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THE CALCULATION OF THE NATURAL FREQUENCY OF A CANTILEVER MONOPLANE WING



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THE CALCULATION OF THE NATURAL FREQUENCY OF A CANTILEVER MONOPLANE WING

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SUMMARY

The purpose of this report is to present a practical application of the calculation of the natural frequency of a stressed skin monoplane wing in bending and in torsion. The theories of the methods employed may be found in the following papers:

Preliminary Study of Fatigue Failures of Metal Propellers Caused by Engine Impulses and Vibration, Air Corps Information Circular, Volume VII, No. 618, by John E. Younger.

Simple Approximate Method of Determining the Natural Frequency of Torsional Vibration (With Particular Reference to Monoplane Wings and Propeller Blades), Airplane Department Memorandum No. 1062, Matériel Division, Air Corps, Wright Field, by John E. Younger.

The calculations are applied to the Fokker C-2A monoplane wing, the final results of which are compared to the experimental results reported in the following paper:

Determination of the Elastic Axis and Natural Periods of Vibration of the C-2A Monoplane Wing, Airplane Department Memorandum No. 1066, Matériel Division, Air Corps, Wright Field, by Charles J. Spere.

The comparison of the experimental and calculated results is as follows:

	Experi- mental	Calcu- lated	Per cent of error
Frequency in bending.....	3.95	4.37	10.60
Frequency in torsion.....	12.00	9.83	18.10

In the Air Corps Information Circular No. 618 mentioned above, the natural frequency of vibration in bending is given as

$$f_b = \frac{\sqrt{g}}{2\pi} \sqrt{\frac{\sum W_1 Y_1 + W_2 Y_2 + \dots + W_n Y_n}{\sum W_1 Y_1^2 + W_2 Y_2^2 + \dots + W_n Y_n^2}} \quad (1)$$

in which—

f_b = Natural frequency of bending in complete cycles per second.

W_n = Weight per running length of span at different intervals.

Y_n = Corresponding deflections due to W_n .

The computations for the deflections Y are based upon the relationship between the loading, shear, moment, slope, and deflection curves expressed in the following:

$$\text{Loading} = w \dots \dots \dots (2)$$

$$\text{Shear} = \int_0^L w \, dx \dots \dots \dots (3)$$

$$\text{Moment} = \int_0^L (\text{Shear}) \, dx \dots \dots \dots (4)$$

$$\text{Slope} = \frac{1}{E} \int_0^L \frac{(\text{Moment}) \, dx}{I} \dots \dots \dots (5)$$

$$\text{Deflection} = \int_0^L (\text{Slope}) \, dx \dots \dots \dots (6)$$

In all the following computations, the graphical method of integration was used in finding the results of the above formulæ. Each curve was plotted against semispan. The semispan was divided into a number of small equal sections from which mean values were found and summed in deriving one curve from another.

The loading curve w was computed from the weight summation of the various component parts of the wing (ribs, spars, and plywood covering) plus a correction factor for each section to make the computed weight of the wing equal to the actual weight. A correction factor is usually necessary because of errors in assumptions and neglect of weight of glue, paint, varnish, nails, fittings, etc.

The weight of the semispan from the computed loading curve, AB, Figure 1, is equal to the area under the curve multiplied by the scale to which it is drawn and is approximately

$$\frac{2.8 \times 460}{2} = 644 \text{ pounds. (See Figure 1.)}$$

The actual weight of the semispan = $\frac{1730.25}{2} = 865$ pounds.

Difference of actual and computed weight = $865 - 644 = 221$ lb./semispan.

This weight difference was divided equally into 10 parts and assumed to be distributed over 10 equal areas in the semispan.

A = area of semispan = 373.3 square feet.

$$\Delta A = \frac{373.3}{10} = 37.33 \text{ square feet} = 5,380 \text{ square inches.}$$

$$\Delta \text{ weight} = \frac{220}{10} = 22 \text{ pounds}/\Delta A \text{ uniformly distributed.}$$

ΔA (equal to Δx times mean chord) was found by scaling from the wing drawing, values of Δx , the incre-

ment of span, and the mean chord to give ΔA equal 5,380 square inches.

$$\frac{\Delta wt}{\Delta x} = \frac{22}{\Delta x}$$

will give the correction factors in pounds per unit length to be applied over increment Δx . The values thus found are given in the following table:

Area	1	2	3	4	5
Mean chord, inches...	149½	149½	145	140	134
Δx , inches.....	36	36	37	38½	40
Correction, pounds per inch.....	0.61	0.61	0.60	0.57	0.55
Area	6	7	8	9	10
Mean chord, inches...	130	125	114	103	81½
Δx , inches.....	41½	43	47	52	66
Correction, pounds per inch.....	0.53	0.51	0.47	0.43	0.39

The corrections were plotted against semi-span in Figure 2 from which values for every 20 inches of span were added to AB to find the loading curve.

This method of finding the loading curve is somewhat different from that used by the United States Army. The Army method is to find—

$$w = \text{weight per square foot} \\ = \frac{\text{weight of wing}}{\text{area}} = \frac{865}{375} = 2.3 \text{ pounds per square foot,}$$

so that the weight per unit length of wing = w times mean chord.

The sections were divided into 2-foot lengths. For section 9, the mean chord is 10.8 feet, so that the weight per running foot of wing is—

$$10.8 \times 2.3 = 24.9 \text{ pounds,}$$

and the weight per running inch of wing is—

$$\frac{24.9}{12} = 2.07 \text{ pounds.}$$

The weight per running inch at the various sections is given in the following table:

Section.....	1	2	3	4	5	6
Mean chord.....	12.50	12.50	12.50	12.50	12.25	11.90
Wt./inch.....	2.40	2.40	2.40	2.40	2.35	2.28
Section.....	7	8	9	10	11	12
Mean chord.....	11.50	11.12	10.80	10.50	10.12	9.75
Wt./inch.....	2.21	2.13	2.07	2.02	1.94	1.87
Section.....	13	14	15	16	17	18
Mean chord.....	9.37	9.00	8.65	8.25	8.00	6.75
Wt./inch.....	1.80	1.73	1.66	1.58	1.53	1.29

This table is plotted in Figure 3 with the loading curve for the purpose of comparison. The former method is more accurate.

The semi-span was divided into small sections for the graphical integration as designated in column 1,

page 7, and the distance between each section is given in column 2. Column 3 gives the mean weight of wing per inch for each section taken from the upper curve of Figure 1.

The mean results at any section (designated by two station numbers) are indicated on the horizontal line between the two stations of that section. Results on the same line as the stations represent the values there and are computed from the mean quantities for that section.

