OTS: 60-11,488

JPRS: 2503

26 April 1960

SELECTED TRANSLATIONS ON SOVIET SPACE RESEARCH

- USSR -

Distributed by:

OFFICE OF TECHNICAL SERVICES
U. S. DEPARTMENT OF COMMERCE
WASHINGTON 25, D. C.

Price: $0.50

U. S. JOINT PUBLICATIONS RESEARCH SERVICE
205 EAST 42nd STREET, SUITE 300
NEW YORK 17, N. Y.

JPRS: 2503
CSO: 3621-N/a

SYNCHROTRONIC RADIATION IN COMETARY NEBULAE

(Submitted by Academician V. A. Ambartsumyan, Sept. 11, 1959)

/Following is a translation of an article by G. A. Gurzadyan which appeared in Doklady Akademii nauk SSSR (Papers of the Academy of Sciences USSR) 1960, Vol. 130, No. 1./

Among the galactic nebulae, cometary nebulae constitute a small, but interesting group. The distinguishing features of these objects are primarily their external shape (comet-shaped or cone-shaped) and the irregular variability of their brightness and structure. The bright part of cometary nebulae is usually of magnitudes comparable to the apparent magnitudes of large planetary nebulae, while the star producing their luminosity belongs, as a rule, to stars of low luminosity (type A and later); it is difficult to reconcile the latter circumstance also with the fact of the presence of emission lines of hydrogen in the spectra of some cometary nebulae. Finally, the emission of a strong continuous spectrum, sometimes interrupted by absorption lines of hydrogen, also characterizes cometary nebulae.

V. A. Ambartsumian was the first to pay serious attention to the peculiarities of radiation of cometary nebulae; he pointed out that the luminosity of these objects is not of a thermal nature (1). Subsequent investigation of this problem indicates that the luminosity of cometary nebulae could be caused by the decelerating radiation of relativistic electrons in the magnetic fields of the nebula (synchronous radiation). It is necessary to note in this connection that an analysis of the proposed hypothesis is by far not limited to a simple calculation of the concentration of relativistic electrons and to finding their spectrum. The hypothesis of relativistic electrons has been found to provide a good explanation of a number of facts pertaining to cometary nebulae, whereas the more widespread hypothesis of the reflection of the light of the nucleus by the dust particles of the nebula does not provide such an explanation. Dome of the results obtained by the author on this problem are cited in this article.

Sometimes, from the reverse side of the cone of a cometary nebula symmetrical to the nucleus, we observe the appearance of the same kind of cometary shape (for example, in NGC 2261, 2245). This causes the nebula to assume the character of a sort of bipolarity which, however, should not be identified with the structural bipolarity of some planetary nebulae (2), but it should also not be considered to be a fortuitious phenomenon.

It can be concluded from this fact that a cometary nebula "rests" with its end upon determinable regions of a star-nucleus, mainly in the region of the magnetic pole.

However, the structure of the magnetic field in this case differs somewhat from the one assumed for stars. It can be shown that in order to explain the shape and the ob served extensiveness of a cometary nebula, its nucleus should possess either a magnetic field situated eccentric to the center of the star, or, and this is difficult to reconcile with our contemporary assumptions of the nature of magnetism, it should possess a unipolar magnetic field.

If we present the magnetic field strength in a given direction in the shape $H \sim r^{-n}$, then $n = 3$ with a dipolar field and $n = 3/2 \div 2$ with a unipolar field, i.e., the gradient of the magnetic field will be much smaller in the latter case than in the former. Another cause of a decrease in the gradient of the magnetic field is that the substance itself, which has been ejected from the region of the star's pole, could carry a magnetic field with it. As a result of these causes, the relativistic electrons emerging from the region of the pole, could produce synchrotronic radiation even at considerable distances from the nucleus, if the intensity of the magnetic field at the pole is of the order of 10^4 gauss.

Magnetic force lines emanating from the pole in the direction of the magnetic axis (or the rotational axis of the star, if they are close to each other), are approximately in the shape of straight rays. The electrons, flying from the pole toward the force lines at some angle, will wind around the latter, describing a spiral trajectory. The trajectory radius should gradually increase as the electron moves away from the pole, since the field intensity decreases with distance.

A change in the radius is tantamount to a change in the radiation frequency of the relativistic electron. Therefore, the electron at a given magnitude of its energy E emits invisible ultrashort waves in regions close to the nucleus, and invisible infrared and radio waves in regions remote from the nucleus.

