NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

PERFORMANCE ANALYSIS OF NONCOHERENT BINARY FREQUENCY SHIFT KEYING USING EQUAL GAIN COMBINING AND POST DETECTION SELECTION COMBINING OVER A NAKAGAMI FADING CHANNEL

by

Pierros Kontodios

September 1998

Thesis Advisor: Tri T. Ha
Co-Advisor: Ralph D. Hippenstiel

Approved for public release; distribution is unlimited.

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY ***(Leave blank)***

2. REPORT DATE
September 1998

3. REPORT TYPE AND DATES COVERED
Master's Thesis

4. TITLE AND SUBTITLE
PERFORMANCE ANALYSIS OF NONCOHERENT BINARY FREQUENCY SHIFT KEYING USING EQUAL GAIN COMBINING AND POST DETECTION SELECTION COMBINING OVER A NAKAGAMI FADING CHANNEL

5. FUNDING NUMBERS

6. AUTHOR(S) Kontodios, Pierros

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Naval Postgraduate School
Monterey, CA 93943-5000

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Research and Development Office

10. SPONSORING / MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES
The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

12a. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT ***(maximum 200 words)***

In this thesis, the performance of a noncoherent Binary Frequency Shift Keying (BFSK) receiver using Equal Gain Combining (EGC) and Post Detection Selection Combining (PDSC) techniques over a frequency nonselective and slowly Nakagami fading channel is investigated.

Analytical and numerical results obtained for EGC are compared to those obtained for first order PDSC (PDSC-1), second order PDSC (PDSC-2) and third order PDSC (PDSC-3).

14. SUBJECT TERMS
Nakagami Fading Channel, Diversity Combining Techniques, Equal Gain Combining (EGC), Post Detection Selection Combining(PDSC).

15. NUMBER OF PAGES
110

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFI- CATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. 239-18

Approved for public release; distribution is unlimited

PERFORMANCE ANALYSIS OF NONCOHERENT BINARY FREQUENCY SHIFT KEYING USING EQUAL GAIN COMBINING AND POST DETECTION SELECTION COMBINING OVER A NAKAGAMI FADING CHANNEL

Pierros Kontodios
Lieutenant Junior Grade, Hellenic Navy
B.S., Hellenic Naval Academy, 1990

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
September 1998

Author: ______________________
Pierros Kontodios

Approved by: ______________________
Tri T. Ha, Thesis Advisor

____
Ralph D. Hippenstiel, Co-Advisor

____
Jeffrey B. Knorr, Chairman
Department of Electrical and
Computer Engineering

ABSTRACT

In this thesis, the performance of a non-coherent Binary Frequency Shift Keying (BFSK) receiver using Equal Gain Combining (EGC) and Post Detection Selection Combining (PDSC) techniques over a frequency nonselective and slowly Nakagami fading channel is investigated.

Analytical and numerical results obtained for EGC are compared to those obtained for first order PDSC (PDSC-1), second order PDSC (PDSC-2) and third order PDSC (PDSC-3).

TABLE OF CONTENTS

I. INTRODUCTION..1

A. NAKAGAMI FADING CHANNEL...3

B. NON-COHERENT BFSK RECEIVER...4

II. EQUAL GAIN COMBINING (EGC)...9

A. BIT ERROR PROBABILITY 9

III. FIRST ORDER POST DETECTION SELECTION COMBINING (PDSC-1).........17

A. PROBABILITY DENSITY FUNCTIONS OF THE DECISION VARIABLES..17

B. BIT ERROR PROBABILITY...23

IV. SECOND ORDER POST DETECTION SELECTION COMBINING (PDSC-2)....27

A. PROBABILITY DENSITY FUNCTIONS OF THE DECISION VARIABLES..28

B. BIT ERROR PROBABILITY...34

V. THIRD ORDER POST DETECTION SELECTION COMBINING (PDSC-3).......39

A. PROBABILITY DENSITY FUNCTIONS OF THE DECISION VARIABLES..40

B. BIT ERROR PROBABILITY...47

VI. NUMERICAL RESULTS..53

VII. CONCLUSIONS...95

LIST OF REFERENCES..97

INITIAL DISTRIBUTION LIST...99

ACKNOWLEDGMENTS

This thesis is dedicated to my wife, Chrisanthi, and my parents, Pericles and Pari, to make them proud, because nothing of this would be possible without their guidance, support and encouragement.

I. INTRODUCTION

Diversity has long been used to improve the performance in communication systems operating in fading environments. Fading is the term used to describe the rapid fluctuations of the amplitude of a transmitted signal over a short period of time. It is caused by the non-coherent recombination of signal components that arrive at the receiver via different paths, at slightly different times. The resulting signal usually varies in amplitude and in phase.

Channel diversity is a powerful technique that is based on a simple concept. L independently fading replicas of the transmitted signal are received, over L different and hence statistically independent channels. This means that if one path experiences a deep fade, another path may have a strong signal, so that the detection is easier. The possibility that all paths have a deep fade at the same time is small, so the average and the instantaneous signal-to-noise ratio (SNR) at the receiver are improved, relative to a single fading channel.

In practice there are several ways in which we can provide the receiver with L replicas of the transmitted signal. One way is by employing frequency diversity, where the signal is simultaneously transmitted at L different frequencies. The carrier frequencies should be separated at least by an amount that equals or exceeds the coherence bandwidth $(\Delta f)_c$ of the channel, so that the independent fading for different paths is ensured. Another way is to transmit the signal in L different time slots (time diversity). Successive time slots should be separated by time that is equal to, or exceeds the coherence time $(\Delta t)_c$ of the channel. For a frequency-non-selective slow fading channel it is assumed that the coherence bandwidth is larger than the bandwidth of the transmitted signal and that the coherence time is larger than the bit interval. A third commonly used way is space (antenna) diversity. This technique employs one transmitting antenna along with several receiving antennas. The receiving antennas are sufficiently separated to ensure that the communication paths are different. As a general rule, a distance of 10 wavelengths separation between two antennas is considered sufficient.

There are other less widely used diversity techniques. For example there is the angle-of-arrival diversity and the polarization diversity. By comparison frequency and time diversity require more bandwidth. On the other hand, they require less hardware. This means less space, so they are used where space availability is a limiting factor in the design of a communication link.

Once the information from the diversity branches is received it can be combined, so that the bit error rate is decreased. In practice there are several methods of diversity combining. In this thesis we will investigate Equal Gain Combining (EGC) and Post Detection Selection Combining (PDSC) of order one and two and three. EGC is a commonly used technique for non-coherent detection. All L branches are equally weighted and combined non-coherently at the receiver. Consequently, EGC requires a complex receiver and is path dependent. In addition, combining more signals does not necessarily improve the performance. This is especially true in environments where the bit error rate is large due to fading or jamming. To overcome these disadvantages other techniques are used, such as selection combining (SC) and PDSC of different orders. In PDSC of order 1 (PDSC-1), a technique invented by professors Tri Ha and Ralph Hippenstiel, the signal with the largest amplitude of the signal branches, is compared to the signal with the largest amplitude among the non-signal branches. In PDSC of order 2 (PDSC-2), the combination of the signals with the two largest amplitudes of the signal branches is compared to the combination of the two signals with the largest amplitudes of the non-signal branches. In PDSC of order 3 (PDSC-3) the combination of the signals with the three largest amplitudes of the signal branches is compared to the combination of the three signals with the largest amplitudes of the non-signal branches. Selection combining is not discussed in this thesis. In selection combining the decision whether the signal is present or not, is based on the predetection signal with the largest amplitude. Both PDSC and SC are combining techniques which are independent of the number of diversity channels L, therefore require less complex receivers.

The communication channel is assumed to be a frequency non-selective, slowly fading Nakagami-m channel, while the additive noise is assumed to be white and Gaussian.

A. NAKAGAMI FADING CHANNEL

The received signal amplitude α of the non-coherent receiver that is employed in this thesis, is a Nakagami-m random variable. Its probability density function is given by [1]

$$f_\alpha(\alpha) = \begin{cases} \dfrac{2}{\Gamma(m)}\left(\dfrac{m}{\Omega}\right)^m \alpha^{2m-1} \exp\left(-\dfrac{m\alpha^2}{\Omega}\right), & \alpha \geq 0 \\ 0, & elsewhere \end{cases} \qquad , \qquad (1)$$

where $\Gamma(m)$ is the gamma function, defined by the integral

$$\Gamma(m) = \int_0^\infty x^{m-1} \exp(-x) dx, \quad m > 0 \qquad , \qquad (2)$$

and

$$m = \frac{\Omega^2}{\mathrm{var}(\alpha^2)} \geq \frac{1}{2} \tag{3}$$

and

$$\Omega = \mathrm{E}\{\alpha^2\} \tag{4}$$

are the two parameters of the distribution. The Nakagami-m distribution is a generalized formula that can model different fading environments using different values of its parameters. For $m = 1$ the channel has Rayleigh fading while as m approaches infinity the channel becomes non-fading. If $m = ½$, the pdf is the one-sided Gaussian distribution [1].

B. NON-COHERENT BFSK RECEIVER

The block diagram of the non-coherent BFSK receiver is shown in Fig. 1.

Since it is very difficult to coherently detect in a fading environment, we use an orthogonal modulation scheme, i.e., Binary Frequency Shift Keying (BFSK), so that non-coherent detection can be performed. In BFSK when the data bit $b_i = 1$ is transmitted, the waveform $v^{(1)}(t)$ is given by

$$v^{(1)}(t) = \begin{cases} A\sqrt{\frac{2}{T}}\cos(2\pi f_1 t + \theta_1), & 0 \leq t \leq T \\ 0, & \textit{elsewhere} \end{cases} , \tag{5}$$

where T is the bit duration, θ_1 is the signal phase, A is the amplitude of the signal and f_1 is the carrier frequency. When the data bit $b_i = 0$ is transmitted, the waveform $v^{(2)}(t)$ is given by

$$v^{(2)}(t) = \begin{cases} A\sqrt{\frac{2}{T}}\cos(2\pi f_2 t + \theta_2), & 0 \leq t \leq T \\ 0, & \textit{elsewhere} \end{cases} , \tag{6}$$

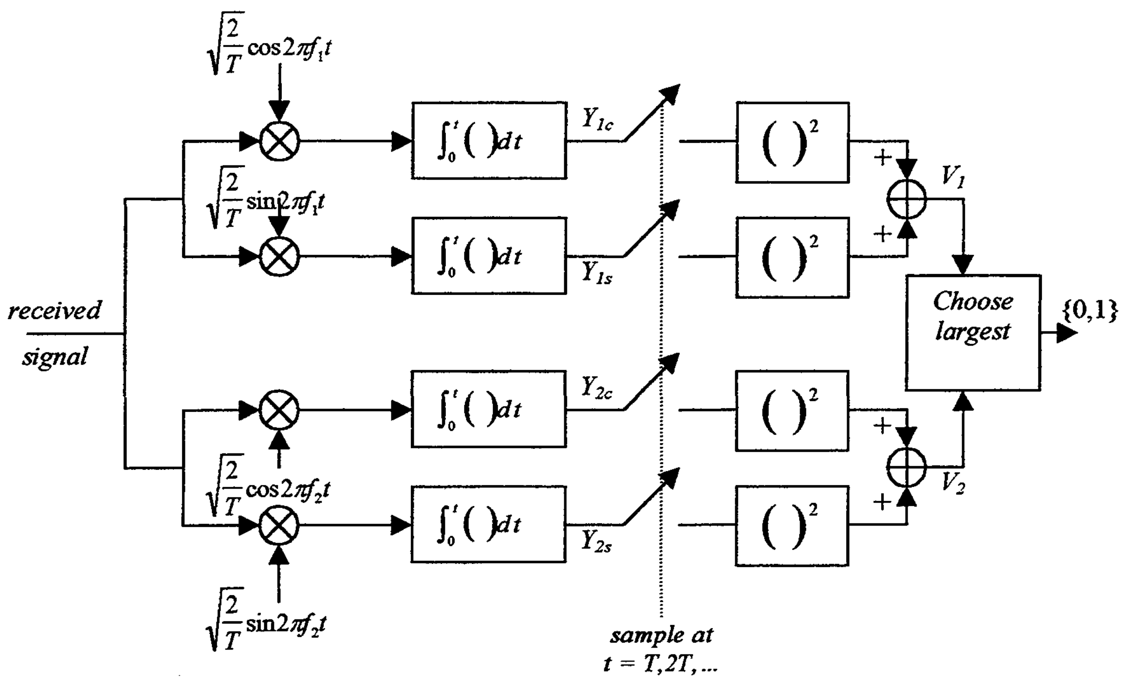

Figure 1. Non-coherent BFSK receiver

where T, θ_2, A are the duration, phase angle, and amplitude, respectively. The carrier frequency f_2 is selected such that the $v^{(1)}(t)$ and $v^{(2)}(t)$ are orthogonal over the interval [0,T]. This implies a minimum frequency spacing of $|f_2 - f_1| = 1/T$.

The received signal in the interval [0,T] has the form:

$$r^{(i)}(t) = \alpha V^{(i)}(t) + n(t), \quad i = 1,2 \qquad , \tag{7}$$

where $n(t)$ is white Gaussian noise with power spectral density $\sigma^2_n = N_o/2$, and α is a Nakagami-m random variable whose probability density function is given by (1). We assume, without loss of generality, that the data bit $b_i = 1$ is transmitted. The in phase outputs of the receiver are given by

$$Y_{1c} = \alpha A \cos\theta_1 + n_{1c} = \alpha\sqrt{E_c}\cos\theta_1 + n_{1c} \qquad , \tag{8}$$

and

$$Y_{2c} = n_{2c} \qquad , \tag{9}$$

where $E_c = A^2$ is the average energy per diversity bit and n_{1c}, n_{2c} are given by

$$n_{1c} = \int_0^T \sqrt{\frac{2}{T}}\, n(t)\cos(2\pi f_1 t)\, dt \tag{10}$$

and

$$n_{2c} = \int_0^T \sqrt{\frac{2}{T}}\, n(t)\cos(2\pi f_2 t)\, dt \qquad . \tag{11}$$

The quadrature outputs of the receiver are given by

$$Y_{1s} = A\cos\theta_1 + n_{1s} = \alpha\sqrt{E_c}\sin\theta_1 + n_{1s} \tag{12}$$

and

$$Y_{2s} = n_{2s} \qquad , \tag{13}$$

where

$$n_{1s} = \int_0^T \sqrt{\frac{2}{T}} n(t) \sin(2\pi f_1 t)\, dt \quad , \tag{14}$$

and

$$n_{2s} = \int_0^T \sqrt{\frac{2}{T}} n(t) \sin(2\pi f_2 t)\, dt \quad . \tag{15}$$

The random variables n_{1c}, n_{2c}, n_{1s}, n_{2s} are independent, identically distributed (iid), zero mean, Gaussian random variables with variances $N_o/2$. Finally the decision variables are defined as

$$V_1 = Y_{1c}^2 + Y_{1s}^2 = \left(\alpha\sqrt{E_c}\cos\theta_1 + n_{1c}\right)^2 + \left(\alpha\sqrt{E_c}\sin\theta_1 + n_{1s}\right)^2 \quad , \tag{16}$$

and

$$V_2 = Y_{2c}^2 + Y_{2s}^2 = n_{2c}^2 + n_{2s}^2 \quad . \tag{17}$$

II. EQUAL GAIN COMBINING (EGC)

Equal Gain Combining (EGC) is a technique that has been widely used in non-coherent receivers. All L branches are equally weighted and added incoherently, to produce a decision variable. A typical non-coherent BFSK receiver employing EGC is shown in Fig.2. If we assume that the data bit $b_i = 1$ is transmitted, then branch 1 of the BFSK non-coherent receiver will be the signal branch and branch 2 will be the non-signal branch. The decision variables V_1 and V_2 in Fig. 2 are given by

$$V_1 = V_{1,1} + V_{1,2} + \ldots + V_{1,L} \tag{18}$$

$$V_2 = V_{2,1} + V_{2,2} + \ldots + V_{2,L} \tag{19}$$

where $V_{1,i}$ and $V_{2,i}$, $i = (1,2,\ldots L)$, are the output random variables. Hence V_1 and V_2 are the sum of L independent random variables.

A. BIT ERROR PROBABILITY

For the non-coherent BFSK receiver with equal gain combining, the decision variables V_1 and V_2 are given by (18) and (19). In [1], the bit error probability for this receiver is derived, and is given by

$$P_e(\gamma_b) = \left(\frac{1}{2}\right)^{2L-1} \exp\left(-\frac{\gamma_b}{2}\right) \sum_{k=0}^{L-1} b_k \left(\frac{\gamma_b}{2}\right)^k \quad , \tag{20}$$

where L is the number of diversity branches, γ_b is the signal-to-noise ratio (SNR) per bit and b_k is given by the following

$$b_k = \frac{1}{k!} \sum_{n=0}^{L-1-k} \binom{2L-1}{n} \quad . \tag{21}$$

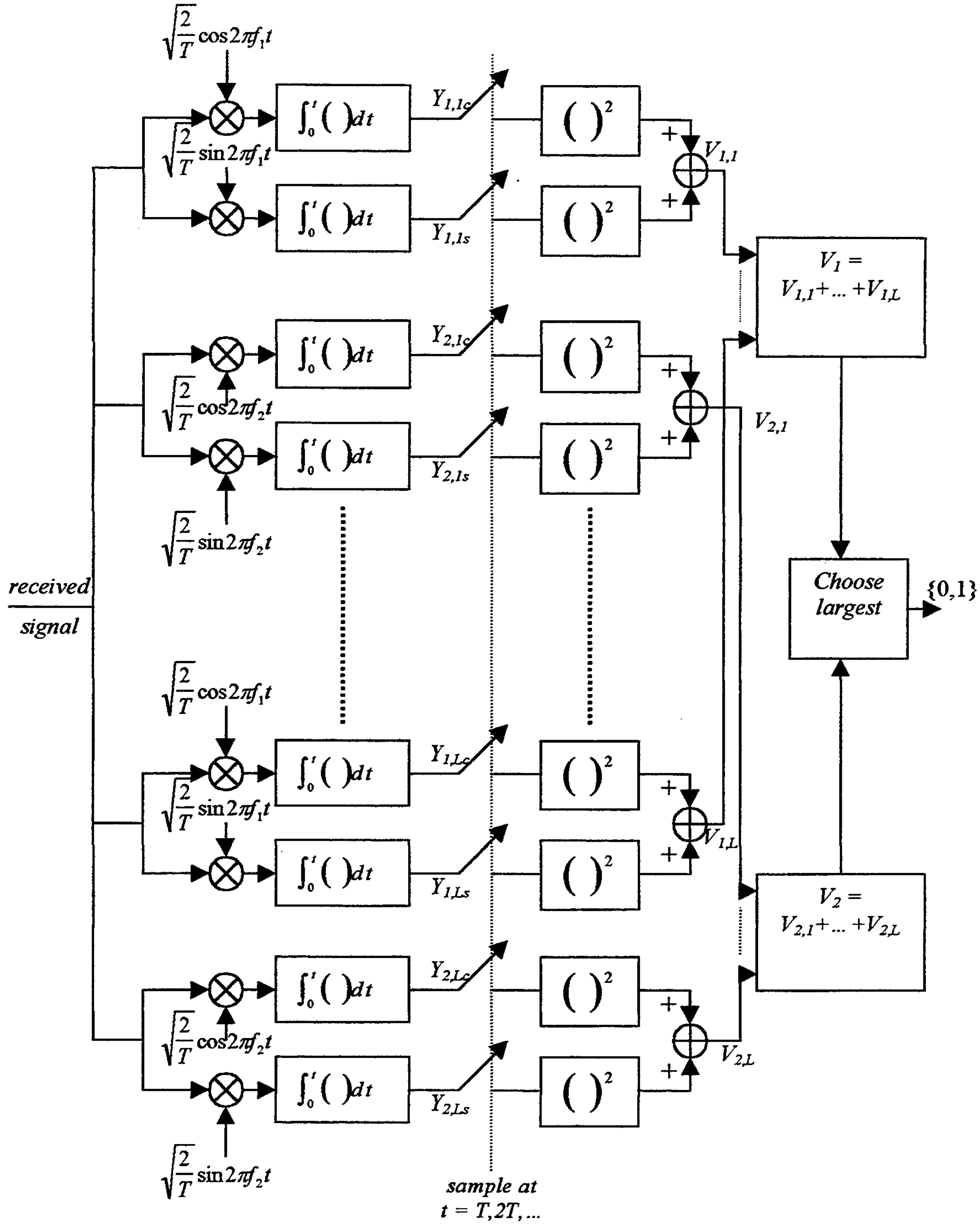

Figure 2. Non-coherent BFSK receiver with equal gain combining

For the non-coherent BFSK receiver the signal-to-noise ratio per bit and b_k is given by

$$\gamma_b = \frac{E}{N_o}\sum_{k=1}^{L}\alpha_k^2 = \sum_{k=1}^{L}\gamma_k \qquad , \qquad (22)$$

where $\gamma_k = \alpha_k^2(E_c/N_o)$ is the instantaneous SNR per diversity bit for the k^{th} channel. Here E_c/N_o is the SNR per diversity bit with no fading. From [1], γ_k is a random variable whose probability density function under Nakagami fading is given by [1]

$$p_{\gamma_k}(\gamma_k) = \frac{m^m}{\Gamma(m)\left(\overline{\gamma_c}\right)^m}\gamma_k^{m-1}\exp\left(-\frac{m\gamma_k}{\overline{\gamma_c}}\right) \qquad , \qquad (23)$$

where $\overline{\gamma_c} = \mathrm{E}\{\alpha_k^2\}(E_c/N_0)$ is the average SNR per diversity bit, which is assumed to be identical for all L channels. The characteristic function of γ_k is given by

$$\Psi_{\gamma_k}(j\omega) = E\left(e^{j\omega\gamma_k}\right) = \int_0^\infty \frac{m^m}{\Gamma(m)\left(\overline{\gamma_c}\right)^m}\gamma_k^{m-1}\exp\left(-\frac{m\gamma_k}{\overline{\gamma_c}}\right)\exp(j\omega\gamma_k)\,d\gamma_k$$

$$= \int_0^\infty \frac{m^m}{\Gamma(m)\left(\overline{\gamma_c}\right)^m}\gamma_k^{m-1}\exp\left[-\left(\frac{m}{\overline{\gamma_c}} - j\omega\right)\gamma_k\right]d\gamma_k$$

$$= \frac{m^m}{\Gamma(m)\left(\overline{\gamma_c}\right)^m}\int_0^\infty \gamma_k^{m-1}\exp\left[-\left(\frac{m}{\overline{\gamma_c}} - j\omega\right)\gamma_k\right]d\gamma_k \quad . \qquad (24)$$

By using the identity [3]

$$\int_0^\infty x^{\nu-1}\exp(-\mu x)\,dx = \frac{\Gamma(\nu)}{\mu^\nu} \quad , \quad \mathrm{Re}(\mu) > 0 \quad , \quad \mathrm{Re}(\nu) > 0 \qquad , \qquad (25)$$

where $\Gamma(\nu)$ is the gamma function, which is defined by

$$\Gamma(v) = \int_{0}^{\infty} x^{v-1} \exp(-x)\,dx \qquad , \quad v > 0 \tag{26}$$

and by letting $m = v$ and $\left(m/\overline{\gamma_c}\right) - j\omega = \mu$, (24) becomes

$$\Psi_{\gamma_k}(j\omega) = \frac{m^m}{\Gamma(m)\left(\overline{\gamma_c}\right)^m} \frac{\Gamma(m)}{\left(\dfrac{m}{\overline{\gamma_c}} - j\omega\right)^m} = \frac{m^m}{\left(\overline{\gamma_c}\right)^m \left(\dfrac{m}{\overline{\gamma_c}} - j\omega\right)^m} \qquad . \tag{27}$$

Since the fading of the L channels is mutually statistically independent, the γ_k's are independent, and the characteristic function of their sum is given by

$$\Psi_{\gamma_b}(j\omega) = \left[\Psi_{\gamma_k}(j\omega)\right]^L = \frac{m^{mL}}{\left(\overline{\gamma_c}\right)^{mL} \left(\dfrac{m}{\overline{\gamma_c}} - j\omega\right)^{mL}} \qquad . \tag{28}$$

Therefore, the probability density function of the random variable γ_b, is given by

$$p_{\gamma_b}(\gamma_b) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Psi_{\gamma_b}(j\omega) \exp(-j\omega\gamma_b)\,d\omega \qquad . \tag{29}$$

Substituting, (28) into (29) leads to

$$p_{\gamma_b}(\gamma_b) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{m^{mL}}{\left(\overline{\gamma_c}\right)^{mL} \left(\dfrac{m}{\overline{\gamma_c}} - j\omega\right)^{mL}} \exp(-j\omega\gamma_b)\,d\omega \qquad . \tag{30}$$

Using $\left(m/\overline{\gamma_c}\right) = \alpha'$ and $mL = \beta'$, (30) becomes

$$p_{\gamma_b}(\gamma_b) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\alpha'^{\beta'}}{(\alpha' - j\omega)^{\beta'}} \exp(-j\omega\gamma_b)\,d\omega \qquad . \tag{31}$$

By using the identity [3]:

$$\int_{-\infty}^{\infty}\frac{\exp(-j\rho x)}{(\beta-jx)^{v}}dx=\frac{2\pi\rho^{v-1}\exp(-\beta\rho)}{\Gamma(v)}\quad,\quad \rho>0 \tag{32}$$

and by letting $\beta = \alpha'$, $v = \beta'$, $\rho = \gamma_b$, we obtain

$$p_{\gamma_b}(\gamma_b)=\frac{1}{2\pi}\alpha'^{\beta'}\frac{2\pi\gamma_b^{\beta'-1}\exp(-\alpha'\gamma_b)}{\Gamma(\beta')}=\left(\frac{m}{\gamma_c}\right)^{mL}\frac{\gamma_b^{mL-1}}{\Gamma(mL)}\exp\left(-\frac{m\gamma_b}{\gamma_c}\right)\quad. \tag{33}$$

The final step is to average the bit error probability given by (20), over the fading channel variable given by (33). The bit error probability will be given by

$$P_e=\int_{-\infty}^{\infty}P_e(\gamma_b)p_{\gamma_b}(\gamma_b)d\gamma_b$$

$$=\int_0^{\infty}\left(\frac{1}{2}\right)^{2L-1}\exp\left(-\frac{\gamma_b}{2}\right)\sum_{k=0}^{L-1}b_k\left(\frac{\gamma_b}{2}\right)^k\left(\frac{m}{\gamma_c}\right)^{mL}\left(\frac{\gamma_b^{mL-1}}{\Gamma(mL)}\right)\exp\left(-\frac{m\gamma_b}{\gamma_c}\right)d\gamma_b$$

$$=\left(\frac{1}{2}\right)^{2L-1}\left(\frac{m}{\gamma_c}\right)^{mL}\frac{1}{\Gamma(m)}\int_0^{\infty}\exp\left(-\frac{\gamma_b}{2}\right)\sum_{k=0}^{L-1}b_k\left(\frac{\gamma_b}{2}\right)^k\gamma_b^{mL-1}\exp\left(-\frac{m\gamma_b}{\gamma_c}\right)d\gamma_b\quad. \tag{34}$$

If we let $(\gamma_b/2) = \gamma$, then $\gamma_b = 2\gamma$ and $d\gamma_b = 2d\gamma$. The limits of the integral do not change, so (34) yields

$$P_e=\left(\frac{1}{2}\right)^{2L-1}\left(\frac{m}{\gamma_c}\right)^{mL}\frac{1}{\Gamma(m)}\int_0^{\infty}\sum_{k=0}^{L-1}b_k\gamma^k 2^{mL-1}\gamma^{mL-1}\exp\left[-\left(\frac{2m}{\gamma_c}+1\right)\gamma\right]2d\gamma$$

$$=\left(\frac{1}{2}\right)^{2L-1}\left(\frac{2m}{\gamma_c}\right)^{mL}\frac{1}{\Gamma(m)}\sum_{k=0}^{L-1}b_k\int_0^{\infty}\gamma^{k+mL-1}\exp\left[-\left(\frac{2m+\gamma_c}{\gamma_c}\right)\gamma\right]d\gamma\quad. \tag{35}$$

Now letting $\left(2m/\overline{\gamma_c}\right)/\overline{\gamma_c} = \delta$ and $\delta\gamma = u$, so $\gamma = u/\delta$ and $d\gamma = (du)/\delta$. The bit error probability becomes

$$P_e = \left(\frac{1}{2}\right)^{2L-1}\left(\frac{2m}{\overline{\gamma_c}}\right)^{mL}\frac{1}{\Gamma(mL)}\sum_{k=0}^{L-1} b_k \int_0^{\infty} \frac{u^{k+mL-1}}{\delta^{k+mL-1}}\exp(-u)\frac{du}{\delta}$$

$$= \left(\frac{1}{2}\right)^{2L-1}\left(\frac{2m}{\overline{\gamma_c}}\right)^{mL}\frac{1}{\Gamma(mL)}\sum_{k=0}^{L-1} \frac{b_k}{\delta^{k+mL}} \int_0^{\infty} u^{k+mL-1}\exp(-u)du \quad . \tag{36}$$

Again using identity [3]

$$\int_0^{\infty} x^{\nu-1}\exp(-\mu x)dx = \frac{\Gamma(\nu)}{\mu^{\nu}} \quad , \quad \mathrm{Re}(\mu) > 0 \quad , \quad \mathrm{Re}(\nu) > 0 \tag{37}$$

and letting $\nu = k+mL$, $\mu = 1$, (36) becomes

$$P_e = \left(\frac{1}{2}\right)^{2L-1}\left(\frac{2m}{\overline{\gamma_c}}\right)^{mL}\frac{1}{\Gamma(mL)}\sum_{k=0}^{L-1} \frac{b_k}{\delta^{k+mL}} \frac{\Gamma(k+mL)}{1^{k+mL-1}}$$

$$= \left(\frac{1}{2}\right)^{2L-1}\left(\frac{2m}{\overline{\gamma_c}}\right)^{mL}\frac{1}{\Gamma(mL)}\sum_{k=0}^{L-1} \frac{b_k\left(\overline{\gamma_c}\right)^{k+mL}}{\left(2m+\overline{\gamma_c}\right)^{k+mL}}\Gamma(k+mL)$$

$$= \left(\frac{1}{2}\right)^{2L-1}\left(\frac{2m}{2m+\overline{\gamma_c}}\right)^{mL}\sum_{k=0}^{L-1} b_k\left(\frac{\overline{\gamma_c}}{2m+\overline{\gamma_c}}\right)^{k}\frac{\Gamma(k+mL)}{\Gamma(mL)} \quad , \tag{38}$$

where b_k is given by (21).

Define the average signal to noise ratio per bit by $\overline{\gamma_b} = L\overline{\gamma_c}$ then (38) becomes

$$P_e = \left(\frac{1}{2}\right)^{2L-1} \left(\frac{2m}{2m + \frac{\overline{\gamma_b}}{L}}\right)^{mL} \sum_{k=0}^{L-1} b_k \left(\frac{\frac{\overline{\gamma_b}}{L}}{2m + \frac{\overline{\gamma_b}}{L}}\right)^{k} \frac{\Gamma(k+mL)}{\Gamma(mL)} \quad . \qquad (39)$$

We note that EGC requires a more complex receiver, since it uses all available information from all L branches. In addition, combining more signals does not necessarily imply that the performance is improved, especially in environments where the bit error rate is large due to fading or jamming. Furthermore, EGC depends on the diversity order L. This is undesirable in applications where L may vary with location, as well as time. Since different diversity channels tend to have different path lengths, the signal-to-noise ratio of each diversity channel is different resulting in an unequal gain combining. This can cause serious performance degradation in an EGC receiver.

III. FIRST ORDER POST DETECTION SELECTION COMBINING (PDSC-1)

In first order Post Detection Selection Combining (PDSC-1), the signal with the largest amplitude of the signal branches is compared to the signal with the largest amplitude of the non-signal branches. If we assume that the data bit $b_i = 1$ is transmitted, then branch 1 of the BFSK non-coherent receiver will be the signal branch and branch 2 will be the non-signal branch. The decision variables V_1 and V_2 are given by

$$\begin{aligned} V_1 &= \max(V_{1,1}, V_{1,2}, \ldots, V_{1,L}) \\ V_2 &= \max(V_{2,1}, V_{2,2}, \ldots, V_{2,L}) \end{aligned} \quad , \tag{40}$$

where $V_{1,1}, V_{1,2}, \ldots, V_{1,L}$ and $V_{2,1}, V_{2,2}, \ldots, V_{2,L}$ are the outputs from the square law detectors of the L signal branches and the L non-signal branches, respectively.

The bit error probability for PDSC-1 is given by

$$P_e = \Pr(V_2 > V_1) = \int_0^{\infty} \left[\int_{V_2 = V_1}^{\infty} f_{V_2}(v_2)\, dv_2 \right] f_{V_1}(v_1)\, dv_1 \quad , \tag{41}$$

where $f_{V_1}(v_1)$ and $f_{V_2}(v_2)$ are the probability density functions of the decision variables from the signal branch and the non-signal branch, respectively.

