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13. ABSTRACT (Maximum 200 words) The Final Technical Progress Report for O. N. R. Grant N00014-92-J-1400 contains brief descriptions of published and to-be-published work that was supported in whole or in part by O. N. R. Grant N00014-92-J-1400 as proposed in the September 1994 Grant Proposal by K. B. Reid. This work resulted in new knowledge concerning several areas of combinatorial mathematics including the aggregation of individual preferences via majority rule with an agenda, the determination of digraphs that arise in population ecology as domination graphs of tournaments representing the relation "to prey" on a collection of species, the determination of subtrees which are "most central" in a tree, the determination of which sequences of integers are score sequences for tournaments and for semicomplete digraphs, the discoveries and investigations of a two-parameter family of central sets in trees and a novel concept of balance vertices in trees, the determination of some bounds for the domination and irredundance numbers for tournaments, and the determination of near-automorphisms of complete multi-partite graphs.			
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FINAL TECHNICAL PROGRESS REPORT

Office of Naval Research GRANT NO. N00014-92-J-1400

Tournaments in Consensus Methods Based on Voting; Dominance
in Tournaments; Centrality in Graphs

June 5, 1998

Principal Investigator: K. B. Reid

K.B. Reid

Founding Faculty

Professor of Mathematics

Social Security Number: [REDACTED]

Mathematical Sciences

College of Arts and Sciences

California State University, San Marcos

San Marcos, California 92096-0001

Office: (760) 750-4088

E-mail: breid@mailhost1.csusm.edu

Fax: (760) 750-4111

Remark: In all of our manuscripts referenced in this report the following statement (or its equivalent) appears: "Supported in part by Office of Naval Research Grant No. N00014-92-J-1400."

Summary of Results 1995-1998

1. Update. In this section we update the progress on work reported in our "Summary of Results 1992-1994" for Office of Naval Research (O. N. R.) Grant No. N00014-92-J-1400. That short summary appeared in our September 1994 O. N. R. Grant Proposal and is appended below for easy reference. Some of the work proposed in the original grant for 1992-1994 was completed, submitted, and published prior to the 1994 proposal; such work was reported and referenced in "Summary of Results 1992-1994" and will not be reported again in the current report. The work proposed in September 1994 was funded for three years, 1995-1997, and extended, at no cost, to May 31, 1998; the grant continued to use the same number, O. N. R. Grant No. N00014-92-J-1400.

Majority rule is a common method to aggregate individual preferences into a group preference. If every voter in a finite set of voters linearly orders n alternatives according to their individual preferences, then a majority digraph arises in which each alternative is a vertex and there is an arc from alternative x to alternative y if and only if a majority of the voters prefer x to y . If there are no ties (e.g., the number of voters is odd), then a majority tournament is obtained. Moreover, it is well known that every tournament with n vertices represents such a majority tournament for some set of voters using majority rule on n alternatives. A common mechanism for determining a single group preference is to adopt an agenda, an ordering of the alternatives from which pairwise comparisons are made by the voters. Our work on the structure of majority tournaments which admit an agenda for which the sincere (non-strategic) and the, so-called, sophisticated voting decisions are identical appeared in a French journal in 1997 ["Equitable agendas: agendas ensuring identical sincere and sophisticated voting decisions," **Social Choice and Welfare** 14 (1997) 363-378]. That work was presented at invited colloquia in 1994 at the University of Colorado, Denver, at the Claremont Colleges in California, and at Southwest Texas State University, and during one of four invited lectures in Fall 1995 at the National University of Singapore.

In 1995 our joint work on plurality preferences based on distances in graphs appeared ["Realization of digraphs by preferences based on distances in graphs." (with Weizhen Gu and Walter Schnyder) **Journal of Graph Theory** 19 (1995) 367-373], and our joint work on plurality preference digraphs realized by trees appeared ["Plurality preference digraphs realized by trees - I. Necessary and sufficient conditions," (with Weizhen Gu) **Discrete Mathematics** 147 (1995) 185-196]; these two studies were motivated in part by preferences of users, based at sites, for locations of facilities on networks. Preliminary

discussion of this work (a follow-up paper had actually appeared in print much earlier) was included in a talk at the Julius Petersen Graph Theory Conference in Hindsø, Denmark, in 1990, at an invited seminar at Simon Fraser University in 1992 (when visiting as an External Examiner for a Ph.D. thesis), and at a colloquium at the National University of Singapore in 1993.