Thus, column 4 gives the weight at each station for the wing section to the right (if the right half of the span is considered) and,

Weight of section at station 19 = mean w for section 19–20 multiplied by $dx = 0.94 \times 20 = 18.8$ pounds.

Column 5 is the evaluation of equation (3) representing the total vertical shear at any points, and is the summation of the shear at all the stations above that point in column 4, since the vertical shear (zero at tip) increases to a maximum at the root. Thus, for station 19,

Vertical shear = $18.8 + 16.6 + 9.2 = 44.6$ pounds, and the mean shear between 19 and 20 = $\frac{44.6 + 25.8}{2} = 35.2$ pounds.

These values are used to find the results in column 7, which is the mean shear times dx .

The total moment given by equation (4),

$$M = \int_0^L (\text{Shear}) dx$$

is represented in column 8, the summation of mean shear times dx . For station 19,

total moment = Σ mean shear times dx of all the sections between station 19 and the tip. = $704 + 350 + 78 + 0 = 1132$ pound-inches.

From the moment the slope is found by equation (5).

$$\text{Slope} = \frac{1}{E} \int_0^L \frac{(\text{Moment}) dx}{I} \\ = \frac{1}{E} \Sigma \frac{M}{I} dx$$

in which—

I = Mean moment of inertia in inches⁴ of both spars and part of the skin for each section.

M = Corresponding mean moment in pound-inches.

dx = Length of section in inches.

E = Modulus of elasticity in pounds per square inch.

In calculating I , the plywood covering approximately four times the width of spar on the top and bottom of both spars was considered as part of the spar. Figure 4 gives the values thus computed, which are listed in column 10.

Column 11 is the mean values of $\frac{M}{I} dx$ and for any station in column 12 the value is for the summation of all the sections below that station in column 11. $\Sigma \frac{M}{I} dx$ for any station in column 12, when divided by E , will give the slope at that point.

The slope at station 3, column 12,

$$\begin{aligned} &= \frac{1}{E} \sum \frac{M}{I} dx \\ &= \frac{1}{E} (321 + 287 + 258) \\ &= \frac{1}{E} 866 \end{aligned}$$

The value E is not substituted until the deflection is found in column 16.

Column 14 is the summation of the average $\frac{M}{I} dx$ in column 13, starting at the root, because the slope there is zero.

Column 15 is the values of column 14 multiplied by dx . In column 16, E (1,300,000 pounds per square inch) is substituted to find the deflection of the wing in inches due to its own weight. The average deflection in feet for each section is given in column 18.

The mean values of WY and WY^2 are listed in columns 20 and 21, respectively, the summation of which is required for formula (1),

$$\begin{aligned} f_b &= \frac{\sqrt{g}}{2\pi} \sqrt{\frac{\sum W_n Y_n}{\sum W_n Y_n^2}} \\ \sum W_n Y_n &= 0.7622 \\ \sum W_n Y_n^2 &= 0.0324 \\ f_b &= \frac{\sqrt{32.2}}{2\pi} \sqrt{\frac{0.7622}{0.0324}} = 0.905 \times 4.85 \\ &= 4.37 \text{ cycles per second.} \end{aligned}$$

The mean experimental value of $f=3.95$ cycles per second.

$$\text{Per cent of error} = \frac{4.37 - 3.95}{3.95} \times 100 = 10.6 \text{ per cent.}$$

The shear, moment, slope, and deflection curves are plotted in Figures 5, 6, 7, and 8. The shear and moment curves are negative; they are zero at the tip of the wing and increase to a maximum at the center. The slope and deflection curves are also negative but are zero at the center with their maximum value at the tip.

Great care must be exercised with the units. The mean deflections Y , in column 18, must be in feet for

the substitution in formula (1). It is usually easier to work in inch units as in these computations. If inch units are employed, use—

- E in pounds per square inch.
- I in inches⁴.
- dx in inches.
- W in pounds per running inch.
- M in pound-inches.

Y will then be in inches, which may be readily converted to feet for substitution in the formula.

