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T-matrix Approach to Array Modeling

by

Clyde L. Scandrett
Steven R. Baker

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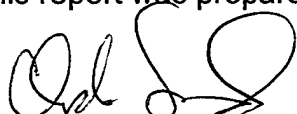Rear Admiral R. C. Chaplin   R. Elster
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T-matrix Approach to Array Modeling

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Abstract

The so-called T-matrix formulation for transducer array calculations is described from a theoretical standpoint. The methodology is then applied to an array problem whose solution can be found analytically, thereby producing a reference solution for comparing array calculations based upon FEM/BEM determined T-matrices. Three numerical codes have been tested for their accuracy in reproducing the exact T-matrix for a spherical shell. These in turn, are used to calculate the near and far field pressure for the reference array. It is demonstrated in the report that the methodology appears to be robust. Provided an "accurate" finite/boundary element model of a transducer is used to produce the transducer's T-matrix, "accurate" field pressure results for both the far and near field of the array can be found.

1 Theoretical Background

The time harmonic ($e^{i\omega t}$) pressure field radiated or scattered from a fluid loaded finite structure can be represented in terms of outgoing spherical Hankel functions applied to the set of spherical harmonics. For a single radiator/scatterer with a local coordinate system written with the index "j", the functional form of the pressure field (p_j) can be written

$$p_j(r_j, \theta_j, \phi_j) = \sum_{n=0}^{N} \sum_{m=-n}^{n} A_{jmn} h_n(kr_j) \Omega_n^m(\theta_j, \phi_j)$$

where

$$\Omega_n^m(\theta, \phi) = P_n^m(\cos\theta) e^{im\phi}$$

are spherical harmonic functions.

A distinction is made between radiated pressures of a structure B_{jmn} and scattered pressures A_{jmn} throughout the analysis used. It is assumed that the coefficients for the radiated amplitudes are known or have been calculated using the finite element method, while the scattered pressures are to be determined when a series of transducers are placed in an array, and are insonified by either an exterior source or from mutual array element interactions when the array is in an active mode of operation.

What the spherical addition theorem allows one to do, is represent a pressure field relative to one coordinate system in terms of coordinates of a second system. The representation of a single outgoing spherical wave is given by the formula [3]

$$h_n(kr_1)\Omega_n^m(\theta_1, \phi_1) = \sum_{\nu=0}^{\infty} \sum_{\mu=-\nu}^{\nu} \sum_{\substack{p=|n-\nu| \\ p \geq |m-\mu|}}^{n+\nu} {}'$$

$$a(\nu, p, n, \mu, m) j_\nu(kr_{2<}) h_p(kr_{2>}) \Omega_p^{\mu+m}(\theta_{21}, \phi_{21}) \Omega_\nu^\mu(\theta_2, \phi_2)$$

(the prime on the summation over p indicates jumps of 2 in the sum)

(r_1, θ_1, ϕ_1) = spherical coordinates relative to system 1

(r_2, θ_2, ϕ_2) = spherical coordinates relative to system 2

$(r_{21}, \theta_{21}, \phi_{21})$ = origin of system 2 relative to 1

$r_{2>} = \max\{r_2, r_{21}\} \qquad r_{2<} = \min\{r_2, r_{21}\}$

and where the coefficients $a(\ldots)$ are related to the Wigner 3-j symbols used in quantum mechanics.

For an array, the translation formula can be used in conjunction with a "T-matrix" (T for transition) *method* to determine the total pressure field of an array of scatterers/radiators relative to theoretically any coordinate system. It is especially useful in cases for which

multiple-scattering and near field pressure calculations are important. A good general reference on the subject is the proceedings of an ONR-sponsored international symposium held in 1979 [6]. The method was originally developed to solve acoustic and electromagnetic scattering from a single obstacle. Later, it was applied to the scattering of elastic waves, and to scattering from arrays of obstacles. It is important to note that the T-matrix method accounts for multiple scattering to all orders, and that the method is capable of solving array problems with arbitrarily dense - randomly packed elements.

The scattering properties of each unique scatterer (transducer) in an array are described by a so-called T-matrix, using a discrete basis set of spherical harmonics. The T-matrix for a particular finite obstacle is determined by analyzing the scattering characteristics of the obstacle subject to "standing" spherical waves of the form

$$p^i(r, \theta, \phi) = j_n(kr)\Omega_n^m(\theta, \phi)$$

The form of these incident pressures is dictated by the spherical addition formula, and the ability to represent an arbitrary incident pressure field in terms of such a basis. The diagram below graphically indicates the T-matrix use. The R_n are essentially reflection coefficients from the obstacle, and constitute entries into the T-matrix.

$$\underbrace{\sum_n I_p j_p(kr)\Omega_p^m(\theta, \phi)}_{\text{standing waves}} \overset{\text{T-matrix}}{\Longrightarrow} \underbrace{\sum_n R_n h_n(kr)\Omega_n^m(\theta, \phi)}_{\text{radiating waves}}$$

To demonstrate the use of the addition formulas with the T-matrix formalism, consider a two element array in which both transducers are active. The two translations of coordinates needed can be found simultaneously due to the simple relationship between angles of one system relative to the other. Let $\mathbf{G}_{ij}$ represent this translation formula from system j to system i (by applying the spherical addition formula). Furthermore, let $\mathbf{T}_k$ be the "T-Matrix" for the kth transducer modeled, and the radiated amplitudes of the two transducers (in the absence of their neighbors) be given by the vectors $\vec{B}_1$ and $\vec{B}_2$. One obtains the following system of equations for the unknown scattering pressure amplitudes $\vec{A}_1$ and $\vec{A}_2$:

$$\mathbf{T}_1 \mathbf{G}_{12}(\vec{A}_2 + \vec{B}_2) = \vec{A}_1$$

$$\mathbf{T}_2 \mathbf{G}_{21}(\vec{A}_1 + \vec{B}_1) = \vec{A}_2$$

or in matrix form:

$$\begin{pmatrix} \mathbf{I} & -\mathbf{T}_1 \mathbf{G}_{12} \\ -\mathbf{T}_2 \mathbf{G}_{21} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \vec{A}_1 \\ \vec{A}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{T}_1 \mathbf{G}_{12}\vec{B}_2 \\ \mathbf{T}_2 \mathbf{G}_{21}\vec{B}_1 \end{pmatrix}$$

If we let

$$\mathcal{G} = \begin{pmatrix} \mathbf{0} & \mathbf{T}_1 \mathbf{G}_{12} \\ \mathbf{T}_2 \mathbf{G}_{21} & \mathbf{0} \end{pmatrix},$$

the matrix equation becomes even simpler:

$$(\mathbf{I} - \mathcal{G})\left(\begin{array}{c}\vec{A}_1 \\ \vec{A}_2\end{array}\right) = \mathcal{G}\left(\begin{array}{c}\vec{B}_1 \\ \vec{B}_2\end{array}\right).$$

Once the scattering amplitudes are found, the total pressure at any field point may be found by first representing that field point in terms of each individual coordinate system $(r, \theta, \phi) \to (r_j, \theta_j, \phi_j)$ and summing the series over each array element:

$$p(r, \theta, \phi) = \sum_j \sum_{n=0}^{N} \sum_{m=-n}^{n} [A_{jmn} + B_{jmn}] h_n(kr_j) \Omega_n^m(\theta_j, \phi_j)$$

