REPORT DOCUMENTATION PAGE

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OMB NO. 0704-0188

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1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
12/2/98

3. REPORT TYPE AND DATES COVERED
Final Report - 8 Aug 95 - 7 Aug 98

4. TITLE AND SUBTITLE
Experimental Control of Chaos

5. FUNDING NUMBERS
DAAH04-95-1-0529

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8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U.S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSORING / MONITORING AGENCY REPORT NUMBER
ARO 34611.13-PH

11. SUPPLEMENTARY NOTES
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12a. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12 b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

We have pursued several research directions on the development, characterization, control and synchronization of nonlinear dynamical systems displaying low-dimensional deterministic chaos. The goal of this program is to uncover fundamental issues related to the control and synchronization of nonlinear systems by conducting experiments with novel devices and making precise comparisons between our observations and theoretical predictions. The results of these studies will eventually lead to improved performance of devices that are based on nonlinear systems and to the development of new classes of communication systems. Specifically, we have investigated the control of high-speed nonlinear systems including electronic circuits and lasers, and we have explored the conditions under which two nonlinear systems will synchronization in an experimental setting.

This research has resulted in 11 publications, 12 invited conference presentations, colloquia, and seminars, 4 distributed conference presentations, one Ph.D. dissertation and several extended contacts with DoD researchers.

14. SUBJECT TERMS
controlling chaos, synchronizing chaos

15. NUMBER IF PAGES
8

16. PRICE CODE

17. SECURITY CLASSIFICATION OR REPORT
UNCLASSIFIED

18. SECURITY CLASSIFICATION OF THIS PAGE
UNCLASSIFIED

19. SECURITY CLASSIFICATION OF ABSTRACT
UNCLASSIFIED

20. LIMITATION OF ABSTRACT
UL

NSN 7540-01-280-5500

Enclosure 1

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. 239-18
298-102

Experimental Control of Chaos

Final Report

Daniel J. Gauthier

Research Period Covered by Report:
8 AUG 95 - 7 AUG 98

U. S. Army Research Office

Grant Number DAAH04-95-1-1-0529

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1 Experimental Control of Chaos

1.1 Statement of the problem

We have pursued several research directions on the development, characterization, control and synchronization of nonlinear dynamical systems displaying low-dimensional deterministic chaos. The goal of this program is to uncover fundamental issues related to the control and synchronization of nonlinear systems by conducting experiments with novel devices and making precise comparisons between our observations and theoretical predictions. The results of these studies will eventually lead to improved performance of devices that are based on nonlinear systems and to the development of new classes of communication systems. Specifically, we have investigated the control of high-speed nonlinear systems including electronic circuits and lasers, and we have explored the conditions under which two nonlinear systems will synchronization in an experimental setting.

This research has resulted in 11 publications, 12 invited conference presentations, colloquia, and seminars, 4 contributed conference presentations, one Ph.D. dissertation, and several extended contacts with DoD researchers, as described in Sec. 1.4. A brief description of the research projects are summarized in the following two sections.

1.2 Summary of important results

1.2.1 Controlling Chaos

Control of chaos in high-speed electronic systems During the first phase of the research program, the PI's group investigated a control scheme that is effective in suppressing deterministic chaos in fast dynamical systems. It is desirable to devise such schemes because the presence of deterministic chaos in devices generally degrades their performance in many applications. The signatures of chaos include erratic, noise-like fluctuations in the temporal evolution of the system variables, broadband features in the power spectrum, and the long-term behavior of the system is extreme sensitivity to applications of small perturbations to the system variables.

