| **REPORT DOCUMENTATION PAGE** | | | *Form Approved* *OMB No. 0704-0188* |
|---|---|---|---|



| **1. AGENCY USE ONLY** *(Leave blank)* | **2. REPORT DATE** 26.Oct.98 | **3. REPORT TYPE AND DATES COVERED** MAJOR REPORT |
|---|---|---|

**4. TITLE AND SUBTITLE**
ORBIT DETERMINATION FROM VISUAL SIGHTINGS: AN INVESTIGATION OF TWO ANGLES-ONLY ORBIT DETERMINATION PROCESSES INCLUDING A SCIENCE ACTIVITY FOR MIDDLE AND HIGH SCHOOL STUDENTS

**5. FUNDING NUMBERS**

**6. AUTHOR(S)**
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**7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)**
UNIVERSITY OF COLORADO AT COLORADO SPRINGS

**8. PERFORMING ORGANIZATION REPORT NUMBER**

**9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)**
THE DEPARTMENT OF THE AIR FORCE
AFIT/CIA, BLDG 125
2950 P STREET
WPAFB OH 45433

**10. SPONSORING/MONITORING AGENCY REPORT NUMBER**
98-029

**11. SUPPLEMENTARY NOTES**

**12a. DISTRIBUTION AVAILABILITY STATEMENT**
Unlimited distribution
In Accordance With AFI 35-205/AFIT Sup 1

**12b. DISTRIBUTION CODE**

**13. ABSTRACT** *(Maximum 200 words)*



| **14. SUBJECT TERMS** | **15. NUMBER OF PAGES** |
|---|---|
| | **16. PRICE CODE** |

| **17. SECURITY CLASSIFICATION OF REPORT** | **18. SECURITY CLASSIFICATION OF THIS PAGE** | **19. SECURITY CLASSIFICATION OF ABSTRACT** | **20. LIMITATION OF ABSTRACT** |
|---|---|---|---|
| | | | |

# Orbit Determination from Visual Sightings:

## An Investigation of Two Angles-only Orbit Determination Processes Including a Science Activity for Middle and High School Students.


By

Lee S. Dewald Jr.

B.S.E.E., The Citadel, 1997


An ivestigative report submitted to the Faculty of the Graduate School of the University of Colorado in partial fulfillment of the requirements for the degree of Master of Engineering in Space Operations.

Spring 1998

## Abstract

This project will investigate the possibilities of determining the orbit of a satellite from the data obtained by observing the satellite passing across the sky. With simple tools such as a stopwatch, compass, and star map or camera, I believe that the orbit of a satellite may be determined by an angles only approach. In addition, young students, of middle school age and older, could be exposed to the fundamentals of orbital mechanics by conducting such an activity. The calculations required to complete this activity will be manipulated into a coded algorithm. This enables the user to enter information which they retrieved during the satellite pass and get back the calculated orbit of the satellite which they observed. This data could then be used to predict future passes of that satellite. This paper will contain a science activity to aid in teaching the fundamentals of orbital mechanics to middle school students. In addition, it will contain a detailed list of instructions for determining when a satellite will be visible to a user's location, as well as how to obtain pertinent data while observing a satellite pass.

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## Introduction

The stars have been used for ages by ancient peoples to observe time and predict events. Centuries ago brilliant mathematicians and astronomers derived methods of predicting the motion of heavenly bodies from observations relating to the night sky. Today these same methods are used in the absence of radar to determine the orbits of satellites both manmade and natural. Now young people can become interested in, as well as knowledgeable of, astrodynamics by gazing into the night sky as many others have thousands of years ago. Although simple, measurements with a stopwatch and star map are all that is necessary to approximate the orbit of a passing satellite and predict the next time that it will be in view. The mathematical method used to do this dates back a few centuries to the days of the greats such as Newton, Halley, Kepler, and many others. In addition, the roots of the mathematics go back even farther to the time of the ancients. To make this orbit determination problem less foreboding to young people, an executable MatLab program will be used to transform the data obtained by the observer into the six orbital elements that define the orbit of the observed satellite. Other programs, such as SkyMap 3.2 and Satellite Tool Kit 4.0, will be used to obtain information regarding the stars and the satellites. Although MatLab will execute the difficult work, special attention must also be drawn to the methods that have been tried to solve this problem. This orbit determination problem has been around for hundreds of years. With the help of computer programming to implement the discoveries of great mathematicians and astronomers, many people young and old will be able to connect themselves with space.

## Background

The universe and its stars have been around for what seems in terms of human years to be an eternity; they have also remained relatively unchanged since the dawn of mankind. All things that we see when we look into the night sky from Earth are happening at incomprehensible rates. One human lifetime is comparable to the blink of an eye in a star's lifetime. It is no wonder that the people of ancient civilizations looked to the stars with curiosity and wonder. In fact, the regular motions of the sun, moon, stars, and passage of day and night did spark curiosity and were even believed to be related to the events in people's lives. This was the dawn of astrology.

When people believed that the orientation and movement of the heavenly bodies affected their personal fortune, they soon desired to know how to predict these movements. Thus, astrology supported further study of space. Stars were used to measure the passage of time and to predict events such as the change of seasons. They were also used by ancient people for long distance navigation. As knowledge and technology advanced, more details were uncovered about space. For example, the Babylonians set out to predict the movement of the sun and moon. In this effort, they discovered that the sun and moon move with increasing speed until a definite maximum is reached. Then their speeds would decrease until a definite minimum was reached at which time the cycle would repeat. As we know today, the velocities referred to above are the velocities of these bodies across the sky. Also, in a Greek poem called Works and Days by Hesiod, the farmer learns of the ideal times to plow, sow, and harvest by the constellations that rise before dawn in the different seasons. The first star catalog had appeared long before 1000 A.D. Thus, astrodynamics, though not by this name, perhaps religion instead, has been a popular if not life dependent science long before the days of Newton and Kepler.

As the centuries passed, more and more data was recorded for all sorts of comet and planetoid observations. By the 16$^{th}$ and 17$^{th}$ centuries, however, man was now determined to explain these observations fully with some sort of mathematical expression. The men to do this were Newton, Halley, and Kepler to name a few of the greatest. In 1609 Johann Kepler published his discovery that orbits are elliptical in shape. Indeed the data recorded of orbiting objects agreed with his discovery. Kepler's laws of planetary motion identified the motion of objects in the solar system, and in 1687 Isaac Newton published Principia. In this publication Newton explains the laws of planetary motion with his laws of motion and gravitation. This publication also includes his development of differential calculus as well as his method of determining the orbit of an object from observations.

By the beginning of the 18th century, Sir Edmund Halley had mastered Newton's method of orbit determination and applied it to many comets of which Halley had many recorded observations. Halley also predicted the return of one of the comets he was studying which later became known as Halley's comet. In 1780, Pierre Simon Laplace, a French astronomer, published a different method of orbit determination from angular data. This method is one of the angles-only approaches for orbit determination that will be explored in this project. Back then these men did not have radar to give range or range rate information. They only had directions, similar to what amateur stargazers today might have if they were to look up into the sky and see an object pass across the sky.

## The Process

A lot of thought must go into the development of the orbit determination process before it is even set in motion. One goal of this project is to keep the necessary field equipment down to a minimum. The first step is to think about what kind of information a person can get about the satellite and its position by just watching it with no instruments. The answer is an exact direction vector to the satellite from the observer's position. This can be obtained by noting the stars that the satellite passes across or near. If this information is coupled with the times at which the direction vectors are valid, the orbit of the satellite can be determined. This is where the findings of the great mathematicians and astronomers of the $16^{th}$ and $17^{th}$ centuries become relevant to the backyard stargazer.

Six independent quantities are required to specify an orbit. These six quantities can be the orbital elements or the six components that make up the position and velocity vectors of a satellite at one time. However, a sighting can yield only two quantities, azimuth and elevation or right ascension and declination of the satellite. Therefore, at least three observations must be recorded yielding three sets of direction descriptions. In 1780, Laplace developed a method to derive a satellite's orbit from three direction vectors and their times of day. This is an iterative process that will be coded into a MatLab program. This program will enable any observer to calculate the orbit of a satellite that they have seen without having to work through all of the math associated with the problem.

Possibly the most valuable resource needed for recording the pass of a satellite is a picture or map of the sky for the time and date of the pass. This brings the need for a source of these star maps. The proposed source for this resource is a computer program called SkyMap 3.2 by the makers of Paint Shop Pro. This program is excellent for this orbit calculation project. The user must only select their location in the world from a database, the time and date of the desired map, and what azimuth and elevation the user desires to look to. Instantly, the program displays the sky as it will be seen at the specified time. SkyMap also offers a number of options including the stick figures for constellations, star names, or planets just to name a fraction of the options. Because this program includes the right ascension and declination of all of the points in the sky, it is a perfect way for the observer to determine the direction vectors to the satellite just by noting where on the star map the satellite appeared.

Another valuable piece of information is the time, date, and direction information for a visible pass. The tool that I used, and suggest for others to use, to determine satellite passes is Satellite Tool Kit 4.0 by Analytical Graphics Inc. This program is very powerful and specialized to do this type of work. In addition, all of the functions required to do this type of analysis are included in the free version of their software. Therefore, additional expensive software modules are not necessary.