A. PROBABILITY DENSITY FUNCTIONS OF THE DECISION VARIABLES

The non-coherent BFSK receiver is shown in Fig.3. For the signal branches, the output random variable from the i^{th} square law detector $V_{1,i}$ before combining is given by

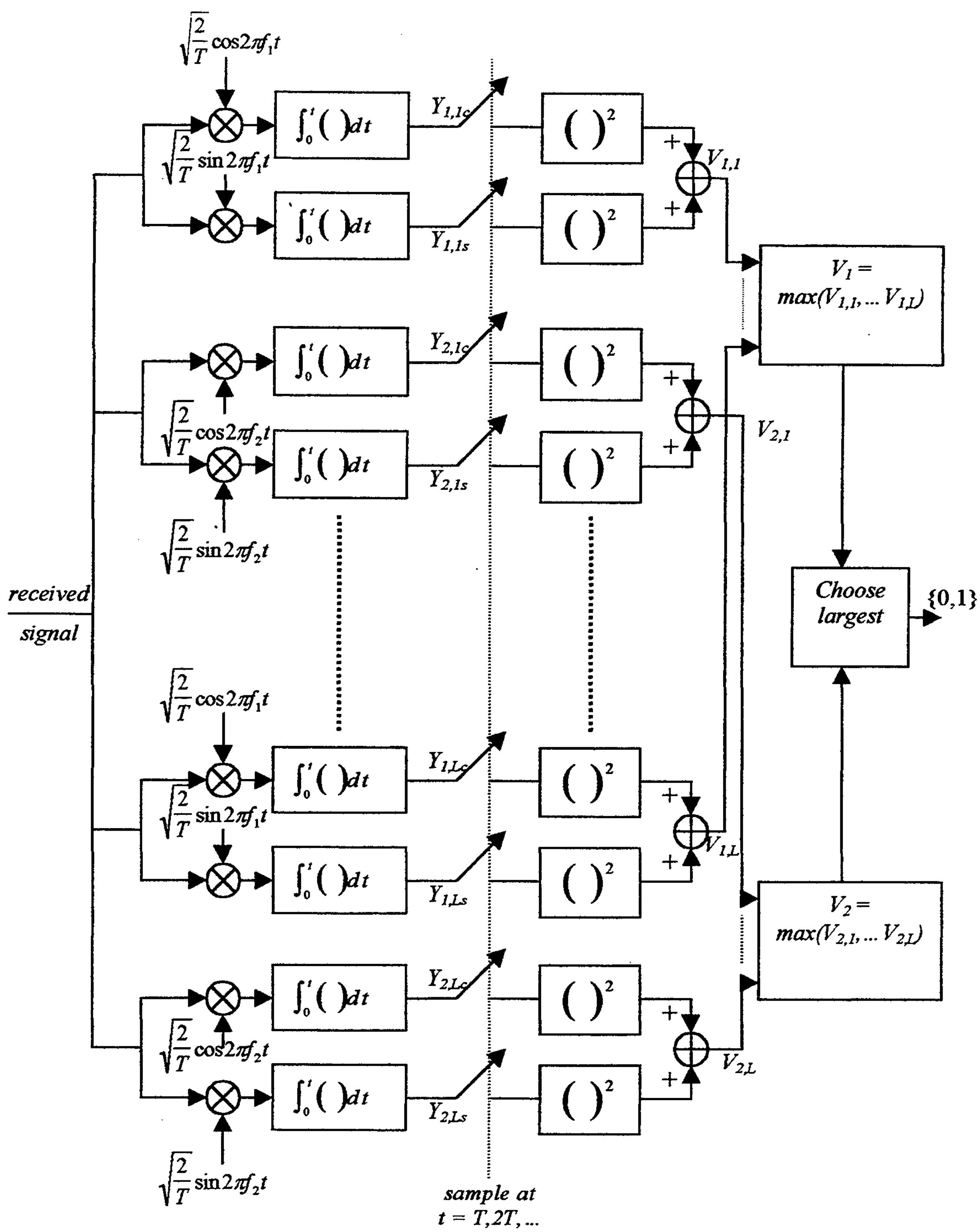

Figure 3. Non-coherent BFSK receiver with first order post detection selection combining

$$V_{1,i} = Y_{1c,i}^2 + Y_{1s,i}^2 = \left(\alpha_i \sqrt{E_c} \cos\theta_i + n_{1c,i}\right)^2 + \left(\alpha_i \sqrt{E_c} \sin\theta_i + n_{1s,i}\right)^2 \quad , \qquad (42)$$

where E_c is the energy per diversity bit without fading. For the non-signal branches the output from the i^{th} square law detector $V_{2,i}$ before combining is given by

$$V_{2,i} = Y_{2c,i}^2 + Y_{2s,i}^2 = n_{2c,i}^2 + n_{2s,i}^2 \quad . \qquad (43)$$

Next, we will derive the probability density functions of the random variables $V_{1,i}$ and $V_{2,i}$, for $i = 1,2,...L$. If we assume, for the moment, that $\alpha_i \sqrt{E_c} \cos\theta_i$ and $\alpha_i \sqrt{E_c} \sin\theta_i$ are fixed values, $V_{1,i}$ is a sum of squares of two Gaussian random variables with means $\alpha_i \sqrt{E_c} \cos\theta_i$ and $\alpha_i \sqrt{E_c} \sin\theta_i$, respectively and variance $N_o/2 = \sigma^2_n$. Therefore the random variable $V_{1,I}$ is a non-central chi-square random variable whose probability density function is given by [1]

$$f_{V_{1,i}}\left(v_{1,i}/\beta\right) = \frac{1}{2\sigma_n^2} \exp\left(-\frac{\beta + v_{1,i}}{2\sigma_n^2}\right) I_0\left(\frac{\sqrt{\beta v_{1,i}}}{\sigma_n^2}\right), \quad v_{1,i} \geq 0 \quad , \qquad (44)$$

where

$$\beta = \left(\alpha_i \sqrt{E_c} \cos\theta_i\right)^2 + \left(\alpha_i \sqrt{E_c} \sin\theta_i\right)^2 = \alpha_i^2 E_c , \quad i = 1,2,\ldots,L \quad . \qquad (45)$$

In [1] the probability density function of β is given by

$$f_\beta(\beta) = \frac{m^m}{\Gamma(m)\overline{\beta}^m} \beta^{m-1} \exp\left(-\frac{m\beta}{\overline{\beta}}\right), \quad \beta \geq 0 \quad , \qquad (46)$$

where $\overline{\beta} = \mathrm{E}\left\{\alpha_i^2\right\} E_c$ and m is the parameter of the Nakagami distribution. To remove the condition on β from (44) we average using (46) as follows

$$f_{V_{1,i}}\left(v_{1,i}\right) = \int_0^\infty f_{V_{1,i}}\left(v_{1,i}/\beta\right) f_\beta(\beta) d\beta$$

$$= \int_0^{\infty} \frac{1}{2\sigma_\eta^2} \exp\left(-\frac{\beta}{2\sigma_\eta^2}\right) \exp\left(-\frac{v_{1,i}}{2\sigma_\eta^2}\right) I_0\left(\frac{\sqrt{\beta v_{1,i}}}{\sigma_\eta^2}\right) \frac{m^m}{\Gamma(m)\overline{\beta}^m} \beta^{m-1} \exp\left(-\frac{m\beta}{\overline{\beta}}\right) d\beta$$

$$= \int_0^{\infty} \frac{1}{N_0} \exp\left(-\frac{\beta}{N_0}\right) \exp\left(-\frac{v_{1,i}}{N_0}\right) I_0\left(\frac{2\sqrt{\beta v_{1,i}}}{N_0}\right) \frac{m^m}{\Gamma(m)\overline{\beta}^m} \beta^{m-1} \exp\left(-\frac{m\beta}{\overline{\beta}}\right) d\beta$$

$$= \int_0^{\infty} \frac{1}{N_0} \exp\left(-\frac{\beta}{N_0}\right) \exp\left(-\frac{v_{1,i}}{N_0}\right) \sum_{k=0}^{\infty} \frac{1}{(k!)^2} \left(\frac{\sqrt{\beta v_{1,i}}}{N_0}\right)^{2k} \frac{m^m}{\Gamma(m)\overline{\beta}^m} \beta^{m-1} \exp\left(-\frac{m\beta}{\overline{\beta}}\right) d\beta$$

$$= \frac{1}{N_0} \exp\left(-\frac{v_{1,i}}{N_0}\right) \frac{m^m}{\Gamma(m)\overline{\beta}^m} \sum_{k=0}^{\infty} \frac{\frac{v_{1,i}^k}{N_0^{2k}}}{(k!)^2} \int_0^{\infty} \beta^{k+m-1} \exp\left[-\left(\frac{1}{N_0} + \frac{m}{\overline{\beta}}\right)\beta\right] d\beta \quad . \tag{47}$$

By using [3]

$$\int_0^{\infty} x^{\nu-1} \exp(-\mu x) dx = \frac{\Gamma(\nu)}{\mu^{\nu}} \text{, } \operatorname{Re}(\mu) > 0 \text{ , } \operatorname{Re}(\nu) > 0 \quad , \tag{48}$$

where $\Gamma(\nu)$ is defined in (26), and by letting $\nu = k+m$ and $\mu = \frac{1}{N_0} + \frac{m}{\overline{\beta}} = \frac{\overline{\beta} + mN_0}{N_0 \overline{\beta}}$, (48)

we obtain

$$f_{V_{1,i}}(v_{1,i}) = \frac{1}{N_0} \exp\left(-\frac{v_{1,i}}{N_0}\right) \frac{m^m}{\Gamma(m)\overline{\beta}^m} \sum_{k=0}^{\infty} \left(\frac{1}{k!}\right)^2 \frac{v_{1,i}^k}{N_0^{2k}} \frac{\Gamma(k+m)}{\left(\frac{\overline{\beta} + mN_0}{N_0 \overline{\beta}}\right)^{k+m}}$$

$$= \frac{1}{N_0} \exp\left(-\frac{v_{1,i}}{N_0}\right) \left(\frac{mN_0}{\overline{\beta} + N_0 m}\right)^m \sum_{k=0}^{\infty} \left(\frac{1}{k!}\right)^2 \left(\frac{v_{1,i}}{N_0}\right)^k \frac{\Gamma(k+m)}{\Gamma(m)} \left(\frac{\overline{\beta}}{\overline{\beta} + N_0 m}\right)^k \quad . \tag{49}$$

By letting $\overline{\gamma_c} = \frac{\overline{\beta}}{N_0}$, (49) becomes

$$f_{V_{1,i}}(v_{1,i}) = \frac{1}{N_0}\exp\left(-\frac{v_{1,i}}{N_0}\right)\left(\frac{m}{\overline{\overline{\gamma}}_c + m}\right)^m \sum_{k=0}^{\infty}\left(\frac{1}{k!}\right)^2\left(\frac{v_{1,i}}{N_0}\right)^k \frac{\Gamma(k+m)}{\Gamma(m)}\left(\frac{\overline{\overline{\gamma}}_c}{\overline{\overline{\gamma}}_c + m}\right)^k \quad . \tag{50}$$

The cumulative distribution function of the random variable $V_{1,i}$ is given by

$$F_{V_{1,i}}(v_{1,i}) = \int_0^{v_{1,i}} f_{V_{1,i}}(u)du \quad . \tag{51}$$

Substituting (50) into (51) leads to

$$F_{V_{1,i}}(v_{1,i}) = \int_0^{v_{1,i}} \frac{1}{N_0}\exp\left(-\frac{u}{N_0}\right)\left(\frac{m}{\overline{\overline{\gamma}}_c + m}\right)^m \sum_{k=0}^{\infty}\left(\frac{1}{k!}\right)^2\left(\frac{u}{N_0}\right)^k \frac{\Gamma(k+m)}{\Gamma(m)}\left(\frac{\overline{\overline{\gamma}}_c}{\overline{\overline{\gamma}}_c + m}\right)^k du$$

$$= \left(\frac{m}{\overline{\overline{\gamma}}_c + m}\right)^m \sum_{k=0}^{\infty}\left(\frac{1}{k!}\right)^2\left(\frac{\overline{\overline{\gamma}}_c}{\overline{\overline{\gamma}}_c + m}\right)^k \frac{\Gamma(k+m)}{\Gamma(m)} \int_0^{v_{1,i}}\left(\frac{u}{N_0}\right)^k \exp\left(-\frac{u}{N_0}\right) d\left(\frac{u}{N_0}\right) \quad . \tag{52}$$

Letting $w = \frac{u}{N_0}$, $dw = d\left(\frac{u}{N_0}\right)$, the upper limit becomes $\frac{v_{1,i}}{N_0}$ and (52) becomes

$$F_{V_{1,i}}(v_{1,i}) = \left(\frac{m}{\overline{\overline{\gamma}}_c + m}\right)^m \sum_{k=0}^{\infty}\left(\frac{1}{k!}\right)^2\left(\frac{\overline{\overline{\gamma}}_c}{\overline{\overline{\gamma}}_c + m}\right)^k \frac{\Gamma(k+m)}{\Gamma(m)} \int_0^{v_{1,i}/N_0} w^k \exp(-w)dw \quad . \tag{53}$$

Using [3]

$$\int_0^u x^n \exp(-\mu x)dx = \frac{n!}{\mu^{n+1}} - \exp(-\mu u)\sum_{\lambda=0}^{n}\frac{n!}{\lambda!}\frac{u^\lambda}{\mu^{n-\lambda+1}} \;, \quad u \geq 0, \quad \mathrm{Re}(\mu) > 0 \tag{54}$$

and letting $n = k$, $\mu = 1$, $u = \frac{v_{1,i}}{N_0}$, results in

$$F_{V_{1,i}}(v_{1,i}) = \left(\frac{m}{\overline{\overline{\gamma}}_c + m}\right)^m \sum_{k=0}^{\infty}\left(\frac{1}{k!}\right)^2\left(\frac{\overline{\overline{\gamma}}_c}{\overline{\overline{\gamma}}_c + m}\right)^k \frac{\Gamma(k+m)}{\Gamma(m)}\left[\frac{k!}{1^{k+1}} - \exp\left(-\frac{v_{1,i}}{N_0}\right)\sum_{\lambda=0}^{k}\frac{k!}{\lambda!}\left(\frac{v_{1,i}}{N_0}\right)^\lambda\right]$$

$$= \left(\frac{m}{\overline{\overline{\gamma}}_c + m}\right)^m \sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{\overline{\overline{\gamma}}_c}{\overline{\overline{\gamma}}_c + m}\right)^k \frac{\Gamma(k+m)}{\Gamma(m)}\left[1 - \exp\left(-\frac{v_{1,i}}{N_0}\right)\sum_{\lambda=0}^{k}\frac{1}{\lambda!}\left(\frac{v_{1,i}}{N_0}\right)^\lambda\right] \quad . \tag{55}$$

Since all L signal branch variables $V_{1,1}$, $V_{1,2}$, ..., $V_{1,L}$ are statistically independent, we can use [8]

$$f_{V_1}(v_1) = L f_{V_{1i}}(v_1) F_{V_{1i}}^{L-1}(v_1) \quad . \tag{56}$$

The probability density function for the signal branch decision variable V_1, is given by

$$f_{V_1}(v_1) = \frac{L}{N_0}\exp\left(-\frac{v_1}{N_0}\right)\left(\frac{m}{\bar{\gamma}_c + m}\right)^m \sum_{k_1=0}^{\infty}\left(\frac{1}{k_1!}\right)^2\left(\frac{v_1}{N_0}\right)^{k_1}\frac{\Gamma(k_1+m)}{\Gamma(m)}\left(\frac{\bar{\gamma}_c}{\bar{\gamma}_c+m}\right)^{k_1}$$

$$\times\left\{\left(\frac{m}{\bar{\gamma}_c+m}\right)^m \sum_{k_2=0}^{\infty}\frac{1}{k_2!}\frac{\Gamma(k_2+m)}{\Gamma(m)}\left(\frac{\bar{\gamma}_c}{\bar{\gamma}_c+m}\right)^{k_2}\left[1-\exp\left(-\frac{v_1}{N_0}\right)\sum_{\lambda=0}^{k_2}\frac{1}{\lambda!}\left(\frac{v_1}{N_0}\right)^{\lambda}\right]\right\}^{L-1}$$

$$= \frac{L}{N_0}\exp\left(-\frac{v_1}{N_0}\right)\left(\frac{m}{\bar{\gamma}_c + m}\right)^{mL} \sum_{k_1=0}^{\infty}\left(\frac{1}{k_1!}\right)^2\left(\frac{v_1}{N_0}\right)^{k_1}\frac{\Gamma(k_1+m)}{\Gamma(m)}\left(\frac{\bar{\gamma}_c}{\bar{\gamma}_c+m}\right)^{k_1}$$

$$\times\left\{\sum_{k_2=0}^{\infty}\frac{1}{k_2!}\frac{\Gamma(k_2+m)}{\Gamma(m)}\left(\frac{\bar{\gamma}_c}{\bar{\gamma}_c+m}\right)^{k_2}\left[1-\exp\left(-\frac{v_1}{N_0}\right)\sum_{\lambda=0}^{k_2}\frac{1}{\lambda!}\left(\frac{v_1}{N_0}\right)^{\lambda}\right]\right\}^{L-1} \quad . \tag{57}$$

For the non-signal branches, the output random variable from the i^{th} square law detector $V_{2,i}$ prior to combining is given by (43). It is a sum of squares of two zero mean Gaussian random variables with identical variances. Therefore, it is a chi-square random variable whose probability density function is given by [1]

$$f_{V_{2,i}}(v_{2,i}) = \frac{1}{2\sigma_n^2}\exp\left(-\frac{v_{2,i}}{2\sigma_n^2}\right) = \frac{1}{N_0}\exp\left(-\frac{v_{2,i}}{N_0}\right), \quad v_{2,i} \geq 0 \quad . \tag{58}$$

The corresponding cumulative distribution function is given by

$$F_{V_{2,i}}\left(v_{2,i}\right)=1-\exp\left(-\frac{v_{2,i}}{N_0}\right) \qquad . \qquad (59)$$

Since all L non-signal branch variables are statistically independent the probability density function of the decision random variable V_2 is given by

$$f_{V_2}\left(v_2\right)=L\,f_{V_{2,i}}\left(v_2\right)F_{V_{2,i}}^{L-1}\left(v_2\right)=\frac{L}{N_0}\exp\left(-\frac{v_2}{N_0}\right)\left[1-\exp\left(-\frac{v_2}{N_0}\right)\right]^{L-1} \qquad . \qquad (60)$$

B. BIT ERROR PROBABILITY

The bit error probability for first order post detection selection combining is given by (41)

$$P_e=\Pr\left(V_2>V_1\right)=\int_0^{\infty}\left[\int_{v_1}^{\infty}f_{V_2}\left(v_2\right)dv_2\right]f_{V_1}\left(v_1\right)dv_1 \qquad , \qquad (61)$$

where

$$\int_{v_1}^{\infty}f_{V_2}\left(v_2\right)dv_2=\int_{v_1}^{\infty}\frac{L}{N_0}\exp\left(-\frac{v_2}{N_0}\right)\left[1-\exp\left(-\frac{v_2}{N_0}\right)\right]^{L-1}dv_2 \qquad . \qquad (62)$$

By using the binomial theorem,

$$(a+b)^N=\sum_{k=0}^{N}\binom{N}{k}a^{N-k}b^k \qquad , \qquad (63)$$

we obtain

$$\left[1-\exp\left(-\frac{v_2}{N_0}\right)\right]^{L-1}=\sum_{k=0}^{L-1}\binom{L-1}{k}(-1)^k\left[\exp\left(-\frac{v_2}{N_0}\right)\right]^k \qquad . \qquad (64)$$

So (62) becomes

$$\int_{v_1}^{\infty}f_{V_2}\left(v_2\right)dv_2=\int_{v_1}^{\infty}\frac{L}{N_0}\exp\left(-\frac{v_2}{N_0}\right)\sum_{k=0}^{L-1}\binom{L-1}{k}(-1)^k\left[\exp\left(-\frac{v_2}{N_0}\right)\right]^k dv_2$$

$$= L\sum_{k=0}^{L-1}\binom{L-1}{N_0}(-1)^k\int_{v_1}^{\infty}\left[\exp\left(-\frac{v_2}{N_0}\right)\right]^{k+1} d\left(\frac{v_2}{N_0}\right) \quad . \tag{65}$$

Letting $x = \frac{v_2}{N_0}$, (65) becomes

$$\int_{v_1}^{\infty} f_{V_2}(v_2)dv_2 = L\sum_{k=0}^{L-1}\binom{L-1}{k}(-1)^k\int_{v_1/N_0}^{\infty}\exp(-(k+1)x)dx$$

$$= L\sum_{k=0}^{L-1}\binom{L-1}{k}(-1)^k\frac{1}{k+1}\exp\left[-(k+1)\frac{v_1}{N_0}\right] \quad . \tag{66}$$

Substituting (66) into (61) leads to

$$P_e = \int_0^{\infty} L\sum_{k_3=0}^{L-1}\binom{L-1}{k_3}\frac{(-1)^{k_3}}{k_3+1}\exp\left[-(k_3+1)\frac{v_1}{N_0}\right]\frac{L}{N_0}\exp\left(-\frac{v_1}{N_0}\right)\left(\frac{m}{\overline{\gamma}_c+m}\right)^{mL}$$

$$\times\sum_{k_1=0}^{\infty}\left(\frac{1}{k_1!}\right)^2\left(\frac{v_1}{N_0}\right)^{k_1}\frac{\Gamma(k_1+m)}{\Gamma(m)}\left(\frac{\overline{\gamma}_c}{\overline{\gamma}_c+m}\right)^{k_1}$$

$$\times\left\{\sum_{k_2=0}^{\infty}\frac{1}{k_2!}\frac{\Gamma(k_2+m)}{\Gamma(m)}\left(\frac{\overline{\gamma}_c}{\overline{\gamma}_c+m}\right)^{k_2}\left[1-\exp\left(-\frac{v_1}{N_0}\right)\sum_{\lambda=0}^{k_1}\frac{1}{\lambda!}\left(\frac{v_1}{N_0}\right)^{\lambda}\right]\right\}^{L-1} dv_1$$

$$= \frac{L^2}{N_0}\left(\frac{m}{\overline{\gamma}_c+m}\right)^{mL}\int_0^{\infty}\sum_{k_3=0}^{L-1}\binom{L-1}{k_3}\frac{(-1)^{k_3}}{k_3+1}\exp\left[-(k_3+2)\frac{v_1}{N_0}\right]$$

$$\times\sum_{k_1=0}^{\infty}\left(\frac{1}{k_1!}\right)^2\left(\frac{v_1}{N_0}\right)^{k_1}\frac{\Gamma(k_1+m)}{\Gamma(m)}\left(\frac{\overline{\gamma}_c}{\overline{\gamma}_c+m}\right)^{k_1}$$

$$\times\left\{\sum_{k_2=0}^{\infty}\frac{1}{k_2!}\frac{\Gamma(k_2+m)}{\Gamma(m)}\left(\frac{\overline{\gamma}_c}{\overline{\gamma}_c+m}\right)^{k_2}\left[1-\exp\left(-\frac{v_1}{N_0}\right)\sum_{\lambda=0}^{k_1}\frac{1}{\lambda!}\left(\frac{v_1}{N_0}\right)^{\lambda}\right]\right\}^{L-1} dv_1 \quad . \tag{67}$$

Changing $u = \frac{v_1}{N_0}$, and realizing that the average signal-to-noise ratio per bit $\overline{\gamma_b} = L\overline{\gamma_c}$,

(67) becomes

$$P_e = \frac{L^2}{N_0}\left(\frac{m}{\frac{1}{L}\overline{\gamma_b}+m}\right)^{mL} \int_0^\infty \left\{\sum_{k_3=0}^{L-1}\binom{L-1}{k_3}\frac{(-1)^{k_3}}{k_3+1}\exp\left[-(k_3+2)u\right]\right\}$$

$$\times \sum_{k_1=0}^{\infty}\left(\frac{1}{k_1!}\right)^2 u^{k_1}\frac{\Gamma(k_1+m)}{\Gamma(m)}\left(\frac{\frac{1}{L}\overline{\gamma_b}}{\frac{1}{L}\overline{\gamma_b}+m}\right)^{k_1}$$

$$\times \left\{\sum_{k_2=0}^{\infty}\frac{1}{k_2!}\frac{\Gamma(k_2+m)}{\Gamma(m)}\left(\frac{\frac{1}{L}\overline{\gamma_b}}{\frac{1}{L}\overline{\gamma_b}+m}\right)^{k_2}\left[1-\exp(-u)\sum_{\lambda=0}^{k_1}\frac{u^\lambda}{\lambda!}\right]\right\}^{L-1} dv_1 \quad . \qquad (68)$$

This expression cannot be simplified any further. It can be evaluated numerically. The results of this evaluation are presented in Chapter VI.

IV. SECOND ORDER POST-DETECTION SELECTION COMBINING (PDSC-2)

The two signals with the largest amplitudes of the signal branches are combined and compared to the combination of the two signals with the largest amplitudes of the non-signal branches. We let branch *1* of the BFSK non- coherent receiver be the signal branch and branch *2* be the non-signal branch. Without loss of generality, let $V_{1,1}$ denote the largest output random variable and $V_{1,2}$ denote the second largest output random variable of the signal branches, then

$$V_{1,1} = \max\left(V_{1,1}, V_{1,2}, \ldots, V_{1,L}\right) \tag{69}$$

$$V_{1,2} = \mathrm{second}\max\left(V_{1,1}, V_{1,2}, \ldots, V_{1,L}\right) \tag{70}$$

Let $V_{2,1}$ denote the largest output random variable, and $V_{2,2}$ denote the second largest output random variable, of the non-signal branches. Then

$$V_{2,1} = \max\left(V_{2,1}, V_{2,2}, \ldots, V_{2,L}\right) \tag{71}$$

$$V_{2,2} = \mathrm{second}\max\left(V_{2,1}, V_{2,2}, \ldots, V_{2,L}\right) \tag{72}$$

Therefore the decision variables V_1 and V_2 are given by

$$V_1 = V_{1,1} + V_{1,2} \tag{73}$$

$$V_2 = V_{2,1} + V_{2,2} \tag{74}$$

The bit error probability is given by

$$P_e = \Pr\left(V_2 > V_1\right) = \int_{v_1=0}^{\infty}\left[\int_{v_2=v_1}^{\infty} f_{V_2}(v_2)\,dv_2\right] f_{V_1}(v_1)\,dv_1 \tag{75}$$

A. PROBABILITY DENSITY FUNCTIONS OF THE DECISION VARIABLES

The non-coherent BFSK receiver, is shown in Fig.4. For the signal branches, the output random variable from the i^{th} square law detector $V_{1,i}$, before combining is given by (42), whereas, for the non- signal branches the output from the i^{th} square law detector $V_{2,i}$, before combining is given by (43).

In [8], the cumulative distribution function of the decision random variable $V_1 = V_{1,1} + V_{1,2}$ is given by

$$F_{V_1}(v_1) = \int_0^{v_1/2} \int_{v_{1,2}}^{v_1 - v_{1,2}} f_{v_{1,1}v_{1,2}}(v_{1,1}, v_{1,2}) dv_{1,1}\, dv_{1,2} \quad , \tag{76}$$

where $f_{V_{1,1}V_{1,2}}(v_{1,1}v_{1,2})$ is the joint probability density function of the random variables $V_{1,1}$ and $V_{1,2}$ which is given by [8]

$$f_{V_{1,1}V_{1,2}}(v_{1,1}v_{1,2}) = L(L-1)\, f_{V_{1,1}}(v_{1,1})\, f_{V_{1,2}}(v_{1,2})\, F_{V_{1,2}}^{L-2}(v_{1,2}) \quad , \tag{77}$$

where $f_{V_{1,1}}(v_{1,1})$ and $f_{V_{1,2}}(v_{1,2})$ are given by (50) and $F_{V_{1,2}}(v_{1,2})$ is given by (55). Substituting (77) into (76) results in

$$F_{V_1}(v_1) = \int_0^{v_1/2} \int_{v_{1,2}}^{v_1 - v_{1,2}} L(L-1)\, f_{V_{1,1}}(v_{1,1})\, f_{V_{1,2}}(v_{1,2})\, F_{V_{1,2}}^{L-2}(v_{1,2})\, dv_{1,1}\, dv_{1,2} \quad . \tag{78}$$

We can obtain the probability density function of the decision random variable for the signal branches V_1, by differentiating (78) with respect to v_1:

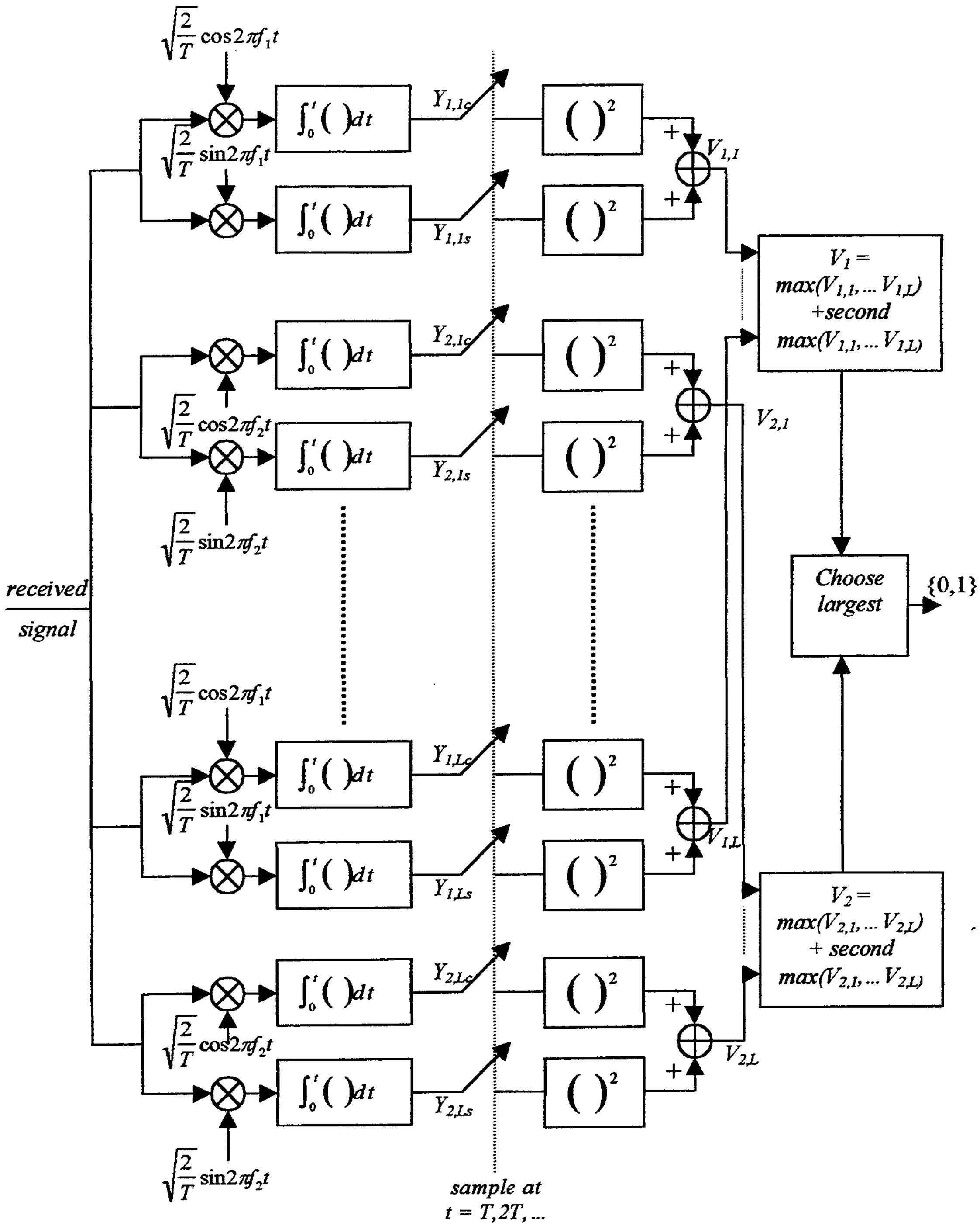

Figure 4. Non-coherent BFSK receiver with second order post detection selection combining

$$f_{V_1}(v_1) = \frac{d}{dv_1} F_{V_1}(v_1) \quad . \tag{79}$$

By using the identity (Leibnitz's rule)

$$\frac{d\left[\int_{a(x)}^{b(x)} f(\lambda, x) d\lambda\right]}{dx} = f(b(x), x)\frac{db(x)}{dx} - f(a(x), x)\frac{da(x)}{dx} + \int_{a(x)}^{b(x)} \frac{df(\lambda, x)}{dx} d\lambda \tag{80}$$

and by letting $x = v_1$, $\lambda = v_{1,2}$, $a(x) = 0$, $b(x) = v_1/2$, and

$$f(\lambda, x) = f(v_{1,2}, v_1) = \int_{v_{1,2}}^{v_1 - v_{1,2}} L(L-1) f_{V_{1,1}}(v_{1,1}) f_{V_{1,2}}(v_{1,2}) F_{V_{1,2}}^{L-2}(v_{1,2})\, dv_{1,1} \quad . \tag{81}$$

Eq. (79) becomes

$$f_{V_1}(v_1) = f(v_1/2, v_1)\frac{d\left(\frac{v_1}{2}\right)}{dv_1} - f(0, v_1)\frac{d(0)}{dv_1} + \int_0^{v_1/2} \frac{df(v_{1,2}, v_1)}{dv_1} dv_{1,2} \quad , \tag{82}$$

where

$$f(v_1/2, v_1) = \int_{v_1/2}^{v_1 - v_1/2} L(L-1) f_{V_{1,1}}(v_{1,1}) f_{V_{1,2}}(v_{1,2}) F_{V_{1,2}}^{L-2}(v_{1,2})\, dv_{1,1} = 0 \tag{83}$$

and

$$\frac{d(a(v_1))}{dv_1} = \frac{d(0)}{dv_1} = 0 \quad , \tag{84}$$

so (82) results in

$$f_{V_1}(v_1) = \int_0^{v_1/2} \frac{df(v_{1,2}, v_1)}{dv_1} dv_{1,2}$$

$$= \int_0^{v_1/2} \left\{ \frac{d\left[\int_{v_{1,2}}^{v_1 - v_{1,2}} L(L-1) f_{V_{1,1}}(v_{1,1}) f_{V_{1,2}}(v_{1,2}) F_{V_{1,2}}^{L-2}(v_{1,2}) dv_{1,1} \right]}{dv_1} \right\} dv_{1,2} \quad . \tag{85}$$