It follows from this that an electron of a given energy can produce radiation in the optical range only at a definite distance from the nucleus.

For example, at $E = 10^{11}$ electron volt, the optical range from 3,200 A to 7,000 A radiates within a range of intensity from $H_1 = 2.2 \cdot 10^{-2}$ gauss to $H_2 = 5 \cdot 10^{-3}$ gauss.

Although we do not know the exact law for the decrease in intensity of the magnetic field of the star's pole with distance, there is no doubt that at some distance from the nucleus r_1 the first condition will be satisfied, and at r_2 the second condition. At a different magnitude of E, we shall obtain different magnitudes of r_1 and r_2.

Thus, depending on the composition and uniformity of the beam of relativistic electrons, maximum brightness may occur at any distance from the nucleus.

When several beams are simultaneously present and they are more or less uniform (monochromatic), but with different values of E, there are several maxima of varying brightness and at different distances from the nucleus. Even during small oscillations in the composition of beams, the bright regions (spots) will drift, and may even disappear.

Finally, if the energy spectrum of the relativistic electrons is continuous, then instead of spots, there is a continuous nebula with a brightness which decreases monotonously with growing distance from the nucleus.

The irregular changes of the brightness and structure of cometary nebulae were considered one of the incomprehensible phenomena of their nature. As we can see, the hypothesis of relativistic electrons ejected from the nucleus of the nebula gives a simple and conclusive explanation of this phenomemon. The picture of a changing brightness and structure will become even clearer when we take into account also the effect of the star's rotation and the effect of possible oscillations of the magnetic field intensity.

The mean concentration of relativistic electrons N_e in the nebula is determined from the assumption that the integral brightness of the nebula is a function of continuous synchronous radiation. Assuming that the spectrum of relativistic electrons is continuous and that it is represented by the form

$$N_e = KE^{-\gamma}$$

for concentrations in the nebula of relativistic electrons which have an energy greater than E_0, we will have

$$N_e(E > E_0) = \frac{1}{\gamma - 1} \frac{K}{E_0^{\gamma - 1}}, \qquad (1)$$

Where K is determined from the relation

$$K = \frac{H^{-\frac{\gamma+1}{2}}}{c(\gamma)} \frac{F_{\odot}}{\Omega \Delta\nu_{pg} R} 10^{-0.4(m_{pg}-m_{\odot})} \nu^{-\frac{1-\gamma}{2}}, \qquad (2)$$

where H is the magnetic field intensity in the middle sections of the nebula; R is the mean linear extension of the nebula along the visual ray; Ω is the apparent surface of the nebula in steradians; $m_{\odot}$ is the apparent bolometric magnitude of the sun; $F_{\odot}$ is the total stream of radiation of the sun; m_{pg} is the integral photographic stellar magnitude of the nebula; $\Delta\nu_{pg}$ is the latitude of the spectral interval in frequency units; ν is the mean frequency of the photographic region of the spectrum.

C(γ) is a function depending on γ and assuming values

γ	2	3	4	5
$C(\gamma)$	$0,47 \cdot 10^{-19}$	$0,95 \cdot 10^{-4}$	$2,85 \cdot 10^{6}$	$8,70 \cdot 10^{14}$

The application of (1) and (2) to a known cometary nebula NGC 2261 yields at $\Omega = 5' \times 5'$, $m_{pg} \approx 10^m$(3), $R = 10^{18}$ cm and $\gamma = 3$ $N_e(E > 10^{12}) \approx 10^{-12} cm^{-3}$ at $H = 10^{-3}$ gauss.

The ionization of hydrogen atoms in cometary nebulae takes place under the effect of ultraviolet synchrotronic radiation which is generated in the nebula. Proceeding from this, it is possible to develop the following formula to determine the electron concentration of common (thermal) electrons in the nebula:

$$n_e^2 = \frac{\omega_\alpha A(\gamma)}{z_3 A_{32} h} \nu_\alpha^{-\frac{1+\gamma}{2}} \qquad (3)$$

where $A(\gamma) = C(\gamma) K H^{\frac{\gamma+1}{2}}$; ω_α and ν_α are the equivalent latitude and frequency of the emission line Hα; $z_3 = n_3/n^+ n_e$ and it is taken from (4); A_{32} is the Einstein coefficient of spontaneous transition of hydrogen $3 \rightarrow 2$. For MGC 2261, this formula gives at w_α = 126A = $0.88 \cdot 10^{13}$ sec^{-1} (5) and γ=3 $\approx$13 cm^{-3}, which is less by two orders than the electron density of planetery nebulae.

