B_Other];

This family of constraints checks to see that no individual bonus option has been
mandated to be offered. These family of constraints correspond with the fourth family
of constraints in Appendix C.

#The constraints for the Army College fund

subject to Force_On_ACF{M in MOS, T in TOS,B_ACF in Bonus_ACF}:Offer_ACF[M,T,B_ACF]>=Force_On_Package_ACF[M,T,B_ACF];

#The constraints for theEnlisted Bonus program

subject to Force_On_EB{M in MOS, T in TOS,B_EB in Bonus_EB}:Offer_EB[M,T,B_EB]>=Force_On_Package_EB[M,T,B_EB];

#The constraints for the Other incentives

subject to Force_On_Other{M in MOS, T in TOS,B_Other in Bonus_Other}:Offer_Other[M,T,B_Other]>=Force_On_Package_Other[M,T,B_Other];

#This family of constraints forces all of the programs to stay within their budgets. This
family corresponds to the fifth family of constraints

#The constraints for the Army College fund

subject to Budget_ACF_Limit: sum{M in MOS, T in TOS, B_ACF in Bonus_ACF}Probability_ACF[M,T,B_ACF]*Population*Money_ACF[B_ACF]*Offer_ACF [M,T,B_ACF]<=Budget_ACF;

#The constraints for theEnlisted Bonus program

subject to Budget_EB_Limit: sum{M in MOS, T in TOS, B_EB in Bonus_EB}Probability_EB[M,T,B_EB]*Population*Money_EB[B_EB]* Offer_EB[M,T,B_EB]<=Budget_EB;

#The constraints for the Other incentives

subject to Budget_Other_Limit: sum{M in MOS, T in TOS, B_Other in Bonus_Other}Probability_Other[M,T,B_Other]*Population*Money_Other[B_Other]*Offer_ Other[M,T,B_Other]<=Budget_Other;

The following series of constraints ensure no lessor length of service has a higher EB
Bonus than a greater length of service (within each MOS). These are the "reasonable"
person logic functions. This family of constraints corresponds with the sixth family of
constraints in Appendix C

This series of equations ensures that all bonus levels from 2 through 5 are less than or
equal to the bonus offered at the 6 year level. These constraints also check to make sure
6 years was an allowed length of service. These constraints are for the Enlistment
Bonus program

#This equation checks 6 years against 5 years

```
subject to Less_Than_6_at_5{M in MOS}: sum{B_EB in Bonus_EB}
Money_EB[B_EB]*Offer_EB[M,"6_years",B_EB]-sum{B_EB in Bonus_EB}
Money_EB[B_EB]*Offer_EB[M,"5_years",B_EB]>=-Big_M*(1-
Shut_Down_TOS[M,"6_years"]);
```

#This equation checks 6 years against 4 years

```
subject to Less_Than_6_at_4{M in MOS}: sum{B_EB in Bonus_EB}
Money_EB[B_EB]*Offer_EB[M,"6_years",B_EB]-sum{B_EB in Bonus_EB}
Money_EB[B_EB]*Offer_EB[M,"4_years",B_EB]>=-Big_M*(1-
Shut_Down_TOS[M,"6_years"]);
```

#This equation checks 6 years against 3 years

```
subject to Less_Than_6_at_3{M in MOS}: sum{B_EB in Bonus_EB}
Money_EB[B_EB]*Offer_EB[M,"6_years",B_EB]-sum{B_EB in Bonus_EB}
Money_EB[B_EB]*Offer_EB[M,"3_years",B_EB]>=-Big_M*(1-
Shut_Down_TOS[M,"6_years"]);
```

#This equation checks 6 years against 2 years

```
subject to Less_Than_6_at_2{M in MOS}: sum{B_EB in Bonus_EB}
Money_EB[B_EB]*Offer_EB[M,"6_years",B_EB]-sum{B_EB in Bonus_EB}
Money_EB[B_EB]*Offer_EB[M,"2_years",B_EB]>=-Big_M*(1-
Shut_Down_TOS[M,"6_years"]);
```

This series of equations ensures that all bonus levels from 2 through 4 are less than or
equal to the bonus offered at the 5 year level. These constraints also check to make sure
5 years was an allowed length of service. These constraints are for the Enlistment
Bonus program

#This equation checks 5 years against 4 years

```
subject to Less_Than_5_at_4{M in MOS}: sum{B_EB in Bonus_EB}
Money_EB[B_EB]*Offer_EB[M,"5_years",B_EB]-sum{B_EB in Bonus_EB}
Money_EB[B_EB]*Offer_EB[M,"4_years",B_EB]>=-Big_M*(1-
Shut_Down_TOS[M,"5_years"]);
```