Specifically, we stabilized unstable periodic orbits of a fast diode resonator driven at 10.1 MHz (corresponding to a drive period under 100 ns) using Extended Time-Delay Autosynchronization (ETDAS). Stabilization is achieved by feedback of an error signal that is proportional to the difference between the value of a state variable and an infinite series of values of the state variable delayed in time by integral multiples of the period of the orbit. The technique is easy to implement electronically and it has an all-optical counterpart that may be useful for stabilizing the dynamics of fast chaotic lasers. We found that increasing the weights given to temporally distant states enlarges the domain of control and reduces the sensitivity of the domain of control on the propagation delays in the feedback loop. We determined the average time to obtain control as a function of the feedback gain and identify the mechanisms that destabilize the system at the boundaries of the domain of control. A theoretical stability analysis of a model of the diode resonator in the presence of time-delay feedback is in good agreement with the experimental results for the size and shape of the domain of control. These results were published in a special focus issue on Control and Synchronization of Chaos appearing in the journal Chaos in December 1997. Figure 1 shows the experimentally observed temporal evolution of the controlled chaotic circuit and the ETDAS error signal as well as the return maps of the controlled and uncontrolled dynamics. These results demonstrate that we control the dynamics about an unstable periodic orbit embedded in the chaotic system using only small perturbations.

Analysis and comparison of multiple-delay schemes for controlling chaos As stated above, one advantage of the ETDAS scheme is that the feedback error signal is generated by making a comparison between the current and past states of the dynamical system; a comparison to a predetermined reference state is never made. In collaboration with Prof. Socolar of Duke University, we compared and contrasted the ETDAS scheme, which uses an infinite series of past iterates with weights that decay by a factor of R with each step, with one that uses an average of N past iterates with equal weights. We determined theoretically the range of feedback parameters that successfully stabilize the system and the robustness of the schemes to noise. It is found that the domain of control of the two schemes is similar for appropriately matched

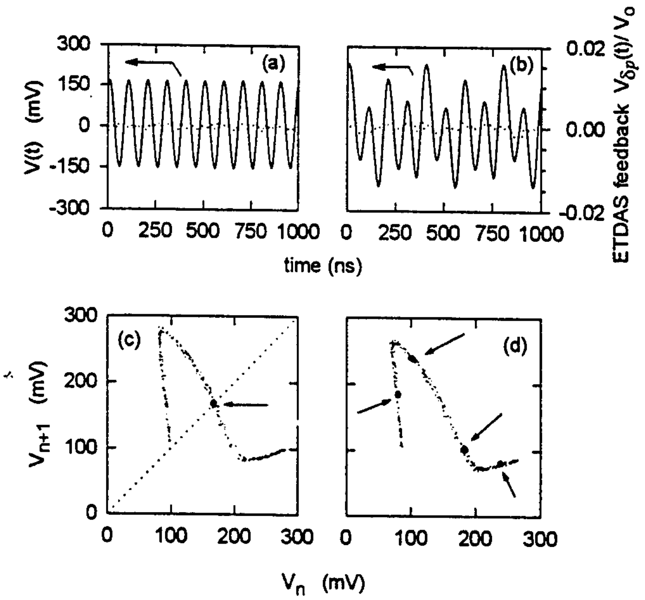

Figure 1: Controlling chaos in a high-speed electronic circuit known as the diode resonator. Control of the period-1 orbit [(a) and (d)] and the period-4 orbit [(b) and (d)] visualized through the temporal evolution of the current flowing through the resonator [(a) and (b)], and from the first return maps [(c) and (d)]. The dashed line in the time series show the size of the feedback signal applied to the resonator expressed as a fraction of the amplitude of the drive signal. In the first return maps, the cluster of dots highlighted by the arrows is the behavior observed when control is applied to the system.

values of R and N, and that the scheme using N equally weighted states tends to be less sensitive to noise, as shown in Fig. 2. The results of this study were published in Physical Review E, and have yet to be tested experimentally.

