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A CONDUCTIVE PLASMA

- USSR -

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AN APPROXIMATE SOLUTION OF THE PROBLEM OF THE MOTION OF
A CONDUCTIVE PLASMA

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Following is the translation of an article
by G. A. Skuridin and K. P. Stanyukovich in
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References [1-4] describe a new method of asymptotic
integration of linear hyperbolic partial differential equa-
tions and show the application of this method for finding
asymptotic solutions of acoustic and Maxwellian equations.
In references [5-8] the indicated method is developed as
it applies to the solution of dynamic problems of the theo-
ry of elasticity.

For the case of linear hyperbolic partial differen-
tial equations (wave equations, for instance), the general
principle of this method is that we attempt to satisfy ap-
proximately the initial conditions by special selection of
the functions, i. e., we seek a solution of the form

$$u(x, y, z, t) = A(x, y, z) \exp(i\omega(t - \Phi(x, y, z))) \quad (1)$$

given the condition that $\omega \rightarrow \infty$.

As a result we obtain the known relationships:

$$\text{grad}^2 \Phi = \frac{1}{c^2}; \quad (2)$$

$$2(\text{grad} A \text{grad} \Phi) + A \Delta \Phi = 0, \quad (3)$$

where $\Phi(x, y, z)$ is the wave Eikonal, and $A(x, y, z)$ is the
oscillation amplitude.

On the other hand, it is well known that the equa-
tion for the jump in the discontinuous solutions of wave
equations coincides with Eq. (3). In other words, the
approximate solution (1) coincides at the wave front (when
 $\omega \rightarrow \infty$) with a discontinuous solution which may exist for

a rigorous solution of the initial equation.

Thus, we can establish the identity between the discontinuity of the nonsteady wave front and the amplitude of the "geometric approximation" that corresponds to the trajectories of rays orthogonal to these wave fronts.

The simplicity of the physical interpretation of the asymptotic method for the case of linear equations unfortunately is not retained in quasilinear and nonlinear equations. Formally, however, this method can be used even here to solve a number of problems.

In this paper the authors apply the approximate method to the problem of integrating equations of plasma oscillations. We investigate the problem of the motion of a gas in a medium of finite conductivity σ . Let us consider that the medium conforms to the equation of state

$$P = \rho e^{\frac{S-S_0}{\gamma}}$$

In this case, if it is considered that $\mathbf{v} \perp \mathbf{H}$, where $\mathbf{v} = (u, 0, 0)$ and $\mathbf{H} = (0, H, 0)$ and the displacement current is neglected, the system of magnetogasdynamic equations in the one-dimensional case will be obtained in the form [9]

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} &= 0, \\ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{\rho} \frac{\partial}{\partial x} \left(P + \frac{H^2}{8\pi} \right) &= 0, \\ \frac{\partial P}{\partial t} + u \frac{\partial P}{\partial x} + \gamma P \frac{\partial u}{\partial x} &= \frac{(\gamma-1)}{4\pi} \kappa \left(\frac{\partial H}{\partial x} \right)^2, \\ \frac{\partial H}{\partial t} + \frac{\partial u H}{\partial x} &= \kappa \frac{\partial^2 H}{\partial x^2}. \end{aligned} \quad (4)$$

Here $\kappa = c^2/4\pi\sigma$ is the magnetic viscosity, c is the conductivity, $\gamma = c_p/c_v$ is the heat capacity ratio; P is the pressure, ρ is the density, u is the gas velocity, c is the speed of light, S is the entropy, and H is the magnetic field strength.

The first equation in System (4) is the continuity equation, the second is the equation of motion, the third is the energy equation, and the last one is the Maxwell equation with Ohm's law taken into account.

Thus the problem is to find the unknowns P , ρ , H and u in a fairly general form, i. e., such that they contain arbitrary functions which can then be determined from the initial and boundary conditions.

Since the energy equation is not exact (heat conduction and radiation are not taken into account) and the

exact equation is too complex [10], when seeking P, ρ, H and u it is advisable to use only three equations from System (4), giving u in a definite form with an accuracy determined by the arbitrary function and constant.