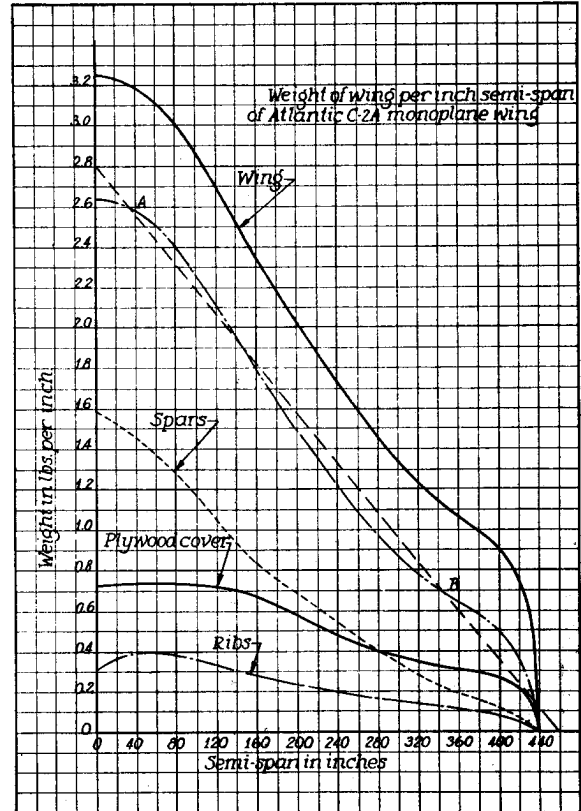


FIGURE 1

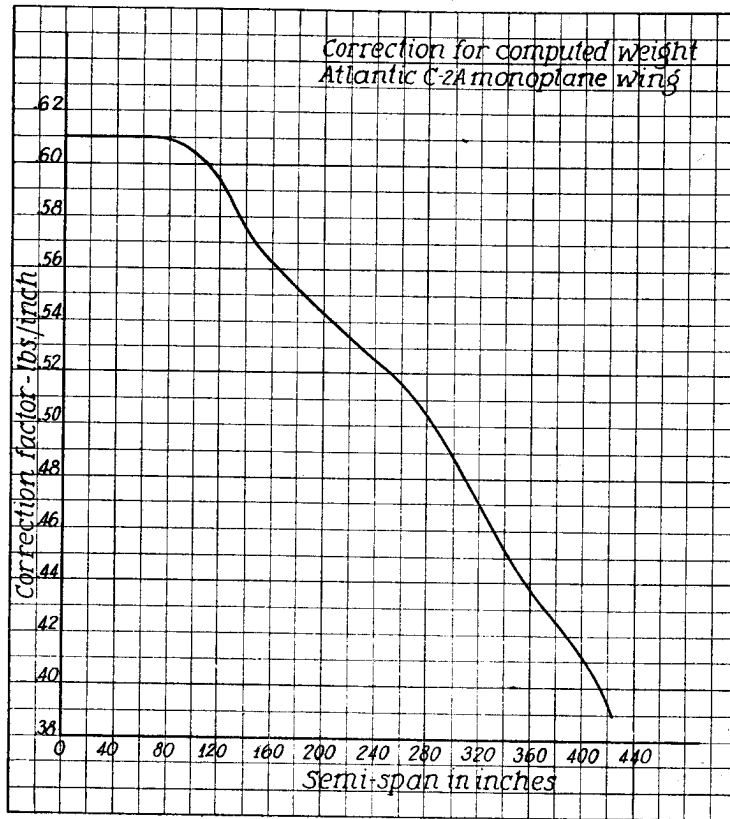


FIGURE 2

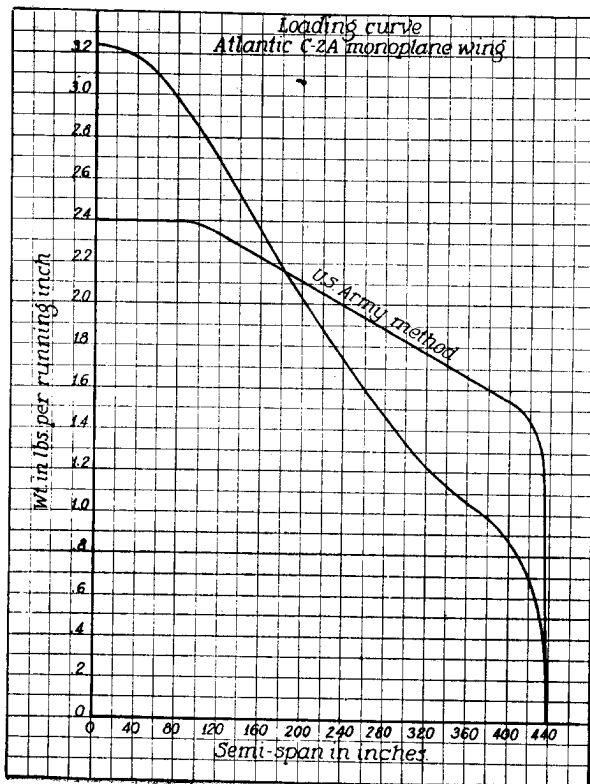


FIGURE 3

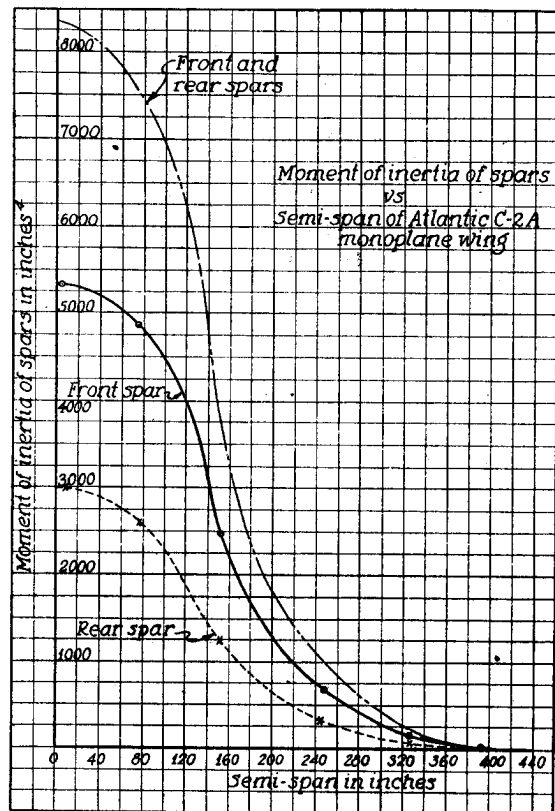


FIGURE 4

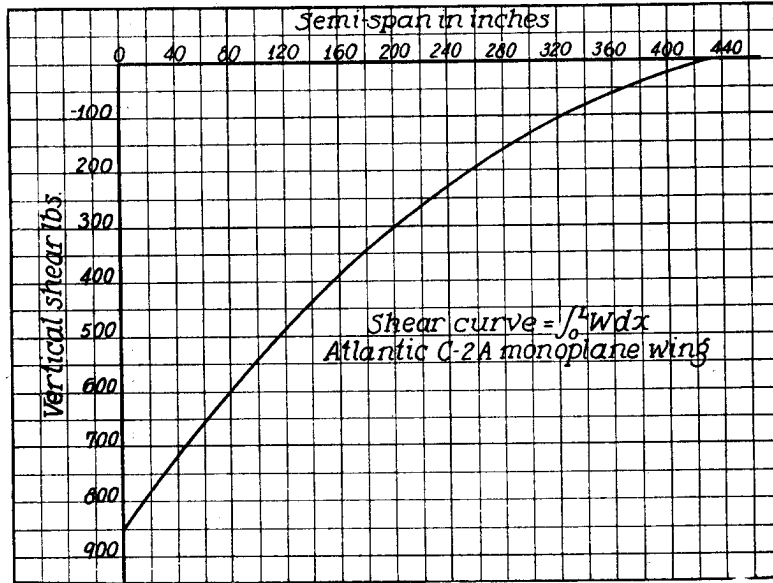


FIGURE 5

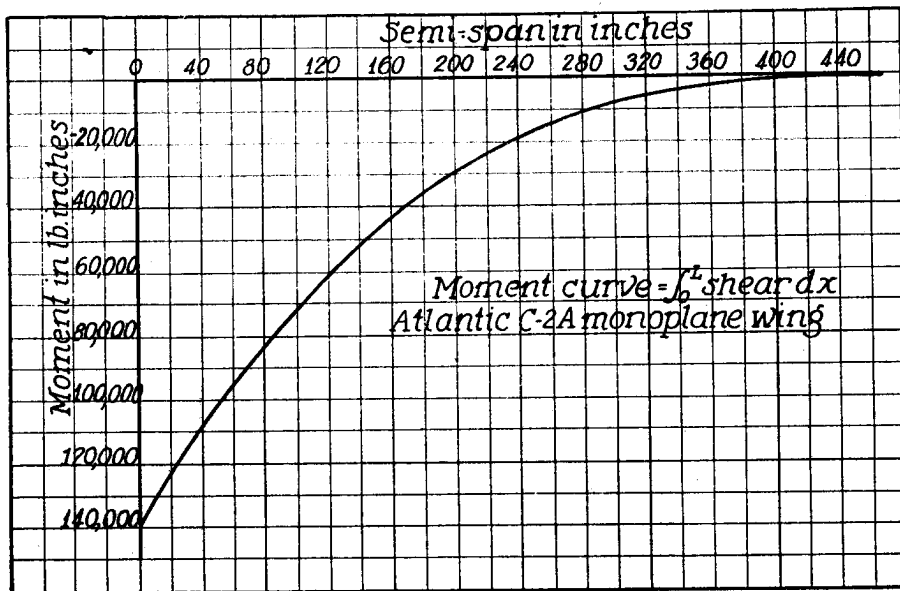


FIGURE 6

Station	dx	W	Wdx	Shear = $\sum Wdx$	Mean shear $\frac{\text{Shear}}{dx}$	Moment = $\sum \text{shear} dx$	M = Mean moment	I	$\frac{M dx}{I}$	$\frac{M}{I} dx$	Mean $\frac{M}{I} dx$	$\frac{\sum \text{Mean slope}}{E}$	$\frac{\sum \text{Mean slope} dx}{E}$	Deflection = $\frac{\sum \text{Mean slope} dx}{1,300,000}$	Deflection	Y = Mean deflection	Y ²	WY	WY ²	
Tip	Inches	Lbs. per in.	Lbs.	Lbs.	Lbs. in.	Lbs. in.	Lbs. in.	Inches ⁴							Feet	Feet				
21	17	0.54	9.2	4.6	0	78	39	5	7,623	506	7,490	72,541	1,450,820	1.11	0.0929	0.0858	0.00735	0.0464	0.00396	
20	20	.83	16.6	17.5	78	428	253	10	6,24	624	6,984	64,985	1,299,700	1.00	.0788	.0764	.00582	.0634	.00433	
19	20	.94	18.8	35.2	350	704	780	25	6,360	624	6,984	57,748	1,154,960	.885	.0740	.0698	.00486	.0656	.00456	
18	20	1.02	20.4	54.8	704	1,132	1,680	60	6,360	560	6,360	51,076	1,021,520	.785	.0655	.0615	.00379	.0626	.00386	
17	20	1.09	21.8	75.9	1,096	2,228	2,987	100	5,800	597	5,800	44,996	899,920	.691	.0576	.0541	.00292	.0590	.00318	
16	20	1.18	23.6	98.6	1,518	3,746	4,732	200	5,204	473	5,204	39,494	789,880	.606	.0505	.0473	.00224	.0539	.00264	
15	20	1.28	25.6	123.2	1,972	5,718	6,950	350	4,731	397	4,731	34,526	690,520	.530	.0442	.0413	.00171	.0528	.00219	
14	20	1.40	28.0	150.0	2,464	8,182	9,682	500	4,334	386	4,334	29,994	599,880	.462	.0384	.0358	.00128	.0501	.00179	
13	20	1.52	30.4	179.2	3,000	11,182	12,974	725	3,948	358	3,948	25,853	517,060	.398	.0331	.0307	.00094	.0466	.00143	
12	20	1.66	33.2	211.0	3,584	14,766	16,876	950	3,590	356	3,590	22,084	441,680	.339	.0282	.0260	.000675	.0431	.00112	
11	20	1.81	36.2	246.0	4,220	18,986	21,446	1,250	3,234	344	3,234	18,672	373,440	.287	.0239	.0220	.000485	.0402	.000878	
10	20	1.95	39.0	283.5	4,920	23,906	26,741	1,600	2,890	334	2,890	15,610	312,200	.240	.0200	.01825	.000333	.0356	.00065	