The results mentioned in the two new collaborations launched during the first three-years of O. N. R. Grant N00014-92-J-1400 have either appeared or will appear shortly. Continued collaboration made possible in large part by the second three years of O. N. R. Grant N00014-92-J-1400 resulted in additional manuscripts (see Section 2 below). Our work with a group at the University of Colorado, Denver, originated from some issues in population ecology. We obtained a general characterization of domination graphs of tournaments while studying competition graphs of tournaments; the results will appear in 1998 ["The domination and competition graphs of a tournament," (with David C. Fisher, J. Richard Lundgren, and Sarah Merz) **Journal of Graph Theory**, to appear, 1998]. We published some auxiliary results on this structural problem in 1995 ["Domination graphs of tournaments and digraphs," (with David C. Fisher, J. Richard Lundgren, and Sarah Merz) **Congressus Numerantium** 108 (1995) 97-107].

Motivated by considerations in the theory of location of facilities on networks, we published work on the central subtrees of trees in 1997 ["Central k-trees of trees," (with Fred McMorris) **Congressus Numerantium** 124 (1997) 139-144]. Our co-author reported these results at the University of Mississippi in 1996 at the Ninth Cumberland Conference.

We also mention that the problem on scores in the square of a tournament, a problem now usually attributed to N. Dean, was solved by D.C. Fisher (one of our co-authors in another context) in 1995 ["Squaring a tournament: A proof of Dean's conjecture," **Journal of Graph Theory** 23 (1996) 43-48].

2. Progress on proposed work. In this section we summarize progress made on problems and issues presented in our September 1994 proposal entitled "Tournaments in Consensus Methods Based on Voting; Dominance in Tournaments; Centrality in Graphs." We address this progress in the same order that the problems were presented in the Introduction of that proposal.

As promised in that proposal, we worked intensely on and published a 40-page, up-to-date survey of tournament results and their applications in order to help the community of discrete mathematics scholars and those interested in potential applications to stay abreast of current activity in the field and to serve as a starting point for students and researchers entering the field ["Tournaments: scores, dominant vertices, generalizations and special topics." (Invited survey) *Surveys in Graph Theory* (eds. G. Chartrand and M. Jacobson) **Congressus Numerantium** 115 (1996) 171-211]. This survey covered topics on scores, including sketches of several published proofs of Landau's Theorem, as well as topics on multipartite tournaments, dominant vertices, generalizations, and several special topics. John Moon's 1968 monograph on tournaments remains a valuable resource, even though it is very out-dated. Our heavily cited 1979 joint survey that appeared in a collection of survey articles on graph theory has also become out-dated. About the same time as our new survey was completed, J. Bang-Jensen and G. Gutin also produced a nice, current survey of a piece of tournament theory, namely paths, trees, cycles, and some other substructures in semicomplete digraphs (spanning complete subgraphs of the complete symmetric digraph). Their survey also appears in the same collection of surveys as our new survey. These surveys helped to fill the information gap on happenings in tournament theory during the previous 15 years. We made a preliminary report on this work at a Special Session during the American Mathematical Society Annual Meeting in San Francisco, California, in 1995. Reports based on part of this work were given at invited seminars at the University of Colorado, Denver, in 1995, at Louisiana State University in 1996, and at an invited colloquium at the University of South Carolina in 1996.