Let us go on to the solution of the stated problem. Introducing the pseudoscalar potential φ by means of the relationship $H = \partial\varphi/\partial x$, we can write the last equation in System (4) in the form

$$\frac{\partial\varphi}{\partial t} + u \frac{\partial\varphi}{\partial x} = x \frac{\partial^2\varphi}{\partial x^2}. \quad (5)$$

Let us find the solution to Eq. (5) by assuming

$$\varphi = A(x, y, z, t) e^{i\omega t(x, y, z, t)}. \quad (6)$$

Substituting (6) in (5) and separating the real and imaginary parts, we find that

$$\frac{\partial A}{\partial t} + u \frac{\partial A}{\partial x} = x \left[\frac{\partial^2 A}{\partial x^2} - \omega^2 A \left(\frac{\partial t}{\partial x} \right)^2 \right]; \quad (7)$$

$$\frac{\partial t}{\partial t} + u \frac{\partial t}{\partial x} = x \left[\frac{\partial^2 t}{\partial x^2} + 2 \frac{\partial t}{\partial x} \frac{\partial}{\partial x} \ln A \right]. \quad (8)$$

When $\omega \gg 1$, it can be shown that

$$\frac{\partial^2 A}{\partial x^2} \ll \omega^2 A \left(\frac{\partial t}{\partial x} \right)^2; \quad (9)$$

here Eq. (7) assumes the form

$$\frac{\partial}{\partial t} (\ln A) + u \frac{\partial}{\partial x} (\ln A) + x \omega^2 \left(\frac{\partial t}{\partial x} \right)^2 = 0. \quad (10)$$

From now on we will deal with the class of solutions (6) which is subject to an additional condition; i. e., we will assume that

$$\omega \sqrt{x} \frac{\partial t}{\partial x} = B = \text{const}. \quad (11)$$

(The more general case $B = B(t)$ could also be considered; here the solution of Eq. (8) will not become more complicated.)

ted in principle.)

Let us assume $\omega\sqrt{x} = \alpha$, and in so doing let us select the order of ω in such a way that the order of α corresponds to the order of the remaining terms in the equation. By means of (11) we obtain

$$f = \frac{B}{\alpha} x + T(t). \quad (12)$$

Substituting (11) into (8) and taking (12) into account, we have

$$u = 2x \frac{\partial}{\partial x} \ln A - \frac{\alpha}{B} \dot{T}. \quad (13)$$

Excluding u , from (10) and (13), we arrive at the equation

$$\dot{T} \frac{\alpha}{B} \frac{\partial}{\partial x} \ln A - \frac{\partial}{\partial t} \ln A - B^2 = 2x \left(\frac{\partial}{\partial x} \ln A \right)^2.$$

Differentiating with respect to x and designating $\theta = (\ln A)_x$, we obtain

$$\frac{\partial \theta}{\partial t} + \left[4x\theta - \frac{\alpha}{B} \dot{T} \right] \frac{\partial \theta}{\partial x} = 0. \quad (14)$$

The solution of this equation has the form

$$x = 4xt\theta - \frac{\alpha}{B} T + F(\theta), \quad (15)$$

where $F(\theta)$ is an arbitrary function.

Thus for the calculation of θ and u we have two arbitrary functions $T(t)$ and $F(\theta)$, and one arbitrary constant B . Let us consider the case in which $F(\theta) = \beta\theta$, where $\beta = \text{const} < 0$. Here we arrive at a linear function for $u = u(x, t)$. Indeed from (15) we have

$$x = (4xt + \beta)\theta - \frac{\alpha}{B} T. \quad (16)$$

Substituting θ from (16) in (13), we obtain

$$u = 2x \frac{x + \frac{\alpha}{B} T}{4xt + \beta} - \frac{\alpha}{B} \dot{T}. \quad (17)$$

Moreover, taking (15) into account, we have

$$A = A_0(t) \exp \left[\frac{x^2 + \frac{\alpha}{B} xT}{4xt + \beta} \right]. \quad (18)$$