9	20	2.11	42.2	324.0	5,670	29,576	32,816	2,050	2,556	320	2,556	12,887	257,740	.198	.0165	.01495	.000224	.0316	.000474	
8	20	2.26	45.2	369.0	6,480	36,056	39,746	2,750	2,236	289	2,236	10,491	209,820	.161	.0134	.01208	.000146	.0273	.00033	
7	20	2.44	48.8	415.0	7,380	43,436	47,586	4,000	1,947	238	1,947	8,399	167,980	.129	.01075	.00959	.000092	.0234	.000224	
6	20	2.61	52.2	465.0	8,300	51,736	56,386	5,700	1,709	198	1,709	6,571	131,420	.101	.00842	.00739	.0000546	.0193	.000143	
5	20	2.77	55.4	519.0	9,300	61,036	66,226	6,650	1,511	199	1,511	4,981	99,220	.076	.00635	.00545	.0000297	.0151	.0000824	
4	20	2.92	58.4	576.0	10,380	71,416	77,173	7,250	1,312	213	1,312	3,549	70,980	.056	.00454	.00377	.0000142	.0110	.0000415	
3	20	3.06	61.2	636.0	11,520	82,936	89,296	7,650	1,099	233	1,099	2,344	46,880	.036	.00300	.00237	.0000056	.00725	.0000171	
2	20	3.15	63.0	698.0	12,720	95,656	102,636	7,950	866	258	866	1,361	27,220	.021	.00174	.00127	.00000161	.00400	.00000507	
1	20	3.21	64.2	761.0	13,960	109,616	117,226	8,175	608	287	608	624	12,480	.001	.0008	.000502	.000000252	.00161	.00000081	
Root	20	3.22	64.4	858.0	15,220	124,836	133,096	8,300	321	321	160	160	3,200	0	.000205	.0001025	.0000001025	.00033	.00000033	
					16,520	141,356	141,356	8,300	0	0	0	0	0	0	0	0	$\sum W_n Y_n = .7622$	$\sum W_n Y_n^2 = .09241$		

CALCULATION OF FREQUENCY IN TORSION

The natural frequency of vibration in torsion of a monoplane wing is shown in A. D. M. 1062 to be—

$$f_t = \frac{1}{2\pi} \sqrt{\frac{\sum J_m \theta_o}{\sum J_m \theta_o^2}} \quad (7)$$

where—

f_t = The natural frequency of vibration in complete cycles per second.

J_m = The mass polar moment of inertia per unit length for each wing section measured in slug inches squared.

θ_o = The total angle of twist of the shell in radians between the root and the section under consideration when subjected to a distributed torsional moment.

$$= \Delta\theta_1 + \Delta\theta_2 + \Delta\theta_3 + \dots + \Delta\theta_n.$$

$\Delta\theta_1, \Delta\theta_2, \Delta\theta_3$, etc., are increments of twist at the various sections.