Controlling unstable steady-states of chaotic systems In many cases of practical interest, it is desirable to stabilize a steady state of a chaotic system where all dynamical variables are constant as a function of time, rather than stabilizing an unstable periodic orbit where the system evolves in a periodic fashion. For example, it is often desirable that lasers produce a constant-intensity beam in some applications rather than a chaotically or periodically fluctuating beam. We have found that the controlling chaos technique we developed for controlling unstable periodic orbits, Extended Time-Delay Autosynchronization, can be adjusted to control unstable steady states of chaotic systems. In a proof-of-concept experiment, we stabilized the unstable steady states of a low-speed chaotic electronic circuit by applying a feedback signal generated by high-pass-filtering in real time the dynamical state of the system to an accessible system variable. Figure 3 shows the experimentally observed temporal evolution of one variable of the circuit when control is switched on suddenly. Once control is achieved, the size of the perturbation applied to the system are much less than 1% of the characteristic signal sizes. The observed behavior is in excellent agreement with a theoretical model of the controlled device. The technique is easy to implement, does not require knowledge of the unstable steady state coordinates in phase space, tracks automatically changes in the system parameters, and is more robust to broadband noise than previous schemes. The simplicity and robustness of the scheme suggests that it is ideally suited for stabilizing unstable steady states in ultra-high-speed systems. The results of this investigation have been accepted for publication in an upcoming issue of Chaos.

Controlling unstable steady-states of chaotic lasers In light of the potential usefulness of the scheme described in the previous section, we conducted a theoretical study to assess whether specific implementations of the protocol might be applicable for stabilizing laser dynamics. Stabilizing the unstable steady states

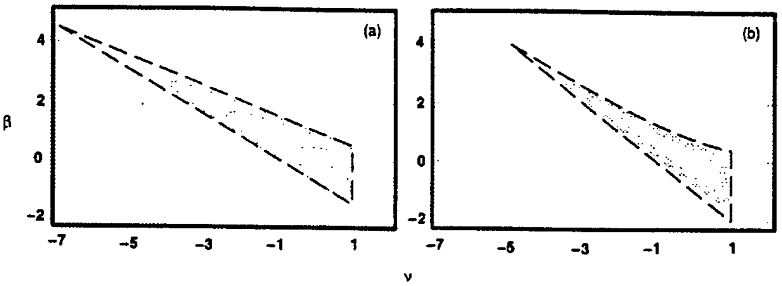

Figure 2: Contour plots of noise sensitivity for (a) ETDAS and (b) NTDAS control for appropriately matched values of the feedback parameters. The parameter ν is a measure of the stability of the underlying fixed point where it is unstable when $|\nu| > 1$. The parameter β is proportional to the feedback gain. The dashed lines indicate the boundary of the domain of control. The contours map indicate the extent to which the controlled system amplifies system noise. In (a), the contours correspond to an amplification factor of $\sqrt{2}$, 8, and 32, and in (b) they correspond to $\sqrt{2}$, 2, and $2\sqrt{2}$, where the smallest contour is for the smallest amplification factor.

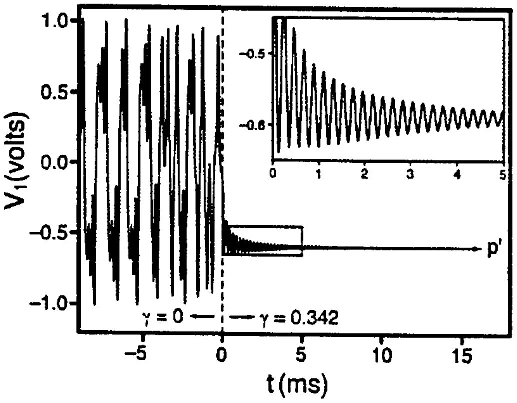

Figure 3: Controlling the unstable steady state of a chaotic electronic circuit using ETDAS feedback that has been optimized for control such states.