There is reason to assume that the optical thickness τ_c in the frequencies of ultraviolet radiation (L_c) is, for some cometary nebulae (NGC 2261) much greater than unity. The formula of ionization of hydrogen in the nebyla, when ionization takes place by means of synchrotronic short-wave radiation, at $\tau_c \gg 1$ in the shape

$$\frac{n^+}{n_1} n_e = \frac{1}{4\pi} \frac{A(\gamma)}{\gamma+3} \frac{c^2 (2\pi\mu)^{3/2} (kT_e)^{1/2}}{n_1 \varkappa_c h^3} \nu_0^{-\frac{\gamma-3}{2}}. \qquad (4)$$

Proceeding from the observations of equivalent latitude of absorption lines H_3 in the spectrum of the cometary nebula, we can determine the concentration of neutral hydrogen atoms within it by means of the formula

$$n_1^2 = 0{,}115 \frac{X_0(w_\lambda)}{R^2 \omega s_1 s_2} \frac{\gamma - 1}{RA(\gamma)} \nu_0^{\frac{\gamma-1}{2}}, \qquad (5)$$

where $X_0(\omega_\lambda)$ is determined from the curve increase formula (6)

$$\frac{w_\lambda}{\lambda} = 2X_0 \frac{\Delta\nu_D}{\lambda} \int_0^\infty (e^{p^2} + X_0)^{-1} dp; \qquad (6)$$

s_1 and s_2 are the coefficients of selective absorption, correspondingly in the frequencies of lines L_α and H_δ; $\omega = 5 \cdot 10^4$; ν_0 is the frequency of ionization of hydrogen; x_0 is the coefficient of continuous absorption per one hydrogen atom.

The application of (4), (5) and (6) to NGC 2261 yields at $w(h_\delta) = 3Å(5)$ and $\gamma = 3$ $n_1 = 170$ cm^{-3}, $n^+/n_1 \approx 0.1$.

Thus, in NGC 2261, the overwhelming part of hydrogen atoms is in a neutral state, and the degree of ionization is very low. Under these conditions, the optical thickness of the nebula in frequencies of L_c-radiation is obtained $\sim 10^3$, and in the frequencies of lines of the Balmer series of hydrogen, of the order of unity.

Synchrotronic radiation should be polarized, and the plane of polarization should be perpendicular to the magnetic intensity line at the given point, and the theoretical degree of polarization very high -- of the order of 70 percent (7). Since the magnetic intensity lines of the unipolar field in the region of the cone of the cometary nebula are in the approximate shape of straight rays, therefore its continuous radiation should, in the first approximation, be polarized radially to the nucleus, and the degree of polarization should be sufficiently high.

What has been said above is confirmed by data of polarimetric investigations made in respect to NGC 2261, by E. Ye. Khachik'yan (3) and N. A. Razmadze (8); they obtained radial polarization. The mean degree of polarization over the entire nebula was found to be 16 and 19 percent, respectively, and in some points of the nebula it reached 50 to 60 percent. Thus, the results of polarimetric investigations also speak in favor of the synchrotronic nature of the luminosity of cometary nebulae.

The hypothesis of synchrotronic radiation of the luminosity of cometary nebulae gives also a satisfactory expoanation of other phenomena observed therein, and in particular of the formation in the spectra of lines of emission and lines of absorbtion.

Byurakansk Astrophysical Observatory of the Academy of Sciences ArmSSR

Received September 3, 1959.

Bibliography

1. Ambartsum'yan, V. A. Soobsh. Byurakansk. obs. (Publications of the Byurakansk Observatory), 13, 3(1954).
2. Gurzadyan, G. A. DAN, 120, No. 4, 734(1958).
3. Khachikyan, E. Ye. Soobshch. Byurakansk. obs. 25, 67 (1958).
4. Cillie, G. M.N., 92, 771(1936).
5. Greenstein, J. Ap. J., 107, 375(1948).
6. Ambartsum'yan, V. A., Mustel', E. R., Severnyy, A. B., Sobolev, V. V. Teoreticheskaya astrofizika (Theoretical Astrophysics), Moscow, 1952.
7. Gaibyan, G. M., Goldman, I.I. Izv. AN ArmSSR, ser. fiz.-matem. cestestv. i tekhn. nauk (Herald of the Acad. Sci., ArmSSR, Series of Physical-Mathematical-Natural and Technical Sciences), 7, 31(1954).
8. Razmadze, N. A. Byull. Abastumansk. obs. (Bulletin of the Abastumansk Observatory), No. 25(1959).