A particular application of the above procedure is that of determining source levels for an array of 16 closely packed spherical shell transducers. Let the array consist of two rows of 8 spherical shell transducers (with outer radius "a" equal 0.5 meters) driven at a frequency such that $ka = 1$, and having separation distances of $\pi/2 = \lambda/2 \approx 1.57$ meters along each row and $\pi/4 = \lambda/4 \approx .785$ meters between rows. Furthermore, one row of transducers is 90° out of phase with that of the other. A graphic of the array is given below:

Orientation of the spherical shells
in the 16 element array
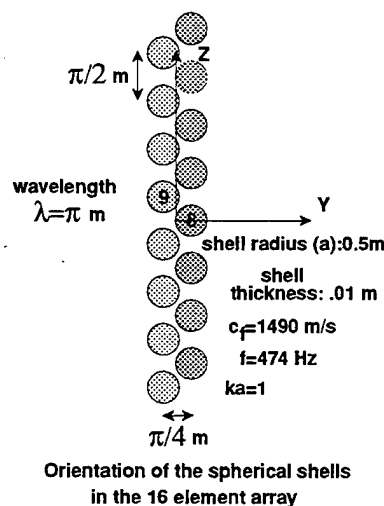
By using the T-matrix for a spherical shell, and the translation formulas, the far field source level can be found for the array operating at a frequency of 474 Hertz which corresponds to an acoustic wavelength of $\pi/2$ ($ka = 1$). The figure below is the far-field pattern for the array, scaled by the far field amplitude of a single spherical shell (or source level of a single element of the array). The graph displays source levels in dB in the plane of the array as well as a superimposed image of the array orientation. Angles are measured from the positive z axis.

Source level of array, ka=1, analytic scaled by FF of a single spherical shell
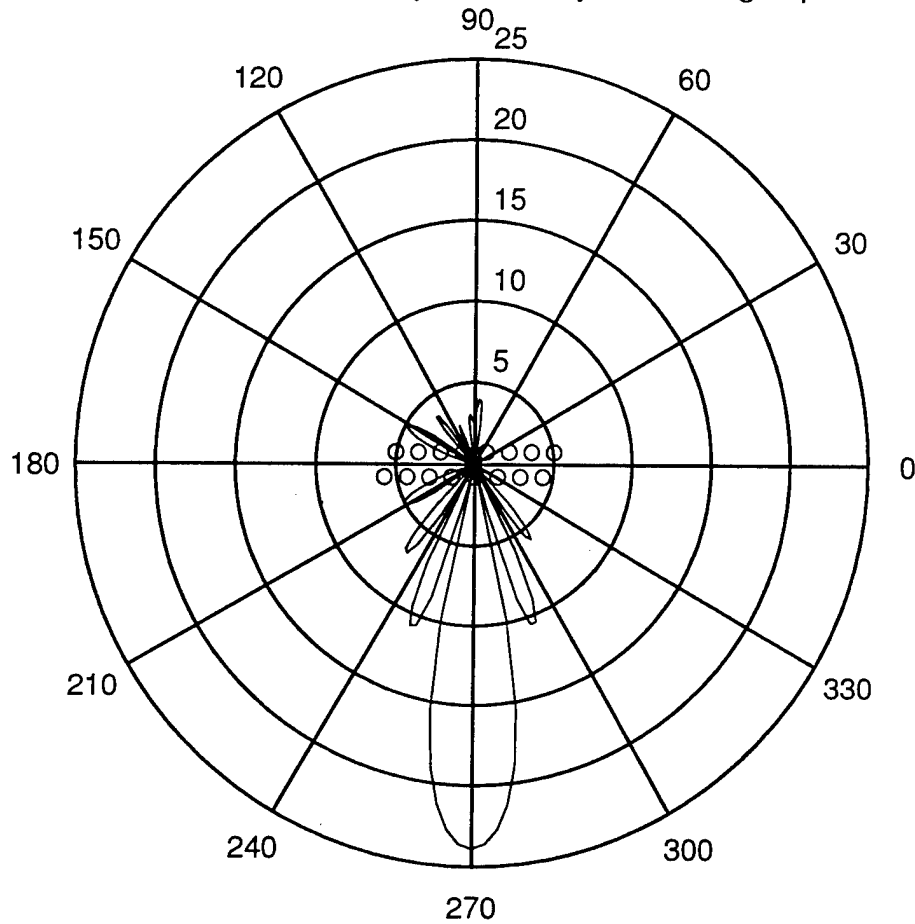
2 Validation Procedure

To test the effectiveness of a boundary element or a finite element code to construct the T-matrix of a particular transducer, comparison is made to an idealized physical problem for which there is an analytical solution. The idealized problem addressed is that of a fluid loaded thin spherical shell. This is a canonical problem which lends itself to analytic treatment due to the simple characterization of the boundaries, and the fact that fluid loading does not affect mode shape (it does affect the eigenfrequencies of the shell however). The analytical solution can be represented in terms of spherical harmonics, and the resulting T-matrix is diagonal. Following Junger and Feit [2] the modal mechanical impedance of a thin spherical shell is:

$$Z_n = \frac{ihc_p\rho_s}{a\Omega} \left\{ \frac{(\Omega^2 - \nu - n(n+1) + 1)(\Omega^2 - 2(1+\nu)) - n(n+1)(1+\nu)^2}{\Omega^2 - \nu - n(n+1) + 1} \right\}$$

a	shell radius
h	shell thickness
E	Young's modulus
ν	Poisson's ratio
Ω	$\omega a / c_p$
c_p	plate wave speed $(\sqrt{E/\rho_s(1-\nu^2)})$
c_f	fluid wave speed
ω	frequency
ρ_s , ρ_f	solid and fluid densities
n	index of the spherical harmonic

(It should be noted that the above impedances are independent of the index "m" which characterizes the longitudinal variation of the field quantities.) From the definition of the modal mechanical impedance we have

$$I_{nm}j_n(ka) + R_{nm}h_n(ka) = i\omega Z_n W_{nm}$$

where I_{nm} and R_{nm} are incident and reflected modal amplitudes, and W_{nm} is the modal normal displacement of the shell. The vector form of the amplitudes requires an ordering of the unknowns, which is as follows: $R_{0,0} \rightarrow R_1$, $R_{1,-1} \rightarrow R_2$, $R_{1,0} \rightarrow R_3$, $R_{1,1} \rightarrow R_4$, $R_{2,-2} \rightarrow R_5$, $R_{2,-1} \rightarrow R_6$, ... A second relationship between the pressures and displacements at the surface of the shell results from Euler's equation (the assumption of no cavitation on the surface of the shell):

$$I_{nm}j_n{}'(ka) + R_{nm}h_n{}'(ka) = \rho_f c_f \omega W_{nm}$$

Combining these and eliminating the normal displacement one can solve for the scattered amplitude in terms of the incident amplitude:

$$R_{nm} = \left[\frac{iZ_n j_n{}'(ka) - \rho_f c_f j_n(ka)}{-iZ_n h_n{}'(ka) + \rho_f c_f h_n(ka)} \right] I_{nm}$$

The bracketed term above is an analytical expression for the diagonal entries of the T-matrix for a thin spherical shell.