The work on the survey reported above spun off two manuscripts that will soon appear in print. In 1996 we found a new proof of Landau's Theorem on score sequences of tournaments and simplified one existing proof. This work will appear in early 1999 ["Landau's Theorem revisited," (with Jerrold Griggs) submitted to the **Australasian Journal of Combinatorics**, March 1998, 5 pp.]. We reported on this work at the 11th Midwestern Conference on Combinatorics, Cryptography, and Computing in 1996 at the University of Nevada, Las Vegas, and at the 28th Southeastern International Conference on Combinatorics, Graph Theory and Computing at Florida Atlantic University in 1997. While reporting on that work at the Las Vegas conference we discovered a general result about out-degree sequences of semicomplete digraphs that is, in fact, equivalent to Landau's Theorem. This work will also appear in late 1998 or

early 1999 ["Score sequences for semicomplete digraphs," (with C. Q. Zhang) **Bulletin of the Institute of Combinatorics and its Applications**, to appear, 1998, 6 pp.].

Motivated by discrete mathematical questions arising from the theory of location of facilities on networks and as promised in our 1994 proposal mentioned above, we focused on issues of centrality in graphs, particularly trees. In the late eighties we discovered a one-parameter family of central sets in trees ["Centroids to centers in trees," **NETWORKS** 21 (1991) 11-17]. During the tenure of this grant, we found (while teaching a graduate course at CSU San Marcos on some graph theory arising from discrete location theory) a new two-parameter family of central sets in trees that includes, as a special case, the earlier one-parameter family. Both families include the classical central sets in trees, the center and the centroid (which is the branch-weight centroid). Let x be a vertex in a tree T . For fixed non-negative integers k and l , and for a path P of length l that has one end at x , consider all vertices that are reachable from x via P , but are more than distance k from x . Choose a P so that this collection of vertices is as large as possible, and label x with the number of reachable vertices in such a largest collection. The set of vertices with smallest labels make up the k -ball l -path branch weight centroid of T . Many, but not all, central sets of vertices in trees consist of a single vertex or two adjacent vertices. We proved that for certain k and l ($0 \leq k \leq \text{radius}(T)$, $0 < l \leq 1 + \text{radius}(T)$), the k -ball l -path branch weight centroid of T always consists of a single vertex or two adjacent vertices. This is not necessarily true for other values of k and l . We also discussed how the proof technique involving orientation of the edges of T gives shorter proofs of some results of Slater on other single-parameter families of central sets in trees. The results appeared in 1997 ["The k -ball l -path branch-weight centroid," **Applied Discrete Mathematics** (80) 2-3 (1997) 243-250]. We reported on this work in 1996 at the Eighth Quadrennial International Conference on Graph Theory, Combinatorics, Algorithms, and Applications in Kalamazoo, Michigan.

In 1997 we studied two new centrality concepts in trees involving what might be called "balance," a concept somewhat analogous to the center of mass of a finite number of point masses along a line. Each vertex x of a tree T is labeled as follows: among all divisions of T into two subtrees T_1 and T_2 so that x is the only vertex in both, choose a division so that the sum of the distances from x to all vertices in T_1 is as nearly equal as possible to the sum of the distances from x to all vertices in T_2 . Label x with the absolute value of the difference of these two sums. If each vertex has the same mass, then these sums may be thought of as the "moments" of x with respect to T_1 and T_2 . The balance

vertices in T are defined to be those vertices with the smallest label. We prove that in any tree T there is either exactly one balance vertex or exactly two adjacent balance vertices. The proof technique involved a "double orientation" of the edges of T . We also mention a balance concept for edges. This work appears in a manuscript which will appear in 1999 ["Balanced bipartitions in trees," invited submission to **NETWORKS**, to appear in 1999, 13 pp.].