(continuous wave state) of laser is often desirable for applications because of the high degree of coherence of such states. For an idealized laser model, we found that there exists a wide range of feedback parameters giving rise to stable behavior (known as the domain of control) and the controller can automatically track slow variation or drift of the laser parameters. We identified two possible ways in which ETDAS control can be applied to lasers including: an incoherent feedback technique where the pump rate is adjusted by a feedback signal proportional to the intensity of the beam generated by the laser and sensed by a square-law detector; and a coherent technique where the control perturbation is an optical field injected into the laser generated by filtering with a Fabry-Perot interferometer the field generated by the laser, as shown schematically in Fig. 4. While it is well known that this idealized laser model does not describe quantitatively the behavior of typical lasers, these observations highlight the potential of the feedback schemes. Detail studies of specific laser systems must be undertaken to ascertain whether the controlling chaos techniques will be useful in real-world applications. The results of this study were published in Optics Letters.

Controlling low-frequency fluctuations in diode lasers During this research period, we have conducted a thorough investigation of the dynamics of semiconductor lasers in the presence of weak optical feedback in the low-frequency fluctuation regime. We find that the 'power drop out events' can be entrained using periodic modulation of the injection current. Under different conditions, the drop out events can be

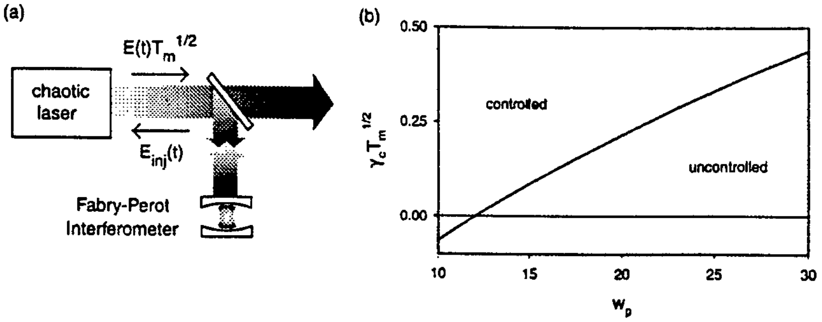

Figure 4: (a) Possible realization of the coherent control scheme with (b) its associated domain of control, where w_p is the laser pump rate, and $\gamma_c T_m^{1/2}$ is a measure of the strength of the feedback perturbations. The steady state operation of the laser is unstable for $w_p > 12$.

forced to occur more frequently while the power spectrum of the laser broadens significantly. Under no conditions have we found that it is possible to suppress the power drop out events using periodic modulation. We have developed a simple theory of the laser with optical injection that is based on a stochastic differential equation. The predictions of this model agree well with the experimental observations when the laser is close to threshold, as shown in Fig. 5. These studies are necessary to understand the dynamics of semiconductor lasers in the presence of optical feedback, which may be crucial for optimizing devices that use diode lasers. The results of part of this study were published in Physical Review A as a Rapid Communication.

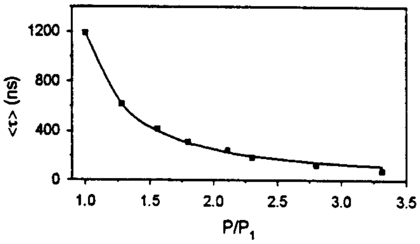

Figure 5: Mean time interval between power dropout events as a function of the laser power above threshold. The square symbols indicate the experimentally observed average time between the beginnings of the events. The solid line illustrates the best fist of the predictions of the Henry and Kasarinov model [C.H. Henry and R.F. Kazarinov, IEEE J. Quantum Electron. **QE-22**, 294 (1986)].