2.1 ATILA modeling for T-matrix determination

T-matrix entries have been tabulated for a spherical shell using ATILA (version 5.11) with the following set of input parameters (these are the same values as those used in the 16 element array given in the preceding section):

a	0.5 meters
h	0.01 meters
E	215 GPa
ν	0.33
$f = \omega/2\pi$	474 Hertz ($ka = 1$)
ρ_s , ρ_f	7500 Kg/m^3 & 1000 Kg/m^3
c_f	1490 meters/sec

The values of the diagonal entries of the T-matrix using ATILA are given in Table II. The analytical values for the diagonal entries of the T-matrix for the same set of parameters are given in Table I.

Table I - T-matrix diagonals - Analytic				
n	Real part	Imag part	Ampl.	Phase(degrees)
1	-0.13465E-01	0.11525E+00	0.11604E+00	0.96663E+02
2	-0.60255E-02	0.77390E-01	0.77624E-01	0.94452E+02
3	-0.60255E-02	0.77390E-01	0.77624E-01	0.94452E+02
4	-0.60255E-02	0.77390E-01	0.77624E-01	0.94452E+02
5	-0.61773E-03	-0.24847E-01	0.24854E-01	-0.91424E+02
6	-0.61773E-03	-0.24847E-01	0.24854E-01	-0.91424E+02
7	-0.61773E-03	-0.24847E-01	0.24854E-01	-0.91424E+02
8	-0.61773E-03	-0.24847E-01	0.24854E-01	-0.91424E+02
9	-0.61773E-03	-0.24847E-01	0.24854E-01	-0.91424E+02

Table II - T-matrix diagonals - ATILA				
n	Real part	Imag part	Ampl.	Phase(degrees)
1	-1.3397e-02	1.3587e-01	1.3652e-01	9.5631e+01
2	-6.0482e-03	8.1155e-02	8.1380e-02	9.4262e+01
3	-3.1121e-03	8.2552e-02	8.2611e-02	9.2159e+01
4	-6.0480e-03	8.1158e-02	8.1383e-02	9.4262e+01
5	5.2933e-04	-2.3758e-02	2.3764e-02	-8.8724e+01
6	8.0858e-04	-2.3813e-02	2.3827e-02	-8.8055e+01
7	1.3738e-03	-2.4142e-02	2.4181e-02	-8.6743e+01
8	8.0907e-04	-2.3814e-02	2.3828e-02	-8.8054e+01
9	5.2948e-04	-2.3759e-02	2.3764e-02	-8.8723e+01

As seen in the tables, the ATILA generated elements suffer from a relatively small error in the phase (within 3 degrees of the analytic), but for the fundamental mode, the amplitude values are off by about 18% of the analytic value (for the remaining terms the relative error is within 6%). In addition, when considering the off diagonal entries of the ATILA generated T-matrix, there appeared to be "leakage" of scattered energy between the $n = 2, m = 2$ and the $n = 2, m = -2$ modes.

The ATILA run made use of the "new" curved shell elements. The spherical shell was divided into 72 elements, while the surrounding fluid contained two fluid layers which were 0.25 meters thick followed by eight fluid layers one-half meter thick for a total fluid thickness of 5 meters. Each fluid layer had 72 fluid "block elements", and the entire finite element mesh was truncated with dipolar damping elements which are consistent with applying the time-harmonic Sommerfeld radiation condition:

$$ikp(r,\theta,\phi) + \frac{\partial p}{\partial r}(r,\theta,\phi) + \frac{1}{r}p(r,\theta,\phi) \to 0 \quad \text{as} \quad r \to \infty$$

One likely source of error is due to the approximate nature of the radiation boundary condition applied at the truncated fluid domain. A second source is the coarseness of the mesh, which has at the poles four elements with longitudinal separation angles of ninety degrees.

The amplitudes of the radiating harmonics are extracted from the ATILA generated scattered pressure at given field points by application of a second code which employs a singular value decomposition of the total scattered field into modal amplitudes. In these runs, the match between ATILA scattered pressure, and the modal representation of the pressure is extremely close, which is considered in checking the fidelity of the modal expansion to the raw data.

2.2   SYSNOISE modeling for T-matrix determination

The currently available version of SYSNOISE is release 5.3A. LMS technologies has agreed to make the Naval Postgraduate School a beta site for the testing of version 5.4, which is anticipated to become commercially available by December of 1998. However, as of October, we have yet to receive a copy of the 5.4 version. This version is necessary since it allows one to use input from ATILA generated impedance matrices for the determination of scattering and radiation from piezoelectric transducers. SYSNOISE version 5.3A is incapable of accepting inputs from ATILA. For these reasons, the focus has been on benchmarking the current version of SYSNOISE to the same problem given above - that of the scattering characteristics of a submerged spherical shell.

Documentation of SYSNOISE 5.3A indicates that it is possible to have user defined input pressures through a special "user" library. A FORTRAN code has been written to provide incident pressures of the form necessary to produce the requisite T-matrix elements (standing spherical harmonics of the form $p^i(r,\theta,\phi) = j_n(kr)\Omega_n^m(\theta,\phi)$). Unfortunately, the makefile necessary to recompile the SYSNOISE source code is incompatible with the operating system (HPUX10.2). The design team in Belgium has been in contact with Dr. Scandrett to try and produce a viable makefile which would allow compilation and linking of the SYSNOISE source code with user defined functions, but as of now, have been unable to do so.

As an alternative to using user defined incident pressures of the form desired, one may use a series of incident plane waves. This requires that the incident plane waves be decomposed into spherical harmonics, which can then be used in combination with their scattered spherical harmonics to produce the T-matrix. Such a methodology can be written in matrix form as follows:

$$TB = S$$

where T is the sought after T-matrix, the columns of B are the amplitudes of the incident plane wave decomposed into standing harmonics, and S are the decomposed scattered pressure amplitudes in terms of radiating spherical harmonics. The equation is transposed and solved in a least squares sense for the T-matrix of the transducer.

Unfortunately, representation of an arbitrary incident plane wave as a sum of standing spherical harmonics, may take several hundred terms, while the scattering amplitudes from the finite obstacle have typically only a few non-negligible components. In particular, if only the first N harmonics are necessary to adequately represent the scattered pressure from the obstacle, only an $N \times N$ T-matrix is needed. Because of the necessity of properly representing a plane wave, we typically require the number of rows in the matrix B to be large (say M where $M >> N$). The number of columns in each of the matrices B and S depend upon the number of distinct incident plane waves needed to determine the T-matrix (say P where we must have $P \geq N$). The full T-matrix given these considerations must be $M \times M$, but we're only interested in the upper left block of this matrix (T_{11}). We have the matrix equation rewritten below

$$\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} S_1 \\ S_2 \end{pmatrix}$$

where the blocks T_{12}, T_{21}, and T_{22}, are respectively $N \times (M - N)$, $(M - N) \times N$, and $(M - N) \times (M - N)$, respectively. The full T-matrix for an arbitrary scatterer could theoretically be found using only plane waves, but it would require the number of columns in S and B to be at least as large as M - the number of harmonics needed to represent the incident plane wave. This highlights our interest in having the ability to arbitrarily specify incident pressures through user defined functions.