CSO: 3621-N/b

ULTRAVIOLET RADIATION AND THE EXCITATION OF OXYGEN LINES IN THE CHROMOSPHERE

[Following is a translation of a paper written by G. M. Nikol'skiy and published in Doklady Akademii nauk SSR (Papers of the Academy of Sciences USSR), Astronomiya (Astronomy), Vol. 130, No. 1, 1960.]

(Presented by Academician V. G. Fesenkov, on August 25, 1959)

Multiplets of OI, (the system of quintets at λλ 7771.95, 7774.18, and 7775.39 and the system of triplets at λλ 8446.35, 8446.73 and 8447.66) are characterized by a fairly high excitation potential (about 10 ev) for their upper states. Figure 1 shows an energy level diagram for OI. The lower states of the multiplets, on coupling with the adjacent ground state, produce lines at λλ 1302.2, 1304.8, and 1306.0 (triplets) and at λλ 1355.6 and 1358.5 -- forbidden lines with respect to spin. American rocket observations(1) have recorded both groups of ultraviolet lines in the solar spectrum. Since the continuous solar emission in the wavelength interval corresponding to 10 ev is negligible, an electron collision must be source of excitation in the initial states for λλ 8446 and 7774 ($^3P_{2,1,0}$ and $^5P_{1,2,3}$).

However, in the case of triplets, a downward cascade process from the next higher level $^3D_{3,2,1}$ may be the mechanism which populates the initial state $^3P_{0,1,2}$ for the λ 8446 emission.

The $^3D_{3,2,1}$ level is excited by the chromospheric emission in the L_β line at 1025.73 A.

For all intents and purposes, here we are dealing with a resonance process: the energy of level ^3D corresponds to λ 1025.77 A, while the width of the L_β line is somewhat less than 1 A.

This mechanism, in its application to the oxygen of the earth's atmosphere, was first examined by P. S. Shklovskiy (2). But for a number of reasons, the attempts to discover the expected (according to (2)) twilight burst at λ 8446 have proved unsuccessful.

Let us examine the action of this mechanism under solar atmosphere conditions. With this in mind, by solving the microstability equation systems separately for the OI triplets and quintets, we will find the occupation number of the initial states for $\lambda\lambda$ 8446 and 7744 [sic], taking into account the $L\beta$ radiation. We shall assign numerals to the states (Fig. 1) and limit our examination to four. For simplicity we can neglect the loosely related processes. The solution takes the form:

$$\left(\frac{n_3}{n_1}\right)_{8446} = \frac{A_{43}}{A_{32}A_{41}}\left(B_{14}\rho_{L\beta} + b_{13}\right), \quad (1)$$

$$\left(\frac{n_3}{n_1}\right)_{7774} = \frac{1}{B_{34}\rho_{8446}} b_{12}. \quad (2)$$

Here, n_1 is the occupation number of the ground states ^3P; A and B are the Einstein coefficients; b is the electron collision excitation factor; and ρ is the radiant excitation energy. Let us note that in deriving (1) and (2), we have taken into account the inequalities $A \gg B\rho \gg b$ which are valid for the chromosphere (with the exception of $\rho = \rho_{L\beta}$ and A for intercombination transitions).

To evaluate the terms in Expression (1), we shall assume that the chromospheric emission in $L\beta$ can be represented as black-body radiation with T about 5,000°; the electron temperature and concentration in the lower chromosphere are T_e = 5,000° and N_e about 10^{12} cm^{-3}; the mean magnitude of the effective cros section, excited by electron collision, is of the gas knnetic order. In general terms then,

$$B_{14}\rho_{L\beta} \sim A_{41} W\, 10^{-\frac{5040}{T}\chi_{14}}, \quad b_{13} \sim N_e\bar{\sigma}\left(\frac{8kT_e}{\pi m}\right)^{1/2} 10^{-\frac{5040}{T_e}\chi_{12}}, \quad B_{14}\rho_{L\beta}/b_{13} \sim 10^3. \quad (3)$$

It should be noted that this evaluation is a strong function of both the assumed values for the $L\beta$ -radiation intensity and the electron temperature T_e of the chromosphere.

