QUALITY CONTROL AND RELIABILITY

TECHNICAL REPORT TR 4

SAMPLING PROCEDURES AND TABLES FOR LIFE AND RELIABILITY TESTING BASED ON THE WEIBULL DISTRIBUTION

(HAZARD RATE CRITERION)

DEPARTMENT OF DEFENSE
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28 FEBRUARY 1962

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The content of this technical report was prepared on behalf of the Office of the Assistant Secretary of Defense (Installations and Logistics) by Professors Henry P. Goode and John H. K. Kao of Cornell University through the cooperation of the Office of Naval Research. It was developed to meet a growing need for the use of mathematically sound sampling plans for life and reliability testing where the Weibull Distribution adequately approximates the test data.

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FOREWORD

This technical report presents a proposed acceptance-sampling procedure together with related sampling-inspection plans for the evaluation of lot quality in terms of the instantaneous failure rate or hazard rate as a function of time. The Weibull distribution, including the exponential distribution as a special case, is used as the underlying lifelength model. The report has been prepared to supplement the procedures and tables of sampling plans for use when lot quality is to be evaluated in terms of mean item life which have been presented in Department of Defense Technical Report No. 3,[15] and have also been discussed elsewhere.[1, 18, 19, 21] The study upon which this report is based was done at Cornell University under a contract sponsored by the Office of Naval Research.

The procedures and plans are for use when inspection of the sample items is by attributes with life testing truncated at some specified time. A set of conversion factors has been prepared from which attribute sampling-inspection plans of any desired form may be designed or from which the operating characteristics of any specified plan may be determined. A comprehensive set of Weibull sampling-inspection plans has also been compiled and included, as well as tables of products for adapting the Military Standard 105C[20] to life testing and reliability applications. In all three of these elements of the study and the report, the exponential model has been included as a special case. As in the case of the previous report, both the procedures and the plans are for use in cases for which the value for the Weibull shape parameter is known or can be assumed. Conversion tables and pages of plans have been provided for a wide range of values for this parameter.

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SECTION 1
INTRODUCTION

1.1 Summary

This technical report outlines an acceptance-sampling procedure and presents tables of related sampling-inspection plans for the evaluation of lot quality in terms of the instantaneous failure or hazard rate as a function of time. The Weibull distribution (including the exponential as a special case) is used as the underlying lifelength model. Inspection of sample items is by attributes with life testing truncated at the end of some specified time. Tables of factors are also provided from which sampling plans for instantaneous failure or hazard rate may be designed to meet given needs or from which the operating characteristics for any specified plan may be evaluated. Also included is a discussion of applications in terms of the average hazard rate as a function of time, a measure which turns out to be distribution-free.

1.2 Introduction

The sampling-inspection procedure and tables of plans presented in the report evaluate the lot in terms of the instantaneous failure or hazard rate at some specified time. They have been designed to match and supplement the procedure and plans for the evaluation of the lot in terms of mean life which have been published as Department of Defense Technical Report No. 3[15] and which will also be found in the proceedings of the Seventh National Symposium on Reliability and Quality Control[1] and in the transactions of the Fifteenth Annual Convention of the American Society for Quality Control, 1961. In both these cases the procedures and plans are for applications for which the Weibull distribution or the exponential distribution, which is a special case of the Weibull, can be assumed as the underlying lifelength model.

The report previously published discussed the nature of the Weibull distribution, the relationships between it and the exponential, and similar points. Hence material on these points will not be repeated here; the report referenced may be consulted. For further information on the Weibull distribution as a statistical model for reliability and lifelength analysis, reference may also be made to a paper by Kao which will be found in the Proceedings of the Sixth National Symposium on Reliability and Quality Control.[2]

It should be noted, however, that the Weibull distribution is a three-parameter distribution; (1) a location or threshold parameter, commonly symbolized by the letter γ, (2) a scale, or characteristic life parameter, symbolized by η, and (3) a shape parameter, symbolized conventionally by the letter β, are all required to completely describe a particular Weibull distribution. For a great many applications the location parameter, γ, can be assumed to equal zero. This means, in effect, there is some probability of item failure right from the start of life or use--there is no initial period of life that is free of risk of failure. The direct application of the factors and sampling-plan tables given in this report assume $\gamma = 0$. However, if γ has some known value other than zero the procedure and tables can be easily and simply modified to allow for this. The method for doing so will be described in another section. The procedure and the information in the tables is independent of the magnitude of η, the scale parameter; the value for this parameter need not be known or estimated. The reason for this will be noted in the section of the report dealing with the mathematical relationships. The Weibull shape parameter, β, is important, however. The sampling plans presented here depend on its magnitude and are for application to product for which the value for this parameter is known or can be assumed to approximate some given value.

Basic factors for the design and evaluation of plans to meet specified needs, and comprehensive tables of single-sampling plans have been prepared and included for each of eleven values for β, values of $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, 1, 1 $\frac{1}{3}$, $1\frac{2}{3}$, 2, $2\frac{1}{2}$, $3\frac{1}{3}$, 4, and 5. This range of values covers the range of shape parameters normally encountered with industrial and military products. Values for β of less than 1 apply to products whose distribution of failures is such that the failure rate** is high in early life and gradually decreases with the passage of time. This seems to be the case for many electronic components such as transistors and resistors. Recognition of the fact that the failure rate decreases (or increases, for that matter) with the passage of time and allowance for this fact is extremely important if acceptance sampling-inspection plans for lifelength and reliability are to be applied realistically. For β = 1 the Weibull distribution is the same as the exponential; the exponential distribution, in effect, being a special case of the Weibull. At β = 1 the failure rate is constant and does not change with the passage of time. Use of exponential sampling plans assumes this constancy--an assumption that may not be warranted for a large proportion of applications. For values of β greater than 1, the failure rate is relatively small at the start of life or service but increases with the passage of time, the rate of increase being larger for larger values of β. Thus β-values larger than 1 may quite logically apply for items for which wear-out or fatigue is an important cause for failure--items such as electron tubes or many mechanical components.

As an illustration of the above comments, reference may be made to Figure 1 which shows, for the Weibull distribution, the relationship between the instantaneous failure or hazard rate (or simply hazard), symbolized by Z (t), and life or time, t , for

**See Appendix A for the definition of failure rate and its relationship to hazard rate.

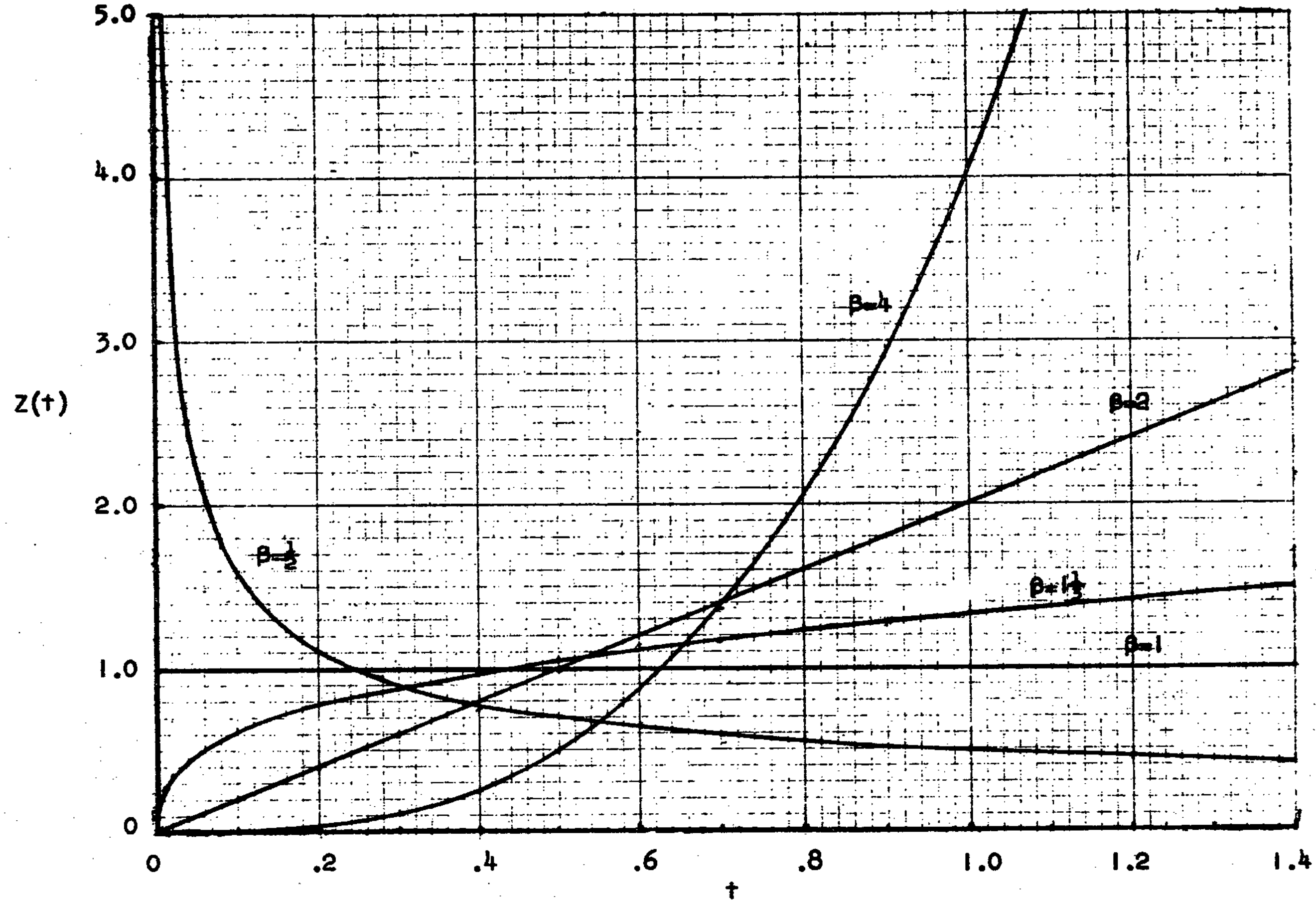


Figure 1. Instantaneous Failure Rate as a Function of Time for Various Values for β (η=1)

various values for β. The value for the location parameter, γ has been taken as zero. Also, in each case a value for the scale parameter, η, of unity has been used.

1.3 The Acceptance Procedure

The factors and the sampling-inspection plans included in this report have been designed for use under the following acceptance procedure:

(a) Select at random a sample of n items from the submitted lot.

(b) Place the selected items on life test for some specified period of time, t.

(c) Determine the number of items that fail during the period of time, t.

(d) Compare the number of items that fail with a specified acceptance number, c.

(e) If the number of sample items that fail is equal to or less than the acceptance number, c, accept the lot; if the number that fail exceeds the acceptance number, reject the lot.

Testing of sample items may be curtailed prior to time t if the lot is to be rejected since it is possible to observe (c + 1) failures in less than t units of time.

It may be noted from this outline that the procedure is for attribute inspection and it takes the form commonly employed for acceptance sampling-inspection when the lot quality of interest is simply the proportion or number defective rather than reliability or item life. The only variation in method is the use of a testing truncation time, t.

Under this procedure the probability of acceptance for a lot, $P(A)$, depends on the probability, p', of item life being less than (or equal to) the test truncation time, t. For applications for which the value for the shape parameter, β, is known (or can be assumed to approximate some known value) and for which the testing truncation time, t, is specified, the probability p' is a function only of the hazard rate $Z(t)$ at time t. Since p' is a function only of t and $Z(t)$, the operating characteristics for any given plan depend only on t and $Z(t)$. In order to provide factors and sampling tables for general use rather than for specific values for t and $Z(t)$, they have been provided in terms of the product $tZ(t)$. Conversion of values for this product to desired values when t or $Z(t)$ is specified, will obviously be quite easy. This will be demonstrated in the discussion of examples that follow later.

To provide a means for the design or evaluation of sampling inspection plans, two tables of factors have been prepared. One, Table 1, lists $tZ(t)$ values for various values of p'. For convenience in tabulation and use, p' (in %) values are used and $tZ(t) \times 100$ rather than $tZ(t)$ values are tabulated. The second, Table 2, tabulates p' (%) values for various values of $tZ(t) \times 100$. In both, values are supplied for each of the eleven selected values for β. Through the use of these tables, acceptance-sampling plans of any desired operating characteristics can be designed, or specified plans can be evaluated using the mathematics and practices ordinarily employed in attribute inspection.

A final point of procedure that should be mentioned is that the factors and tables of plans are for direct application in cases for which the time t at which the hazard is specified or is to be tested is the same as the time t at which life-testing of sample items is to be truncated. However, a table of factors has been prepared, (Table 5) to use in a simple modification which allows the test truncation time to differ from the time at which the hazard rate is specified. The life test time for sample items can be one-half or one-fifth, for example, of the time at which the hazard rate is specified.

SECTION 2

THE BASIC CONVERSION FACTORS

2.1 Computation of the Conversion Factors.

The instantaneous failure or hazard rate or simply hazard at any specified time t, which may be symbolized by Z(t), may be expressed by the relationship

$$Z(t) = f(t) / R(t) \qquad (1)$$

where f(t) is the population density function (p.d.f.) and where

$$R(t) = 1 - F(t), \qquad (2)$$

for which F(t) is the cumulative distribution function (c.d.f.).

For the Weibull distribution (and for the case for which the value for γ, the location parameter is 0), the expression for f(t), the population density fuction, is

$$f(t) = (\beta/\eta)\,(t/\eta)^{\beta-1} \exp\left[- (t/\eta)^{\beta} \right]. \qquad (3)$$

Again for the Weibull distribution and for $\gamma = 0$, the expression for F(t), the cumulative distribution fuction, is

$$F(t) = 1 - \exp\left[- (t/\eta)^{\beta} \right]. \qquad (4)$$

Using these expressions the hazard can now be given by dividing Equation (3) by the unit compliment of Equation (4), thus,

$$Z(t) = (\beta/\eta)\,(t/\eta)^{\beta-1}. \qquad (5)$$

For the steps to follow, it will be useful to multiply each side of the above equation by (t/β). This step will give the relationship

$$\frac{tZ(t)}{\beta} = (t/\eta)^{\beta}. \tag{6}$$

In the design or evaluation of attribute sampling plans, one is concerned with the probability of a sample item failing before the end of the test time, t. This probability, which may be symbolized by p', is given by the cumulative distribution function (c.d.f.); thus,

$$p' = F(t) = 1 - \exp[-(t/\eta)^{\beta}]. \tag{7}$$

Combining Equations (6) and (7), p' in terms of Z(t) becomes,

$$p' = 1 - \exp\left[-\frac{tZ(t)}{\beta}\right]. \tag{8}$$

From this expression it may be noted that upon transposing and taking the natural logarithm that

$$\frac{-tZ(t)}{\beta} = \ln(1-p') \text{ or}$$

$$tZ(t) = -\beta \ln(1-p'). \tag{9}$$

The two equations, Equations (8) and (9) furnish the basic relationships for computing the factors required for the design or evaluation of the attribute sampling-inspection plans for lifelength and reliability being considered in this study. Equation (8) may be used for computing values for p' corresponding to given values for tZ(t) and Equation (9) may be used for computing values for tZ(t) corresponding to given values for p'.

For convenience in computation, Equation (9) can be rewritten as,

$$tZ(t) = \beta \exp\{\ln[-\ln(1-p')]\}. \tag{10}$$

Values for the expression

$$-\ln [-\ln (1-p')] \qquad (11)$$

are available from a table of the inverse of the cumulative probability function of extremes, compiled by the National Bureau of Standards.[3] For both the relationship equations values for e raised to the powers indicated were read from the National Bureau of Standards tables of the exponential function.[4]

From the relationship equations an important point may be noted, which is that for the attribute form of inspection used in the acceptance-sampling procedure, the Weibull scale parameter, η, is not directly involved With the value for the shape parameter, β, known or given, only the product of test time, t, and the hazard of interest, $Z(t)$, are of concern. Attribute plans may be designed or evaluated in terms of $tZ(t)$ and with one element of this product given or assumed, the other element may readily be determined.

In the above analysis it has been assumed that the value for γ, the Weibull location parameter, is zero. For a large proportion of possible applications this will actually be the case. However, if in an application γ has some non-zero value, adaptation of the procedure to allow for this is quite easy. All that must be done is to subtract the value for γ from the value used for t to get a converted value, t_0. This converted value is then applied to form a converted product $t_0Z(t_0)$ which can be used for all computations and readings from the table. Any solution in terms of t_0 can then be converted back to real values by simply adding the value for γ.

2.2 The Tables of Factors and Their Use.

With the use of Equation (9) a table of values for $tZ(t)$ for various values of p' has been prepared. It is presented at the end of this report as Table 1. For convenience in tabulation and use, p' values have been multiplied by 100 and expressed as percentages and values for tZ(t) have likewise been multiplied by 100 to give tZ(t) x 100 values. The values used for p' range from .010% to 80% with the selection made in accordance with a standard preferred number series. Also, through the use of Equation (8), a table of values for p' (%) for various values for tZ(t) x 100 has been prepared. It will be found presented as Table 2. With this table, values for p' (%) may be found without interpolation when rounded values for tZ(t) x 100 are given. The two tables provide the basic factors necessary for the design or evaluation of attribute sampling inspection plans of the form described in the previous section of this report. Two examples of their use follow.

Example (1)

One possibility of use for the conversion factors is in the evaluation of specified acceptance sampling plans. Suppose, to consider a simple example, a single-sampling attribute plan has been specified with a sample size, n, of 115 items and an acceptance number c, of 3 items. The test time for the sample items is to be 500 hours. A value for β of 2 and a value for γ of 0 seem reasonable to assume. The operating characteristics for the plan under these conditions must be determined. Lot quality is to be evaluated in terms of the hazard at 500 hours of use.

Under plans of the form presented here, the probability of acceptance for a lot, P(A), depends on the probability, p', of an item failing before the end of time, t, which in this case is 500 hours. The first step, then, in determining the operating characteristics is to determine the probability of acceptance, P(A) associated with each of an appropriate range of values for p'. These probabilities may be determined by one of the methods commonly used in the evaluation of ordinary attribute sampling inspection plans. It is most convenient to make use of the readily available tables of the hypergeometric distribution, of the binomial distribution, or of the Poisson distribution--the choice depending on the ratio of the sample size to the lot size, the size of the sample, the range of p' values involved, and on the accuracy desired. If the plan happens to be one matching an existing attributes plan of the ordinary form (for defectives) for which an operating characteristic curve has been prepared, the required values for the probability of acceptance may be read from this curve to sufficient accuracy for ordinary use. For the example under discussion, the values for P(A) corresponding to each of a suitable series of p' values are listed in the tabulation below (in the second and first columns, respectively). Note that the p' values have been selected from those used in the construction of the table of conversion factors (Table 1) and that the range of p' values used is the range required to give values for P(A) for most of the range from 1.00 to 0 so that a complete picture of operating characteristics may be obtained.

The next step is to make use of the first of the tables of conversion factors, Table 1, and read off the value for tZ(t) x 100 corresponding to each of the listed p' (in %) values. These are found in the seventh column of factors in Table 1, the column for $\beta = 2$. The values for this example are listed in the third column of the tabulation below.

The final step is to divide each of the values tabulated for tZ(t) by the value specified for t, which is 500 hours, to give the associated hazard, Z(t). These computations have been made with the results as listed in the last column of the tabulation below.