For the special case of a spherical shell, we know that the off diagonal entries of the T-matrix should be zero, and hence $T_{12} = T_{21} = 0$. Exploiting this property, we can rewrite the first N rows of the above matrix equation in the form

$$T_{11}B_1 = S_1$$

Now the number of columns in the S matrix need only be greater than or equal to N the row/column size of the sought after T-matrix. For a general obstacle/transducer, this trick cannot be used due to mode coupling of the spherical harmonics.

Before the above methodology is exploited, a check is made on the ability of SYSNOISE to accurately determine the plane wave scattering from a fluid loaded structure. To this end, the plane wave scattering from the same spherical shell (using the same input parameters as above) is found and compared to an analytical solution. The incident plane wave in this instance is propagating in the positive 'z' direction. The analytical solution can be written for the scattered pressure in terms of radiating spherical harmonics:

$$p_s(r, \theta, \phi) = \sum_{n=0}^{\infty} A_n h_n(kr) P_n(\cos \theta)$$

$$A_n = (-i)^n (2n+1) \frac{-Z_n j'_n(ka) + i\rho_f c_f j_n(ka)}{Z_n h'_n(ka) - i\rho_f c_f h_n(ka)}$$

and Z_n is the modal mechanical impedance given previously. Comparison of SYSNOISE output with the analytic solution is shown in the figures below.

Plane wave scattering, SYSNOISE & analytic, Freq=474Hz, ka=1
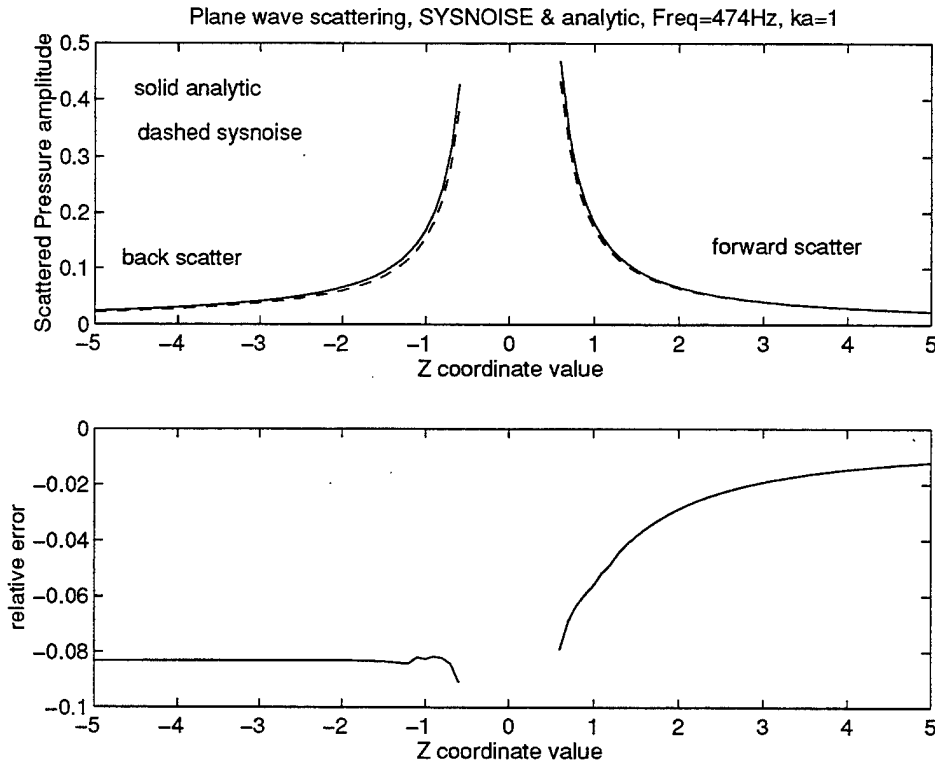
The discretization used in SYSNOISE consisted of 216 planar quadrilateral elements to model the shell. It can be seen that the relative error is greatest in the backscatter direction, and is particularly bad if field points too close to the shell are employed. In total however, the results are favorable, being by and large within a relative error of 10 percent. We are now in a position to try and determine the 9×9 T-matrix elements of the spherical shell using 9 incident plane waves(this will include spherical harmonics through the quadrupole term), and their resulting scattered pressures. The B matrix has column entries (one column for each plane wave) which are the analytic values of the coefficients used in expanding the given incident plane wave in terms of standing harmonics through the formula [5]

$$e^{-ikr \cos \gamma} = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} B_{nm} j_n(kr) \Omega_n^m(\theta, \phi)$$

$$B_{nm} = \frac{(-i)^n(2n+1)(n-m)!}{(n+m)!} P_n^m(\cos\alpha)e^{-im\beta}$$

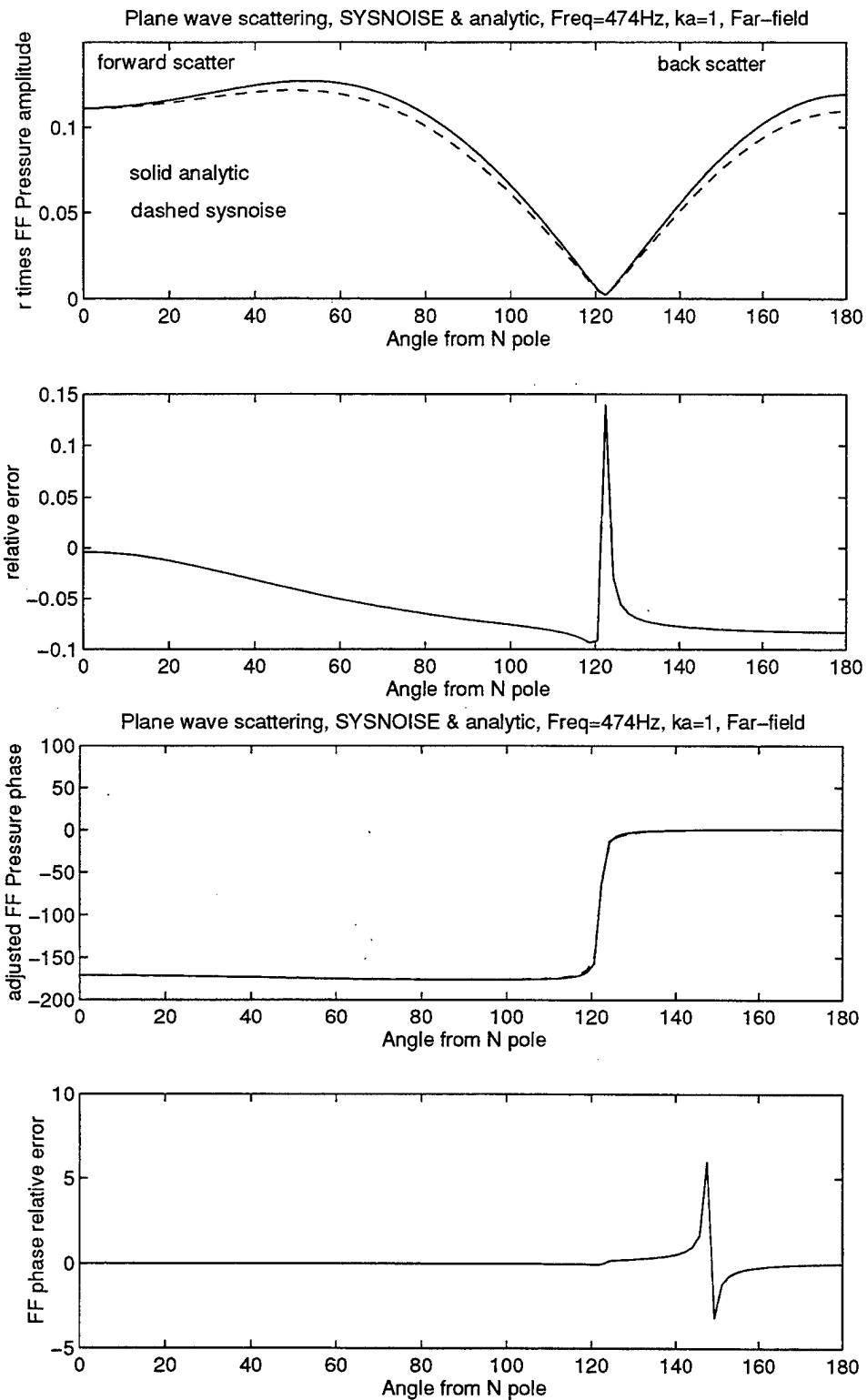$$\cos\gamma = \sin\alpha\sin\theta\cos(\phi-\beta) + \cos\alpha\cos\theta$$