Controlling a chaotic point process As part of our efforts on controlling the dynamics of nonlinear systems, we have investigated systems that are well described as a point process. Such a system evolves so that long periods of inactivity are punctuated by brief, nearly identical bursts of activity. The semiconductor laser in the low-frequency fluctuations regime is approximately of this type, where the power drop out events are the brief bursts of activity. In a proof-of-concept experiment, we created a low-speed physical point process by passing a continuous, deterministic, chaotic signal from a chaotic electronic circuit through an integrate-and-fire device, as shown in Fig. 6. The timing between the discrete events was controlled using proportional feedback incorporating only the time intervals between events. This system is unique in that the mean time between events can be adjusted independent of the dynamics of the underlying chaotic system. It is found that the range of feedback parameters giving rise to control as a function of the mean firing time exhibits surprisingly complex structure, and control is not possible when the mean interspike interval is comparable to or larger than the underlying system memory time. The results of this investigation were published in Physical Review E.

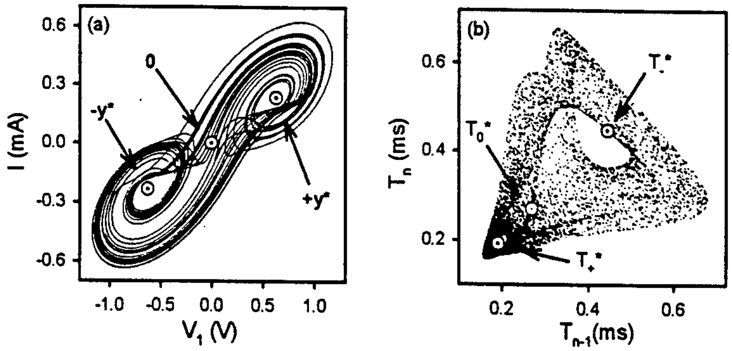

Figure 6: (a) Projection in phase space of the chaotic attractor of the electronic circuit used in the experiments. (b) Reconstruction of the chaotic attractor using the interspike intervals. ETDAS feedback successful stabilized the fixed point denoted by T_+^*.

1.2.2 Synchronizing Chaos

We have performed a simple experiment to characterize the conditions under which chaotic oscillators synchronize. Surprisingly, we find that the oscillators do not synchronize when expected based on the standard, widely-used criterion for synchronization of chaotic oscillators, as shown in Fig. 7. We proposed a new criterion for the robust synchronization of the chaotic oscillators that agrees well with our experimental observations. Our results clearly demonstrate that the assumptions used in many theoretical papers over the last ten years may have to be reinvestigated when considering the experimental applicability of the predictions of the theories. Publication and discussion of these results has spurred new research in this area; we collaborated with Prof. Ed Ott's theoretical group at the University of Maryland to extend our work, leading to a joint publication. We are currently collaborating with Dr. Lou Pecora of the Naval Research Laboratory and Prof. Reggie Brown to explore these issues further. The results of these studies were published in Physical Review Letters and in the Proceedings of the 4th Experimental Chaos Conference.

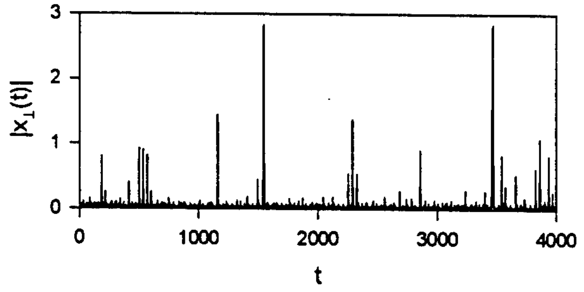

Figure 7: Experimentally observed temporal evolution of the distance in phase space between two coupled chaotic electronic circuits in a regime where high-quality synchronization is expected. Long intervals of high-quality synchronization are interrupted by brief, large-scale (comparable ot the size of the synchronization manifold) desynchronization events. This behavior is now know as 'attractor bubbling.'

Recently, we have investigate the synchronization of periodic oscillators and find that our understanding of when they synchronize is also not well understood. Our studies indicate that the synchronization criterion proposed by our group appears to be the most useful in an experimental setting. The results of this study has been submitted for publication to Physical Review Letters.

1.3 List of publications and technical reports

J.N. Blakely, D.J. Gauthier, G. Johnson, T.L. Carroll, and L.M. Pecora, 'Bursting and Synchronization Criteria in Coupled Oscillators,' submitted for publication (1998).