It may now be noted by scanning the figures in this last column that for a lot made up of product whose hazard is 4.83×10^{-5} at 500 hours, the probability of acceptance will be .95: if, on the other hand, the hazard is 26.88×10^{-5}, the probability of its acceptance is only .06, for example . If desirable, each P(A) value and its associated Z(t) value may be plotted to construct an operating characteristic curve. This has been done for this application with the resulting curve being the one shown for $\beta = 2$ in Figure 2. To provide some indication of the sensitivity of the acceptance-inspection procedure to changes in the value for β, the shape parameter, operating characteristic curves have been prepared and presented in Figure 2 using the same plan and the same value for t, but with other values for β, values of 1 and 4 with the value 1 representing the exponential distribution commonly assumed in reliability work.

p' (in %)	P(A)	tZ(t) x 100	Z(t)
.5	.99	1.002	2.00×10^{-5}
.8	.98	1.606	3.21×10^{-5}
1.0	.97	2.010	4.02×10^{-5}
1.2	.95	2.414	4.83×10^{-5}
1.5	.90	3.022	6.04×10^{-5}
2.0	.80	4.040	8.08×10^{-5}
2.5	.68	5.064	10.13×10^{-5}
3.0	.55	6.092	12.18×10^{-5}
4.0	.33	8.164	16.33×10^{-5}
5.0	.17	10.258	20.52×10^{-5}
6.5	.06	13.442	26.88×10^{-5}
8.0	.02	16.676	33.35×10^{-5}

$\beta = 2$ $n = 115$ $c = 3$ $t = 500$

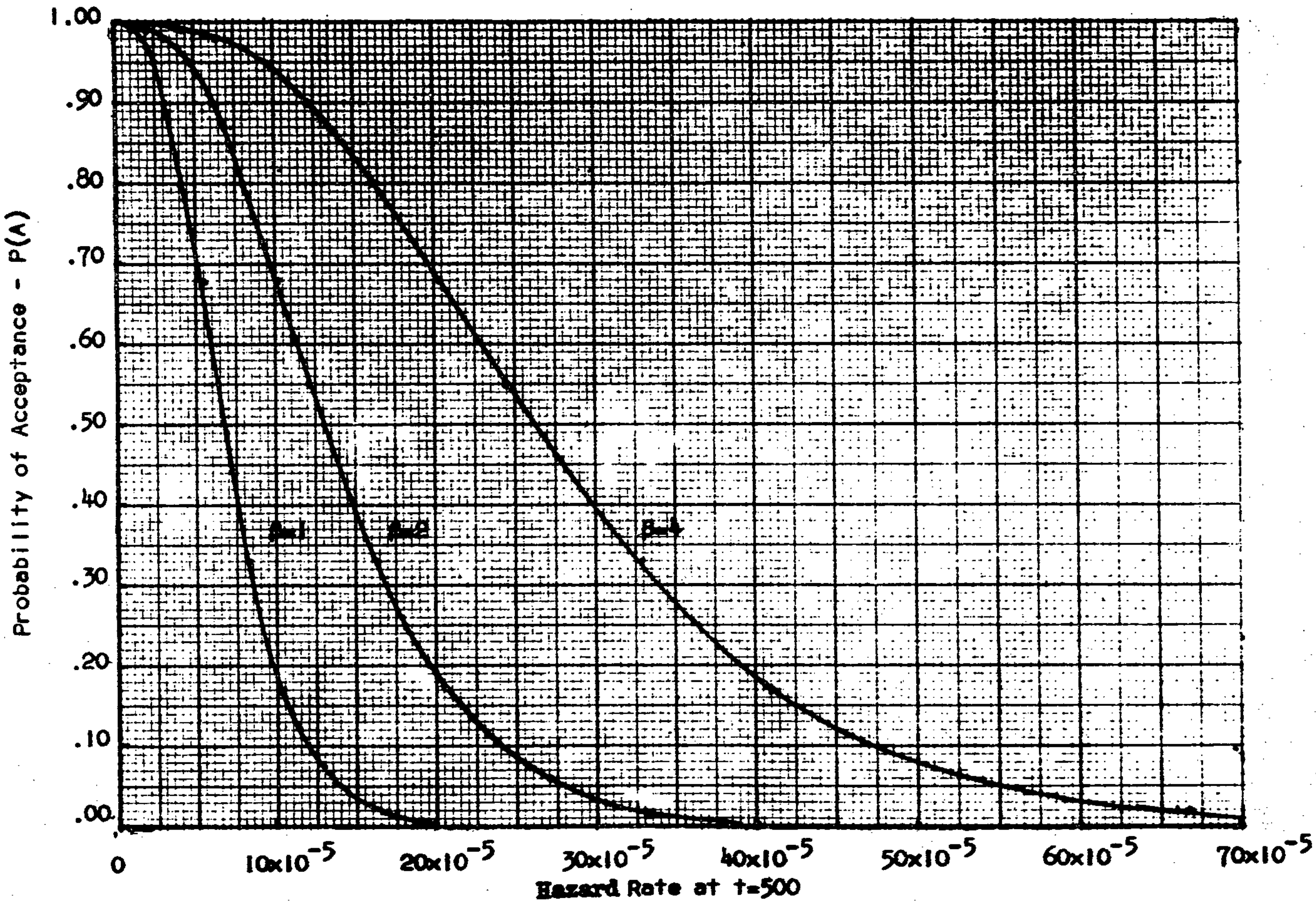


Figure 2. Operating Characteristic Curves for n=115 c=3

In connection with this example it may be well to note that any double-sampling or multiple-sampling plan that "matches" the single-sampling plan in ordinary attributes sampling--that is, for which the same P(A) values are associated with each value for p'--will provide the same operating characteristics in terms of hazard, Z(t). For the plan used in this illustration, for example, a matching double-sampling plan is one for which the first sample size is 75 and and the second 150, with an acceptance number of 1 and a rejection number of 6 for the first sample and an acceptance number of 5 and a rejection number of 6 for the combined samples. The only changes in procedure required will be those normally associated with multiple sampling plans. The value for t specified for single sampling will be used for both the first and the second sample. Double-sampling and multiple-sampling offer excellent opportunities for reducing, over the long run, the average amount of acceptance inspection. However, in testing for life and reliability the added elapsed time required for testing when a second or subsequent sample is required may be a serious obstacle to application of this form of sampling.

Example (2)

Another possibility for use of the basic factors is in the design of an acceptance sampling plan to meet specified requirements. Consider the case of a manufacturer who is to purchase in large quantities a certain electronic component. Experience in the testing and use of this component has indicated the Weibull distribution applies well as a statistical model with a value for β, the shape parameter, of 1 1/3 and for γ, the location parameter, of 400 hours. A test period for sample items of 1200 hours has been agreed upon. Experience with the product has further indicated that a competent supplier should be able to submit lots for which the hazard at 1200 hours is .000044 (per hour).

Accordingly this value for $Z(t)$ is to be adopted as the Acceptable Hazard Rate (AHR) and a plan is desired for which acceptance is virtually certain, say with $P(A) = .99$, if a lot is this good or better. That is, $P(A \mid AHR) \geq .99$ where AHR = .000044 at 1200 hours. Furthermore, the user has ascertained that if the hazard for a lot or shipment is .000155 (per hour) or more, use of the components will lead to much difficulty. Thus a low probability of acceptance, say .05 or less, is desired. Accordingly the Rejectable Hazard Rate (RHR) is .000155 at $t = 1200$ and a $P(A|RHR) \leq .05$ is specified.

The first step in the design of the plan is to subtract the value for γ, the location parameter, from the time t to give a new time, t_0, in terms of $\gamma = 0$. Accordingly, $t_0 = t - \gamma$ or $t_0 = 1200 - 400 = 800$ hours. The next step is to compute the $t_0 Z(t_0) \times 100$ product at both the Acceptable Hazard Rate and the Rejectable Hazard Rate, using the value determined for t_0. These are:

$$t_0 Z(t_0) \times 100 = 800 \times .000044 \times 100 = 3.52 \text{ (at the AHR)},$$

$$t_0 Z(t_0) \times 100 = 800 \times .000155 \times 100 = 12.4 \text{ (at the RHR)}. \quad (12)$$

With the use of Table 2 these computed values can now be applied to determine the probability, p', of an item failing before the end of 1200 hours ($t_0 = 800$) under both the above conditions. These values (for $\beta = 1\ 1/3$) may be read, through linear interpolation, as:

$$p' = 2.605\ \% \text{ or roughly, } 2.6\% \text{ (at the AHR)},$$

$$p' = 8.87\ \% \text{ or roughly, } 8.9\% \text{ (at the RHR)} \quad (13)$$

Finally, a sampling plan may be determined, using procedures commonly employed to design sampling plans of the usual form for attribute inspection. For this case a plan is required for which

$P(A) \geq .99$ when $p' = 2.6\%$ and for which $P(A) \leq .05$ when $p' = 8.9\%$. Making use of factors prepared by Cameron[5] (which are based on the Poisson distribution) it will be found that an acceptance number, c, of 10 will provide most closely the required characteristics and that a sample size of 184 will give a $P(A) = .99$ if $p' = 2.6\%$. A check using other factors supplied at the same source indicates $P(A) = .05$ if $p' = 9.2\%$. Similar results for n and c may be obtained by using the beta probability chart (which is based on the binomial distribution) given by Kao[6]. Thus this plan, $n = 184$, $c = 10$, $t = 1200$, meets closely the specifications.

SECTION 3

THE TABLES OF SAMPLING PLANS

3.1 Construction and Use of the Basic Tables.

An extensive collection of sampling plans has been designed, with a separate table prepared for each of the eleven representative values for β for which the relationship between $tZ(t)$ and p' has been determined. These tables of plans will be found at the end of this report as Tables 3a through 3k.

Each table lists 208 single-sampling plans, with the acceptance number, c, and the corresponding sample size, n, given for each. The plans have been cataloged in terms of a Rejectable Hazard Rate (RHR) under the assumption that for components for which the quality of interest is reliability or lifelength, consumer protection will be of most conern. The figure in each column heading is the $tZ(t) \times 100$ product for which the probability of acceptance under the sampling plans in that column is .10 or less. Expressed otherwise, the column headings are percent values of $t \times$ RHR, and for the plans below each the $P(A \mid RHR) \leq .10$.

However, the tables also provide guidance for selection and evaluation of plans in terms of the Acceptable Hazard Rate, (AHR).

The figure in parentheses below the sample size number, n, for each plan is the per cent value of t x AHR product for which the probability of acceptance is .95 or more. With the use of these values, a plan can be selected in terms of an Acceptable Hazard Rate if one so desires. In any case the two tZ(t) x 100 products broadly describe the operating characteristics of the plan with which they are associated. They thus provide a basis for the appropriate choice of a plan, or for determining the operating characteristics of a plan that has been specified or is being applied. Illustrative examples of the use of the tables of plans will be found below. When tZ(t) x 100 table values to match computed or given values cannot be found, linear interpolation between table values that are available will give solutions that are precise enough for most practical purposes.

These plans have been designed under the assumption that the size of the sample will be relatively small compared to the size of the lot (an assumption imposed under almost all other published sets of plans). Under this assumption the number of failures prior to time t approximates the binomial distribution.. Binomial tables compiled by Grubbs[7] were used to prepare some of the plans, those for values for c from 0 to 9 and for values for n up to 150. For values for c from 10 to 15 and for n up to 60 or so, Pearson's tables of the incomplete-beta function[8] were used. Plans requiring higher values for n were determined by using the Poisson approximation, making use of tables prepared by Cameron.[5] At each point of change from the binomial to the Poisson the match in values for n was checked and found to be close. The slight differences, usually 1 or 2 items, that were found were on the conservative side, the value for the sample size being slightly larger than the size theoretically required.

Example (3)

A receiving inspection plan is required for a certain product. Experience has indicated a value for β of 2/3 can be assumed and a value for γ of 0. It has been determined that lots with a hazard rate of .00005 (per hour) at 1000 hours of use for the product will be considered an unsatisfactory rate and ~ can be used as an RHR value. The test time for sample items will be 1000 hours, the same value for t used in specifying the RHR. Testing facilities are available to test as many as 200 items at one time.

Computation of the tZ(t) x 100 product at the RHR gives 1000 x .00005 x 100 or 5.0. Plans for this value must now be found in Table 3c, the table of plans for β = 2/3. Any plan under the column headed by the value 5 will give a probability of acceptance of not more than .10 at the RHR value of .00005. If full use is to be made of the testing facilities, the plan with an acceptance number, c, of 9 and a sample size, n, of 197 may be employed. With this plan, the tZ(t) x 100 product for a P(A) = .95 is 1.8. Simple substitution in the product gives 1000 x Z(t) x 100 = 1.8 from which it will be found that Z(t) = .000018. If the producer submits product with this hazard rate or less (at 1000 hours), his risk is low; the probability of acceptance will be high, namely .95 or more.

Example (4)

Consider another application for which a plan is to be selected. In this case, β = 1 1/3 and γ = 1250 hours. A Rejectable Hazard Rate of .000020 and an Acceptable Hazard Rate of .000008, both measured at 2000 hours seem to be most suitable. The test time, t, is also to be 2000 hours.

Since $\beta = 1\ 1/3$ can be assumed, a plan from Table 3e can be used. First, a new time, t_0, in terms of $\gamma = 0$ must be found. From $t_0 = t - \gamma$ or $t_0 = 2000 - 1250$, a value for t_0 of 750 hours is found. Next, the $t_0 Z(t_0) x100$ products at the AHR and the RHR can be computed, using t_0 or 750 hours. They are:

$$750 \times .000020 \times 100 = 1.5 \quad \text{(at RHR)}$$
$$750 \times .000008 \times 100 = .6. \quad \text{(at AHR)} \qquad (14)$$

Any plan in Table 3e under the column headed by the product 1.5 will meet the RHR requirement. Scanning this column in terms of the products in parentheses, the plan using an acceptance number, c, of 10 and a sample size, n, of 1380 is selected. For this plan, the $P(A|RHR) = .10$ and the $P(A|AHR) = .95$, the probabilities of acceptance being these figures at the designated hazard rates and at a life of 2000 hours.

3.2 The Dependence of Operating Characteristics on Sample Size

An interesting and somewhat useful characteristic of the form of plans presented in this report is that the ability of a plan to discriminate between good and bad lots depends on the size of the acceptance number rather than on the size of the sample; operating characteristic curves become steeper as the magnitude of the acceptance number is increased rather than becoming steeper as the size of the sample is increased as under the usual forms of sampling inspection. For given magnitudes for consumer and for producer risks at the Rejectable Hazard Rate and the Acceptable Hazard Rate respectively, a nearly constant ratio between the Rejectable Hazard Rate and the Acceptable Hazard Rate will be found for any given value for the acceptance number. For the collection of plans presented here, the ratio is constant (for all practical purposes) for all plans for which the $tZ(t) \times 100$ value at the RHR is more than 5. For values less than 5, there is a slight decrease in the magnitude of the ratio as sample sizes become increasingly small, but not enough to be significant for practical purposes.

Not only is the ratio between the Rejectable Hazard Rate and the Acceptable Hazard Rate constant for any given value for c, but any ratio computed for any given pair of consumer and producer risks applies for all values for β; the ratio does not depend on β as is the case for the Weibull sampling plans that test lots in terms of mean item life.

A table of hazard rate ratios for values of c from 0 to 15 has been prepared for a number of alternatives for consumer risk and producer risk values. They are presented as Table 4. As indicated, the figures in the body of the table are approximate values for RHR/AHR for each value for c for the acceptance probabilities indicated in the table headings. The use of this table will be described by giving two examples.

Example (5)

Consider the case described in Example 2 in which the desired sampling plan was to give a probability of acceptance of .05 or less if the hazard rate was .000155 and a probability of acceptance of .99 or more if the hazard rate was .000044. That is the $P(A \mid RHR) \leq .05$ and $P(A \mid AHR) \geq .99$. In using this table (Table 4) one first determines the value for the ratio RHR/AHR which is .000155/.000044 or 3.52. Next, the table is scanned in the column giving ratios for $P(A \mid RHR) = .05$ and $P(A \mid AHR) = .99$, looking for a value close to the computed ratio. A value close to it, 3.56, is found opposite c = 10. Thus an acceptance number of 10 will be required. This, it will be noted, is the acceptance number that was determined in other ways in the previous example. Table 4 provides a quick, short-cut way of finding c, the acceptance number for any application (within the limits of the table).

Example (6).

A plan is required for a product for which $\beta = 3\ 1/3$ and $\gamma = 0$, for which a Rejectable Hazard Rate of .0005 per hour has been established, and for which a $P(A \mid RHR) \leqq .10$ will be satisfactory. The test time, t, is to be 50 hours. Furthermore, a high probability of acceptance, .99, is required at the Acceptable Hazard Rate, which has been established as .0001 per hour.

The $tZ(t) \times 100$ product at the RHR is $50 \times .0005 \times 100$ or 2.5. Referring to Table 3i which contains plans for $\beta = 3\frac{1}{3}$, any plan in the column with the 2.5 value heading will give the RHR risk of $\leqq .10$ specified. The one to select can be determined by use of Table 4. The RHR/AHR ratio for this case is .0005/.0001 or 5. At a $P(A \mid AHR) \leqq .10$ and a $P(A \mid AHR) \geqq .99$, Table 4 indicates an acceptance number, c, of 5 will give most closely the operating characteristics desired. The RHR/AHR ratio has a value of 5.20 which is close to the value of 5 obtained for the specified rates. Thus the plan to use is the sixth one down in the column (headed 2.5) in Table 3i, the plan for which $c = 5$ and $n = 1240$.

3.3 Differences in Lifetesting Time

In order to prepare tables of factors and sampling plans for general use, it has been necessary to assume that the lifetesting time for sample items will be the same as the time, t, at which hazard rates are specified or are to be determined. However, a simple modification of the procedure can be made to allow for cases in which the two times do differ. All that must be done is to determine the hazard rate at the test truncation time which corresponds (for the value for β that applies) to the hazard rate at the specification time.

A table of hazard rate ratios, Table 5, has been prepared for making this conversion. The table gives for various values of t_2/t_1 values for the ratio $Z(t_2) / Z(t_1)$ for all the β-values concerned in this study. Also, a chart has been prepared, Figure 3, showing the relationship between hazard rate and time for the same values for β. The two may be used interchangeably, the chart being useful for cases in which table values are not available and the table being useful when more precision is required and one of the tabulated table values applies to the case at hand. If $Z(t_2)$ represents a specified hazard rate and t_2 the time at which it is specified, then $Z(t_1)$ can be used to represent the corresponding hazard rate at some other time, t_1. The table can also be used for cases in which the test-truncation time is greater than the time at which the hazard rate is specified (if such cases are encountered). All that need be done in such cases is to reverse the meanings given to the subscripts 1 and 2. Conversion from one hazard rate to another or from one time to another when necessary to make use of the factors and plans presented here is quite easy, as will be shown in the example which follows.

Example (7)

A plan is required for a case for which a hazard rate specified as an RHR (at a P(A) = .10) is .000375 at 4000 hours. However, a test truncation time of 1000 hours must be used. A sampling plan in terms of the specification but using the shorter testing time is required. A value for β of 1 2/3 and for γ of 0 can be assumed.

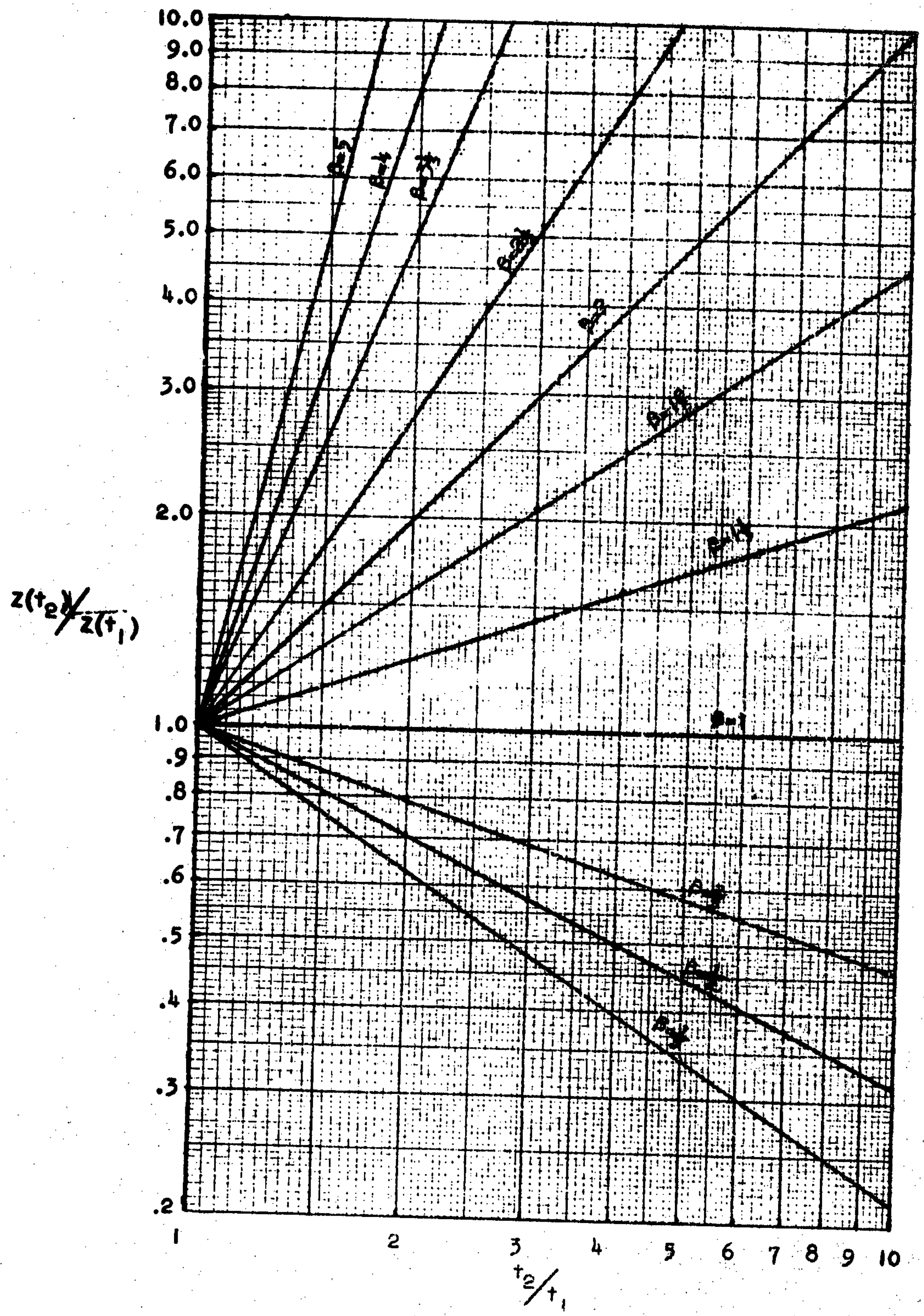


Figure 3. The Relationship Between Hazard Rate and Time for Various Values for β

The value for t_2/t_1 in this case is 4000/1000 = 4. Reference to Table 5 indicates for this t_2/t_1 ratio and for $\beta = 1\ 2/3$, that the $Z(t_2)\ /\ Z(t_1)$ ratio is 2.52. Substitution of .000375 (the RHR) for $Z(t_2)$ gives $.000375\ /\ Z(t_1) = 2.52$ or $Z(t_1) = .00015$. This equivalent value, $Z(t_1)$, of .00015 may now be used to select a plan to use with the 1000 hour testing time. The product $tZ(t) \times 100$ to apply is thus $1000 \times .00015 \times 100 = 15$. A plan may now be selected from those given in Table 3f for $\beta = 1\ 2/3$. Any plan under the column with a heading value of 15 will meet the RHR requirement. Suppose testing facilities are available for as many as 150 sample items and so the plan for which c = 8 and n = 149 is to be applied. A reversal of the above procedure can be used to make use of the $tZ(t) \times 100$ product given (in parentheses) for this plan in Table 3f for a $P(A) \geqq .95$ to determine the Acceptable Hazard Rate. The product $tZ(t) \times 100$ is 5.4. At the shortened testing time of 1000 hours, $1000\ Z(t) \times 100 = 5.4$ or $Z(t) = .000054$. For lots with this hazard rate at 1000 hours, the $P(A) \geqq .95$. The corresponding hazard rate at 4000 hours can be found from the hazard ratio obtained from Table 5. Since $Z(t_2)\ /\ Z(t_1) = 2.52$, $Z(t_2)\ /\ .000054 = 2.52$ or $Z(t_2) = .000136$. Lots with this hazard rate at 4000 hours have a probability of .95 or greater of acceptance; the rate can be used as the Acceptable Hazard Rate at this specified time.