Nevertheless, the emission lines at λ 8446 must be substantially stronger in the regions of the chromosphere with increased radiant energy of $L\beta$ radiation.

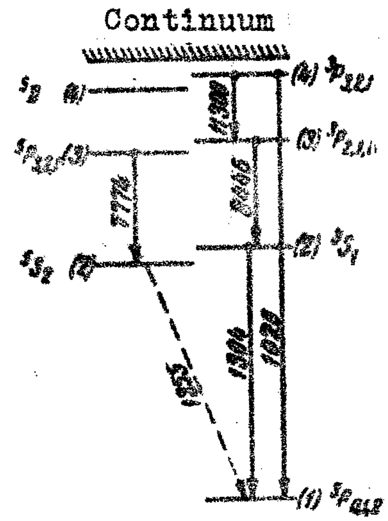

Fig. 1. OI energy level diagram. Triplets and quintets.

How great is the ratio of the intensities in the chro mospheric lines I_{8446}/I_{7774}? When λ 8446 is not excited by $L\beta$ radiation, this ratio must be small. This follows from the metastability of the lower level ^5S for λ 7774 and from the effective discharge -- the permitted transition of excited atoms from the analogous state ^3S for λ 8446 to the ground state ^3P.

When the absorption lines $\lambda\lambda$ 8446 and 7774 appear in the dense layers of the solar atmosphere, excitation is determined primarily by collisions and I_{8556}/I_{7774}8 must be close to unity. This is confirmed by an evaluation of the equivalent widths, using (4 to 6).

The features of the excitation of chromospheric lines at 8446 make possible the measurement of the intensity of $L\beta$ in various regions of the chromosphere. Despite certain definite problems in observing the extremely weak infrared lines of OI in the spectrum of the chromosphere when there is no eclipse, this method has a number of advantages when compared to rocket observations of $L\beta$.

Let us note that because of the great optical density of the chromosphere in the lines of the Lyman series this method only provides data on the radiant energy of the $L\beta$ radiation in those layers of the chromosphere where the emission line λ 8446 is formed.

In setting up this type of observation, it is convenient to record $\lambda\lambda$ 8446 and 7774 at the same time, since the ratio of intensities according to (1), (2) and (3), is independent of electron concentration and is a considerably weaker function of electron temperature than each line by *[sic].

itself.

The condition $I_{8446}/I_{7774} > 1$ is a direct indication of the additional excitation at λ 8446 due to L_β radiation.

Institute of Terrestrial
Magnetism, Ionosphere and
Radiowave Propagation of
the Academy of Sciences USSR.

Submitted
August 20, 1959

Bibliography

1. Johnson, F. S., Maltison, H. H., et al. Ap. J., 127, No. 1(1958).
2. Shklovskiy, I. S. Astr. Zhurn., (Astronomical Journal), 34, No. 1, 127(1957).
3. Van de Hulst, Solntse (The Sun), Chapter V, Foreign Literature Press, 1957.
4. Minnaert, M., Mulders, G. F. W., Houtgast, J. Photometric Atlas of the Solar Spectrum, Amsterdam, 1940.
5. St. John, Ch. E., Moore, Ch. E., et al. Revision of Rowland's Preliminary Table of Solar Spectrum, Carnegie Inst. of Washington, 1928.
6. Mulders, G. F. W. Zs. Astrophys., 10, 306(1935).

CSO: 3621-N/c

GRAVITATIONAL INSTABILITY IN PLANE ROTATING SYSTEMS HAVING AXIAL SYMMETRY

(Submitted by Academician L. I. Sedov, Aug. 31, 1959)

[Following is a translation of an article by V. S. Safronov which appeared in Doklady Akademii nauk SSSR (Papers of the Academy of Sciences USSR), 1960, Vol. 130, No. 1.]

The generalization of Jeans'(1) classical condition of gravitational instability was made by Chandrasekhar (2). It was an analysis of an infinite uniform medium rotating about a certain axis z at an angular velocity which is not a function of the distance from the axis. Chandrasekhar found that the Jeans criterion remains valid for disturbances spreading out in all directions, except those which are strictly perpendicular to the axis of rotation. In the case of disturbances perpendicular to the axis, gravitational instability occurs only if the density exceeds the critical value

$$\rho > \rho_{\kappa p} = \frac{\omega^2}{\pi G}. \qquad (1)$$

This result is, however, inapplicable to actual rotating systems which commonly appear to be very flat.