Again using (80) and by letting $x = v_1$, $\lambda = v_{1,1}$, $\alpha(x) = v_{1,2}$, $b(x) = v_1 - v_{1,2}$ we have

$$f(\lambda, x) = f(v_{1,1}, v_1) = L(L-1) f_{V_{1,1}}(v_{1,1}) f_{V_{1,2}}(v_{1,2}) F_{V_{1,2}}^{L-2}(v_{1,2}) = f(v_{1,1}) \quad . \tag{86}$$

Eq. (85) becomes

$$f_{V_1}(v_1) = \int_0^{v_1/2} \left\{ \frac{d\left[\int_{v_{1,2}}^{v_1 - v_{1,2}} L(L-1) f_{V_{1,1}}(v_{1,1}) f_{V_{1,2}}(v_{1,2}) F_{V_{1,2}}^{L-2}(v_{1,2}) dv_{1,1} \right]}{dv_1} \right\} dv_{1,2}$$

$$= \int_0^{v_1/2} \left[f(v_1 - v_{1,2}, v_1) \frac{d(v_1 - v_{1,2})}{dv_1} - f(v_{1,2}, v_1) \frac{d(v_{1,2})}{dv_1} + \int_{v_{1,2}}^{v_1 - v_{1,2}} \frac{df(v_{1,1})}{dv_1} dv_{1,1} \right] dv_{1,2} \quad . \tag{87}$$

We note that the second and the third terms of the summation inside the brackets of (87) are equal to zero, hence (87) becomes

$$f_{V_1}(v_1) = \int_0^{v_1/2} \left[f(v_1 - v_{1,2}, v_1) \frac{d(v_1 - v_{1,2})}{dv_1} \right] dv_{1,2} = \int_0^{v_1/2} f(v_1 - v_{1,2}, v_1) dv_{1,2} \quad . \tag{88}$$

If we substitute (86) into (88) we obtain

$$f_{V_1}(v_1) = \int_0^{v_1/2} \left[L(L-1) f_{V_{1,1}}(v_1 - v_{1,2}) f_{V_{1,2}}(v_{1,2}) F_{V_{1,2}}^{L-2}(v_{1,2}) \right] dv_{1,2} \quad . \tag{89}$$

Finally, if we substitute (50) and (55) into (89) we get the expression for the probability density function of the decision random variable as

$$f_{V_1}(v_1)=\int_0^{v_1/2} L(L-1)\left[\frac{1}{N_0}\exp\left(-\frac{v_1-v_{1,2}}{N_0}\right)\left(\frac{m}{m+\overline{\gamma_c}}\right)^m \sum_{k_1=0}^{\infty}\left(\frac{1}{k_1!}\right)^2\left(\frac{v_1-v_{1,2}}{N_0}\right)^{k_1}\frac{\Gamma(k_1+m)}{\Gamma(m)}\right.$$

$$\left.\times\left(\frac{\overline{\gamma_c}}{m+\overline{\gamma_c}}\right)^{k_1}\right]\left[\frac{1}{N_0}\exp\left(-\frac{v_{1,2}}{N_0}\right)\left(\frac{m}{m+\overline{\gamma_c}}\right)^m\sum_{k_2=0}^{\infty}\left(\frac{1}{k_2!}\right)^2\left(\frac{v_{1,2}}{N_0}\right)^{k_2}\frac{\Gamma(k_2+m)}{\Gamma(m)}\left(\frac{\overline{\gamma_c}}{m+\overline{\gamma_c}}\right)^{k_2}\right]$$

$$\times\left\{\left(\frac{m}{m+\overline{\gamma_c}}\right)^m\sum_{k_3=0}^{\infty}\frac{1}{k_3!}\left(\frac{\overline{\gamma_c}}{m+\overline{\gamma_c}}\right)^{k_3}\frac{\Gamma(k_3+m)}{\Gamma(m)}\left[1-\exp\left(-\frac{v_{1,2}}{N_0}\right)\sum_{\lambda=0}^{k_3}\frac{1}{\lambda!}\left(\frac{v_{1,2}}{N_0}\right)^{\lambda_3}\right]\right\}^{L-2} dv_{1,2}$$

$$=\frac{L(L-1)}{N_0}\left(\frac{m}{m+\overline{\gamma_c}}\right)^{mL}\exp\left(-\frac{v_1}{N_0}\right)\int_0^{v_1/2}\left[\sum_{k_2=0}^{\infty}\left(\frac{v_{1,2}}{N_0}\right)^{k_2}\left(\frac{1}{k_2!}\right)^2\frac{\Gamma(k_2+m)}{\Gamma(m)}\left(\frac{\overline{\gamma_c}}{m+\overline{\gamma_c}}\right)^{k_2}\right]$$

$$\times\left[\sum_{k_1=0}^{\infty}\left(\frac{v_1-v_{1,2}}{N_0}\right)^{k_1}\left(\frac{1}{k_1!}\right)^2\frac{\Gamma(k_1+m)}{\Gamma(m)}\left(\frac{\overline{\gamma_c}}{m+\overline{\gamma_c}}\right)^{k_1}\right]$$

$$\times\left\{\sum_{k_3=0}^{\infty}\frac{1}{k_3!}\left(\frac{\overline{\gamma_c}}{m+\overline{\gamma_c}}\right)^{k_3}\frac{\Gamma(k_3+m)}{\Gamma(m)}\left[1-\exp\left(-\frac{v_{1,2}}{N_0}\right)\sum_{\lambda=0}^{k_3}\frac{1}{\lambda!}\left(\frac{v_{1,2}}{N_0}\right)^{\lambda}\right]\right\}^{L-2} d\left(\frac{v_{1,2}}{N_0}\right) \quad . \qquad (90)$$

For the non-signal branches the output random variable from the i^{th} square law detector $V_{2,i}$ before combining is given by (58). The cumulative distribution function of the decision variable V_2 defined in (74) is given by

$$F_{V_2}(v_2)=\int_0^{v_2/2}\int_{v_{2,2}}^{v_2-v_{2,2}} f_{v_{2,1}v_{2,2}}(v_{2,1},v_{2,2})dv_{2,1}\,dv_{2,2} \quad , \qquad (91)$$

where $f_{V_{2,1}V_{2,2}}\left(v_{2,1}v_{2,2}\right)$ is the joint probability density function of the random variables $V_{2,1}$ and $V_{2,2}$,which is given by [8]

$$f_{V_{2,1}V_{2,1}}\left(v_{2,1}v_{2,2}\right)=L(L-1)f_{V_{2,1}}\left(v_{2,1}\right)f_{V_{2,2}}\left(v_{2,2}\right)F_{V_{2,2}}^{L-2}\left(v_{2,2}\right) \quad , \tag{92}$$

where $f_{V_{2,1}}\left(v_{2,1}\right)$ and $f_{V_{2,2}}\left(v_{2,2}\right)$ are given by (58) and $F_{V_{2,2}}\left(v_{2,2}\right)$ is given by (59).Substituting (92) in (91) results in

$$F_{V_2}\left(v_2\right)=\int_0^{v_2/2}\int_{v_{2,2}}^{v_2-v_{2,2}} L(L-1)f_{V_{2,1}}\left(v_{2,1}\right)f_{V_{2,2}}\left(v_{2,2}\right)F_{V_{2,2}}^{L-2}\left(v_{2,2}\right)dv_{2,1}\,dv_{2,2} \quad . \tag{93}$$

If we substitute (58) and (59) in (93) we obtain

$$F_{V_2}\left(v_2\right)=\int_0^{v_2/2}\int_{v_{2,2}}^{v_2-v_{2,2}} \frac{L(L-1)}{N_0^2}\exp\left(-\frac{v_{2,1}}{N_0}\right)\exp\left(-\frac{v_{2,2}}{N_0}\right)\left[1-\exp\left(-\frac{v_{2,2}}{N_0}\right)\right]^{L-2} dv_{2,1}\,dv_{2,2} \quad . \tag{94}$$

Performing this integration yields [8]

$$F_{V_2}\left(v_2\right)=L(L-1)\left\{\frac{1}{2}\left[1-\left(1+\frac{v_2}{N_0}\right)\exp\left(-\frac{v_2}{N_0}\right)\right]+\sum_{k=1}^{L-2}\binom{L-2}{k}(-1)^k W_2(k)\right\} \quad , \tag{95}$$

where

$$W_2(k)=\frac{1}{2+k}-\frac{1}{k}\exp\left(-\frac{v_2}{N_0}\right)+\frac{2}{k(k+2)}\exp\left[-\frac{k+2}{2}\left(\frac{v_2}{N_0}\right)\right] \quad . \tag{96}$$

Finally, if we differentiate (95) with respect to v_2 we obtain the probability density function of the decision variable of the non-signal branches [8]

$$f_{V_2}\left(v_2\right)=\frac{L(L-1)}{N_0}\exp\left(-\frac{v_2}{N_0}\right)\left\{\frac{v_2}{2N_0}+\sum_{k=1}^{L-2}\binom{L-2}{k}\frac{(-1)^k}{k}\left[1-\exp\left(-\frac{k}{2}\left(\frac{v_2}{N_0}\right)\right)\right]\right\} \quad . \tag{97}$$

B. BIT ERROR PROBABILITY

The bit error probability is given by (75)

$$P_e = \Pr(V_2 > V_1) = \int_{v_1=0}^{\infty}\left[\int_{v_2=v_1}^{\infty} f_{V_2}(v_2)dv_2\right] f_{V_1}(v_1)dv_1 \quad , \tag{98}$$

where

$$\int_{v_1}^{\infty} f_{V_2}(v_2)dv_2 = \int_{v_1}^{\infty}\frac{L(L-1)}{N_0}\exp\left(-\frac{v_2}{N_0}\right)\left\{\frac{v_2}{2N_0}+\sum_{k=1}^{L-2}\binom{L-2}{k}\frac{(-1)^k}{k}\left[1-\exp\left(-\frac{k}{2}\left(\frac{v_2}{N_0}\right)\right)\right]\right\}dv_2$$

$$= \int_{v_1}^{\infty} L(L-1)\exp\left(-\frac{v_2}{N_0}\right)\left\{\frac{v_2}{2N_0}+\sum_{k=1}^{L-2}\binom{L-2}{k}\frac{(-1)^k}{k}\left[1-\exp\left(-\frac{k}{2}\left(\frac{v_2}{N_0}\right)\right)\right]\right\}d\left(\frac{v_2}{N_0}\right) \quad . \tag{99}$$

By a change of variable $u = \frac{v_2}{N_0}, du = d\left(\frac{v_2}{N_0}\right)$,the limit of the integral changes to $\frac{v_1}{N_0}$ and (99) yields

$$\int_{v_1}^{\infty} f_{V_2}(v_2)dv_2 = L(L-1)\int_{v_1/N_0}^{\infty}\frac{u}{2}\exp(-u)du$$

$$+L(L-1)\int_{v_1/N_0}^{\infty}\exp(-u)\sum_{k=1}^{L-2}\binom{L-2}{k}\frac{(-1)^k}{k}\left[1-\exp\left(-\frac{k}{2}\right)u\right]du$$

$$= \frac{L(L-1)}{2}\int_{v_1/N_0}^{\infty} u\exp(-u)du$$

$$+L(L-1)\sum_{k=1}^{L-2}\binom{L-2}{k}\frac{(-1)^k}{k}\int_{v_1/N_0}^{\infty}\left[\exp(-u)-\exp\left(-\frac{k+2}{2}u\right)\right]du \quad . \tag{100}$$

Using [3]

$$\int_{w}^{\infty} x^n \exp(-\mu x)dx = \exp(-\mu w)\sum_{k=0}^{n} \frac{n!}{k!}\frac{w^k}{\mu^{n-k+1}}, \quad w>0, \quad \mathrm{Re}(\mu)>0 \tag{101}$$

and by letting $w = \frac{v_1}{N_0}$, $n = 1$, $\mu = 1$ the first integral of (100) results in

$$\frac{L(L-1)}{2}\int_{v_1/N_0}^{\infty} u\exp(-u)du = \frac{L(L-1)}{2}\exp\left(-\frac{v_1}{N_0}\right)\sum_{k=0}^{1}\frac{1}{k!}\left(\frac{v_1}{N_0}\right)^k$$

$$= \frac{L(L-1)}{2}\left(1+\frac{v_1}{N_0}\right)\exp\left(-\frac{v_1}{N_0}\right) \quad , \tag{102}$$

whereas the second integral of (100) results in

$$L(L-1)\sum_{k=1}^{L-2}\binom{L-2}{k}\frac{(-1)^k}{k}\int_{v_1/N_0}^{\infty}\left[\exp(-u)-\exp\left(-\frac{k+2}{2}u\right)\right]du$$

$$= L(L-1)\sum_{k=1}^{L-2}\binom{L-2}{k}\frac{(-1)^k}{k}\left[\exp\left(-\frac{v_1}{N_0}\right)-\frac{2}{2+k}\exp\left(-\frac{k+2}{2}\left(\frac{v_1}{N_0}\right)\right)\right] \quad . \tag{103}$$

Substituting (102) and (103) in (100) results in

$$\int_{v_1}^{\infty} f_{V_2}(v_2)dv_2 = L(L-1)\exp\left(-\frac{v_1}{N_0}\right)$$

$$\times\left\{\frac{1+\frac{v_1}{N_0}}{2}+\sum_{k=1}^{L-2}\binom{L-2}{k}\frac{(-1)^k}{k}\left[1-\frac{2}{2+k}\exp\left(-\frac{k}{2}\frac{v_1}{N_0}\right)\right]\right\} \quad . \tag{104}$$

Finally substituting (90) and (104) in (98) yields

$$P_e = \int_0^\infty L(L-1)\exp\left(-\frac{v_1}{N_0}\right)\left\{\frac{1+\frac{v_1}{N_0}}{2} + \sum_{k=1}^{L-2}\binom{L-2}{k}\frac{(-1)^k}{k}\left[1-\frac{2}{2+k}\exp\left(-\frac{k}{2}\frac{v_1}{N_0}\right)\right]\right\}$$

$$\times \frac{L(L-1)}{N_0}\left(\frac{m}{m+\overline{\overline{\gamma_c}}}\right)^{mL}\exp\left(-\frac{v_1}{N_0}\right)\int_0^{v_1/2}\left[\sum_{k_2=0}^{\infty}\left(\frac{v_{1,2}}{N_0}\right)^{k_2}\left(\frac{1}{k_2!}\right)^2\frac{\Gamma(k_2+m)}{\Gamma(m)}\left(\frac{\overline{\gamma_c}}{m+\overline{\overline{\gamma_c}}}\right)^{k_2}\right]$$

$$\times\left[\sum_{k_1=0}^{\infty}\left(\frac{v_1-v_{1,2}}{N_0}\right)^{k_1}\left(\frac{1}{k_1!}\right)^2\frac{\Gamma(k_1+m)}{\Gamma(m)}\left(\frac{\overline{\gamma_c}}{m+\overline{\overline{\gamma_c}}}\right)^{k_1}\right]$$

$$\times\left\{\sum_{k_3=0}^{\infty}\frac{1}{k_3!}\left(\frac{\overline{\gamma_c}}{m+\overline{\overline{\gamma_c}}}\right)^{k_3}\frac{\Gamma(k_3+m)}{\Gamma(m)}\left[1-\exp\left(-\frac{v_{1,2}}{N_0}\right)\sum_{\lambda=0}^{k_3}\frac{1}{\lambda!}\left(\frac{v_{1,2}}{N_0}\right)^{\lambda}\right]\right\}^{L-2} d\left(\frac{v_{1,2}}{N_0}\right)dv_1 \quad .(105)$$

By changing variables $x=\frac{v_{1,2}}{N_0}, dx = d\left(\frac{v_{1,2}}{N_0}\right), u=\frac{v_1}{N_0}, v_1 = uN_0$ and $dv_1 = N_0 du$, the limits of both integrals do not change and (105) results in the final expression for the bit error probability

$$P_e = L^2(L-1)^2\left(\frac{m}{m+\overline{\overline{\gamma_c}}}\right)^{mL}\int_0^\infty \exp(-2u)\left\{\frac{1+u}{2}+\sum_{k=1}^{L-2}\binom{L-2}{k}\frac{(-1)^k}{k}\left[1-\frac{2}{2+k}\exp\left(-\frac{k}{2}u\right)\right]\right\}$$

$$\times\int_0^{u/2}\left[\sum_{k_2=0}^{\infty}x^{k_2}\left(\frac{1}{k_2!}\right)^2\frac{\Gamma(k_2+m)}{\Gamma(m)}\left(\frac{\overline{\gamma_c}}{m+\overline{\overline{\gamma_c}}}\right)^{k_2}\right]\left[\sum_{k_1=0}^{\infty}(u-x)^{k_1}\left(\frac{1}{k_1!}\right)^2\frac{\Gamma(k_1+m)}{\Gamma(m)}\left(\frac{\overline{\gamma_c}}{m+\overline{\overline{\gamma_c}}}\right)^{k_1}\right]$$

$$\times\left\{\sum_{k_3=0}^{\infty}\frac{1}{k_3!}\left(\frac{\overline{\gamma_c}}{m+\overline{\overline{\gamma_c}}}\right)^{k_3}\frac{\Gamma(k_3+m)}{\Gamma(m)}\left[1-\exp(-x)\sum_{\lambda=0}^{k_3}\frac{1}{\lambda!}x^{\lambda}\right]\right\}^{L-2} dx\,du \quad . \quad (106)$$

Using the average signal-to-noise ratio per bit $\overline{\gamma_b} = L\overline{\gamma_c}$, (106) yields in

$$P_e = L^2(L-1)^2\left(\frac{m}{m+\frac{1}{L}\overline{\gamma_b}}\right)^{mL}\int_0^{\infty}\exp(-2u)\left\{\frac{1+u}{2}+\sum_{k=1}^{L-2}\binom{L-2}{k}\frac{(-1)^k}{k}\left[1-\frac{2}{2+k}\exp\left(-\frac{k}{2}u\right)\right]\right\}$$

$$\times\int_0^{u/2}\left[\left[\sum_{k_2=0}^{\infty}x^{k_2}\left(\frac{1}{k_2!}\right)^2\frac{\Gamma(k_2+m)}{\Gamma(m)}\left(\frac{\frac{1}{L}\overline{\gamma_b}}{m+\frac{1}{L}\overline{\gamma_b}}\right)^{k_2}\right]\left[\sum_{k_1=0}^{\infty}(u-x)^{k_1}\left(\frac{1}{k_1!}\right)^2\frac{\Gamma(k_1+m)}{\Gamma(m)}\left(\frac{\frac{1}{L}\overline{\gamma_b}}{m+\frac{1}{L}\overline{\gamma_b}}\right)^{k_1}\right]\right]$$

$$\times\left\{\sum_{k_3=0}^{\infty}\frac{1}{k_3!}\left(\frac{\frac{1}{L}\overline{\gamma_b}}{m+\frac{1}{L}\overline{\gamma_b}}\right)^{k_3}\frac{\Gamma(k_3+m)}{\Gamma(m)}\left[1-\exp(-x)\sum_{\lambda=0}^{k_3}\frac{1}{\lambda!}x^{\lambda}\right]\right\}^{L-2}dx\,du \quad . \tag{107}$$

This expression cannot be simplified any further. It can be evaluated numerically. The results of this evaluation are presented in Chapter VI.

V. THIRD ORDER POST-DETECTION SELECTION COMBINING (PDSC-3)

The three signals with the largest amplitudes of the signal branches are combined and compared to the combination of the three signals with the largest amplitudes of the non-signal branches. In particular, we let branch *1* be the signal branch and branch *2* be the non-signal branch. Without loss of generality let $V_{1,1}$ denote the largest output random variable, $V_{1,2}$ denote the second largest output random variable, and $V_{1,3}$ denote the third largest output random variable of the signal branches:

$$V_{1,1} = \max\left(V_{1,1}, V_{1,2}, V_{1,3} \cdots, V_{1,L}\right) \tag{108}$$

$$V_{1,2} = second \max\left(V_{1,1}, V_{1,2}, V_{1,3} \cdots, V_{1,L}\right) \tag{109}$$

$$V_{1,3} = third \max\left(V_{1,1}, V_{1,2}, V_{1,3} \cdots, V_{1,L}\right) \quad . \tag{110}$$

Let also $V_{2,1}$ denote the largest output random variable, $V_{2,2}$ denote the second largest output random variable, and $V_{2,3}$ denote the third largest output random variable of the non-signal branches:

$$V_{2,1} = \max\left(V_{2,1}, V_{2,2}, V_{2,3} \cdots, V_{2,L}\right) \tag{111}$$

$$V_{2,2} = second \max\left(V_{2,1}, V_{2,2}, V_{2,3} \cdots, V_{2,L}\right) \tag{112}$$

$$V_{2,3} = third \max\left(V_{2,1}, V_{2,2}, V_{2,3} \cdots, V_{2,L}\right) \quad . \tag{113}$$

The decision variables V_1 and V_2 are given by

$$V_1 = V_{1,1} + V_{1,2} + V_{1,3} \tag{114}$$

$$V_2 = V_{2,1} + V_{2,2} + V_{2,3} \quad . \tag{115}$$

The bit error probability is given by

$$P_e = \Pr(V_2 > V_1) = \int_{v_1=0}^{\infty}\left[\int_{v_2=v_1}^{\infty} f_{V_2}(v_2)dv_2\right]f_{V_1}(v_1)dv_1 \quad . \quad (116)$$

A. PROBABILITY DENSITY FUNCTIONS OF THE DECISION VARIABLES

The non-coherent BFSK receiver is shown in Fig.5. For the signal branches, the output random variable from the i^{th} square law detector $V_{1,i}$ prior to combining is given by (42), whereas, for the non- signal branches the output from the i^{th} square law detector $V_{2,i}$ prior to combining is given by (43).

In [8], the cumulative distribution function of the decision random variable $V_1=V_{1,1}+V_{1,2}+V_{1,3}$, defined in (114) is given by

$$F_{V_1}(v_1) = \int_{0}^{v_1/3}\int_{v_{1,3}}^{(v_1-v_{1,3})/2}\int_{v_{1,2}}^{v_1-v_{1,2}-v_{1,3}} f_{V_{1,1}V_{1,2}V_{1,3}}(v_{1,1}v_{1,2}v_{1,3})dv_{1,1}\,dv_{1,2}\,dv_{1,3} \quad , \quad (117)$$

where $f_{V_{1,1}V_{1,2}V_{1,3}}(v_{1,1}v_{1,2}v_{1,3})$ is the joint probability density function of the random variables $V_{1,1}$, $V_{1,2}$ and $V_{1,3}$ which is given by [8]

$$f_{V_{1,1}V_{1,2}V_{1,3}}(v_{1,1}v_{1,2}v_{1,3}) = L(L-1)(L-2)f_{V_{1,1}}(v_{1,1})f_{V_{1,2}}(v_{1,2})f_{V_{1,3}}(v_{1,3})F_{V_{1,3}}^{L-3}(v_{1,3}) \quad , (118)$$

where $f_{V_{1,1}}(v_{1,1})$, $f_{V_{1,2}}(v_{1,2})$ and $f_{V_{1,3}}(v_{1,3})$ are given by (50) and $F_{V_{1,3}}(v_{1,3})$ is given by (55). Substituting (118) into (117) becomes

$$F_{V_1}(v_1)=\int_0^{v_1/3}\int_{v_{1,3}}^{(v_1-v_{1,3})/2}\int_{v_{1,2}}^{v_1-v_{1,2}-v_{1,3}} L(L-1)(L-2)\,f_{V_{1,1}}(v_{1,1})f_{V_{1,2}}(v_{1,2})f_{V_{1,3}}(v_{1,3})F_{V_{1,3}}^{L-3}(v_{1,3})\,dv_{1,1}\,dv_{1,2}\,dv_{1,3} \quad . \tag{119}$$

We can obtain the probability density function of the decision random variable for the signal branches V_1, by differentiating (119) with respect to v_1

$$f_{V_1}(v_1)=\frac{d}{dv_1}F_{V_1}(v_1) \quad . \tag{120}$$

Using Leibnitz's rule

$$\frac{d\left[\int_{a(x)}^{b(x)} f(\lambda,x)\,d\lambda\right]}{dx}=f(b(x),x)\frac{db(x)}{dx}-f(a(x),x)\frac{da(x)}{dx}+\int_{a(x)}^{b(x)}\frac{df(\lambda,x)}{dx}d\lambda \quad , \tag{121}$$

and by letting $x = v_1$, $\lambda = v_{1,3}$, $a(x) = 0$, $b(x) = v_1/3$, and

$$f(\lambda,x)=f(v_{1,3},v_1)=\int_{v_{1,3}}^{(v_1-v_{1,3})/2}\int_{v_{1,2}}^{v_1-v_{1,2}-v_{1,3}} L(L-1)(L-2)\,f_{V_{1,1}}(v_{1,1})f_{V_{1,2}}(v_{1,2})f_{V_{1,3}}(v_{1,3})F_{V_{1,3}}^{L-3}(v_{1,3})\,dv_{1,1}\,dv_{1,2} \tag{122}$$

eq. (121) becomes

$$f_{V_1}(v_1)=f(v_1/3,v_1)\frac{d\left(\frac{v_1}{3}\right)}{dv_1}-f(0,v_1)\frac{d(0)}{dv_1}+\int_0^{v_1/3}\frac{df(v_{1,3},v_1)}{dv_1}dv_{1,3} \quad . \tag{123}$$

Now

$$f(v_1/3,v_1)=\int_{v_1/3}^{v_1/3}\int_{v_{1,2}}^{v_1-v_{1,2}-v_1/3} L(L-1)(L-2)f_{V_{1,1}}(v_{1,1})f_{V_{1,2}}(v_{1,2})f_{V_{1,3}}\left(\frac{v_1}{3}\right)F_{V_{1,3}}^{L-3}\left(\frac{v_1}{3}\right)dv_{1,1}\,dv_{1,2}=0 \tag{124}$$

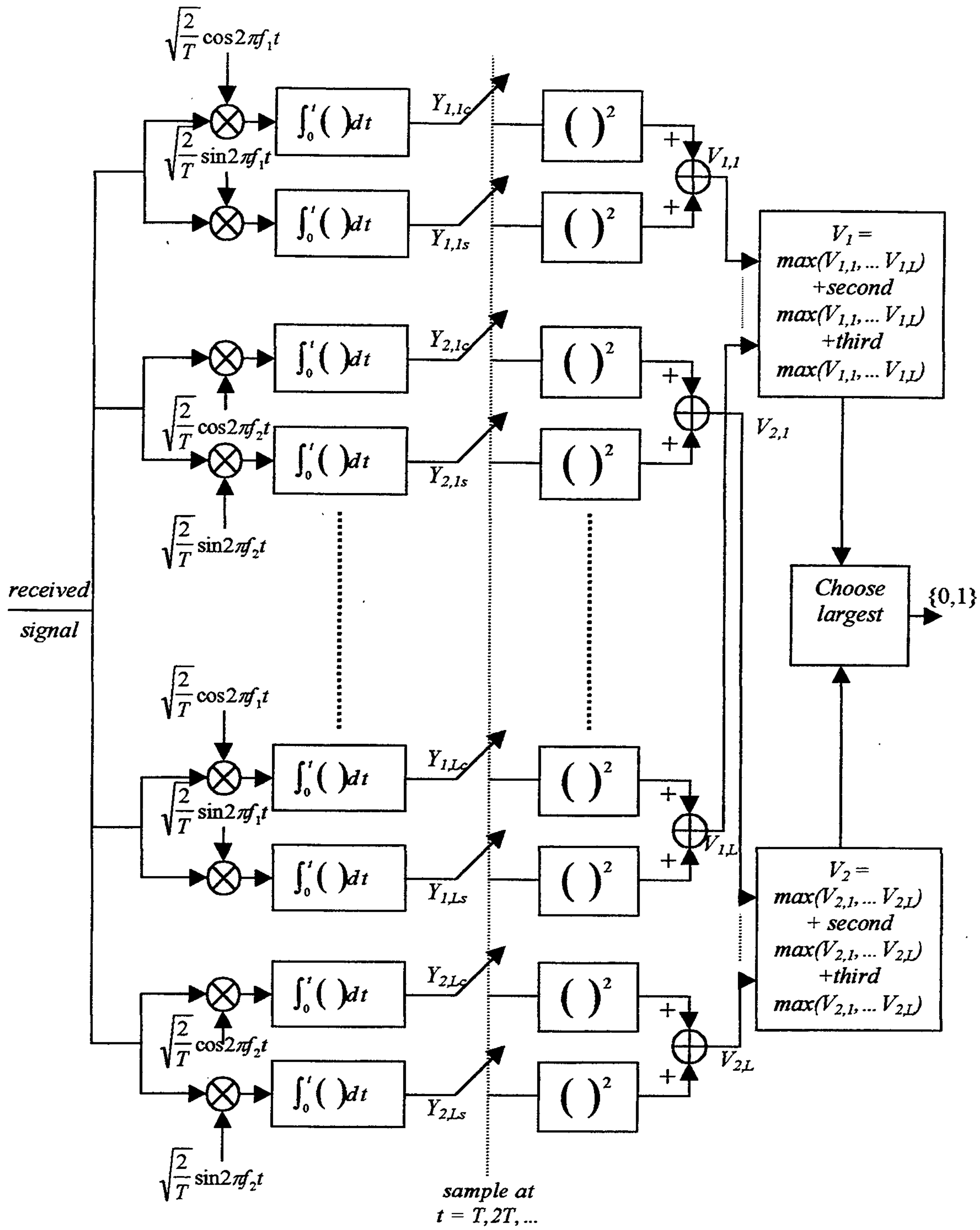

Figure 5. Non-coherent BFSK receiver with third order post detection selection combining

and

$$\frac{d(a(v_1))}{dv_1} = \frac{d(0)}{dv_1} = 0 \qquad , \qquad (125)$$

so (123) results in

$$f_{V_1}(v_1) = \int_0^{v_1/3} \frac{df(v_{1,3}, v_1)}{dv_1} dv_{1,3}$$

$$= \int_0^{v_1/3} \left\{ \frac{d\left[\int_{v_{1,3}}^{(v_1-v_{1,3})/2} \int_{v_{1,2}}^{v_1-v_{1,2}-v_{1,3}} L(L-1)(L-2) f_{V_{1,1}}(v_{1,1}) f_{V_{1,2}}(v_{1,2}) f_{V_{1,3}}(v_{1,3}) F_{V_{1,3}}^{L-3}(v_{1,3}) dv_{1,1}\, dv_{1,2} \right]}{dv_1} \right\} dv_{1,3} \ .(126)$$

Again by using (121) and letting $x = v_1$, $\lambda = v_{1,2}$, $\alpha(x) = v_{1,3}$, $b(x) = (v_1 - v_{1,3})/2$, and

$$f(\lambda, x) = f(v_{1,2}, v_1) = \int_{v_{1,2}}^{v_1-v_{1,2}-v_{1,3}} L(L-1)(L-2) f_{V_{1,1}}(v_{1,1}) f_{V_{1,2}}(v_{1,2}) f_{V_{1,3}}(v_{1,3}) F_{V_{1,3}}^{L-3}(v_{1,3}) dv_{1,1} \quad ,(127)$$

eq. (126) becomes

$$f_{V_1}(v_1) = \int_0^{v_1/3} \left\{ \frac{d\left[\int_{v_{1,3}}^{(v_1-v_{1,3})/2} \int_{v_{1,2}}^{v_1-v_{1,2}-v_{1,3}} L(L-1)(L-2) f_{V_{1,1}}(v_{1,1}) f_{V_{1,2}}(v_{1,2}) f_{V_{1,3}}(v_{1,3}) F_{V_{1,3}}^{L-3}(v_{1,3}) dv_{1,1}\, dv_{1,2} \right]}{dv_1} \right\} dv_{1,3}$$

$$= \int_0^{v_1/3} \left[f\left(\frac{v_1 - v_{1,3}}{2}, v_1 \right) \frac{d\left(\frac{v_1 - v_{1,3}}{2} \right)}{dv_1} - f(v_{1,3}, v_1) \frac{d(v_{1,3})}{dv_1} + \int_{v_{1,3}}^{(v_1-v_{1,3})/2} \frac{d[f(v_{1,2}, v_1)]}{dv_1} dv_{1,2} \right] dv_{1,3} \ .(128)$$

Now

$$f\left(\frac{v_1 - v_{1,3}}{2}, v_1\right) = \int_{(v_1-v_{1,3})/2}^{(v_1-v_{1,3})/2} L(L-1)(L-2) f_{V_{1,1}}(v_{1,1}) f_{V_{1,2}}(v_{1,2}) f_{V_{1,3}}(v_{1,3}) F_{V_{1,3}}^{L-3}(v_{1,3}) dv_{1,1} = 0 \quad (129)$$

and

$$\frac{d(v_{1,3})}{dv_1} = 0 \qquad , \qquad (130)$$

so (128) becomes

$$f_{V_1}(v_1) = \int_0^{v_1/3} \int_{v_{1,3}}^{(v_1-v_{1,3})/2} \frac{d\left[\int_{v_{1,2}}^{v_1-v_{1,2}-v_{1,3}} L(L-1)(L-2) f_{V_{1,1}}(v_{1,1}) f_{V_{1,2}}(v_{1,2}) f_{V_{1,3}}(v_{1,3}) F_{V_{1,3}}^{L-3}(v_{1,3}) dv_{1,1}\right]}{dv_1} dv_{1,2}\, dv_{1,3} \quad .(131)$$