SECTION 4

AVERAGE HAZARD PLANS

The average hazard over any specified time period up to t, which is denoted by m(t), may be expressed as,

$$m(t) = \frac{1}{t} \int_{\infty}^{t} Z(y)dy = M(t)/_{t} \tag{15}$$

where Z is the hazard function defined by Equation (1) and M(t) is called the cumulative hazard over t. For a lifelength distribution which starts at time γ (the threshold or location parameter), the lower limit of the integral M(t) should be γ which is assumed to be zero in this report.

Following from Equation (A14) of Appendix A, the probability of a sample item failing before the end of the test time t is then,

$$p' = F(t) = 1\text{-}\exp\left[-\ tm(t)\ \right],\ t>0. \tag{16}$$

From this, it may be noted that

$$tm(t) = -\ln(1-p'),\ 0 < p' < 1. \tag{17}$$

These two equations form the basic relationship for the design or evaluation of the attribute sampling-inspection plans using the average hazard, m(t) as a life quality criterion. By direct comparison of these two equations with Equations (8) and (9), it may be noted that m(t) plays the role of Z(t) with $\beta = 1$. For this reason, no additonal tables need be prepared for sampling plans where the average hazard, m(t) is specified. Furthermore, since Equations (16) and (17) are valid for any arbitrary distribution, the sampling plan based upon the average hazard is non-parametric, i.e., distribution-free.

Nevertheless it is useful to equate Equation (16) with the exponential c.d.f., $F(t) = 1-e^{t/\theta} = 1-e^{-t\lambda}$, for $t, \theta, \lambda > 0$, and to note that $m(t) = \lambda$. Writing this conversely by letting m^{-1} be the inverse function of m,

$$t = m^{-1}(\lambda) = \tau_\lambda \tag{18}$$

where τ_λ is called the hazard-breakeven time. For any lifelength distribution with a monotonically decreasing (or increasing) failure rate or hazard rate in time, the τ_λ , the hazard-breakeven time represents the point in time when the average hazard of the lifelength distribution under consideration is equal to the specified constant hazard (for the exponential case). But when the monotonicity assumption is removed, then there may be multiple solutions for Equation (18), depending upon the nature of $m(t)$.

For maintainable equipment, in contrast to the so-called one-shot equipment such as a fired missile, the average hazard for the component therein represents the average amount of replacement necessary to keep the equipment in operation. Hence it is significant to consider the hazard-breakeven time. If λ is the tolerable average hazard, then for a component with a monotonically decreasing failure rate, τ_λ means the equipment break-in time, while for a component with monotonically increasing failure rate, τ_λ means equipment retirement time. Two numerical examples will follow.

Example (8)

Suppose the lifelength distribution is of the Weibull form given by Equations (3) and (4), then Equation (17) becomes,

$$tm(t) = (t/\eta)^\beta \tag{19}$$

and Equation (18) becomes,

$$\tau_\lambda = [\lambda\alpha]^{1/(\beta-1)} \quad (20)$$

where $\alpha = \eta^\beta$ is the quasi-scale parameter of the Weibull distribution[2]

Consider the case of a manufacturer who is producing a certain electronic component. Experience has indicated that a Weibull distribution with $\gamma = 0$, $\beta = 1/3$, and $\eta = 42.875 \times 10^9$ (or $\alpha = 3{,}500$) applies. A customer specifies a value for λ = 6% per thousand hours. The hazard-breakeven time τ_λ in this case is computed as follows. The per hour value for λ is, $\lambda = 0.06/1000 = 6 \times 10^{-5}$ (per hour). Substituting this into Equation (20) gives,

$$\tau_\lambda = (3{,}500 \times 6 \times 10^{-5})^{-(3/2)} = 10.39 \text{ hours.}$$

That is, if the equipment in which the component is to be installed is allowed to break-in for a period of approximately ten hours, then thereafter the average hazard of the component will be less than that specified by the customer--a very comforting assurance.

<u>Example (9)</u>

Suppose the manufacturer in Example (8) has decided to truncate the lifetest at 1000 hours and desires a sampling plan which will accept with high probability (say .98), lots having an average hazard of 0.6% / 1000-hrs. and reject with high probability (say .90), lots having an average hazard of one order of magnitude higher (i.e., 6%/ 1000-hrs.). In this case, the producers risk is 2% and the consumer's risk is 10% which gives a 90% confidence coefficient of shipping products with 6% / 1000-hrs. or better average hazard.

The values for $tm(t) \times 100$ are 0.6 and 6.0 at $P(A) = .98$ and $P(A) = .10$ respectively. By linear interpolation in Table 2 for $\beta = 1$, the following values for p' are found:

$$\text{At } P(A) = .98,\ p' = .00597 \text{ or approx. } 0.6\%$$
$$\text{At } P(A) = .10,\ p' = .05821 \text{ or approx. } 5.8\% \qquad (21)$$

The sample size, n, and the acceptance number, c, of the desired sampling plan may be determined from the requirements specified by Equation (21). The use of the beta probability chart[6] gives $c + 1 = 3$, $n - c = 90$, or $n = 92$ and $c = 2$.

SECTION 5

ADAPTATION OF THE MIL-STD-105C PLANS

5.1 Use of the MIL-STD-105C Plans for Life and Reliability Testing

In some cases it may be advantageous to employ the familar Military Standard 105-C[20] plans for reliability and lifetesting applications. To make this possible, the basic conversion factors described in Section 2 have been employed to find tZ(t) x 100 products for all the plans in the MIL-STD-105C collection. Just as for the basic tables of plans described in Section 3, separate tables of products have been prepared for each of a number of selected values for β, the Weibull shape parameter. The special case for $\beta = 1$, which is the exponential case, has been included.

The acceptance procedure to be used is the same as that employed for the basic plans and as outlined in Section 1.3. For single sampling the steps are (a) select a suitable sampling inspection plan, making use of the tables of products provided, (b) draw at random a sample of n items, as specified by the plan, (c) place the sample items on life test for the specified period of time, t, (d) determine the number of sample items that fail during the test period, (e) compare the number of items that fail with the acceptance number, c, specified for the plan, and (f) if the number that fail is equal to or less than the acceptance number, accept the lot; if the number failing exceeds the acceptance number, reject the lot.

Both the sample sizes and the acceptance numbers used will be those specified for the MIL-STD-105C plans. For convenience, the single-sample sizes and the corresponding acceptance numbers have been included in this report and will be found in Table 7. Single sampling will presumably be usdd, but by simple modification double sampling or multiple sampling can be used if desired. It may be noted that the acceptance procedure for any form of sampling is the same as that specified for the MIL-STD-105C plans with the single exception of the use of a test truncation time, t.

The selection of a suitable plan, the determination of operating characteristics in terms of hazard rate for a specified plan, and the determination of an appropriate life-testing time are all made through use of the tables of products which will be found at the end of this report. Ways for making use of these tables will be discussed in the section that follows and in the accompanying illustrative examples.

It may be noted that the probability of acceptance for a lot under the procedure outlined above depends only on the probability, which may be designated by p', of an item failing before the end of the test period, t. The actual life at which an item fails need not be determined; inspection is on a attribute basis. For this reason it is possible to use the 105B plans to evaluate submitted lots in terms of hazard rate, Z(t), at some specified time, t. The operating characteristics for any sampling plan specified by c and n depend only on t and Z(t). A brief outline of the mathematics involved in the procedure will be found in Section 2.1.

To provide a procedure of simple form and one suitable for general use, the tables of factors for adapting the 105C plans to use in terms of hazard rates have been prepared in terms of dimensionless products of t times Z(t). Actually, in order to give figures that may be more conveniently used, the tables are composed of tZ(t) x 100 products. Each of the 105C plans is cataloged in terms of such tZ(t) x 100 products. These product values are to be used in much the same way that percent defective values are used in the selection and application of plans for ordinary attribute inspection. With t and Z(t) specified, all that must be done is to compute their product and then select a plan in its terms. If on the other hand, a plan (n and c) and a test truncation time (t) are specified, the product may be used to give an evaluation of operating characteristics in terms of hazard rate, Z(t). Or, alternatively, the product may be used to find a suitable test truncation time, t. Examples of such application of the factors will be found in the latter part of this section.

The Weibull distribution, one will recall, is a three-parameter distribution, requiring a location or threshold parameter, a scale parameter, and a shape parameter for complete description. For the procedure and plans presented here, the scale parameter need not be ascertained or known; the tZ(t) x 100 product contains information on its magnitude. For the location or threshold parameter (commonly symbolized by the letter γ), a value of zero is to be assumed in the direct application of the products and the procedure. For many applications a γ value of 0 will apply; there will be no initial period of life free of risk of failure. This assumption is equivalent to knowing the value for gamma. If γ has some non-zero value, all that is necessary is to subtract the value for γ from t to obtain a value t_0 and then use t_0 rather than t in working with the tables of products. The third parameter, the Weibull shape parameter (which is commonly symbolized by the letter β)

must be known. The products depend on the value for β so that its magnitude must be known or estimated from past research, engineering, or inspection data. Separate tables of products have been provided for each of eleven values for β, values ranging from 1/3 to 5. The assumed value for β=1 represents the special case of the exponential distribution and so may be used when this distribution seems to be the most appropriate statistical model. Estimation procedures for β or γ are available and may be found in the papers by Kao [(2), (22)].

One final point of procedure must be mentioned. It is that for the direct use of the tables of products, the item life at which the hazard rate is measured or specified and the life at which the testing of sample items is truncated are assumed to be the same. That is, the t used in the tZ(t) x 100 products is the same as the test truncation time, t. However, if the life at which hazard rates are to be specified or evaluated must differ from the test truncation time, a simple variation in procedure may be made to allow for any desired departure. All that must be done is to find (for the value for β assumed) the hazard rate at the test truncation time that corresponds to the hazard rate at the time used in the specification or at which lots are to be evaluated. A table of hazard rate ratios has been compiled for this purpose. It will be found as Table 5 at the end of this report. The method for its use will be described in one of the illustrative examples which will follow.

5.2 The Tables of Products and Their Use

To provide the necessary data for using the 105C plans in life and reliability testing in terms of hazard rate, eleven tables of tZ(t) x 100 products have been prepared. One is available for each of the following values for β: $\frac{1}{3}$.$\frac{1}{2}$,$\frac{2}{3}$, 1, $1\frac{1}{3}$, $1\frac{2}{3}$, 2, $2\frac{1}{2}$, $3\frac{1}{3}$, 4, and 5. These values encompass the range of shape parameter values that will commonly be encountered. For values of interest for which no table has been provided (but within the above range), linear interpolation will give factors accurate enough for most practical purposes. The tables of factors have been included at the end of this report as Tables 6a through 6k.

Each table provides tZ(t) x 100 products for each 105C plan, that is for each combination of Sample Size Code Letter and Acceptable Quality Level the 105C Standard utilizes. The factors apply not only to the single-sampling plans, but also to the matched double-sampling and multiple-sampling plans. The sample sizes corresponding to each Sample Size Code Letter and the acceptance and rejection numbers applying to each Acceptable Quality Level as found in the 105C Standard are thus to be used in the procedure presented in this paper.

Across the top of each table will be found tZ(t) x 100 products corresponding to each of the Acceptable Quality Level values utilized in the 105C tables. Each of these matched tZ(t) x 100 products provides for all of the 105C plans of the corresponding Acceptable Quality Level a measure of lot quality for which the probability of acceptance is high. The producer's risk will be low with its actual magnitude being the same as that experienced under normal use of the 105C plans. It will be recalled that this risk varies, being as low as .01 for large sample sizes and as high as .20 for small sample sizes. The risk associated with a specific plan may be obtained from the operating characteristic curves provided in the 105C standard.

Within the body of each table will be found tZ(t) x 100 products for each plan for which the probability of acceptance is low. In each case this probability is .10 or less. These products may be used as a means of estimating the consumer's risk when a Rejectable Hazard Rate is to be determined or when a plan is to be selected in such terms. Thus with pairs of factors provided for use in terms of both an Acceptable Hazard Rate as found across the top of each table and a Rejectable Hazard Rate as found within the body of each table, plans may be selected or evaluated in terms of either the producer's risk or the consumer's risk or both. Recently a revision"C" of MIL-STD-105 included, among other things, the listing of LTPD values as consumer's protection measures as well as the AQL values. (23)

The interpretation of these $tZ(t) \times 100$ products may be demonstrated by means of a simple example. Suppose that for a product to be submitted to acceptance inspection β can be assumed to have the value of $\frac{1}{2}$ and γ a value of 0. Life testing of sample items is to be truncated at 500 hours. A single-sampling plan for Sample Size Code Letter H and an AQL of 1.5% has been specified for use (n, the sample, size will thus be 35 and c, the acceptance number, 1). Reference to Table 6b which contains products for $\beta = 1/2$ shows that the $tZ(t) \times 100$ product at the Acceptable Quality Level of 1.5% is .756. With t = 500 thus $500Z(t) \times 100 = .756$. Z(t), the hazard rate, can thus be computed as $Z(t) = .756/(500 \times 100) = .0000151$ (per hour). Accordingly, this figure, .0000151, can be considered as the Acceptable Hazard Rate (as measured at 500 hours of Life). Next, by entering the body of the table at Sample Size Code Letter H and an AQL of 1.5%, the $tZ(t) \times 100$ product for which the probability of acceptance is low (.10) is 5.5. Again with t = 500, $500Z(t) \times 100 = 5.5$ or $Z(t) = 5.5/(500 \times 100) = .00011$. Thus the Rejectable Hazard Rate (at 500 hours of life) is .00011 (per hour). These two figures found for Z(t) may be interpreted thus, (a) lots for which the hazard rate for items at 500 hours of life is .0000151 per hour or less will have a high probability of acceptance (examination of the OC curves in the 105C Standard indicates the probability is slightly more than .90); lots for which the hazard rate for items at 500 hours of life is .00011 will have a low probability of acceptance (namely .10 or less).

Example (10):

A purchased electronic component is to be inspected, lot by lot, for lifelength by sampling inspection. The MIL-STD-105C plans are to be employed. The lot quality of interest is the hazard rate at a life of 500 hours. From past experience with the component it has been determined that the Weibull distribution can be applied as a statistical model and that a value for β, the shape parameter, of 2/3 can be assumed and a value for γ, the location or threshold parameter, of 0 can be expected. Normal inspection is to be employed and the lot size in each case will be 3,000 items. A plan employing an Acceptable Quality Level (in terms of the percent defective as used in the standard) of 4.0% and a test period of 500 hours have been tentatively selected, with single sampling to be utilized Under these given conditions, the acceptance procedure and the operating characteristics in terms of hazard rate at 500 hours are to be determined.

By referring to Table III of MIL-STD-105C it will be found that for Inspection Level II (which is customarily employed unless there are special reasons for doing otherwise) and for a lot size of 3,000, Sample Size Code Letter L should be employed. Next, reference to Table IV-A of the same Military Standard or to Table 7 of this report shows that for Sample Size Code Letter L the sample size for single sampling is 150 items. Further reference to this latter Table shows that for the 4.0% Acceptable Quality Level selected the acceptance number to use is 11 and the rejection number 12. The acceptance-rejection procedure will accordingly be:

(a) Draw at random from the lot a sample of 150 items.

(b) Place the sample items on life test for 500 hours.

(c) Determine the number of sample items that fail prior to the end of this test period.

(d) If the number that fail is 11 or less, accept the lot; if the number that fail is 12 or more, reject it.

The operating characteristics of the above acceptance procedure in terms of hazard rate can be determined by reference to Table 6c which will be found at the end of this report. The table lists tZ(t) x 100 products for application when $\beta = \frac{2}{3}$. Examination of the factors across the top of the table (immediately below the AQL figures in p'(%)) shows that at 4.0% the corresponding tZ(t) x 100 ratio is 2.72. With t = 500 and with tZ(t) x 100 = 2.72, the hazard rate is thus determined by:

$$500Z(t) \times 100 = 2.72$$

$$Z(t) = \frac{2.72}{500 \times 100} = .0000544 \quad .$$

This figure may be considered as an Acceptable Hazard Rate (AHR). Thus if the quality of the submitted lot is such that the hazard rate at 500 hours is .0000544 (per hour), the probability of its acceptance will be high. By reference to Table VI-L of the Military standard, operating characteristic curves for single sampling for Code letter L may be found. The curve for a 4.0% AQL indicates that at 4.0% the probability of acceptance is slightly more than .98, the producer's risk, then for lots for which the hazard rate is .0000544 at 500 hours is 1.00-.98 or .02.

A Rejectable Hazard Rate (RHR), a rate at which the probability of acceptance will be low under the above plan, may also be determined by further reference to Table 6c of this report. In the body of this table at Sample Size Code Letter L and an AQL of 4.0%, a second tZ(t) x 100 product for the application, one whose value is 7.9, will be found. This is the product for which the probability of acceptance is .10 or less. Using t = 500 and making another simple computation gives:

$$500Z(t) \times 100 = 7.9$$

$$Z(t) = \frac{7.9}{500 \times 100} = .00016 \quad .$$

This answer may be considered as a Rejectable Hazard Rate (RHR). If the lot quality is such that the hazard rate is .00016 (per hour) at 500 hours of life, the probability of acceptance for the lot will be low, namely .10 or less. This figure quantifies the consumer's risk at the Rejectable Hazard Rate of .00016.

Example (11):

For another illustration of use, consider a sampling inspection application for which an Acceptable Hazard Rate of .000025 (per hour) and a Rejectable Hazard Rate of .00015 (per hour) with both at 1000 hours of life is required. A value for β of $1\frac{2}{3}$ and for γ of 0 can be assumed. The test time for sample items is to be 1,000 hours with double sampling to be employed. The problem is to select from the MIL-STD-105C plans the one that will meet most closely these requirements.

Computations at the Acceptable Hazard Rate give tZ(t) x 100 = 1,000 x .000025 x 100 = 2.5. Reference may now be made to Table 6f of this report which lists tZ(t) x 100 products for application when $\beta = 1\frac{2}{3}$. In the line of factors across the top of the table (the column headings) the factor 2.52 will be found corresponding to a 1.5% AQL. Use of any plan with this AQL percentage will thus meet closely the Acceptable Hazard Rate requirement; the probability of acceptance for lots meeting this specified figure will be high.

Computations at the Rejectable Hazard Rate give tZ(t) x 100 = 1,000 x .00015 x 100 = 15. Examination of the column of products for the 1.5% AQL figure show that for Sample Size Code Letter J the tZ(t) x 100 product is 15. Thus use of this Code Letter (with the AQL value selected) will provide a low probability of acceptance (.10 or less) for lots at which the hazard rate is .00015 at 1000 hours. Any 105C plan (single, double, or multiple) with a 1.5 AQL and a Sample Size Code Letter J will meet closely the operational requirements.

For double sampling, reference to Table IV-B of the 105C Standard indicates the first sample size is 50 and the second sample size is 100 for Letter J. At the AQL value of 1.5 the acceptance number under normal inspection for the first sample must be 1 and the rejection number 6; for combined samples the acceptance number must be 5 and the rejection number 6. Other details of the acceptance-rejection procedure will be those customarily employed in double sampling. It should be noted that the life testing time for the first sample must be 1,000 hours; likewise, it must be 1,000 hours for the second sample when testing of a second sample becomes necessary.

Example (12):

Consider a case for which a β value of 2-1/2 and a γ value of 0 can be assumed. A Rejectable Hazard Rate of .00090 (per hour) at a life of 2,000 hours has been specified. However, a test period of only 500 hours for sample items is, for practical reasons, the longest that can be utilized. The problem is to determine a single-sampling 105C plan that will meet the RHR requirement.

A suitable plan in terms of this required time for the lot quality specification but one that uses the reduced testing time can be found through application of data in Table 5 of this report, a table of hazard rate ratios. If we let t_2 represent the life at which the hazard rate is specified and t_1 represent the testing time to be employed, then the ratio between the two which is required for the use of the table may be determined. It is t_2/t_1 or 2000/500 which is 4. Entering the table at the 4.00 value for this ratio and reading across to values for use when $\beta = 2\text{-}1/2$, a $Z(t_2)/Z(t_1)$ ratio of 8.00 may be found. Letting $Z(t_2)$ represent the hazard rate specified at time t_2 of 2,000 hours (which is .00090), a corresponding hazard rate $Z(t_1)$ at time t_1 of 500 hours can be computed. Since $Z(t_2)/Z(t_1) = 8.00$, $Z(t_1) = .00090/8.00 = .00011$. A $tZ(t) \times 100$ ratio can now be computed using this new rate to apply at 500 hours. It is $500 \times .00011 \times 100 = 5.5$.

With this value for the RHR product, 5.5, one may now make reference to Table 6h which lists products for $\beta = 2\text{-}1/2$. Any plan for which a value at or close to 5.5 may be found in the body of this table will meet the RHR specifications. For example, a plan with an AQL of 0.25% and with Sample Size Code Letter N will do; likewise, one with an AQL of 1.0%

and with Sample Size Code Letter Q meets the requirement. A choice from these or other suitable alternatives will depend on the Acceptable Hazard Rate that seems most suitable for the case. Use of the $tZ(t) \times 100$ product at the tope of a column from which a plan is selected will enable one to determine the hazard rate for which the probability of acceptance will be high. For the first alternative mentioned above (0.25% and N), $tZ(t) \times 100 = .625$. Thus the AHR $= .625/(500 \times 100) = .0000125$ at 500 hours. At 2000 hours it depends on the ratio $Z(t_2)/Z(t_1) = 8.00$. Thus at 2000 hours the AHR, $Z(t_2) = .0000125 \times 8.00 = .00010$.