They cannot possess instability about z, and we can only speak of a spread of disturbances and of the emergence of instability in a plane perpendicular to the axis of rotation, i.e., precisely when the Jeans criterion is inapplicable.

However, even for that instance the quantitative estimate of Chandrasekhar (1) should be subjected to a reappraisal. First of all, because in actual systems the velocity of rotation is not constant and depends on r, if only for the reason that there is usually a concentration of matter toward the center, and secondly because the dimension of the system in the direction of axis z is considerably smaller than the dimensions in directions perpendicular to z. The physical unreality of systems extending infinitely in the direction of the axis of rotation is particularly emphasized by K. F. Ogorodnikov (3).

The following approximation has actually been made by Bel and Schatzman (4). They assume that the velocity of rotation of a system is a function of r. Analysis was made of disturbances which spread in a plane perpendicular to the axis of rotation and which are summetrical relative to this axis.

However, the authors only analyzed a two-dimensional instance and the Poisson equation for the potential was written in the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\delta\varphi\right) = -4\pi G\delta\rho. \qquad (2)$$

This means in reality that the system here considered was also extending infinitely along axis z. Since the gravitational force of a flat circle is considerably lower than the gravitation of the corresponding infinite cylinder, it should be expected that the condition of gravitational instability obtained by the authors

$$4\pi G\rho > \frac{2\omega}{r}(\omega r^2)' + \frac{4\pi^2 c^2}{\lambda^2} + \frac{c^2}{4r^2} \qquad (3)$$

would give a lowered value of the critical density.

It is the purpose of this article to find the value of the critical density for a real flat rotating cloud. We shall assume, like the aforementioned authors, that the axial symmetry of the cloud is maintained at all times, and that consequently the disturbances are radial (circular). An analysis of condition (3) shows that it represents a balance of forces acting upon an element which was displaced, as a result of a wave disturbance, in a radial direction by a magnitude $\delta r = 1$, without a change in its moment of quantitative motion relative to the center of the system. The left-hand member characterizes the gravitation of the cylindrical circle in agreement with the Poisson equation (2), the first member on the right-hand side is a force which returns the displaced element to the former orbit and which is associated with the stability of circular orbits, the second member on the right-hand side is the gradient of gas pressure produced as a result of the disturbance, i.e., also a force returning the element, and the last member is very small compared to the prior, and can be neglected.

Going over from a system which is infinite along z to flat systems, only the left-hand member in the inequality (3) will change which is associated with gravitation. In order to determine the component force of gravitation down the r of the circle and having a density $\delta\rho$, it is now inconvenient to use the Poisson equation, since the additional component

$d^2 \delta\psi/dz^2$ will enter into the left-hand side (2). It is therefore expedient to calculate δF_r directly.

Let r be the distance from the axis of rotation, h -- the distance from the central plane of the cloud. The component of the gravitational force along r at point r_0 in the central plane, the force being the result of the disturbance $\delta\rho$, equals

$$\delta F_r = G \int_{r_0-\lambda/4}^{r_0+\lambda/4} \int_{-h_0}^{+h_0} \int_{0}^{2\pi} \frac{\delta\rho\,(r\cos\varphi - r_0)\, r\, dr\, dh\, d\varphi}{(r^2 + r_0^2 + h^2 - 2rr_0\cos\varphi)^{3/2}}. \tag{4}$$

Integrating (4) along φ brings us to elliptical integrals of the first and second order

$$\delta F_r = 4Gr_0 \int_{-y_0}^{+y_0} \int_{0}^{z_0} \frac{\delta\rho\,(1+y)}{[(2+y)^2 + z^2]^{1/2}} \left[\frac{2y + y^2 + z^2}{y^2 + z^2} E - K\right] dy\, dz, \tag{5}$$

where

$$y = \frac{r - r_0}{r_0}, \quad y_0 = \frac{\lambda}{4r_0}, \quad z = \frac{h}{r_0}, \quad z_0 = \frac{h_0}{r_0},$$

$$K = \int_0^{\pi/2} \frac{d\psi}{\sqrt{1 - k^2\sin^2\psi}}, \quad E = \int_0^{\pi/2} \sqrt{1 - k^2\sin^2\psi}\, d\psi, \quad k = \frac{4(1+y)}{(2+y)^2 + z^2}.$$