Using again (121), and letting $x = v_1$, $\lambda = v_{1,1}$, $\alpha(x) = v_{1,2}$, $b(x) = v_1 - v_{1,2} - v_{1,3}$, and

$$f(\lambda, x) = f(v_{1,1}, v_1) = L(L-1)(L-2) f_{V_{1,1}}(v_{1,1}) f_{V_{1,2}}(v_{1,2}) f_{V_{1,3}}(v_{1,3}) F_{V_{1,3}}^{L-3}(v_{1,3}) = f(v_{1,1}) \quad ,(132)$$

eq. (131) results in

$$f_{V_1}(v_1) = \int_0^{v_1/3} \int_{v_{1,3}}^{(v_1-v_{1,3})/2} \frac{d\left[\int_{v_{1,2}}^{v_1-v_{1,2}-v_{1,3}} L(L-1)(L-2) f_{V_{1,1}}(v_{1,1}) f_{V_{1,2}}(v_{1,2}) f_{V_{1,3}}(v_{1,3}) F_{V_{1,3}}^{L-3}(v_{1,3}) dv_{1,1}\right]}{dv_1} dv_{1,2}\, dv_{1,3}$$

$$= \int_0^{v_1/3} \int_{v_{1,3}}^{(v_1-v_{1,3})/2} \left[f(v_1 - v_{1,2} - v_{1,3}, v_1) \frac{d(v_1 - v_{1,2} - v_{1,3})}{dv_1} - f(v_{1,2}, v_1) \frac{d(v_{1,2})}{dv_1} + \right.$$

$$\left. + \int_{v_{1,2}}^{v_1-v_{1,2}-v_{1,3}} \frac{d[f(v_{1,1}, v_1)]}{dv_1} dv_{1,1} \right] dv_{1,2} dv_{1,3} \quad . \qquad (133)$$

The second and the third terms of the summation inside the brackets in (133) are equal to zero, therefore (133) becomes

$$f_{V_1}(v_1)=\int_0^{v_1/3}\int_{v_{1,3}}^{(v_1-v_{1,3})/2}\left[f(v_1-v_{1,2}-v_{1,3},v_1)\frac{d(v_1-v_{1,2}-v_{1,3})}{dv_1}\right]dv_{1,2}\,dv_{1,3}$$

$$=\int_0^{v_1/3}\int_{v_{1,3}}^{(v_1-v_{1,3})/2}f(v_1-v_{1,2}-v_{1,3},v_1)dv_{1,2}\,dv_{1,3}\quad . \tag{134}$$

If we substitute (132) into (134) we obtain

$$f_{V_1}(v_1)=\int_0^{v_1/3}\int_{v_{1,3}}^{(v_1-v_{1,3})/2}L(L-1)(L-2)f_{V_{1,1}}(v_1-v_{1,2}-v_{1,3})f_{V_{1,2}}(v_{1,2})f_{V_{1,3}}(v_{1,3})F_{V_{1,3}}^{L-3}(v_{1,3})dv_{1,2}\,dv_{1,3}\ . \tag{135}$$

Finally if we substitute (50) and (55) into (135) we get the expression for the probability density function of the decision random variable in third order post detection selection combining

$$f_{V_1}(v_1)=\int_0^{v_1/3}\int_{v_{1,3}}^{(v_1-v_{1,3})/2}L(L-1)(L-2)$$

$$\times\left[\frac{1}{N_0}\exp\left(-\frac{v_1-v_{1,2}-v_{1,3}}{N_0}\right)\left(\frac{m}{m+\overline{\gamma_c}}\right)^m\sum_{k_1=0}^{\infty}\left(\frac{1}{k_1!}\right)^2\left(\frac{v_1-v_{1,2}-v_{1,3}}{N_0}\right)^{k_1}\frac{\Gamma(k_1+m)}{\Gamma(m)}\left(\frac{\overline{\gamma_c}}{m+\overline{\gamma_c}}\right)^{k_1}\right]$$

$$\times\left[\frac{1}{N_0}\exp\left(-\frac{v_{1,2}}{N_0}\right)\left(\frac{m}{m+\overline{\gamma_c}}\right)^m\sum_{k_2=0}^{\infty}\left(\frac{1}{k_2!}\right)^2\left(\frac{v_{1,2}}{N_0}\right)^{k_2}\frac{\Gamma(k_2+m)}{\Gamma(m)}\left(\frac{\overline{\gamma_c}}{m+\overline{\gamma_c}}\right)^{k_2}\right]$$

$$\times\left[\frac{1}{N_0}\exp\left(-\frac{v_{1,3}}{N_0}\right)\left(\frac{m}{m+\overline{\gamma_c}}\right)^m\sum_{k_3=0}^{\infty}\left(\frac{1}{k_3!}\right)^2\left(\frac{v_{1,3}}{N_0}\right)^{k_3}\frac{\Gamma(k_3+m)}{\Gamma(m)}\left(\frac{\overline{\gamma_c}}{m+\overline{\gamma_c}}\right)^{k_3}\right]$$

$$\times\left\{\left(\frac{m}{m+\overline{\gamma_c}}\right)^m\sum_{k_4=0}^{\infty}\frac{1}{k_4!}\left(\frac{\overline{\gamma_c}}{m+\overline{\gamma_c}}\right)^{k_4}\frac{\Gamma(k_4+m)}{\Gamma(m)}\left[1-\exp\left(-\frac{v_{1,3}}{N_0}\right)\sum_{\lambda=0}^{k_4}\frac{1}{\lambda!}\left(\frac{v_{1,3}}{N_0}\right)^{\lambda}\right]\right\}^{L-3}dv_{1,2}dv_{1,3}$$

$$=\frac{L(L-1)(L-2)}{N_0}\left(\frac{m}{m+\overline{\gamma_c}}\right)^{mL}\exp\left(-\frac{v_1}{N_0}\right)$$

$$\times\int_0^{v_1/3}\int_{v_{1,3}}^{(v_1-v_{1,3})/2}\left[\sum_{k_1=0}^{\infty}\left(\frac{v_1-v_{1,2}-v_{1,3}}{N_0}\right)^{k_1}\left(\frac{1}{k_1!}\right)^2\frac{\Gamma(k_1+m)}{\Gamma(m)}\left(\frac{\overline{\gamma_c}}{m+\overline{\gamma_c}}\right)^{k_1}\right]$$

$$\times\left[\sum_{k_2=0}^{\infty}\left(\frac{v_{1,2}}{N_0}\right)^{k_2}\left(\frac{1}{k_2!}\right)^2\frac{\Gamma(k_2+m)}{\Gamma(m)}\left(\frac{\overline{\gamma_c}}{m+\overline{\gamma_c}}\right)^{k_2}\right]$$

$$\times\left[\sum_{k_3=0}^{\infty}\left(\frac{v_{1,3}}{N_0}\right)^{k_3}\left(\frac{1}{k_3!}\right)^2\frac{\Gamma(k_3+m)}{\Gamma(m)}\left(\frac{\overline{\gamma_c}}{m+\overline{\gamma_c}}\right)^{k_3}\right].$$

$$\times\left\{\sum_{k_4=0}^{\infty}\frac{1}{k_4!}\left(\frac{\overline{\gamma_c}}{m+\overline{\gamma_c}}\right)^{k_4}\frac{\Gamma(k_4+m)}{\Gamma(m)}\left[1-\exp\left(-\frac{v_{1,3}}{N_0}\right)\sum_{\lambda=0}^{k_4}\frac{1}{\lambda!}\left(\frac{v_{1,3}}{N_0}\right)^{\lambda}\right]\right\}^{L-3}d\left(\frac{v_{1,2}}{N_0}\right)d\left(\frac{v_{1,3}}{N_0}\right)\quad. \tag{136}$$

For the non-signal branches the output random variable from the i^{th} square law detector $V_{2,i}$ prior to combining is given by (58). The cumulative distribution function of the decision variable V_2 defined in (115) is given by

$$F_{V_2}(v_2)=\int_0^{v_2/3}\int_{v_{2,3}}^{(v_2-v_{2,3})/2}\int_{v_{2,2}}^{v_2-v_{2,2}-v_{2,3}}f_{V_{2,1}V_{2,2}V_{2,3}}(v_{2,1}v_{2,2}v_{2,3})dv_{2,1}\,dv_{2,2}\,dv_{2,3}\quad, \tag{137}$$

where $f_{V_{2,1}V_{2,2}V_{2,3}}(v_{2,1}v_{2,2}v_{2,3})$ is the joint probability density function of the random variables $V_{2,1}$, $V_{2,2}$ and $V_{2,3}$, which is given by [8]

$$f_{V_{2,1}V_{2,2}V_{2,3}}(v_{2,1}v_{2,2}v_{2,3})=L(L-1)(L-2)f_{V_{2,1}}(v_{2,1})f_{V_{2,2}}(v_{2,2})f_{V_{2,3}}(v_{2,3})F_{V_{2,3}}^{L-3}(v_{2,3})\,, \tag{138}$$

where $f_{V_{2,1}}(v_{2,1})$, $f_{V_{2,2}}(v_{2,2})$ and $f_{V_{2,3}}(v_{2,3})$ are given by (58) and $F_{V_{2,3}}(v_{2,3})$ is given by (59). Substituting (138) into (137) results in

$$F_{V_2}(v_2)=\int_0^{v_2/3}\int_{v_{2,3}}^{(v_2-v_{2,3})/2}\int_{v_{2,2}}^{v_2-v_{2,2}-v_{2,3}} L(L-1)(L-2) f_{V_{2,1}}(v_{2,1}) f_{V_{2,2}}(v_{2,2}) f_{V_{2,3}}(v_{2,3}) F_{V_{2,3}}^{L-3}(v_{2,3})\, dv_{2,1}\, dv_{2,2}\, dv_{2,3} \,. \quad (139)$$

Substituting (58) and (59) into (139) we obtain

$$F_{V_2}(v_2)=\int_0^{v_2/3}\int_{v_{2,3}}^{(v_2-v_{2,3})/2}\int_{v_{2,2}}^{v_2-v_{2,2}-v_{2,3}} \frac{L(L-1)(L-2)}{N_0^3}\exp\left(-\frac{v_{2,1}}{N_0}\right)\exp\left(-\frac{v_{2,2}}{N_0}\right)$$

$$\times\left[1-\exp\left(-\frac{v_{2,3}}{N_0}\right)\right]^{L-3}\exp\left(-\frac{v_{2,3}}{N_0}\right) dv_{2,1}\, dv_{2,2}\, dv_{2,3} \quad . \quad (140)$$

Performing the integration and differentiating the result with respect to v_2, the probability density function of the decision variable for the non-signal branches is given by [8]

$$f_{V_2}(v_2)=\frac{L(L-1)(L-2)}{2N_0}\exp\left(-\frac{v_2}{N_0}\right)\left\{\frac{v_2^2}{6N_0^2}+\sum_{k=1}^{L-3}\binom{L-3}{k}\frac{(-1)^k}{k^2}\left[\frac{kv_2}{N_0}-3\left(1-\exp\left(-\frac{kv_2}{3N_0}\right)\right)\right]\right\} \,. \quad (141)$$

B. BIT ERROR PROBABILITY

The bit error probability is given by (116)

$$P_e=\Pr(V_2>V_1)=\int_{v_1=0}^{\infty}\left[\int_{v_2=v_1}^{\infty} f_{V_2}(v_2)\,dv_2\right] f_{V_1}(v_1)\,dv_1 \quad , \quad (142)$$

where

$$\int_{v_1}^{\infty} f_{V_2}(v_2)dv_2 = \int_{v_1}^{\infty} \frac{L(L-1)(L-2)}{2N_0} \exp\left(-\frac{v_2}{N_0}\right)$$

$$\times \left\{ \frac{v_2^2}{6N_0^2} + \sum_{k=1}^{L-3} \binom{L-3}{k} \frac{(-1)^k}{k^2} \left[\frac{kv_2}{N_0} - 3\left(1 - \exp\left(-\frac{kv_2}{3N_0}\right)\right)\right]\right\} dv_2$$

$$= \int_{v_1}^{\infty} \frac{L(L-1)(L-2)}{2} \exp\left(-\frac{v_2}{N_0}\right)$$

$$\times \left\{ \frac{v_2^2}{6N_0^2} + \sum_{k=1}^{L-3} \binom{L-3}{k} \frac{(-1)^k}{k^2} \left[\frac{kv_2}{N_0} - 3\left(1 - \exp\left(-\frac{kv_2}{3N_0}\right)\right)\right]\right\} d\left(\frac{v_2}{N_0}\right) \quad . \tag{143}$$

Changing variable $u = \frac{v_2}{N_0}$, $du = d\left(\frac{v_2}{N_0}\right)$, (143) becomes

$$\int_{v_1}^{\infty} f_{V_2}(v_2)dv_2 = \frac{L(L-1)(L-2)}{2} \int_{v_1/N_0}^{\infty} \exp(-u)$$

$$\times \left\{ \frac{u^2}{6} + \sum_{k=1}^{L-3} \binom{L-3}{k} \frac{(-1)^k}{k^2} \left[ku - 3\left(1 - \exp\left(-\frac{k}{3}u\right)\right)\right]\right\} du \quad . \tag{144}$$

Finally substituting (136) and (144) into (142) yields

$$P_e = \int_0^{\infty}\left\{\frac{L(L-1)(L-2)}{2}\int_{v_1/N_0}^{\infty}\exp(-u)\left\{\frac{u^2}{6}+\sum_{k=1}^{L-3}\binom{L-3}{k}\frac{(-1)^k}{k^2}\left[ku-3\left(1-\exp\left(-\frac{k}{3}u\right)\right)\right]\right\}du\right\}$$

$$\times\frac{L(L-1)(L-2)}{N_0}\left(\frac{m}{m+\overline{\gamma_c}}\right)^{mL}\exp\left(-\frac{v_1}{N_0}\right)$$

$$\times\int_0^{v_1/3}\int_{v_{1,3}}^{(v_1-v_{1,3})/2}\left[\sum_{k_1=0}^{\infty}\left(\frac{v_1-v_{1,2}-v_{1,3}}{N_0}\right)^{k_1}\left(\frac{1}{k_1!}\right)^2\frac{\Gamma(k_1+m)}{\Gamma(m)}\left(\frac{\overline{\gamma_c}}{m+\overline{\gamma_c}}\right)^{k_1}\right]$$

$$\times\left[\sum_{k_2=0}^{\infty}\left(\frac{v_{1,2}}{N_0}\right)^{k_2}\left(\frac{1}{k_2!}\right)^2\frac{\Gamma(k_2+m)}{\Gamma(m)}\left(\frac{\overline{\gamma_c}}{m+\overline{\gamma_c}}\right)^{k_2}\right]$$

$$\times\left[\sum_{k_3=0}^{\infty}\left(\frac{v_{1,3}}{N_0}\right)^{k_3}\left(\frac{1}{k_3!}\right)^2\frac{\Gamma(k_3+m)}{\Gamma(m)}\left(\frac{\overline{\gamma_c}}{m+\overline{\gamma_c}}\right)^{k_3}\right]$$

$$\times\left\{\sum_{k_4=0}^{\infty}\frac{1}{k_4!}\left(\frac{\overline{\gamma_c}}{m+\overline{\gamma_c}}\right)^{k_4}\frac{\Gamma(k_4+m)}{\Gamma(m)}\left[1-\exp\left(-\frac{v_{1,3}}{N_0}\right)\sum_{\lambda=0}^{k_4}\frac{1}{\lambda!}\left(\frac{v_{1,3}}{N_0}\right)^{\lambda}\right]\right\}^{L-3} d\left(\frac{v_{1,2}}{N_0}\right)d\left(\frac{v_{1,3}}{N_0}\right)dv_1. \quad (145)$$

Changing variables $x=\frac{v_{1,2}}{N_0}$, $dx=d\left(\frac{v_{1,2}}{N_0}\right)$, $w=\frac{v_{1,3}}{N_0}$, $dw=d\left(\frac{v_{1,3}}{N_0}\right)$, $z=\frac{v_1}{N_0}$, $v_1=zN_0$ and $dv_1=N_0dz$, (145) results in the final expression for the bit error probability

$$P_e = \frac{L^2(L-1)^2(L-2)^2}{2}\left(\frac{m}{m+\overline{\gamma_c}}\right)^{mL}\int_0^{\infty}\left\{\int_z^{\infty}\exp(-u)\right.$$

$$\times\left\{\frac{u^2}{6}+\sum_{k=1}^{L-3}\binom{L-3}{k}\frac{(-1)^k}{k^2}\left[ku-3\left(1-\exp\left(-\frac{k}{3}u\right)\right)\right]\right\}du\Bigg\}$$

$$\times\exp(-z)\int_0^{z/3}\int_w^{(z-w)/2}\left[\sum_{k_1=0}^{\infty}(z-x-w)^{k_1}\left(\frac{1}{k_1!}\right)^2\frac{\Gamma(k_1+m)}{\Gamma(m)}\left(\frac{\overline{\gamma_c}}{m+\overline{\gamma_c}}\right)^{k_1}\right]$$

$$\times\left[\sum_{k_2=0}^{\infty}x^{k_2}\left(\frac{1}{k_2!}\right)^2\frac{\Gamma(k_2+m)}{\Gamma(m)}\left(\frac{\overline{\gamma_c}}{m+\overline{\gamma_c}}\right)^{k_2}\right]\left[\sum_{k_3=0}^{\infty}w^{k_3}\left(\frac{1}{k_3!}\right)^3\frac{\Gamma(k_3+m)}{\Gamma(m)}\left(\frac{\overline{\gamma_c}}{m+\overline{\gamma_c}}\right)^{k_3}\right]$$

$$\times\left\{\sum_{k_4=0}^{\infty}\frac{1}{k_4!}\left(\frac{\overline{\gamma_c}}{m+\overline{\gamma_c}}\right)^{k_4}\frac{\Gamma(k_4+m)}{\Gamma(m)}\left[1-\exp(-w)\sum_{\lambda=0}^{k_4}\frac{w^{\lambda}}{\lambda!}\right]\right\}^{L-3}dx\,dw\,dz \quad . \qquad (146)$$

Using the average signal-to-noise ratio per bit $\overline{\gamma_b} = L\overline{\gamma_c}$, (146) yields

$$
\begin{aligned}
P_e = {} & \frac{L^2(L-1)^2(L-2)^2}{2}\left(\frac{m}{m+\frac{1}{L}\bar{\gamma}_b}\right)^{mL}\int_0^{\infty}\left\{\int_z^{\infty}\exp(-u)\right. \\
& \times\left\{\frac{u^2}{6}+\sum_{k=1}^{L-3}\binom{L-3}{k}\frac{(-1)^k}{k^2}\left[ku-3\left(1-\exp\left(-\frac{k}{3}u\right)\right)\right]\right\}du\Bigg\} \\
& \times\exp(-z)\int_0^{z/3}\int_w^{(z-w)/2}\left[\sum_{k_1=0}^{\infty}(z-x-w)^{k_1}\left(\frac{1}{k_1!}\right)^2\frac{\Gamma(k_1+m)}{\Gamma(m)}\left(\frac{\frac{1}{L}\bar{\gamma}_b}{m+\frac{1}{L}\bar{\gamma}_b}\right)^{k_1}\right] \\
& \times\left[\sum_{k_2=0}^{\infty}x^{k_2}\left(\frac{1}{k_2!}\right)^2\frac{\Gamma(k_2+m)}{\Gamma(m)}\left(\frac{\frac{1}{L}\bar{\gamma}_b}{m+\frac{1}{L}\bar{\gamma}_b}\right)^{k_2}\right]\left[\sum_{k_3=0}^{\infty}w^{k_3}\left(\frac{1}{k_3!}\right)^3\frac{\Gamma(k_3+m)}{\Gamma(m)}\left(\frac{\frac{1}{L}\bar{\gamma}_b}{m+\frac{1}{L}\bar{\gamma}_b}\right)^{k_3}\right] \\
& \times\left\{\sum_{k_4=0}^{\infty}\frac{1}{k_4!}\left(\frac{\frac{1}{L}\bar{\gamma}_b}{m+\frac{1}{L}\bar{\gamma}_b}\right)^{k_4}\frac{\Gamma(k_4+m)}{\Gamma(m)}\left[1-\exp(-w)\sum_{\lambda=0}^{k_4}\frac{w^{\lambda}}{\lambda!}\right]\right\}^{L-3} dx\,dw\,dz \quad . \qquad (147)
\end{aligned}
$$

This expression cannot be simplified any further. It should be evaluated numerically. The results of this evaluation are presented in the following chapter.

VI. NUMERICAL RESULTS

In Chapters II, III, IV and V the bit error rate (BER) expressions for non-coherent BFSK signals operating in a frequency non-selective, slowly fading Nakagami channel, are analytically obtained. This is done for EGC, PDSC-1, PDSC-2 and PDSC-3. The main objective of this thesis is to compare the performance of BFSK signals using EGC and PDSC techniques. The numerical analysis and evaluation of the bit error rate expressions is performed using Mathcad 7 [7], and Matlab 5.1 [6]. The results are shown in figures 6 – 43. The average bit energy-to-noise ratio per bit is chosen to be in the range of 6 – 20 dB. All results are expressed in terms of the parameter m of the Nakagami distribution and the number of diversity order L. For $m = 1$, the channel becomes a Rayleigh fading channel, and as m approaches infinity the channel becomes non-fading. An $m = ½$ results in the one-sided Gaussian fading distribution [1]. Hence, in numerical evaluations, values of $m = 0.5, 0.75, 1, 1.5, 2$ and 3 should provide sufficient detail in order to illustrate the performance differences.

In Figs 6 –11, the performance of the receiver with EGC is demonstrated for $m = 0.5, 0.75, 1, 1.5, 2$ and 3, respectively. In each figure, the diversity order L ranges from 1 to 5.

In Figs 12 –17, the performance of the receiver with PDSC-1 is demonstrated for $m = 0.5, 0.75, 1, 1.5, 2$ and 3, respectively. Again in each figure the diversity order L ranges from 1 to 5.

In Figs 18 –23, the performance of the receiver, for PDSC-2, is shown for m = *0.5, 0.75, 1, 1.5, 2* and *3*, respectively. In each figure the diversity order L ranges from *2* to *5*.

In Figs 24 –29, the performance of the receiver, for PDSC-3, is illustrated for m = *0.5, 0.75, 1, 1.5, 2* and *3*, respectively. The diversity order L ranges from *3* to *5*.

In Figs 30 – 37, the performance of the receiver with EGC and PDSC techniques is illustrated, for different values of the diversity order L. In each figure the parameter m of the Nakagami distribution has values of *0.5, 0.75, 1, 1.5, 2* and *3*.

In Figs 38 – 43, the performance of the non-coherent BFSK receiver with EGC is compared to its performance with PDSC, for diversity order $L = 5$ and $m = 0.5, 0.75, 1, 1.5, 2$ and *3*.

In Figs 6 – 11, the performance of the receiver with EGC is demonstrated for m = *0.5, 0.75, 1, 1.5, 2* and *3* respectively. For $m = 0.5$, which means that the channel fading is one-sided Gaussian, as L increases the receiver performance improves. The same applies for all values of m. However, as m increases, which means that the channel becomes non-fading, for small values of the average bit energy-to-noise density ratio, the system performance with smaller diversity order L is observed to be superior to those with larger diversity order L. This trend reverses as the average bit energy-to-noise density ratio increases. This phenomenon is due to the non-coherent combining loss and is present in all four techniques analyzed in this thesis. The performance improvement effect of increasing L is noticeable after a certain bit energy-to-noise density ratio, especially because the noise in each diversity channel is more dominant at low energy-to-noise density ratios, and becomes less significant as the ratio increases. Therefore each diversity branch contributes positively to the overall SNR resulting in a better performance for systems with larger diversity order L.

Figs 12 – 29 show the receiver performance for PDSC for $m = 0.5, 0.75, 1, 1.5, 2,$ and 3. As with EGC, as L increases, performance improvement is observed.

In Figs 30 – 37 the performances of the four techniques are demonstrated for different values of the diversity order L and for different values of the Nakagami parameter m. As expected, as m increases, for the same value of L, the performance of the receiver is improved, because the channel becomes less fading.

In Figs 38 – 43 the techniques analyzed in this thesis are compared to each other for the same values of L and m. Note that in all cases the performance with the EGC technique is superior to the others. PDSC-1 performs worse in all cases. When the order of the PDSC technique is increased (from PDSC-1 to PDSC-3), the performance is improved. The difference in the performance between the different techniques is more obvious as m becomes larger, whereas when m is small the performances of all techniques are similar, with the EGC technique always being the superior one and the PDSC-1 always being the inferior one. Finally there is no difference in the performances between the EGC and PDSC-1 for $L = 1$. The same applies for the EGC and PDSC-2 for $L = 2$ as well as for the EGC and PDSC-3 for $L = 3$. This happens because for these values of L the EGC and PDSC techniques are virtually the same technique.

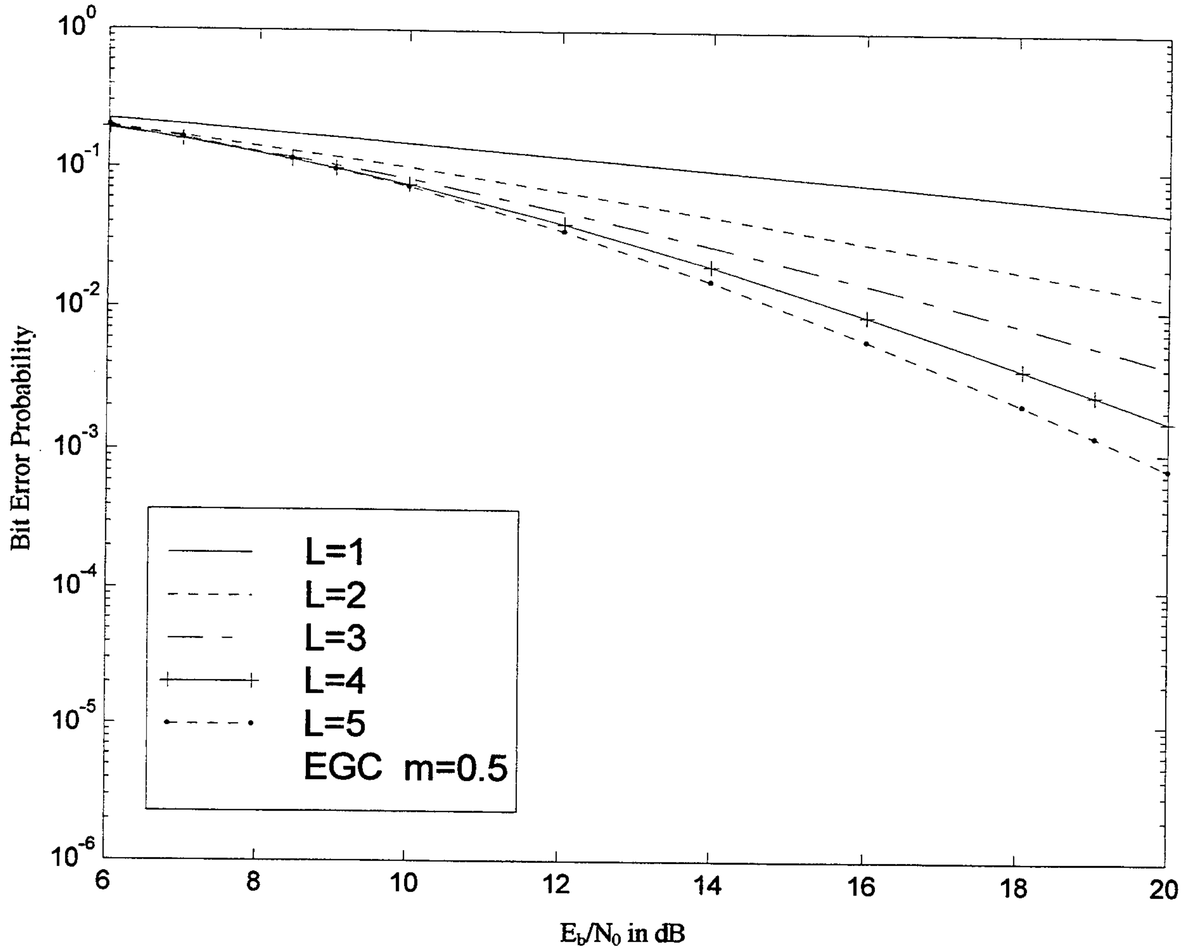

Figure 6. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 0.5$, using equal gain combining (EGC) for diversity orders $L = 1,2,3,4$ and 5.

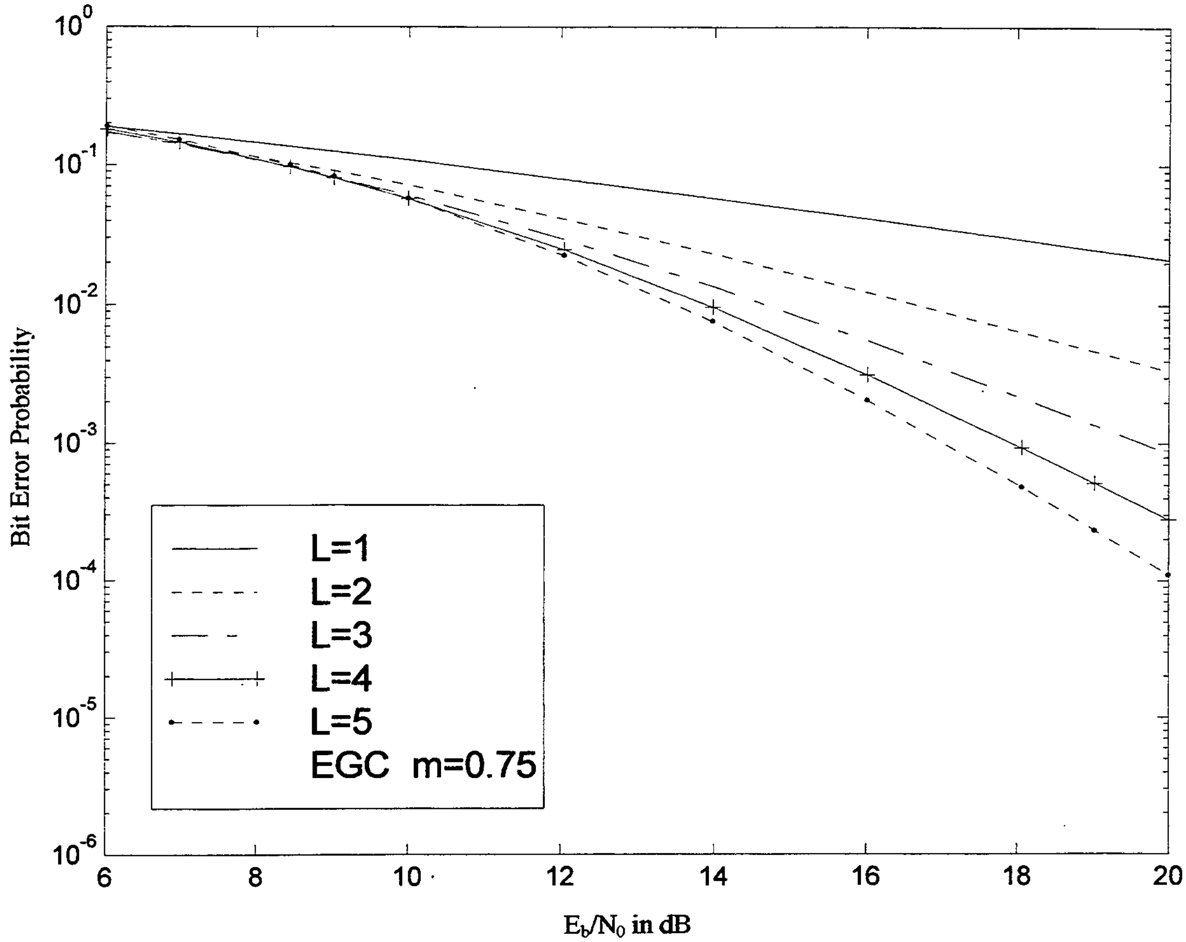

Figure 7. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 0.75$, using equal gain combining (EGC) for diversity orders $L = 1,2,3,4$ and 5.