<u>Example (13):</u>

For an example of a case for which the location or threshold parameter is not zero, consider an application for which one can assume $\beta = 4$ and $\gamma = 200$ hours. A sample size of 50 has been specified (single sampling is to be employed). A life testing time of 600 hours for sample items has been agreed upon. It seems reasonable to expect a hazard rate of .00070 at 600 hours and so this figure is selected as an Acceptable Hazard Rate and a plan is accordingly to be selected in these terms. The problem is to determine what acceptance number must be used and also what measure of consumer protection will be afforded.

The first step is to subtract the value for γ from the value for t, the test and specification time value, to get t_o, a value in zero threshold-parameter terms. Accordingly, $t - \gamma = 600 - 200 = 400 = t_o$. This converted value may now be used in the normal manner for the tables and procedures being described in this report.

The $t_o Z(t_o) \times 100$ product at the Acceptable Hazard Rate will thus be $400 \times .00070 \times 100$ or 28. Reference may now be made to Table 6j which lists products for $\beta = 4$. Examination of the column headings indicates for an AQL of 6.5% a corresponding $tZ(t) \times 100$ product of 26.9, a figure reasonably close to 28. Reference next to table IV-A of MIL-STD-105C or Table 7 of this report shows that for single sampling with a sample of size 50, and with an AQL of 6.5%, the acceptance number must be 6. Also, it may be noted that a single sample of size 50 corresponds to Sample Size Code Letter I.

To determine a measure of consumer protection when the sample size is 50 (Letter I) and the acceptance number is 6 (AQL = 6.5%), reference may now be made again to Table 6j of this report. At Letter I and an AQL of 6.5%, it will be found that the $tZ(t) \times 100$ product is 89 at the Rejectable Hazard Rate (for which the probability of acceptance is low, namely .10 or less). With this value one may determine that $t_o Z(t) \times 100 = 400\ Z(t) \times 100 = 89$, or $Z(t) = 89/(400 \times 100) = .0022$. Note again for these computations the converted time value, t_o, is used. The Rejectable Hazard Rate of .0022 as computed above actually applies, however, at a real life of $(t_o + \gamma)$ or 600 hours.

TABLE 1

Table of tZ(t) x 100 Values for Various Values of p'

p' (in %)	Shape Parameter - β										
	1/3	1/2	2/3	1	1 1/3	1 2/3	2	2 1/2	3 1/3	4	5
.010	.003	.005	.007	.010	.013	.017	.020	.025	.033	.040	.050
.012	.004	.006	.008	.012	.016	.020	.024	.030	.040	.048	.060
.015	.005	.007	.010	.015	.020	.025	.030	.038	.050	.060	.075
.020	.007	.010	.013	.020	.027	.033	.040	.050	.067	.080	.100
.025	.008	.012	.017	.025	.033	.042	.050	.063	.083	.100	.125
.030	.010	.015	.020	.030	.040	.050	.060	.075	.100	.120	.150
.040	.013	.020	.027	.040	.053	.067	.080	.100	.133	.160	.200
.050	.017	.025	.033	.050	.067	.083	.100	.125	.167	.200	.250
.065	.022	.032	.043	.065	.087	.108	.130	.163	.217	.260	.325
.080	.027	.040	.053	.080	.107	.133	.160	.200	.267	.320	.400
.100	.033	.050	.067	.100	.133	.167	.200	.250	.333	.400	.500
.12	.040	.060	.080	.120	.160	.200	.240	.300	.400	.480	.600
.15	.050	.075	.100	.150	.200	.250	.300	.375	.500	.600	.750
.20	.067	.100	.133	.200	.266	.333	.400	.500	.666	.800	1.000
.25	.083	.125	.167	.250	.333	.417	.500	.625	.833	1.000	1.250
.30	.100	.150	.200	.300	.400	.500	.600	.750	1.000	1.200	1.500
.40	.134	.201	.267	.401	.535	.668	.802	1.003	1.337	1.604	2.005
.50	.167	.251	.334	.501	.668	.835	1.002	1.253	1.670	2.004	2.505
.65	.217	.326	.435	.652	.869	1.087	1.304	1.630	2.173	2.608	3.260
.80	.268	.402	.535	.803	1.071	1.338	1.606	2.008	2.677	3.212	4.015
1.00	.335	.503	.670	1.005	1.340	1.675	2.010	2.513	3.350	4.020	5.025
1.2	.402	.604	.805	1.207	1.609	2.012	2.414	3.018	4.023	4.828	6.035
1.5	.504	.756	1.007	1.511	2.015	2.518	3.022	3.778	5.037	6.044	7.555
2.0	.673	1.010	1.347	2.020	2.693	3.367	4.040	5.050	6.733	8.080	10.100
2.5	.844	1.266	1.688	2.532	3.376	4.220	5.064	6.330	8.440	10.128	12.660
3.0	1,015	1.523	2.031	3.046	4.061	5.077	6.092	7.615	10.153	12.184	15.230
4.0	1.361	2.041	2.721	4.082	5.443	6.803	8.164	10.205	13.607	16.328	20.410
5.0	1.710	2.565	3.419	5.129	6.839	8.548	10.258	12.823	17.097	20.516	25.645
6.5	2.240	3.360	4.481	6.721	8.961	11.201	13.442	16.802	22.403	26.884	33.605
8.0	2.779	4.169	5.559	8.338	11.117	13.897	16.676	20.845	27.794	33.352	41.691
10.0	3.512	5.268	7.024	10.536	14.048	17.560	21.072	26.340	35.120	42.144	52.680
12	4.261	6.392	8.522	12.783	17.044	21.305	25.567	31.958	42.611	51.133	63.917
15	5.417	8.126	10.835	16.252	21.669	27.086	32.504	40.630	54.173	65.008	81.260
20	7.438	11.157	14.876	22.314	29.753	37.191	44.629	55.786	74.381	89.258	111.57
25	9.589	14.384	19.179	28.768	38.358	47.947	57.536	71.921	95.894	115.07	143.84
30	11.889	17.834	23.778	35.668	47.557	59.446	71.335	89.169	118.89	142.67	178.34
40	17.027	25.541	34.055	51.082	68.110	85.137	102.16	127.71	170.27	204.33	255.41
50	23.105	34.657	46.210	69.315	92.420	115.52	138.63	173.29	231.05	277.26	346.57
65	34.994	52.491	69.988	104.98	139.98	174.97	209.96	262.46	349.94	419.93	524.91
80	53.648	80.472	107.29	160.94	214.59	268.24	321.89	402.36	536.48	643.77	804.72

TABLE 2

Table of p' (%) Values for Various Values of tZ(t) x 100

tZ(t) x 100	Shape Parameter - β										
	1/3	1/2	2/3	1	1 1/3	1 2/3	2	2 1/2	3 1/3	4	5
.010	.030	.020	.015	.010	.008	.006	.005	.004	.003	.003	.002
.012	.036	.024	.018	.012	.009	.007	.006	.005	.004	.003	.002
.015	.045	.030	.022	.015	.011	.009	.008	.006	.005	.004	.003
.020	.060	.040	.030	.020	.015	.012	.010	.008	.006	.005	.004
.025	.075	.050	.037	.025	.019	.015	.013	.010	.008	.006	.005
.030	.090	.060	.044	.030	.022	.018	.015	.012	.009	.008	.006
.040	.120	.080	.060	.040	.030	.024	.020	.016	.012	.010	.008
.050	.150	.100	.074	.050	.038	.030	.025	.020	.015	.013	.010
.065	.195	.130	.097	.065	.049	.039	.032	.026	.020	.016	.013
.080	.240	.150	.120	.080	.060	.048	.040	.032	.024	.020	.016
.100	.300	.200	.150	.100	.075	.060	.050	.040	.030	.025	.020
.12	.359	.240	.180	.120	.090	.072	.060	.048	.036	.030	.024
.15	.449	.300	.225	.150	.112	.090	.075	.060	.045	.038	.030
.20	.598	.400	.300	.200	.150	.120	.100	.080	.060	.050	.040
.25	.747	.499	.370	.250	.187	.150	.125	.100	.075	.063	.050
.30	.896	.598	.449	.300	.225	.180	.150	.120	.090	.075	.060
.40	1.193	.797	.598	.399	.300	.240	.200	.160	.120	.100	.080
.50	1.489	.995	.747	.499	.374	.300	.250	.200	.150	.125	.100
.65	1.931	1.292	.970	.648	.481	.389	.324	.260	.195	.163	.130
.80	2.371	1.587	1.193	.797	.598	.479	.399	.319	.240	.200	.160
1.00	2.955	1.980	1.489	.995	.747	.598	.499	.399	.300	.250	.200
1.2	3.536	2.371	1.784	1.193	.896	.717	.598	.479	.359	.300	.240
1.5	4.400	2.955	2.225	1.489	1.119	.896	.747	.598	.449	.374	.300
2.0	5.824	3.921	2.955	1.980	1.489	1.193	.995	.797	.598	.499	.399
2.5	7.226	4.877	3.681	2.469	1.858	1.489	1.242	.995	.747	.623	.499
3.0	8.607	5.824	4.400	2.955	2.225	1.784	1.489	1.193	.896	.747	.598
4.0	11.308	7.688	5.824	3.921	2.955	2.371	1.980	1.587	1.193	.995	.797
5.0	13.929	9.516	7.226	4.877	3.681	2.955	2.469	1.980	1.489	1.242	.995
6.5	17.717	12.190	9.290	6.293	4.758	3.825	3.198	2.566	1.931	1.612	1.292
8.0	21.337	14.786	11.308	7.688	5.824	4.687	3.921	3.149	2.371	1.980	1.587
10.0	25.918	18.127	13.929	9.516	7.226	5.824	4.877	3.921	2.955	2.469	1.980
12	30.232	21.337	16.473	11.308	8.607	6.947	5.824	4.687	3.536	2.955	2.371
15	36.237	25.918	20.148	13.929	10.640	8.607	7.226	5.824	4.400	3.681	2.955
20	45.119	32.968	25.918	18.127	13.929	11.308	9.516	7.688	5.824	4.877	3.921
25	52.763	39.347	31.271	22.120	17.097	13.929	11.750	9.516	7.226	6.059	4.877
30	59.343	45.119	36.237	25.918	20.148	16.473	13.929	11.308	8.607	7.226	5.824
40	69.881	55.067	45.119	32.968	25.918	21.337	18.127	14.786	11.308	9.516	7.688
50	77.687	63.212	52.763	39.347	31.271	25.918	22.120	18.127	13.929	11.750	9.516
65	85.773	72.747	62.281	47.795	38.584	32.294	27.747	22.895	17.717	14.998	12.190
80	90.928	79.810	69.881	55.067	45.119	38.122	32.968	27.385	21.337	18.127	14.786
100	95.021	86.466	77.587	63.212	52.763	45.119	39.347	32.968	25.918	22.120	18.127

TABLE 3a

Table of Sampling Plans for $\beta = 1/3$

c	n tZ(t) x 100 Product for Which P(A) = .10 (or less) 50	25	15	10	5	2.5	1.5	1.0	0.5	0.25	0.15	0.10	0.05
0	2 (.85)	4 (.42)	6 (.29)	8 (.21)	16 (.10)	31 (.05)	52 (.03)	77 (.02)	155 (.01)	308 --	513 --	768 --	1540 --
1	4 (3.4)	6 (2.1)	10 (1.2)	14 (.88)	27 (.44)	53 (.23)	87 (.13)	130 (.09)	261 (.04)	521 (.02)	866 (.01)	1300 (.01)	2600 --
2	5 (6.9)	9 (3.4)	13 (2.2)	19 (1.5)	37 (.75)	72 (.38)	120 (.23)	180 (.15)	357 (.08)	712 (.04)	1190 (.02)	1770 (.01)	3550 (.01)
3	7 (8.4)	11 (4.8)	17 (2.9)	24 (2.0)	47 (.99)	91 (.51)	150 (.30)	226 (.20)	447 (.10)	894 (.05)	1490 (.03)	2230 (.02)	4450 (.01)
4	8 (11)	13 (6.0)	20 (3.7)	29 (2.4)	56 (1.2)	109 (.61)	182 (.37)	270 (.25)	537 (.12)	1070 (.06)	1770 (.04)	2670 (.02)	5330 (.01)
5	10 (12)	16 (6.5)	24 (4.1)	34 (2.7)	65 (1.4)	127 (.70)	211 (.42)	314 (.28)	623 (.14)	1240 (.07)	2070 (.04)	3090 (.03)	6180 (.01)
6	11 (14)	18 (7.3)	27 (4.6)	39 (3.0)	74 (1.5)	144 (.77)	239 (.46)	356 (.31)	707 (.15)	1410 (.08)	2350 (.05)	3510 (.03)	7020 (.01)
7	13 (15)	20 (8.1)	30 (5.0)	43 (3.3)	82 (1.6)	163 (.83)	266 (.50)	398 (.34)	791 (.17)	1580 (.08)	2620 (.05)	3920 (.03)	7850 (.02)
8	14 (16)	22 (8.8)	34 (5.2)	48 (3.5)	91 (1.7)	180 (.89)	295 (.54)	440 (.36)	873 (.18)	1740 (.09)	2890 (.05)	4330 (.04)	8660 (.02)
9	15 (18)	24 (9.3)	37 (5.6)	52 (3.8)	100 (1.8)	197 (.94)	323 (.57)	481 (.38)	954 (.19)	1900 (.09)	3160 (.06)	4740 (.04)	9470 (.02)
10	17 (18)	26 (9.6)	41 (5.8)	57 (3.8)	111 (1.9)	213 (.99)	350 (.60)	521 (.40)	1040 (.20)	2060 (.10)	3430 (.06)	5130 (.04)	10300 (.02)
11	18 (19)	29 (9.6)	44 (5.8)	61 (3.8)	119 (2.0)	230 (1.0)	377 (.62)	562 (.42)	1120 (.21)	2220 (.10)	3700 (.06)	5530 (.04)	11100 (.02)
12	20 (19)	31 (10)	47 (6.2)	69 (3.9)	127 (2.1)	246 (1.0)	404 (.64)	602 (.43)	1190 (.22)	2380 (.11)	3960 (.06)	5930 (.04)	11900 (.02)
13	21 (20)	33 (10)	50 (6.2)	73 (4.1)	136 (2.1)	262 (1.1)	431 (.66)	642 (.45)	1250 (.22)	2540 (.11)	4220 (.07)	6320 (.04)	12600 (.02)
14	22 (21)	35 (10)	53 (6.6)	78 (4.2)	145 (2.2)	279 (1.1)	457 (.68)	681 (.46)	1350 (.23)	2700 (.11)	4480 (.07)	6710 (.04)	13400 (.02)
15	24 (21)	37 (11)	56 (6.6)	82 (4.3)	153 (2.2)	295 (1.1)	484 (.70)	721 (.47)	1430 (.23)	2850 (.12)	4740 (.07)	7100 (.05)	14200 (.02)

tZ(t) x 100 products in parentheses are for P(A) = .95 (or more)

TABLE 3b

Table of Sampling Plans for $\beta = 1/2$

c	n												
	tZ(t) x 100 Product for Which P(A) = .10 (or less)												
	50	25	15	10	5	2.5	1.5	1.0	0.5	0.25	0.15	0.10	0.05
0	3 (.85)	5 (.50)	8 (.31)	12 (.21)	23 (.10)	46 (.05)	77 (.03)	115 (.02)	231 (.01)	462 --	768 --	1150 --	2300 --
1	5 (3.9)	9 (2.0)	14 (1.3)	20 (.90)	40 (.44)	79 (.23)	130 (.13)	196 (.09)	391 (.04)	780 (.02)	1300 (.01)	1950 (.01)	3890 --
2	7 (6.8)	12 (3.6)	19 (2.2)	28 (1.4)	55 (.75)	108 (.38)	180 (.23)	269 (.15)	535 (.08)	1070 (.04)	1770 (.02)	2660 (.01)	5320 (.01)
3	9 (9.1)	15 (5.0)	24 (3.0)	35 (2.0)	69 (1.0)	135 (.51)	226 (.30)	337 (.20)	670 (.10)	1340 (.05)	2230 (.03)	3340 (.02)	6680 (.01)
4	11 (11)	19 (5.8)	29 (3.6)	42 (2.4)	82 (1.2)	164 (.61)	271 (.37)	404 (.25)	803 (.12)	1600 (.06)	2670 (.04)	4000 (.02)	7990 (.01)
5	13 (12)	22 (6.6)	34 (4.1)	49 (2.8)	96 (1.4)	190 (.70)	314 (.42)	468 (.28)	932 (.14)	1860 (.07)	3090 (.04)	4640 (.03)	9280 (.01)
6	14 (15)	25 (7.4)	39 (4.5)	56 (3.1)	109 (1.5)	216 (.77)	356 (.46)	532 (.31)	1060 (.15)	2110 (.08)	3510 (.05)	5270 (.03)	10500 (.01)
7	16 (16)	28 (8.0)	43 (5.0)	63 (3.3)	122 (1.6)	241 (.83)	398 (.50)	594 (.34)	1180 (.17)	2360 (.08)	3920 (.05)	5880 (.03)	11800 (.02)
8	18 (16)	31 (8.6)	48 (5.2)	70 (3.5)	134 (1.8)	266 (.89)	440 (.54)	656 (.36)	1310 (.18)	2600 (.09)	4330 (.05)	6500 (.04)	13000 (.02)
9	20 (17)	34 (9.1)	52 (5.6)	76 (3.7)	147 (1.9)	291 (.94)	481 (.57)	718 (.38)	1430 (.19)	2850 (.09)	4740 (.06)	7100 (.04)	14200 (.02)
10	21 (19)	37 (9.2)	57 (5.7)	85 (3.7)	162 (1.9)	316 (.99)	521 (.60)	778 (.40)	1550 (.20)	3090 (.10)	5140 (.06)	7700 (.04)	15400 (.02)
11	23 (20)	40 (9.9)	61 (6.0)	92 (3.8)	174 (2.0)	340 (1.0)	562 (.62)	838 (.42)	1670 (.21)	3330 (.10)	5530 (.06)	8300 (.04)	16600 (.02)
12	25 (20)	43 (9.9)	68 (6.0)	98 (4.0)	187 (2.1)	365 (1.0)	602 (.64)	898 (.43)	1790 (.22)	3560 (.11)	5930 (.06)	8890 (.04)	17800 (.02)
13	27 (20)	46 (10)	73 (6.0)	105 (4.1)	199 (2.1)	389 (1.1)	642 (.66)	957 (.45)	1910 (.22)	3800 (.11)	6320 (.07)	9480 (.04)	19000 (.02)
14	29 (21)	49 (10)	78 (6.2)	111 (4.2)	212 (2.2)	413 (1.1)	681 (.68)	1020 (.46)	2020 (.23)	4030 (.11)	6710 (.07)	10100 (.04)	20100 (.02)
15	31 (21)	52 (11)	82 (6.5)	117 (4.4)	224 (2.3)	437 (1.1)	721 (.70)	1080 (.47)	2140 (.23)	4270 (.12)	7100 (.07)	10700 (.05)	21300 (.02)

tZ(t) x 100 products in parentheses are for P(A) = .95 (or more)

TABLE 3c

Table of Sampling Plans for $\beta = 2/3$

c	n: tZ(t) x 100 Product for Which P(A) = .10 (or less) 100	50	25	15	10	5	2.5	1.5	1.0	0.5	0.25	0.15	0.10
0	2 (1.6)	4 (.86)	7 (.49)	11 (.31)	16 (.21)	31 (.10)	62 (.05)	102 (.03)	155 (.02)	308 (.01)	622 --	1020 --	1540 --
1	4 (6.7)	6 (4.2)	11 (2.2)	18 (1.3)	27 (.89)	53 (.44)	105 (.23)	175 (.13)	261 (.09)	521 (.04)	1050 (.02)	1730 (.01)	2590 (.01)
2	5 (13)	9 (6.7)	16 (3.6)	25 (2.2)	37 (1.5)	72 (.77)	143 (.38)	239 (.23)	357 (.15)	712 (.08)	1440 (.04)	2370 (.02)	3550 (.01)
3	7 (16)	11 (9.6)	20 (4.9)	32 (3.0)	47 (1.9)	91 (1.0)	181 (.51)	300 (.30)	449 (.20)	894 (.10)	1810 (.05)	2970 (.03)	4450 (.02)
4	8 (22)	13 (12)	24 (5.8)	38 (3.7)	56 (2.4)	109 (1.2)	217 (.61)	359 (.37)	537 (.25)	1070 (.12)	2160 (.06)	3550 (.04)	5330 (.02)
5	10 (23)	16 (13)	28 (6.7)	44 (4.1)	65 (2.7)	127 (1.4)	252 (.70)	417 (.42)	623 (.28)	1240 (.14)	2510 (.07)	4120 (.04)	6180 (.03)
6	11 (28)	18 (14)	32 (7.4)	50 (4.6)	74 (3.0)	144 (1.5)	286 (.77)	473 (.46)	707 (.31)	1410 (.15)	2850 (.08)	4680 (.05)	7020 (.03)
7	13 (29)	20 (16)	35 (8.4)	56 (5.0)	82 (3.3)	163 (1.6)	320 (.83)	529 (.50)	791 (.34)	1580 (.17)	3180 (.08)	5230 (.05)	7850 (.03)
8	14 (33)	22 (17)	39 (8.8)	63 (5.2)	91 (3.5)	180 (1.7)	353 (.89)	584 (.54)	873 (.36)	1740 (.18)	3510 (.09)	5780 (.05)	8660 (.04)
9	15 (36)	24 (18)	43 (9.2)	68 (5.6)	100 (3.7)	197 (1.8)	386 (.94)	638 (.57)	954 (.38)	1900 (.19)	3840 (.09)	6310 (.06)	9470 (.04)
10	17 (36)	26 (19)	47 (9.2)	76 (5.6)	111 (3.8)	213 (1.9)	419 (.99)	692 (.60)	1030 (.40)	2060 (.20)	4160 (.10)	6850 (.06)	10300 (.04)
11	18 (38)	29 (19)	51 (10)	82 (5.8)	119 (3.9)	230 (2.0)	451 (1.0)	746 (.62)	1120 (.42)	2220 (.21)	4490 (.10)	7380 (.06)	11100 (.04)
12	20 (38)	31 (20)	55 (10)	88 (6.0)	128 (4.1)	246 (2.1)	483 (1.0)	799 (.64)	1190 (.43)	2380 (.22)	4810 (.11)	7900 (.06)	11900 (.04)
13	21 (40)	33 (20)	58 (10)	94 (6.2)	136 (4.2)	262 (2.2)	515 (1.1)	852 (.66)	1270 (.45)	2540 (.22)	5120 (.11)	8430 (.07)	12600 (.04)
14	22 (43)	35 (21)	62 (10)	100 (6.4)	145 (4.3)	279 (2.2)	547 (1.1)	905 (.68)	1350 (.46)	2700 (.23)	5440 (.11)	8950 (.07)	13400 (.04)
15	24 (43)	37 (22)	68 (11)	106 (6.6)	153 (4.4)	295 (2.3)	578 (1.1)	957 (.70)	1430 (.47)	2850 (.23)	5760 (.12)	9460 (.07)	14200 (.05)

tZ(t) x 100 products in parentheses are for P(A) = .95 (or more)