There are several constraints that must be considered to determine a visible satellite pass. STK can quickly factor in these constraints. Foremost, the satellite must travel over or near the observer's position on Earth. The next most important constraint for a visible pass is the lighting constraint. In order to view a satellite without a telescope or other equipment, the observer's location must be in the umbra of the Earth. This constraint ensures that the sky will be dark enough to see the stars and the passing satellite. At the same time the satellite that the observer wishes to view must receive direct sunlight. For low Earth orbiting satellites or LEOs, this generally means that the satellite will be getting direct sunlight as the observer either has just lost sunlight or is just about to receive sunlight. In simpler terms, visible passes for LEOs generally occur within a few hours of sunrise or sunset. Another constraint that may apply to a visible satellite pass is an elevation mask constraint. For example, an observer in Colorado Springs, Colorado will have a ten degree elevation mask due to Pikes Peak. Other locations may have a similar constraint due to trees or a lack of open area to stand in to observe a pass. After entering these constraints into STK, a report can be generated that lists the time and date of the passes that fit the user's constraints. The report also gives the azimuth and elevation angles of the pass with respect to the location that the user specified. The azimuth angle is the angle measured from true North to the satellite's location. The elevation angle is the angle measured up from the horizon to the satellite showing the observer how high in the sky to look. This report is useful in finding times that the satellite will be visible and in determining a fairly specific direction in which to look to acquire the satellite. An excerpt from an access report is shown below.

**Facility-Colorado_Springs-To-Satellite-MIR: Inview Azimuth, & Elevation**

| Time (LCLG) | Azimuth (deg) | Elevation (deg) |
|---|---|---|
| 2 Apr 1998 20:04:30.67 | 265.547 | 10.000 |
| 2 Apr 1998 20:05:30.00 | 277.001 | 16.899 |
| 2 Apr 1998 20:06:30.00 | 299.280 | 25.546 |
| 2 Apr 1998 20:07:30.00 | 336.514 | 29.304 |
| 2 Apr 1998 20:08:30.00 | 9.140 | 22.469 |
| 2 Apr 1998 20:09:30.00 | 26.659 | 14.208 |
| 2 Apr 1998 20:10:08.45 | 33.242 | 10.000 |

(Above) Excerpt from STK4.0 Access Report

Now that I have a time and date of a pass and a map of the sky at that time, I need to be able to get a description of a point in the sky should the satellite pass across it during the pass. I checked a star atlas and other astronomy books but found all of the information that the calculations require in the SkyMap program. This software provides the right ascension and declination of all of the points in the sky in minute and second precision. This is important because the formula for determining the direction unit vectors from the observer to the satellite uses only the right ascension and declination. The process for getting these values is as simple as calling up the star map for the time of the pass and using the mouse to point to the place on the map where the satellite was observed. A bar at the bottom of the window indicates the azimuth and elevation as well as the right ascension and declination of the point that the mouse is pointing to on the map.

Some prepatory work must be done prior to the pass to provide for a smooth pass observation. The first step is to determine the time and date of a visible pass using STK. With the information included in the access report, a sky map can be generated. Another helpful step is to indicate to SkyMap the azimuth and elevation angles that observer will be using. The software will then in a sort of virtual reality sense provide the observer with the sky on the screen as the observer will see it at the time of the pass along with an azimuth and elevation grid option laid over the sky. This map and view is then printed or saved as a windows bitmap file. In working with the program I learned that it is best to save the file and open the file in Paint Shop Pro or some other bitmap editor. There I greyscale the image and take the negative image of it so that I am not printing a black or dark sky with a few stars. Rather, I am printing a white or light sky with a few dark stars. Following are examples of both the SkyMap 3.2 display and the edited version of the map:

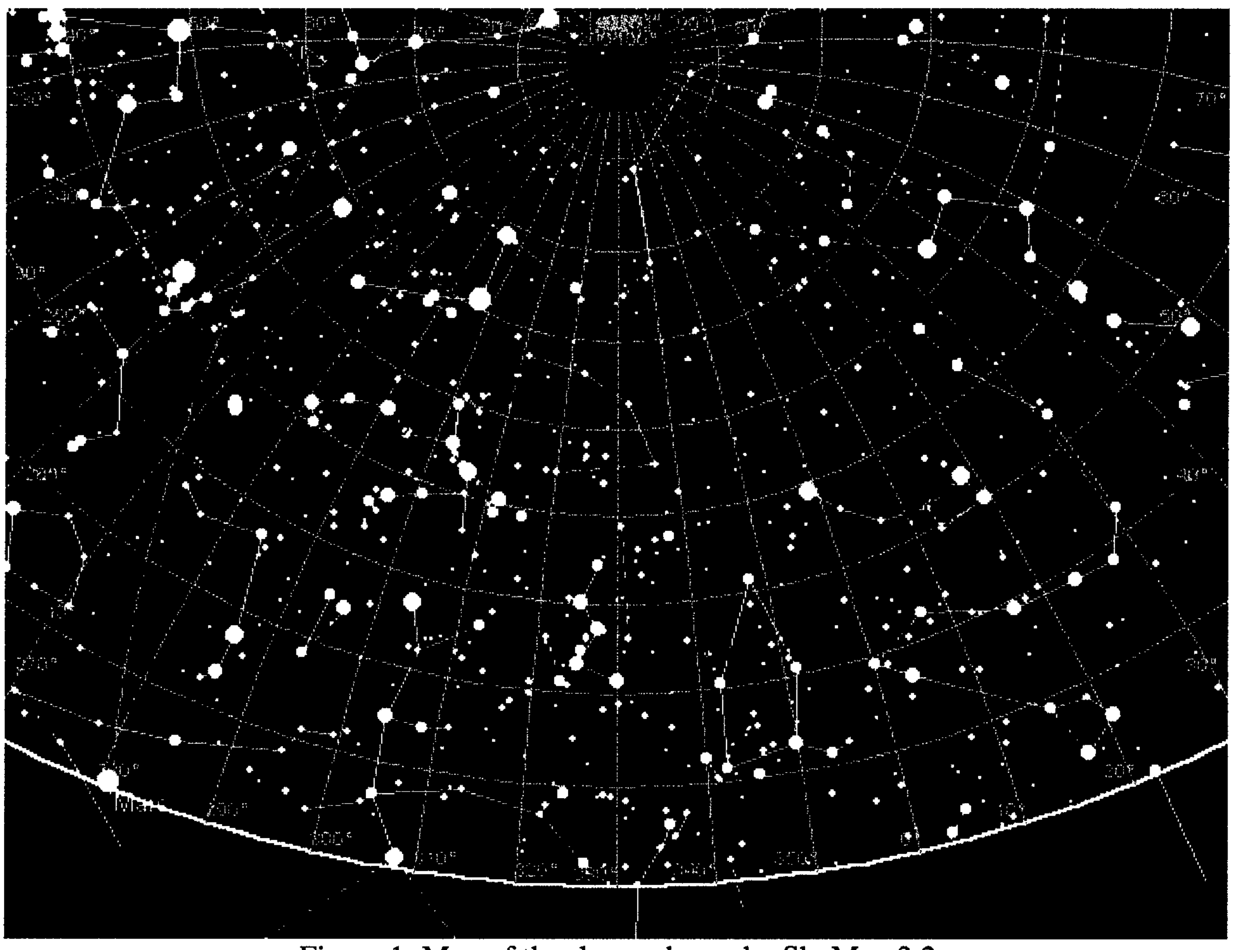


Figure 1: Map of the sky as shown by SkyMap 3.2

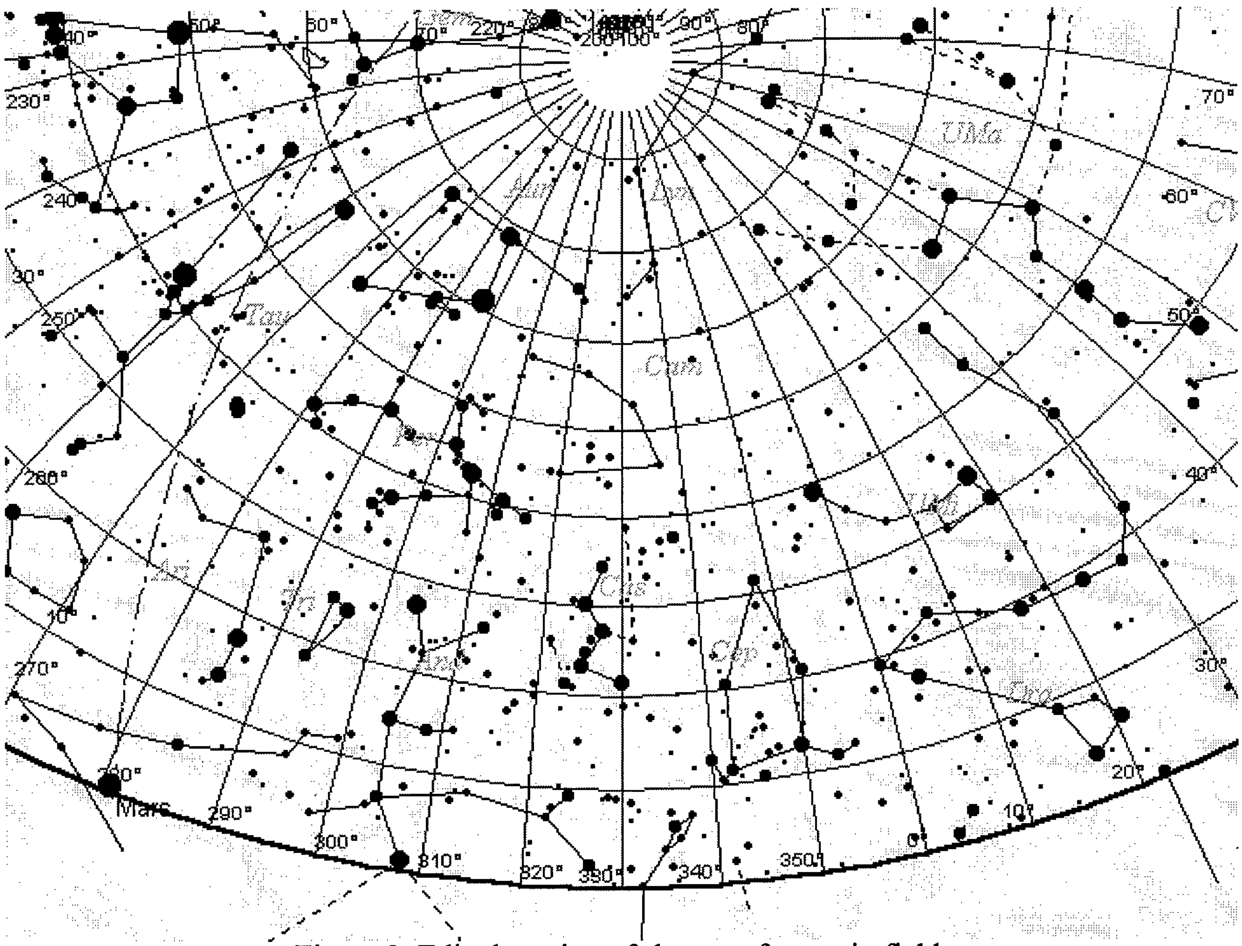



Figure 2: Edited version of sky map for use in field.

This edited version is much more friendly for sketching the satellite's path. Once I have the hard copy of the sky, I sit down with the STK satellite pass report and roughly sketch the predicted satellite pass on it using the azimuth and elevation angles given in the STK report and the azimuth and elevation grid provided on the sky map. This provides me with a better idea of where to look in the sky when the pass draws near.

In order to determine what materials are needed to observe the pass properly, the important information that the pass needs to yield must be kept in mind. First of all, the three direction vectors must be measured. This requires the sky map for the correct time and a pencil and clipboard for marking the map with locations. These materials can be replaced in the field by a camera that will capture the image of the satellite and the stars around it. However, this method might be quite costly. The time of day of the first location is also mandatory. Thus, a watch is necessary. Finally, the times between the locations are needed and can be determined with a stopwatch. All of the information needed to calculate the orbit of the observed satellite can be recorded with a sky map, pencil, watch, and stop watch. A minimum of three directions must be recorded, but as with all experiments, the more data the better. If more than three directions are recorded, the orbit can be calculated a number of times using different combinations of the direction vectors.

After going outside a number of times for satellite passes, I have developed a process that makes viewing the satellite very easy. The first thing that I do is to find a big shadow to stand in where I can see the necessary portion of the sky and my eyes can be free of unnecessary light from street lights, porch lights, or head lights. Sometimes it is also useful to shield against bright moonlight. In town even though I am standing in the shadows, there is still enough light to see the map quite well.

Once I am in place, I orient myself with the sky. I find a constellation on the map and then find it in the sky. This orientation also gives me a good idea of about how many of the stars shown on the map will actually be visible. This is something that actually changes quite frequently with the time of day or the direction that I am looking. It is also useful in finding out if patchy clouds will be in the ideal position for me not to see the satellite at all, which happens quite a large percentage of the time. Once I have matched a few star patterns on the map with those in the sky, I know where to watch and wait for the satellite to come into view. It usually comes into view like clockwork. This process, from being in the place where I will watch the satellite until actually just waiting for the pass, takes between five and ten minutes.

Theoretically, after the satellite comes into view, it will probably pass across or near several stars or constellations. I should now add that having at least two people to observe a pass is more than helpful. At the first location the time should be recorded and a stopwatch started. Also, the location must be marked on the map as accurately as possible. At the second location the lap time function on the stopwatch should be used and this lap time recorded. Once more, this second location must be accurately marked on the map. This process is repeated for the third or more location(s). At the end of the pass, this method will yield the time of day of the first location, at least three satellite locations marked on the map, and the time duration between each location.

After the pass observation and data collection is over, some post-pass data processing is required before the orbit can be determined. First, the time of day of the first location must be converted into Universal time as well as its accuracy offset corrected. Next, the time between locations must be converted into seconds. Finally, the locations marked on the map must be identified by their right ascension and declination. This requires the use of SkyMap 3.2. For the first location, the sky map must be viewed at the local time of day of the first location. Then the mouse must be pointed in the program window to the location specified on the map. With the mouse in position, the window border will indicate the right ascension and declination of the first location. Next the sky map must be propagated through the time interval between the first and second locations. Then the right ascension and declination of the second location can be recorded. This process is repeated for the remaining location(s). The reason for the propagation between locations is to obtain the most accurate direction vectors possible even though the directions are only improved by a fraction of a degree over a one minute period. Now the raw data recorded during the satellite's pass is ready to be entered into the orbit determination program.

## Analysis

One of the important categories that I needed to research was orbit determination methods. Many methods exist and vary greatly because of the different observations that can made of a satellite's motion and the different approaches available to determine an orbit from those observations. Most of the orbit determination methods in text books use range, range rate, and time information to solve the problem. However, in my scenario, that information is unavailable just by looking up into the sky. Therefore, I had to search some more for an orbit determination method that uses only direction vectors and times to determine the satellite's orbit. During my research I found two methods for determining the orbit of a satellite from three direction observations and the time between the observations. I found both of these methods in Fundamentals of Astrodynamics by Roger R. Bate, Donald D. Mueller, and Jerry E. White.

### Method 1: Laplace Method

The first method that I implemented with a MatLab program was a method developed by Laplace in 1780. First, the three sets of angular data must be put into the form of direction vectors, $\mathbf{L}_i$. The right ascension, $\alpha_i$, and declination, $\delta_i$, angles can be used directly to obtain the direction vectors in the Earth Centered Inertia reference frame, or ECI coordinate system. This is shown below:

$$\vec{L}_i = \begin{bmatrix} L_I \\ L_J \\ L_K \end{bmatrix} = \begin{bmatrix} \cos(\delta_i)\cdot\cos(\alpha_i) \\ \cos(\delta_i)\cdot\sin(\alpha_i) \\ \sin(\delta_i) \end{bmatrix}, i = 1,2,3$$

Because these are the direction vectors from the observer to the satellite, they are unit vectors that when multiplied by the slant range from the observer to the satellite, $\rho$, give us the slant range vectors to the satellite. We also know the site vector, **R**, which is the vector from the center of the Earth to the observer. By vector addition, we can then determine the vector, **r**, from the center of the Earth to the satellite. This is shown below:

$$\vec{r} = \rho\vec{L} + \vec{R}$$

The key to the problem now is determining the slant ranges for each observation. In order to do this, the first and second derivatives of the direction vectors must be calculated. This is done using the Lagrange interpolation formula to describe **L** as a function of time with a general formula:

$$\vec{L}(t) = \frac{(t-t_2)(t-t_3)}{(t_1-t_2)(t_1-t_3)}\vec{L}_1 + \frac{(t-t_1)(t-t_3)}{(t_2-t_1)(t_2-t_3)}\vec{L}_2 + \frac{(t-t_1)(t-t_2)}{(t_3-t_1)(t_3-t_2)}\vec{L}_3$$

This equation is differentiated twice to get the first and second derivatives of **L**. The second derivative of the site vector is also required for these matrices. The rotation rate of the Earth must be taken into consideration for this calculation. As shown below, ω is the vector representing the rotation rate of the Earth in the ECI reference frame:

$$\vec{\omega} = 0\hat{I} + 0\hat{J} + \omega_E\hat{K}$$

I had to use the following equation to calculate the derivative of the site vector:

$$\dot{\vec{R}}\Big|_{Fixed} = \dot{\vec{R}}\Big|_{Rotating} + \vec{\omega} \times \vec{R}\Big|_{Fixed}$$

Here the derivative of the site vector in the rotating frame is zero. The equation below is used to compute the second derivative of the site vector:

$$\ddot{\vec{R}} = \vec{\omega} \times (\vec{\omega} \times \vec{R})$$

These vectors are all used to form the matrices used to solve for ρ and $\dot{\rho}$.

$$D = 2\begin{bmatrix} L_{I2} & \dot{L}_{I2} & \ddot{L}_{I2} \\ L_{J2} & \dot{L}_{J2} & \ddot{L}_{J2} \\ L_{K2} & \dot{L}_{K2} & \ddot{L}_{K2} \end{bmatrix}$$