The collaboration begun with the University of Colorado group on some issues arising from population ecology continued successfully. Our general thrust has been to study some of the combinatorial possibilities of collections of species in which some pairs have a common prey. Sharpened structural results describing exactly which connected graphs are domination graphs of tournaments were obtained and the results will appear in 1998 ["Connected domination graphs of tournaments," (with David C. Fisher, J. Richard Lundgren, and Sarah Merz) **Journal of Combinatorial Mathematics and Combinatorial Computing**, to appear, 1998, 5 pp.]. Study during 1996 showed that graphs that are not connected and contain isolated vertices are quite difficult to classify as domination graphs of tournaments. But, if isolated vertices are not allowed, then we were able to arrive at a characterization in late 1996 and summer 1997. That work has been submitted for publication ["Domination graphs with nontrivial components," (with David C. Fisher, David Guichard, J. Richard Lundgren, and Sarah Merz) submitted to **Graphs and Combinatorics**, April 1998]. Work is underway to try to completely settle this project by considering disconnected graphs with isolated vertices. Some of this work was presented at an invited seminar at Simon Fraser University in 1994 (when visiting again as an External Examiner for another Ph.D. thesis), during one of four invited lectures in Fall 1995 at the National University of Singapore, at invited colloquia at the University of Alabama in Huntsville and at Clemson University in 1996, and at a contributed talk at the University of Mississippi in 1996 at the Ninth Cumberland Conference.

The determination of the domination number of a tournament is a very difficult problem in general. We have launched a new investigation into the domination number of a tournament by looking at other "domination-like" parameters in tournaments. We expect to find many new directions given the considerable activity on such ideas in undirected graph theory over the past dozen years as evidenced by the new books Fundamentals of Dominance in Graphs and Domination in Graphs by Hayes, Hedetniemi and Slater. We concentrate on tournament analogues of the domination number and the

irredundance number of undirected graphs. We have produced a preliminary manuscript that will be reworked in summer 1998 ["Domination and irredundance parameters in tournaments "(with Steven and Sandra Hedetniemi and Alice McRae), **draft** in revision].

We made very modest progress on the issue of monochromatic reachability in arc-colored tournaments in an attempt to attack the intriguing 15 year old conjecture of Erdős, Sands, Saurer, and Woodrow. That conjecture states that for every positive integer m , there is a positive integer $f(m)$, so that every finite tournament whose arc set has been arbitrarily partitioned into m classes (i.e., whose arcs are colored with m colors) contains a set S of $f(m)$ vertices with the property that for every vertex x not in S there is a path from x to a vertex in S , all of whose arcs lie in the same class (i.e., x can reach S via some monochromatic path). They showed that $f(2)=1$ (we gave an independent proof in 1984), but even the existence of $f(3)$ remains unknown! The few results on the original problem (for $m=2$ and special cases of $m=3$), together with some unpublished work (with B. Bollobás) on tournament Ramsey theory, were presented at an invited colloquium at the University of Alaska in Fall 1996. We have been able to show that for $m=3$ colors there is such a finite set if the colors on the allowed paths are allowed to vary according to certain partial orderings on the colors. We reported on this work at an invited seminar at the University of Colorado, Denver, in 1997, but more remains to be done.

3. Other Work. During our investigations in summer 1997 we found new results in a new direction of study in algebraic graph theory on "near-automorphisms" of graphs, i.e., permutations of the vertices of graphs that are very near to preserving adjacency. This general area of study promises to shed new and important light on symmetry and graphical structure with potential to carry over to new insights in applications. For the purposes of this report, a near-automorphism ϕ of a graph G is a permutation of the vertices of G which is not an automorphism of G , but is such that $\sum |d(u,v) - d(\phi u, \phi v)|$ is as small as possible, where the sum is over all $n(n-1)/2$ pairs $\{u, v\}$ of distinct vertices of G . The value of this smallest sum is denoted $\pi(G)$. G. Chartrand presented some very preliminary (joint) results on near-automorphisms at the Annual Joint Meeting of the American Mathematical Society and the Mathematical Association of America in San Diego in January 1997. Chartrand's group proved that for a tree T of order n the smallest such sum when only transposition permutations (which are not automorphisms) are allowed is less than or equal to $2n-4$. They conjectured that for a path G with n vertices,

$\pi(G) = 2n-4$. One of our colleagues at CSU San Marcos, W. Aitken, proved this in early summer 1997 ["Total relative displacement of permutations," **Journal of Combinatorial Theory A**, to appear]. Chartrand, et al., proved that for $r \leq s$ and $s \geq 2$, $\pi(K_{r,s}) = 2(r+s-2)$. We have extended this result to all complete multi-partite graphs by reducing the problem to a combinatorial number theory problem involving sums of squares of entries in certain square integral matrices, then solving the reduced problem. The manuscript containing this result, to be titled "Near-automorphisms of complete multipartite graphs," is in preparation. In future work we aim to treat other general classes of graphs.