TABLE 3d

Table of Sampling Plans for $\beta = 1$

c	n												
	tZ(t) x 100 Product for Which P(A) = .10 (or less)												
	100	50	25	15	10	5	2.5	1.5	1.0	0.5	0.25	0.15	0.10
0	3 (1.7)	5 (1.0)	10 (.51)	16 (.32)	24 (.20)	46 (.11)	92 (.06)	155 (.03)	231 (.02)	461 (.01)	922 --	1540 --	2300 --
1	5 (8.0)	9 (4.2)	17 (2.1)	27 (1.3)	40 (.90)	79 (.45)	158 (.22)	261 (.13)	389 (.09)	778 (.05)	1560 (.02)	2590 (.01)	3890 (.01)
2	7 (14)	12 (7.4)	23 (3.7)	37 (2.3)	55 (1.5)	108 (.76)	216 (.38)	357 (.23)	533 (.15)	1070 (.08)	2130 (.04)	3550 (.02)	5320 (.02)
3	9 (19)	15 (10)	29 (5.0)	47 (3.0)	69 (2.0)	135 (1.0)	271 (.50)	449 (.30)	669 (.20)	1340 (.10)	2670 (.05)	4450 (.03)	6680 (.02)
4	11 (22)	19 (12)	34 (6.2)	56 (3.6)	82 (2.4)	164 (1.2)	324 (.61)	537 (.37)	800 (.24)	1600 (.12)	3200 (.06)	5330 (.04)	8000 (.02)
5	13 (25)	22 (13)	40 (7.0)	65 (4.2)	96 (2.8)	191 (1.4)	376 (.69)	623 (.42)	928 (.28)	1860 (.14)	3710 (.07)	6180 (.04)	9280 (.03)
6	14 (30)	25 (15)	46 (7.5)	74 (4.6)	109 (3.0)	216 (1.5)	427 (.77)	707 (.46)	1050 (.31)	2110 (.16)	4210 (.08)	7020 (.05)	10500 (.03)
7	16 (33)	28 (16)	51 (8.2)	82 (5.0)	122 (3.3)	242 (1.7)	477 (.83)	791 (.50)	1180 (.34)	2360 (.17)	4710 (.08)	7850 (.05)	11800 (.03)
8	18 (35)	31 (17)	57 (9.0)	91 (5.4)	135 (3.5)	267 (1.8)	527 (.89)	872 (.54)	1300 (.36)	2600 (.18)	5200 (.09)	8660 (.05)	13000 (.04)
9	20 (36)	34 (18)	62 (9.3)	100 (5.7)	147 (3.7)	292 (1.9)	576 (.94)	954 (.57)	1420 (.38)	2840 (.19)	5680 (.10)	9470 (.06)	14200 (.04)
10	22 (38)	37 (19)	70 (9.5)	111 (5.7)	162 (3.9)	316 (2.0)	624 (1.0)	1030 (.60)	1540 (.40)	3080 (.20)	6160 (.10)	10300 (.06)	15400 (.04)
11	23 (40)	40 (20)	76 (9.8)	119 (6.0)	175 (4.0)	341 (2.1)	672 (1.1)	1110 (.62)	1660 (.42)	3320 (.21)	6640 (.10)	11100 (.06)	16600 (.04)
12	25 (42)	43 (20)	81 (10)	128 (6.2)	187 (4.2)	365 (2.2)	720 (1.1)	1190 (.64)	1780 (.43)	3560 (.22)	7110 (.11)	11900 (.06)	17800 (.04)
13	27 (42)	45 (21)	86 (10)	136 (6.4)	200 (4.3)	389 (2.2)	768 (1.1)	1270 (.66)	1900 (.45)	3790 (.22)	7580 (.11)	12600 (.07)	19000 (.05)
14	29 (44)	48 (22)	91 (11)	145 (6.5)	212 (4.4)	413 (2.3)	815 (1.1)	1350 (.68)	2010 (.46)	4030 (.23)	8050 (.11)	13400 (.07)	20100 (.05)
15	31 (44)	51 (22)	97 (11)	153 (6.7)	224 (4.6)	437 (2.4)	863 (1.2)	1430 (.70)	2130 (.47)	4260 (.24)	8520 (.12)	14200 (.07)	21300 (.05)

tZ(t) x 100 products in parentheses are for P(A) = .95 (or more)

TABLE 3e

Table of Sampling Plans for $\beta = 1\ 1/3$

c	n												
	tZ(t) x 100 Product for Which P(A) = .10 (or less)												
	100	50	25	15	10	5	2.5	1.5	1.0	0.5	0.25	0.15	0.10
0	4 (1.6)	7 (.97)	13 (.52)	21 (.32)	31 (.22)	62 (.11)	123 (.05)	206 (.03)	308 (.02)	616 (.01)	1230 --	2060 --	3070 --
1	6 (8.6)	11 (4.3)	22 (2.1)	36 (1.3)	53 (.90)	105 (.45)	209 (.23)	348 (.13)	521 (.09)	1040 (.04)	2080 (.02)	3470 (.01)	5190 (.01)
2	9 (13)	16 (7.2)	30 (3.8)	49 (2.3)	72 (1.5)	144 (.75)	286 (.38)	476 (.23)	712 (.15)	1420 (.08)	2850 (.04)	4750 (.02)	7100 (.01)
3	11 (19)	20 (9.8)	38 (5.0)	61 (3.0)	91 (2.0)	181 (.99)	360 (.51)	597 (.30)	894 (.20)	1790 (.10)	3570 (.05)	5970 (.03)	8910 (.02)
4	13 (24)	24 (12)	45 (6.0)	74 (3.6)	109 (2.4)	217 (1.2)	430 (.61)	714 (.37)	1070 (.25)	2140 (.12)	4270 (.06)	7140 (.04)	10700 (.02)
5	16 (26)	28 (13)	52 (6.9)	85 (4.2)	127 (2.7)	252 (1.4)	499 (.70)	829 (.42)	1240 (.28)	2480 (.14)	4960 (.07)	8280 (.04)	12400 (.03)
6	18 (29)	32 (14)	60 (7.6)	97 (4.6)	144 (3.0)	286 (1.5)	567 (.77)	941 (.46)	1410 (.31)	2820 (.15)	5630 (.08)	9400 (.05)	14000 (.03)
7	20 (32)	35 (16)	67 (8.3)	109 (5.0)	163 (3.3)	320 (1.6)	634 (.83)	1050 (.50)	1580 (.34)	3150 (.17)	6290 (.08)	10500 (.05)	15700 (.03)
8	22 (35)	39 (17)	74 (8.8)	120 (5.3)	180 (3.5)	359 (1.7)	699 (.89)	1160 (.54)	1740 (.36)	3470 (.18)	6950 (.09)	11600 (.05)	17300 (.04)
9	24 (37)	43 (18)	81 (9.2)	131 (5.7)	197 (3.7)	386 (1.8)	765 (.94)	1270 (.57)	1900 (.38)	3800 (.19)	7600 (.09)	12700 (.06)	18900 (.04)
10	26 (38)	47 (18)	90 (9.4)	145 (5.7)	213 (3.9)	419 (1.9)	829 (.99)	1380 (.60)	2060 (.40)	4120 (.20)	8240 (.10)	13800 (.06)	20500 (.04)
11	29 (38)	51 (19)	97 (9.7)	156 (6.1)	230 (4.0)	451 (2.0)	893 (1.0)	1480 (.62)	2220 (.42)	4440 (.21)	8880 (.10)	14800 (.06)	22100 (.04)
12	31 (40)	55 (20)	104 (10)	167 (6.3)	246 (4.2)	483 (2.1)	957 (1.0)	1590 (.64)	2380 (.43)	4750 (.22)	9510 (.11)	15900 (.06)	23700 (.04)
13	33 (42)	58 (21)	111 (10)	178 (6.5)	262 (4.4)	515 (2.2)	1020 (1.1)	1690 (.66)	2540 (.45)	5070 (.22)	10100 (.11)	16900 (.07)	25300 (.04)
14	35 (44)	62 (21)	118 (11)	189 (6.6)	279 (4.5)	547 (2.2)	1080 (1.1)	1800 (.68)	2690 (.46)	5380 (.23)	10800 (.11)	18000 (.07)	26800 (.04)
15	37 (45)	68 (22)	125 (11)	200 (6.8)	295 (4.6)	578 (2.3)	1150 (1.1)	1900 (.70)	2850 (.47)	5690 (.23)	11400 (.12)	19000 (.07)	28400 (.05)

tZ(t) x 100 products in parentheses are for P(A) = .95 (or more)

TABLE 3f

Table of Sampling Plans for β = 1 2/3

c	n												
	tZ(t) x 100 Product for Which P(A) = .10 (or less)												
	100	50	25	15	10	6.5	5.0	4.0	2.5	1.5	1.0	0.5	0.25
0	4 (2.1)	8 (1.0)	16 (.53)	26 (.32)	39 (.21)	59 (.14)	77 (.11)	96 (.09)	155 (.05)	257 (.03)	385 (.02)	768 (.01)	1540 --
1	7 (9.0)	14 (4.4)	27 (2.2)	44 (1.3)	66 (.89)	100 (.58)	130 (.44)	164 (.36)	261 (.23)	434 (.13)	651 (.09)	1300 (.04)	2590 (.02)
2	10 (15)	19 (7.5)	37 (3.8)	60 (2.3)	90 (1.5)	138 (.99)	180 (.75)	224 (.61)	357 (.38)	594 (.23)	890 (.15)	1770 (.08)	3550 (.04)
3	13 (20)	24 (10)	47 (5.0)	76 (3.0)	113 (2.0)	175 (1.2)	226 (1.0)	282 (.82)	449 (.51)	746 (.30)	1120 (.20)	2230 (.10)	4450 (.05)
4	16 (23)	29 (12)	56 (6.1)	91 (3.7)	136 (2.4)	209 (1.5)	271 (1.2)	337 (.98)	537 (.61)	892 (.37)	1340 (.25)	2660 (.12)	5330 (.06)
5	19 (26)	34 (13)	65 (7.0)	106 (4.1)	159 (2.7)	242 (1.7)	314 (1.4)	391 (1.1)	623 (.70)	1040 (.42)	1550 (.28)	3090 (.14)	6180 (.07)
6	21 (30)	39 (15)	74 (7.6)	120 (4.7)	181 (3.0)	275 (2.0)	356 (1.5)	444 (1.2)	707 (.77)	1180 (.46)	1760 (.31)	3510 (.15)	7020 (.08)
7	24 (32)	43 (16)	82 (8.5)	135 (4.9)	202 (3.3)	308 (2.1)	398 (1.6)	496 (1.3)	791 (.83)	1310 (.50)	1970 (.34)	3930 (.17)	7850 (.08)
8	26 (35)	48 (17)	91 (9.0)	149 (5.4)	223 (3.5)	340 (2.2)	440 (1.7)	558 (1.4)	873 (.89)	1450 (.54)	2170 (.36)	4330 (.18)	8630 (.09)
9	29 (37)	52 (18)	100 (9.4)	165 (5.5)	244 (3.7)	371 (2.4)	481 (1.8)	599 (1.5)	954 (.94)	1590 (.57)	2380 (.38)	4740 (.19)	9470 (.09)
10	32 (37)	57 (19)	111 (9.4)	179 (5.9)	265 (3.9)	403 (2.5)	521 (1.9)	650 (1.5)	1030 (.99)	1720 (.60)	2580 (.40)	5140 (.20)	10300 (.10)
11	34 (39)	61 (19)	119 (9.9)	193 (6.1)	285 (4.0)	434 (2.6)	562 (2.0)	700 (1.6)	1110 (1.0)	1850 (.62)	2780 (.42)	5530 (.21)	11100 (.10)
12	37 (41)	69 (20)	128 (10)	207 (6.3)	305 (4.2)	465 (2.7)	602 (2.1)	750 (1.7)	1190 (1.0)	1980 (.64)	2970 (.43)	5930 (.22)	11900 (.11)
13	39 (43)	73 (20)	136 (10)	220 (6.5)	326 (4.4)	496 (2.8)	642 (2.2)	800 (1.7)	1270 (1.1)	2120 (.66)	3170 (.45)	6320 (.22)	12600 (.11)
14	42 (43)	78 (21)	145 (11)	234 (6.7)	346 (4.5)	526 (2.9)	681 (2.2)	849 (1.8)	1350 (1.1)	2250 (.68)	3370 (.46)	6710 (.23)	13400 (.11)
15	44 (45)	82 (21)	153 (11)	247 (6.9)	366 (4.6)	557 (3.0)	721 (2.3)	898 (1.8)	1430 (1.1)	2380 (.70)	3560 (.47)	7100 (.23)	14200 (.12)

tZ(t) x 100 products in parentheses are for P(A) = .95 (or more)

TABLE 3g

Table of Sampling Plans for $\beta = 2$

c	n												
	tZ(t) x 100 Product for which P(A) = .10 (or less)												
	100	50	25	15	10	6.5	5.0	4.0	2.5	1.5	1.0	0.5	0.25
0	5 (2.0)	10 (1.0)	19 (.54)	31 (.33)	46 (.22)	73 (.14)	92 (.11)	115 (.09)	185 (.05)	308 (.03)	462 (.02)	921 (.01)	1840 --
1	9 (8.4)	16 (4.5)	32 (2.2)	53 (1.3)	79 (.90)	120 (.59)	158 (.44)	196 (.36)	313 (.23)	521 (.13)	780 (.09)	1560 (.04)	3120 (.02)
2	12 (14)	23 (7.5)	44 (3.8)	72 (2.3)	108 (1.5)	166 (.98)	216 (.75)	269 (.61)	429 (.38)	712 (.23)	1070 (.15)	2130 (.08)	4260 (.04)
3	15 (20)	29 (10)	55 (5.0)	91 (3.0)	135 (2.0)	209 (1.3)	271 (1.0)	337 (.82)	538 (.51)	894 (.30)	1340 (.20)	2670 (.10)	5340 (.05)
4	18 (24)	34 (12)	66 (6.2)	109 (3.6)	163 (2.4)	250 (1.5)	324 (1.2)	404 (.98)	644 (.61)	1070 (.37)	1600 (.25)	3200 (.12)	6400 (.06)
5	22 (27)	40 (13)	77 (7.1)	127 (4.2)	190 (2.7)	290 (1.8)	376 (1.4)	468 (1.1)	747 (.70)	1240 (.42)	1860 (.28)	3710 (.14)	7420 (.07)
6	25 (29)	45 (15)	88 (7.7)	143 (4.7)	216 (3.0)	329 (2.0)	427 (1.5)	532 (1.2)	848 (.77)	1410 (.46)	2110 (.31)	4210 (.15)	8430 (.08)
7	28 (33)	51 (16)	98 (8.4)	163 (5.0)	241 (3.3)	368 (2.1)	477 (1.6)	594 (1.3)	948 (.83)	1580 (.50)	2360 (.34)	4710 (.17)	9420 (.08)
8	31 (35)	57 (17)	109 (8.9)	180 (5.2)	266 (3.5)	406 (2.3)	526 (1.7)	656 (1.4)	1050 (.89)	1740 (.54)	2600 (.36)	5200 (.18)	10400 (.09)
9	34 (37)	62 (19)	119 (9.5)	197 (5.6)	291 (3.7)	444 (2.4)	575 (1.9)	717 (1.5)	1140 (.94)	1900 (.57)	2850 (.38)	5680 (.19)	11400 (.09)
10	37 (37)	70 (18)	131 (9.7)	213 (5.9)	316 (3.9)	482 (2.5)	624 (1.9)	778 (1.5)	1240 (.99)	2060 (.60)	3090 (.40)	6160 (.20)	12300 (.10)
11	40 (40)	75 (19)	141 (9.9)	230 (6.1)	340 (4.1)	519 (2.6)	672 (2.0)	838 (1.6)	1340 (1.0)	2220 (.62)	3330 (.42)	6640 (.21)	13300 (.10)
12	43 (40)	80 (20)	151 (10)	246 (6.4)	365 (4.2)	556 (2.7)	720 (2.1)	898 (1.7)	1430 (1.0)	2380 (.64)	3560 (.43)	7110 (.22)	14200 (.11)
13	46 (42)	86 (20)	161 (10)	262 (6.6)	389 (4.4)	593 (2.8)	768 (2.2)	957 (1.7)	1530 (1.1)	2540 (.66)	3800 (.45)	7580 (.22)	15200 (.11)
14	49 (42)	91 (21)	171 (11)	279 (6.8)	413 (4.5)	629 (2.9)	815 (2.2)	1020 (1.8)	1620 (1.1)	2700 (.68)	4030 (.46)	8050 (.23)	16100 (.11)
15	52 (44)	96 (22)	181 (11)	295 (7.0)	437 (4.6)	666 (3.0)	862 (2.3)	1080 (1.8)	1710 (1.1)	2850 (.70)	4270 (.47)	8520 (.23)	17000 (.12)

t(Z) x 100 products in parentheses are for P(A) = .95 (or more)

TABLE 3h

Table of Sampling Plans for $\beta = 2\ 1/2$

c	n												
	tZ(t) x 100 Product for Which P(A) = .10 (or less)												
	100	50	25	15	10	6.5	5.0	4.0	2.5	1.5	1.0	0.5	0.25
0	6 (2.1)	12 (1.0)	23 (.54)	39 (.33)	58 (.21)	89 (.14)	115 (.11)	144 (.09)	231 (.05)	385 (.03)	577 (.02)	1150 (.01)	2300 --
1	11 (8.4)	20 (4.5)	40 (2.2)	66 (1.3)	98 (.90)	150 (.58)	196 (.45)	245 (.36)	391 (.23)	650 (.13)	975 (.09)	1950 (.04)	3890 (.02)
2	15 (14)	28 (7.5)	55 (3.8)	90 (2.3)	134 (1.5)	207 (.99)	269 (.76)	335 (.61)	535 (.38)	890 (.23)	1330 (.15)	2660 (.08)	5320 (.04)
3	19 (19)	35 (10)	69 (5.0)	113 (3.0)	170 (2.0)	260 (1.3)	337 (1.0)	421 (.82)	671 (.51)	1120 (.30)	1670 (.20)	3340 (.10)	6680 (.05)
4	23 (23)	42 (12)	82 (6.1)	136 (3.6)	204 (2.4)	312 (1.5)	404 (1.2)	504 (.98)	803 (.61)	1340 (.37)	2000 (.25)	4000 (.12)	7990 (.06)
5	26 (28)	49 (13)	96 (6.9)	159 (4.1)	237 (2.7)	361 (1.8)	468 (1.4)	584 (1.1)	932 (.70)	1550 (.42)	2320 (.28)	4640 (.14)	9280 (.07)
6	30 (31)	56 (15)	109 (7.7)	181 (4.6)	269 (3.0)	410 (2.0)	532 (1.5)	664 (1.2)	1060 (.77)	1760 (.46)	2640 (.31)	5270 (.15)	10500 (.08)
7	34 (32)	63 (16)	122 (8.3)	202 (4.9)	300 (3.3)	459 (2.1)	594 (1.6)	742 (1.3)	1180 (.83)	1970 (.50)	2950 (.34)	5890 (.17)	11800 (.08)
8	37 (35)	70 (17)	134 (9.0)	223 (5.2)	331 (3.5)	506 (2.3)	656 (1.7)	819 (1.4)	1310 (.89)	2170 (.54)	3260 (.36)	6500 (.18)	13000 (.09)
9	41 (37)	76 (18)	147 (9.5)	244 (5.5)	362 (3.7)	554 (2.4)	717 (1.8)	895 (1.5)	1430 (.94)	2380 (.57)	3560 (.38)	7100 (.19)	14200 (.09)
10	44 (37)	85 (18)	162 (9.5)	265 (5.8)	393 (3.9)	600 (2.5)	778 (1.9)	971 (1.5)	1550 (.99)	2580 (.60)	3860 (.40)	7700 (.20)	15400 (.10)
11	48 (40)	92 (19)	174 (9.9)	285 (6.1)	423 (4.1)	647 (2.6)	838 (2.0)	1050 (1.6)	1670 (1.0)	2780 (.62)	4160 (.42)	8300 (.21)	16600 (.10)
12	51 (40)	98 (20)	187 (10)	305 (6.3)	454 (4.2)	693 (2.7)	898 (2.1)	1120 (1.7)	1790 (1.0)	2970 (.64)	4460 (.43)	8890 (.22)	17800 (.11)
13	55 (43)	105 (20)	199 (10)	326 (6.5)	483 (4.4)	739 (2.8)	957 (2.2)	1190 (1.7)	1910 (1.1)	3170 (.66)	4750 (.45)	9480 (.22)	19000 (.11)
14	58 (43)	111 (21)	212 (11)	346 (6.6)	513 (4.5)	784 (2.9)	1020 (2.2)	1270 (1.8)	2020 (1.1)	3370 (.68)	5040 (.46)	10100 (.23)	20100 (.11)
15	62 (43)	117 (22)	224 (11)	366 (6.8)	543 (4.6)	830 (3.0)	1080 (2.3)	1340 (1.8)	2140 (1.1)	3560 (.70)	5340 (.47)	10600 (.23)	21300 (.12)

tZ(t) x 100 products in parentheses are for P(A) = .95 (or more)