#This equation checks 5 years against 3 years

subject to Less_Than_5_at_3{M in MOS}: sum{B_EB in Bonus_EB} Money_EB[B_EB]*Offer_EB[M,"5_years",B_EB]-sum{B_EB in Bonus_EB} Money_EB[B_EB]*Offer_EB[M,"3_years",B_EB]>=-Big_M*(1-Shut_Down_TOS[M,"5_years"]);

#This equation checks 5 years against 2 years

subject to Less_Than_5_at_2{M in MOS}: sum{B_EB in Bonus_EB} Money_EB[B_EB]*Offer_EB[M,"5_years",B_EB]-sum{B_EB in Bonus_EB} Money_EB[B_EB]*Offer_EB[M,"2_years",B_EB]>=-Big_M*(1-Shut_Down_TOS[M,"5_years"]);

#This series of equations ensures that all bonus levels from 2 through 3 are less than or
equal to the bonus offered at the 4 year level. These constraints also check to make sure
4 years was an allowed length of service. These constraints are for the Enlistment
Bonus program

#This equation checks 4 years against 3 years

subject to Less_Than_4_at_3{M in MOS}: sum{B_EB in Bonus_EB} Money_EB[B_EB]*Offer_EB[M,"4_years",B_EB]-sum{B_EB in Bonus_EB} Money_EB[B_EB]*Offer_EB[M,"3_years",B_EB]>=-Big_M*(1-Shut_Down_TOS[M,"4_years"]);

#This equation checks 4 years against 2 years

subject to Less_Than_4_at_2{M in MOS}: sum{B_EB in Bonus_EB} Money_EB[B_EB]*Offer_EB[M,"4_years",B_EB]-sum{B_EB in Bonus_EB} Money_EB[B_EB]*Offer_EB[M,"2_years",B_EB]>=-Big_M*(1-Shut_Down_TOS[M,"4_years"]);

This series of equations ensures that all bonus levels at 2 years are less than or equal to
the bonus offered at the 3 year level. These constraints also check to make sure 3 years
was an allowed length of service. This constraint is for the Enlistment
Bonus program

#This equation checks 3 years against 2 years

subject to Less_Than_3_at_2{M in MOS}: sum{B_EB in Bonus_EB} Money_EB[B_EB]*Offer_EB[M,"3_years",B_EB]-sum{B_EB in Bonus_EB} Money_EB[B_EB]*Offer_EB[M,"2_years",B_EB]>=-Big_M*(1-Shut_Down_TOS[M,"3_years"]);

This series of constraints is used to ensure no lessor length of service has a higher ACF
Bonus than a greater length of service (within each MOS)

This series of equations ensures that all bonus levels from 2 through 5 are less than or
equal to the bonus offered at the 6 year level. These constraints also check to make sure
6 years was an allowed length of service. These constraints are for the ACF
program

#This equation checks 6 years against 5 years

subject to Less_Than_6_at_5_ACF{M in MOS}: sum{B_ACF in Bonus_ACF} Money_ACF[B_ACF]*Offer_ACF[M,"6_years",B_ACF]-sum{B_ACF in Bonus_ACF} Money_ACF[B_ACF]*Offer_ACF[M,"5_years",B_ACF]>=-Big_M*(1-Shut_Down_TOS[M,"6_years"]);

#This equation checks 6 years against 4 years

subject to Less_Than_6_at_4_ACF{M in MOS}: sum{B_ACF in Bonus_ACF} Money_ACF[B_ACF]*Offer_ACF[M,"6_years",B_ACF]-sum{B_ACF in Bonus_ACF} Money_ACF[B_ACF]*Offer_ACF[M,"4_years",B_ACF]>=-Big_M*(1-Shut_Down_TOS[M,"6_years"]);

#This equation checks 6 years against 3 years

subject to Less_Than_6_at_3_ACF{M in MOS}: sum{B_ACF in Bonus_ACF} Money_ACF[B_ACF]*Offer_ACF[M,"6_years",B_ACF]-sum{B_ACF in Bonus_ACF} Money_ACF[B_ACF]*Offer_ACF[M,"3_years",B_ACF]>=-Big_M*(1-Shut_Down_TOS[M,"6_years"]);