A. Chang, J.C. Bienfang, G.M. Hall, J.R. Gardner, and D.J. Gauthier, 'Stabilizing unstable steady states using extended time-delay autosynchronization,' to appear in Chaos (1998).

G.M. Hall, S. Bahar, and D.J. Gauthier, 'Experimental control of a chaotic integrate-and-fire system using interspike intervals,' Phys. Rev. E **58**, 1685 (1998).

J.E.S. Socolar and D.J. Gauthier, 'Analysis and comparison of multiple-delay schemes for controlling unstable fixed points of discrete maps,' Phys. Rev. E **75**, 6589 (1998).

D.J. Gauthier, 'Controlling lasers by use of extended time-delay autosynchronization,' Opt. Lett. **23**, 703 (1998).

D.J. Gauthier, 'Intermittent loss of synchronized chaos under conditions when high-quality synchronization is expected,' to appear in the Proceedings of the 4^{th} Experimental Chaos Conference (1997).

D.W. Sukow, J.R. Gardner, and D.J. Gauthier, 'Statisitics of power dropout events in semiconductor lasers with time-delayed optical feedback,' Phys. Rev. A (Rapid Communications) **56**, R3370 (1997).

D.W. Sukow, M.E. Bleich, D.J. Gauthier, and J.E.S. Socolar, 'Controlling chaos in fast dynamical systems: Experimental results and theoretical analysis,' Chaos **7**, 560 (1997).

D.W. Sukow, 'Experimental control of instabilities and chaos in fast dynamical systems,' Duke University Ph.D. dissertation, unpublished.

S.C. Venkataramani, B.R. Hunt, E. Ott, D.J. Gauthier, and J.C. Bienfang, 'Bubbling transitions of chaotic systems,' Phys. Rev. Lett. **77**, 5361 (1996).

D.J. Gauthier and D.W. Sukow, 'Controlling chaos and instabilities in fast optical systems,' LEOS Newsletter **6**, 15 (1996).

D.J. Gauthier and J.C. Bienfang, 'Intermittent loss of synchronization in coupled chaotic oscillators: toward a new criterion for high-quality synchronization,' Phys. Rev. Lett. **77**, 1751 (1996).

1.4 DoD Contacts and Technology Transfer

The PI delivered the following presentations at DOD facilities:
'Controlling the Dynamics of High-Speed Optical Systems', US ARO workshop on Communicating by Chaos: Digital Signal Generation by Simple Nonlinear Devices, U.S. Army Research Office, RTP, NC, 5 JUN 96. As an outgrowth of this workshop, the PI worked closely with Dr. John Lavery of the ARO on preparing a section of a Strategic Assessment Report titled 'Toward a New Digital Communication Technology Based on Nonlinear Dynamics and Chaos.'

'Linear and Nonlinear Chaos Control without Reference States,' at the Workshop on Theory, Diagnostics and Control of Chaos, December 5, 1997, Redstone Arsenal, Huntsville, AL.

'Intermittent loss of synchronization in coupled dynamical systems,' Nonlinear Dynamics Seminar, December 13, 1995, Naval Research Laboratory, Washington, DC.

'Tutorial on chaos and fractals,' Workshop on Chaos, Fractals and Wavelets in Data Links, November 14, 1995, Redstone Arsenal, AL.

1.5 List of participating personnel

Dr. Jeff Gardner, Dr. David Sukow, Mr. Mark Steen, and Mr. Jonathan Blakely were partially supported by this grant under the supervision of Prof. Daniel Gauthier. Dr. Sukow was awarded a Ph.D. in Physics for his work on the control of chaos in high-speed dynamical systems. Mr. Blakely was awarded a M.A. in Physics for his work on the synchronization of chaos.

1.6 Inventions

No inventions have been disclosed as part of this grant.