TABLE 31

Table of Sampling Plans for $\beta = 3\ 1/3$

c	n: tZ(t) x 100 Product for Which P(A) = .10 (or less)												
	100	50	25	15	10	6.5	5.0	4.0	2.5	1.5	1.0	0.5	0.25
0	8 (2.1)	16 (1.0)	31 (.55)	52 (.33)	78 (.21)	118 (.14)	155 (.11)	193 (.09)	308 (.05)	513 (.03)	768 (.02)	1540 (.01)	3070 --
1	14 (8.8)	27 (4.4)	53 (2.2)	87 (1.3)	132 (.90)	201 (.57)	261 (.45)	326 (.36)	521 (.23)	866 (.13)	1300 (.09)	2590 (.04)	5190 (.02)
2	19 (15)	37 (7.6)	72 (3.8)	120 (2.3)	180 (1.5)	276 (.99)	357 (.76)	446 (.61)	712 (.38)	1190 (.23)	1770 (.15)	3550 (.08)	7100 (.04)
3	24 (20)	47 (9.9)	91 (5.0)	150 (3.0)	226 (2.0)	350 (1.3)	449 (1.0)	560 (.82)	894 (.51)	1490 (.30)	2230 (.20)	4450 (.10)	8910 (.05)
4	29 (24)	56 (12)	109 (6.0)	182 (3.6)	271 (2.4)	414 (1.5)	537 (1.2)	670 (.98)	1070 (.61)	1780 (.37)	2660 (.25)	5330 (.12)	10700 (.06)
5	34 (27)	65 (14)	127 (6.9)	211 (4.1)	314 (2.7)	480 (1.8)	623 (1.4)	777 (1.1)	1240 (.70)	2070 (.42)	3090 (.28)	6180 (.14)	12400 (.07)
6	39 (30)	74 (15)	143 (7.9)	239 (4.6)	356 (3.1)	545 (2.0)	707 (1.5)	883 (1.2)	1410 (.77)	2350 (.46)	3510 (.31)	7020 (.15)	14000 (.08)
7	43 (33)	82 (16)	163 (8.2)	268 (4.9)	398 (3.3)	610 (2.1)	791 (1.7)	990 (1.3)	1580 (.83)	2620 (.50)	3920 (.34)	7850 (.17)	15700 (.08)
8	48 (35)	91 (17)	180 (8.8)	295 (5.3)	440 (3.5)	673 (2.3)	873 (1.7)	1090 (1.4)	1740 (.89)	2890 (.54)	4330 (.36)	8660 (.18)	17300 (.09)
9	52 (38)	100 (18)	196 (9.4)	323 (5.6)	481 (3.7)	736 (2.4)	954 (1.9)	1190 (1.5)	1900 (.94)	3160 (.57)	4740 (.38)	9470 (.19)	18900 (.09)
10	57 (38)	111 (18)	213 (9.8)	350 (5.9)	521 (3.9)	798 (2.5)	1030 (1.9)	1290 (1.5)	2060 (.99)	3430 (.60)	5140 (.40)	10300 (.20)	20500 (.10)
11	61 (38)	119 (19)	230 (10)	377 (6.2)	562 (4.1)	860 (2.6)	1110 (2.0)	1390 (1.6)	2220 (1.0)	3700 (.62)	5530 (.42)	11100 (.21)	22100 (.10)
12	69 (39)	128 (20)	246 (10)	404 (6.4)	602 (4.3)	921 (2.7)	1190 (2.1)	1490 (1.7)	2380 (1.0)	3960 (.64)	5930 (.43)	11900 (.22)	23700 (.11)
13	73 (40)	136 (21)	262 (10)	430 (6.6)	642 (4.4)	982 (2.8)	1270 (2.2)	1590 (1.7)	2540 (1.1)	4220 (.66)	6320 (.45)	12600 (.22)	25300 (.11)
14	78 (41)	145 (21)	279 (11)	457 (6.7)	681 (4.5)	1040 (2.9)	1350 (2.3)	1690 (1.8)	2690 (1.1)	4480 (.68)	6710 (.46)	13400 (.23)	26800 (.11)
15	82 (43)	153 (22)	295 (11)	484 (6.9)	721 (4.6)	1100 (3.0)	1430 (2.3)	1780 (1.8)	2850 (1.1)	4740 (.70)	7100 (.47)	14200 (.23)	28400 (.12)

tZ(t) x 100 products in parentheses are for P(A) = .95 (or more)

TABLE 3j

Table of Sampling Plans for $\beta = 4$

c	n: tZ(t) x 100 Product for Which P(A) = .10 (or less)												
	100	65	50	25	15	10	6.5	5.0	4.0	2.5	1.5	1.0	0.5
0	10 (2.0)	15 (1.3)	19 (1.0)	37 (.55)	62 (.33)	92 (.22)	142 (.14)	185 (.11)	231 (.09)	370 (.05)	616 (.03)	921 (.02)	1840 (.01)
1	16 (9.0)	25 (5.7)	32 (4.5)	63 (2.2)	105 (1.3)	158 (.89)	241 (.58)	313 (.45)	391 (.36)	624 (.23)	1040 (.13)	1560 (.09)	3110 (.04)
2	23 (14)	34 (9.8)	44 (7.4)	87 (3.8)	144 (2.3)	216 (1.5)	330 (.99)	429 (.76)	535 (.61)	854 (.38)	1420 (.23)	2130 (.15)	4260 (.08)
3	29 (20)	43 (13)	55 (10)	109 (5.0)	181 (3.0)	271 (2.0)	414 (1.3)	538 (1.0)	671 (.82)	1070 (.51)	1790 (.30)	2670 (.20)	5340 (.10)
4	34 (24)	52 (15)	66 (12)	130 (6.0)	217 (3.6)	324 (2.4)	496 (1.5)	644 (1.2)	803 (.98)	1280 (.61)	2140 (.37)	3200 (.25)	6400 (.12)
5	40 (27)	60 (18)	77 (14)	153 (6.8)	252 (4.2)	376 (2.8)	575 (1.8)	747 (1.4)	932 (1.1)	1490 (.70)	2480 (.42)	3710 (.28)	7420 (.14)
6	45 (31)	68 (20)	88 (15)	174 (7.5)	286 (4.7)	427 (3.1)	653 (2.0)	848 (1.5)	1060 (1.2)	1690 (.77)	2820 (.46)	4210 (.31)	8430 (.15)
7	51 (33)	76 (21)	98 (16)	194 (8.3)	320 (5.0)	477 (3.3)	730 (2.2)	948 (1.7)	1180 (1.3)	1890 (.83)	3150 (.50)	4710 (.34)	9420 (.17)
8	57 (35)	84 (23)	109 (17)	214 (8.7)	353 (5.3)	526 (3.6)	806 (2.3)	1050 (1.8)	1310 (1.4)	2090 (.89)	3480 (.54)	5200 (.36)	10400 (.18)
9	62 (37)	92 (25)	119 (19)	234 (9.3)	386 (5.6)	575 (3.8)	881 (2.4)	1140 (1.9)	1430 (1.5)	2280 (.94)	3800 (.57)	5680 (.38)	11400 (.19)
10	70 (37)	103 (25)	131 (19)	254 (9.8)	419 (5.8)	624 (4.0)	956 (2.6)	1240 (1.9)	1550 (1.5)	2470 (.99)	4120 (.60)	6160 (.40)	12300 (.20)
11	75 (38)	111 (26)	141 (20)	274 (10)	451 (6.1)	672 (4.1)	1030 (2.7)	1340 (2.0)	1670 (1.6)	2660 (1.0)	4440 (.62)	6640 (.42)	13300 (.21)
12	80 (40)	119 (26)	151 (20)	293 (10)	483 (6.3)	720 (4.3)	1100 (2.8)	1430 (2.1)	1790 (1.7)	2850 (1.0)	4750 (.64)	7110 (.43)	14200 (.22)
13	86 (41)	126 (27)	161 (21)	313 (10)	515 (6.5)	768 (4.4)	1180 (2.9)	1530 (2.2)	1910 (1.7)	3040 (1.1)	5070 (.66)	7580 (.45)	15200 (.22)
14	91 (42)	134 (28)	171 (22)	332 (11)	547 (6.7)	815 (4.5)	1250 (3.0)	1620 (2.3)	2020 (1.8)	3230 (1.1)	5380 (.68)	8050 (.46)	16100 (.23)
15	96 (44)	142 (29)	181 (22)	351 (11)	578 (6.9)	862 (4.6)	1320 (3.1)	1710 (2.4)	2140 (1.8)	3420 (1.1)	5690 (.70)	8520 (.47)	17000 (.23)

tZ(t) x 100 products in parentheses are for P(A) = .95 (or more)

TABLE 3k

Table of Sampling Plans for β = 5

c	n tZ(t) x 100 Product for Which P(A) = .10 (or less) 100	65	50	40	25	15	10	6.5	5.0	4.0	2.5	1.5	1.0
0	12 (2.1)	18 (1.4)	23 (1.1)	29 (.88)	46 (.55)	77 (.33)	115 (.22)	178 (.14)	231 (.11)	289 (.09)	462 (.05)	768 (.03)	1150 (.02)
1	20 (9.1)	31 (5.8)	40 (4.5)	50 (3.6)	79 (2.2)	130 (1.3)	196 (.90)	301 (.59)	391 (.46)	488 (.36)	780 (.23)	1300 (.13)	1950 (.09)
2	28 (15)	42 (9.9)	55 (7.5)	68 (6.1)	108 (3.8)	180 (2.3)	269 (1.5)	412 (.99)	535 (.76)	668 (.61)	1070 (.38)	1770 (.23)	2660 (.15)
3	35 (20)	53 (13)	69 (10)	85 (8.1)	135 (5.0)	226 (3.0)	337 (2.0)	517 (1.3)	671 (1.0)	838 (.82)	1340 (.51)	2230 (.30)	3340 (.20)
4	42 (24)	64 (15)	82 (12)	102 (9.8)	164 (6.0)	271 (3.6)	404 (2.4)	619 (1.6)	803 (1.2)	1000 (.98)	1600 (.61)	2660 (.37)	4000 (.25)
5	49 (28)	74 (18)	96 (14)	119 (11)	190 (6.8)	314 (4.2)	468 (2.8)	718 (1.8)	932 (1.4)	1160 (1.1)	1860 (.70)	3090 (.42)	4640 (.28)
6	56 (31)	84 (20)	109 (15)	135 (12)	216 (7.6)	356 (4.7)	532 (3.0)	815 (2.0)	1060 (1.5)	1320 (1.2)	2110 (.77)	3510 (.46)	5270 (.31)
7	63 (33)	94 (22)	122 (16)	153 (13)	241 (8.2)	398 (5.0)	594 (3.3)	911 (2.2)	1180 (1.7)	1480 (1.3)	2360 (.83)	3920 (.50)	5890 (.34)
8	70 (35)	104 (23)	134 (18)	169 (14)	266 (8.8)	440 (5.3)	656 (3.6)	1010 (2.3)	1310 (1.8)	1630 (1.4)	2600 (.89)	4330 (.54)	6500 (.36)
9	76 (37)	114 (25)	147 (19)	185 (14)	291 (9.3)	481 (5.6)	717 (3.8)	1100 (2.5)	1430 (1.9)	1780 (1.5)	2850 (.94)	4740 (.57)	7100 (.38)
10	85 (37)	126 (25)	162 (19)	200 (15)	316 (9.7)	521 (5.9)	778 (4.0)	1190 (2.6)	1550 (1.9)	1930 (1.5)	3090 (.99)	5140 (.60)	7700 (.40)
11	92 (39)	136 (26)	174 (20)	216 (16)	340 (10)	562 (6.1)	838 (4.1)	1280 (2.7)	1670 (2.0)	2080 (1.6)	3330 (1.0)	5530 (.62)	8250 (.42)
12	98 (40)	146 (27)	187 (21)	231 (16)	365 (10)	602 (6.4)	898 (4.3)	1380 (2.8)	1790 (2.1)	2230 (1.7)	3560 (1.0)	5930 (.64)	8890 (.43)
13	105 (41)	156 (27)	199 (21)	247 (17)	389 (10)	642 (6.6)	957 (4.4)	1470 (2.9)	1910 (2.2)	2380 (1.7)	3800 (1.1)	6320 (.66)	9480 (.45)
14	111 (43)	165 (28)	212 (22)	262 (18)	413 (11)	681 (6.8)	1020 (4.5)	1560 (3.0)	2020 (2.3)	2530 (1.8)	4030 (1.1)	6710 (.68)	10100 (.46)
15	117 (43)	175 (29)	224 (23)	277 (18)	437 (11)	721 (6.9)	1080 (4.6)	1650 (3.1)	2140 (2.4)	2670 (1.8)	4270 (1.1)	7100 (.70)	10600 (.47)

tZ(t) x 100 products in parentheses are for P(A) = .95 (or more)

TABLE 4

Table of Hazard Rate Ratios for Values of c

Approximate Values for RHR/AHR

(These ratios apply for all values of β)

c	P(A|RHR)								
	.10			.05			.01		
	P(A|AHR)			P(A|AHR)			P(A|AHR)		
	.99	.95	.90	.99	.95	.90	.99	.95	.90
0	228	44.9	21.9	297	58.4	28.5	456	89.8	43.9
1	26.1	11.0	7.31	31.8	13.4	8.92	44.6	18.7	12.5
2	12.2	6.51	4.83	14.4	7.70	5.71	19.3	10.3	7.63
3	8.12	4.89	3.83	9.42	5.68	4.44	12.2	7.35	5.76
4	6.25	4.06	3.29	7.16	4.65	3.76	9.07	5.89	4.77
5	5.20	3.55	2.94	5.89	4.02	3.34	7.34	5.02	4.16
6	4.52	3.21	2.70	5.08	3.60	3.04	6.25	4.43	3.74
7	4.05	2.96	2.53	4.52	3.30	2.82	5.51	4.02	3.44
8	3.71	2.77	2.39	4.12	3.07	2.66	4.96	3.71	3.20
9	3.44	2.62	2.28	3.80	2.89	2.52	4.55	3.46	3.02
10	3.23	2.50	2.19	3.56	2.75	2.42	4.22	3.27	2.87
11	3.06	2.40	2.12	3.35	2.63	2.33	3.96	3.10	2.74
12	2.92	2.31	2.06	3.19	2.53	2.25	3.74	2.97	2.64
13	2.80	2.24	2.00	3.05	2.44	2.18	3.56	2.85	2.55
14	2.69	2.18	1.95	2.93	2.37	2.12	3.40	2.75	2.47
15	2.60	2.12	1.91	2.82	2.30	2.07	3.27	2.66	2.40

TABLE 5

Table of Hazard Rate Ratios for t_2/t_1

t_2/t_1	$Z(t_2)/Z(t_1)$										
	β										
	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	1	$1\frac{1}{3}$	$1\frac{2}{3}$	2	$2\frac{1}{2}$	$3\frac{1}{3}$	4	5
1.25	.862	.894	.928	1.00	1.08	1.16	1.25	1.40	1.68	1.95	2.44
1.50	.763	.816	.873	1.00	1.14	1.31	1.50	1.84	2.57	3.38	5.06
1.75	.689	.756	.823	1.00	1.21	1.45	1.75	2.32	3.69	5.36	9.38
2.00	.630	.707	.794	1.00	1.26	1.59	2.00	2.83	5.04	8.00	16.0
2.25	.583	.667	.763	1.00	1.31	1.72	2.25	2.38	6.64	11.4	25.6
2.50	.543	.632	.734	1.00	1.36	1.84	2.50	3.95	8.49	15.6	39.1
2.75	.510	.603	.714	1.00	1.40	1.96	2.75	4.56	10.6	20.8	57.2
3.00	.481	.577	.694	1.00	1.44	2.08	3.00	5.20	13.0	27.0	81.0
3.25	.456	.555	.675	1.00	1.48	2.19	3.25	5.86	15.6	34.3	112
3.50	.434	.534	.659	1.00	1.52	2.30	3.50	6.55	18.4	42.9	148
3.75	.414	.516	.644	1.00	1.55	2.42	3.75	7.26	21.8	52.7	198
4.00	.397	.500	.630	1.00	1.59	2.52	4.00	8.00	25.4	64.0	256
4.25	.381	.485	.617	1.00	1.62	2.62	4.25	8.76	29.3	76.8	326
4.50	.367	.472	.606	1.00	1.65	2.73	4.50	9.54	33.4	91.1	410
4.75	.354	.459	.595	1.00	1.68	2.83	4.75	10.4	37.9	107	509
5.00	.342	.447	.585	1.00	1.71	2.92	5.00	11.2	42.8	125	625

TABLE 5a

Table of tZ(t) x 100 Products for β =1/3

Sample Size Code Letter	Acceptable Quality Level (AQL) p'(%)	0.015	0.035	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10.0
	tZ(t) x 100	005	.012	.022	.033	.050	.083	.134	.217	.335	.504	.844	1.36	2.24	3.51
A	tZ(t) x 100 Product at RHR [P(A) = 10]													↓	38
B													↓	24	↑
C												↓	15	↑	↓
D											↓	10	↑	↓	19
E										↓	7.6	↑	↓	13	19
F									↓	5.0	↑	↓	8.9	12	16
G								↓	3.0	↑	↓	5.3	7.3	9.6	13
H							↓	2.1	↑	↓	3.7	5.2	6.6	9.6	12
I						↓	1.5	↑	↓	2.5	3.6	4.4	5.6	7.4	10
J					↓	1.0	↑	↓	1.7	2.3	3.0	3.6	4.9	6.7	9.7
K				↓	.69	↑	↓	1.1	1.6	2.0	2.4	3.2	4.0	5.9	8.4
L			↓	.50	↑	↓	.85	1.1	1.5	1.8	2.0	2.9	3.9	5.6	7.8
M		↓	.36	↑	↓	.57	.79	.99	1.2	1.4	1.9	2.5	3.7	5.0	7.0
N		.25	↑	↓	.43	.59	.73	.88	1.0	1.3	1.7	2.3	3.1	4.9	6.6
O		↑	↓	.28	.39	.49	.59	.68	.87	1.1	1.5	2.0	2.8	4.1	5.9
P		↓	.17	.23	.29	.35	.46	.57	.73	.94	1.2	1.8	2.5	3.7	5.3
Q		.086	.11	.15	.20	.26	.31	.42	.55	.72	.97	1.5	2.1	3.2	4.9

TABLE 5b

Table of tZ(t) x 100 Products for β = 1/2

Sample Size Code Letter	Acceptable Quality Level (AQL)													
	p'(%)	0.015	0.035	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10.0
	tZ(t) x 100	.007	.017	.032	.050	.075	.125	.201	.326	.503	.756	1.27	2.04	3.36	5.27
	tZ(t) x 100 Product at RHR [P(A) = .10]														
A														↓	56
B													↓	34	↑
C												↓	22	↑	↓
D											↓	16	↑	↓	29
E										↓	11	↑	↓	20	29
F									↓	7.6	↑	↓	13	18	25
G								↓	4.5	↑	↓	7.9	10	14	20
H							↓	3.2	↑	↓	5.5	7.8	10	14	18
I						↓	2.2	↑	↓	3.8	5.4	6.9	8.2	11	15
J					↓	1.5	↑	↓	2.6	3.5	4.4	5.4	7.2	10	14
K				↓	1.0	↑	↓	1.7	2.4	3.0	3.6	4.8	6.0	8.8	12
L			↓	.76	↑	↓	1.2	1.7	2.2	2.7	3.1	4.2	5.8	8.3	11
M		↓	.51	↑	↓	.86	1.1	1.4	1.8	2.1	2.9	3.7	5.5	7.4	10
N		.38	↑	↓	.65	.88	1.1	1.3	1.5	1.9	2.6	3.4	4.7	7.2	9.8
O		↑	↓	.43	.59	.74	.88	1.0	1.3	1.7	2.2	3.1	4.1	6.0	8.8
P		↓	.26	.35	.49	.53	.70	.86	1.1	1.4	1.8	2.7	3.7	5.5	8.0
Q		.13	.17	.22	.31	.39	.47	.63	.82	1.0	1.4	2.2	3.2	4.8	7.2

TABLE [illegible]