#This equation checks 6 years against 2 years

subject to Less_Than_6_at_2_ACF{M in MOS}: sum{B_ACF in Bonus_ACF} Money_ACF[B_ACF]*Offer_ACF[M,"6_years",B_ACF]-sum{B_ACF in Bonus_ACF} Money_ACF[B_ACF]*Offer_ACF[M,"2_years",B_ACF]>=-Big_M*(1-Shut_Down_TOS[M,"6_years"]);

This series of equations ensures that all bonus levels from 2 through 4 are less than or
equal to the bonus offered at the 5 year level. These constraints also check to make sure
5 years was an allowed length of service. These constraints are for the Army College
Fund program

#This equation checks 5 years against 4 years

subject to Less_Than_5_at_4_ACF{M in MOS}: sum{B_ACF in Bonus_ACF} Money_ACF[B_ACF]*Offer_ACF[M,"5_years",B_ACF]-sum{B_ACF in Bonus_ACF} Money_ACF[B_ACF]*Offer_ACF[M,"4_years",B_ACF]>=-Big_M*(1-Shut_Down_TOS[M,"5_years"]);

#This equation checks 5 years against 3 years

subject to Less_Than_5_at_3_ACF{M in MOS}: sum{B_ACF in Bonus_ACF} Money_ACF[B_ACF]*Offer_ACF[M,"5_years",B_ACF]-sum{B_ACF in Bonus_ACF} Money_ACF[B_ACF]*Offer_ACF[M,"3_years",B_ACF]>=-Big_M*(1-Shut_Down_TOS[M,"5_years"]);

#This equation checks 5 years against 2 years

subject to Less_Than_5_at_2_ACF{M in MOS}: sum{B_ACF in Bonus_ACF} Money_ACF[B_ACF]*Offer_ACF[M,"5_years",B_ACF]-sum{B_ACF in Bonus_ACF} Money_ACF[B_ACF]*Offer_ACF[M,"2_years",B_ACF]>=-Big_M*(1-Shut_Down_TOS[M,"5_years"]);

#This series of equations ensures that all bonus levels from 2 through 3 are less than or
equal to the bonus offered at the 4 year level. These constraints also check to make sure
4 years was an allowed length of service. These constraints are for the ACF program.

#This equation checks 4 years against 3 years

subject to Less_Than_4_at_3_ACF{M in MOS}: sum{B_ACF in Bonus_ACF} Money_ACF[B_ACF]*Offer_ACF[M,"4_years",B_ACF]-sum{B_ACF in Bonus_ACF} Money_ACF[B_ACF]*Offer_ACF[M,"3_years",B_ACF]>=-Big_M*(1-Shut_Down_TOS[M,"4_years"]);

#This equation checks 4 years against 2 years

subject to Less_Than_4_at_2_ACF{M in MOS}: sum{B_ACF in Bonus_ACF} Money_ACF[B_ACF]*Offer_ACF[M,"4_years",B_ACF]-sum{B_ACF in Bonus_ACF} Money_ACF[B_ACF]*Offer_ACF[M,"2_years",B_ACF]>=-Big_M*(1-Shut_Down_TOS[M,"4_years"]);

This series of equations ensures that all bonus levels at 2 years are less than or equal to
the bonus offered at the 3 year level. These constraints also check to make sure 3 years
was an allowed length of service. This constraint is for the ACF program.

#This equation checks 3 years against 2 years

subject to Less_Than_3_at_2_ACF{M in MOS}: sum{B_ACF in Bonus_ACF} Money_ACF[B_ACF]*Offer_ACF[M,"3_years",B_ACF]-sum{B_ACF in Bonus_ACF} Money_ACF[B_ACF]*Offer_ACF[M,"2_years",B_ACF]>=-Big_M*(1-Shut_Down_TOS[M,"3_years"]);