The equation of θ_o given in A. D. M. 1058 is

$$\theta_o = \frac{QLdx}{4A^2tE_s} \quad (8)$$

in which—

$$Q = \sum J_m.$$

L = The mean length of periphery of the section in inches.

J_m has the same meaning as in equation (7).

dx = The length of section in inches subjected to torque.

t = The mean thickness of stressed cover in inches.

A = The average area in square inches bounded by periphery, L .

E_s = The modulus of elasticity of the cover in shear in pounds per square inch.

With reference to A. D. M. 1059, the total area of the airfoil section, A , is—

$$A = 0.725 hc \text{ to } 0.785 hc \quad (9)$$

For these computations the area of the airfoil is taken—

$$A = 0.731 hc.$$

$$L = \left[2.7 \left(\frac{h}{c} \right)^2 + 2 \right] c \quad (10)$$

$$I_x = (0.119h + 0.256c) h^2 t \quad (11)$$

$$I_y = 0.0435 (c + 6h) c^2 t \quad (12)$$

$$J_m = \frac{\rho}{32.2} J dx \quad (13)$$

In which—

t = thickness of cover in inches.

h = maximum ordinate of section in inches.

c = chord of section in inches.

J = static polar moment of inertia of airfoil shell in inches.⁴

$$= I_x + I_y$$

J_m = mass polar moment of inertia in slug inches squared.

ρ = density in pounds per cubic inch for shell.

L = length of periphery of airfoil section in inches.

I_x and I_y are respectively the moments of inertia in inches⁴ of the shell about the X and the Y axis through the center of gravity of the section.

The same sections were used as in the case of bending.

The mean values of h , t , and c are taken respectively from Figures 9, 10, and 11 and are tabulated in columns 24, 25, and 26. These results are used in the evaluation of A , L , I_x , and I_y from equations (9), (10), (11), and (12), the results of which are listed in columns 27, 28, 29, and 30. A and L are plotted in Figures 12 and 13.

The mean value of J for any section is tabulated in column 31, and is found from columns 29 and 30 at that section. $\sum J$ for any section is given in column 32, found by summing all the J 's in column 31 between that section and the wing tip. $\sum J$ for section 19-20 equals $322 + 1969 + 2877 = 5168$ inches.⁴

$\sum J$, however, must be multiplied by a factor 3.22 since J is composed of not only the shell alone, but also the ribs and spars. It was assumed that the proportion of J contributed by the ribs, spars, and shell was the same throughout the wing. By comparing the total moment of inertia of a section with that of the wing covering alone, the factor 3.22 was determined for the section between stations 9 and 10.

Thus, for ribs—

$$I_{major} = 0.0418 c^3 h \quad (14)$$

$$I_{minor} = 0.454 ch^3 \quad (15)$$

(See Air Corps Information Circular No. 597, Volume VI.)

I_{major} and I_{minor} are, respectively, the moments of inertia of the rib about the major and minor axis through the center of gravity.

$$I_{major} = 0.0418 [132 - (2 \times 0.070)]^3 \times [22.9 - (2 \times 0.070)].$$

$$= 0.0418 (131.86)^3 22.76 = 2,170,000 \text{ inches.}^4$$

$$I_{minor} = 0.454 \times 131.86 (22.76)^3$$

$$= 704,000 \text{ inches.}^4$$

$$J \text{ for rib} = 2,170,000 + 704,000 = 2,874,000 \text{ inches.}^4$$

For each section,

$$J \text{ for rib} = 2,874,000 \times \frac{14}{22} = 1,825,000 \text{ inches.}^4 \text{ since}$$

there are 14 ribs in 22 sections. From equation (13)

$$J_m \text{ for ribs} = \frac{\rho}{32.2} 1,825,000 dx \text{ slug inches.}^2 \text{ in which—}$$

ρ = density of spruce in pounds per cubic inch.

$$= 0.0156 \text{ lbs. per cubic inch.}$$

dx = width of rib = 0.062 inch.

$$J_m \text{ for ribs} = \frac{0.0156}{32.2} \times 1,825,000 \times 0.062 = 55 \text{ slug inches.}^2$$

In finding the mass polar moment of inertia of the front and rear spars, it was assumed that they were concentrated masses, M_1 and M_2 , located on the principal axis distant r_1 and r_2 from the center of gravity of the airfoil section. Then—

$$J_m \text{ for spars} = M_1 r_1^2 + M_2 r_2^2.$$

Weight of both spars at section 9-10 = 0.7 lbs. per inch span.

$$\text{Weight of each spar per 20 inches} = 0.35 \times 20 = 7 \text{ lbs.}$$

$$r_1 = 43.7 \text{ inches; } M_1 = \frac{7}{32.2}$$

$$r_2 = 19.3 \text{ inches; } M_2 = \frac{7}{32.2}$$

hence—

$$J_m \text{ for spars} = \frac{7}{32.2} \times (43.7)^2 + \frac{7}{32.2} \times (19.3)^2$$

$$= 415 + 81 = 496 \text{ slug inches squared.}$$

This value of J_m is not strictly correct as the weight of the spars is not equally divided according to the assumption. The proportion of front and rear spar weights is given in Appendix A by Mr. G. A. Zink.

$$J_m \text{ for shell} = \frac{\rho}{32.2} J dx$$

in which—

$$J = 15,642 \text{ inches}^4. \text{ (See column 31, sec. 9-10.)}$$

$$\rho = \text{density 3-ply birch} = 0.0257 \text{ pound per cubic inch.}$$

$$dx = \text{length of shell} = 20 \text{ inches.}$$

$$J_m \text{ for shell} = \frac{.0255}{32.2} \times 15642 \times 20.$$

$$= 248 \text{ slug inches}^2 \text{ at section 9-10.}$$

The total mass polar moment of inertia at section 9-10 is the sum of—

$$\text{Shell} = 248 \text{ slug inches}^2.$$

$$\text{Ribs} = 55 \text{ slug inches}^2.$$

$$\text{Spars} = 496 \text{ slug inches}^2.$$

$$\text{Total } J_m \text{ for section 9-10} = 799 \text{ slug inches}^2.$$

$$\text{Ratio of } \frac{J_m \text{ of total}}{J_m \text{ of shell}} = \frac{799}{248} = 3.22$$

which, when multiplied by the static polar moment of inertia, J , gives the total static polar moment of inertia at that section. Thus, equation (13) becomes

$$J_m = J \cdot 3.22 \cdot \frac{\rho}{32.2} dx \text{----- (16)}$$

In the actual computations, it is not necessary to change J to J_m for each section by equation (16) since ρ , the density of plywood covering, is a constant in ordinary airplane construction. Equation (7) can be modified to use J , which simplifies the computations. From equations (8) and (16)—

