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The objective of this project was to study dynamics and control of jetengine compression systems within the new framework that we have developed for the infinite-dimensional Moore-Greitzer model. In this framework, theMoore-Greitzer model of compressor dynamics was reformulated as a set of nonlinear evolution equations (one partial differential equation, and two ordinary differential equations). We designed control laws for the infinite-dimensional Moore-Greitzer model that can be truncated to finite dimensional laws for the purpose of implementation. Finite dimensionality is thus not a control architecture issue but an implementation issue. Stability and dynamics of stall cells was investigated numerically and analytically within the new framework. We have investigated the Moore-Greitzer model in cases when the number of stages is not very large. We have shown that the solution in this case is still a travelling wave that travels around the annulus with the speed that is equal to $1/2$ of the rotor speed. We have also shown that there are no other travelling wave solutions, i.e. the stall cell can not travel at any other speed. We have developed a large-scale, averaged model that reduces to the Moore-Greitzer model in a certain limit and investigated its properties. The model is based on measurable quantities and thus avoids the problem of having the phenomenological compressor characteristic as one of the assumptions. Based on symmetry considerations, we have shown that the assumption of Moore and Greitzer that the ``disturbance travels through the compressor'' is indeed valid rigorously in the asymptotic limit where the rotational effects due to rotors dominate the compressor dynamics

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Final Report on the Project:

Dynamics and Control of Infinite-Dimensional Models of Jet Engine Compression Systems

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University of California, Santa Barbara

Grant number: F49620-97-1-0293

Objectives

The objective of this project was to study dynamics and control of jet engine compression systems within the new framework that we have developed for the infinite-dimensional Moore-Greitzer model. In this framework, the Moore-Greitzer model of compressor dynamics was reformulated as a set of nonlinear evolution equations (one partial differential equation, and two ordinary differential equations). We designed control laws for the infinite-dimensional Moore-Greitzer model that can be truncated to finite dimensional laws for the purpose of implementation. Finite dimensionality is thus not a control architecture issue but an implementation issue. Stability and dynamics of stall cells was investigated numerically and analytically within the new framework. We have investigated the Moore-Greitzer model in cases when the number of stages is not very large. We have shown that the solution in this case is still a travelling wave that travels around the annulus with the speed that is equal to 1/2 of the rotor speed. We have also shown that there are no other travelling wave solutions, i.e. the stall cell can not travel at any other speed. We have developed a large-scale, averaged model that reduces to the Moore-Greitzer model in the limit and investigated its properties. As suggested in the proposal, the model is based on measurable quantities and thus avoids the problem of having phenomenological compressor characteristic as one of the assumptions. Based on symmetry considerations, we have shown that the assumption of Moore and Greitzer that the "disturbance travels through the compressor" is indeed valid rigorously in the asymptotic limit where the rotational effects due to rotors dominate the compressor dynamics.

Accomplishments/New Findings:

The accomplishments of the project were:

- **New models and control of axial compression system dynamics.** In the paper [6] we have described a large-scale theory that predicts the crucial features of deep stall dynamics, building on the ideas presented in [7, 8]. The consequence of our analysis based on the averaging of incompressible Navier-Stokes equations is that, in the deep stall regime, the flow in the compressor, away from the hub and casing, is two-dimensional, as observed in experiments ("...the disturbance extends through the engine..."). We have shown that the stall instability is a consequence of the competition between the flow acceleration due to blade forces and momentum redistribution due to velocity fluctuations.

  The compression system setup that we considered is presented in figure 1. The system consists of an inlet duct, the axial compressor with rotor and stator rows denoted by **R** and **S** respectively, the outlet duct and the plenum that has uniform pressure p_p. Our treatment is based on incompressible Navier-Stokes equations, as are all the other studies