The determinant above is used in the following equations to solve for the slant range at the second location:

$$D\rho = -2\begin{bmatrix} L_I & \dot{L}_I & \ddot{R}_I \\ L_J & \dot{L}_J & \ddot{R}_J \\ L_K & \dot{L}_K & \ddot{R}_K \end{bmatrix} - 2\frac{\mu}{r^3}\begin{bmatrix} L_I & \dot{L}_I & R_I \\ L_J & \dot{L}_J & R_J \\ L_K & \dot{L}_K & R_K \end{bmatrix}$$

In solving for ρ, the determinants of these matrices are used. One restriction is that the determinant, D, cannot be zero. This implies that the orbit cannot be determined if the satellite passes directly overhead of the observer. When this equation is solved for ρ, it can then be substituted into the following equation:

$$r^2 = \rho^2 + 2\rho\vec{L} \bullet \vec{R} + R^2$$

In this equation r must be solved for by an iterative process. Once r has been determined, ρ can be determined. Knowing the slant range at the central date, the vector, **r**, at the second location can be determined. Now, the velocity vector for this position vector must be determined. The slant range rate is solved for just as the slant range was solved for previously. Using the value of r at the second time, the following equation can be solved to find the slant range rate.

$$D\dot{\rho} = -\begin{vmatrix} L_I & \ddot{R}_I & \ddot{L}_I \\ L_J & \ddot{R}_J & \ddot{L}_J \\ L_K & \ddot{R}_K & \ddot{L}_K \end{vmatrix} - \frac{\mu}{r^3}\begin{vmatrix} L_I & R_I & \ddot{L}_I \\ L_J & R_J & \ddot{L}_J \\ L_K & R_K & \ddot{L}_K \end{vmatrix}$$

The velocity vector, **v**, at the second time can then be determined using the following equation:

$$\vec{v} = \dot{\rho}\vec{L} + \rho\dot{\vec{L}} + \dot{\vec{R}}$$

The orbital elements of the satellite's orbit are then determined from these **r** and **v** vectors.

Method 2: f and g Series Method

The second method that I tried for determining the orbit of a satellite with angular data only uses the f and g series. The f and g series are functions of time. These functions are used to describe the position of any satellite in its orbital plane as a linear combination of its initial **r** and **v** vectors. The same information that is required for the previous method is required for this method. Just as with Laplace's method, the **L** vectors are calculated the same way. The site vector, **R**, from the center of the Earth to the observation site is also calculated. This method is quite a bit easier to implement as it is a system of six linear equations. The following is the system of equations that must be solved in order to find the components of the **r** and **v** vectors:

$$
\begin{aligned}
&f_1 L_{1z} x - f_1 L_{1x} z + g_1 L_{1z} \dot{x} - g_1 L_{1x} \dot{z} = R_{1x} L_{1z} - R_{1z} L_{1x} \\
&f_1 L_{1z} y - f_1 L_{1y} z + g_1 L_{1z} \dot{y} - g_1 L_{1y} \dot{z} = R_{1y} L_{1z} - R_{1z} L_{1y} \\
&L_{2z} x - L_{2x} z = R_{2x} L_{2z} - R_{2z} L_{2x} \\
&L_{2z} y - L_{2y} z = R_{2y} L_{2z} - R_{2z} L_{2y} \\
&f_3 L_{3z} x - f_3 L_{3x} z + g_3 L_{3z} \dot{x} - g_3 L_{3x} \dot{z} = R_{3x} L_{3z} - R_{3z} L_{3x} \\
&f_3 L_{3z} y - f_3 L_{3y} z + g_3 L_{3z} \dot{y} - g_3 L_{3y} \dot{z} = R_{3y} L_{3z} - R_{3z} L_{3y}
\end{aligned}
$$

The x, y, z, x, y, and z values are the components of the **r** and **v** vectors respectively. The textbook also recommends a process for solving this system of equations.

1. Estimate the magnitude of **r** at the second time.
2. Use this magnitude to compute $u_2$.

$$u_2 = \frac{\mu}{r_2^3}$$

3. Compute the values of $f_1$, $g_1$, $f_3$, $g_3$ using the terms of the f and g equations which are independent of $p_2$ and $q_2$.

$$f_i = 1 - \frac{1}{2} u_2 \tau_i^{\,2} + \frac{1}{2} u_2 p_2 \tau_i^{\,3} + \frac{1}{24} u_2 (u_2 - 15 p_2^{\,2} + 3 q_2) \tau_i^{\,4} + \frac{1}{8} u_2 p_2 (7 p_2^{\,2} - u_2 - 3 q_2) \tau_i^{\,5} + \ldots$$

$$g_i = \tau_i - \frac{1}{6} u_2 \tau_i^{\,3} + \frac{1}{4} u_2 p_2 \tau_i^{\,4} + \frac{1}{120} u_2 (u_2 - 45 p_2^{\,2} + 9 q_2) \tau_i^{\,5} + \ldots$$

4. Substitute these values and the components of the three **L** and **R** vectors into the system of equations. Solve the system for x, y, z, and the rates for these components.

5. Compute new values for $u_2$, $p_2$, and $q_2$ using the following equations. Then compute new values for $f_1$, $g_1$, $f_3$, $g_3$.

$$u_2 = \frac{\mu}{r_2^3}$$

$$p_2 = \frac{1}{r_2^2}(\vec{r} \bullet \vec{v})$$

$$q_2 = \frac{v_2^2}{r_2^2} - u_2$$

6. Continue substituting, solving, and recalculating until the solution converges.

## MatLab Programming

Not only did I find challenges in applying these processes, but I also found challenges in coding these calculations into a MatLab program. Nonetheless, I succeeded in coding both the Laplace and the f and g series methods. This work is transparent to the user with the exception that the program returns the orbital elements for the time and direction inputs. However, there is some work that will be quite handy for the user who is less experienced with orbital mechanics or mathematics in general. The user will have to fill in a data file with the necessary information for the computer to calculate the orbital elements. This information includes the data taken during the pass as well as the information regarding the location of the observer and the time of year. The SkyMap program provides right ascension in hours, minutes, and seconds. In addition, SkyMap 3.2 as well as many atlases give latitude and longitude in degrees, minutes, and seconds. These are the same units that SkyMap 3.2 uses for declination. Many people who would conduct this science activity might not have the necessary experience with manipulating numbers to convert these units to degrees. For this reason, the MatLab program that I have written will do the necessary conversions for the user. This way a user can just input the information just as SkyMap 3.2 or an atlas would provide the information. This reduces the chances of a user error in the whole orbit determination process.

## Conclusion

After conducting this experiment several times, I consistently found that the f and g series method of orbit determination generated orbits that were closer to that of the mir space station. In comparing these methods for one pass, I worked both methods by hand and implemented both methods using MatLab. When my hand work and programs generated the same results for each method, I knew that I had correctly coded the methods into MatLab.

From my results, I have determined that the time elapsed between the direction vectors has a large impact on the orbit determination. However, I have not determined the pattern that the accuracy follows as the time interval between satellite observations changes. I started testing these angles-only orbit determination processes with data that was spaced evenly at one minute between each direction vector. However, when I increased the time interval to two, three, and four minutes the accuracy of the calculated orbits varied a great deal. The f and g series method, overall, yielded the closest orbits even though they still differed from the actual orbit. Below are the ground traces of the calculated orbits from both methods and the Mir space station. Note the change in the calculated accuracies as the time interval between observations changes. The white ground traces indicate the orbit calculated by the Laplace method. The yellow ground traces indicate the orbit calculated by the f and g series method. The actual Mir orbit is shown in red.