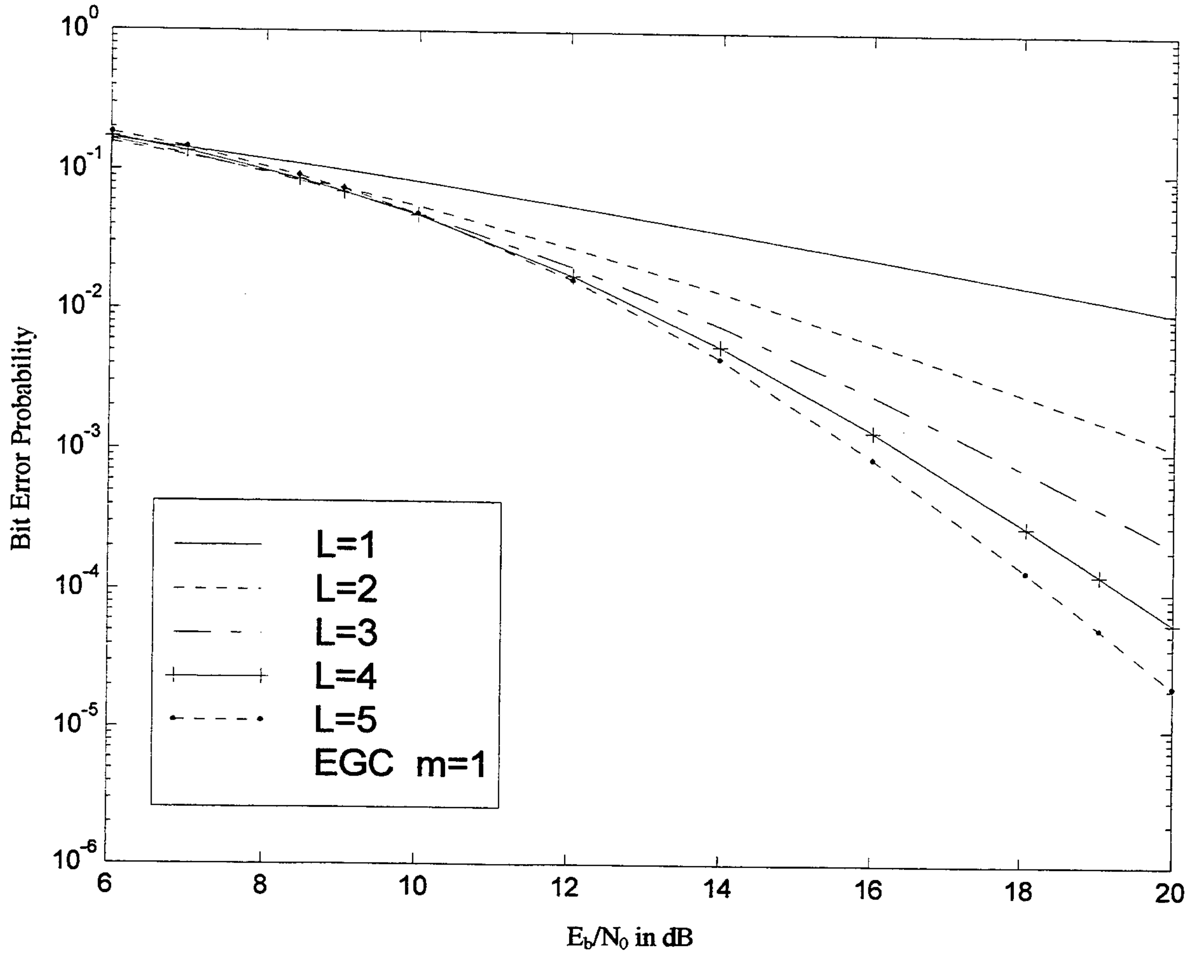

Figure 8. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 1$, using equal gain combining (EGC) for diversity orders $L = 1,2,3,4$ and 5.

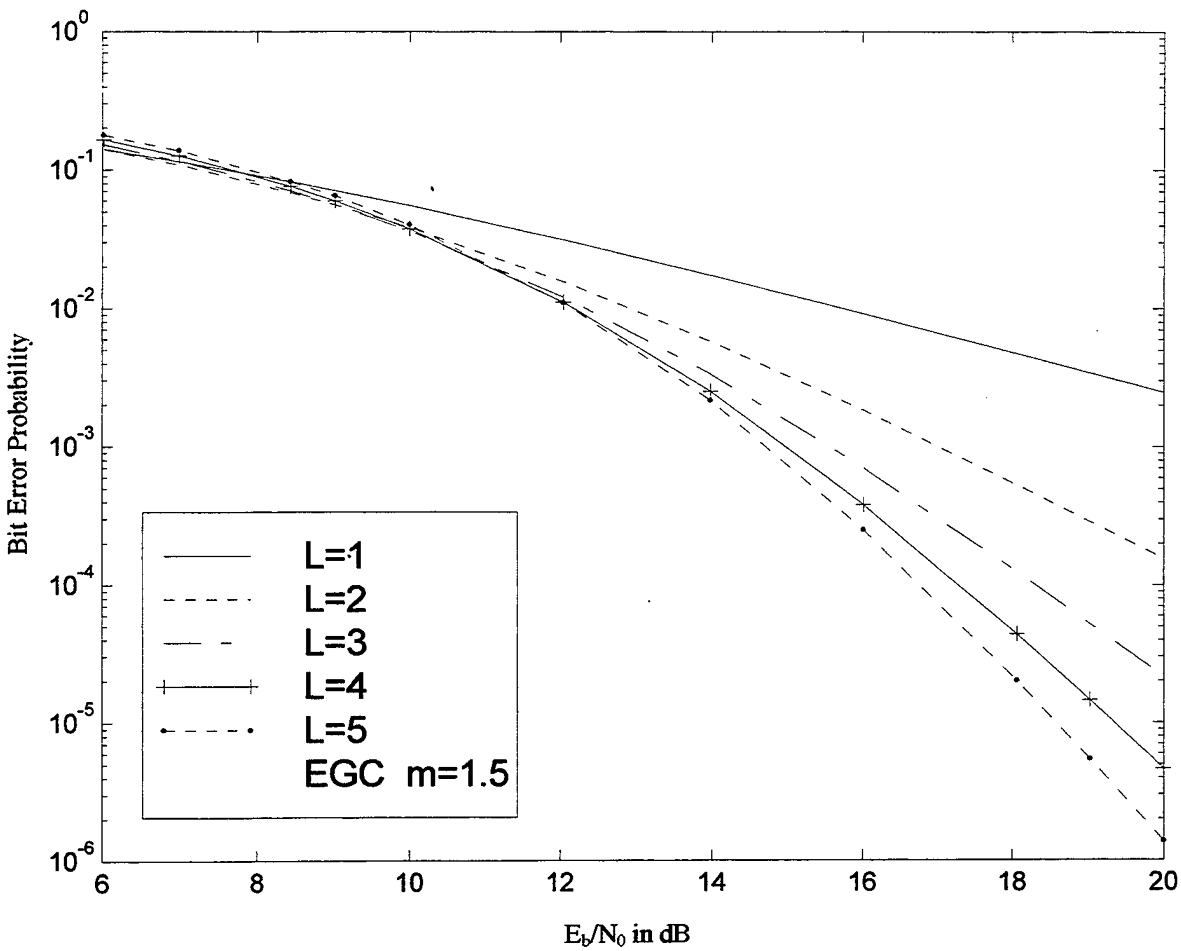

Figure 9. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 1.5$, using equal gain combining (EGC) for diversity orders $L = 1,2,3,4$ and 5.

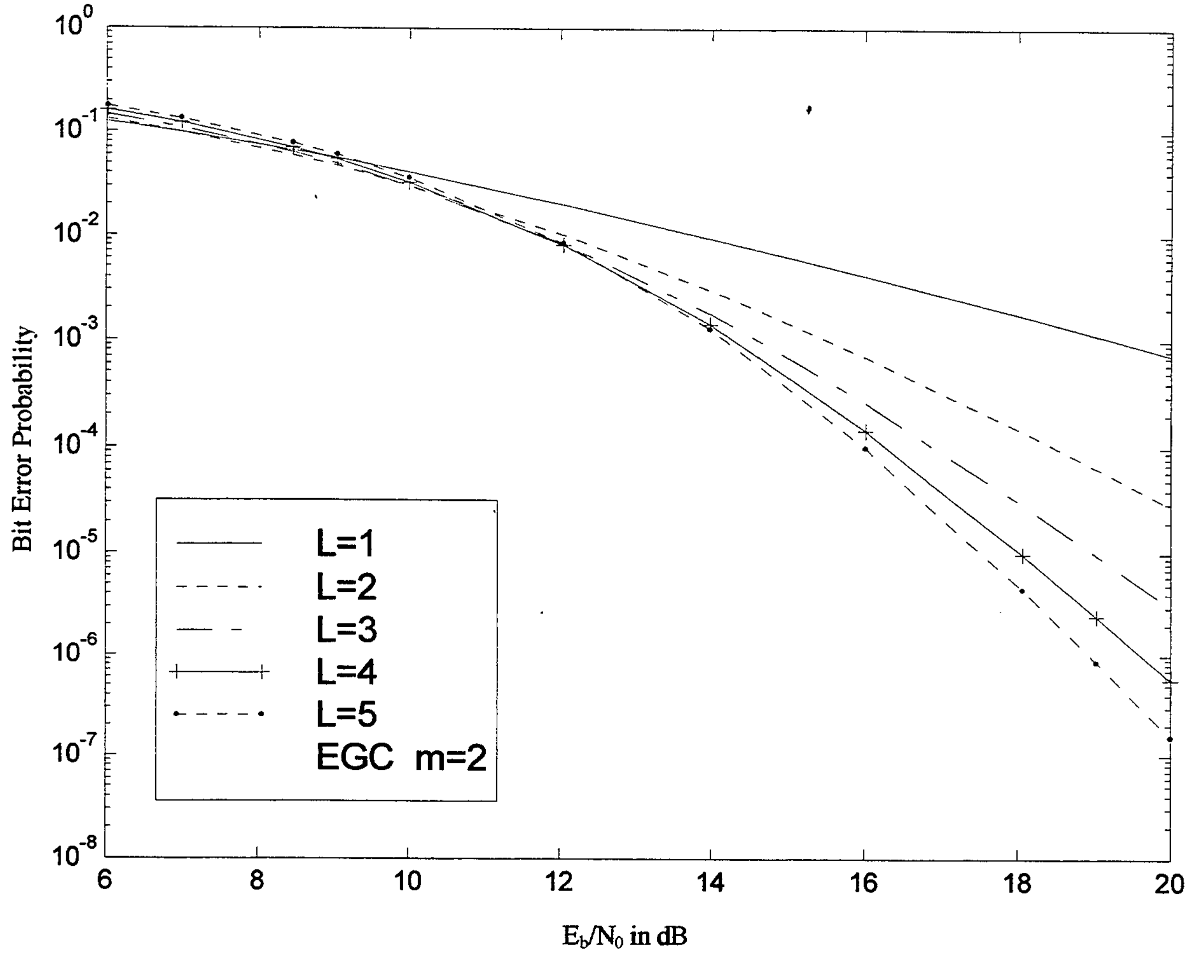

Figure 10. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 2$, using equal gain combining (EGC) for diversity orders $L = 1,2,3,4$ and 5.

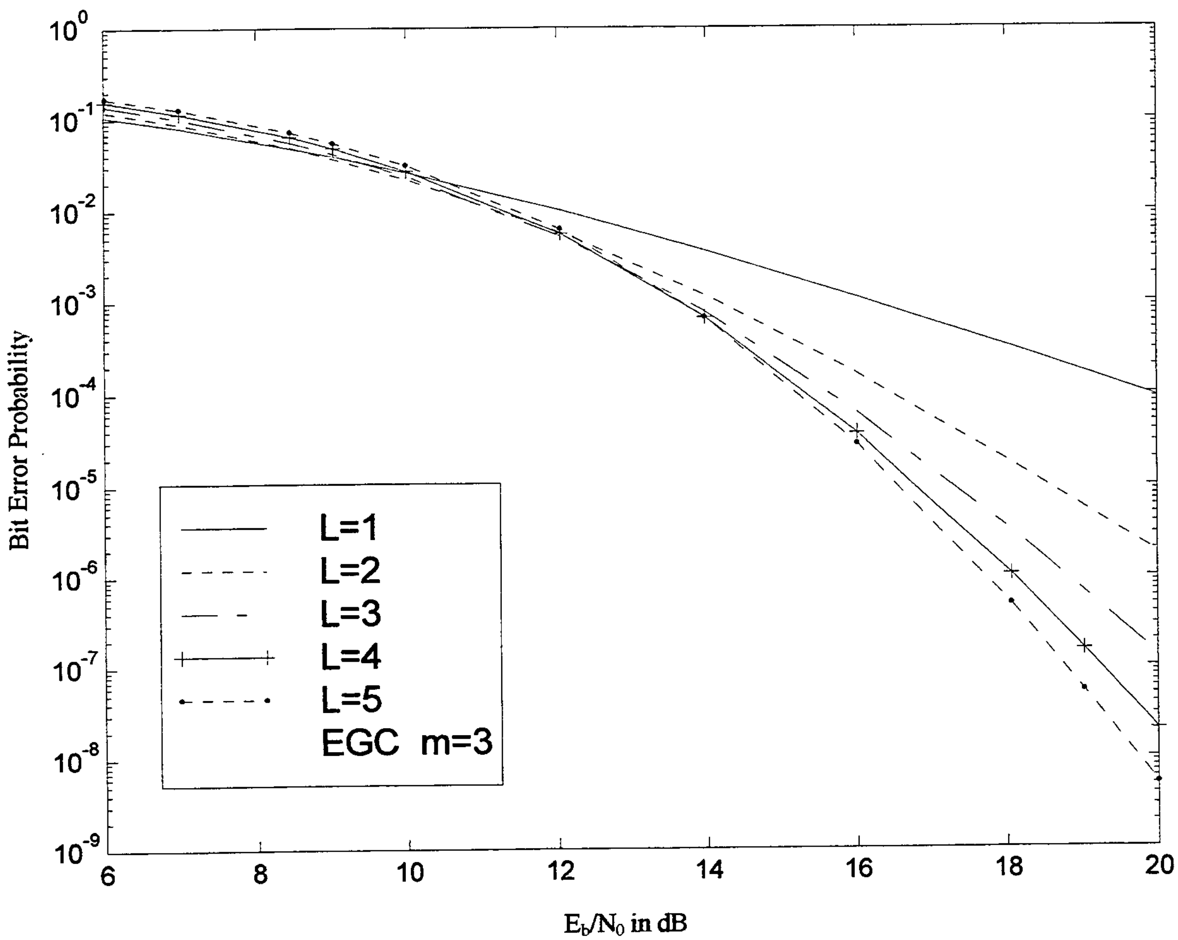

Figure 11. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 3$, using equal gain combining (EGC) for diversity orders $L = 1,2,3,4$ and 5.

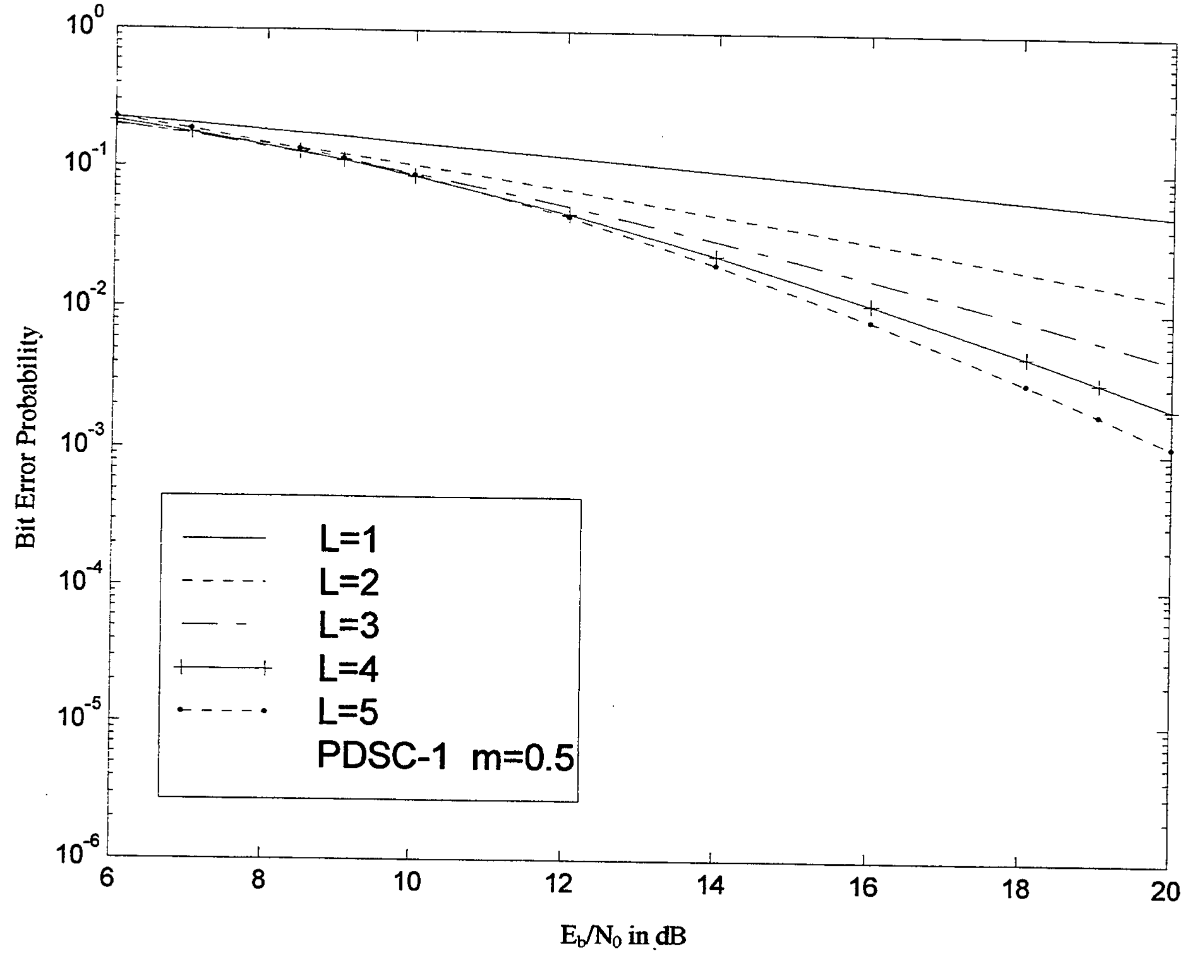

Figure 12. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 0.5$, using first order post detection selection combining (PDSC-1) for diversity orders $L = 1,2,3,4$ and 5.

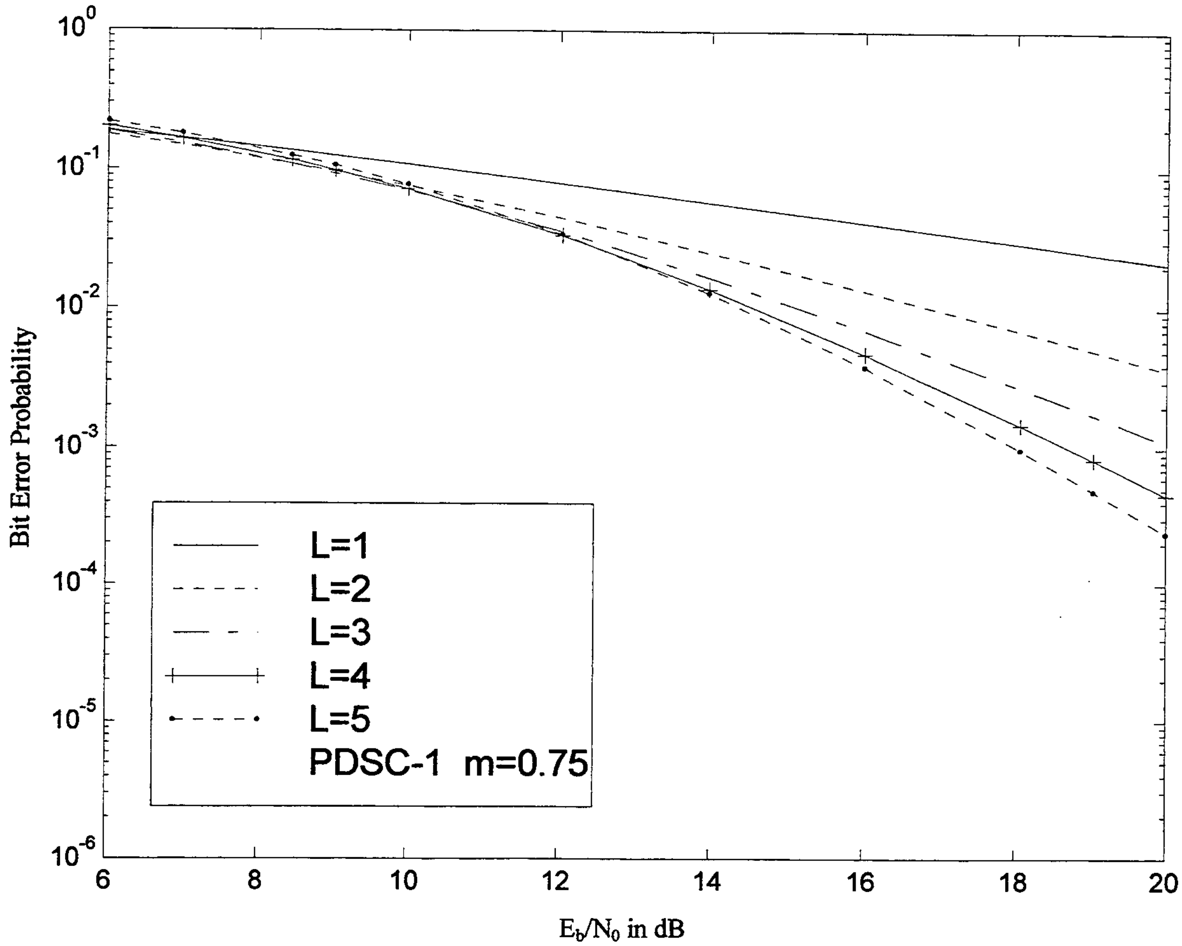

Figure 13. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 0.75$, using first order post detection selection combining (PDSC-1) for diversity orders $L = 1,2,3,4$ and 5.

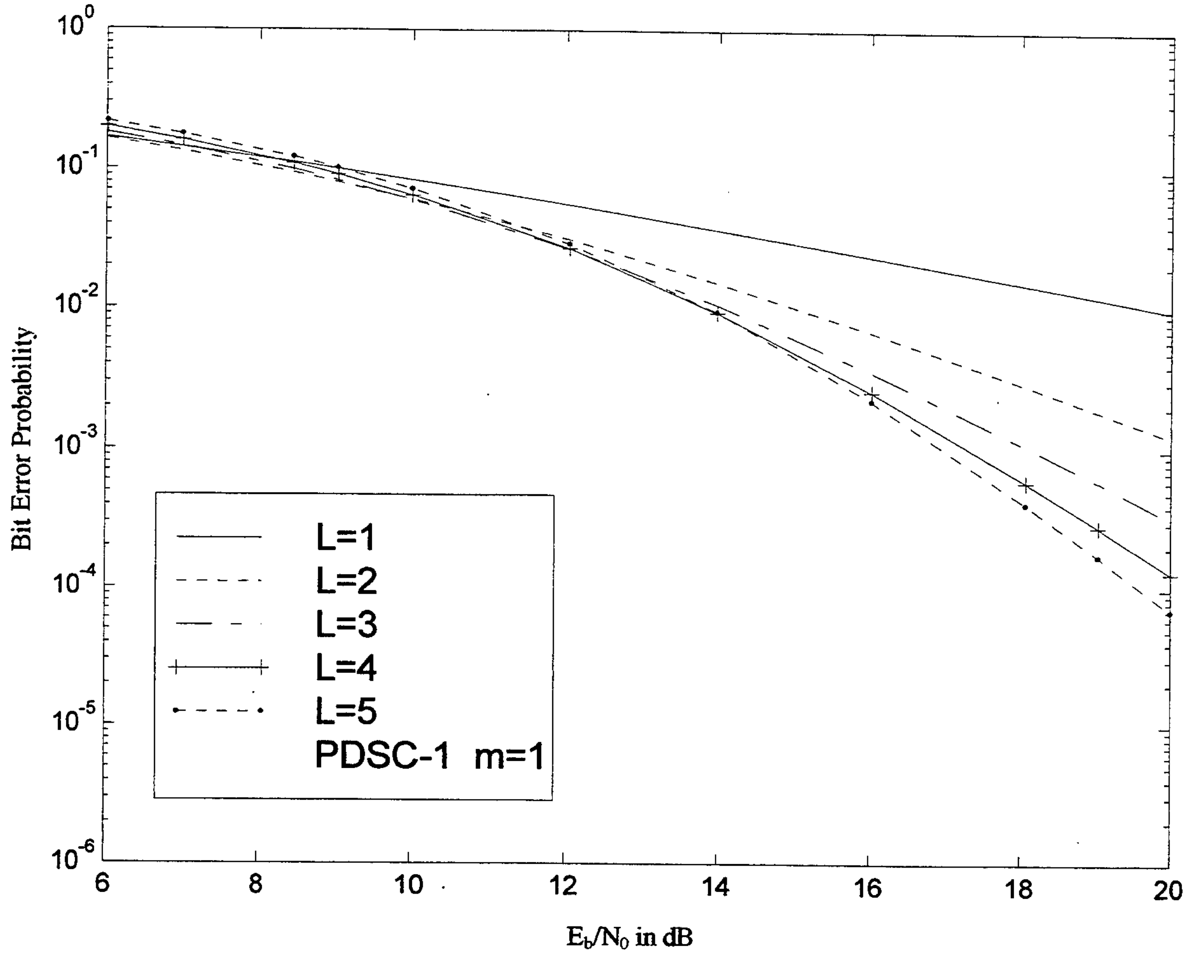

Figure 14. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 1$, using first order post detection selection combining (PDSC-1) for diversity orders $L = 1,2,3,4$ and 5.

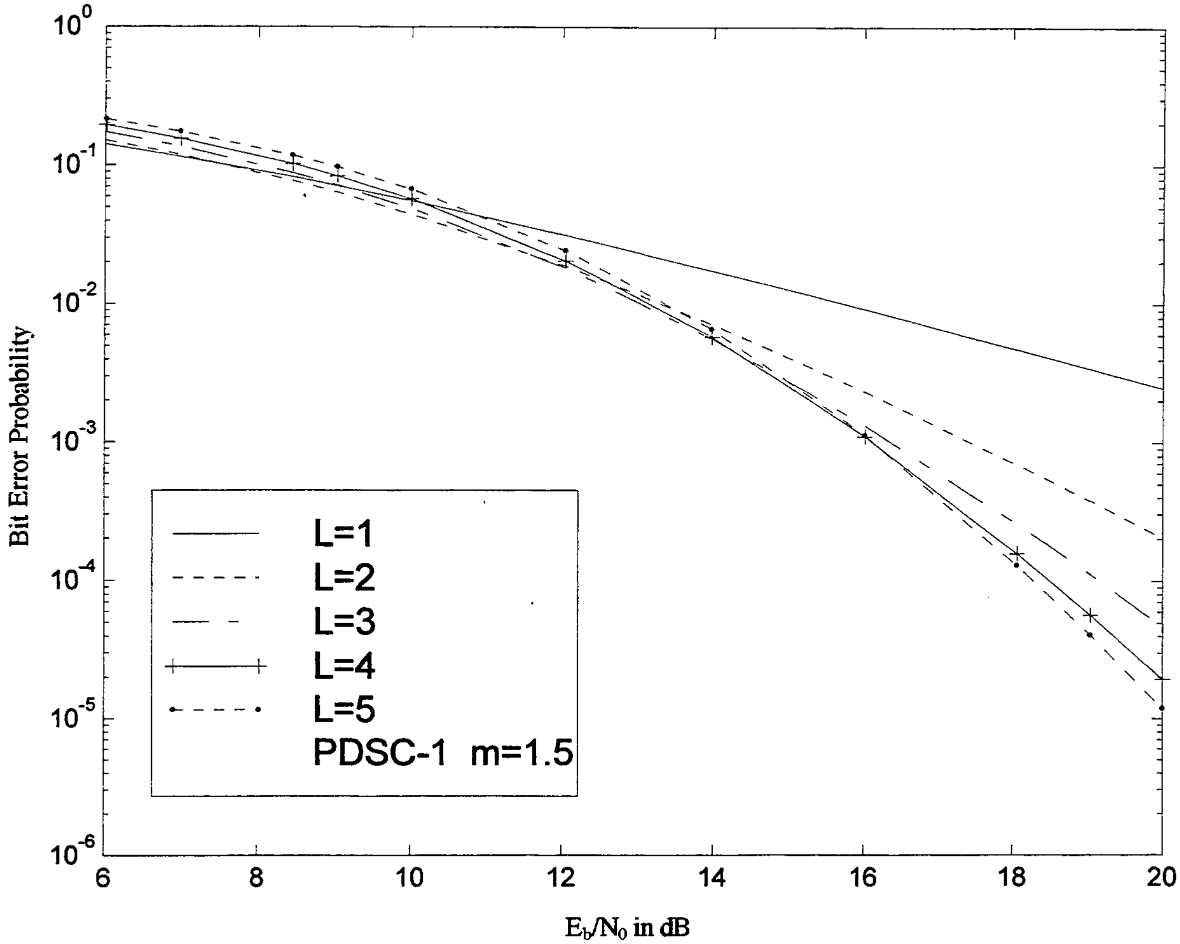

Figure 15. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 1.5$, using first order post detection selection combining (PDSC-1) for diversity orders $L = 1,2,3,4$ and 5.

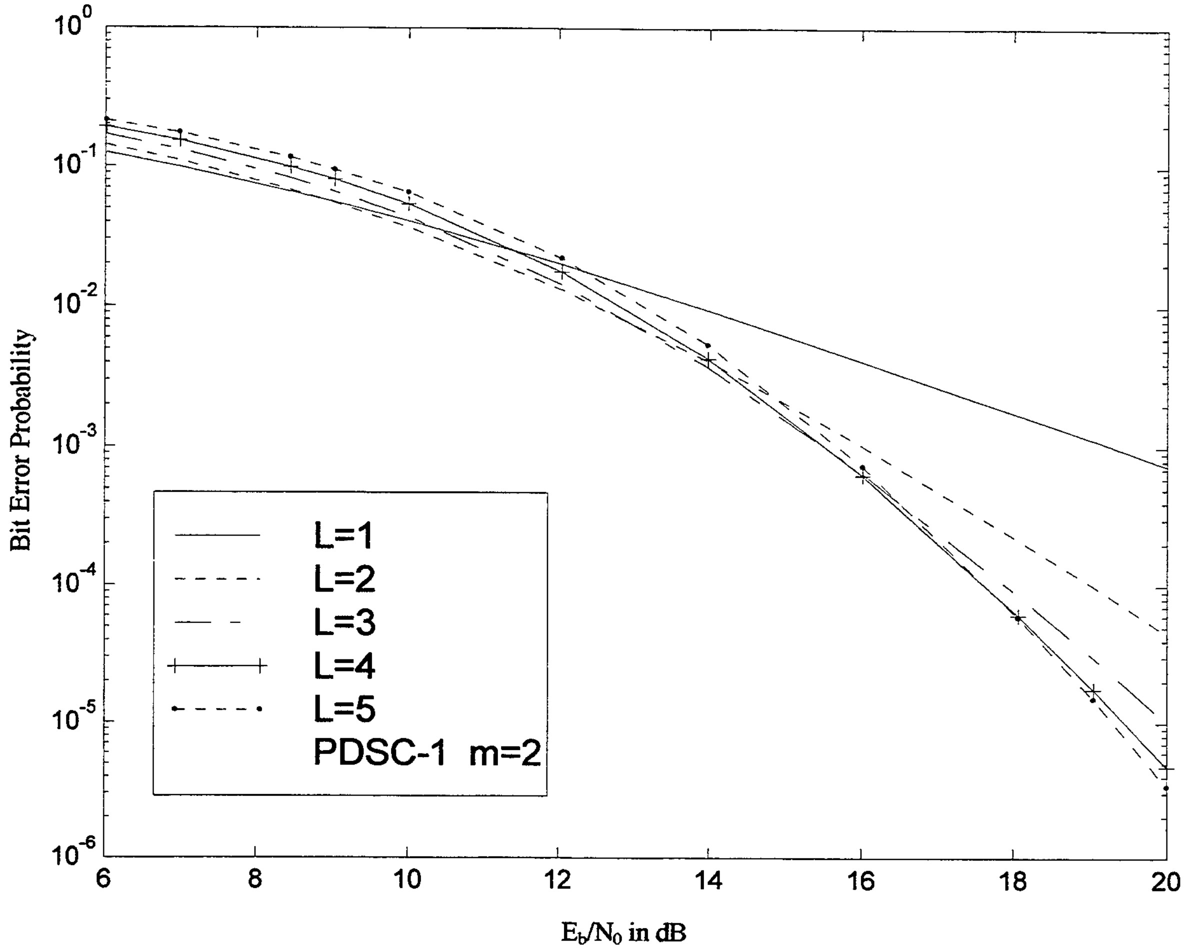

Figure 16. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 2$, using first order post detection selection combining (PDSC-1) for diversity orders $L = 1,2,3,4$ and 5.

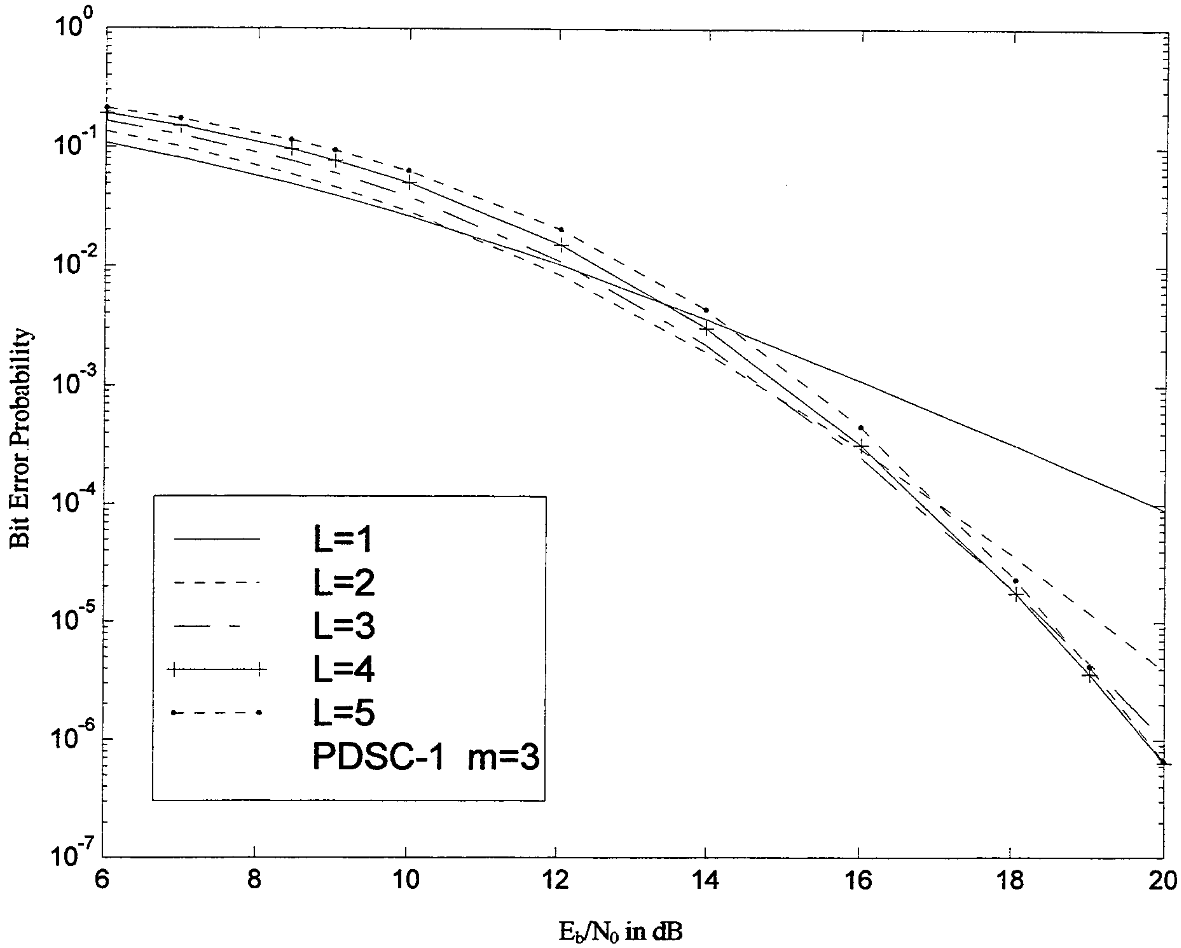

Figure 17. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 3$, using first order post detection selection combining (PDSC-1) for diversity orders $L = 1,2,3,4$ and 5.