4. Acknowledgment. We are very appreciative to the Office of Naval Research for the research support that O. N. R. Grant No. N00014-92-J-1400 made possible. It provided, in part, the most valuable resource for a mathematician, time. The summer research time was most important for obtaining the results on the work proposed. The grant also provided generous support for travel to several important research conferences and meetings, for several visiting scholars to CSUSM with whom valuable research collaboration took place and who also delivered colloquia, for some valuable computer hardware, for a bit of computer software, for some reference tools, and for some very valuable student assistance during the writing of the survey mentioned above. In addition, since the grant was carried out during the formative years of our new university, the local administration of the grant by the CSU San Marcos Foundation (the unit that administers external grants at CSU San Marcos) was an exercise that helped both the Foundation and the University to address and to formulate various procedures and policies that inform the administration of other external grants and contracts. This was not part of the proposed outcome of the grant, but it was an exercise that will pay dividends in the management of this institution into the near future.

ADDENDUM

Summary of Results 1992-1994

(This summary appeared in our September 1994 O. N. R. Grant Proposal which was funded for 1995-1997 then extended to May 31, 1998; the grant continued with the same number as the 1992-1994 grant, Grant No. N00014-92-J-1400. The references here point to the list of references in that September 1994 Proposal)

Perhaps the major accomplishment we have made to date while receiving research support from O. N. R. Grant No. N00014-92-J-1400 was the determination in summer 1993, after a search for several years, of the structure of tournaments (thought of as representing the outcome of majority voting) which admit an agenda (a voting order of the alternatives) for which the sincere (non-strategic) and the, so-called, sophisticated voting decisions are identical. Such tournaments are exactly those for which the initial strong component is a 3-cycle (see [Reid, 1993]). This result has implications for agenda control. In 1993 we polished our joint work [Gu, Reid, and Schnyder, 1994] on plurality preferences based on distances in graphs and our joint work [Reid and Gu, 1994] on plurality preference digraphs realized by trees; these studies were motivated by preferences of users (based at sites) for locations of facilities on networks and will appear in the literature in 1994. Our joint work [Gimble and Reid, 1994] on independent edges in certain bipartite graphs appeared in summer 1994¹.

In late 1993 we launched two new collaborations made possible by O. N. R. Grant N00014-92-J-1400. As a result of work initiated by a November 1993 visit to CSUSM by R. Lundgren, we [Fisher, Lundgren, Merz, and Reid] obtained a first characterization of domination graphs of tournaments in summer 1994 while studying competition graphs of tournaments. We are now sharpening that result, among other things, and plan to have our working manuscript polished and submitted before the end of the year. We also have determined the minimum number of edges possible in the competition graph of a tournament that will form the basis of another manuscript. As a result of work initiated by a December 1993 visit to CSUSM by F. McMorris, we [McMorris and Reid, 1994] obtained some preliminary results on central subtrees of trees and on another centrality idea in trees based on "balanced partitions" of the vertices. This work continues, and we

¹ "Independent edges in bipartite graphs obtained from orientations of graphs," **Journal of Graph Theory** 18 (1994) 515-533.

expect to have a manuscript underway by the end of the year (we are to visit U. of Louisville in October 1994).

In summer 1994, as a result of conversations at the Albuquerque SIAM Meeting with N. Dean concerning a problem of Seymour and Winkler, we [Reid, 1994] constructed an infinite family of tournaments T in which NO vertex has out-degree in T^2 , the square of T , equal to twice its out-degree in T . The tournament version of the Seymour and Winkler conjecture is that every tournament T contains a vertex whose out-degree in T^2 , is AT LEAST twice its out-degree in T . Our result casts only slight reservation about its truth.