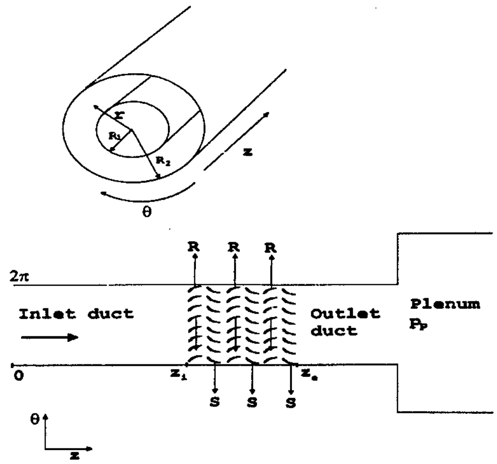

Figure 1: The compression system.

of low-speed compressor dynamics [9] due to the low Mach number for the flows considered. The integro-differential reaction-diffusion equation governing the evolution in time t of the axial velocity $u_z(r, \theta, t)$ in the compressor is shown to be

$$\boxed{\frac{\partial u_z}{\partial t} = \Psi(u_z, r) - \langle \Psi(u_z, r) \rangle + \left(\frac{\partial}{\partial r} \frac{1}{Re_r(r)} \frac{\partial u_z}{\partial r} + \frac{1}{Re_\theta(r)} \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2} \right),} \tag{1}$$

where Ψ is a cubic-type nonlinearity and angular brackets denote averaging over the compressor annulus. In deriving (1) the rotating stall was treated explicitly and formally as a large-scale phenomenon, arising as a feature of the average flow. The averaging volume was extended over rotor and stator rows and many blades. The mean behavior of the flow was extracted from the rapidly oscillating flow that is changing on the scale of blades. The small parameter in the problem was taken to be the angular distance between the blades divided by the typical scale of the flow ϵ/l. The limit when this number is small was considered. The two-dimensional nature of the axial flow was shown to be the consequence of the rotational nature of the average flow in the spirit similar to Taylor-Proudman theorem [10]. The theory of fluid flow symmetries necessary to derive this result was pursued in [3]. While the flow in the inlet duct has previously been treated as irrotational [7, 8], we present a rotational treatment. This is necessary as even when the inlet guide vanes are present, the rotational stall cell disturbance propagates backwards to the inlet duct [2], and results both in the simplification of the final result and better comparison with the experimental results. The role of velocity fluctuations in deep stall instability was emphasized. It was shown that the instability arises due to competition between blade forces and velocity fluctuations. In the case when the axial force exerted on the fluid by the blades increases with the axial velocity, a local perturbation towards bigger velocity results in the acceleration of the axial flow which thus becomes even bigger. In the same way, a local decrease in the axial velocity results in the decceleration of the axial flow which thus becomes even smaller. This leads to instability amplification. This amplification is counteracted by the redistribution of axial momentum that is due to turbulent velocity fluctuations. We derived the cross-sectional, averaged flow in the compressor thus completing the three-dimensional picture of the motion in the compressor. The azimuthal component was shown to obey the law of radial equilibrium which was previously concluded for stalled flows based on experimental evidence [1]. Other physical effects were taken into account in the cases when some of the parameters considered small in the above analysis are not small. Numerical simulations of (1) were performed (see figure 2), showing good comparison with the experimental data from ([5]). Analytical results on (1) such as the prediction of the growth of the stall cell size with the decrease in the mean mass flow compare favorably with experiments. We have

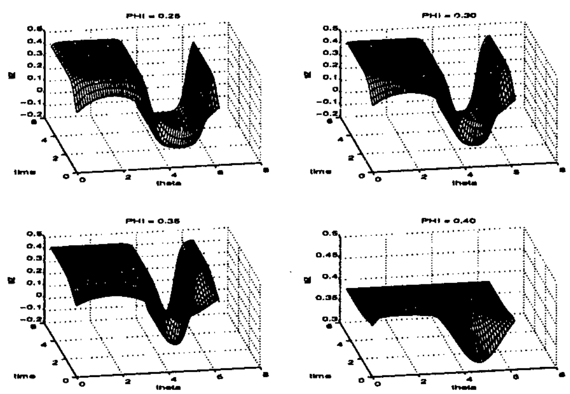

Figure 2: The stall cell solutions of (1), mean flow $\Phi = 0.40$ (lower right) to $\Phi = 0.25$ (upper left).

pursued an investigation of efficient numerical schemes for compressor dynamics models and shown that artificial viscosity methods stabilize the inherently unstable dynamics in the limit when the number of compressor stages becomes very large.