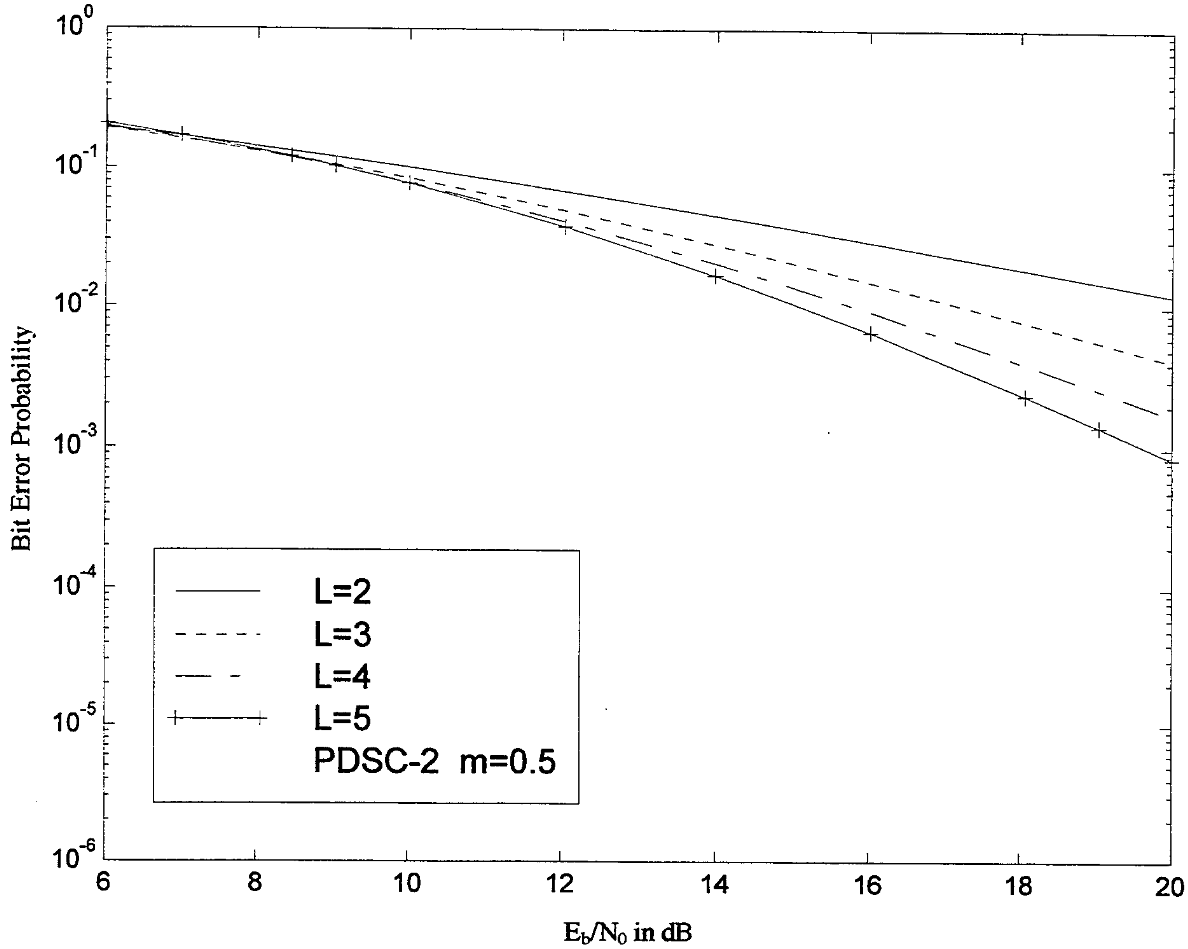

Figure 18. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 0.5$, using second order post detection selection combining (PDSC-2) for diversity orders $L = 2,3,4$ and 5.

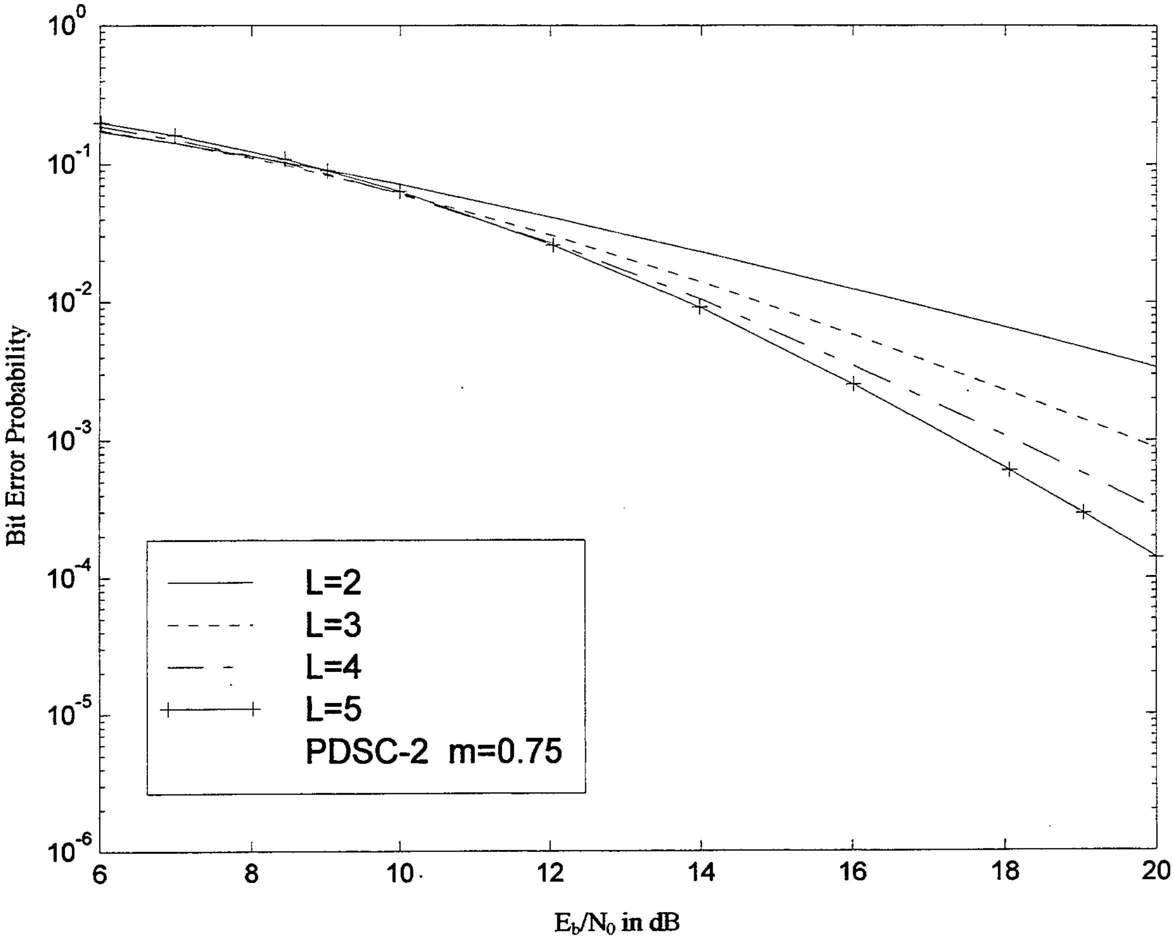

Figure 19. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 0.75$, using second order post detection selection combining (PDSC-2) for diversity orders $L = 2,3,4$ and 5.

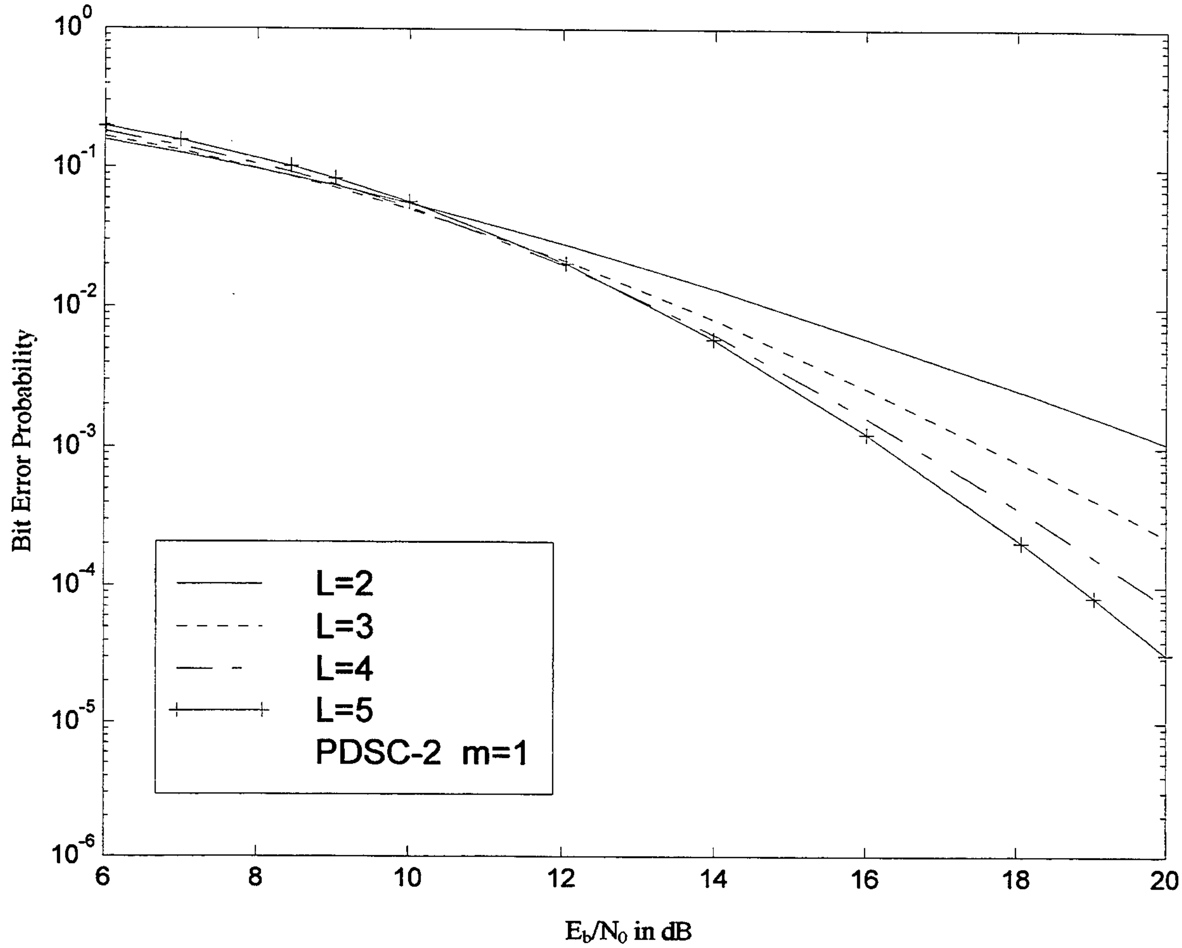

Figure 20. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 1$, using second order post detection selection combining (PDSC-2) for diversity orders $L = 2,3,4$ and 5.

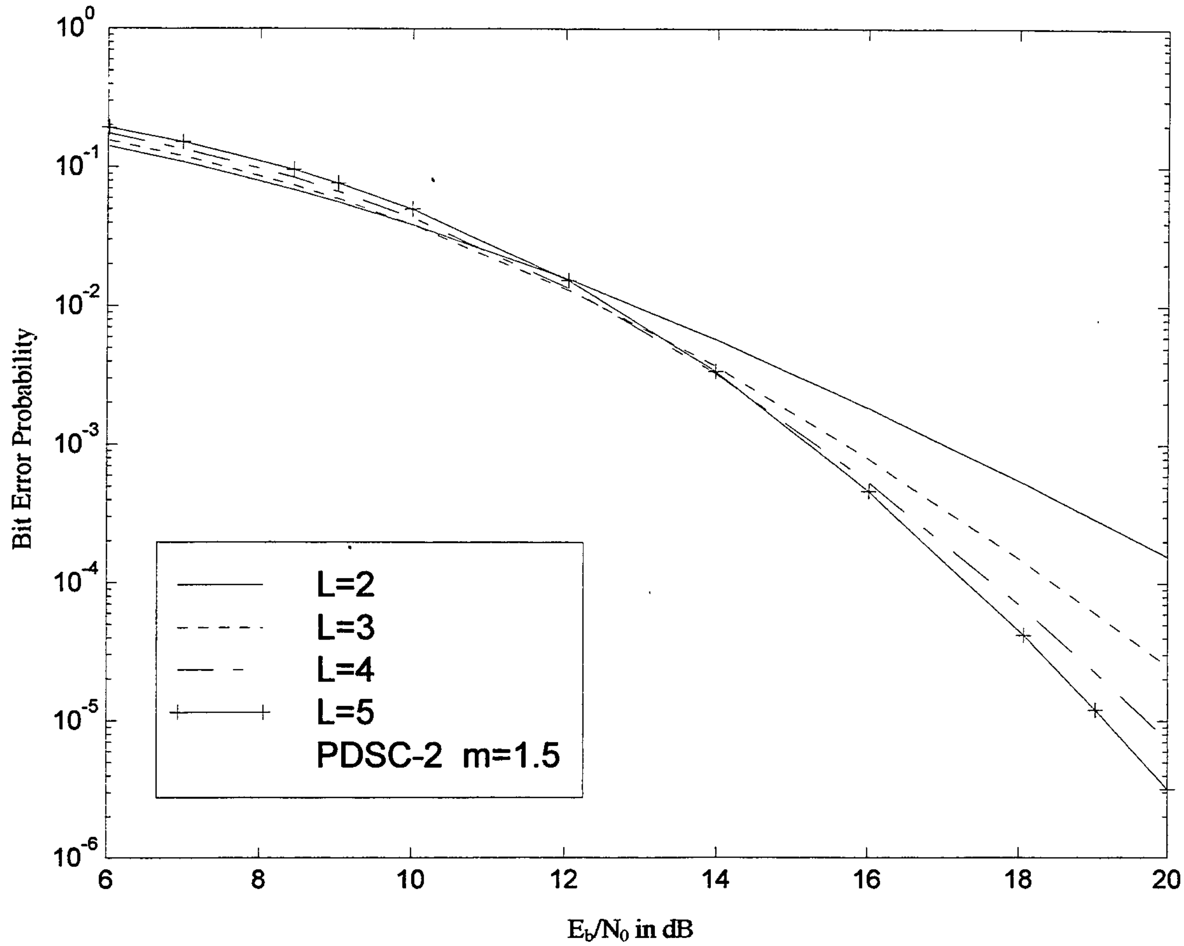

Figure 21. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 1.5$, using second order post detection selection combining (PDSC-2) for diversity orders $L = 2,3,4$ and 5.

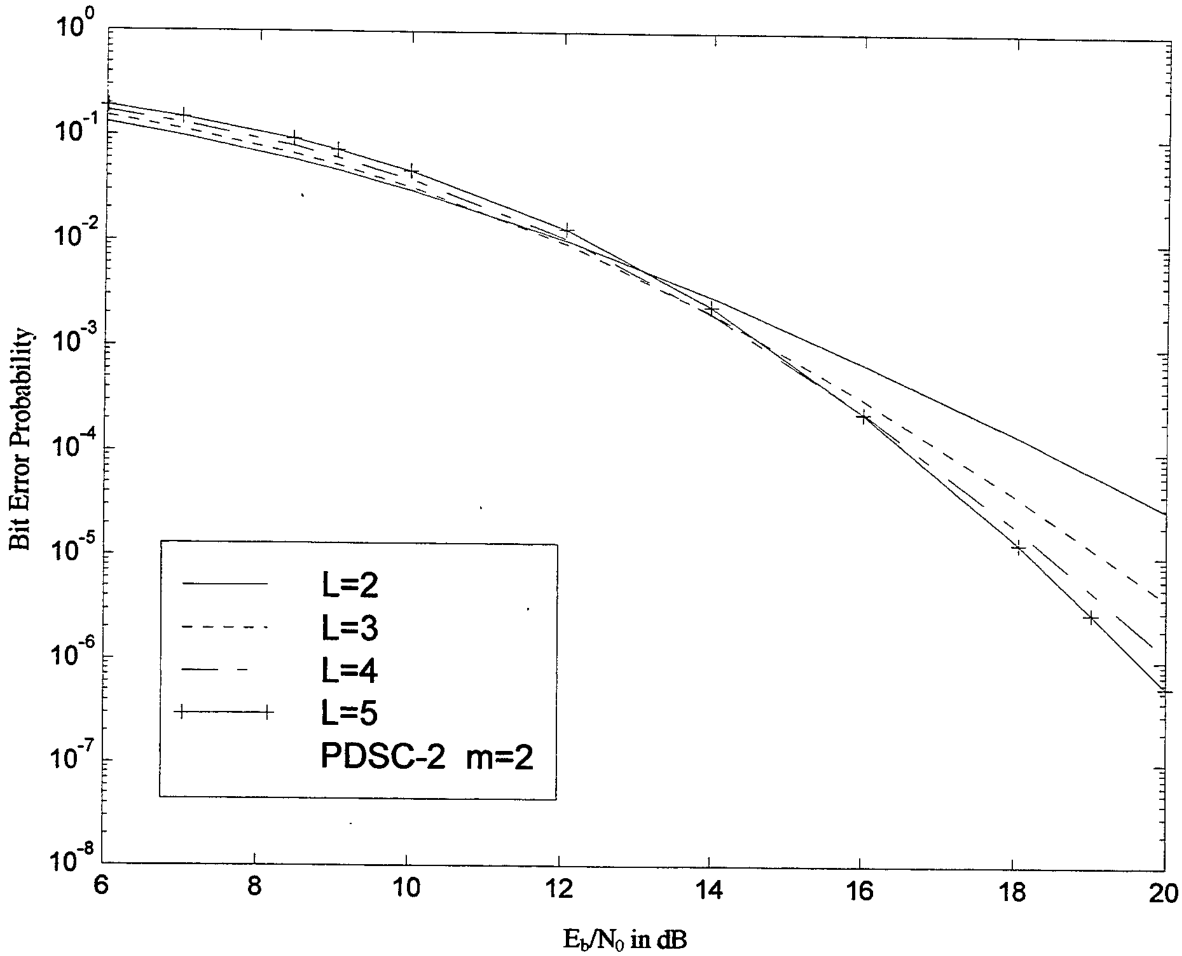

Figure 22. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 2$, using second order post detection selection combining (PDSC-2) for diversity orders $L = 2,3,4$ and 5.

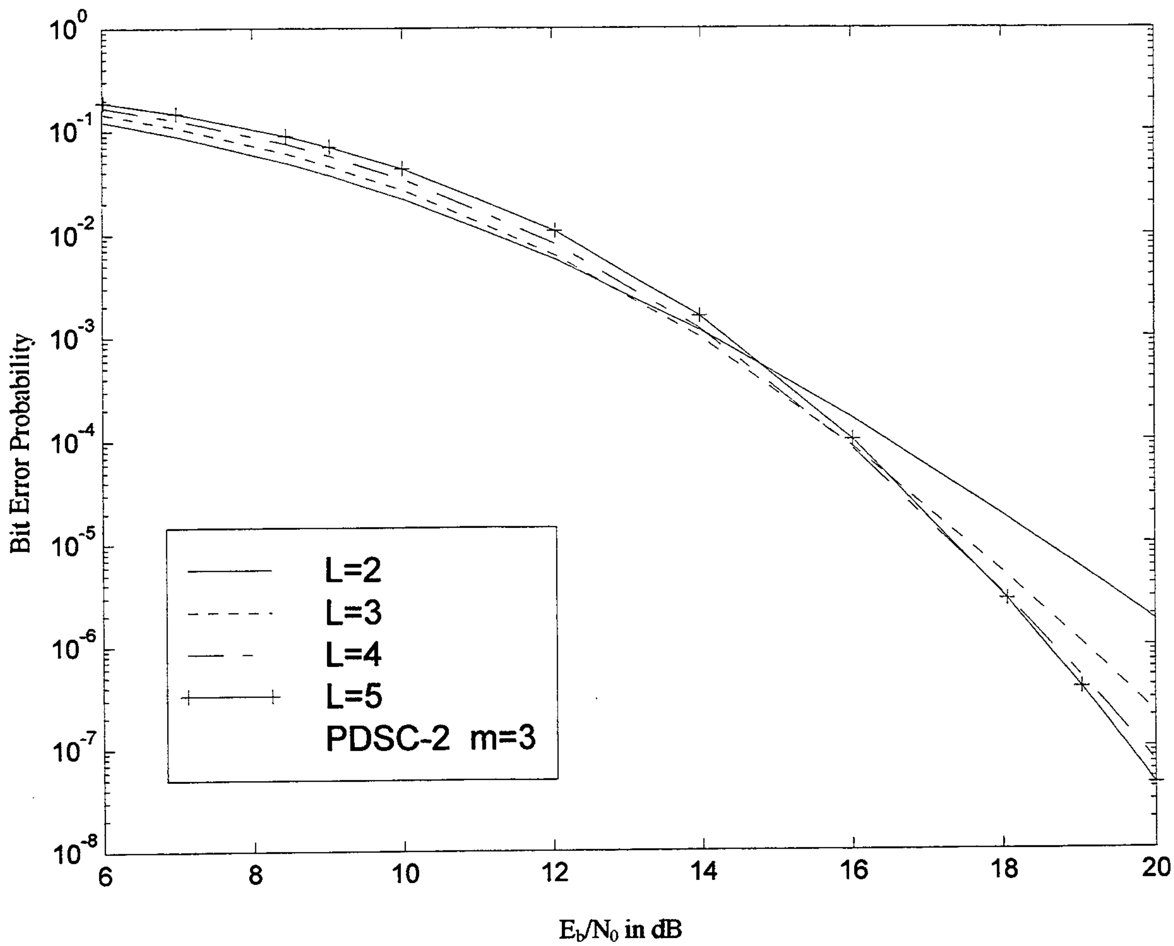

Figure 23. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 3$, using second order post detection selection combining (PDSC-2) for diversity orders $L = 2,3,4$ and 5.

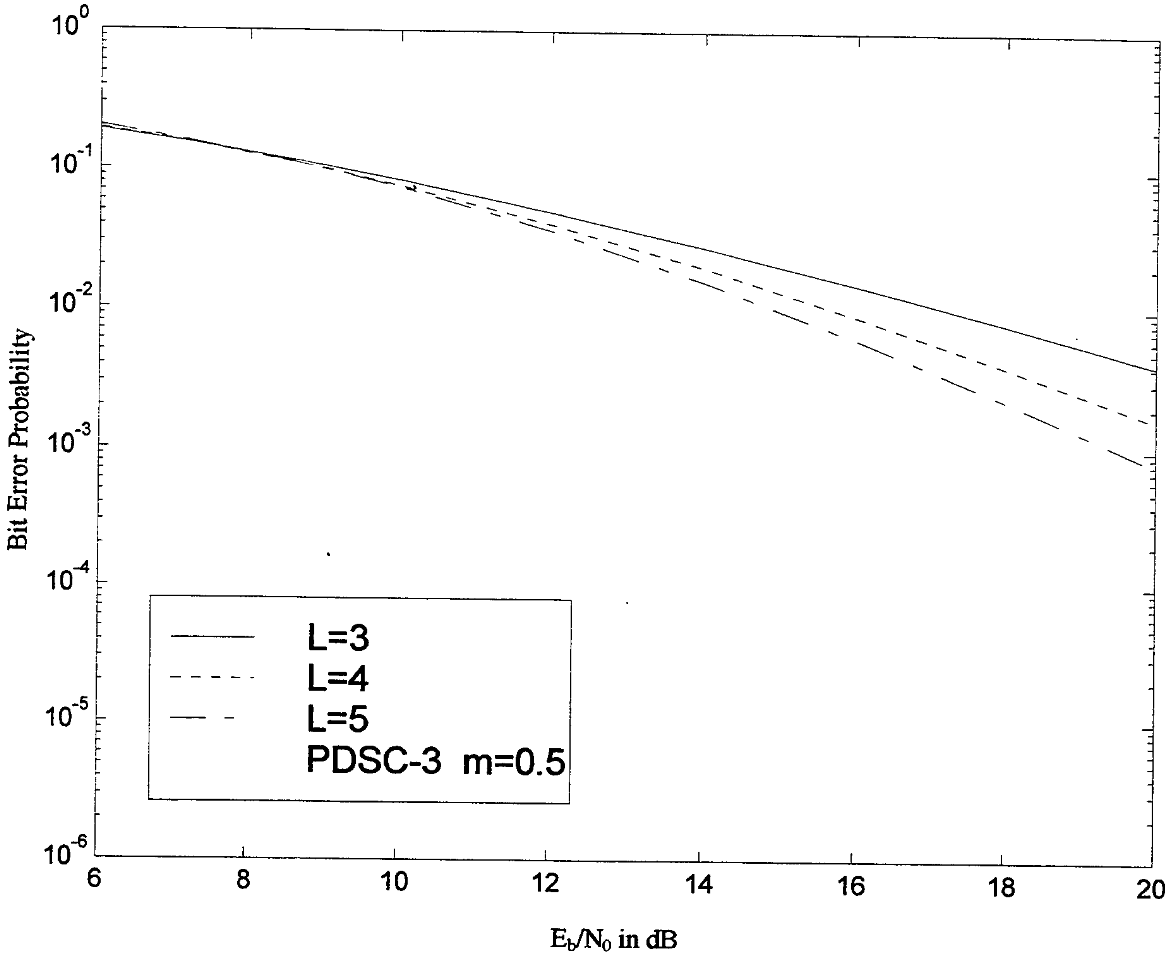

Figure 24. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 0.5$, using third order post detection selection combining (PDSC-3) for diversity orders $L = 3,4$ and 5.

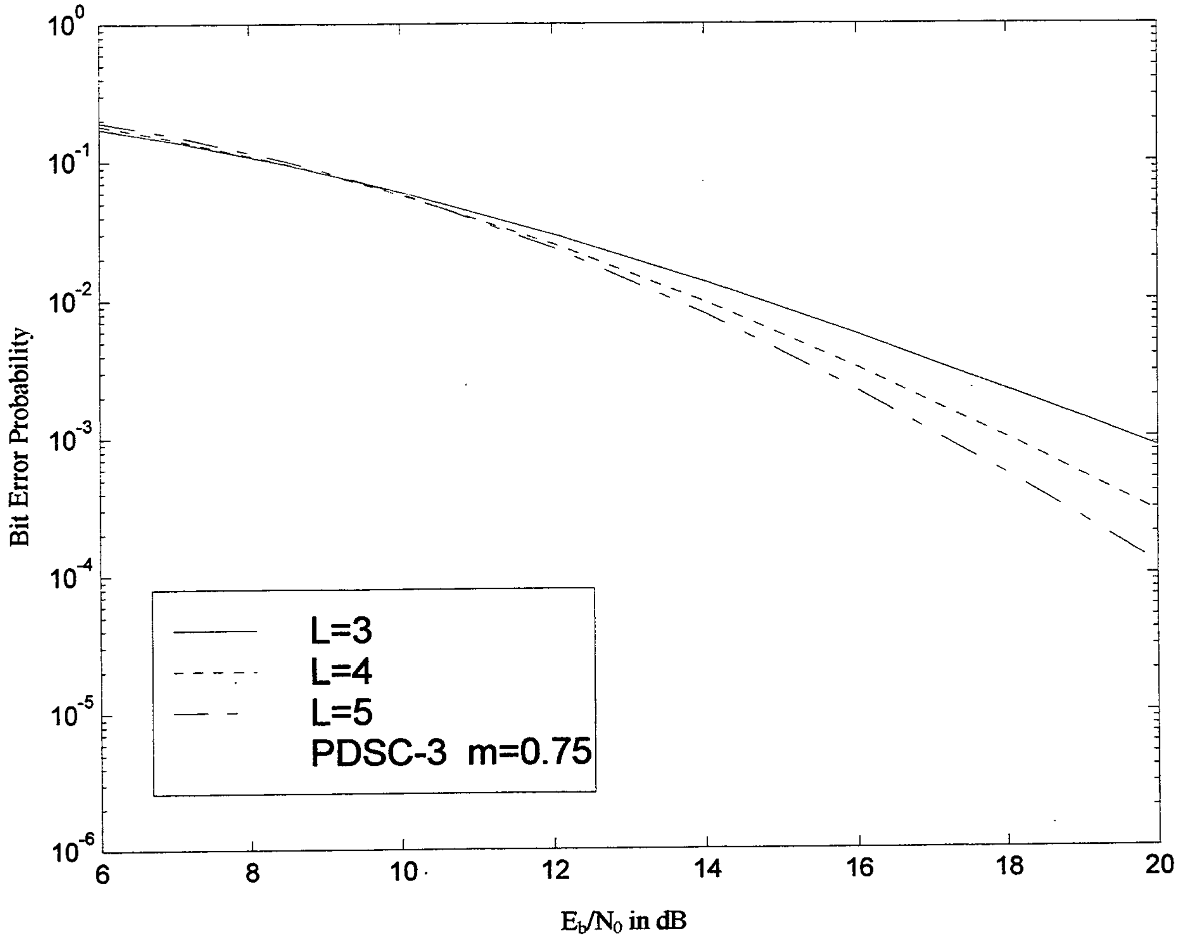

Figure 25. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 0.75$, using third order post detection selection combining (PDSC-3) for diversity orders $L = 3, 4$ and 5.

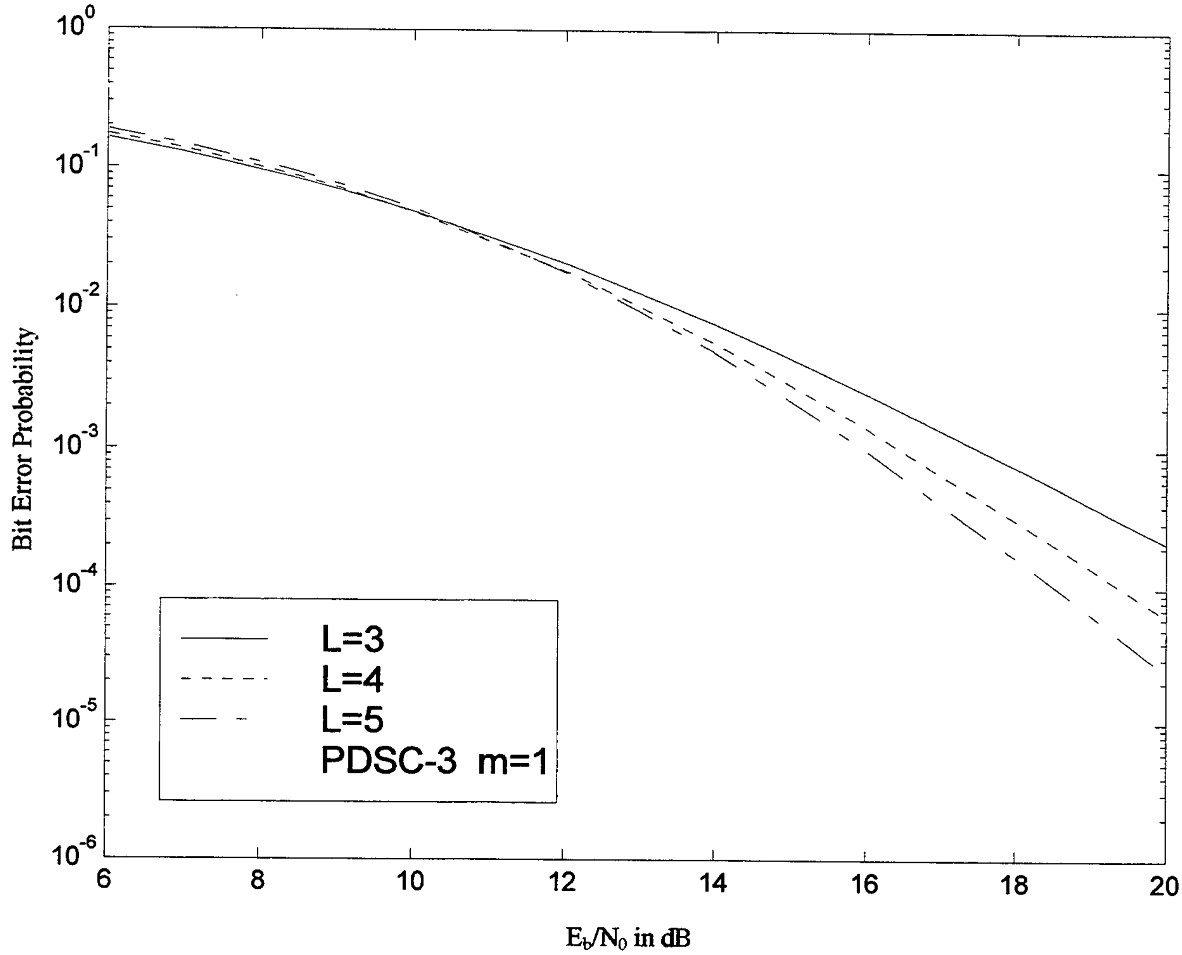

Figure 26. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 1$, using third order post detection selection combining (PDSC-3) for diversity orders $L = 3,4$ and 5.