$$\theta_o = \frac{\sum J L dx}{4A^2 t E_s} \cdot 3.22 \cdot \frac{\rho}{32.2} dx$$

$$\theta_o = \frac{\sum J L}{4A^2 t E_s} \times 3.22 \times \frac{\rho}{32.2} dx^2$$

$$\theta_o = \theta \frac{1}{E} \times 3.22 \times \frac{\rho}{32.2} dx^2 \text{----- (17)}$$

where—

$$\theta_o = \frac{\sum J L}{4A^2 t} \text{----- (18)}$$

$\sum J$ is taken over the part of the wing between the tip and the section considered since the formula for θ_o was derived on the assumption that J is equal to the distributed torque. If J is used instead of J_m , equation (7) becomes

$$f_t = \frac{1}{2\pi} \sqrt{E_s} \sqrt{\frac{\sum J \theta}{\sum J \theta^2}} \sqrt{\frac{32.2}{3.22 \rho dx^2}} \text{----- (19)}$$

where ρ = density of plywood covering in pounds per cubic inch.

The average values of $\Delta\theta$ are found from columns 25, 27, 28, and 32 which are listed in column 33. For example—

$$\Delta\theta \text{ at section 19-20} = \frac{\sum J L}{4A^2 t} \frac{5165 \times 192}{4 \times 640^2 \times 0.047} = 12.87$$

The $\Delta\theta$'s are proportional to the increments of twist in length dx of the wing. $\sum \Delta\theta$ in column 34 is the sum of $\Delta\theta$'s in column 33, starting with zero at the

root, since the wing there is considered rigidly fixed. $\sum \Delta\theta$ at any station gives the total angle of twist, θ_o , in radians at that point when multiplied by $\frac{3.22 \rho dx^2}{32.2 E_s}$, in which case dx is 20 inches, E_s equals 685,500, and ρ equals 0.0257. (See equations (17) and (18).) $\sum \Delta\theta$ plotted in Fig. 14 shows the shape of the θ_o curve. $\sum J \times \sum \Delta\theta$ and $\sum J \times \sum \Delta\theta^2$ are found from columns 32, 34, and 35, and are tabulated in 36 and 37 the sum of which columns are used in the substitution of equation (19).

$$f_t = \frac{1}{2\pi} \sqrt{E_s} \sqrt{\frac{\sum J \theta}{\sum J \theta^2}} \sqrt{\frac{32.2}{1.82 \rho dx^2}}$$

$$\sum J \theta = 293,343 \times 10^3.$$

$$\sum J \theta^2 = 5182 \times 10^7.$$

$$\rho = 0.0255 \text{ lbs. per cubic inch.}$$

$$dx = 20 \text{ inches.}$$

From tests by the Materials Branch on shear of birch plywood, page 24,

$$E_s = 685,500 \text{ pounds per square inch.}$$

$$f_t = \frac{1}{2\pi} \sqrt{685,500} \sqrt{\frac{293,343 \times 10^3}{5182 \times 10^7}} \sqrt{\frac{32.2}{3.22 \times 0.0255 \times 20^2}}$$

$$= \frac{1}{2\pi} \times 829 \times \frac{75.3}{10^2} \times 0.99.$$

$$= 9.83 \text{ cycles per second.}$$

$$\text{Experimental } f_t = 12 \text{ cycles per second.}$$

$$\text{Per cent error} = \frac{2.17}{12} = 18.1 \text{ per cent.}$$

The calculated frequency of torsional vibration is affected by the rigidity of the spars and the modulus of shear of birch plywood. The rigidity of the spars has been neglected, which, if considered, would give a higher calculated frequency. Little testing has been carried out to determine the modulus of shear of birch plywood, and until more extensive tests are made the modulus of shear must be accepted as given.

SHEAR TESTS ON BIRCH PLYWOOD

(Copy of Data Furnished by Materials Branch)

1. The values for the shear test parallel to the face grain on three equal ply, all birch plywood of 1/16 inch thickness for the beam of the spruce flanges are given as follows:

Web No.	Ultimate shear	Modulus of elasticity in shear	Remarks
1.....	Lbs. per sq. in. 2,140	Lbs. per sq. in. 715,000	This web, located on upper face at east end, failed first.
2.....	2,606	656,000	This web, upper face at west end, failed second.
Average....	2,480	685,500	

2. The results were obtained from tests on specimens $3 \times 5\frac{1}{4}$ inches with a gap of $\frac{1}{4}$ inch between the shear tools.

3. The same average values for modulus of elasticity and shear may be used for the web of beams with plywood flanges. The values for the modulus of elasticity in shear as given above are only approximate inasmuch as the number of specimens tested is limited and accurate values are difficult to obtain.