When the length of the disturbing wave λ and of the semi-thickness of layer h_0, which are small in comparison with distance r_0 from the axis of rotation, magnitudes y and z are small, and k is close to unity. Elliptical integrals and other functions entering (5) can be expanded into a series and limited to the first members (5):

$$E = 1 + \frac{1}{2}\left(\Lambda - \frac{1}{2}\right)k'^2 + \ldots, \quad K = \Lambda + \frac{\Lambda - 1}{4}k'^2 + \ldots,$$

where $k'^2 = 1 - k^2$, $\Lambda = \ln\frac{4}{k'}$. With an accuracy to the infinitesimals of the second order y^2, z^2

$$E \approx 1 + \frac{1}{16}[6\ln 2 - 1 - \ln(y^2 + z^2)](y^2 + z^2),$$

$$K \approx 3\ln 2 - \frac{1}{2}\ln(y^2 + z^2) + \frac{1}{16}\left[3\ln 2 - 1 - \frac{1}{2}\ln(y^2 + z^2)\right](y^2 + z^2).$$

$$\frac{1+y}{\sqrt{(2+y)^2 + z^2}} \approx \frac{1}{2}\left(1 + \frac{y}{2} - \frac{y^2}{4} - \frac{z^2}{8}\right).$$

Then, the subintegral function in (5) will equal

$$\delta\rho f(y,\ z)=\frac{\delta\rho}{2}\left(1+\frac{y}{2}-\frac{y^2}{4}-\frac{z^2}{8}\right)\Big\{1-3\ln 2+\frac{1}{2}\ln(y^2+z^2)+$$
$$+\frac{1}{16}(y^2+z^2)\left[3\ln 2-\frac{1}{2}\ln(y^2+z^2)\right]+$$
$$+\frac{2y}{y^2+z^2}\left[1+\frac{1}{8}\left\{3\ \ln 2-\frac{1}{2}-\frac{1}{2}\ln(y^2+z^2)\right\}(y^2+z^2)\right]\Big\}. \qquad (6)$$

Since $\delta\rho\ (-y) = -\ \delta\rho\ (y)$, the sum total of values (6) at points +y and -y is changed into the difference f(y,z) -f (-y,z). Hence

$$\delta F_r = 4G r_0 \int_0^{y_0}\int_0^{z_0} \delta\rho\left\{\frac{2y}{y^2+z^2}+\right.$$
$$\left.+\,y\left[\frac{1}{8}-\frac{3}{4}\ \ln 2+\frac{1}{8}\ln(y^2+z^2)-\frac{y^2}{4(y^2+z^2)}\right]\right\}dy\,dz\,. \qquad (7)$$

The second right-hand member is small compared with the first, and we can neglect it. We integrate (7) along z, taking out as a sign of the integral the value of $\delta\rho$, averaged along z:

$$\delta F_r = 8G r_0 \int_0^{y_0} \delta\rho \operatorname{arc\,tg}\frac{z_0}{y}\,dy\,. \qquad (8)$$

In the case of a usual sinusiodal disturbance with a maximum displacement δr at point r_0 and at a constant thickness of the layer $2h_0$

$$\delta\rho \approx \left[\frac{2\pi\rho}{\lambda}\sin\frac{\pi y}{2y_0}-\frac{\rho}{r_0(1+y)}\cos\frac{\pi y}{2y_0}\right]\delta r\,. \qquad (9)$$

When $\lambda \ll r_0$ the second member can also be neglected. Then

$$\delta F_r = 4\pi G\rho f(\xi)\,\delta r, \qquad (10)$$

where

$$f(\xi)=\int_0^1 \sin\frac{\pi x}{2}\ \operatorname{arc\,ctg}\frac{\xi x}{2}\,dx, \quad \xi=\frac{\lambda}{H}\,. \qquad (11)$$

The condition of gravitational instability (3) can then be written in the form

$$4\pi G\rho f(\xi) > \frac{2\omega}{r}(\omega r^2)' + \frac{4\pi^2 c^2}{\lambda^2}\,. \qquad (12)$$

The function f(ξ) has the following values:

ξ	0,2	2	4	6	8	10	14	20
$f(\xi)$	0,96	0,64	0,43	0,34	0,28	0,23	0,172	0,124

Hence, it can be seen that a correction toward the magnitude of the critical density is considerable and that it is a function of the ratio of the length of the disturbing wave to the thickness of the layer.