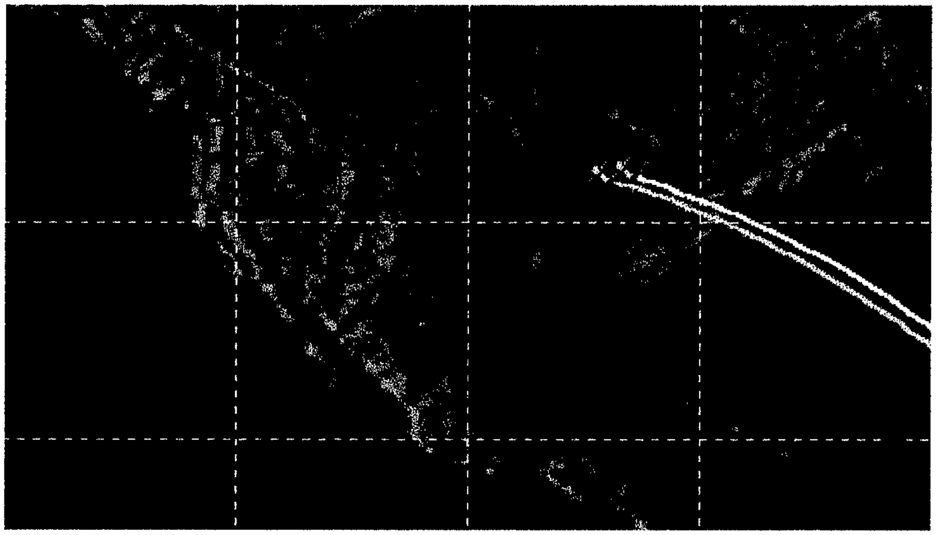


Figure 3: Orbits calculated with one minute separation between observations

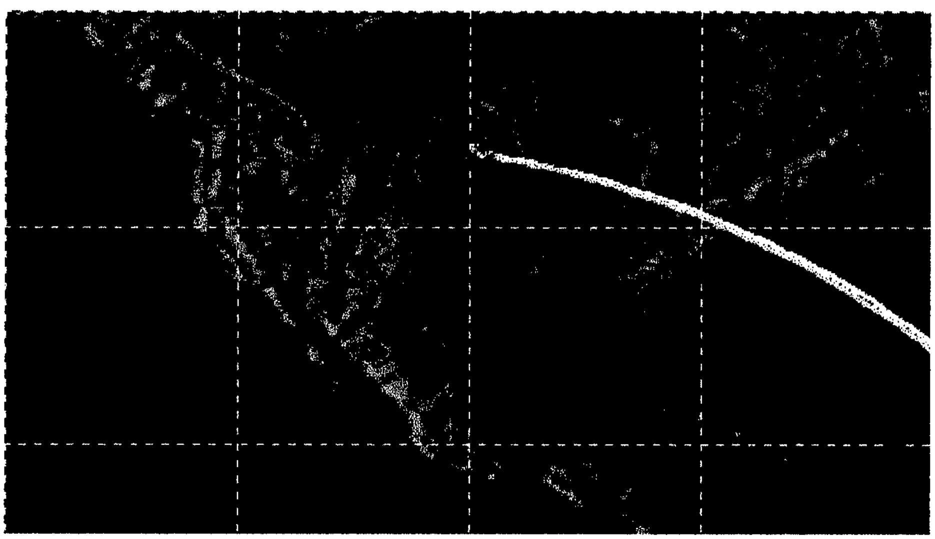


Figure 4: Orbits calculated with two minutes separation between observations

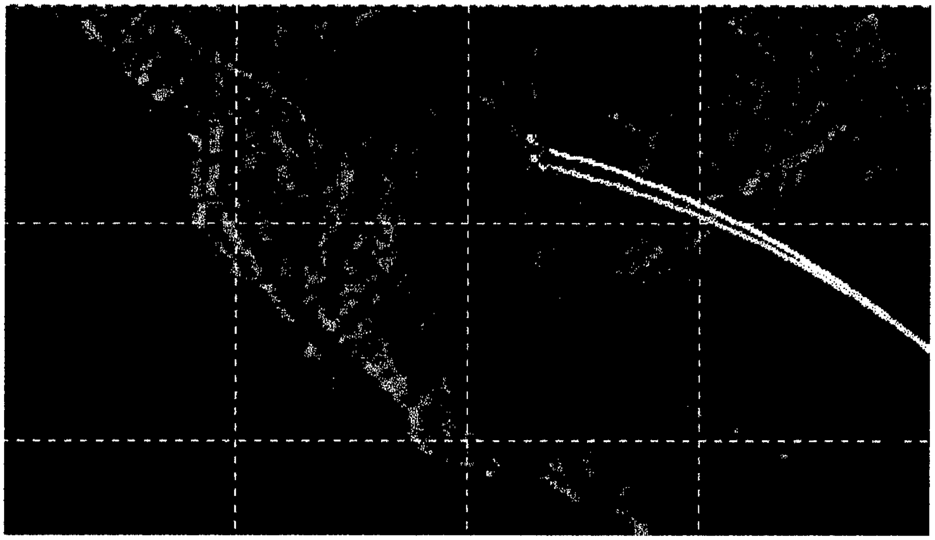


Figure 5: Orbits calculated with three minutes separation between observations

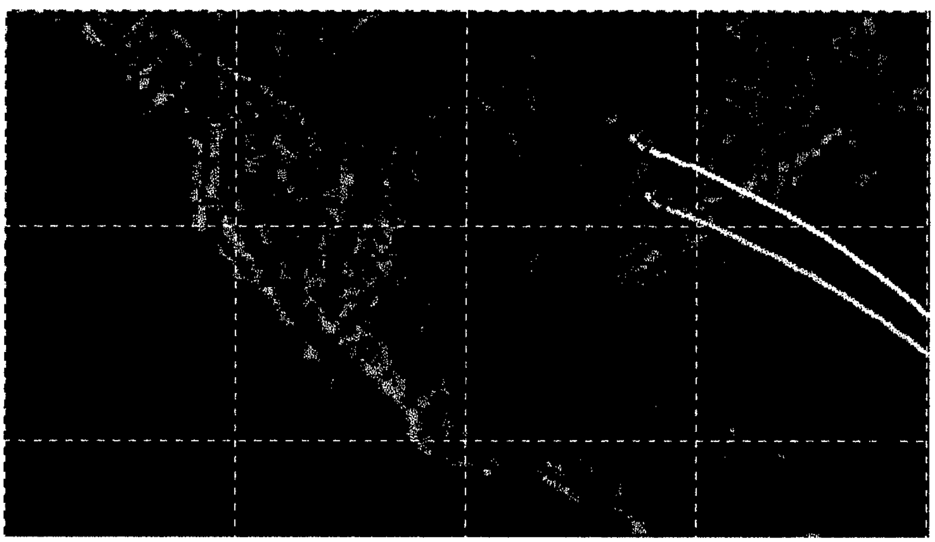


Figure 6: Orbits calculated with four minutes separation between observations

In calculating these orbits, the f and g series method relies on convergence for accuracy and the Laplace method on matrix determinants. I believe that the inaccuracies of these methods can be attributed to these facts. Because the f and g series method has trouble converging as the time interval between observations increases, the accuracy of this method also fluctuates as the time interval increases. The results of the Laplace method depend on determinants of nearly singular matrices. Because these determinants are so small, along the order of $10^{-8}$, the accuracy of the end result relies on the accuracy of direction vectors which were determined during the satellite viewing. These accuracies are only accurate to the ten thousandths place. For this reason, unless the accuracy of the satellite's angular position can be increased, the accuracy of these methods cannot improve. In addition, because the positions are determined by visual sightings with no equipment, the accuracy will generally remain the same. However, in defense of the results, this is something to marvel at that by just watching a satellite pass across the sky, a calculated orbit can be determined that is this close to the actual satellite.

I have written a science activity that may be used by middle school children and older to help develop their interest in space. Having young people become more interested in space is important, because space affects all people. For example, children living in the central United States benefit directly from remote sensing satellites that provide valuable information about the soil and land use. In addition, weather satellites provide information for the prediction of severe storms. People living in urban areas are frequently using satellites that let them use cellular phones. In addition, navigation satellites like the Global Positioning System aid people living in the high country areas and rescue services.

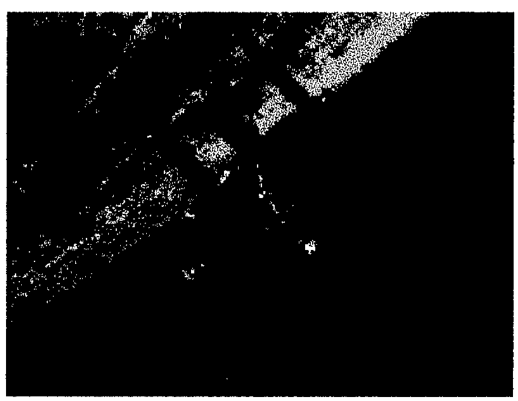
This activity will actually have them repeat the experiment that I have already done. However, they will only have to enter their data into my programs in order to calculate the orbit that applies to their data. Before students perform this activity, some preliminary work must be done. This work includes using STK to determine visible satellite passes as well as using SkyMap to print star maps for these passes. This information is provided in the Appendix to the Science Activity attached to the back of the science activity itself.

**Science Activity:**

**Orbit Determination by Satellite Viewing**

Introduction:

Right now there are thousands of satellites orbiting the Earth. These satellites may be large or small, active or inactive, useful pieces of equipment or space junk. The satellites that have been designed to be operational for a long time and transmit information to people on Earth have a vast range of purposes. Some of these satellites are used for communications purposes such as telephone services. Others are needed for navigation like the Global Positioning Satellites and still others for picture taking like the Landsat remote sensing satellites. We also get valuable information about our weather from satellites like the GOES weather satellites. The MIR space station is also a satellite that is capable of supporting human life. This will be the primary satellite that we will be observing.



Mir Space Station Orbiting over the Pacific Ocean

The satellites classified as space junk can be pieces of machinery that were at one time operational satellites like those just described that have merely reached the end of their lifetime and are now useless. Other space junk may be large parts of the rockets used to launch satellites into space. Once these rockets have served their purpose, they spend some time orbiting the Earth before they burn up upon re-entering the Earth's atmosphere. The presence of these satellites can be detected by radar systems; however, some of these satellites can be seen some of the time by the human eye.

This experiment will rely on the times that you can go outside one evening or early morning and actually see the satellite pass across the sky. The only time that you can see the stars is between sunset and sunrise. This is also the only time that you can see a satellite. There is also another restriction for seeing a satellite. Where stars emit there own light, satellites rely on reflected light to be seen. For this reason, we can only see the satellite if the sun's light reflects off of it. Therefore, a satellite can only be seen by a person if it passes within line of sight of the observer after sunset and while the satellite is still receiving direct sunlight. This occurs either within a few hours after sunset or within a few hours of sunrise. Determining when these satellite viewing times will be is a very complicated process by hand. For this reason we will use a very powerful program called Satellite Tool Kit 4.0 to determine these times for us.