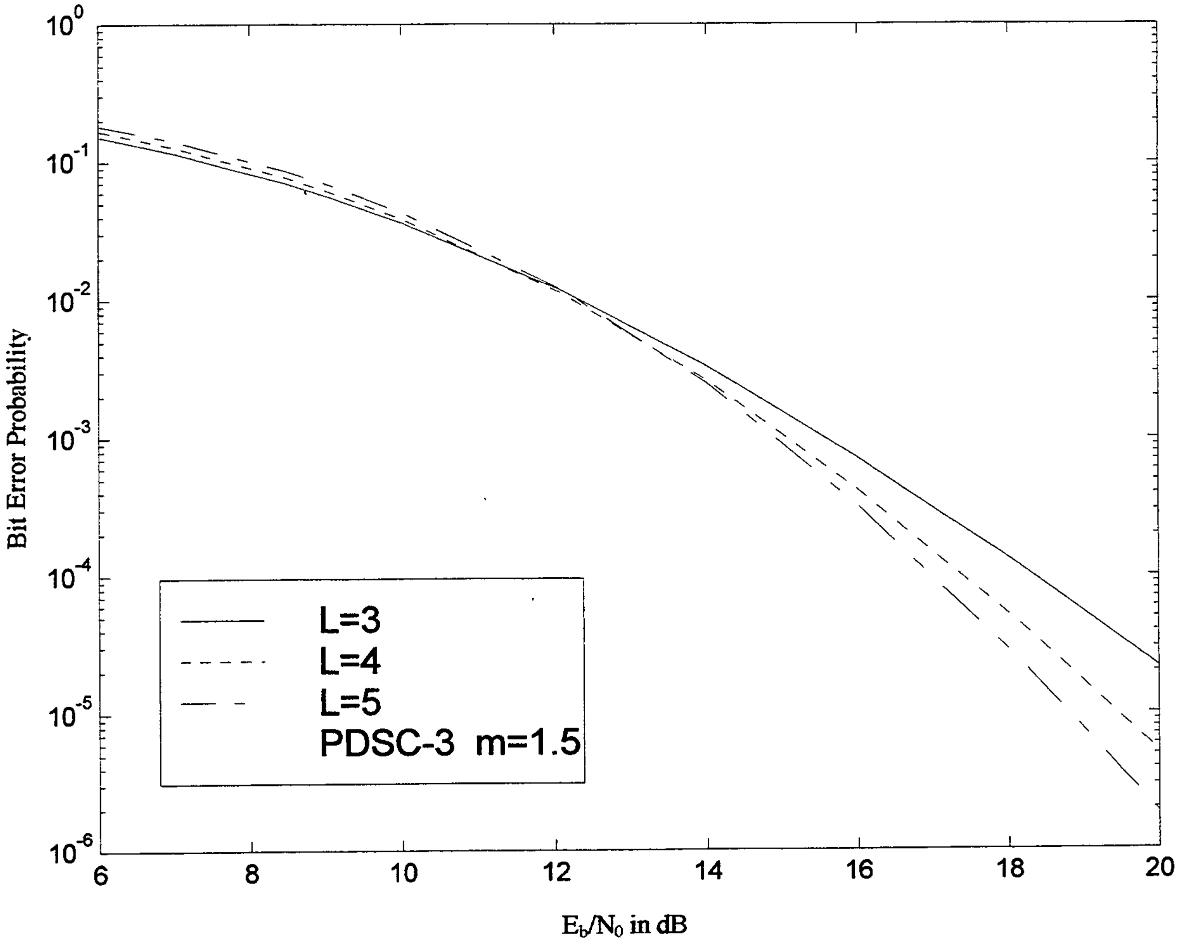

Figure 27. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 1.5$, using third order post detection selection combining (PDSC-3) for diversity orders $L = 3,4$ and 5.

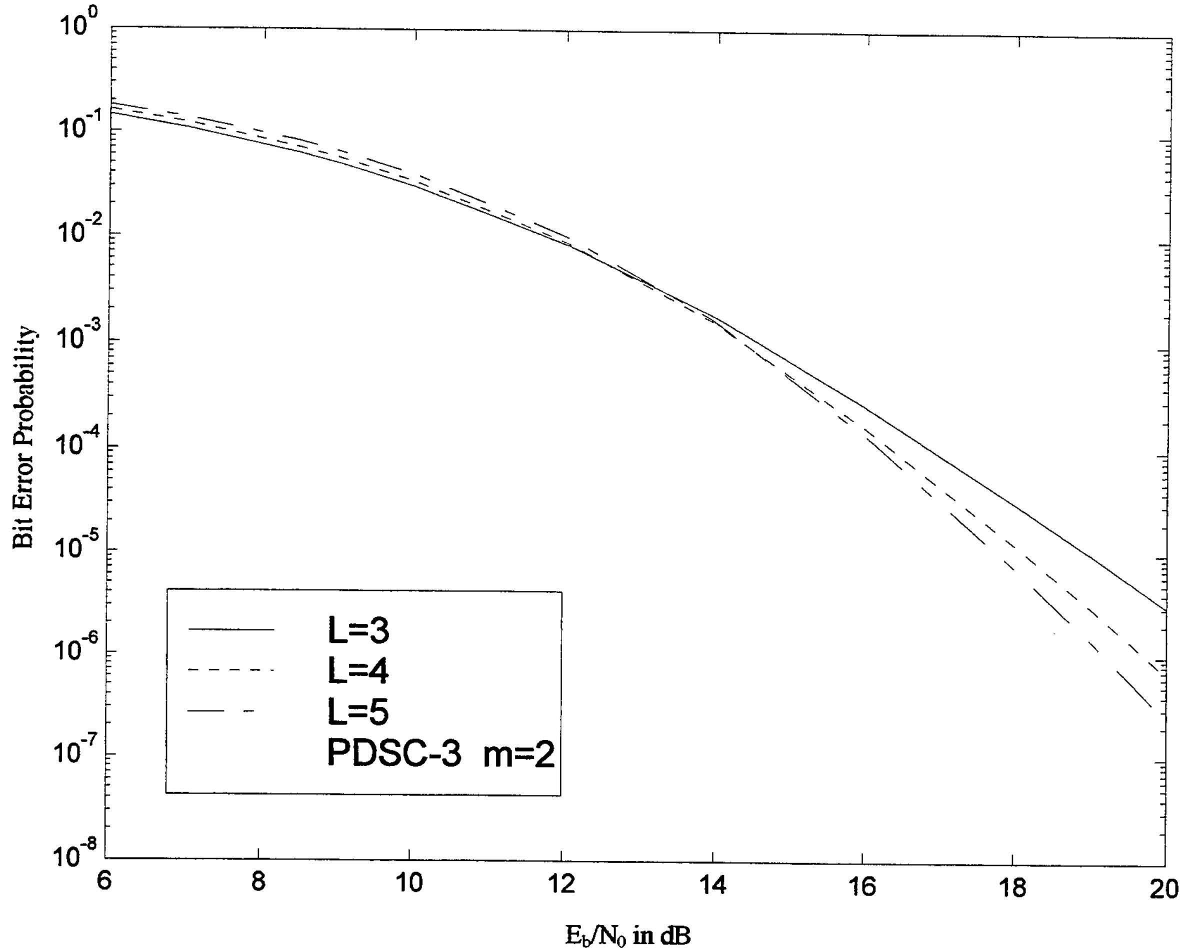

Figure 28. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 2$, using third order post detection selection combining (PDSC-3) for diversity orders $L = 3, 4$ and 5.

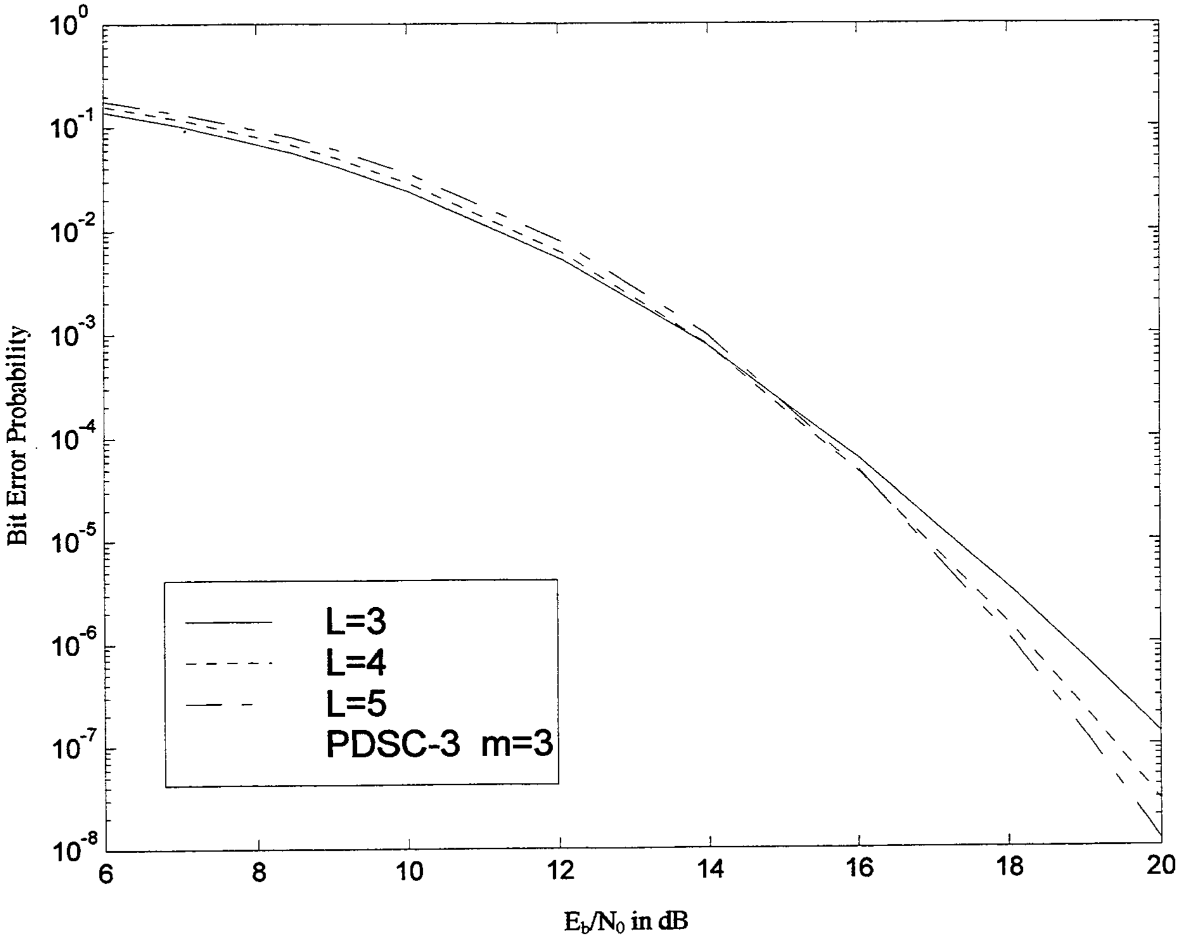

Figure 29. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 3$, using third order post detection selection combining (PDSC-3) for diversity orders $L = 3,4$ and 5.

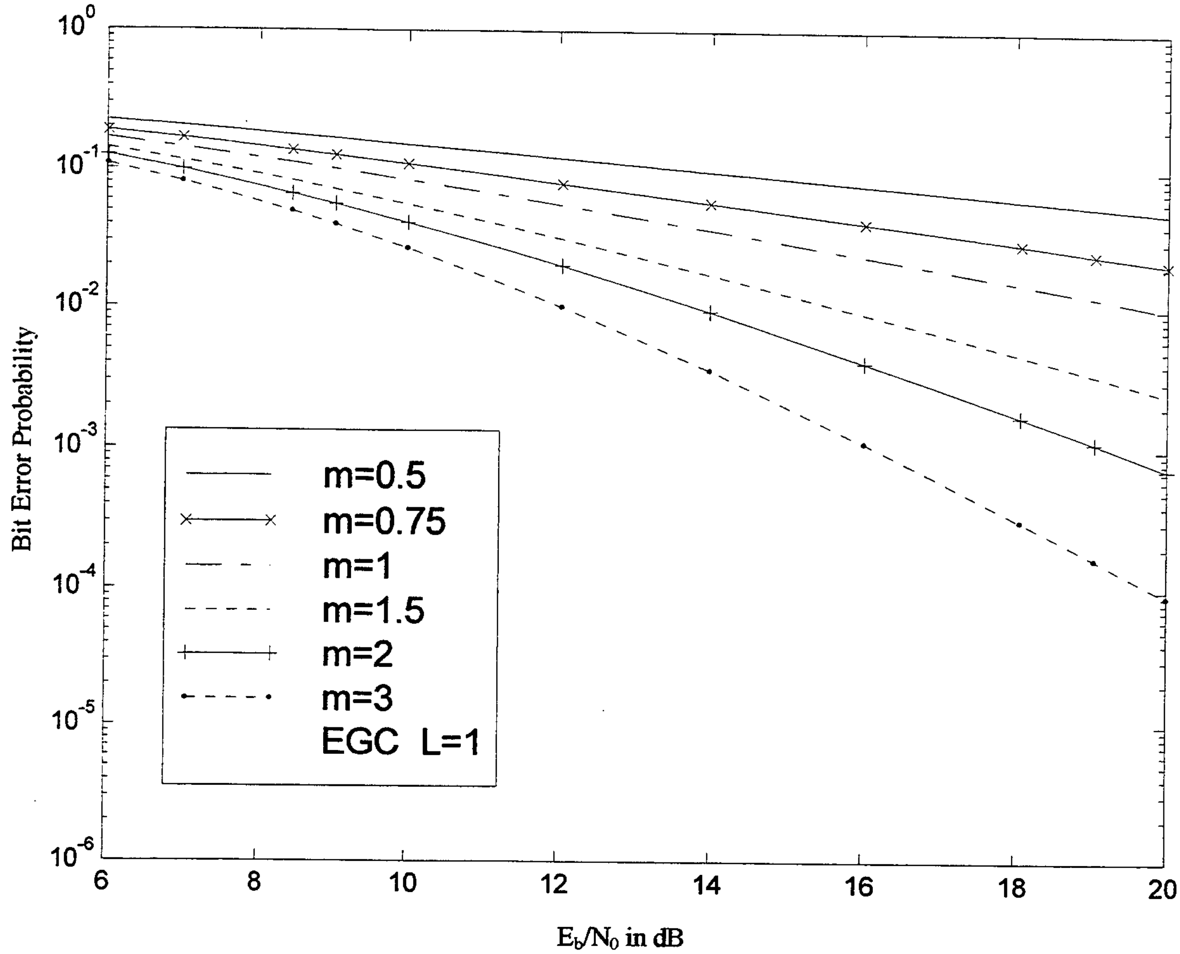

Figure 30. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with diversity order $L = 1$, using equal gain combining (EGC), for $m = 0.5, 0.75, 1, 1.5, 2$ and 3.

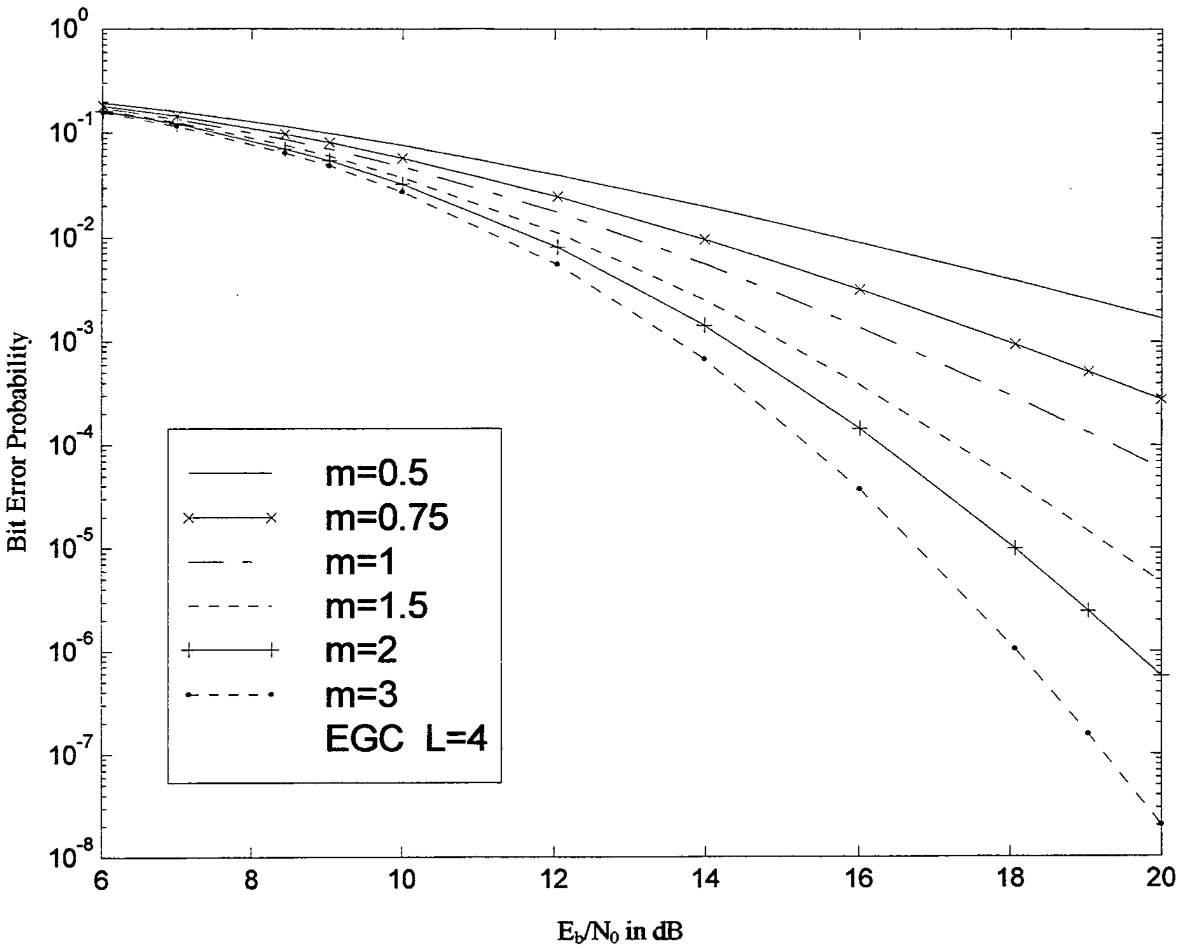

Figure 31. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with diversity order $L = 4$, using equal gain combining (EGC), for $m = 0.5, 0.75, 1, 1.5, 2$ and 3.

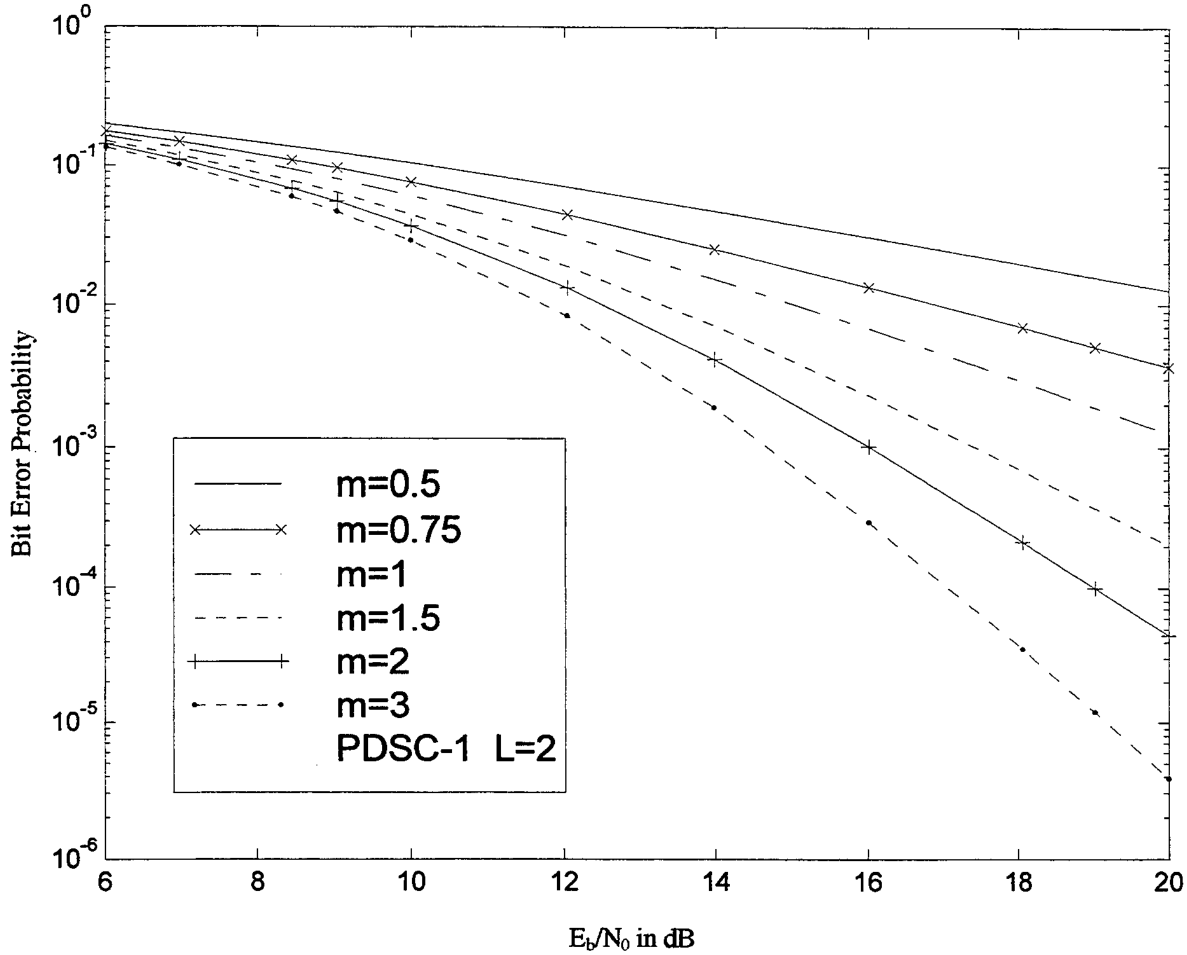

Figure 32. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with diversity order $L = 2$, using first order post detection selection combining (PDSC-1), for $m = 0.5, 0.75, 1, 1.5, 2$ and 3.

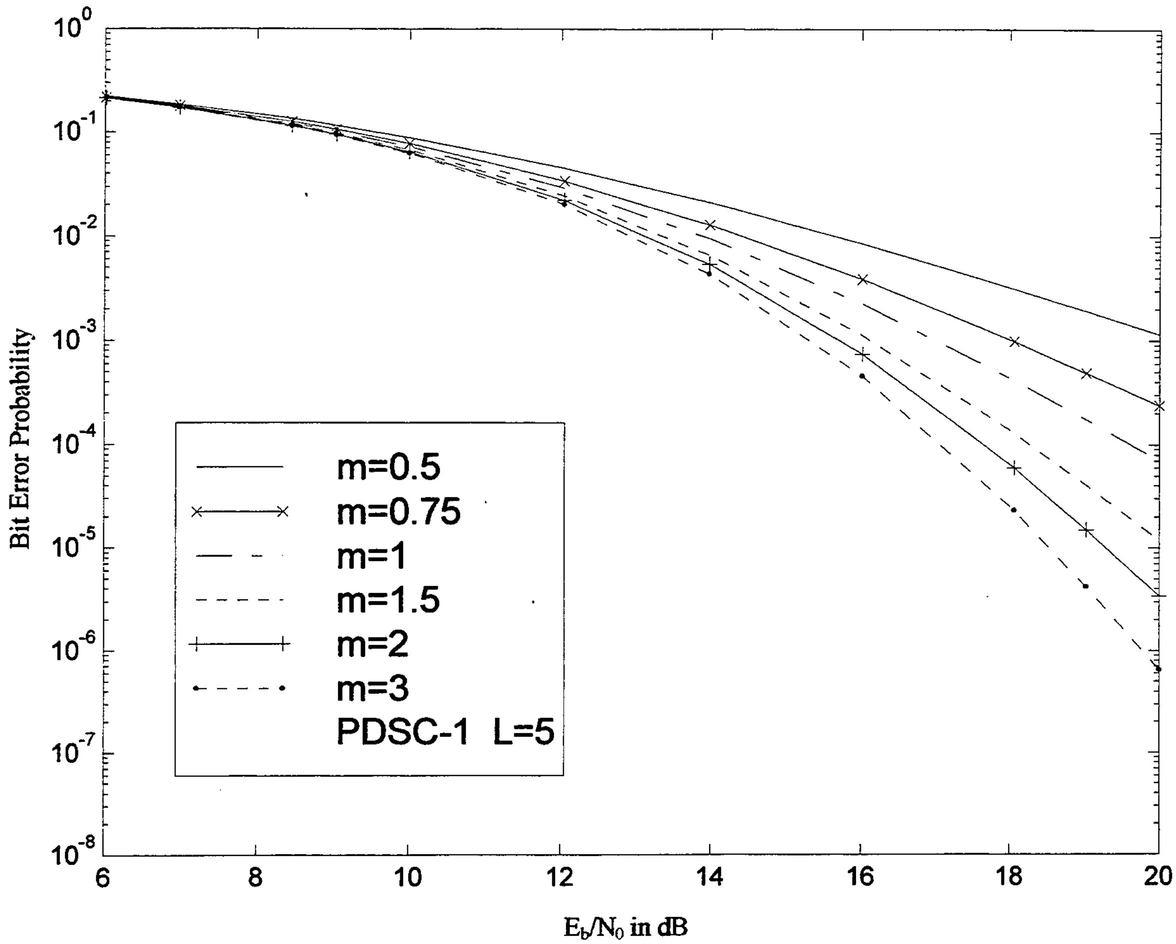

Figure 33. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with diversity order $L = 5$, using first order post detection selection combining (PDSC-1), for $m = 0.5, 0.75, 1, 1.5, 2$ and 3.

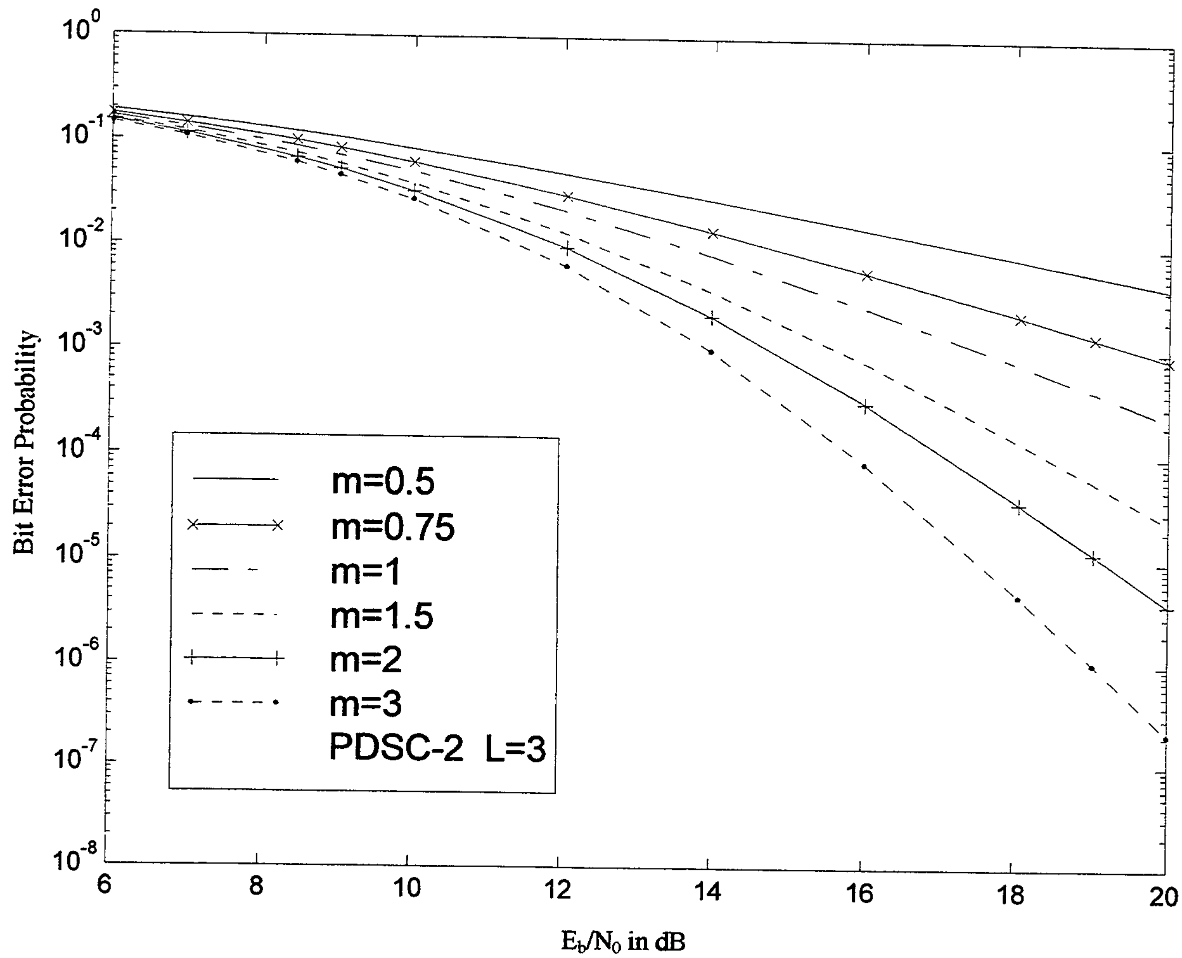

Figure 34. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with diversity order $L = 3$, using second order post detection selection combining (PDSC-2), for $m = 0.5, 0.75, 1, 1.5, 2$ and 3.

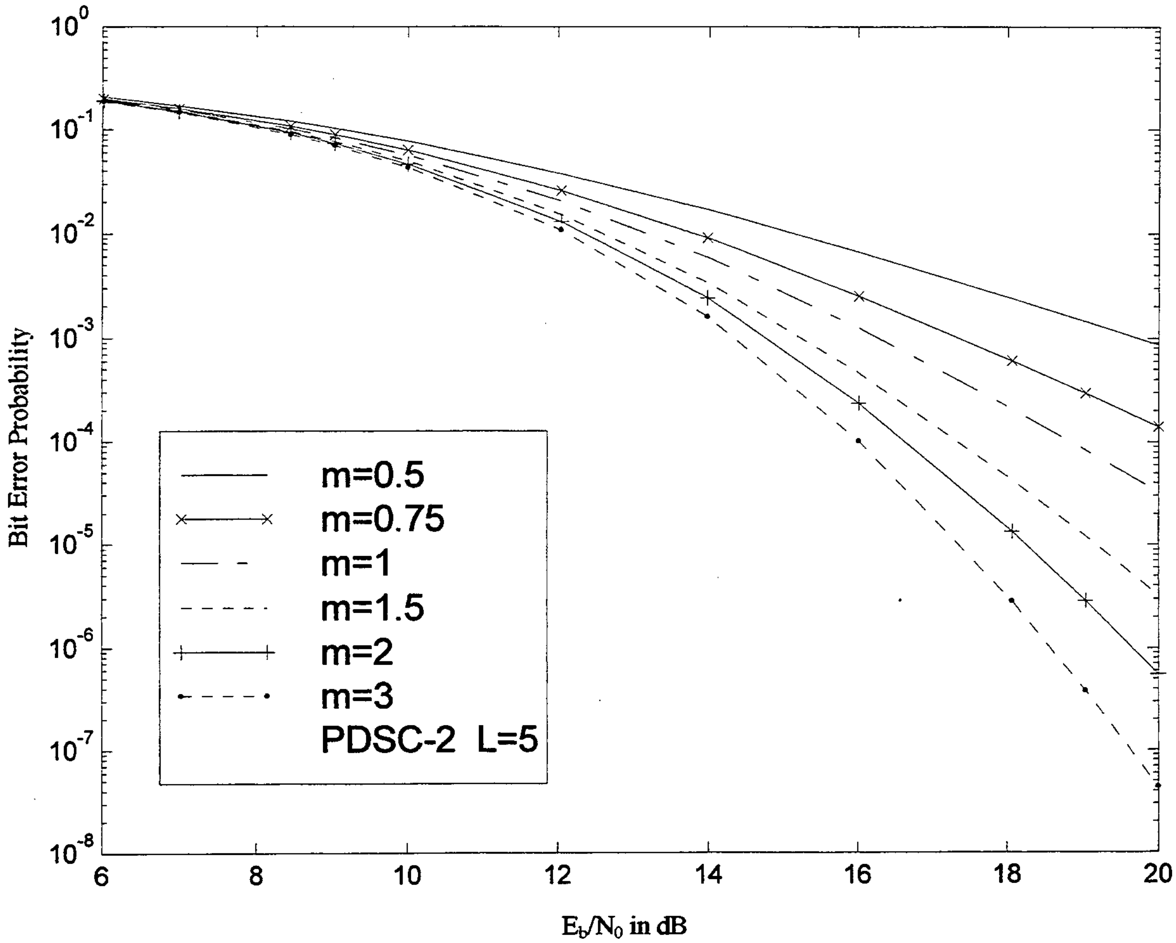

Figure 35. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with diversity order $L = 5$, using second order post detection selection combining (PDSC-2), for $m = 0.5, 0.75, 1, 1.5, 2$ and 3.