and the values of α and β are the azimuthal and longitudinal angles of the plane wave's propagation direction vector. The components of the S matrix are found using the scattered
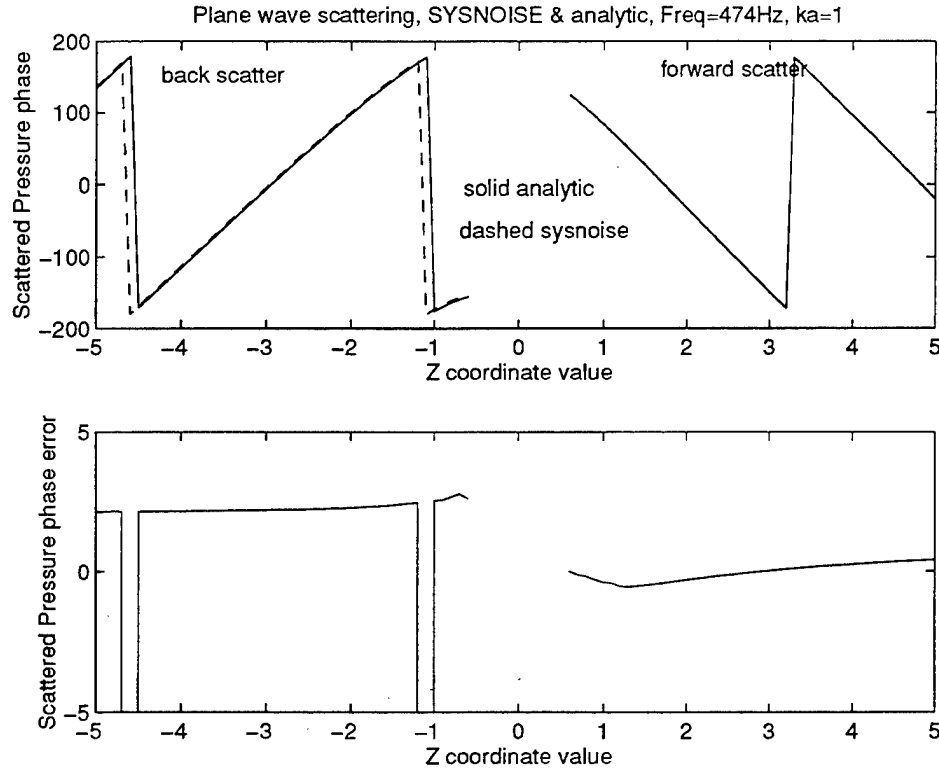
pressures at field points at a distance of 5 meters from the sphere center generated by SYSNOISE, and analyzed using a least squares program. The *relative* residual error was in every case on the order of 6×10^{-5}. (This relative error, is the length of the error vector in comparing the SYSNOISE generated data to its computed spherical harmonic representation, divided by the length of the original data vector. It is symbolically represented as

$$\text{relative error} = \frac{\|\vec{S} - T\vec{H}\|}{\|\vec{S}\|}$$

where the vector $\vec{H}$ corresponds to the set of radiating spherical harmonics at each of the prescribed field points.)

Plane wave scattering, SYSNOISE & analytic, Freq=474Hz, ka=1, Far–field

forward scatter   back scatter

solid analytic

dashed sysnoise

Plane wave scattering, SYSNOISE & analytic, Freq=474Hz, ka=1, Far–field

The final results of the T-matrix determination are given in Table III.

Table III - T-matrix diagonals - SYSNOISE				
n	Real part	Imag part	Ampl.	Phase(degrees)
1	-1.2453e-02	1.1093e-01	1.1163e-01	9.6405e+01
2	-5.5062e-03	7.3775e-02	7.3981e-02	9.4268e+01
3	-5.5539e-03	7.3788e-02	7.3996e-02	9.4304e+01
4	-5.5532e-03	7.3784e-02	7.3993e-02	9.4304e+01
5	-5.5229e-04	-2.2295e-02	2.2301e-02	-9.1419e+01
6	-5.3767e-04	-2.2251e-02	2.2258e-02	-9.1384e+01
7	-5.3716e-04	-2.2120e-02	2.2127e-02	-9.1391e+01
8	-5.4100e-04	-2.2240e-02	2.2246e-02	-9.1393e+01
9	-5.4614e-04	-2.2257e-02	2.2263e-02	-9.1406e+01

As can be seen in the above, there is a much better match to the analytic values than was found in using ATILA alone. In particular, the relative errors in the magnitude are about 4 percent for the breathing mode, nearly 5% for the dipole terms, and increase to about 10 percent for the quadrupole modes. The phase error is uniformly less than one-half of 1 percent. There are however, off diagonal entries in the computed T-matrix which are non-negligible in the above analysis. For the first 7 modes of excitation, the amplitudes of T-matrix entries in the corresponding column are roughly two orders of magnitude less that of the diagonal entry. For the n=2 m=1 and m=2 columns, it is found that the n=2, m=-1 and m=-2 have amplitudes only one order of magnitude less than the diagonal entry. It is suspected that this is the result of two phenomenon. The first of these is that the relationship between associated Legendre functions tends to magnify those Legendre functions with negative index m as seen by the identity

$$P_n^{-m}(x) = \frac{(n-m)!}{(n+m)!} P_n^m(x)$$

A second reason for the larger than expected magnitudes of these off-diagonal entries is that perhaps these modes are preferentially magnified by the mesh used (in this case a set of 216 flat quadrilateral shapes generated by SYSNOISE).