Once we know when to look at the sky to see a satellite, we need to know the general area where the satellite will be travelling. We do not want to miss the satellite's pass, because we had to search the entire sky for it. STK 4.0 will also provide these directions. These directions are given by azimuth and elevation angles. The ***azimuth*** is the angle measured from true North where the observer should look. The ***elevation*** is the angle measured skyward from the horizon showing the observer how high in the sky to look. An approximation of five degrees elevation can be made be extending your arm out in front of you and making a fist. With the pinky side of your fist on the horizon, the top of your thumb shows about a five degree elevation angle. By watching a satellite pass across the sky, the orbit of the satellite can be determined just by recording a few pieces of information.

The purpose of this activity is to determine the orbit of a visible satellite. An orbit is defined by a set of numbers. Just as words can describe a picture, a set of numbers describe an orbit. One of these numbers is called the ***eccentricity***, e. This number tells the shape of the orbit. The shape can be circular, elliptical, parabolic, or hyperbolic. The satellites that we will attempt to view will have circular orbits, which means the eccentricity of the orbit is zero. The ***semimajor axis***, a, of the orbit is half of the orbits entire length. For the satellites that you will be watching, this value will typically be around 6700 kilometers. For circular orbits the semimajor axis is the radius of the Earth, 6378 kilometers, plus the satellite's altitude. The satellites that you will be watching are travelling at about 300 kilometers above the Earth's surface. Derived from the semimajor axis is the satellite's ***period***. This value is the amount of time required for the satellite to complete one orbit, or revolution about the Earth. The ***right ascension of the ascending node*** or ***longitude of the ascending node***, $\Omega$, is the right ascension where the satellite passes from the southern hemisphere to the northern hemisphere of the Earth. The right ascension and declination values of the celestial sphere are similar to the latitude and longitude on Earth. Just as latitude and longitude describe points on Earth, right ascension and declination describe points in space. The ***argument of perigee***, $\omega$, is the angle between the equator at the longitude of ascending node and the periapsis of the orbit. The periapsis point is the point in the orbit where the satellite is closest to the Earth. The ***inclination***, i, is the angle between the Earth's equator and the satellite's orbital plane. The ***true anomaly***, $\nu$, is an angle describing the location of the satellite in its orbit. Once these values are determined, we can determine when and where we will be able to see the satellite again.

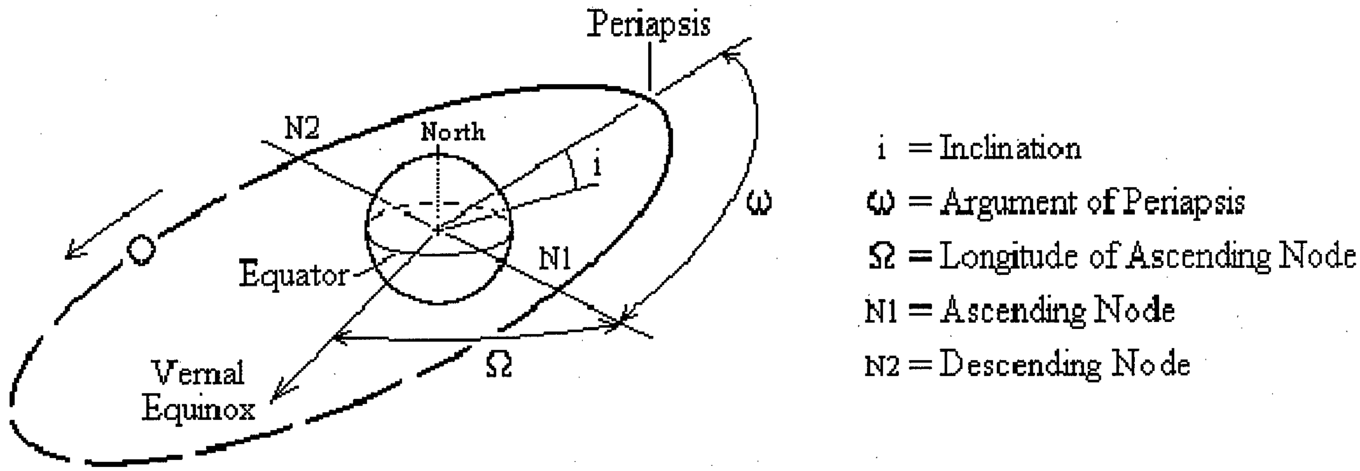



Figure 7: Diagram showing the orbital plane, Earth and the orbital elements.

Materials:

In order to record the necessary information, you will need a few materials outside with you while you observe the satellite pass. Two methods, which will be described later, can be used to get the same information. Here are the necessary materials for both methods

Method 1: Data collection without a camera

- watch
- stopwatch
- pencil
- star map for the appropriate time and date

Method 2: Data collection with a camera

- watch
- stopwatch
- star map of sky during pass or compass
- camera with film capable of capturing starlight

## Procedure:

### Method 1: Data collection without a camera

The objective of this process is to mark on the star map the stars that the satellite passes in front of or the approximate location of the satellite. On the time and date of the visible pass, find a location where you will be able to see the predicted location of the satellite. Take about five to ten minutes prior to the satellite pass to orient yourself with the sky using your star map. In addition, take a moment to familiarize yourself with the Observation Data Table where you will be entering all of the data that will be recorded during the pass. When you first see the satellite, record the time of day from the watch when the satellite was at this location. Also, start the stopwatch. This first location can either be recorded when the satellite passes near a star, or you can approximate its location using the stars on your starmap. When the satellite comes to another location that will be marked, record the lap time from the stopwatch making sure not to stop the stopwatch. Then mark the location on the star map. At the third location, mark the location on the map and stop the stop watch. Record this last time. Now all of the data collection is complete.

Method 2: Data collection with a camera

This process will result in the same information as the first method. However, some of the information that the other method yielded will not appear until the film has been developed. First, find a location that will let you stay in a shadow of the night lights as well as see the satellite pass. Then use either the star map or a compass to identify the area of the sky where the predicted pass will begin. Also identify the areas on the table where you will be recording the time information. When the satellite first comes into view, photograph the satellite making sure that the satellite is in the center of the camera view. Once the photograph is taken, immediately record the time of day of the photograph and start the stop watch. If possible, space the photos at least one minute apart. At least three photos of the satellite must be taken and the more time elapsed between the photographs the better. When the second photograph is taken, record the lap time on the stopwatch. After the third satellite, stop the stopwatch and record the time. The required information has now been recorded. Now have the film developed so that the required information may be extracted from the images that were recorded during the pass. A Polaroid instant camera may be beneficial because the film can give you the necessary data right away.

**Observation Data Table**

| | | |
|---|---|---|
| **24 hour universal time of first satellite location→** | | time = |
| **Altitude of observer location→** | | H = |
| **Latitude of observer location: (degrees, minutes, and seconds)** | | |
| latd = | latm = | lats = |
| **Longitude of observer location: (degrees, minutes, and seconds)** | | |
| longd = | longm = | longs = |
| **Direction of Longitude: ('e' or 'w')→** | | direct = |
| **Date of satellite pass: (month, day, and year)** | | |
| m = | d = | y = |
| **Right ascension and declination for 3 satellite locations:**<br>**(right ascension in hours, minutes, and seconds)**<br>**(declination in degrees, minutes, and seconds)** | | |
| ra1h = | ra1m = | ra1s = |
| dec1d = | dec1m = | dec1s = |
| ra2h = | ra2m = | ra2s = |
| dec2d = | dec2m = | dec2s = |
| ra3h = | ra3m = | ra3s = |
| dec3d = | dec3m = | dec3s = |
| **Time duration between locations 1 and 2 (seconds)→** | | t1_2 = |
| **Time duration between locations 2 and 3 (seconds)→** | | t2_3 = |

## Data Processing:

In this section you will extract pertinent information from the raw data that was recorded during the satellite pass. The following steps will require a computer program called SkyMap 3.2.

1. Open SkyMap 3.2 and select the proper observer location from the list of locations provided. To do this, click on the Location button with a picture of the globe on it. This can also be accessed by selecting the Observer choice under the View menu.

2. In the Observer Location window, check next to daylight savings time if this applies.

3. Press the Time button next to the Location button or select the Time option under the View menu. Enter the 24 hour time that you recorded at the first satellite location.

4. Select the Field of View button next to the Time button or select the Field of View choice under the View menu. Enter the approximate azimuth and altitude of the first satellite location. Note: The altitude is the same as the elevation angle that STK 4.0 gives in the satellite view report.

5. Consulting either the star map with the recorded satellite locations or the photograph of the first location, use the mouse pointer to point to the place on the screen showing the first location of the satellite. In the space provided in the Observation Data Table, record the right ascension and declination of the mouse pointer location. This value is found in the bottom right hand corner of the SkyMap 3.2 window. The right ascension is given in hours, minutes, and seconds. The declination is given in degrees, minutes, and seconds.

6. Propagate the stars to the second location time. Do this by adjusting the time of the star map by the time between the first and second location. Repeat step 5 for this time.