Table of tZ(t) x 100 Products for β = 2/3

Sample Size Code letter		Acceptable Quality Level (AQL)													
	p'(%)	0.015	0.035	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10.0
	tZ(t) x 100	.010	.023	.043	.067	.100	.167	.267	.435	.670	1.01	1.69	2.72	4.48	7.02
A	tZ(t) x 100 Product at RHR [P(A) = .10]	-	-	-	-	-	-	-	-	-	-	-	-	↓	76
B		-	-	-	-	-	-	-	-	-	-	-	-	50	↑
C		-	-	-	-	-	-	-	-	-	-	-	↓ 30	↑	·
D		-	-	-	-	-	-	-	-	-	↓	↓ 21	↑	↓	↓ 39
E		-	-	-	-	-	-	-	-	↓	15	↑	·	27	39
F		-	-	-	-	-	-	-	-	10	↑	·	↓ 17	25	33
G		-	-	-	-	-	-	↓	↓ 6.0	↑	↓	↓ 10	14	18	27
H		-	-	-	-	-	↓	4.3	↑	↓	7.4	10	13	18	24
I		-	-	-	-	-	3.0	↑	·	5.1	7.2	9.2	11	15	20
J		-	-	-	↓	↓ 2.0	↑	↓	↓ 3.4	4.7	6.0	7.2	9.6	13	19
K		-	-	↓	1.3	↑	↓	2.3	3.2	4.0	4.9	6.5	8.0	11	16
L		-	-	1.0	↑	·	1.7	2.3	3.0	3.5	4.1	5.8	7.9	11	15
M		↓	↓ .68	↑	↓	↓ 1.1	1.5	1.9	2.3	2.7	3.9	5.0	7.4	10	14
N		.51	↑	↓	.86	1.1	1.5	1.7	2.0	2.6	3.4	4.6	6.2	9.6	13
O		↑	·	.57	.78	.98	1.1	1.3	1.7	2.2	3.0	4.1	5.6	8.2	11
P		↓	↓ .34	.47	.59	.71	.93	1.1	1.5	1.9	2.4	3.5	5.0	7.4	10
Q		.17	.23	.30	.41	.52	.63	.84	1.1	1.4	1.9	3.0	4.2	6.5	9.6

TABLE 6d

Table of tZ(t) x 100 Products for β = 1

Sample Size Code Letter		Acceptable Quality Level (AQL)													
	p'(%)	0.015	0.035	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10.0
	tZ(t) x 100	.015	.035	.065	.100	.150	.250	401	.652	1.01	1.51	2.53	4.08	6.72	10.5
A	tZ(t) x 100 Product at RHR [P(A) = .10]													↓	115
B													↓	77	↑
C												↓	46	↑	↓
D											↓	33	↑	↓	61
E										↓	23	↑	↓	41	60
F									↓	15	↑	↓	27	38	50
G								↓	9.2	↑	↓	16	22	28	42
H							↓	6.5	↑	↓	11	16	20	28	38
I						↓	4.6	↑	↓	7.8	11	14	17	23	31
J					↓	3.1	↑	↓	5.3	7.2	9.1	11	15	20	29
K				↓	2.1	↑	↓	3.6	4.9	6.2	7.4	9.9	12	18	26
L			↓	1.5	↑	↓	2.6	3.6	4.5	5.4	6.3	9.0	12	17	24
M		↓	1.0	↑	↓	1.8	2.4	3.0	3.6	4.2	6.0	7.6	11	15	21
N		.78	↑	↓	1.3	1.8	2.3	2.7	3.1	4.0	5.3	6.9	9.5	14	20
O		↑	↓	.86	1.2	1.5	1.8	2.1	2.7	3.5	4.5	6.2	8.7	13	18
P		↓	52	.72	.91	1.1	1.4	1.8	2.3	2.9	3.7	5.4	7.6	11	16
Q		.26	.35	.45	.63	.79	.97	1.3	1.7	2.2	3.0	4.5	6.5	9.9	15

TABLE 6e

Table of tZ(t) x 100 Products for β = 1-1/3

Sample Size Code Letter		Acceptable Quality Level (AQL)													
	p'(%)	0.015	0.035	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10.0
	tZ(t) x 100	.020	.047	.087	.133	.200	.333	.535	.869	1.34	2.02	3.38	5.44	8.96	14.0
A	tZ(t) x 100 Product of RHR [P(A) = .10]													↓	150
B													↓	100	
C												↓	61	↑	↓
D											↓	44	↑	↓	80
E										↓	30	↑	↓	55	80
F									↓	20	↑	↓	35	50	67
G								↓	12	↑	↓	21	29	38	56
H							↓	8.6	↑	↓	14	20	26	38	50
I						↓	6.0	↑	↓	10	14	18	22	30	41
J					↓	4.0	↑	↓	7.0	9.4	12	14	19	27	38
K				↓	2.7	↑	↓	4.7	6.5	8.2	9.7	13	16	23	33
L				2.0	↑	↓	3.4	4.7	6.0	7.2	8.2	11	15	22	31
M		↓	1.3	↑	↓	2.3	3.2	4.0	4.8	5.5	7.8	10	14	20	28
N		1.0	↑	↓	1.7	2.3	2.9	3.5	4.1	5.2	7.0	9.2	12	19	26
O		↑	↓	1.1	1.5	1.9	2.3	2.7	3.5	4.6	6.0	8.2	11	16	23
P		↓	.68	.94	1.1	1.4	1.8	2.3	2.9	3.8	4.9	7.6	10	14	21
Q		.34	.45	.60	.82	1.0	1.2	1.6	2.2	2.9	3.9	6.0	8.5	13	19

TABLE 6f

Table of tZ(t) x 100 Products for β = 1-2/3

Sample Size Code Letter		Acceptable Quality Level (AQL)													
	p'(%)	0.015	0.035	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10.0
	tZ(t) x 100	.025	.058	.108	.167	.250	.417	.668	1.09	1.68	2.52	4.22	6.80	11.2	17.6
A	tZ(t) x 100 Product at RHR [P(A) = 10]	-	-	-	-	-	-	-	-	-	-	-	-	↓	190
B		-	-	-	-	-	-	-	-	-	-	-	↓	120	↑
C		-	-	-	-	-	-	-	-	-	-	↓	76	↑	-
D		-	-	-	-	-	-	-	-	-	↓	54	↑	↓	↓ 99
E		-	-	-	-	-	-	-	-	↓	38	↑	↓	68	99
F		-	-	-	-	-	-	-	-	25	↑	↓	44	63	84
G		-	-	-	-	-	-	↓	↓ 15	↑	↓	26	36	47	69
H		-	-	-	-	-	↓	10	↑	↓	18	26	33	47	62
I		-	-	-	-	↓	7.6	↑	↓	13	18	23	27	37	52
J		-	-	-	↓	5.0	↑	-	8.6	11	15	18	24	33	48
K		-	-	↓	3.4	↑	↓	↓ 5.9	8.0	10	12	16	20	29	42
L		-	↓	2.5	↑	↓	4.2	6.0	7.5	9.0	10	14	19	28	39
M		↓	1.7	↑	↓	2.8	4.0	5.0	6.0	7.0	9.8	12	18	25	35
N		1 2	↑	↓	2.1	2.9	3.7	4.4	5.2	6.6	8.7	11	15	24	32
O		↑	↓	1.4	1.9	2.4	2.9	3.4	4.4	5.8	7.6	10	14	20	29
P		↓	.86	1.1	1.4	1.7	2.3	2.8	3.7	4.8	6.1	9.0	12	18	26
Q		.43	.58	.75	1.0	1.3	1.5	2.1	2.7	3.6	4.9	7.6	10	16	24

TABLE 6g

Table of tZ(t) x 100 Products for $\beta = 2$

Sample Size Code Letter		Acceptable Quality Level (AQL)													
	p'(%)	0.015	0.035	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10.0
	tZ(t) x 100	.030	.070	.130	.200	.300	.500	.802	1.30	2.01	3.02	5.06	8.16	13.4	21.1
A	tZ(t) ≈ 100 Product at RHR [P(A) = .10]	—	—	—	—	—	—	—	—	—	—	—	—	↓	230
B		—	—	—	—	—	—	—	—	—	—	—	↓	150	↑
C		—	—	—	—	—	—	—	—	—	—	↓	92	↑	↓
D		—	—	—	—	—	—	—	—	—	↓	66	↑	↓	120
E		—	—	—	—	—	—	—	—	↓	46	↑	↓	82	120
F		—	—	—	—	—	—	—	↓	30	↑	↓	54	77	100
G		—	—	—	—	—	—	↓	18	↑	↓	32	44	56	84
H		—	—	—	—	—	↓	12	↑	↓	22	31	40	56	75
I		—	—	—	—	↓	9.1	↑	↓	15	22	28	33	44	63
J		—	—	—	↓	6.0	↑	↓	10	14	18	22	29	40	58
K		—	—	↓	4.1	↑	↓	7.1	9.8	12	14	20	24	35	51
L		—	↓	3.0	↑	↓	5.1	7.2	9.0	10	12	17	23	34	48
M		↓	2.0	↑	↓	3.4	4.8	6.1	7.3	8.4	11	15	22	30	42
N		1.5	↑	↓	2.6	3.5	4.4	5.4	6.3	8.0	10	13	19	29	39
O		↑	↓	1.7	2.3	2.9	3.5	4.1	5.3	7.0	9.1	12	17	25	35
P		↓	1.0	1.4	11.7	2.1	2.8	3.4	4.4	5.8	7.4	10	15	22	32
Q		.50	.70	.90	1.2	1.5	1.9	2.5	3.3	4.4	6.0	9.1	12	20	29

TABLE 6h

Table of tZ(t) x 100 Products for $\beta = 2\text{-}1/2$

Sample Size Code Letter	Acceptable Quality Level (AQL)														
	p'(%)	0.015	0.035	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10.0
	tZ(t) x 100	.038	.087	.163	.250	.375	.625	1.00	1.63	2.51	3.78	6.33	7.62	16.8	26.3
A	tZ(t) x 100 Product at RHR [P(A) = .10]													↓	290
B													↓	190	↑
C												↓	110	↑	↓
D											↓	82	↑	↓	140
E										↓	57	↑	↓	100	140
F									↓	36	↑	↓	67	95	120
G								↓	23	↑	↓	40	54	71	100
H							↓	16	↑	↓	28	39	50	71	93
I						↓	11	↑	↓	19	27	35	42	55	78
J					↓	7.7	↑	↓	13	17	22	27	36	50	73
K				↓	5.1	↑	↓	8.8	12	15	18	25	32	44	64
L			↓	3.8	↑	↓	6.3	8.9	11	13	15	22	29	42	59
M		↓	2.5	↑	↓	4.3	5.9	7.5	9.0	10	14	19	28	37	52
N		1.9	↑	↓	3.2	4.4	5.5	6.6	7.8	9.9	13	17	23	36	49
O		↑	↓	2.1	2.9	3.7	4.4	5.1	6.5	8.6	11	15	21	31	45
P		↓	1.3	1.7	2.2	2.6	3.5	4.3	5.5	7.1	9.0	13	18	28	40
Q		.65	.87	1.1	1.5	1.9	2.3	3.1	4.2	5.5	7.4	11	16	25	36

TABLE 6i

Table of tZ(t) x 100 Products for β = 3-1/3

Sample Size Code Letter	Acceptable Quality Level (AQL)													
	p'(%)	0.015	0.035	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10.0
	tZ(t) x 100	.050	.117	.217	.333	.500	.833	1.34	2.17	3.35	5.04	8.44	13.6	22.4	35.1
A	tZ(t) x 100 Product at RHR [P(A) = .10]													↓	380
B													↓	250	↑
C												↓	150	↑	↓
D											↓	100	↑	↓	190
E										↓	76	↑	↓	130	190
F									↓	51	↑	↓	88	120	160
G								↓	30	↑	↓	53	73	94	130
H							↓	21	↑	↓	37	52	66	94	120
I						↓	15	↑	↓	25	36	46	56	74	100
J					↓	10	↑	↓	17	23	30	36	48	67	97
K				↓	6.9	↑	↓	11	16	20	24	33	40	59	84
L			↓	5.0	↑	↓	8.4	12	15	17	20	29	37	56	78
M		↓	3.4	↑	↓	5.7	8.0	10	12	14	19	25	37	50	70
N		2.5	↑	↓	4.3	5.8	7.4	8.8	10	13	17	22	31	48	65
O		↑	↓	2.8	3.9	4.9	5.9	6.8	8.8	11	15	20	28	41	59
P		↓	1.7	2.3	2.9	3.5	4.6	5.7	7.4	9.6	12	17	25	37	53
Q		.86	1.1	1.5	2.0	2.6	3.1	4.2	5.6	7.3	9.9	15	21	33	48

TABLE 6j

Table of tZ(t) x 100 Products for β = 4

Sample Size Code Letter		Acceptable Quality Level (AQL)													
	p'(%)	0.015	0.035	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10.0
	tZ(t) x 100	.060	.140	.260	.400	.600	1.00	1.60	2.61	4.02	6.04	10.1	16.3	26.9	42.1
A	tZ(t) x 100 Product at RHR [P(A) = .10]	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	470
B		↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	310	↑
C		↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	180	↑	↓
D		↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	130	↑	↓	230
E		↓	↓	↓	↓	↓	↓	↓	↓	↓	92	↑	↓	160	230
F		↓	↓	↓	↓	↓	↓	↓	↓	62	↑	↓	100	150	200
G		↓	↓	↓	↓	↓	↓	↓	36	↑	↓	65	89	110	160
H		↓	↓	↓	↓	↓	↓	26	↑	↓	45	64	80	110	150
I		↓	↓	↓	↓	↓	18	↑	↓	31	44	56	68	89	120
J		↓	↓	↓	↓	12	↑	↓	21	28	36	44	58	81	110
K		↓	↓	↓	8.2	↑	↓	14	19	25	29	40	49	71	100
L		↓	↓	6.0	↑	↓	10	14	18	21	25	35	47	69	96
M		↓	4.0	↑	↓	6.9	9.5	12	14	16	23	30	45	60	85
N		3.0	↑	↓	5.2	7.0	8.8	10	12	15	21	27	38	58	79
O		↑	↓	3.4	4.7	5.9	7.0	8.2	10	13	18	25	34	50	71
P		↓	2.0	2.8	3.5	4.2	5.6	6.9	8.8	11	14	21	30	45	66
Q		1.0	1.4	1.8	2.4	3.1	3.8	5.0	6.8	8.8	11	18	25	40	58

TABLE 6k

Table of tZ(t) x 100 Products for $\beta = 5$

Sample Size Code Letter	Acceptable Quality Level (AQL) p'(%)	0.015	0.035	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10.0
	tZ(t) x 100	.075	.175	.325	.500	.750	1.25	2.01	3.26	5.03	7.56	12.7	20.4	33.6	52.7
A														↓	580
B													↓	380	↑
C												↓	220	↑	↓
D											↓	160	↑	↓	290
E										↓	110	↑	↓	200	290
F									↓	76	↑	↓	130	180	250
G								↓	45	↑	↓	80	110	140	200
H							↓	32	↑	↓	56	79	100	140	180
I						↓	23	↑	↓	38	54	70	84	110	150
J					↓	15	↑	↓	26	35	45	55	72	100	140
K				↓	10	↑	↓	17	24	31	36	49	60	88	120
L		↓	↓	7.6	↑	↓	12	17	22	27	31	44	58	84	110
M			5.1	↑	↓	8.6	11	15	18	21	29	37	55	75	100
N		3.8	↑	↓	6.5	8.8	11	13	15	19	26	34	47	72	98
O		↑	↓	4.3	5.9	7.4	8.8	10	13	17	23	31	43	62	88
P		↓	2.6	3.5	4.9	5.3	7.0	8.6	11	14	18	27	38	55	80
Q		1.3	1.7	2.2	3.1	3.9	4.7	6.3	8.4	11	14	23	32	49	72

TABLE 7

Single Sample Sizes and Acceptance Numbers for the 105B Plans

Sample size code letter	Sample size n	Acceptance Number - c													
		Acceptable Quality Level (AQL)													
		0.015	0.035	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10.0
A	2														0
B	3													0	
C	5												0		
D	7											0			1
E	10										0			1	2
F	15									0			1	2	3
G	25								0			1	2	3	5
H	35							0			1	2	3	5	7
I	50						0			1	2	3	4	6	9
J	75					0			1	2	3	4	6	9	13
K	110				0			1	2	3	4	6	8	12	18
L	150			0			1	2	3	4	5	8	11	17	24
M	225		0			1	2	3	4	5	8	11	17	24	34
N	300	0			1	2	3	4	5	7	10	14	20	32	44
O	450			1	2	3	4	5	7	10	14	20	29	43	62
P	750		1	2	3	4	6	8	11	15	20	31	45	68	98
Q	1500	1	2	3	5	7	9	13	18	25	35	56	81	124	184

Appendix A

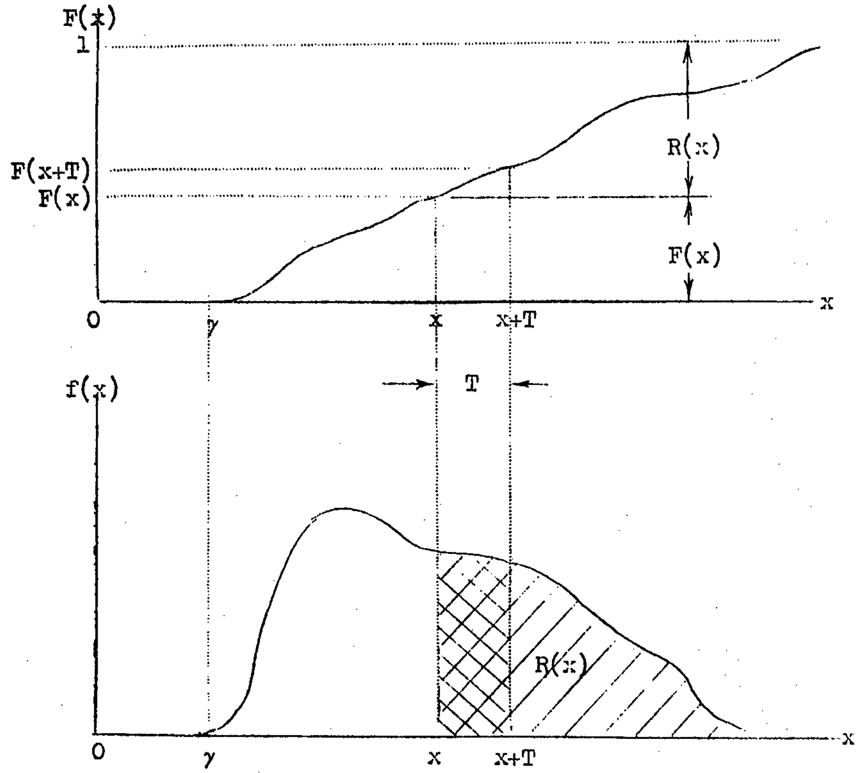
Instantaneous Failure Rate as a Life-quality Criterion.

This appendix describes the instantaneous failure or hazard rate which is used as the life-quality criterion for items subject to the testing procedures given in this report.

For an arbitrary lifelength distribution defined over $\gamma \leq x < \infty$ (γ is the threshold or location parameter) with c.d.f. = F(x), and p.d.f. = f(x), the failure rate, G(x,T), at time x in proportion or fraction (multiply by 100 if "percent" instead of "fraction" is desired) per T time units is (see sketch),

$$G(x,T) = \frac{\int_x^{x+T} f(y)dy}{T\,[\,1-F(x)\,]} = \frac{F(x+T) - F(x)}{T} \cdot \frac{1}{R(x)} \;. \quad \text{(A1)}$$

In Equation (A1): (i) The integral represents the proportion of the original items expected to fail from time x to time (x + T) and (ii) 1 - F(x) = R(x) represents the proportion of the original items expected to survive to time x.



Hence (i) /[(ii) · T]given by Equation (A1) represents the expected rate of failure at time x for the next T time units. When $x = \gamma$, the threshold or location parameter, $G(\gamma,T)$ is called the initial failure rate. Since $F(\gamma) = 0$, $R(\gamma) = 1$,

$$G(\gamma,T) = F(\gamma + T) / T. \tag{A2}$$

For $\gamma = 0$, the initial failure rate is,

$$G(0,T) = F(T) / T, \tag{A3}$$

which is a very special kind of failure rate. But Equation (A3) is used widely to define the failure rate. (Cf. e.g, Altman[9], Davis[10], H-108[11], Section 1 A3-2, p. 12, and Meyers[12]. However, this definition of failure rate is correct if the underlying lifelength distribution is exponential, and incorrect for any other distribution.

To show the above statement is so, Equation (A1) may be re-written as,

$$G(x,T) = \frac{R(x) - R(x + T)}{T \cdot R(x)} = \frac{1}{T} \left[1 - \frac{R(x + T)}{R(x)} \right]. \qquad (A4)$$

For the exponential case, $R(x) = \exp[-x/\theta]$, for $x, \theta > 0$; then,

$$\text{Exponential, } G(x,T) = [1 - e^{-T/\theta}] / T = F(T)/T, \qquad (A5)$$

which depends only on T and is independant of x. Furthermore, for small values of T/θ, by use of the exponential series expansion and neglecting terms of second and higher orders for T/θ, $F(T) = 1 - e[^{-T/\theta}] \cong T/\theta$. Then Exponential, $G(x,T) \cong (T/\theta) / T = 1/\theta = \lambda$, which is often misquoted as the exponential failure rate. (See comments on the instantaneous failure rate below).

