

One also has the well-known expression for the mutual information, $I_{AB}(Q)$, in the Alice-Bob channel in terms of the error rate, Q , in the Alice-Bob channel [1],

$$I_{AB}(Q) = 1 + Q \log_2 Q + (1 - Q) \log_2 (1 - Q) . \quad (9)$$

In the present work, I prove a significant new result, namely, that in the entangled translucent eavesdropping scenario, the unsafe error rate based on Eq. (2) is equivalent to the maximum allowable error rate based on Eq. (3). In this case, the unsafe error rate is not in fact overly conservative, as is commonly supposed.

Maximum Allowable Error Rate

First define

$$\alpha = \frac{\pi}{4} - \frac{\theta}{2} . \quad (10)$$

Next, evaluating Eq. (7) for $Q = \sin^2 \alpha$, one obtains

$$Q_{BE}(\sin^2 \alpha, \theta) = 0 . \quad (11)$$

If one next substitutes Eq. (11) in Eq. (8), one obtains

$$I_{BE}(Q_{BE}(\sin^2 \alpha, \theta)) = 1 . \quad (12)$$

Therefore, comparing Eqs. (12) and (3), one concludes that

$$Q_{\max} = \sin^2 \alpha , \quad (13)$$

in agreement with Eq. (41) of Ekert et al [1]. The maximum allowable error rate is given by Eqs. (13) and (10) in terms of the angle between the two photon polarization states.

Proof that Unsafe Error Rate is Maximum Allowable Error Rate

Evaluating Eq. (4) for $Q = \sin^2 \alpha$, one obtains

$$Q_{AE}(\sin^2 \alpha, \theta) = \sin^2 \alpha . \quad (14)$$

If one next substitutes Eq. (14) in Eq. (6), one obtains

$$I_{AE}(Q_{AE}(\sin^2 \alpha, \theta)) = 1 + \sin^2 \alpha \log_2 (\sin^2 \alpha) + \cos^2 \alpha \log_2 (\cos^2 \alpha) . \quad (15)$$

Next, using Eq. (9), one obtains

$$I_{AB}(\sin^2 \alpha) = 1 + \sin^2 \alpha \log_2 (\sin^2 \alpha) + \cos^2 \alpha \log_2 (\cos^2 \alpha) . \quad (16)$$

Comparing Eq. (16) with Eq. (15), one can conclude that

$$I_{AE}(Q_{AE}(\sin^2 \alpha, \theta)) = I_{AB}(\sin^2 \alpha) . \quad (17)$$

Using Eq. (12), one obtains

$$\begin{aligned} \min(I_{AE}(Q_{AE}(\sin^2 \alpha, \theta)), I_{BE}(Q_{BE}(\sin^2 \alpha, \theta))) \\ = \min(I_{AE}(Q_{AE}(\sin^2 \alpha, \theta)), 1) \\ = I_{AE}(Q_{AE}(\sin^2 \alpha, \theta)) . \end{aligned} \quad (18)$$

Therefore, substituting Eq. (17) in Eq. (18), one has

$$\min(I_{AE}(Q_{AE}(\sin^2 \alpha, \theta)), I_{BE}(Q_{BE}(\sin^2 \alpha, \theta))) = I_{AB}(\sin^2 \alpha) . \quad (19)$$

Next comparing Eq. (19) with Eq. (2), one can conclude that

$$Q_u = \sin^2 \alpha . \quad (20)$$

Finally, comparing Eqs. (13) and (20), one has

$$Q_u = Q_{\max} , \quad (21)$$

which was the claim. If one substitutes Eq.(10) in Eqs. (20) and (21), one obtains

$$Q_u = Q_{\max} = \frac{1}{2} (1 - \sin \theta) , \quad (22)$$

written explicitly in terms of the angle between the photon polarization states.

Conclusion

I conclude that for the entangled translucent eavesdropping scenario of Ekert et al [1], the unsafe error rate defined by Eq. (2) is in fact equal to the maximum allowable error rate defined by Eq. (3). These rates are given by Eq. (22).

For this scenario, the unsafe error rate is not in fact overly conservative, as is commonly supposed.

References

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4. H. E. Brandt, J. M. Myers, and S. J. Lomonaco, Jr., *New Results in Entangled Translucent Eavesdropping in Quantum Cryptography*, to appear in Proceedings of SPIE Conference, Photonic Quantum Computing, 21-25 April 1997, Orlando, FL, S. P. Hotaling and A. R. Pirich, editors (1997).

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**C. Report's Point of Contact: (Name, Organization, Address,
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Howard E. Brandt (301) 394-4143
2800 Powder Mill Road
Adelphi, MD 20783

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