2.3 ATILA modeling with EQI for T-matrix determination

In an entirely analogous fashion, the T-matrix can be determined from ATILA coupled with the EQI code which is a boundary integral method based upon Jones' technique [1] for handling anomalous interior eigenfrequencies. The discretization used for the full ATILA code was employed on just the surface of the spherical shell, and coupled to EQI to find the results of scattering calculations. This work was accomplished with the help of Régis Bossut, (an ISEN employee) since NPS does not currently have the rights to use EQI.

At present, EQI can not handle arbitrary incident pressures in its boundary integral formulation, so the same methodology used with SYSNOISE will be employed here. It

should be noted, that the designers of ATILA/EQI are open to modifications of their code which would allow user defined incident pressures should this research continue, and this report supplies adequate justification for purchasing such a development.

ATILA & analytic,Freq=474Hz, Ka=1, PW scatter at 0.5 meter
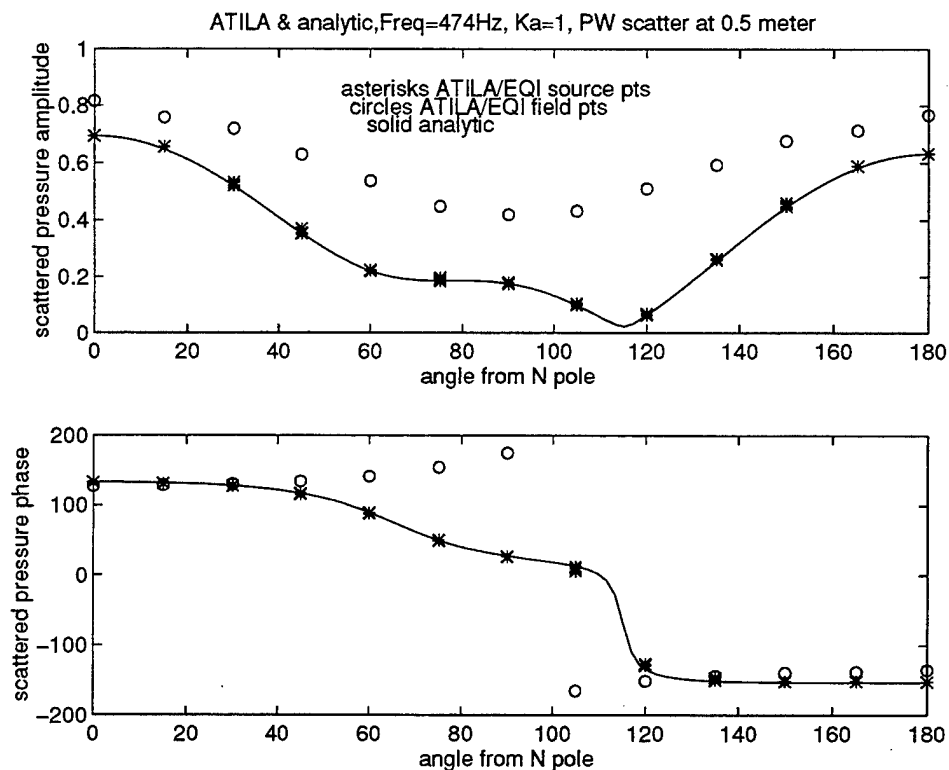
As with SYSNOISE, the EQI boundary element code is tested with plane wave scattering from a spherical shell. The first graph compares the analytic scattered pressure to the EQI results. The asterisks are points on the surface of the scatterer used to determine field point pressures elsewhere in the fluid. As can be seen, these are quite accurate, but in using these values to determine the field point pressures on the surface of the shell would lead to large errors. A second graph compares the analytic solution with ATILA/EQI using field points at 1 meter from the center of the shell (1/2 meter from the shell surface). At this distance, error from field point calculations appears to have been avoided. It is at this distance, that field point scattered pressures will be computed for the T-matrix determination given the same set of incident plane waves used in the SYSNOISE analysis.
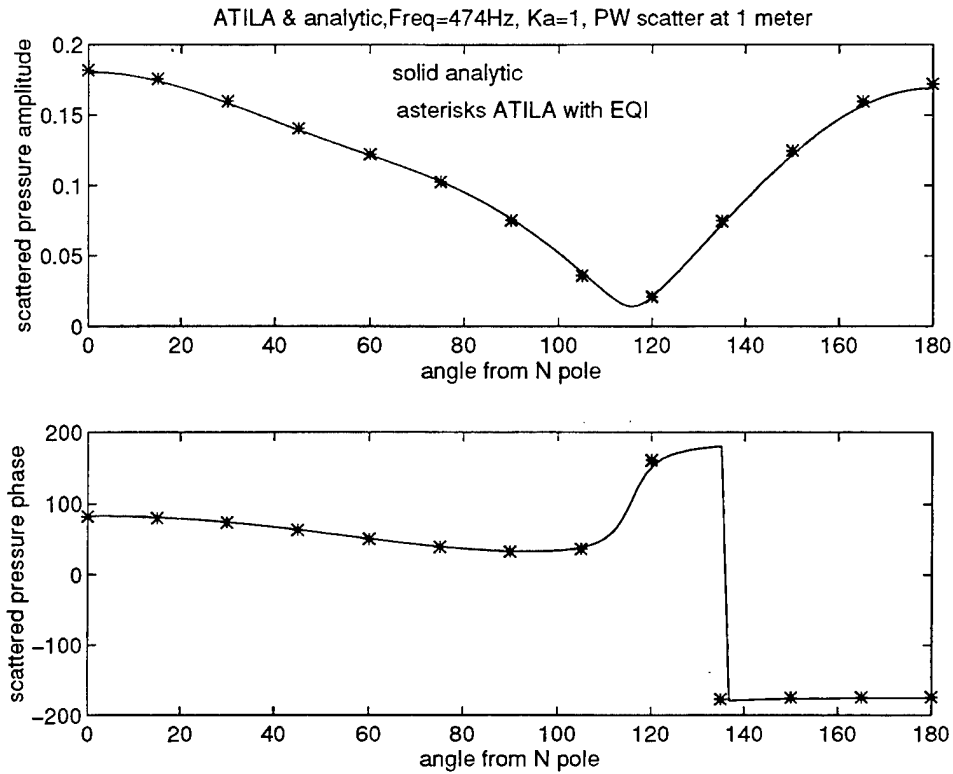
The results of this study lead to the following table of diagonal entries for the T-matrix.

n	Real part	Imag part	Ampl.	Phase(degrees)
Table IV - T-matrix diagonals - ATILA/EQI
1	-1.2869e-02	1.1277e-01	1.1350e-01	9.6510e+01
2	-6.4242e-03	7.9990e-02	8.0248e-02	9.4592e+01
3	-6.4859e-03	8.0094e-02	8.0356e-02	9.4630e+01
4	-6.4356e-03	7.9975e-02	8.0234e-02	9.4601e+01
5	-5.9398e-04	-2.4592e-02	2.4599e-02	-9.1384e+01
6	-6.0821e-04	-2.4650e-02	2.4657e-02	-9.1413e+01
7	-6.2987e-04	-2.5082e-02	2.5090e-02	-9.1439e+01
8	-6.0554e-04	-2.4649e-02	2.4656e-02	-9.1407e+01
9	-5.9399e-04	-2.4592e-02	2.4600e-02	-9.1384e+01

In the above table we have the best results so far. The monopole term has a relative error of about 2%, the dipole about 4%, and the quadrupole about 1%. It is believed that a direct cause of the larger relative error in the dipole term is due to the rather coarse discretization at the two poles of the surface mesh. The EQI result also suffered less from energy leakage in the quadrupole terms than SYSNOISE or ATILA alone, but there is still some occurring

from the n=2, m=2 mode to the n=2, m=-2 mode. In the seventh column, the entry in the fifth row (n=2,m=-2) is only one order of magnitude less than the diagonal entry.