Consequently, our solution of Eq. (3) will assume the form

$$\varphi = A_0(t) \exp \left[\frac{x^2 + \frac{\alpha}{B} xT}{4xt + \beta} \right] \cos \omega \left(\frac{B}{\alpha} x + T \right). \quad (19)$$

We obtain from this for the magnetic field strength

$$H = A_0(t) \left\{ \frac{x + \frac{\alpha}{B} xT}{4xt + \beta} \cos \omega \left(\frac{B}{\alpha} x + T \right) - \frac{B\omega}{\alpha} \sin \omega \left(\frac{B}{\alpha} x + T \right) \right\} \exp \left[\frac{x^2 + \frac{\alpha}{B} xT}{4xt + \beta} \right]. \quad (20)$$

Thus one of the unknowns in System (4) has been found.

The arbitrary function $T(t)$ can be determined by introducing some condition for u ; for example, we can assume that at $x = x_0$, $u = 0$ (at the wall), or in a more general case we can assume that the wall moves according to the law $x = \phi(t)$. Then $u = \dot{x} = \dot{\phi}(t)$. We obtain from Eq. (17)

$$\dot{T} = \frac{2xT}{4xt + \beta} - \frac{B}{\alpha} \left[\dot{\phi} - \frac{2x\dot{\phi}}{4xt + \beta} \right] \quad (21)$$

or

$$T = \sqrt{4xt + \beta} \left\{ \text{const} + \frac{\beta}{\alpha} \int \left[\frac{2x\dot{\phi}}{4xt + \beta} - \dot{\phi} \right] \frac{dt}{\sqrt{4xt + \beta}} \right\}. \quad (22)$$

The constant may be determined by assuming that $T = 0$ at $t = 0$.

If the value \dot{t} from (21) is substituted in (17), the last expression takes the form

$$u = \frac{2x(x-\psi)}{4xt + \beta} + \dot{\phi}. \quad (23)$$

Further, we find the density ρ from the first equation in System (4). Since, taking (23) into account

$$\frac{2x}{4xt + \beta} + \frac{\partial}{\partial t} (\ln \rho) + \left[\frac{2x(x-\psi)}{4xt + \beta} + \dot{\phi} \right] \frac{\partial}{\partial x} (\ln \rho) = 0,$$

then

$$\rho = \sqrt{4xt + \beta} \Phi(z), \quad (24)$$

where $\Phi(z)$ is the arbitrary function, and

$$z = \frac{x}{\sqrt{4xt + \beta}} + \int \left[\frac{2x\psi}{4xt + \beta} - \dot{\phi} \right] \frac{dt}{\sqrt{4xt + \beta}}. \quad (25)$$

After this, we find the pressure P from the second equation in System (4)

$$P + \frac{H^2}{8\pi} = P_0(t) - \int \rho(u_t + uu_x) dx. \quad (26)$$

where $P_0(t)$ is the arbitrary time function.

The arbitrary functions $\Phi(z)$ and $P_0(t)$ should be determined from the boundary conditions -- by assuming (in the case of motion with a shock wave, for example) that the known relationships between $\rho = \rho(u)$ and $P = P(u)$ are satisfied at the shock-wave front.

Solution of the problem in the concrete form $\phi = \phi(t)$ does not present any difficulty.

Thus, by means of Relationships (20), (23), (24) and (26) we determine all the quantities comprising System (4).

In conclusion we should point out that in integrating Eq. (7) we have neglected the term $\frac{1}{A} \frac{\partial^2 A}{\partial x^2}$ as small

compared with $\omega^2 \left(\frac{df}{dx} \right)^2 = \frac{B^2}{x}$. It is not hard to see that

$$\frac{x}{B^2} \frac{1}{A} \frac{\partial^2 A}{\partial x^2} = \frac{x}{B^2} \left[\left(x + \frac{\alpha}{B} T \right)^2 + \frac{1}{4xI + \beta} \right] \approx \frac{1}{\omega^2 \left(\frac{df}{dx} \right)^2} \ll 1,$$

i. e., our assumption is borne out.

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