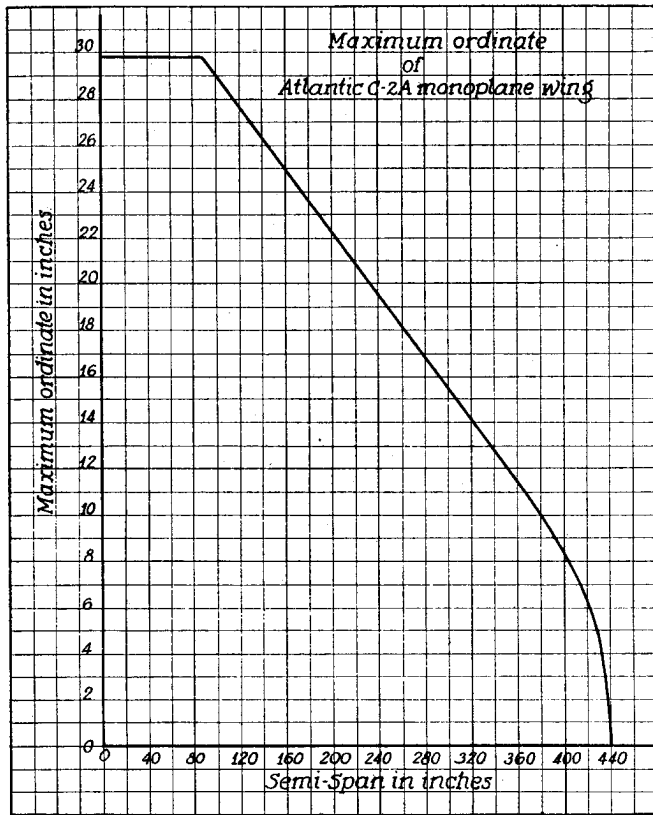


FIGURE 9

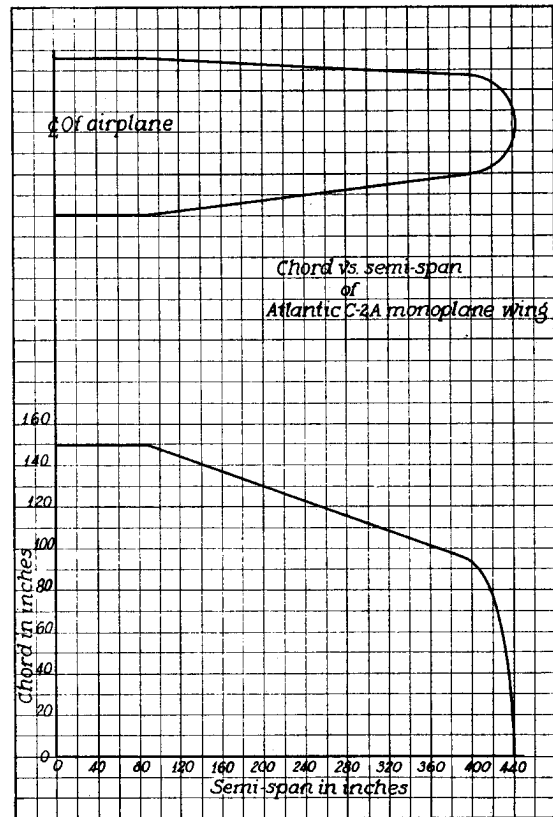


FIGURE 11

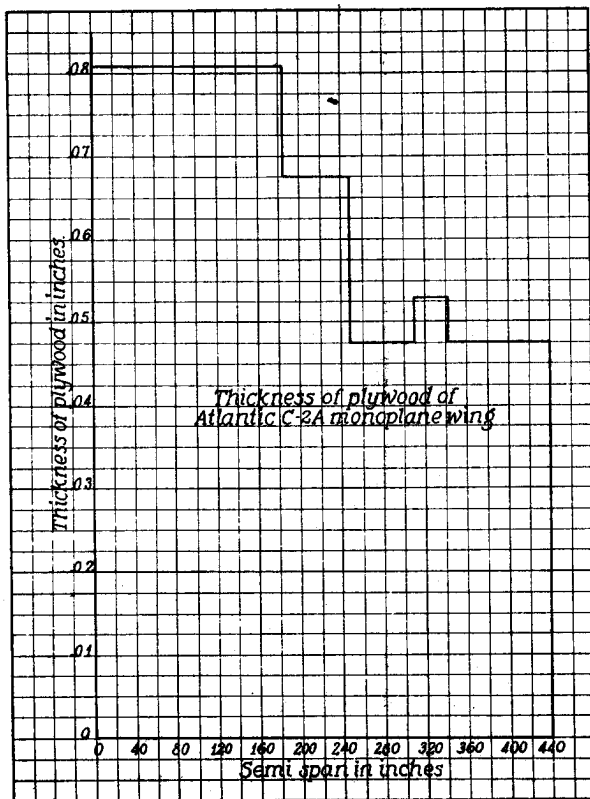


FIGURE 10

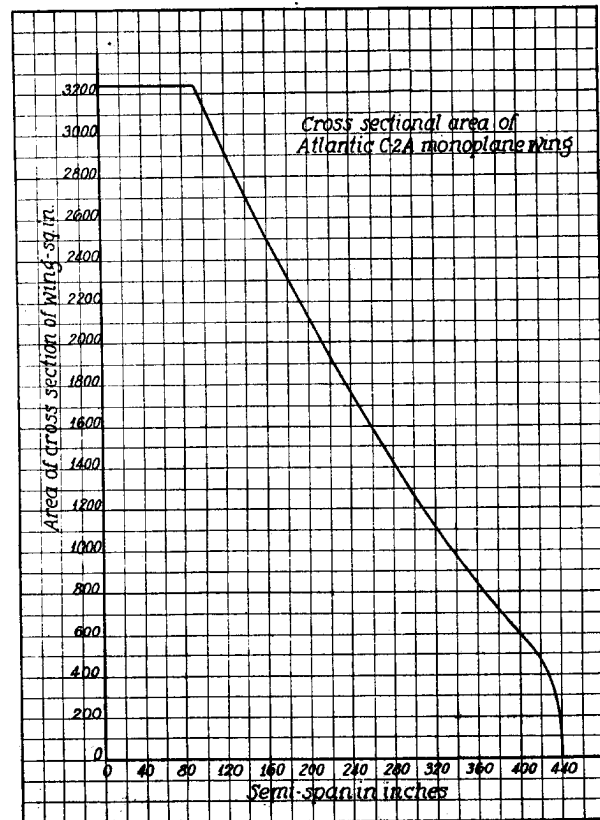


FIGURE 12

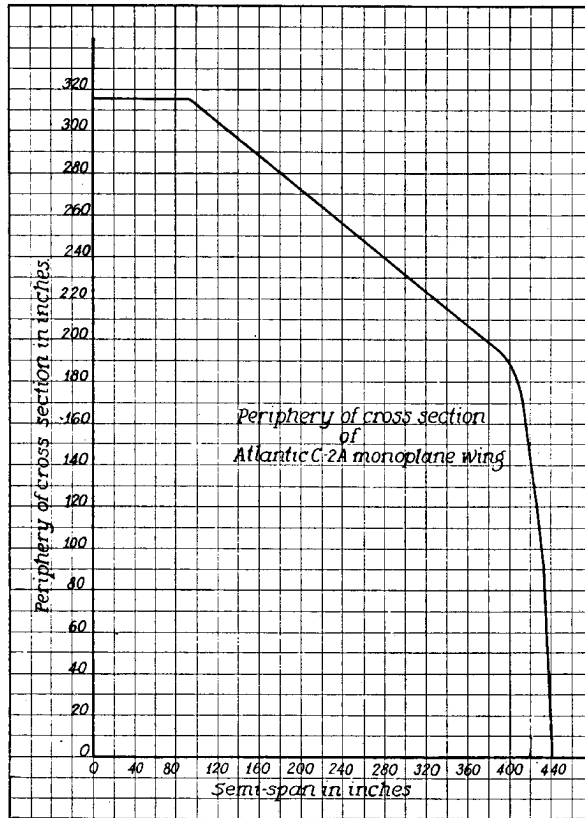


FIGURE 13

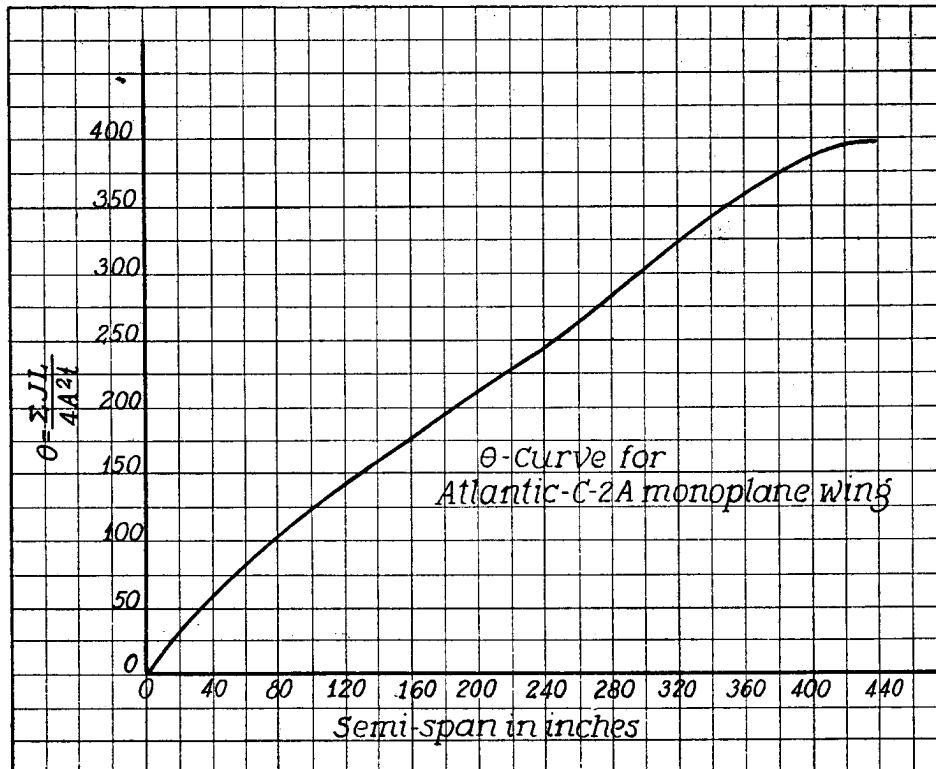


FIGURE 14

APPENDIX A

The weights of the rear and front spars, as calculated from the details of the spars between stations 9 and 10, are 5.84 and 7.08 pounds respectively. These weights were based on a density of 0.0156 pounds per cubic inch for the spar material.

The dimensions referred to are at a distance of 190.0 inches from the fuselage, and are as follows:

$$c \text{ (chord)} = 132.0 \text{ inches.}$$

$$h \text{ (maximum ordinate)} = 22.9 \text{ inches.}$$

$$t \text{ (thickness of plywood shell)} = 0.070 \text{ inches.}$$

The formulas used in the following computations are the same as those used in the discussion. A sample computation for the determining of J_m for the wing cell follows:

$$I_x = [0.119 (22.9) + 0.256 (132.0)](22.9)^2 (0.070) \\ = 1,342 \text{ inches}^4.$$

$$I_y = (0.0435)[132.0 + 6 (22.9)](132)^2 (0.070) \\ = 14,300 \text{ inches}^4.$$

$$J = I_x + I_y \\ = 15,642 \text{ inches}^4.$$

$$J_m = \frac{(0.0255)}{32.2} (20) (15,642) \\ = 248.0 \text{ pounds-inches}^2.$$

J_m for the ribs is found as follows:

$$c_1 = c - 2 (0.070); c_1 = 131.86 \text{ inches.}$$

$$h_1 = h - 2 (0.070); h_1 = 22.76 \text{ inches.}$$

$$\text{Center distance of spars} = 63.0 \text{ inches.}$$

$$I_y = 0.0418 c_1^3 h_1.$$

$$I_x = 0.454 c_1 h_1^3.$$

$$I_y = 0.0418 (131.86)^3 (22.76) \\ = 2,170,000 \text{ inches}^4.$$