Appendix E: AMPL Data File

This appendix shows and discusses the data file that was used with the prototype EB model. Lines that begin with the pound sign (#) are comments; all other lines are AMPL data statements.

```
# These are the indices used in the variables.

set MOS: = Medical_Jobs, Military_Intelligence, Administrative_Jobs, Combat_arms,
        Electronic_Repair, Engineering, Maintenance;
set TOS: = 2_years, 3_years, 4_years, 5_years, 6_years;
set Bonus_ACF: = 20K_ACF, 40K_ACF, 60K_ACF;
set Bonus_EB: = 0K_EB,04K_EB, 10K_EB, 16K_EB;
set Bonus_Other: = Loan_Paid, Unit_Location;

#These are the weights and targets for each MOS. Underweight is the UWi coefficient
# and Overweight is the OWi coefficient used in the objective function in Appendix C.
# Target is the USAREC goal for each MOS used as the RHS for the first family of
# constraints in Appendix C

param:                  Underweight   Overweight   Target:=
Medical_Jobs                 1             1          4765
Military_Intelligence        2             1          8347
Administrative_Jobs         50           100         14714
Combat_arms                  1             1          8315
Electronic_Repair           10             5          7496
Engineering                  3             1          4698
Maintenance                  5             2         12456;

# These are the bonus amounts in dollars. Money_ACF corresponds with matrix MA,
# Money_EB corresponds with matrix ME, and Money_Other corresponds with matrix
# MO in Appendix C.

param          Money_ACF:=
20K_ACF        10000
40K_ACF        20000
60K_ACF        30000;

param          Money_EB:=
0K_EB          0
04K_EB         4000
10K_EB         10000
16K_EB         16000;
```

```
param           Money_Other:=
Loan_Paid       15000
Unit_Location   2500;

# This is the large number used in the reasonable person logic constraints. It corresponds
# with L in the sixth family of constraints in Appendix C.

param Big_M: = 20000;

#This is the total population of propensed enlistees. It corresponds with P in Appendix C.

param Population: = 64000;

# This is the percent leeway allowed by USAREC for MOS targets

param Leeway: = .02;

#This is the total budget for each bonus category

param Budget_ACF: = 32000000;
param Budget_EB: = 61000000;
param Budget_Other: = 50000000;

# These are the marginal probabilities of a person selecting an Army College Fund Bonus
# Package given that it is offered. This 3 dimensional matrix corresponds with matrix
# B_{i,j,m} in appendix C

param Probability_ACF:=

#This is the probability matrix for the $20K College fund

[*,*,20K_ACF]:            2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs              0.0172  0.017   0.0158  0.0193  0.0162
Military_Intelligence     0.0186  0.0184  0.0171  0.0209  0.0176
Administrative_Jobs       0.0134  0.0132  0.0123  0.015   0.0126
Combat_arms               0.0155  0.0153  0.0143  0.0174  0.0147
Electronic_Repair         0.0142  0.014   0.0131  0.016   0.0134
Engineering               0.0141  0.0139  0.013   0.0158  0.0133
Maintenance               0.0152  0.015   0.014   0.0171  0.0143

#This is the probability matrix for the $40K College fund

[*,*,40K_ACF]:            2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs              0.018   0.0178  0.0166  0.0203  0.017
Military_Intelligence     0.0195  0.0192  0.0179  0.0219  0.0184
Administrative_Jobs       0.014   0.0138  0.0129  0.0158  0.0133
Combat_arms               0.0162  0.016   0.015   0.0183  0.0154
Electronic_Repair         0.0149  0.0147  0.0137  0.0168  0.0141
Engineering               0.0148  0.0146  0.0136  0.0166  0.014
Maintenance               0.0159  0.0157  0.0146  0.0179  0.015
```

```
#This is the probability matrix for the $60K College fund

[*,*,60K_ACF]:             2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs               0.0192  0.019   0.0177  0.0216  0.0182
Military_Intelligence      0.0208  0.0205  0.0192  0.0234  0.0197
Administrative_Jobs        0.015   0.0148  0.0138  0.0168  0.0141
Combat_arms                0.0173  0.0171  0.016   0.0195  0.0164
Electronic_Repair          0.0159  0.0157  0.0146  0.0179  0.015
Engineering                0.0157  0.0156  0.0145  0.0177  0.0149
Maintenance                0.017   0.0168  0.0156  0.0191  0.0161;
```