7. Propagate the stars to the third location time. Repeat step 5 again for this time.

All of the data has now been refined to enable the final data analysis. The Observation Data Table is now complete, and this information can now be used to determine the orbit of the satellite that you had viewed.

Data Analysis:

The information that you have just put into the Observation Data Table will now be entered into the program that will determine the orbit of the satellite. The following steps will dictate how to complete the data analysis.

1. Start the computer program called MatLab. Under the File menu choose the Open option. Select the file called VIEWDATA.m. Follow the instructions for entering the Observation Data Table information into the VIEWDATA program shown below.

```
;%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
;% This program is enters the inputs from          %
;% the Observation Data Table to the view.m         %
;% MATLAB programs.                                 %
;%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

;% Enter the 24 hour universal time of the 1st location. %
;%------------------------------------------------------%
time=0721.95;

;% Enter the altitude above sea level of the observation %
;% location in feet                                      %
;%-------------------------------------------------------%
H=0;

;% Enter the latitude and longitude of the observation  %
;% location.  The 'd' is for degrees.  The 'm' is for     %
;% minutes.  The 's' is for seconds.                      %
;%-------------------------------------------------------%
latd=38.84;
latm=0;
lats=0;
longd=104.82;
longm=0;
longs=0;
```

```
;% Enter the direction in which the longitude is        %
;% measured. 'e' or 'w'                                 %
;%------------------------------------------------------%
direct='w';

;% Enter the month, day, and year of the satellite pass. %
;%-------------------------------------------------------%
m=4;
d=1;
y=1998;

;% Enter the right ascension and declination of the three   %
;% satellite locations. The 'h' is for hours. The 'd'       %
;% is for degrees. The 'm' is for minutes. The 's' is       %
;% for seconds.                                             %
;%----------------------------------------------------------%
ra1h=1;
ra1m=4;
ra1s=52.3;
dec1d=67;
dec1m=19;
dec1s=46;
ra2h=21;
ra2m=58;
ra2s=1.2;
dec2d=55;
dec2m=46;
dec2s=30;
ra3h=20;
ra3m=47;
ra3s=31;
dec3d=36;
dec3m=40;
dec3s=48;

;% Enter the time duration between the first and second     %
;% locations in seconds.                                    %
;%----------------------------------------------------------%
t1_2=60;

;% Enter the time duration between the second and third     %
;% locations in seconds.                                    %
;%----------------------------------------------------------%
t2_3=60;
```

2. Once all of the information has been entered, choose the Save option from the File menu.
3. In the MatLab workspace type the word "view" to determine the orbit described by the data that was entered in the VIEWDATA file.
4. Finally, record the orbit information that MatLab has returned in the Orbit Data Table.

## Results and Conclusion:

The values that MatLab has returned describe an orbit that would have generated the type of satellite pass that you witnessed and recorded. You now know its orbital elements: semimajor axis, eccentricity, right ascension of the ascending node, argument of periapsis, inclination, and true anomaly. These are the same values that scientists and engineers use to calculate orbits that they will launch satellites into. Enter these values into the Results Data Table. Because our backyard measurements are not completely accurate, this calculated orbit may not be exactly that of the Mir space station; however, it should be quite close.

**Results Data Table**

| **Calculated orbital elements of observed satellite** | |
|---|---|
| **semimajor axis→** | a = |
| **eccentricity→** | e = |
| **inclination→** | inc = |
| **argument of perigee→** | w = |
| **right ascension of the ascending node→** | omega = |
| **true anomaly→** | ta = |

## Appendix to the Science Activity:

## Teacher's Reference

Before this science activity can be carried out, some preliminary work must be done. The times and dates for visible satellite passes must be determined. Also star maps need to be printed out for these times and dates. The processes below will direct step by step how this will be done. Paint Shop Pro and SkyMap free 30 day evaluation versions can be downloaded from the following internet sites:

| | |
|---|---|
| SkyMap: | http://www.skymap.com/skymap_eval.htm |
| Paint Shop Pro: | http://www.jasc.com/pspdl.html |

Following the activity the students' data can be used to create a new satellite in STK. The ground tracks shown by STK will show how close or how far off the calculated data was from the Mir's actual orbit.

Determining MIR Spacestation viewing times:

1. Open the Satellite Tool Kit 4.0 program.
2. Select New Scenario under the File menu.
3. Rename the scenario to 'Viewtime' by clicking on the name and typing the new name.
4. With Viewtime highlighted, select the Basic option under the Properties menu.
5. In the Basic Properties window select the Units tab. Under this tab highlight the DateFormat option. In the Change Unit Value column highlight Gregorian LCL (LCLG). Click OK.
6. Open the Basic Properties window for Viewtime again. In the Period box, enter the time period for which you are interested in seeing a satellite. Click OK

7. Under the Tools menu, select the Satellite Database. Put a check next to Common Name. In the space type the word, mir. Then click on Perform Search.

8. When the Search Results window comes up, highlight the MIR satellite and click OK. Then Cancel the database window.

9. With MIR highlighted, select the Constraints option under the Properties menu. Select the Sun tab, and place a check next to Lighting. Make sure the space displays Direct Sun. Then click OK.

10. Press the New Facility button in the Create New Objects tool bar. This is located between the New Area Target and New Ground Vehicle buttons.

11. Rename the new facility, Home. With Home highlighted, select the Basic Properties option. Enter the latitude, longitude, altitude of the observer's location and the local time offset from Universal time. Click OK.

12. Next, select the Constraints window for Home. Place a check next to the minimum Elevation Angle constraint. Then enter at least a five degree elevation angle minimum. Now select the Lighting tab. Put a check next to the Lighting constraint and select Umbra. Click OK.

13. With Home highlighted, select the Access option under the Tools menu. In the Access window under Associated Objects highlight MIR. Click Compute. After asterisk appears next to MIR, Select the AER option in the Reports section. This will give time, azimuth, and elevation angle information for visible satellite passes.

14. In the Satellite Access Report window select the Units option in the File menu. Remove the check from the Use Default Report Units option. Now highlight the Date Format under Units. Choose Gregorian LCL in the Change Unit Value box. Place a check next to Make Default Report Units. Click OK.

15. Lastly after dismissing the Report and Access windows, highlight the scenario Viewtime. Now select the save option under the File menu. For calculating viewing times for different dates, open the Viewtime scenario, change the scenario time and date, and compute the new accesses.

Printing a Star Map for a Predetermined Time:

1. Open the SkyMap Program. Click the Location button; it has a picture of the globe on it. Select the proper location from the list and check next to Daylight Savings Time if applicable. Click OK.

2. Click the Time button next to the Location button. Select the desired local time and date of the star map. Click OK.

3. Press the Field of View button next to the Time Button. Enter the azimuth and elevation angles for the middle satellite location of the satellite pass. Consult the STK viewing report for these values. Adjust the Map Size to ensure that the entire pass will fit on the map view. Click OK. Note: the altitude angle in SkyMap is the elevation angle.

4. Once the desired map view is achieved, select Write BMP File from the File menu. Enter a name for the file, click OK, and select a 640X480 file size.

5. Close the SkyMap program and open the Paint Shop Pro or other .bmp editor. Open the star map .bmp file that you just saved. Select the Greyscale option from the Colors menu. Then select the Negative Image option from the Colors menu.

6. This map should be printed in a landscape format to make use of the full page.

Creating a New Satellite with Calculated Data:

1. Open the Viewtime scenario in STK. Press the New Satellite button on the Create New Objects toolbar. Cancel the Orbit Wizard.

2. With the new satellite highlighted, Select the Basic option under the Properties menu.

3. For the Orbit Epoch, enter the time and date of the second satellite location.

4. Enter the value for the semimajor axis, a, that was returned by Matlab.

5. Enter the eccentricity, e, that Matlab calculated.

6. Enter the inclination, i, from the MatLab results.

7. Enter the argument of perigee, $\omega$, from MatLab.

8. Next to RAAN, right ascension of the ascending node, enter OMEGA from MatLab.

9. Next enter the true anomaly.

10. Click OK. You have now created a satellite with the orbital elements calculated by MatLab.

11. In the scenario window call this satellite 'themir' and use the method previously described to calculate future visible passes for this satellite.