$$I_x = 0.454 (131.86) (22.76)^3 \\ = 704,000 \text{ inches}^4.$$

$$J = I_x + I_y \\ = 2,874,000 \text{ inches}^4.$$

$$J_m = \frac{(0.0156)}{32.2} (0.062) (2,874,000) \left(\frac{14}{22}\right) \\ = 55.0 \text{ pounds-inches}^2.$$

J_m for the spars.

Distance of C. G. from leading edge of wing = 62.7 inches.

$$r_1 = 19.3 \text{ inches, } r_2 = 43.7 \text{ inches.}$$

$$M_1 = \frac{5.84}{32.2}, M_2 = \frac{7.08}{32.2}$$

$$J_m = \frac{5.84}{32.2} (19.3)^2 + \frac{7.08}{32.2} (43.7)^2 \\ = 487.5 \text{ pounds-inches}^2.$$

Weight of rear spar/20 inches of length, stations 9-10.
2.92 pounds weight of web.

2.92 pounds weight of beam.

Total weight of spar, rear, 20 inches length = 5.84 pounds.

Weight of front spar/20 inches of length, stations 9-10.

4.21 pounds weight of web.

2.87 pounds weight of beam.

Total weight of spar, front, 20 inches length = 7.08 pounds.

C. G. along x axis = 132.0 (0.475) = 62.7 inches.

C. G. along y axis = 22.9 (0.37) = 8.47 inches.

Shell—

$$J = I_x + I_y.$$

$$I_x = (0.119h + 0.256c)h^2t.$$

$$I_x = [0.119 (22.9) + 0.256 (132.0)](22.9)^2 (0.070).$$

$$I_x = 1,342.$$

$$I_y = 14,300.$$

$$J = 15,642.$$

$$J_m = \frac{0.0255}{32.2} (20) (15,642) = 248.0.$$

Rib—

$$I_y = 0.0418 c_1^3 h_1.$$

$$I_x = 0.454 c_1 h_1^3.$$

$$c_1 = c - 2 (0.070) = 131.86 \text{ inches.}$$

$$h_1 = h - 2 (0.070) = 22.76 \text{ inches.}$$

$$\text{C. L. rear spar to C. L. front} = 63.0 \text{ inches.}$$

The total J_m for section considered is equal to the sum of the J_m 's of the shell, rib, and spars, or—

$$J_m = 487.5 + 55.0 + 248.0$$

$$= 790.5 \text{ pounds-inches}^2.$$

$$\text{Ratio of } \frac{J_m \text{ of total } 790.5}{J_m \text{ of shell } 248.0} = 3.19.$$