These are the marginal probabilities of a person selecting an Enlistment Bonus
Package given that it is offered. This 3 dimensional matrix corresponds with matrix
$A_{i,j,k}$ in appendix C

```
param Probability_EB:=

#This is the probability matrix for the $0K Enlisted Bonus (Nothing Offered)

[*,*,0K_EB]:               2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs               0.008   0.0079  0.0073  0.009   0.0075
Military_Intelligence      0.0086  0.0085  0.008   0.0097  0.0082
Administrative_Jobs        0.0062  0.0061  0.0057  0.007   0.0059
Combat_arms                0.0072  0.0071  0.0066  0.0081  0.0068
Electronic_Repair          0.0066  0.0065  0.0061  0.0074  0.0062
Engineering                0.0065  0.0065  0.006   0.0074  0.0062
Maintenance                0.0071  0.007   0.0065  0.0079  0.0067

#This is the probability matrix for the $4K Enlisted Bonus

[*,*,04K_EB]:              2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs               0.0136  0.0134  0.0125  0.0153  0.0129
Military_Intelligence      0.0147  0.0145  0.0136  0.0166  0.0139
Administrative_Jobs        0.0106  0.0104  0.0097  0.0119  0.01
Combat_arms                0.0123  0.0121  0.0113  0.0138  0.0116
Electronic_Repair          0.0112  0.0111  0.0104  0.0127  0.0106
Engineering                0.0111  0.011   0.0103  0.0125  0.0105
Maintenance                0.012   0.0119  0.0111  0.0135  0.0114
```

```
#This is the probability matrix for the $10K Enlisted Bonus

[*,*,10K_EB]:               2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs                0.014   0.0139  0.0129  0.0158  0.0133
Military_Intelligence       0.0152  0.015   0.014   0.0171  0.0144
Administrative_Jobs         0.0109  0.0108  0.0101  0.0123  0.0103
Combat_arms                 0.0127  0.0125  0.0117  0.0143  0.012
Electronic_Repair           0.0116  0.0115  0.0107  0.0131  0.011
Engineering                 0.0115  0.0114  0.0106  0.0129  0.0109
Maintenance                 0.0124  0.0123  0.0114  0.014   0.0117

#This is the probability matrix for the $16K Enlisted Bonus

[*,*,16K_EB]:               2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs                0.014   0.0139  0.0129  0.0158  0.0133
Military_Intelligence       0.0152  0.015   0.014   0.0171  0.0144
Administrative_Jobs         0.0109  0.0108  0.0101  0.0123  0.0103
Combat_arms                 0.0127  0.0125  0.0117  0.0143  0.012
Electronic_Repair           0.0116  0.0115  0.0107  0.0131  0.011
Engineering                 0.0115  0.0114  0.0106  0.0129  0.0109
Maintenance                 0.0124  0.0123  0.0114  0.014   0.0117;

# These are the marginal probabilities of a person selecting an "Other Incentive Package
# given that it is offered.  This 3 dimensional matrix corresponds with matrix
# C_{i,j,n} in appendix C

param Probability_Other:=

#This is the probability matrix associated with selecting school loans getting paid off

[*,*,Loan_Paid]:        2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs                0.0123  0.0121  0.0113  0.0138  0.0116
Military_Intelligence       0.0133  0.0131  0.0122  0.0149  0.0126
Administrative_Jobs         0.0095  0.0094  0.0088  0.0107  0.009
Combat_arms                 0.0111  0.0109  0.0102  0.0124  0.0105
Electronic_Repair           0.0101  0.01    0.0093  0.0114  0.0096
Engineering                 0.01    0.0099  0.0093  0.0113  0.0095
Maintenance                 0.0108  0.0107  0.01    0.0122  0.0102

#This is the probability matrix associated with selecting a unit or location

[*,*,Unit_Location]:        2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs                0.013   0.0129  0.012   0.0147  0.0123
Military_Intelligence       0.0141  0.0139  0.013   0.0159  0.0133
Administrative_Jobs         0.0101  0.01    0.0093  0.0114  0.0096
Combat_arms                 0.0118  0.0116  0.0108  0.0132  0.0111
Electronic_Repair           0.0108  0.0106  0.0099  0.0121  0.0102
Engineering                 0.0107  0.0105  0.0098  0.012   0.0101
Maintenance                 0.0115  0.0114  0.0106  0.0129  0.0109;
```

```
# This matrix allows you to close off entire [MOS,TOS] options.  For instance if you
# never wanted to offer any package to the medical MOS field for the two year enlistment
# option, you would place a zero in its respective location within the matrix.  To make
# every option available, all values in the matrix should be a one (1).  This 2 dimensional
# matrix corresponds with D in Appendix C

param Shut_Down_TOS:  2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs              1       1       1       1       1
Military_Intelligence     1       1       1       1       1
Administrative_Jobs       1       1       1       1       1
Combat_arms               1       1       1       1       1
Electronic_Repair         1       1       1       1       1
Engineering               1       1       1       1       1
Maintenance               1       1       1       1       1;

# This set of matrices allows you to turn off individual [MOS,TOS, Bonus] options. For
# instance if you did not want to offer a two year $20,000 ACF bonus to a person entering
# the medical MOS, you would place a zero in its respective location within the
# [*,*,20K_ACF] matrix.  Generally all options should be one (1).  This matrix
# corresponds with matrix G in Appendix C

param Shut_Down_Package_ACF:=