## Appendix B:
## MatLab Program, view.m, Used To Approximate Orbit

```
;%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
;%% Lee Dewald                                         March 27, 1998 %%
;%%                 Orbit Determination by 3 direction vectors              %%
;%%                                                                         %%
;%% -This program will take the right ascension and declination of three stars that a satellite%%
;%%  has passed in front of and the time intervals between the stars and return the orbital  %%
;%%  elements of that satellite as computed from the observations.            %%
;%%                                                                         %%
;%%                     variable name              units                    %%
;%%                     -----------------          ------                   %%
;%%   Inputs:               ra1h                   hours                    %%
;%%                         ra1m                   minutes                  %%
;%%                         ra1s                   seconds                  %%
;%%                         dec1d                  degrees                  %%
;%%                         dec1m                  minutes                  %%
;%%                         dec1s                  seconds                  %%
;%%                         ra2h                   hours                    %%
;%%                         ra2m                   minutes                  %%
;%%                         ra2s                   seconds                  %%
;%%                         dec2d                  degrees                  %%
;%%                         dec2m                  minutes                  %%
;%%                         dec2s                  seconds                  %%
;%%                         ra3h                   hours                    %%
;%%                         ra3m                   minutes                  %%
;%%                         ra3s                   seconds                  %%
;%%                         dec3d                  degrees                  %%
;%%                         dec3m                  minutes                  %%
;%%                         dec3s                  seconds                  %%
;%%                         time                   24hr time                %%
;%%                         t1_2                   seconds                  %%
;%%                         t2_3                   seconds                  %%
;%%                          H                     feet                     %%
;%%                         latd                   degrees                  %%
;%%                         latm                   minutes                  %%
;%%                         lats                   seconds                  %%
;%%                         longd                  degrees                  %%
;%%                         longm                  minutes                  %%
;%%                         longs                  seconds                  %%
;%%                         direct                 'e' or 'w'               %%
;%%                          m                     month                    %%
;%%                          d                     day                      %%
;%%                          y                     year                     %%
;%%                                                                         %%
;%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
viewdata
u=398601.2;
ra1=((ra1s/60+ra1m)/60+ra1h)*15*pi/180;
ra2=((ra2s/60+ra2m)/60+ra2h)*15*pi/180;
ra3=((ra3s/60+ra3m)/60+ra3h)*15*pi/180;
dec1=((dec1s/60+dec1m)/60+dec1d)*pi/180;
dec2=((dec2s/60+dec2m)/60+dec2d)*pi/180;
dec3=((dec3s/60+dec3m)/60+dec3d)*pi/180;
lat=((lats/60+latm)/60+latd)*pi/180;
long=((longs/60+longm)/60+longd)*pi/180;
H=H*0.0003048;
t1=0;
t(1)=t1;
t2=t1_2;
t(2)=t2;
t3=t1_2+t2_3;
t(3)=t3;
tao1=-t1_2;
tao3=t2_3;
k=[0 0 1];
ivector=[1 0 0];
jvector=[0 1 0];
we=7.292115856E-5;
WE=[0;0;we];
ee=0.08182;
ae=6378.145;
X=abs((ae/sqrt(1-ee^2*(sin(lat))^2))+H)*cos(lat);
Z=abs(((ae*(1-ee^2))/sqrt(1-ee^2*(sin(lat))^2))+H)*sin(lat);
hr=fix(time/100);
mn=time-(hr*100);
for s=1:3,
 minute(s)=mn+t(s)/60;
 if minute(s)>60
  hr=hr+1;
  minute(s)=minute(s)-60;
  if hr>24
   hr=hr-24;
   d=d+1;
  end
 end
 x=fix((7*(y+fix((m+9)/12)))/4);
 jd=367*y-x+fix(275*m/9)+d+1721013.5+((minute(s)/60)+hr)/24;
 tu=(fix(jd)+0.5-2451545)/36525;
 b=1.0027379093*2*pi*(jd-fix(jd)-0.5);
 gst=1.753368560+628.3319706889*tu+(6.7707e-6)*tu^2-(4.5e-10)*tu^3+b;
 if direct=='e'
```

```
 lst0=gst+long;
else
 lst0=gst-long;
end
n=fix(lst0/(2*pi));
lst=lst0-(n*2*pi);
if lst<0
 lst=lst+(2*pi);
else
 lst=lst;
end
Rsite=[X*cos(lst); X*sin(lst); Z];
if s==1
 rsite1x=dot(Rsite,ivector);
 rsite1y=dot(Rsite,jvector);
 rsite1z=dot(Rsite,k);
end
if s==2
 rsite2x=dot(Rsite,ivector);
 rsite2y=dot(Rsite,jvector);
 rsite2z=dot(Rsite,k);
end
if s==3
 rsite3x=dot(Rsite,ivector);
 rsite3y=dot(Rsite,jvector);
 rsite3z=dot(Rsite,k);
end
end
L1=[cos(dec1)*cos(ra1);cos(dec1)*sin(ra1);sin(dec1)];
L2=[cos(dec2)*cos(ra2);cos(dec2)*sin(ra2);sin(dec2)];
L3=[cos(dec3)*cos(ra3);cos(dec3)*sin(ra3);sin(dec3)];
l1x=dot(L1,ivector);
l1y=dot(L1,jvector);
l1z=dot(L1,k);
l2x=dot(L2,ivector);
l2y=dot(L2,jvector);
l2z=dot(L2,k);
l3x=dot(L3,ivector);
l3y=dot(L3,jvector);
l3z=dot(L3,k);
rmag=6378;
for s=1:100,
 u2=u/rmag^3;
 if s>1
  P=dot(rvector,vvector)/rmag^2;
  Q=velocity^2/rmag^2-u2;
```

```
 f1=1-.5*u2*tao1^2+.5*u2*P*tao1^3+(1/24)*u2*(u2-15*P^2+3*Q)*tao1^4+(1/8)*u2*P*(7*P^2-u2-3*Q)*tao1^5;
 f3=1-.5*u2*tao3^2+.5*u2*P*tao3^3+(1/24)*u2*(u2-15*P^2+3*Q)*tao3^4+(1/8)*u2*P*(7*P^2-u2-3*Q)*tao3^5;
 g1=tao1-(1/6)*u2*tao1^3+.25*u2*P*tao1^4+(1/120)*u2*(u2-45*P^2+9*Q)*tao1^5;
 g3=tao3-(1/6)*u2*tao3^3+.25*u2*P*tao3^4+(1/120)*u2*(u2-45*P^2+9*Q)*tao3^5;
 else
 f1=1-.5*u2*tao1^2;
 f3=1-.5*u2*tao3^2;
 g1=tao1-(1/6)*u2*tao1^3;
 g3=tao3-(1/6)*u2*tao3^3;
 end
 A=[f1*l1z 0 -f1*l1x g1*l1z 0 -g1*l1x;0 f1*l1z -f1*l1y 0 g1*l1z -g1*l1y;l2z 0 -l2x 0 0 0;0 l2z -l2y 0 0 0;f3*l3z 0 -f3*l3x g3*l3z 0 -g3*l3x;0 f3*l3z -f3*l3y 0 g3*l3z -g3*l3y];
 B=[rsite1x*l1z-rsite1z*l1x;rsite1y*l1z-rsite1z*l1y;rsite2x*l2z-rsite2z*l2x;rsite2y*l2z-rsite2z*l2y;rsite3x*l3z-rsite3z*l3x;rsite3y*l3z-rsite3z*l3y];
 answer=inv(A)*B;
 rvector=[dot(answer,[1 0 0 0 0 0]);dot(answer,[0 1 0 0 0 0]);dot(answer,[0 0 1 0 0 0])];
 vvector=[dot(answer,[0 0 0 1 0 0]);dot(answer,[0 0 0 0 1 0]);dot(answer,[0 0 0 0 0 1])];
 hvector=cross(rvector,vvector);
 velocity=sqrt(dot(vvector,vvector));
 rmag=sqrt(dot(rvector,rvector));
end
h=sqrt(dot(hvector,hvector));
inc=acos(dot(hvector,k)/h)*180/pi;
nvector=cross(k,hvector);
n=sqrt(dot(nvector,nvector));
omega=acos(dot(nvector,ivector)/n)*180/pi;
if dot(nvector,jvector)<0
 omega=360-omega;
else
 omega=omega;
end
evector=(velocity^2/u-1/rmag)*rvector-(dot(rvector,vvector)/u)*vvector;
e=sqrt(dot(evector,evector));
w=acos(dot(nvector,evector)/(n*e))*180/pi;
if dot(evector,k)<0
 w=360-w;
else
 w=w;
end
ta=acos(dot(evector,rvector)/(e*rmag))*180/pi;
if dot(rvector,vvector)<0
 ta=360-ta;
else
 ta=ta;
```

```
end
p=h^2/u;
a=p/(1-e^2);
period=2*pi*sqrt(a^3/u)/60;
a
e
inc
w
omega
ta
end
```

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Fundamentals of Astrodynamics, Roger R. Bate, Donald D. Mueller, and Jerry E. White. Dover Publications, INC., New York, New York. 1971.

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## An Investigation of Two Angles-only Orbit Determination Processes Including a Science Activity for Middle and High School Students.

By

Lee S. Dewald Jr.

B.S.E.E., The Citadel, 1997

An ivestigative report submitted to
the Faculty of the Graduate School of
the University of Colorado in partial
fulfillment of the requirements for
the degree of Master of Engineering in
Space Operations.

Spring 1998

Abstract

This project will investigate the possibilities of determining the orbit of a satellite from the data obtained by observing the satellite passing across the sky. With simple tools such as a stopwatch, compass, and star map or camera, I believe that the orbit of a satellite may be determined by an angles only approach. In addition, young students, of middle school age and older, could be exposed to the fundamentals of orbital mechanics by conducting such an activity. The calculations required to complete this activity will be manipulated into a coded algorithm. This enables the user to enter information which they retrieved during the satellite pass and get back the calculated orbit of the satellite which they observed. This data could then be used to predict future passes of that satellite. This paper will contain a science activity to aid in teaching the fundamentals of orbital mechanics to middle school students. In addition, it will contain a detailed list of instructions for determining when a satellite will be visible to a user's location, as well as how to obtain pertinent data while observing a satellite pass.

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