We shall now find the value ξ, at which the critical density, which is a prerequisite of gravitational instability, is minimal. We shall take advantage of the relation between the thickness of layer H and density ρ_0 in its central plane, found by Ye. L. Ruskol (6):

$$H = \sqrt{\frac{2RT}{\pi G \mu \rho_0}}\, I, \tag{13}$$

where

$$I = \frac{1}{2}\int_0^1 \frac{du}{\sqrt{1 - u - {}^1/_3 u^* \ln u}}, \quad u^* = \frac{\rho^*}{\rho_0}; \tag{14}$$

ρ^* is a density which will be obtained if the mass of the central body is equally distributed within the sphere of radius r. Tabulating this integral yields the following dependence of I on ρ_0/ρ^*:

ρ_0/ρ^*	$^1/_3$	$^4/_3$	$^{10}/_3$	5	10
I	0,66	0,87	0,94	0,96	0,975

From (13), we obtain

$$\frac{4\pi^2 c^2}{\lambda^2} = 4\pi G \frac{\Upsilon \pi^2 \rho_0}{2 I^2 \xi^2}. \tag{15}$$

For a system whose rotation is determined chiefly by the gravitation of the central mass (the solar system, outer parts of the Galaxy):

$$\omega r^2 = \sqrt{GMr}, \quad \frac{2\omega}{r}(\omega r^2)' = \omega^2 = \frac{4}{3}\pi G \rho^*. \tag{16}$$

Then the condition of instability will be written in the form

$$\rho > f^{-1}(\xi)\left(\frac{\rho^*}{3} + \frac{\pi^2 \Upsilon}{2 I^2}\frac{\rho_0}{\xi^2}\right). \tag{17}$$

Value ρ represents density averaged along z, and hence it is smaller than ρ_0. However, if we integrate (7) only along the layer of thickness $2h_0 = H$, which does not include all the matter of the cloud, we have also lowered δF_r in (8) to a certain extent. In order to take into account the gravitation of the entire layer, the value ρ in (17) should be somewhat increased. Numerical estimates show that $\rho \approx 0.9\ _0$ depends little on ξ.

Taking this into account and using numerical values for f(ξ) and I, referred to above, we can find the critical value of ρ_0, which will satisfy the condition of instability (17).

The results of calculations at $\gamma = 1$, yield:

ξ	4	6	8	10	15
$\rho_{окр}/\rho^*$	6,8	2,3	2,1	2,2	2,4

Thus, the critical density requisite for gravitational instability which, as is well known, depends on λ, proves to be minimal at a length of the wave of disturbance equal to 8 times the thickness of the cloud H. In decreasing λ its increase will depend on the increase of the second component in (17), associated with the usual criterion of Jeans. In increasing λ, the main part in (17) will be played by the first component associated with the rotation of the system. In this instance, the critical density increases as a result of an increase of $f^{-1}(\xi)$, which indicates by how many factors the gravitation of the flat circle is smaller than the gravitation of the cylindrical circle, extending infinitely along z. The minimal value of the critical density $\rho_{cr} = 2.1\rho^*$ is greater than the critical density $1/3\rho^*$ by a factor of 6, which had been obtained by Bel and Schatzman for a two-dimensional instance, and it is also greater than the instance found by Chandrasekhar. Thus, the conditions for gravitational instability in the interstellar matter of the Galaxy prove to be more rigid than obtained before.

Institute of the Earth's
Physics imeni O. Yu. Shmidt of
of the Academy of Sciences USSR
USSR

Received
August 14, 1959.

Bibliography

1. Jeans, J. Phil. Trans., A.199, 1(1902); Astronomy and Cosmogony, Cambridge, 1929, p. 345.
2. Chandrasekhar, S. Vistas in Astronomy, 1, 1955, p. 344.
3. Ogorodnikov, K. F. Dinamika vrashchayushchikhsya zvyezdnikh sistem (Dynamics of Rotating Stellar Systems), 1958, p. 536.
4. Bel, N., Schatzman, E. Rev. Mod. Phys., 30, 1015(1958).
5. Yanke, Ye., Emde, F. Tablitsy funktsiy (Tables of Functions), Moscow-Leningrad, 1948, p. 172.
6. Ruskol, Ye. L. Voprosy kosmogonii (Problems of Cosmogony), 7, 1960.

END