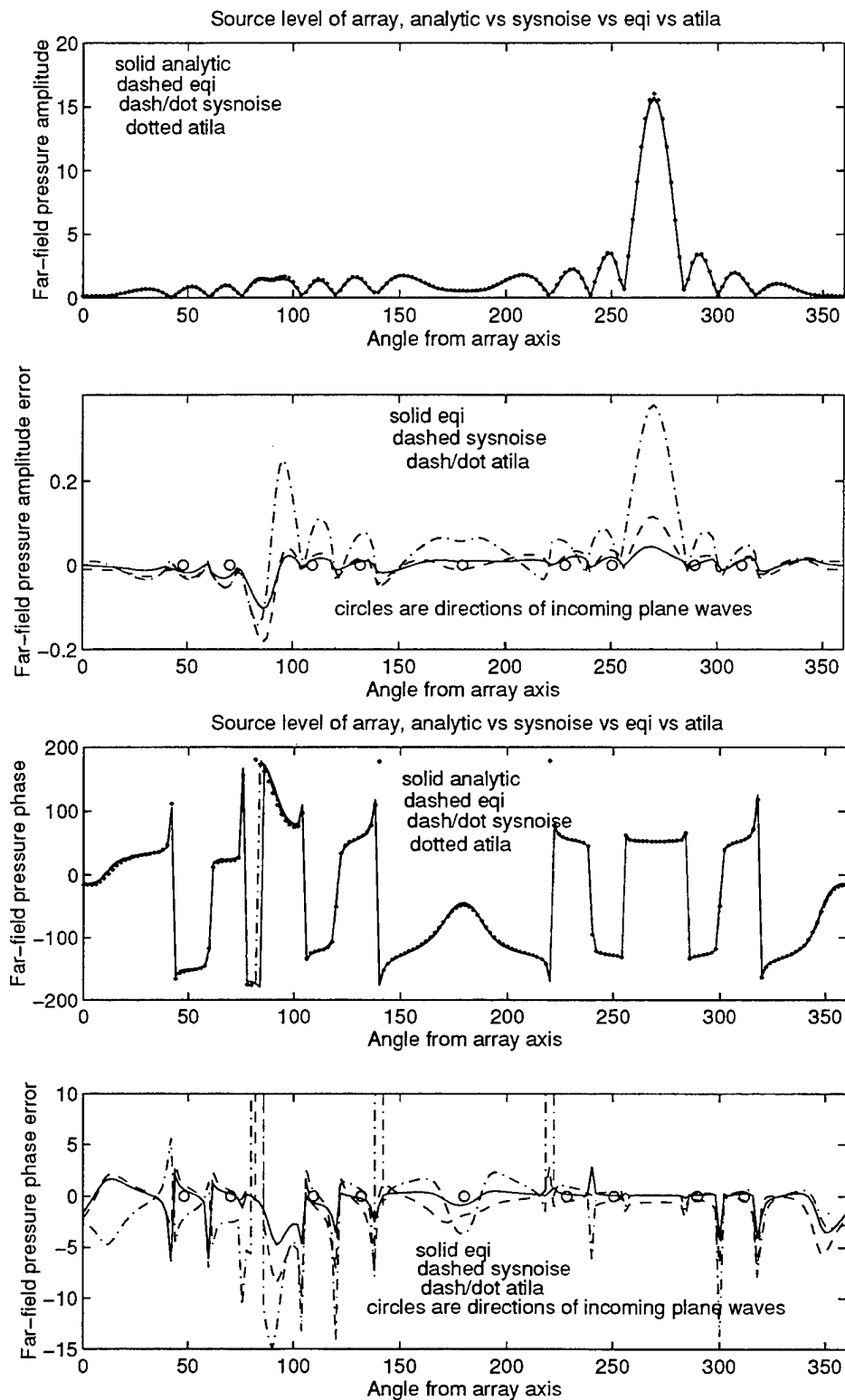## 2.4 Comparison of methods in Array calculations

Using the model 16 element array, for which we have an analytically determined solution, we compare each of the three computed T-matrices. The computed 9×9 T-matrices are read into the array code, and a comparison of source levels and surface pressures were undertaken. The polar plot of the array source level was unable to distinguish the four curves, and so is not repeated here. Instead, line graphs indicated the amplitude (absolute pressure and pressure in dB) and phase of the different solutions are given below.

For the far-field calculations in dB, we see that the three methods all do quite well. The error in dB is uniformly less than 2 for all methods, with ATILA/EQI best, and ATILA alone worst. The results are scaled by the magnitude of a single shell radiating in a breathing mode. It can be seen in the graph, that the ATILA/EQI results are within 1 dB of the analytic, and for much of the angles where the dominant lobes of the pressure amplitude reside, the error is well below 1 dB.
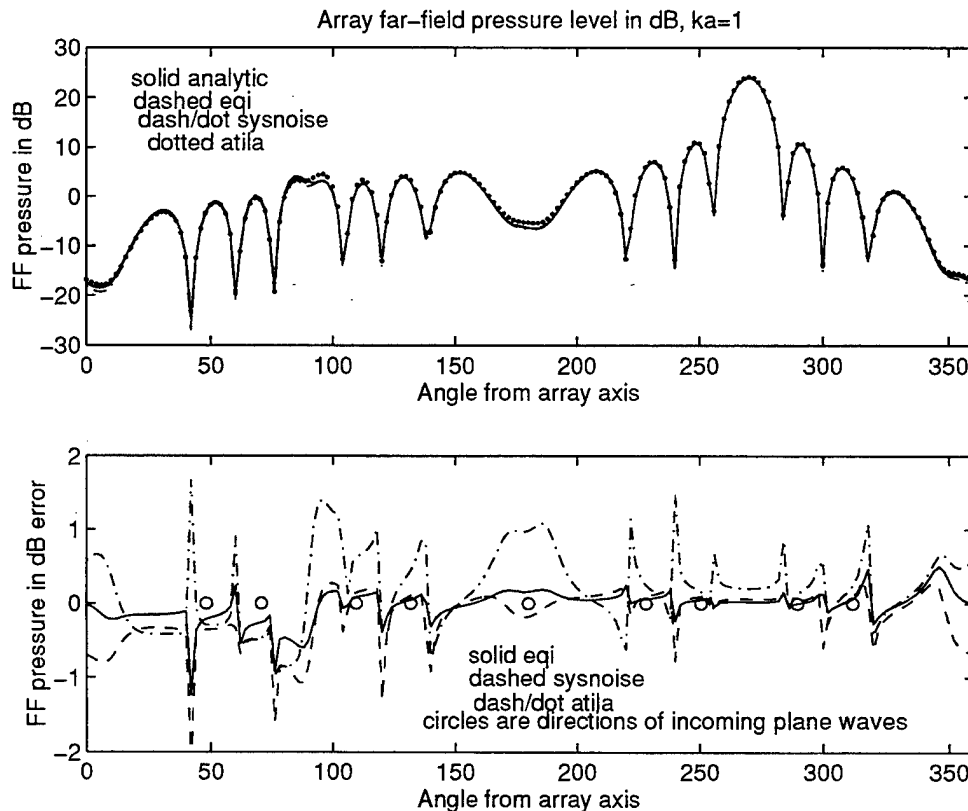
An analogous graph using the absolute pressure in the far field clearly indicates the differences in solutions. Note that the circles in the lower graph represent angles from which incident plane waves were coming in the T-matrix determination for SYSNOISE and ATILA/EQI. However, except for the 180° circle, the incident plane waves were not propa-

gating in the plane of the array, which is the plane for which the results have been reported.