[*,*,20K_ACF]:        2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs              1       1       1       1       1
Military_Intelligence     1       1       1       1       1
Administrative_Jobs       1       1       1       1       1
Combat_arms               1       1       1       1       1
Electronic_Repair         1       1       1       1       1
Engineering               1       1       1       1       1
Maintenance               1       1       1       1       1

 [*,*,40K_ACF]:       2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs              1       1       1       1       1
Military_Intelligence     1       1       1       1       1
Administrative_Jobs       1       1       1       1       1
Combat_arms               1       1       1       1       1
Electronic_Repair         1       1       1       1       1
Engineering               1       1       1       1       1
Maintenance               1       1       1       1       1

[*,*,60K_ACF]:        2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs              1       1       1       1       1
Military_Intelligence     1       1       1       1       1
Administrative_Jobs       1       1       1       1       1
Combat_arms               1       1       1       1       1
Electronic_Repair         1       1       1       1       1
Engineering               1       1       1       1       1
Maintenance               1       1       1       1       1;
```

```
# This set of matrices allows you to turn off individual [MOS,TOS, Bonus] options. For
# instance if you did not want to offer a two year $16,000 enlistment bonus to a person
# entering the medical MOS, you would place a zero in its respective location within the
# [*,*,16K_EB] matrix.  Generally all options should be one (1).  This matrix corresponds
# with matrix F in Appendix C

param Shut_Down_Package_EB:=
[*,*,0K_EB]:            2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs              1       1       1       1       1
Military_Intelligence     1       1       1       1       1
Administrative_Jobs       1       1       1       1       1
Combat_arms               1       1       1       1       1
Electronic_Repair         1       1       1       1       1
Engineering               1       1       1       1       1
Maintenance               1       1       1       1       1

[*,*,04K_EB]:           2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs              1       1       1       1       1
Military_Intelligence     1       1       1       1       1
Administrative_Jobs       1       1       1       1       1
Combat_arms               1       1       1       1       1
Electronic_Repair         1       1       1       1       1
Engineering               1       1       1       1       1
Maintenance               1       1       1       1       1

[*,*,10K_EB]:           2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs              1       1       1       1       1
Military_Intelligence     1       1       1       1       1
Administrative_Jobs       1       1       1       1       1
Combat_arms               1       1       1       1       1
Electronic_Repair         1       1       1       1       1
Engineering               1       1       1       1       1
Maintenance               1       1       1       1       1

[*,*,16K_EB]:           2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs              1       1       1       1       1
Military_Intelligence     1       1       1       1       1
Administrative_Jobs       1       1       1       1       1
Combat_arms               1       1       1       1       1
Electronic_Repair         1       1       1       1       1
Engineering               1       1       1       1       1
Maintenance               1       1       1       1       1;
```

```
# This set of matrices allows you to turn off individual [MOS,TOS, Bonus] options. For
# instance if you did not want to offer a two year Loan Paid bonus to a person
# entering the medical MOS, you would place a zero in its respective location within the
# [*,*,Loan_Paid] matrix.  Generally all options should be one (1).  This matrix corresponds
# with matrix H in Appendix C

param Shut_Down_Package_Other:=
[*,*,Loan_Paid]:    2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs             1       1       1       1       1
Military_Intelligence    1       1       1       1       1
Administrative_Jobs      1       1       1       1       1
Combat_arms              1       1       1       1       1
Electronic_Repair        1       1       1       1       1
Engineering              1       1       1       1       1
Maintenance              1       1       1       1       1

[*,*,Unit_Location]: 2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs             1       1       1       1       1
Military_Intelligence    1       1       1       1       1
Administrative_Jobs      1       1       1       1       1
Combat_arms              1       1       1       1       1
Electronic_Repair        1       1       1       1       1
Engineering              1       1       1       1       1
Maintenance              1       1       1       1       1;
```