- **The findings for an infinite number of stages were extended to the case of the finite number of stages in the compressor**. The dynamics and nature of stall cells was investigated. We found that, for a large number of stages stall cells are similar in shape to square stall cells that were discovered in the infinite number of stages case. These stall cell solutions have the form of a travelling wave. We found that the speed of the stall cells in the Moore-Greitzer model can be only one-half of the rotor speed. This was found to be a consequence of physical assumptions of the Moore-Greitzer model. New models that we have developed (described above) remove this restriction by making the stall cell speed dependent on the microparameters (i.e. the shape) of the rotors and stators. We found that these stall cell solutions are stable in the Moore-Greitzer set-up. It is the consequence of these investigations that new methods for development of low-order models need to be found.

- **We have pursued investigation of control laws for the infinite-dimensional Moore-Greitzer models.** In this study, with A. Banaszuk, we have shown that a backstepping controler can be devised for the full Moore-Greitzer model using the min norm instead of the smooth H^1 norm used in our previous investigations. We used non-smooth analysis in this study. The new controler can be useful as it does not require the knowledge of the whole disturbance field, but only its minimum.

- **We have investigated the efficiency of different numerical schemes for the reformulated Moore-Greitzer model.** Different artificial viscosity schemes were employed to quench the numerical instabilities occuring when the simulation of axial compression systems with a large number of stages is pursued. We now have a stable and reliable scheme based on artificial viscosity. We are pursuing further investigation in this direction, in analogy with the methods used for simulating systems of conservation laws.

- **Symmetries of fluid flows and their application in compression systems dynamics.** In this study we developed a general, coordinate-free theory for the reduction of volume preserving flows with a volume preserving symmetry on three-manifolds. The reduced flow is generated by a one-degree-of-freedom Hamiltonian which is the generalization of the Bernoulli invariant from hydrodynamics. The reduction procedure also provides global coordinates for the study of symmetry-breaking perturbations. Our theory gives a unified geometric treatment of the integrability of three-dimensional, steady Euler flows and two-dimensional, unsteady Euler flows, as well as quasi-geostrophic and magnetohydrodynamic flows. We have applied these methods in our study of axial compression systems dynamics.

Personnel Supported

Faculty: Igor Mezić, Department of Mechanical and Environmental Engineering, University of California, Santa Barbara.

Postdoctoral fellows: Kaixia Zheng, Department of Mechanical and Environmental Engineering, University of California, Santa Barbara.

Graduate students: Gregory Hagen, Department of Mechanical and Environmental Engineering, University of California, Santa Barbara.

Publications:

“A Backstepping Controller for a Nonlinear Partial Differential Equation Model of Compression System Instabilities”. (with A. Banaszuk and H. A. Hauksson) (1997). To appear SIAM J. of Control and Optimization.

“Reduction of three-dimensional, volume-preserving flows with symmetry”. (with G. Haller) (199). Nonlinearity **11**, 319-339 (1998).

“A Large-Scale Theory of Axial Compression System Dynamics”. (1997) Submitted to the Journal of Fluid Mechanics.

“Dynamics of the Moore-Greitzer model of axial compression system instabilities”. (with A. Banaszuk and H. A. Hauksson) (1997). In preparation.

Interactions/transitions:

Igor Mezić has presented a number of lectures on the dynamics of axial compression systems. He has interacted with the Dynamics and Control Group at the United Technologies Research Center, in particular with Dr. Andrzej Banaszuk on the prospect of using the ideas from the above described work for the control of stall in jet engines. The Caltech group (Murray, Doyle, Marsden) has used these results to start their own investigation in optimal ways of truncating the Moore-Greitzer model and Proper Orthogonal Decomposition of axial compression systems data.

New discoveries:

- Travelling wave solutions of the Moore-Greitzer equations.
- New models of axial compression system dynamics.
- New backstepping controller using the minimum norm of the disturbances.
- New numerical schemes that provide stable simulation of compressor models and are based on artificial viscosity.

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