Source level of array, analytic vs sysnoise vs eqi vs atila

solid analytic
dashed eqi
dash/dot sysnoise
dotted atila

solid eqi
dashed sysnoise
dash/dot atila

circles are directions of incoming plane waves

Source level of array, analytic vs sysnoise vs eqi vs atila

solid analytic
dashed eqi
dash/dot sysnoise
dotted atila

solid eqi
dashed sysnoise
dash/dot atila
circles are directions of incoming plane waves

An important aspect of our methodology is the ability to find near surface pressure values

in addition to the far-field source level values of a given array. To test this ability, two of the spherical shells near the acoustic center of the array, were analyzed for their surface pressure

Total surface pressures in plane of array for shell #8

solid analytic
dashed eqi
dash/dot sysnoise
dotted atila

solid eqi
dashed sysnoise
dash/dot atila

Total surface pressures (phase) in plane of array for shell #8

solid analytic
dashed eqi
dash/dot sysnoise
dotted atila

solid eqi
dashed sysnoise
dash/dot atila
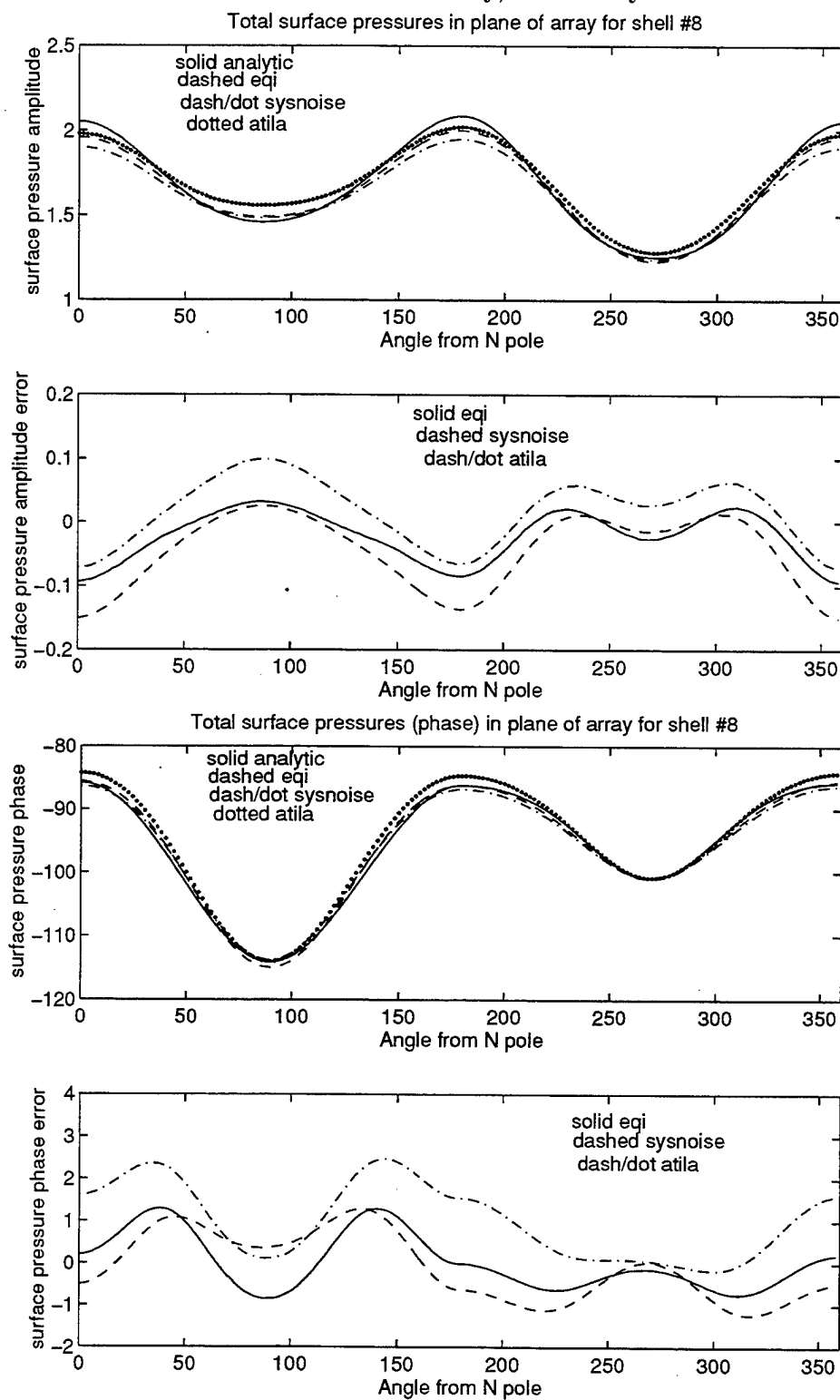
(amplitude and phase). They are the two shells numbered 8 and 9 in the array graphic given previously. Spherical shell 8 is on the "shadow" side of the array while 9 is on the "lit" side.

Total surface pressures in plane of array for shell #9

solid analytic
dashed eqi
dash/dot sysnoise

dotted atila

surface pressure amplitude vs *Angle from N pole*

solid eqi
dashed sysnoise

dash/dot atila

surface pressure amplitude error vs *Angle from N pole*

Total surface pressures (phase) in plane of array for shell #9

solid analytic

dashed eqi

dash/dot sysnoise

dotted atila

surface pressure phase vs *Angle from N pole*

solid eqi

dashed sysnoise

dash/dot atila

surface pressure phase error vs *Angle from N pole*

The graphs indicate an error (relative to the max pressure amplitude) which is less than 5% for nearly the whole surface. The analytic values of the total surface pressure used spherical harmonics up to a value of n=6 for the array. Only the total surface pressure in the plane of the array (y-z plane) are shown, but this is the plane where the anticipated largest and smallest values of the total pressure should occur. In fact, the pressure max points appear to lie on the north and south poles of the shells as one would expect, since along these directions there are more shells. An interesting result is the surface pressure phase on shell 9. In this instance, ATILA/EQI appears to do the worst of all three methodologies. It is assumed that this result is due to the relative error in calculating the dipole T-matrix elements

The important thing to note is the accuracy of the actual surface pressure for this array which has shells of radius 50 cm, and nearest neighbors only 11 cm away.

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