This set of matrices allows you to force on individual [MOS,TOS, Bonus] options. For
instance if you required a two year $20,000 ACF bonus to a person entering the medical
MOS, you would place a one (1) in its respective location within the [*,*,20K_ACF]
matrix. Generally all options should be zero (0). This matrix corresponds with the R
matrix in appendix C.

```
param Force_On_Package_ACF:=
[*,*,20K_ACF]:         2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs               0       0       0       0       0
Military_Intelligence      0       0       0       0       0
Administrative_Jobs        0       0       0       0       0
Combat_arms                0       0       0       0       0
Electronic_Repair          0       0       0       0       0
Engineering                0       0       0       0       0
Maintenance                0       0       0       0       0


[*,*,40K_ACF]:         2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs               0       0       0       0       0
Military_Intelligence      0       0       0       0       0
Administrative_Jobs        0       0       0       0       0
Combat_arms                0       0       0       0       0
Electronic_Repair          0       0       0       0       0
Engineering                0       0       0       0       0
Maintenance                0       0       0       0       0

[*,*,60K_ACF]:         2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs               0       0       0       0       0
Military_Intelligence      0       0       0       0       0
Administrative_Jobs        0       0       0       0       0
Combat_arms                0       0       0       0       0
Electronic_Repair          0       0       0       0       0
Engineering                0       0       0       0       0
Maintenance                0       0       0       0       0;
```

```
# This set of matrices allows you to force on individual [MOS,TOS, Bonus] options. For
# instance if you required a two year $16,000 bonus to a person entering the medical
# MOS, you would place a one (1) in its respective location within the [*,*,16K_EB]
# matrix.  Generally all options should be zero (0).  This matrix corresponds with the Q
# matrix in appendix C.

param Force_On_Package_EB:=
[*,*,0K_EB]:          2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs                0       0       0       0       0
Military_Intelligence       0       0       0       0       0
Administrative_Jobs         0       0       0       0       0
Combat_arms                 0       0       0       0       0
Electronic_Repair           0       0       0       0       0
Engineering                 0       0       0       0       0
Maintenance                 0       0       0       0       0

[*,*,04K_EB]:         2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs                0       0       0       0       0
Military_Intelligence       0       0       0       0       0
Administrative_Jobs         0       0       0       0       0
Combat_arms                 0       0       0       0       0
Electronic_Repair           0       0       0       0       0
Engineering                 0       0       0       0       0
Maintenance                 0       0       0       0       0

[*,*,10K_EB]:         2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs                0       0       0       0       0
Military_Intelligence       0       0       0       0       0
Administrative_Jobs         0       0       0       0       0
Combat_arms                 0       0       0       0       0
Electronic_Repair           0       0       0       0       0
Engineering                 0       0       0       0       0
Maintenance                 0       0       0       0       0

[*,*,16K_EB]:         2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs                0       0       0       0       0
Military_Intelligence       0       0       0       0       0
Administrative_Jobs         0       0       0       0       0
Combat_arms                 0       0       0       0       0
Electronic_Repair           0       0       0       0       0
Engineering                 0       0       0       0       0
Maintenance                 0       0       0       0       0;
```

```
# This set of matrices allows you to force on individual [MOS,TOS, Bonus] options. For
# instance if you required a two year Loan Paid bonus to a person entering the medical
# MOS, you would place a one (1) in its respective location within the [*,*,Loan_Paid]
# matrix.  Generally all options should be zero (0).  This matrix corresponds with the S
# matrix in appendix C.

param Force_On_Package_Other:=
[*,*,Loan_Paid]:    2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs              0       0       0       0       0
Military_Intelligence     0       0       0       0       0
Administrative_Jobs       0       0       0       0       0
Combat_arms               0       0       0       0       0
Electronic_Repair         0       0       0       0       0
Engineering               0       0       0       0       0
Maintenance               0       0       0       0       0

[*,*,Unit_Location]: 2_years 3_years 4_years 5_years 6_years:=
Medical_Jobs              0       0       0       0       0
Military_Intelligence     0       0       0       0       0
Administrative_Jobs       0       0       0       0       0
Combat_arms               0       0       0       0       0
Electronic_Repair         0       0       0       0       0
Engineering               0       0       0       0       0
Maintenance               0       0       0       0       0;
```