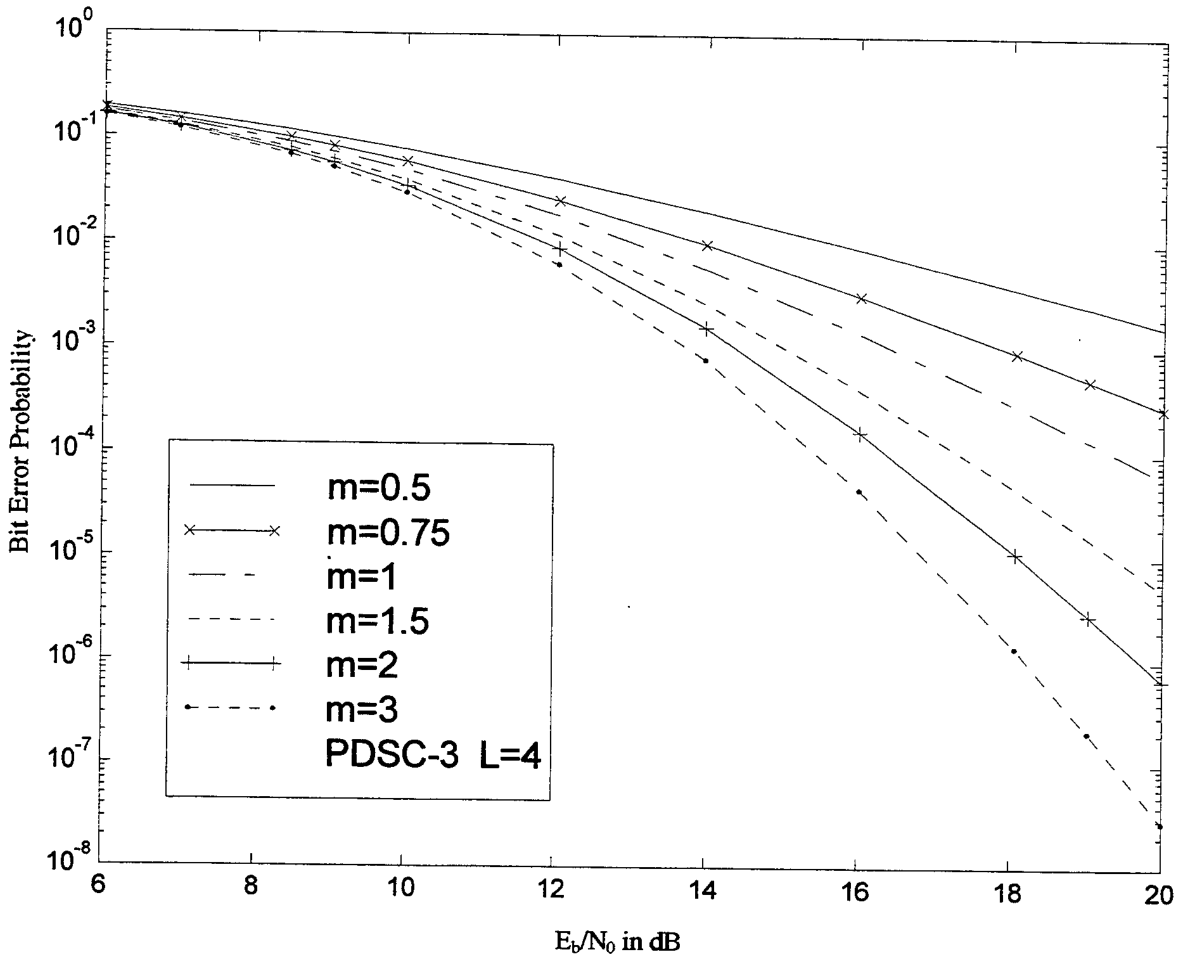

Figure 36. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with diversity order $L = 4$, using third order post detection selection combining (PDSC-3), for $m = 0.5, 0.75, 1, 1.5, 2$ and 3.

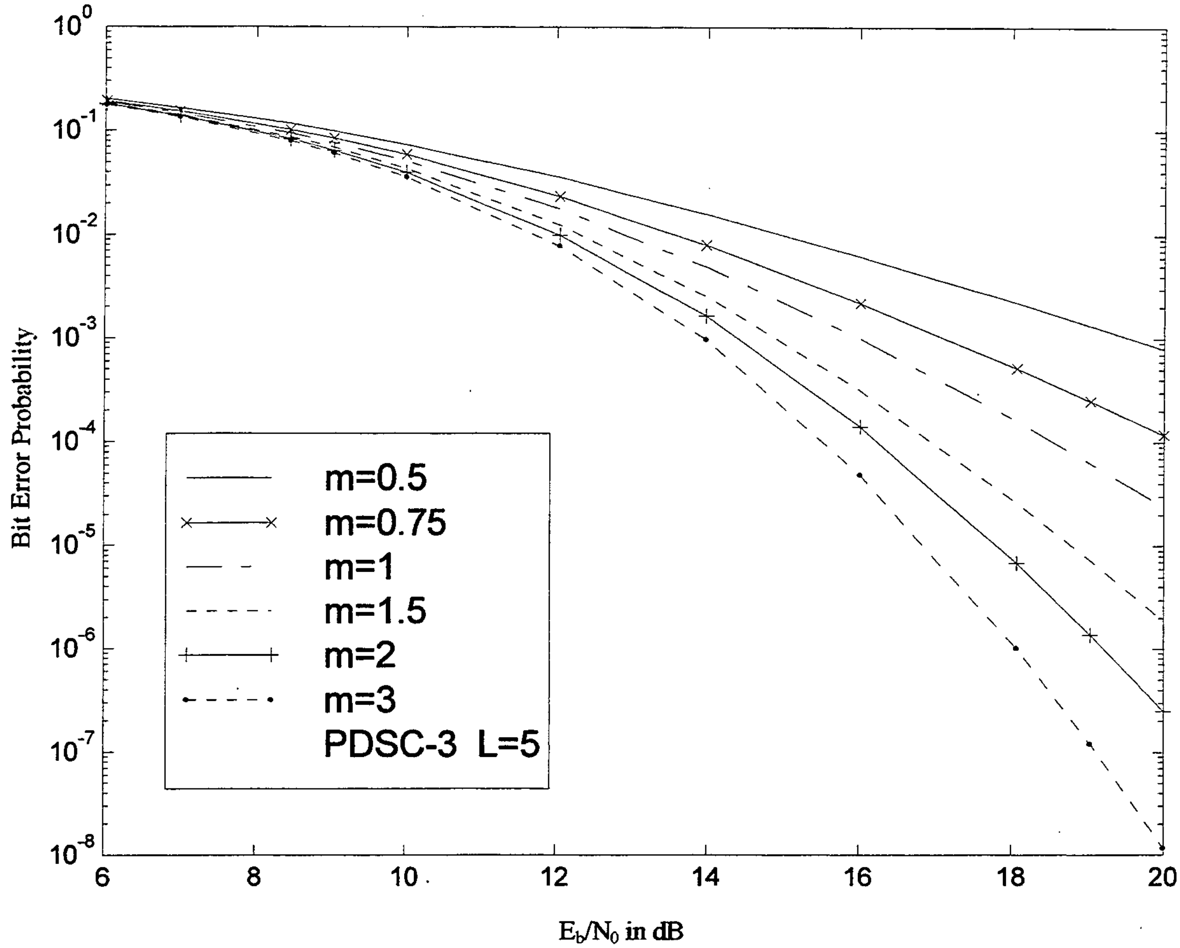

Figure 37. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with diversity order $L = 5$, using third order post detection selection combining (PDSC-3), for $m = 0.5, 0.75, 1, 1.5, 2$ and 3.

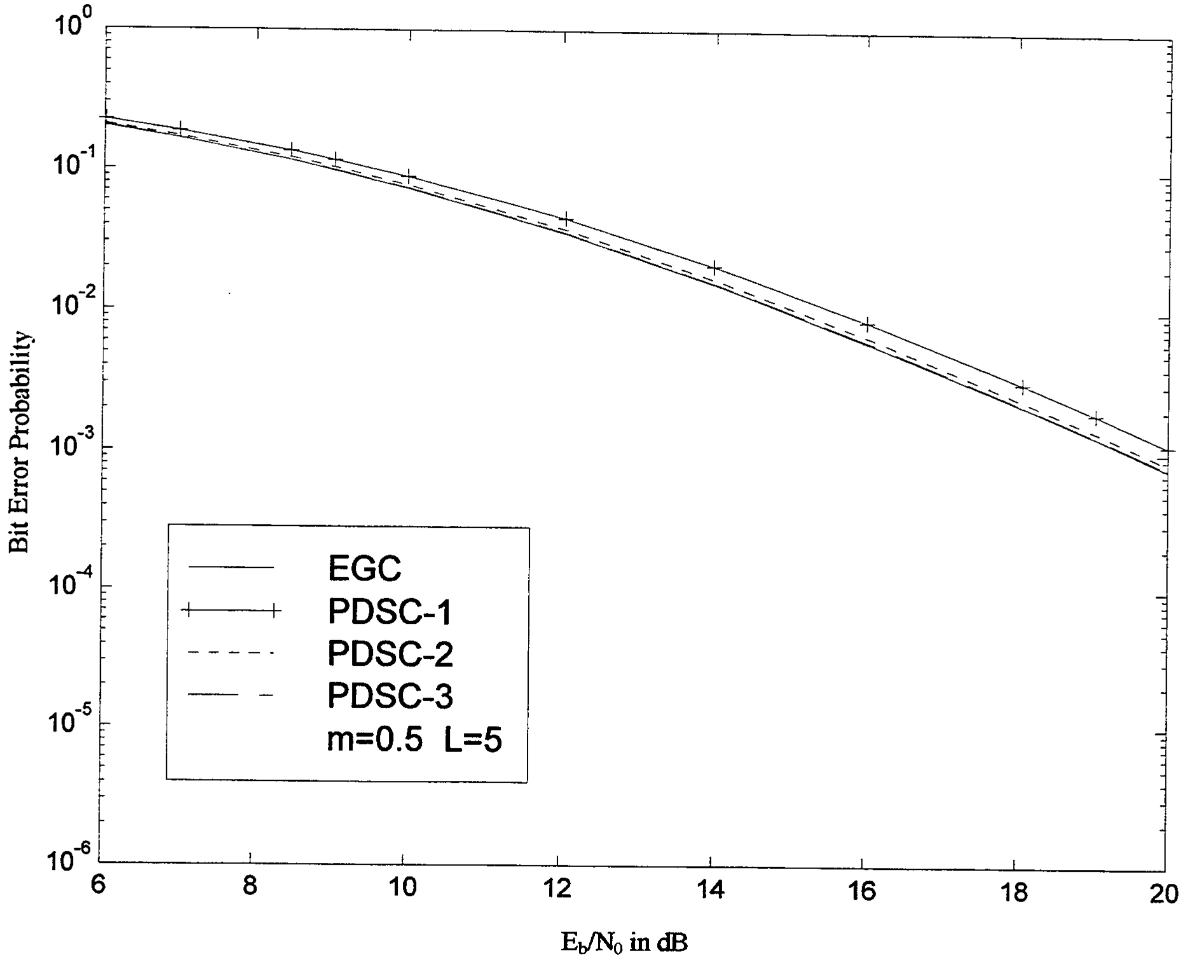

Figure 38. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with diversity order $L = 5$ and $m = 0.5$, using EGC, PDSC-1, PDSC-2 and PDSC-3.

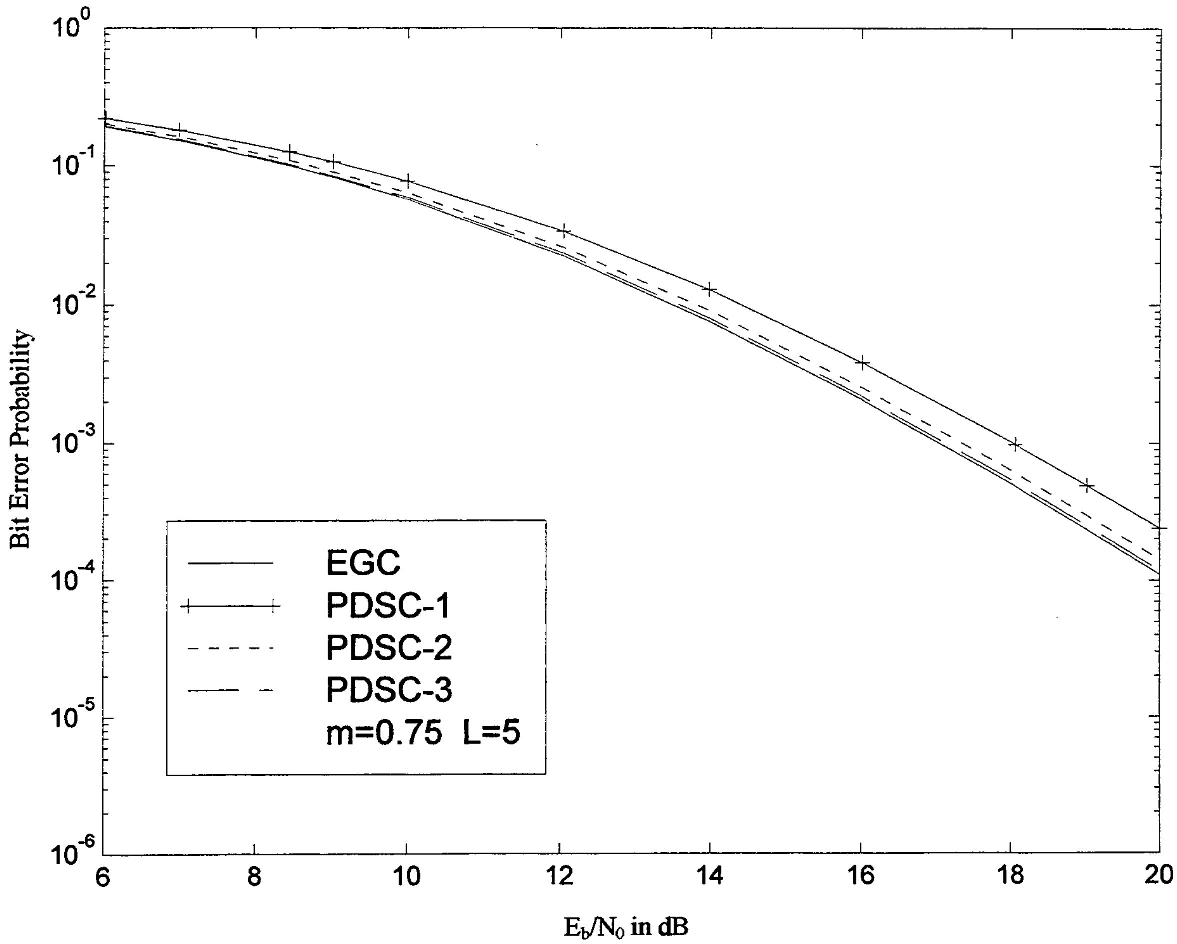

Figure 39. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with diversity order $L = 5$ and $m = 0.75$, using EGC, PDSC-1, PDSC-2 and PDSC-3.

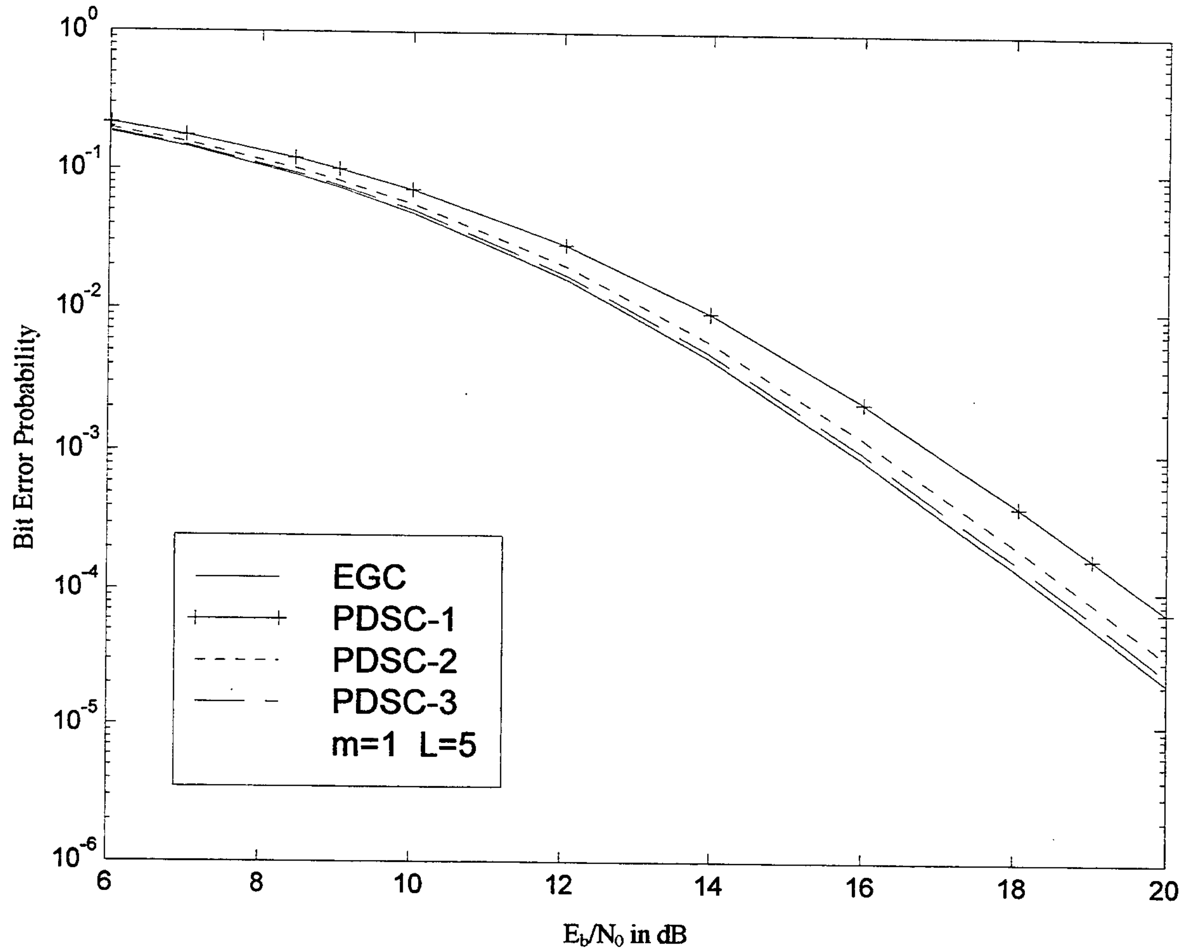

Figure 40. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with diversity order $L = 5$ and $m = 1$ using EGC, PDSC-1, PDSC-2 and PDSC-3.

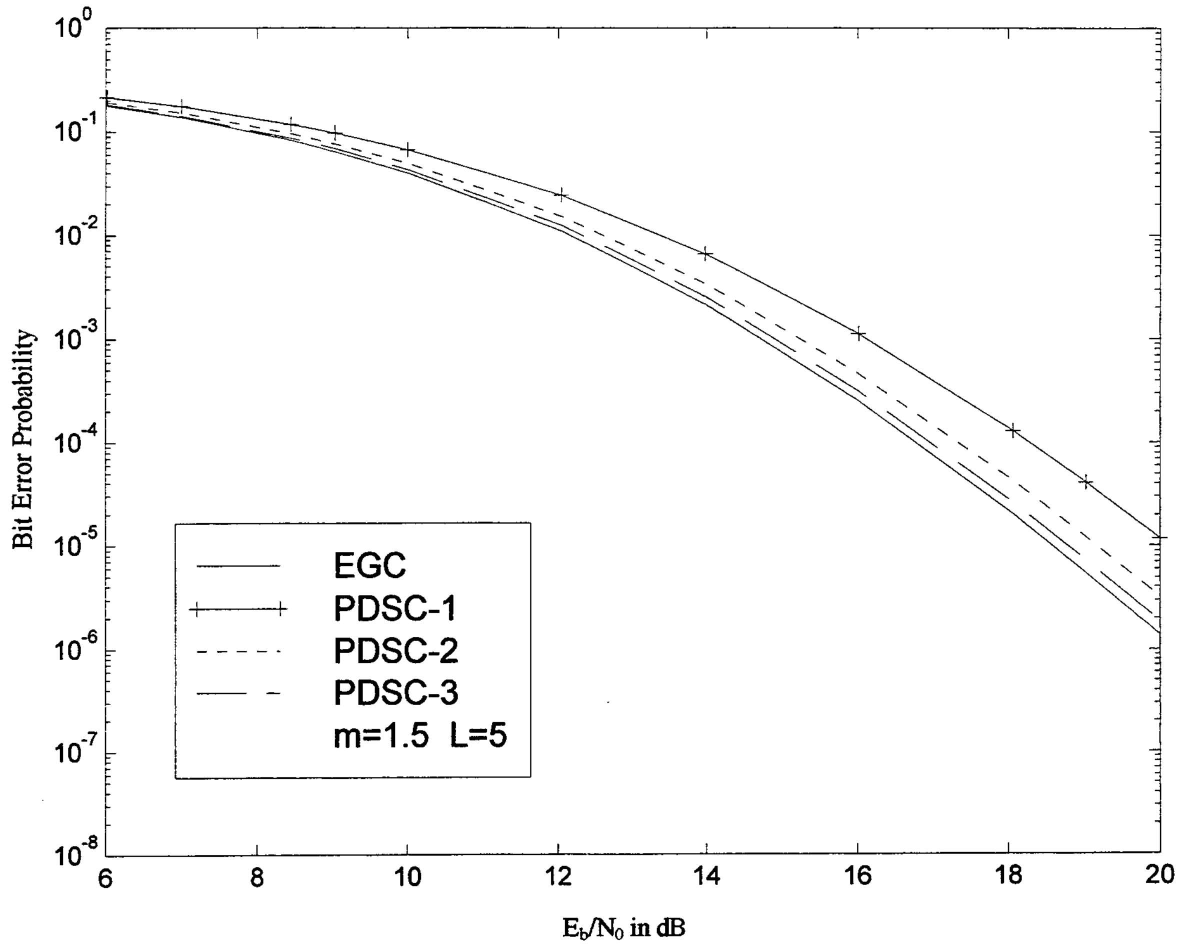

Figure 41. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with diversity order $L = 5$ and $m = 1.5$, using EGC, PDSC-1, PDSC-2 and PDSC-3.

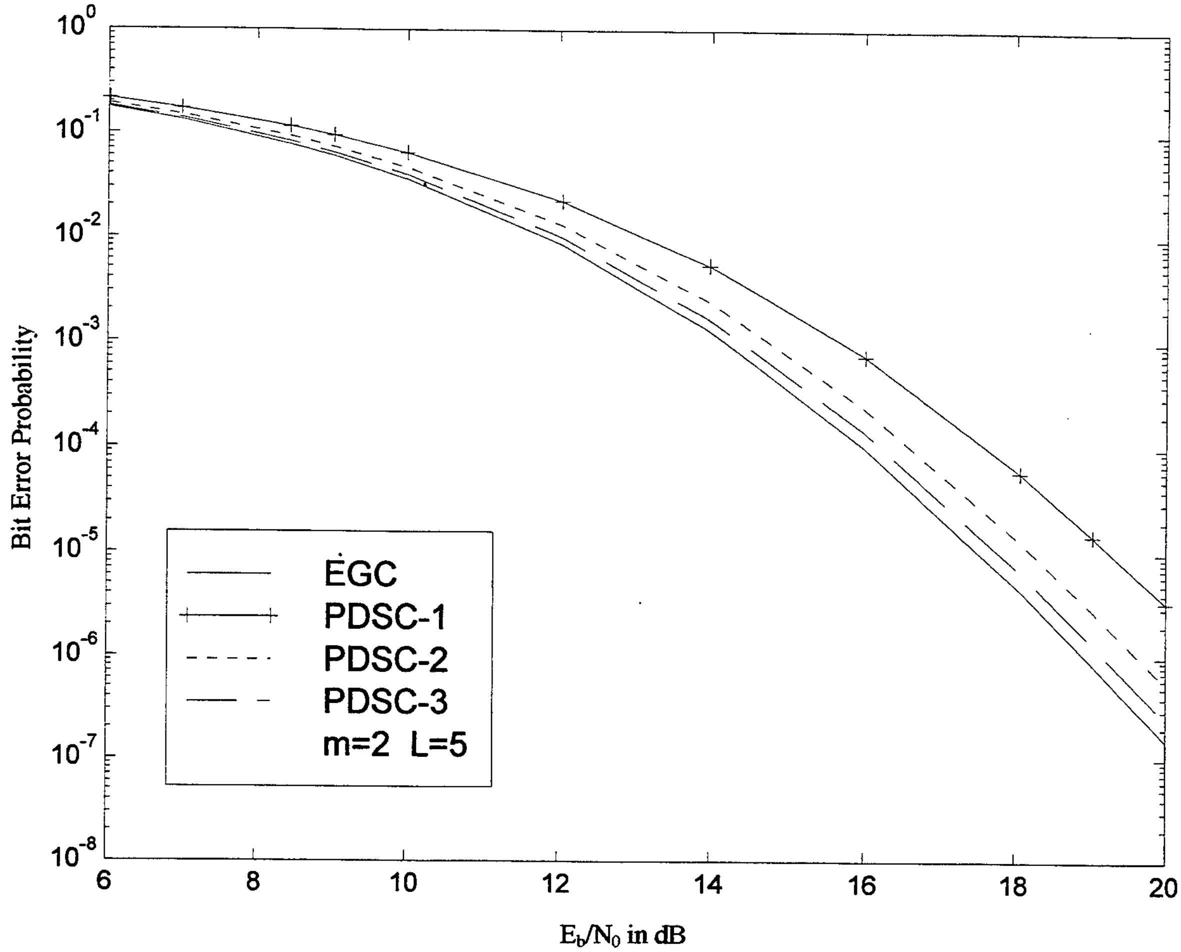

Figure 42. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with diversity order $L = 5$ and $m = 2$ using EGC, PDSC-1, PDSC-2 and PDSC-3.

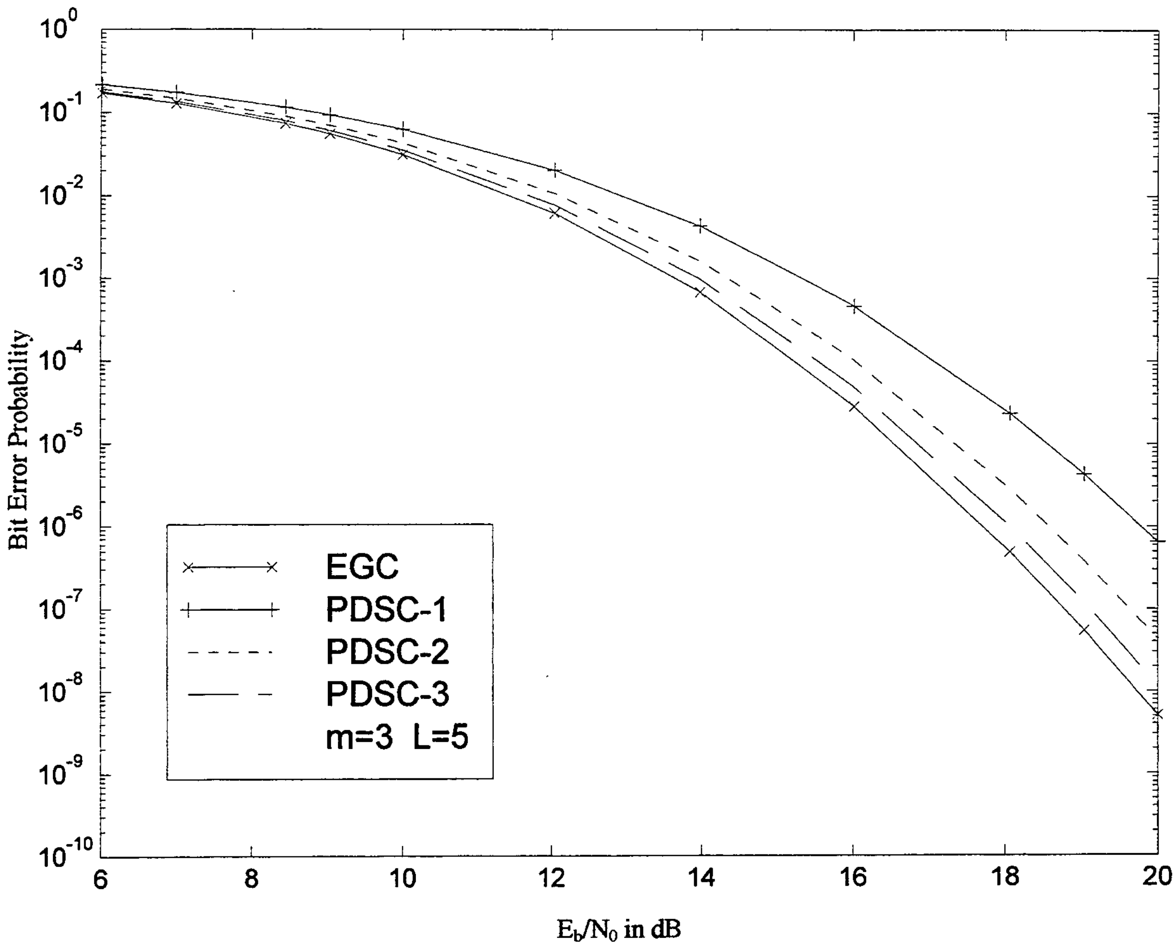

Figure 43. Performance of the non-coherent BFSK receiver over a Nakagami fading channel with diversity order $L = 5$ and $m = 3$ using EGC, PDSC-1, PDSC-2 and PDSC-3.

VII. CONCLUSIONS

In this thesis the bit error probability performance of a non-coherent BFSK receiver with L order diversity in a frequency non-selective slowly fading Nakagami fading channel is compared using EGC, PDSC-1, PDSC-2 and PDSC-3.

The EGC technique is widely used in communication systems that use non-coherent demodulation, but as we have already seen, it suffers from non-coherent combining loss as L increases and is path dependent. In addition to these, it requires complex receivers.

The PDSC techniques have been suggested by professors Tri T. Ha and Ralph D. Hippenstiel as simpler techniques that can provide adequate performance without an explicit L dependency. We have shown that as the order of the PDSC technique increases the performance of the receiver improves, so that it is comparable to that of the EGC technique. However, the receiver complexity is also increased. The PDSC techniques are not optimal combining techniques since they do not use all available information from all L diversity branches. On the other hand, since they are not L dependent, they may be preferable in applications where the value of the diversity order L varies with time or location.

LIST OF REFERENCES

[1] J.G. Proakis, *Digital Communications*, 3rd ed. New York, NY: McGraw-Hill, 1995.

[2] A. Papoulis, *Probability, Random Variables and Stochastic Processes*, New York, NY: McGraw-Hill, 1991.

[3] I.S. Gradshteyn and I.M. Ryzhic, *Table of Integrals, Series and Products*, Corrected and Enlarged ed., New York, NY: Academic Press, 1980.

[4] Tri T. Ha, Communications Engineering, Class Notes from EC3510, Naval Postgraduate School, Monterey, CA, January 1997.

[5] Tri T. Ha, Digital Communications, Class Notes from EC4550, Naval Postgraduate School, Monterey, CA, July 1997.

[6] MATLAB 5.1, The MathWorks, Inc., Natick, MA, 1996.

[7] MATHCAD 7, The MathSoft, Inc., MA, 1997.

[8] T. Eng, N. Kong and L.B. Milstein, "Comparison of Diversity Combining Techniques for Rayleigh-Fading Channels," *IEEE Trans. Commun.*, vol. 44, no. 9, pp. 1117-1129, September 1996.

[9] T.S. Rappaport, T.S., *Wireless Communications, Principles and Practice,* Upper Saddle River, NJ: Prentice-Hall, 1996.

INITIAL DISTRIBUTION LIST

No. Copies

1. Defense Technical Information Center....................................2
   8725 John J. Kingman Rd., Ste 0944
   Ft. Belvoir, VA 22060-6218

2. Dudley Knox Library..2
   Naval Postgraduate School
   411 Dyer Rd.
   Monterey, CA 93943-5101

3. Chairman, Code EC..1
   Department of Electrical and Computer Engineering
   Naval Postgraduate School
   Monterey, CA 93943-5121

4. Professor Tri T. Ha, Code EC/Ha..2
   Department of Electrical and Computer Engineering
   Naval Postgraduate School
   Monterey, CA 93943-5121

5. Professor Ralph D. Hippenstiel, Code EC/Hi.............................2
   Department of Electrical and Computer Engineering
   Naval Postgraduate School
   Monterey, CA 93943-5121

6. Embassy of Greece..1
   Naval Attache
   2228 Massachusetts Ave. N.W.
   Washington DC 20008

7. Pierros Kontodios..3
   4 Ellinwn Olympionikwn, Alimos 17455
   Athens, Greece