The instantaneous failure or hazard rate (or simply hazard), $Z(x)$, also known as the intensity function, is the limiting value of $G(x,T)$ as $T \to 0$. Hence,

$$Z(x) = \lim_{T \to 0} G(x,T), \qquad (A6)$$

which may be worked out for an arbitrary lifelength distribution by taking such a limit. By recalling the definition of the first derivative of a function,

$$\lim_{T \to 0} \left[\frac{F(x + T) - F(x)}{T} \right] = F'(x) = f(x). \qquad (A7)$$

Hence,

$$Z(x) = f(x)/R(x), \text{ for } x \geq \gamma ;$$
$$= 0, \text{ otherwise.} \qquad (A8)$$

The physical meaning of the instantaneous failure or hazard rate lies in the fact that $Z(x)$ represents the relative failure-propensity or failure density of an item at age x. In this report, it is adopted as a measure of life-quality (Cf. Eq. (1)). By its definition, Equation (A8), since $R(x) \leq 1$, $Z(x) \geq f(x) \geq 0$ for $x \geq \gamma$. As a matter of fact, it can be larger than unity. Thus the term hazard "rate" (which implies a proportion or fraction) given to $Z(x)$ refers only to a "per unit time" figure, and is not necessarily a fractional number.

From Equation (A8), one may derive the original arbitrary life-length distribution by the steps that follow [Equations (A9) through (A15)] . Rewrite Equation (A8) into:

$$Z(x) = F'(x) / R(x) = - R'(x) / R(x) = - \frac{d}{dx} [\ln R(x)], \qquad (A9)$$

which is,

$$d [\ln R(x)] = - Z(x) dx. \qquad (A10)$$

Integrating both sides over the interval $(-\infty , x)$ gives,

$$\int_{-\infty}^{\gamma} d [\ln R(y)] + \int_{\gamma}^{x} d [\ln R(y)] = - \int_{-\infty}^{\gamma} Z(y) dy - \int_{\gamma}^{x} Z(y) dy. \qquad (A11)$$

Since for $-\infty < x \leq \gamma$, $R(x) = 1$, or $\ln R(x) = 0$ and $Z(x) = 0$, Equation (A11) becomes,

$$\ln R(x) = -\int_{\gamma}^{x} Z(y)\, dy, \text{ for } x \geq \gamma, \qquad \text{(A12)}$$

or,

$$R(x) = \exp\left[-\int_{\gamma}^{x} Z(y)\, dy \right] = 1-F(x) \text{ for } x \geq \gamma . \qquad \text{(A13)}$$

Hence, the lifelength c.d.f. is,

$$F(x) = 1 - \exp\left[-\int_{\gamma}^{x} Z(y)\, dy \right] \text{ for } x \geq \gamma;$$

$$= 0, \text{ otherwise}, \qquad \text{(A14)}$$

and its first derivative with respect to x,

$$f(x) = Z(x) \exp\left[-\int_{\gamma}^{x} Z(y)\, dy \right] \text{ for } x \geq \gamma;$$

$$= 0, \text{ otherwise}. \qquad \text{(A15)}$$

The foregoing is true for all distributions including the exponential case. We now turn to our assumed mathematical model, the Weibull distribution of the form:

$$F(x) = 1 - \exp\left[-\left(\frac{x-\gamma}{\eta}\right)^{\beta} \right], \; x \geq \gamma; \; \eta, \beta > 0;$$

$$= 0, \text{ otherwise}, \qquad \text{(A16)}$$

and its p.d.f.,

$$f(x) = \frac{\beta}{\eta} \left(\frac{x-\gamma}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{x-\gamma}{\eta}\right)^{\beta} \right], \; x \geq \gamma, \; \eta,\beta > 0;$$

$$= 0, \text{ otherwise}. \qquad \text{(A17)}$$

In this report, since γ is assumed to be known, Equation (A16) and (A17) are not used. Instead, Equations (A18) and (A19) are referred to throughout this report. For γ known, there is no loss of generality by assuming $\gamma = 0$. In this case, the Weibull c.d.f. and p.d.f. are, for $x > 0$, η, $\beta > 0$,

$$F(x) = 1 - \exp[-(x/\eta)^{\beta}], \text{ and} \tag{A18}$$

$$f(x) = \frac{\beta}{\eta}(x/\eta)^{\beta} \exp[-(x/\eta)^{\beta}]. \tag{A19}$$

These were referred to earlier as Equations(4) and (3) respectively. For this Weibull model, the expression for the failure rate, by Equation (A4), is,

$$\text{Weibull, } G(x,T) = \frac{1}{T}\left\{1 - \frac{\exp[-(\frac{x+T}{\eta})^{\beta}]}{\exp[-(x/\eta)^{\beta}]}\right\}. \tag{A20}$$

The expression for the instantaneous failure or hazard rate, by Equation (A8), is,

$$\text{Weibull, } Z(x) = \frac{\beta}{\eta}(x/\eta)^{\beta-1}, \text{ for } x \geq 0, \tag{A21}$$

which was given before by Equation (5).

When the shape parameter $\beta = 1$, the Weibull distribution, Equation (A18), reduces to the exponential distribution, (see Equation(A5)) with $\eta = \theta$. Also, Equation (A20) becomes Equation (A5) and Equation (A21) will be equal to $1/\theta$ as remarked earlier. Furthermore, under the Weibull case, both Equation (A20) for $G(x,T)$ for any fixed T and Equation (A21) for $Z(x)$ can now be seen as increasing (decreasing) functions in x when the shape parameter β is greater (less) than unity.

Appendix B

Average Hazard Rate as a Life-quality Criterion

This appendix describes the average hazard rate which is useful as the life-quality criterion for any arbitrary lifelength distribution whether it be Weibull or otherwise.

The average hazard rate, $m(x)$ is defined as $M(x)/x$ where $M(x)$ is the cumulative hazard rate defined as, (Cf. Broadbent[13]),

$$M(x) = \int_{-\infty}^{x} Z(y)\, dy. \tag{B1}$$

Suppose, as before, the arbitrary lifelength distribution is defined over $\gamma \leq x < \infty$, then the average hazard rate would be,

$$\begin{aligned} m(x) &= \frac{1}{x} \int_{\gamma}^{x} Z(y)\, dx \quad \text{for } x > \gamma, \\ &= 0, \text{ otherwise.} \end{aligned} \tag{B2}$$

Garvin[14] made good use of $m(x)$ for the Weibull case with γ, the threshold or location parameter, equal to zero and devised a novel graphical method for analysing among other things the failure data of gas turbine blades for jet engines.

In acceptance sampling, the notion of average hazard rate is particularly useful when the instantaneous failure or hazard rate, $Z(x)$, of the lifelength distribution changes monotonically in time-- such as does a Weibull distribution with shape parameter β other than unity. This was seen in Example (8) of this report.

Combining Equation (B1) with Equation (A14), we have,

$$F(x) = 1 - \exp[-M(x)] = 1 - \exp[-x \cdot m(x)]. \tag{B3}$$

From Equation (A5), the exponential $F(x) = 1\text{-}\exp[-x/\theta] = 1\text{-}\exp[-\lambda x]$, for $x, \theta, \lambda > 0$, one may interpret $m(x)$ by comparing the exponential c.d.f. with Equation (B3) and noting that $m(x)$ is that average hazard over the time period x such that,

$$m(x) = 1/\theta = \lambda. \tag{B4}$$

Or inversely, we may define the hazard-breakeven time, t_λ , as the solution of Equation (B4) for any specified value of λ, thus,

$$\tau_\lambda = m^{-1}(\lambda), \tag{B5}$$

where m^{-1} is the usual inverse function of m. This was also illustrated in Example (8).

Equation (B3) may also be re-written as Equation (B6), and Equation (B1) as Equation (B7), thus,

$$M(x) = -\ln[1\text{-}F(x)] = -\ln R(x) \tag{B6}$$

and,

$$Z(x) = \frac{d}{dx}[M(x)] = m(x) + x\frac{d}{dx}[m(x)] \tag{B7}$$

These relationships facilitate the estimation of the cumulative hazard rate, $M(x)$, and hence the average hazard rate, $m(x)$, as well as the hazard rate, $Z(x)$, from the failure data under any arbitrary lifelength distribution by writing Equation (B6) in the following form:

$$\begin{aligned} x \cdot m(x) = M(x) &= \exp[\ln M(x)] \\ &= e^{\ln[-\ln(1-F(x))]}. \end{aligned} \tag{B8}$$

The negative values for the exponent $\ln[-\ln(1-p')]$ appearing in the last member of Equation (B8) are tabulated in Table 2 of the National Bureau of Standards Tables[3] for p' = .0001 (.0001) .0050(.001) .9880 (.0001) .99940(.00001) .99999. Estimates for $p' = F(x)$ from failure data are discussed in Appendix B of Goode and Kao.[15] A useful approximation which can be derived from the logarithmic expansion of Equation (B6), is $M(x) \cong F(x)$ for small values of $F(x)$, say 10% or less, with two decimal places in accuracy.

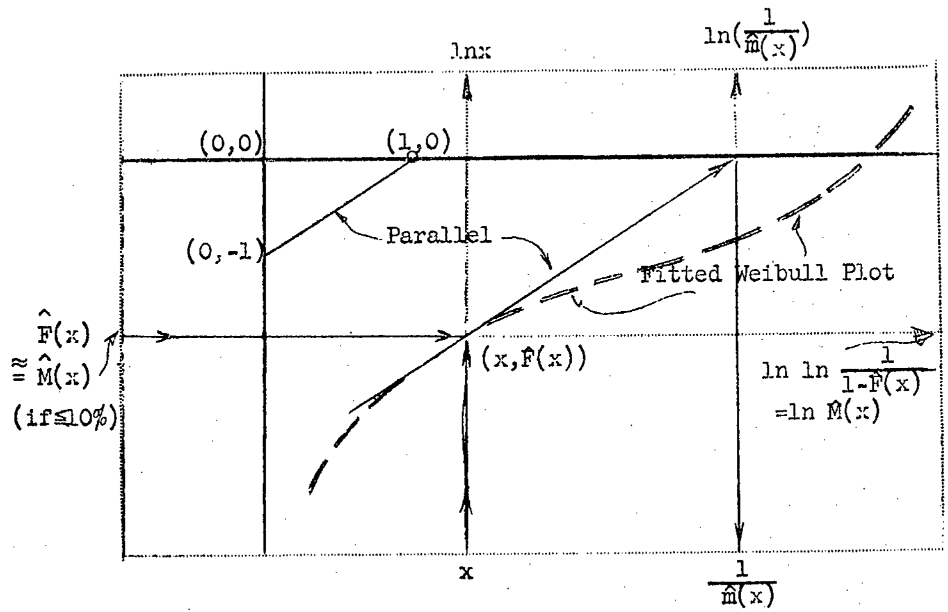
A graphical method for estimating the average hazard rate, $m(x)$ as a function of x for any arbitrary lifelength distribution is available. The method will involve the use of Weibull probability paper of the first kind mentioned in the reference (for γ known)[15]. After having plotted the failure data (grouped or ungrouped) into the so-called Weibull plot (not necessarily a straight line, since the distribution is arbitrary) the graphical estimates of $m(x)$ for any x may be obtained in the manner to be described.

The reason that the Weibull probability paper can be used non-parametrically is the following:

Equation (B3) giving the arbitrary lifelength c.d.f. in terms of $m(x)$, may be re-written as:

$$\ln \ln \left[\frac{1}{1-F(x)}\right] = -\ln\left[\frac{1}{m(x)}\right] + \ln x \qquad \text{(B9)}$$

which is in the form of $y = -a + z$, an equation giving a straight line with slope = 1 on the Weibull probability paper[2] with the Y-intercept being the estimate of $-\ln\left[\frac{1}{m(x)}\right]$. Since the slope of Equation (B9) is unity, its X-intercept will also estimate $\ln\left[\frac{1}{m(x)}\right]$. Furthermore, since the auxilliary bottom X-scale is calibrated in natural logarithms, the estimate [1/m(x)] may be read off the auxilliary scale directly. This procedure is illustrated by the following sketch.



The steps for getting the estimate of m(x) for a given value of x are (Refer to Kao[2] for scale designations of the Weibull probability paper):

1) Choose a point X along the auxilliary (bottom) X-axis and erect over it a vertical line cutting across the fitted Weibull plot at point $(x, \hat{F}(x))$. The coordinates of this point are in reference to the auxilliary scales.

2) Through this point pass a straight line parallel to a line adjoining pts. = (0,1) and (1,0), both in reference to the principal scales.

3) The X-intercept of the above straight line estimates $\ln\left(\frac{1}{m(x)}\right)$, with values to be read off the top principal scale.

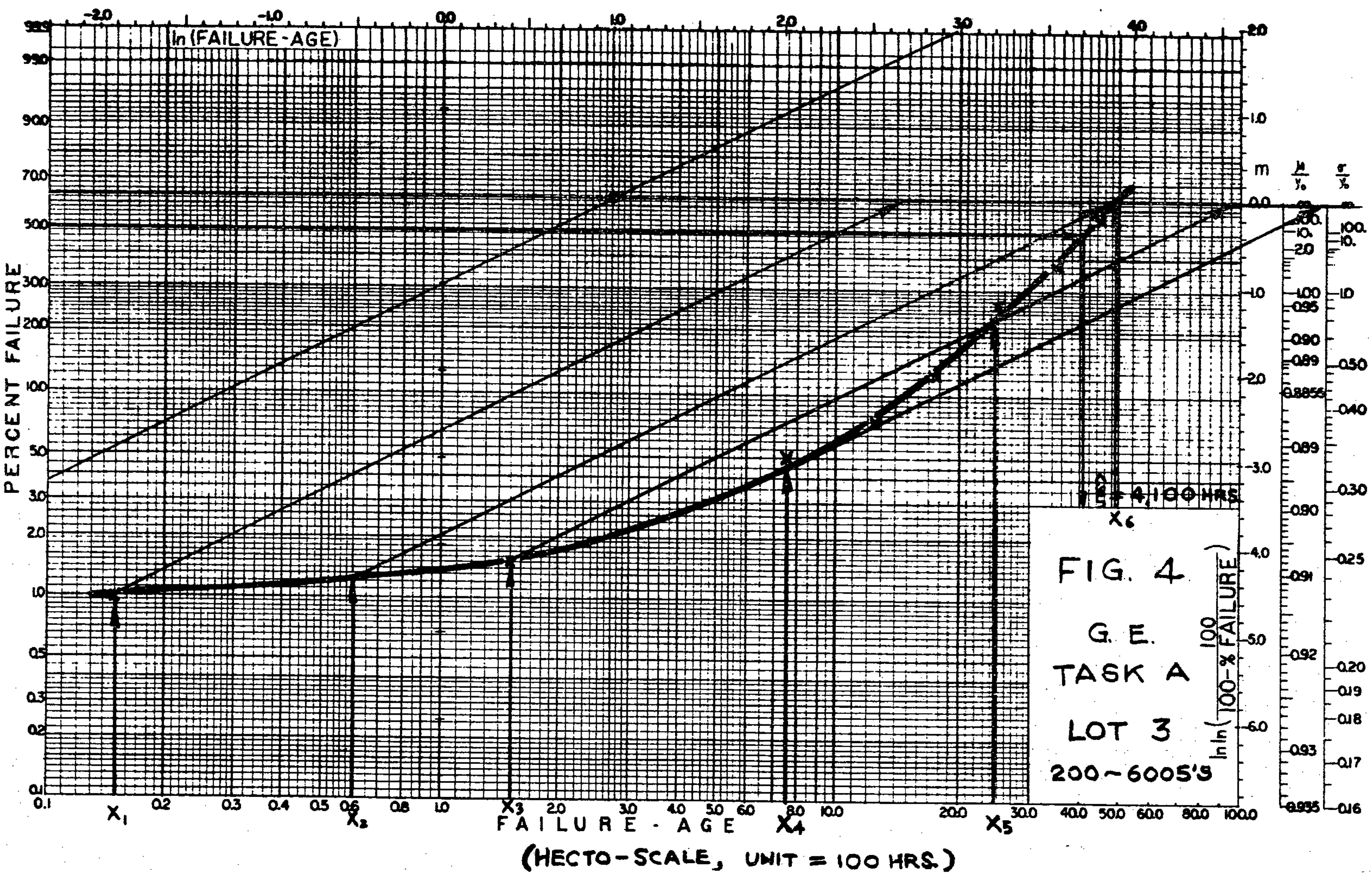
4) Dropping a vertical line from this X-intercept cutting across the bottom auxilliary scale gives the value $1/\hat{m}(x)$, which is the reciprocal of the estimate for $m(x)$.

5) Repeat Steps (1) through (4) for each value of x for which $\hat{m}(x)$, the estimate of $m(x)$ is desired.

It will be noted that if the Weibull plot happens to be a straight line with slope equal to unity, i.e., the exponential case, then the value $\hat{m}(x)$ produced by the above steps will be the same value independent of x, a result which is to be expected of the exponential distribution. Also the value $1/\hat{m}(x)$ read off the graph is the estimated mean life by the interpretation given by Kao[2]. This latter result is also unique for the exponential distribution. Figure 4 shows a Weibull plot given by Dalman and Kao[16] which also appeared in Walsh and Tsao[17] (p. 21), with the estimate of $1/m(x)$ for six values of x.

i	x_i (in hrs.)	$1/\hat{m}(x_i)$ (in hrs.)	$\hat{m}(x)$ (in failure/ 1000 hrs.)
1	15	1,450	.6897
2	60	4,900	.2041
3	150	9,600	.1042
4	750	16,000	.0625
5	2,450	9,600	.1042
6	4,900	4,900	.2041

Figure 5 shows a plot of $\hat{m}(x)$ vs. x for the four intermediate points. It is interesting to note the "bath-tub" curve which is so characteristic of the failure pattern of many electronic and mechanical components and systems.


ln (FAILURE-AGE)
PERCENT FAILURE
FAILURE - AGE
(HECTO-SCALE, UNIT = 100 HRS.)
X1
X2
X3
X4
X5
X6
4,100 HRS.
m
ln ln (100 / (100-% FAILURE))
FIG. 4
G. E.
TASK A
LOT 3
200~6005'S

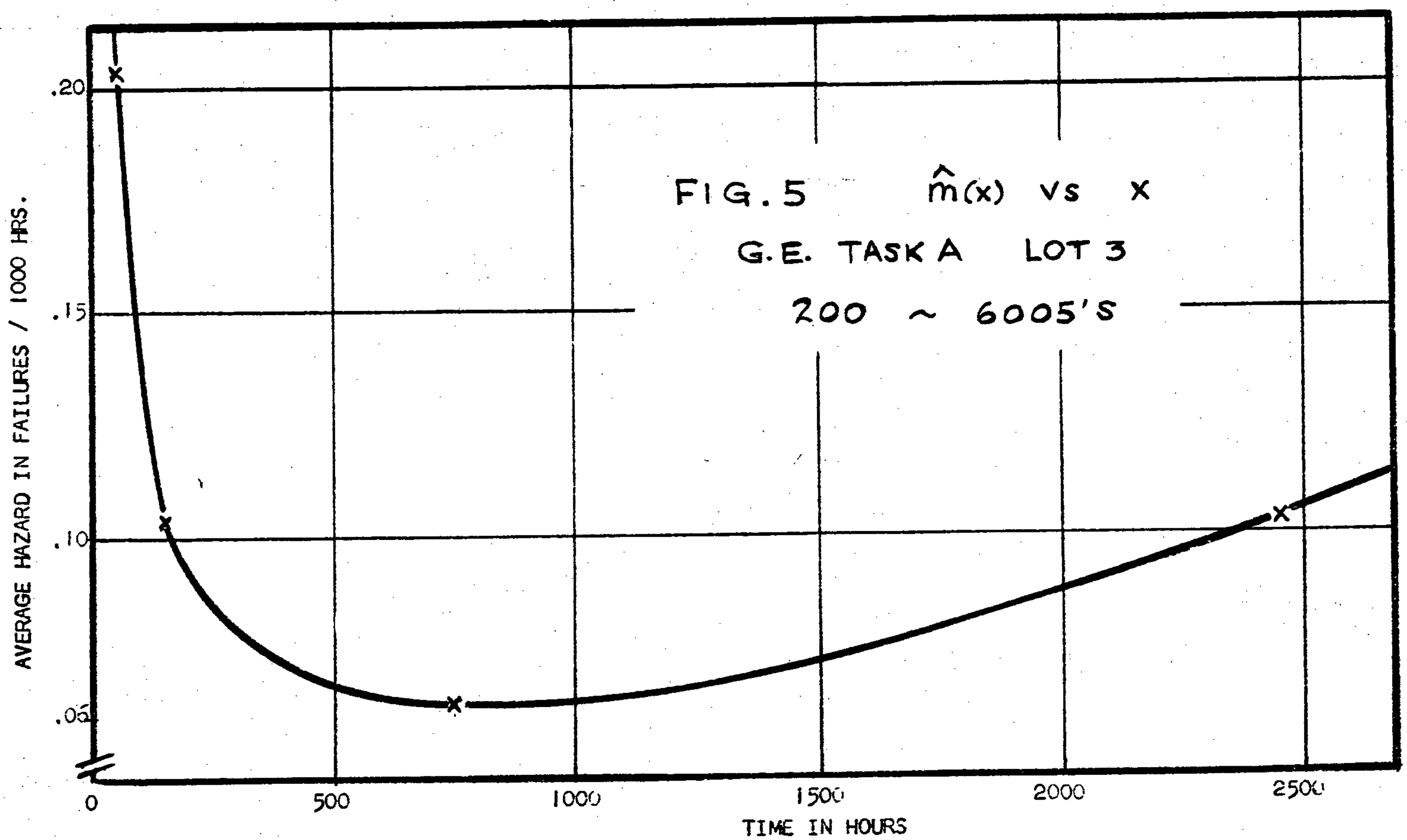


FIG. 5 $\hat{m}(x)$ vs x
G.E. TASK A LOT 3
200 ~ 6005'S
.20
.15
.10
.05
0
500
1000
1500
2000
2500
TIME IN HOURS
AVERAGE HAZARD IN FAILURES / 1000 HRS.

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