Appendix F: Run File

The file EB_dist2.run is used by AMPLPlus to automate the procedures used to run the EB Distribution model. Specifically, to view the results of any model created in AMPLPlus, the user must develop parameter queries using the model window in the AMPLPlus application. While this is not a difficult task, it is tedious and produces a less than informative presentation of the model results. By writing the model results to a text file, the data may be easily manipulated into a more meaningful presentation. The run file automates this process and relieves the user of this task. Every time a user runs the model, the run file is executed.

The run file is broken into four parts:

- The first part limits run time until completion to 10 minutes or 4 integer solutions. By changing the runtime (currently set to 600 seconds) or the number of feasible solutions until (currently set to 4), the user can modify the limits used to stop Cplex. The EB Distribution model is such a large problem, it can run for hours without finding the optimal solution. By setting these stopping parameters, the user is guaranteed a good solution within a reasonable amount of time. The higher the stopping limits, the better the final answer will be.
- The second part of the file writes the binary decision variable results to a file called Solset1.txt.
- The third part of the file writes the set of probabilities used to solve the problem to a file called Solset2.txt. This data is used in Report.xls to generate a report of the model results.
- The forth and final portion of the file writes the additional parameter results to the file Solset3.txt. Again, this data is used by Report.xls to generate a report of the model results.

```
option cplex_options 'time=600 mipsolutions=4';
solve;
```

Part 1

```
csvdisplay Offer_EB > EB_dist\Sol_set1.txt;
csvdisplay Offer_ACF >> EB_dist\Sol_set1.txt;
csvdisplay Offer_Other >> EB_dist\Sol_set1.txt;
close EB_dist\Sol_set1.txt;
```

Part 2

```
csvdisplay Probability_EB > EB_dist\Sol_set2.txt;
csvdisplay Probability_ACF >> EB_dist\Sol_set2.txt;
csvdisplay Probability_Other >> EB_dist\Sol_set2.txt;
close EB_dist\Sol_set2.txt;
```

Part 3

```
csvdisplay Overgoal > EB_dist\Sol_set3.txt;
csvdisplay Undergoal >> EB_dist\Sol_set3.txt;
csvdisplay Target >> EB_dist\Sol_set3.txt;
csvdisplay Population >> EB_dist\Sol_set3.txt;
csvdisplay EB_Budget_Amount >> EB_dist\Sol_set3.txt;
csvdisplay Budget_EB >> EB_dist\Sol_set3.txt;
csvdisplay ACF_Budget_Amount >> EB_dist\Sol_set3.txt;
csvdisplay Budget_ACF >> EB_dist\Sol_set3.txt;
csvdisplay Other_Budget_Amount >> EB_dist\Sol_set3.txt;
csvdisplay Budget_Other >> EB_dist\Sol_set3.txt;
close EB_dist\Sol_set3.txt;
```

Part 4

Appendix G: Output Files

This appendix discusses and explains the Excel file (Report.xls) that formats the three output files generated by the EB Distribution model discussed in Appendix F.

REPORT.xls

The run file discussed in Appendix F produces three text files: Solset1.txt, Solset2.txt, and Solset3.txt. Report.xls reads the text files and converts them to a report that can be easily analyzed and understood. The report consists of a series of tables that present the following information:

- which bonus package was offered sorted by MOS category and length of service,
- the expected number of enlistees assessed based on that specific bonus option,
- the expected total number of recruits for a specific MOS as compared to the target for each specific MOS,
- the expected shortfall or overage for each specific MOS,
- whether or not the difference was within the allowable leeway,
- and budget data for each bonus program.

Administrative_Jobs

Duration of Enlistment	Bonus Package	Expected Enlistees
2 Year Enlistment	Soldier Selects Unit or Location	2355
4 Year Enlistment	$10,000 Enlistment Bonus	1308
4 Year Enlistment	Soldier Selects Unit or Location	1590
5 Year Enlistment	$16,000 Enlistment Bonus	2435
5 Year Enlistment	$20,000 ACF Contribution	2294
6 Year Enlistment	$16,000 Enlistment Bonus	2133
6 Year Enlistment	$20,000 ACF Contribution	2012
	Total MOS Enlistment	14127
	Total MOS Target	14714
	Enlistees Short of Target	-587

Results by MOS

Expected Enlistees by option

Sorted by length of service

Bonus package offered

Red lettering indicates in excess of leeway, black lettering indicates within leeway

Table G- 1: Sample Output From Report.xls

Table G-1 on the previous page depicts sample data from the report that presents information concerning the bonus packages offered for a specific MOS. A similar table will be generated for each MOS considered.

Program Name	Money Used	Money Allocated	Money Remaining
Enlistment Bonus	$ 36,180,400.00	$ 61,000,000.00	$ 24,819,600.00
Army College Fund	$ 35,902,000.00	$ 40,000,000.00	$ 4,098,000.00
Other Incentives	$ 21,982,000.00	$ 25,000,000.00	$ 3,018,000.00

Expected budget balance by program

Budget by program

Expected budget balance by program

Table G- 2: Sample Output From Report.xls

Table G-2 shows the budget data developed by Report.xls. All values except for the Money Allocated values are expected results based on the EB Distribution Model output.

Appendix H: References

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